Risk Minimization in Portfolio Optimization

This brief summary outlines the
***methods**, **results**, **visualizations & insights**, and **discussion**.*
For a more detailed introduction, see the [GitHub repository] and README.

Data and Preprocessing

To calculate our variables, we used 5 years of historical stock prices, monthly, for the 10 companies, with varied sector labels. The sectors are divided as follows: Consumer Discretionary (Disney, Home Depot, Nike), Financials (Goldman Sachs and JPMorgan), Healthcare (UNH), Industrials (Boeing and Coca Cola), and Technology (Amazon and Apple).

The expected return E[R] is essentially the annual mean of returns R over some years to predict returns in the future. First, we calculate, returns of each stock per month from 2020 to 2025 with the formula below,

$$R = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Where P represents the price of stock in month t. We calculate E[R] by using the Average() function in Excel for each individual stock.

Then, we proceed to calculate our cost variable, risk. A common measure of risk is variance, Var(R), because variance measures the deviation of return from the average return. This is calculated by the formula,

$$Var(R) = E[R^2] - (E[R])^2.$$

Methodology

Optimization Problem Formulation

We approach portfolio selection as a **constrained optimization problem**, where the goal is to determine the optimal allocation of capital across a set of assets to minimize total portfolio risk, subject to a set of investment constraints.
Each asset in our dataset is assigned a **weight**, representing the proportion of the total portfolio invested in that asset. These weights are the decision variables of our optimization model.

The optimization relies on two key inputs derived from historical data: risk & expected return.

Objective Function

Our objective is to **minimize the total portfolio risk**. This can be expressed as:

$$\min \quad \sum_{a \in A} Risk_a \cdot \ w_a$$

Where w_a is the weight of asset a, and $Risk_a$ is its associated volatility. This formulation aims to provide a solution consisting of less volatile assets, provided they meet the return requirements and other constraints.

Constraints

We impose the following constraints on this objective function to ensure financial feasibility, progressively expanding the model's complexity:

Core Constraints (Baseline Model):

1. Budget Constraint: The total investment must equal the entire portfolio value, which means the sum of all asset weights must equal 1:

$$\sum_{a \in \mathrm{A}} w_a = 1$$

2. Return Constraint: The expected return of the portfolio must meet or exceed a user-specified target:

$$\sum_{a \in \mathrm{A}} \mathrm{ExpectedReturn}_a \cdot w_a \geq \mathrm{TargetReturn}$$

3. Non-Negativity Constraint: No short-selling is allowed in this model, meaning all weights must be non-negative:

$$w_a \geq 0 \quad \forall a \in \mathrm{A}$$

Enhanced Constraints:
To improve diversification and mitigate sector-specific risks, we introduce additional restrictions:

4. Sector Constraints:
   To ensure portfolio diversification and manage sector-specific exposure in our model, we introduce:

- Minimum and Maximum Sector Weights: Each sector must receive between user-specified minimum sector weight and maximum sector weight of the portfolio's total investment,

$$\sum_{a \in \mathrm{A},\ \mathrm{S}_a = s} w_a \geq \mathrm{MinSectorWeight}_s \quad \forall s \in \mathrm{S}$$

$$\sum_{a \in \mathrm{A},\ \mathrm{S}_a = s} w_a \leq \mathrm{MaxSectorWeight}_s \quad \forall s \in \mathrm{S}$$

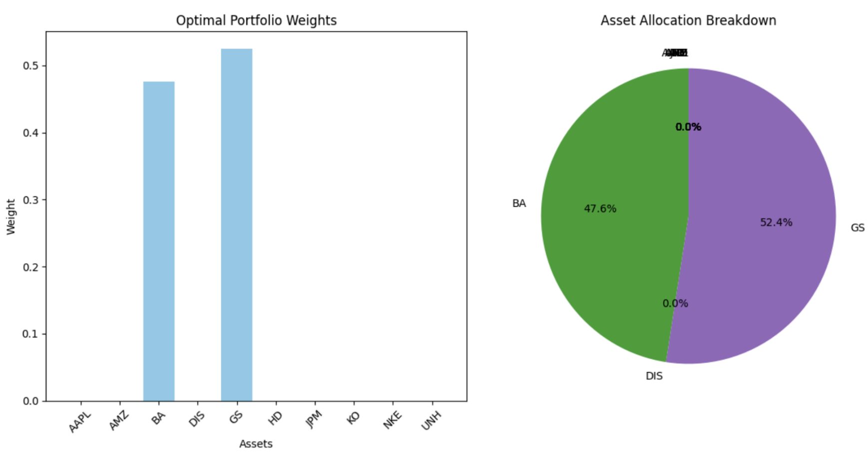
5. Maximum Asset Weight: No single asset may account for more than user-defined percantage of the portfolio.

$$\sum_{a \in A, S_a = s} w_a \leq \text{MaxAssetWeight}$$

The baseline model (Model 1) incorporates only the core constraints (1–3), while the extended model (Model 2) integrates all constraints (1–5) to enforce diversification.

Results & Insights

In the baseline model (model 1), we place no restrictions on sector exposure or individual asset weight. The linear program achieves the target return of 20% by allocating the entire portfolio to just two companies: Boeing (BA)and Goldman Sachs (GS). Model 1 recommends the investor to allocate approximately 47.5% of their budget to Boeing (BA) and approximately 52.4% to Goldman Sachs (GS). This outcome reflects the optimizer's freedom to concentrate capital in the assets with the highest return-to-risk efficiency.

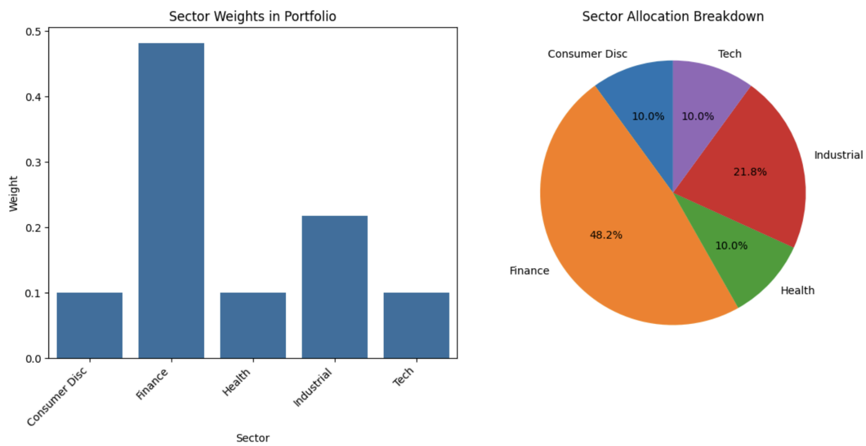


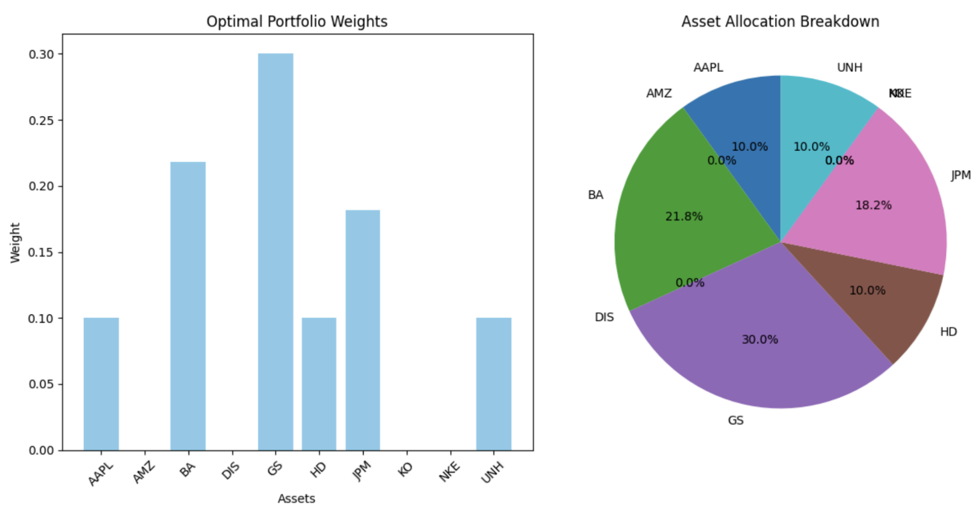
Model 1: Baseline Model

(No restriction on any Asset or Sectors)

In the sector-constrained and weight-constrained model (model 2), the target return is the same at 20%, but the investor seeks to diversify their investment to decrease risk by including restrictions on maximum sector weight (50% per sector), minimum sector weight (10% per sector) and maximum asset weight (30% per asset). These additional restrictions/constraints spread investment across five sectors, including safer alternatives within each. For example, within the Consumer Discretionary sector, the model prefers Home Depot (HD) over Nike (NKE), likely due to HD's lower associated risk.

Overall, the model result recommends the investor to allocate approximately 10% of their budget into AAPL, 21.8% into BA, 30% into GS, 10% into HD, 18% into JPM, and 10% into UNH (visualization attached below).





Model 2: Sector-Constrained and Weight-Constrained Model

(Min & max for each sector & Max 30% for each asset).

Quantitatively, we observe:

- Baseline Model Risk (TotalRisk): **0.0066**
- Sector-Constrained Model Risk (TotalRisk): **0.0175**

This aligns with a key concept in linear programming and portfolio theory: **adding constraints tends to increase the risk or reduce the optimality of the solution**. While the expected return remains fixed, the feasible region shrinks, often leading to higher measured risk or lower mathematical optimality. Although Model 2 exhibits greater risk than the unconstrained baseline, this does not necessarily make it the inferior choice. In practice, a more diversified portfolio better insulates investors from market volatility and idiosyncratic shocks (factors our simplified LP framework cannot fully capture here).

Further, this result supports the idea that higher risk does not always guarantee higher return. For example, Nike has a relatively high risk but a low expected return, and thus was excluded from the optimal allocation.

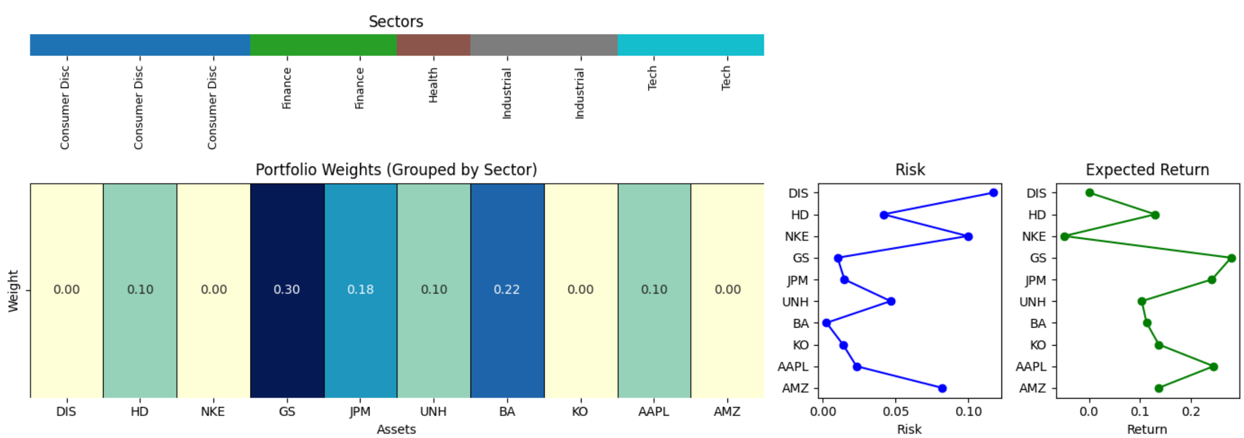
Discussion

I. Strength:

Strengths of the Overall LP Framework

- **Reproducibility:** Clearly defined decision variables, objectives, constraints, data processing methods, and publicly available data sources make the approach easy to implement and replicate.
- **Flexibility:** New constraints (e.g., turnover limits, minimum holdings) or objectives can be added with minimal reformulation.
- **Pedagogical Clarity:** Illustrates foundational portfolio-optimization concepts such as the trade-off between risk and return, effect of constraints, etc., within a unified mathematical model.
- **Foundation for Extensions:** Provides a point of reference to build more advanced models using advanced techniques (stochastic programming, dynamic rebalancing, interaction effects) in future work.

II. Limitations and Extensions:



1. No asset interaction

Despite providing useful insights, our model has a key limitation: it does **not account for the covariance** between assets. Covariance is the measure of one asset's effect or correlation to another. As a result, the computed portfolio risk may not accurately capture real-world volatility.

To address this, future extensions of this project could involve formulating the problem using **quadratic programming**, where the objective would minimize the **portfolio variance** (using a covariance matrix), allowing for a more realistic risk assessment.

2. **Static return assumptions**

The model is based on past returns, assuming they predict future performance, which is often not true in practice. A more robust approach would be to apply stochastic optimization, which incorporates uncertainty and variability in asset returns.

These limitations highlight the value of more sophisticated modeling techniques in capturing real-world investment behavior.

Conclusion

In this project, we developed and implemented a linear programming framework to construct an optimal stock portfolio that minimizes overall risk while meeting a target return of 20%. Two LP models are formulated: the baseline model and the sector-constrained model with additional constraints.

The baseline model, unconstrained by diversification requirements, concentrated investments in a few high-performing assets to meet the return target with the lowest possible risk. In contrast, the sector-constrained model introduced real-world limitations on sector and individual asset weights, resulting in a more diversified portfolio.

The sector-constrained model introduced additional constraints to enforce diversification. Mathematically, diversification slightly increased total risk compared to the baseline. However, the diversified portfolio still represented the lowest-risk allocation within the imposed constraints and was better aligned with practical investment strategies that prioritize resilience against market volatility. This comparison illustrates how real-world constraints shape portfolio composition and influence the trade-off between risk and diversification.

Overall, our models offer a simple yet effective and reproducible approach to portfolio optimization using linear programming. While our analysis is simplified, it provides valuable insights into how return goals and investment policies interact, and it lays the foundation for future extensions using more advanced techniques such as stochastic optimization or dynamic modeling.

References

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