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Analysis II  
Research Paper Topic Proposal  
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## **Topic Proposal**

For my research paper, I would like to explore Ordinary Differential Equations, comparing first order ODE's to higher order ODE's as well as proving the Global Picard Theorem. The bulk of my paper will be spent introducing key terms and theorems that will be used to set up and prove the Global Picard theorem, and I will take ownership over the proof of the theorem by combining two or three other proofs of the Global Picard Theorem that I have read. I would then like to work through a couple of examples, demonstrating the usefulness of being able to apply the Global Picard Theorem.

## **Tentative Outline**

### Introduction

- Brief description of ODE's, when and how to use them, why they are useful.
- Brief introduction to solutions to differential equations, why we care about their existence and uniqueness
- Introduce the Global Picard Theorem

### Goal

The goal of my paper is to setup and prove the Global Picard Theorem and explain how it is a valuable theorem when working with ODE's, but to also discuss its limitations.

### Key Terms and Theorems

These are the terms and theorems that I will use in my introduction and proof of the Global Picard Theorem:

- Lipschitz constant
- Fixed point
- Initial Value Problem
- Equicontinuous
- Taylor's Theorem
- Weierstrass M-test
- Banach Contraction Principle
- Peano's Theorem (tentative)
- Arzela-Ascoli (tentative)

### Setup

Here I state the Global Picard Theorem and go over any assumptions/hypotheses as well as provide an outline of my proof, naming the steps and goals.

### Proof

Here I will claim ownership over my own thorough proof inspired by the terser proofs I have come across in my research.

### Examples

I will pull examples from the Davidson and Donsig text to demonstrate how to apply the Global Picard Theorem, and I would also like to either make or find an example demonstrating the limitations of the Global Picard Theorem. (i.e example of existence without uniqueness, or an example where a unique solution exists, but the Global Picard Theorem does not apply).

### Conclusion

Here I will rephrase my introduction, drawing the reader back to the Global Picard Theorem as a useful existence theorem with limitations when dealing with ODE's.

### **Sources**

Cronin, J. (1994). *Differential equations: introduction and qualitative theory*. New York: Marcel Dekker, Inc.

This book is a fairly comprehensive introduction to differential equations so it will serve as a useful reference throughout my research since it contains most of the definitions and theorems I need for my paper. It's very different than my other sources in the sense that it's much explanative. The proofs in this book don't show as many steps, but Cronin explains the proof, but she's very descriptive and explains some of the nuances in her proofs. I think this source is a good balance with my others since my other sources don't provide nearly as much written explanations in their proofs.

Davidson, K. R., & Donsig, A. P. (2010). *Real analysis and applications: theory in practice*. New York, NY: Springer.

This textbook is my starting point for my research paper. It contains the most information on the Global Picard Theorem and several useful examples. I also find it to be very straight forward and the proof to be very thorough. Combined with my other sources, I should have a firm handle on definitions and theorems in my paper. This textbook will likely be the main source I utilize in my paper.

Murray, F. J., & Miller, K. S. (1954). *Existence theorems for ordinary differential equations*. New York: New York University Press; distributed by Interscience Publishers.

This book is composed of just existence theorems for ordinary differential equations. Not only does it contain information on the Global Picard Theorem, but it also provides me with several other existence theorems that I can use for comparison to help me better understand the importance of the Global Picard Theorem. This source is also much older than my other two sources so it is interesting to compare it to my other sources since the proofs are written differently with some slightly different notation, but I think it could give me a different perspective.