LAB 4: Centroidal floating base dynamics

This lab is to acquire confidence to some invariant properties of floating base dynamics. The base frame B of a floating base robot is usually placed at the robot waist/trunk because this choice induces a branch sparsity of the Jacobians. However, by defining a different parametrization for the floating base, it is possible to obtain a *block diagonal* joint space inertia matrix. This is achieved by defining a variable changes for the state of the system using the Center of Mass velocity and the *average* angular velocity. It can be shown that the mechanical power is invariant with respect to the change of coordinate frames used to represent the floating base.

1) *Center of mass computation:* Compute the Center of Mass of the system performing a weighted average of the Center of Masses of all the links.

2) *Kinetic energy:* Compute the kinetic energy of the system in two different ways: one by the generalized velocities and system mass matrix, another summing up the contributions from each joint, showing the result is the same.

3) *Centroidal coordinates transform:* Build the transformation matrix ${}_gT_b$ to change coordinates from base to com frame.

$$ {}_GT_B = \begin{bmatrix} {}_GX_B & I_G^{-1}\,{}_GX_B^{*}F_B \\ 0_{n\times 6} & I_{n\times n} \end{bmatrix} \tag{1} $$

where:

$$ {}_GX_B = \begin{bmatrix} I_{n\times n} & -[x_{b,com}]_\times \\ 0_{3\times 3} & I_{3\times 3} \end{bmatrix} \tag{2} $$

4) *Block-diagonal joint space inertia matrix:* Show the mass matrix is decoupled if you apply coordinate transform to com variables:

$$ M_G = ({}_GT_B)^{-T} M_B ({}_GT_B)^{-1} \tag{3} $$

5) *Joint gravity vector nullifies in com coordinates:*

$$ G_G = -M_g \begin{bmatrix} g_w \\ 0_{n+3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -mg \\ 0_{3x1} \\ 0_{nx1} \end{bmatrix} \tag{4} $$

$$ G_G = ({}_GT_B)^{-T} G_B \tag{5} $$