LAB 3: Contact consistent (fixed) base dynamics

1) *Simulation of contact consistent dynamics:* Generate a sinusoidal reference for Shoulder Lift joint with amplitude 0.6 rad and frequency 1 Hz. Implement a constraint consistent dynamics where a point (3D) contact is possible only at the end-effector (ee_link). Consider the appropriate projection of the dynamics before integrating the accelerations. Use the pre-implemented PD controller with gravity compensation and the logic to deal with the contact. What is the best way to compute the term $\dot{J}\dot{q}$? (hint: since $\ddot{x} = J\ddot{q} + \dot{J}\dot{q}$ compute the acceleration at the end-effector while setting $\ddot{q} = 0$). Hint: consider the instantaneous correction of the joint velocity at the occurrence of impact. Verify that the linear part of the twist at the end-effector equals $J\dot{q}$.

2) *Contact forces disappear when projecting the dynamics:* Check contact force disappeared when projected after the projection in the null-space of J^T (i.e. $N_c^T J^T f = 0$) What happens if you use the Moore-penrose pseudo-inverse to compute the projector? Plot also the torques in the row-space of J^T, check they are barely zero during the contact because there is almost no internal joint motion. Compare them with a plot of the joint torques. The motion during the contact depends mainly only on the null-space projector N_c^T that "cuts-out" the torques that generate contact forces, leaving only the ones that generate internal motions (in this case very small).

3) *Constraint consistent joint reference:* Try to double the amplitude of the reference of Shoulder List joint to 1.2 rad. Design a reference trajectory that is consistent with the contact (hint: compute $\dot{q}^d$ project with N_c and integrate to get q^d. Are there some internal motions now? are the torque in the row space different than zero?

4) *Gauss principle of least effort*: Verify that the Gauss principle of least constraint is satisfied (e.g. solve the QP where you minimize the distance w.r.t the unconstraint accelerations under the contact constraint).

5) *Change in the contact location*: Modify the code in order to allow the contact at a different location (e.g. origin of wrist_3_link frame). Verify that the end-effector penetrates the ground.

6) *Check the shifting law (Optional):* The twist at ee_link is v_e, twist at origin of wrist_3_joint is: v_o. Since they belong to the same rigid body (wrist_3_link) they are linked the shifting law, through by time-invariant *motion* transform ${}_eX_o \in \mathbb{R}^{6\times6}$:

$$v_e = {}_{ee}X_o v_o \tag{1}$$

$$J_e\dot{q} = {}_{ee}X_o J_o\dot{q} \tag{2}$$

therefore, also for the jacobians holds:

$$J_e = {}_eX_o J_o \tag{3}$$

where:

$$
{}_eX_o = \begin{bmatrix} {}_oR_e & -{}_oR_e[{}_et]_\times \\ 0 & {}_oR_e \end{bmatrix}^{-1} \tag{4}
$$

Where ${}_et$ is the relative position of the origin of frame o w.r.t. frame e expressed in frame e. By activating the TF function in *rviz* check the relative location of the frames.