## **Propensity Score Matching**

#### Outline

- Describe the problem
- Introduce propensity score matching as one solution
- Present empirical tests of propensity score estimates as unbiased
- Illustrate special challenges in practice
- Discuss any applications from your work

### "Work Horse" Design: Description

- O \_X\_O\_ O O
- Two components, pretest and control group not formed at random and hence non-equivalent on expectation
- These two components make up one slope that one wants to treat as though it were a perfect counterfactual
- But it isn't known to be, and is not likely to be

# Chief Internal Validity Threats with Design

With or without differential attrition, we struggle to rule out:

- Selection–Maturation
- Selection-History (Local History)
- Selection—Instrumentation
- Selection-Statistical Regression
- So why not match to eliminate all these different faces of selection? If groups can be made equivalent to start with, does not the problem go away, as it does with random assignment?

### Bad Matching for Comparability

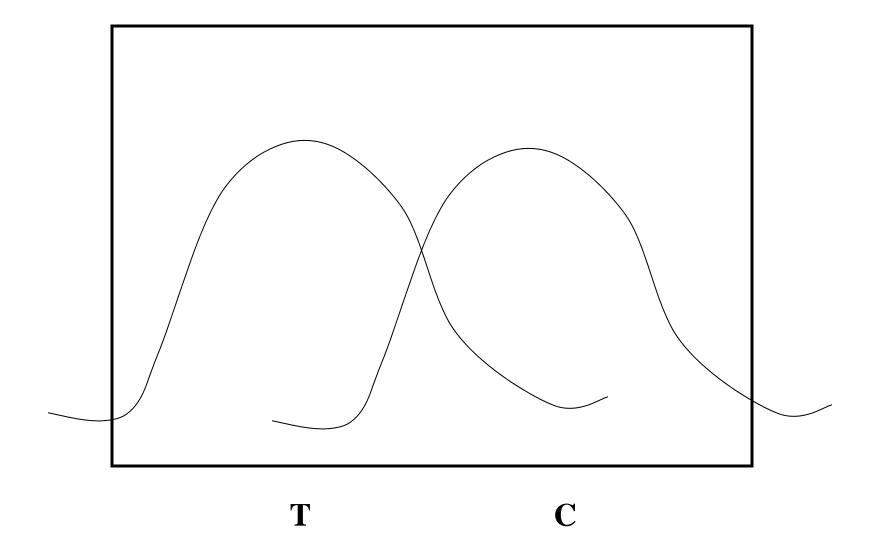
Simple Regression illustrated with one group

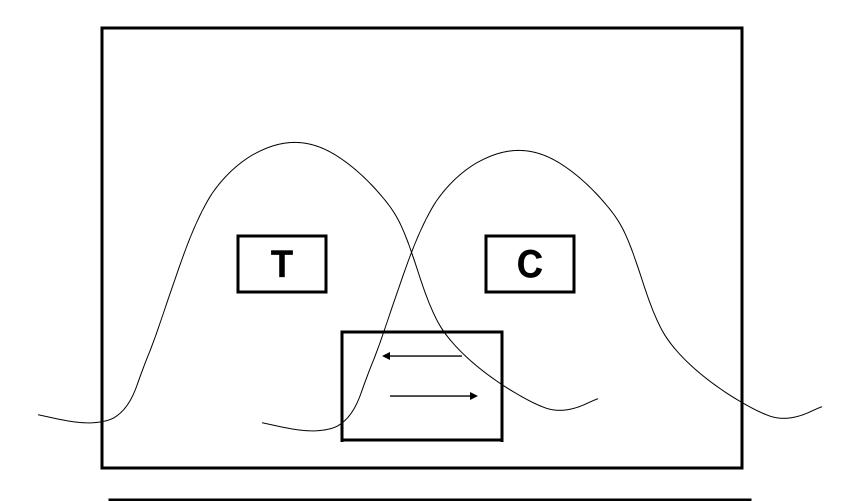
Frequency of such regression in our society

It is a function of all imperfect correlations

Size of Regression gross f(unreliability/population difference)

Simple one-on-one case matching from overlap visually described





T: Decreases

C: Increases

Both forces makes T look ineffective

#### The Net Effect is...

- If either treatment decreases or controls increase due to regression, then bias results
- If both change in opposite directions, then the bias is exacerbated
- Matching individual units from extremes is not recommended

#### In this Predicament You Might...

- The Cicirelli Head Start Evaluation had this problem, concluding Head Start was harmful
- LISREL reanalyses by Magidson using multiple measures at pretest led to different conclusion, likely by reducing unreliability.
- Reliability is higher using aggregate scores like schools--but beware here as with effective schools literature.

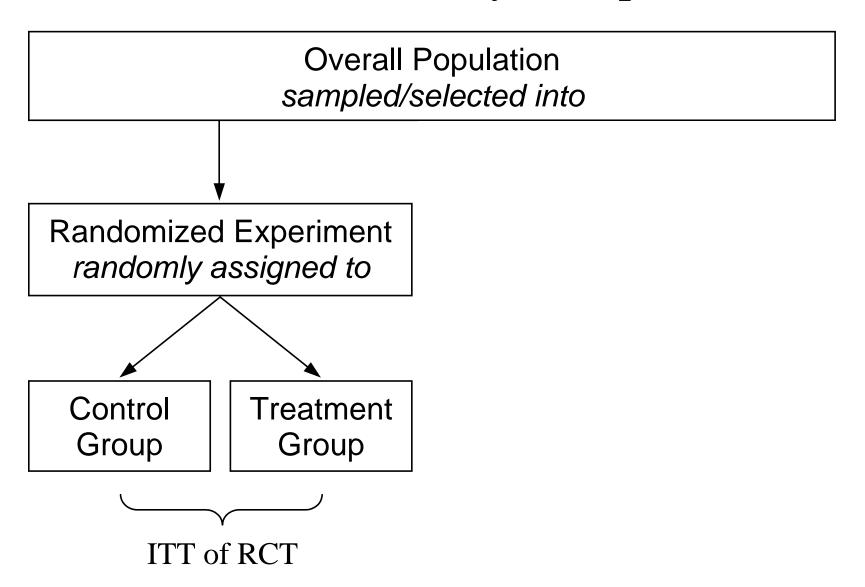
#### Better is to get out of the Pickle

- Don't match from extremes! Use intact groups instead, selecting for comparability on pretest
- Comer Detroit study as an example
- Sample schools in same district; match by multiple years of prior achievement and by race composition of school body--why?
- Choose multiple matches per intervention school, bracketing so that one close match above and the other below intervention schools

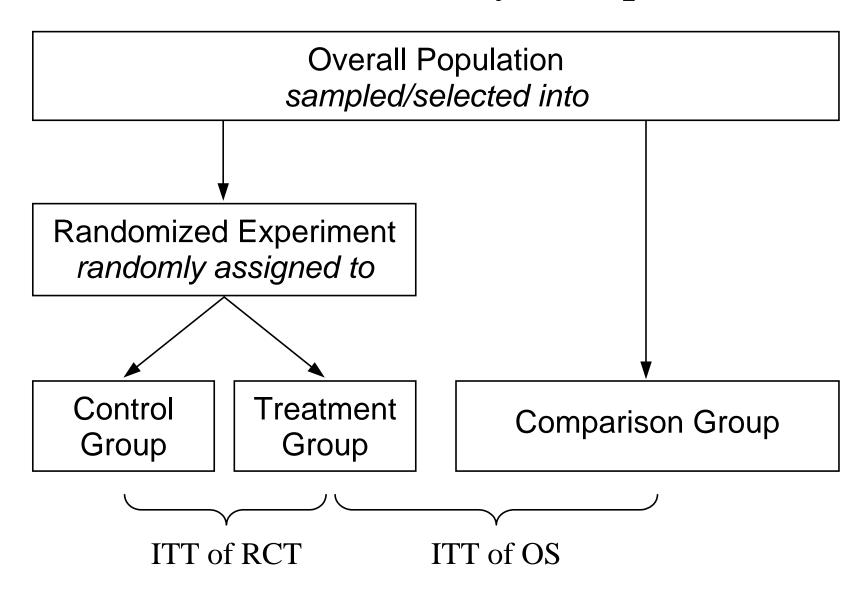
### The Value of Intact Group Matching?

Exemplified through within-study comparisons

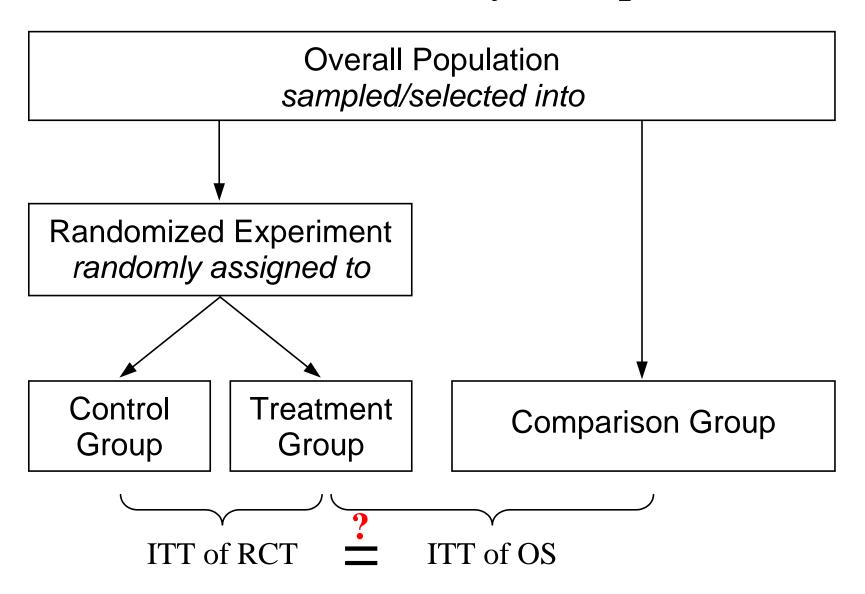
### What is a Within-Study Comparison?



#### What is a Within-Study Comparison?



#### What is a Within-Study Comparison?



# Criteria for Comparing Experiments and Q-Es

- Clear variation in mode of forming control group-random or not
- RCT merits being considered a "gold standard" because it demonstrably meets assumptions
- Experiment and non-experiment difference is not confounded with 3rd variables like measurement
- The quasi-experiment should be a good example of its type--otherwise one compares a good experiment to a poor quasi-experiment

#### Criteria continued

- The experiment and quasi-experiment should estimate the same causal quantity--not LATE vs ATE or ITT vs TOT
- Criteria for inferring correspondence of results should be clear
- The non-experimental analyses should be done blind to the experimental results
- Historical change in meeting of criteria

### Consider 3 examples

■ Bloom, Michaelopoulos et al.

■ Aiken, West et al.

■ Diaz & Handa

#### Bloom, Michaelopoulos et al

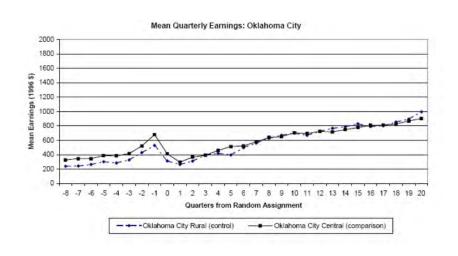
Logic of the design is to compare ES from a randomly created control group with ES from a non-randomly formed comparison that shares the same treatment group

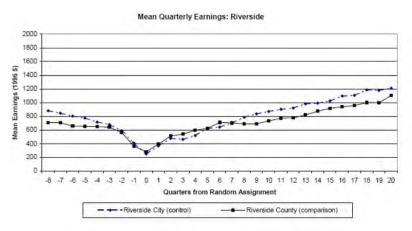
- Treatment group is a constant and can be ignored, comparing only the two types of control
- Issue is: Will the randomly and non-randomly formed control groups differ over 8 pretest observation points after standard statistical adjustments for any differences in mean or slope

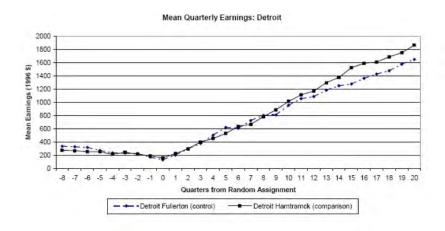
#### The Context

- RCT is on job training at 11 sites
- Bloom et al restrict the ITS to 5 within-state comparisons, 4 of them within-city
- Non-random comparison cases chosen from job training centers in same city
- Measured in the same ways as treated at same times

## Results: 3 within-city Samples







#### What you see in Graphs

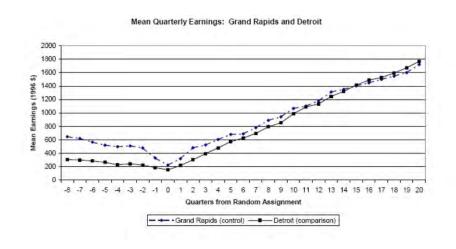
- Hardly differ at all --- one advantage of TS is that we can see group differences
- Statistical tests confirm no differences in intercept or slope
- In these cases, equivalence of randomly and non-randomly formed comparison groups is achieved thru sampling design alone
- Thus, no need for statistical tests to render them "equivalent"

#### Two other Sites

Portland--sample size smallest and least stable

Detroit vs Grand Rapids--a within-state but not withincity comparison. Hence, this is not a very local
comparison

## Bloom et al. Results (2)





#### Here you see

- TS are not equivalent overall
- TS especially not equivalent around the crucial intervention point
- Thus use of a random or non-random control group would produce different results
- 10 types of statistical analyses were used to make the series equivalent:

#### Results of these Analyses

- OLS, propensity scores, Heckman selection models, random growth models--all failed to give the same results as the experiment under these conditions
- But the more the pretest time points, the less the bias
- Only the random growth model took advantage of the TS nature of the data
- Why did it fail too?

# Selecting Intact Groups locally matched on pretest outcomes

- Without intending it, Bloom et al's choice of withincity non-equivalent controls achieved comparability with the randomly formed experimental controls. That is, there was
- No bias across 3 of the 4 within-city samples; nor for the weighted average of all 4 sites
- So, overlap on observables was achieved through the sampling design alone, precluding need for statistical adjustments
- Remember: There was bias in across-state comparisons, and it could not be adjusted away statistically with the data and models used

# Selecting Intact Groups with Maximal Overlap: 2nd Example

- Aiken et al. ASU--effects of remedial writing
- Sample selection in their Quasi-Experiment was from the same range of ACTs and SATs as in their experiment
- Differed by failure of researchers to contact them over summer and later registration
- What will the role of unobserved variables be that are correlated with these two features that differentiate randomly and non-randomly formed control units?
- Measurement framework the same in the experiment and quasi-experiment, as were the intervention and control group experiences

#### Results

- On SAT/CAT, 2 pretest writing measures, the randomly and non-randomly formed comparison groups did not differ
- So close correspondence on observables w/o any need for statistical adjustment; and
- In Q-E, OLS test controls for pretest to add power and not to reduce bias
- Results for multiple choice writing test in SD units = .59 and .57--both sig.
- Results for essay = .06 and .16 both non-sig

## 3<sup>rd</sup> Example: Diaz & Handa (2006)

- Progresa: Matched Villages with and without the program
- One sample of villages had to meet the village eligibility standards—in bottom quintile on test of material resources, but for a variety of reasons not in experiment
- The eligible no-treatment comparison families in these villages were not different on outcomes from the randomly created comparison group

# But there were different on a few family characteristics

Nonetheless, the results of the matched village analyses were similar whether covariates were added to control for these differences or not

#### Implications of all Three Studies

- Aiken et al and Bloom et al. created non-equivalent control groups that were not different on observables from the treatment group.
- These observables included a pretest on the same scale as the outcome
- Diaz and Handa created much overlap but some differences that did not affect outcome
- But what about unobservables? We never know. But if there are real differences, or real unadjusted differences, then we know to worry

## What is a Local, Focal, Non-Equivalent Intact Control Group 1

- Local because...
- Focal because...
- Non-Equivalent because...
- Intact because...

## What is a Local, Focal, Non-Equivalent Intact Control Group 2

- Identical Twins
- Fraternal Twins
- Siblings
- Successive Grade Cohorts within the same School
- Same Cohort within different Schools in same District
- Same Cohort within different Schools in different districts in same state
- Same Cohort within different schools in different states, etc.

#### The Trade Offs here are...

- Identity vs. Comparability. We cannot assume that siblings are identical, for example. They have some elements of non-shared genes and environments.
- Comparability vs. Contamination. Closer they are in terms of space and presumed receptivity to the intervention, the greater the risk of contamination.
- To reduce an inferential threat is not to prevent it entirely.

# Analysis of Workhorse Design Data when group Differences

- Modeling the outcome, like covariance analysis
- Modeling selection, like Propensity Scores
- Empirical Validation literature

### Propensity Score Matching

- Begin with design of empirical test of propensity score methods
- Implementation of a PS analysis
  - 1. Causal estimand of interest
  - 2. Selection of covariates
  - 3. PS estimation
  - 4. Estimation of treatment effect
  - 5. Sensitivity analysis
- Results of empirical test

# Nonrandomized Experiments (Quasi-Experiments; Observational Studies)

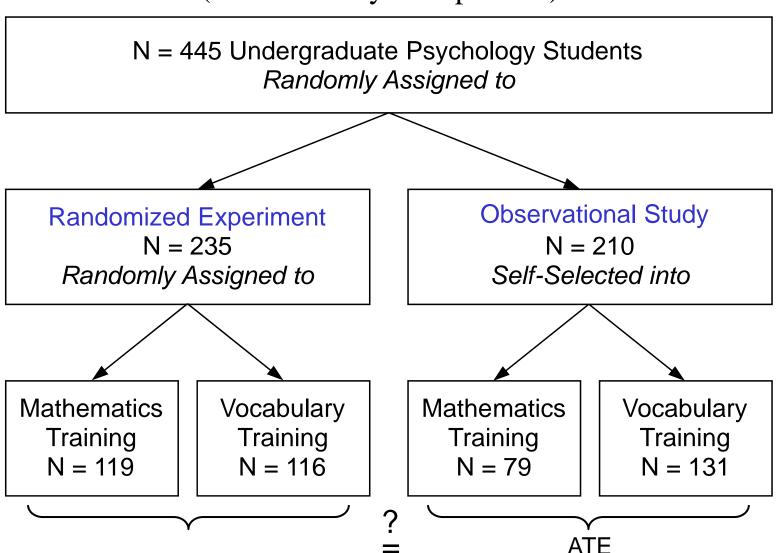
- A central hypothesis about the use of nonrandomized experiments is that their results can well-approximate results from randomized experiments
  - □ especially when the results of the nonrandomized experiment are appropriately adjusted by, for example, selection bias modeling or propensity score analysis.
  - □ I take the goal of such adjustments to be: to estimate what the effect would have been if the nonrandomly assigned participants had instead been randomly assigned to the same conditions and assessed on the same outcome measures.
  - □ The latter is a counterfactual that cannot actually be observed
  - □ So how is it possible to study whether these adjustments work?

# Randomly Assign People to Random or Nonrandom Assigment

- One way to test this is to randomly assign participants to being in a randomized or nonrandomized experiment in which they are otherwise treated identically.
- Then one can adjust the quasi-experimental results to see how well they approximate the randomized results.
- Here is the design as we implemented it:

### Shadish, Clark & Steiner (2008)

(Within-Study Comparison)



### Shadish et al.: Treatments & Outcomes

- Two treatments and outcomes
  - □ *Two treatments*: short training either in *Vocabulary* (advanced vocabulary terms) or *Mathematics* (exponential equations)
    - → All participants were treated together without knowledge of the different conditions
  - □ *Two outcomes*: *Vocabulary* (30-item posttest) and *Mathematics* (20-item posttest)
- Treatment effect:
  - □ ATE: *average treatment effect* for the overall population in the observational study

### Shadish et al.: Covariates

- Extensive measurement of constructs (in the hope that they would establish strong ignorability):
  - □ 5 construct *domains* with
  - □ 23 *constructs* based on
  - □ 156 questionnaire *items*!
- Measured *before* students were randomly assigned to randomized or quasi-experiment
  - ☐ Hence, measurements are not influenced by assignment or treatment

### Shadish et al.: Construct Domains

#### 23 constructs in 5 domains

- Demographics (5 single-item constructs):
  Student's age, sex, race (Caucasian, Afro-American, Hispanic), marital status, credit hours
- *Proxy-pretests* (2 multi-item constructs): 36-item Vocabulary Test II, 15-item Arithmetic Aptitude Test
- Prior academic achievement (3 multi-item constructs):
   High school GPA, current college GPA, ACT college admission score

#### Shadish et al.: Construct Domains

- Topic preference (6 multi-item constructs):
  Liking literature, liking mathematics, preferring mathematics over literature, number of prior mathematics courses, major field of study (math-intensive or not), 25-item mathematics anxiety scale
- *Psychological predisposition* (6 multi-item constructs):

Big five personality factors (50 items on extroversion, emotional stability, agreeableness, openness to experience, conscientiousness), Short Beck Depression Inventory (13 items)

## Shadish et al.: Unadjusted Results Effects of Math Training on Math Outcome

	Math	Vocab	Mean	Absolute
	Tng	Tng	Diff	Bias
	Mean	Mean		
Unadjusted Randomized Experiment	11.35	7.16	4.19	
Unadjusted Quasi-Experiment	12.38	7.37	5.01	.82

#### Conclusions:

- □ The effect of math training on math scores was larger when participants could self-select into math training.
- □ The 4.19 point effect (out of 18 possible points) in the randomized experiment was overestimated by 19.6% (.82 points) in the nonrandomized experiment

## Shadish et al.: Unadjusted Results Effects of Vocab Training on Vocab Outcome

	Vocab	Math	Mean	Absolute
	Tng	Tng	Diff	Bias
	Mean	Mean		
Unadjusted Randomized Experiment	16.19	8.08	8.11	
Unadjusted Quasi-Experiment	16.75	7.75	9.00	.89

#### **Conclusions:**

- □ The effect of vocab training on vocab scores was larger (9 of 30 points) when participants could self-select into vocab training.
- □ The 8.11 point effect (out of 30 possible points) in the randomized experiment was overestimated by 11% (.89 points) in the nonrandomized experiment.

## Adjusted Quasi-Experiments

- It is no surprise that randomized and nonrandomized experiments might yield different answers.
- The more important question is whether statistical adjustments can improve the quasi-experimental estimate, i.e., remove the selection bias
- Selection bias can be removed if
  - □ The treatment selection is *strongly ignorable*, i.e., all the covariates that simultaneously affect treatment selection and potential outcome (confounders) are measured

## Theory of Strong Ignorability (SI)

- If all covariates  $\mathbf{X} = (X_1, ..., X_p)'$  related to both treatment assignment and potential outcomes are observed, and
- If the selection probabilities, given  $\mathbf{X}$ , are strictly between zero and one  $0 < P(Z = 1 | \mathbf{X}) < 1$  holds
- then, potential outcomes are independent of treatment assignment given observed covariates **X**:

$$(Y^0,Y^1)\perp Z\mid \mathbf{X}$$

and treatment assignment is said to be strongly ignorable (*strong ignorability*; Rosenbaum & Rubin 1983).

### Practice of Strong Ignorability (SI)

- Need to measure all confounding covariates! If *not* all covariates, that are simultaneously related to treatment selection and potential outcomes, are observed
  - □ the strong ignorability assumption is not met and
  - □ the average treatment effect will remain biased!
- Need to measure covariates reliably (with respect to the selection mechanism)
- Each subject must have a positive probability of being in the treatment group (overlap).

## How to Estimate an Unbiased Treatment Effect?

- Assume that we observe all covariates **X** such that SI holds, then selection bias can be removed with different approaches
- With original covariates **X** 
  - □ Covariance adjustments (standard regression methods)
  - □ Case matching on observed covariates
  - □ Multivariate stratification
- With a composite of original covariates  $b = f(\mathbf{X})$ 
  - □ *Propensity scores* (Rosenbaum & Rubin, 1983)
  - □ Other approaches we will not consider: first discriminant, prognosis scores

## Propensity Scores (PS)

■ The *propensity score* is the conditional probability that a subject belongs to the treatment group given the *observed* covariates **X**:

$$e(\mathbf{X}) = P(Z = 1 | \mathbf{X})$$

If treatment selection is *strongly ignorable* given an observed set of individual covariates X, then it is also strongly ignorable when these individual covariates are combined into a propensity score e(X), i.e.

$$(Y^0, Y^1) \perp Z \mid e(\mathbf{X})$$
 with  $0 < e(\mathbf{X}) < 1$  (proved Rosenbaum & Rubin 1983)

## Propensity Scores (PS)

- The propensity score reduces all the information in the predictors to one number
  - □ This can make it easier to do matching or stratifying when there are multiple matching variables available.
- In a randomized experiment, the true propensity score is .50 for each person
- In a quasi-experiment, the true propensity score is unknown

### **Estimated Propensity Scores**

- PS needs to be *estimated* from observed data, e.g., using logistic regression
  - □ *Weighted composite* of all covariates affecting treatment selection (covariates that do not affect selection are typically not included in the PS model)
  - □ Estimated PS is *used to match* treatment and control cases with the same or very similar values
  - □ Therefore, the matched treatment and comparison group are *equivalent* on the PS (as it would be in a RCT)
  - □ Interpretation: if treatment = 1 and control = 0, then a propensity score closer to 1.00 (e.g., .83) is a prediction that the person is more likely to be in the treatment group

## Example (treatment, covariates, PS)

Colle	treatment	vocabpre	mathpre	numbmath	likemath	likelit	preflit	pextra	pagree	collgpaa	PS
	.00	24.00	7.00		6.00	7.00	2	16.00	39.00	2.73	.51168
	.00	26.00	3.00	2.00	2.00	10.00	3	22.00	41.00	3.60	.81662
	.00	17.00	5.00		3.00	8.00	3	31.00	39.00	2.76	.53052
	1.00	23.00	4.00	2.00	8.00	10.00	2	22.00	46.00	1.64	.68402
-	1.00	23.00	5.00		2.00	7.00	3	29.00	48.00	3.66	.70982
	1.00	28.00	7.00		2.00	9.00	3	28.00	43.00	3.42	.84256
	.00	12.00	5.00	4.00	6.00	6.00	1	31.00	32.00	2.20	.27809
	1.00	25.00	10.00	3.00	4.00	3.00	2	23.00	36.00	2.40	.54592
)	.00	17.00	9.00	1.00	7.00	2.00	1	20.00	50.00	3.00	.30746
0	1.00	23.00	8.00		4.00	7.00	3	34.00	42.00	3.65	.79574
1	1.00	26.00	3.00	1.00	4.00	8.00	3	21.00	40.00	2.56	.8259
2	.00	19.00	7.00		3.00	8.00	3	35.00	44.00	3.69	.77284
3	1.00	17.00	9.00	1.00	3.00	5.00	3	29.00	35.00	2.62	.72132
4	1.00	23.00	10.00		2.00	9.00	3	35.00	32.00	3.15	.83434
5	1.00	24.00	7.00		7.00	10.00	3	37.00	40.00	3.43	.66448
6	.00	6.00	5.00	2.00	8.00	6.00	1	22.00	35.00	3.16	.17003
7	1.00	28.00	9.00	1.00	3.00	8.00	3	37.00	43.00	3.16	.81817
8	.00	19.00	6.00	1.00	8.00	10.00	3	35.25	41.49	2.31	.43690
9	1.00	13.00	5.00	2.00	6.00	4.00	2	32.00	36.00	2.88	.52160
0	1.00	21.00	4.00		2.00	4.00	3	18.00	31.00	3.70	75800

## Steps in Conducting a PS Analysis

### Steps in conducting a PS analysis

- 1. Choice of the treatment effect of interest
- 2. Assessing strong ignorability
- 3. Estimation of the PS
- 4. Estimation of the treatment effect & standard errors
- 5. Sensitivity analysis
- Use data from Shadish et al. (2008) to illustrate

## PS Analysis

#### Steps in conducting a PS analysis

- 1. Choice of the treatment effect of interest
- 2. Assessing strong ignorability
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- 4. Estimation of the treatment effect & standard errors
- 5. Sensitivity analysis

#### Choice of Treatment effect

- Types of treatment effects:
  - □ Average treatment effect (ATE):

    Treatment effect for the *overall target population* in the study (treated and untreated subjects together)
  - □ Average treatment effect for the treated (ATT): Treatment effect for the *treated* subjects only
  - □ Average treatment effect for the untreated (ATU): Treatment effect for the *untreated* subjects only
- ATE, ATT, and ATU differ if the treatment effect is not constant (heterogeneous) across subjects

#### Choice of Treatment effect

- Choice depends on the research question (and sometimes on the data at hand)
- Example: Effect of retaining students on achievement scores (one or more years after retaining):
  - □ ATT investigates the effect of retaining on the retained students (What would have happened if the retained students would have not been retained?)
  - □ ATE investigates the effect of retaining if all students would have been retained vs. all would have passed
- Shadish et al.: ATE
  - □ What is the treatment effect if all students would have received the vocabulary (math) training?

## PS Analysis

### Steps in conducting a PS analysis

- 1. Choice of the treatment effect of interest
- 2. Assessing strong ignorability
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- 5. Sensitivity analysis

## Assessing Covariates & SI

- If a *causal* treatment effect should be estimated, we need to assess whether strong ignorability holds
  - □ Are at least *all* constructs that simultaneously affect treatment selection and potential outcomes measured?
  - □ If selection is on *latent constructs* (e.g., self-selection dependent on ability) instead of *observed ones* (e.g., administrator selection on reported achievement scores), are those constructs *reliably* measured?
- If some important covariates are missing or highly unreliable causal claims are probably not warranted!
  - □ *Hidden bias*, i.e., bias due to unobserved covariates, may remain.

## Assessing Covariates & SI

- How can we guess that SI holds?
  - □ Strong theories on selection process and outcome model
  - □ Expert knowledge
  - □ Direct investigation of the selection process
  - ☐ Having a large number of covariates from heterogeneous domains
- Are there groups of covariates that justify SI more than others?
  - □ *Pretest measures* on the outcome do usually better
  - □ Direct measures of the *selection process*
  - □ Easily available data like demographic are typically not sufficient for warranting SI!

## Assessing Covariates & SI

- Shadish et al. (2008):
  - Extensive & reliable measurement of different construct domains and constructs
    - □ *Pretest measures*, though only proxy-pretest measures (assumed to be highly correlated with potential outcomes)
    - □ *Direct measures* of the assumed *selection process*, e.g., like mathematics, prefer literature over math (assumed to be highly correlated with the selection process)
    - □ *Broad range of other measures* on prior academic achievement, demographics, psychological predisposition
- Would we have been confident in assuming SI—without having our experimental benchmark?

## PS Analysis

### Steps in conducting a PS analysis

- 1. Choice of the treatment effect of interest
- 2. Assessing strong ignorability
- 3. Estimation of the PS
- 4. Estimation of the treatment effect & standard errors
- 5. Sensitivity analysis

#### Estimation of PS

- Different methods for estimating PS can be used:
  - □ *Binomial models*: logistic regression, probit regression
    - Most widely used
    - Rely on functional form assumption
  - □ *Statistical learning algorithms* (data mining methods): classification trees, boosting, bagging, random forests
    - Do not rely on functional form assumptions
    - It is not yet clear, whether these methods are on average better than binomial models

### Estimation of PS

- Logistic regression of treatment Z on observed predictors  $\mathbf{W}$  (covariates  $\mathbf{X}$  or transformations thereof, e.g. polynomials, log, interactions)
  - □ Logistic model:  $logit(Z) \sim W\gamma$
  - $\square$  Estimated *PS logit*:  $\hat{l} = \mathbf{W}\hat{\gamma}$
  - $\exists \text{ Estimated } PS: \qquad \hat{e} = \frac{\exp(\hat{l})}{1 + \exp(\hat{l})}$
  - □ PS (logit) is the predicted value from the logistic regression
- The distribution of the estimated PS-logit shows how similar/different treatment and control group are

## Sample SPSS Syntax

#### LOGISTIC REGRESSION VAR=vm

/METHOD=ENTER vocabpre numbma\_r likema\_r likeli\_r prefli\_r pextra pconsc

beck cauc afram age momdeg\_r daddeg\_r majormi liked avoided selfimpr

/SAVE PRED (ps).

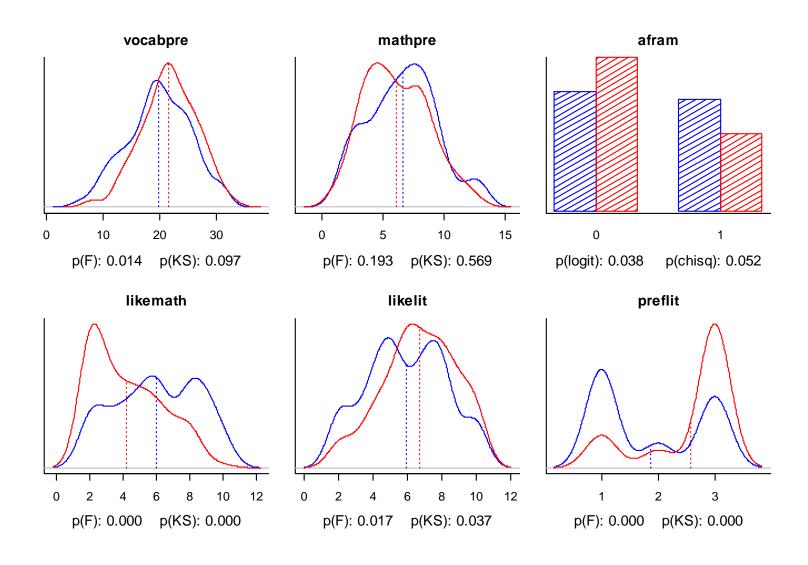
### Balance

- PS-model is specified according to balancing criteria
  - □ good prediction or optimal model fit is not necessarily required—balance in observed covariates matters!
- *Balance* refers to the equivalence of the treatment and control groups' distribution on all observed covariates
  - □ *Joint distribution* of all covariates (in RCT balance is achieved by randomization)
  - □ In practice, balance on the joint distribution is hard to check (curse of dimensionality) -- focus on each covariates distribution separately
- Categories of balancing criteria
  - □ Descriptive criteria & Inferential criteria

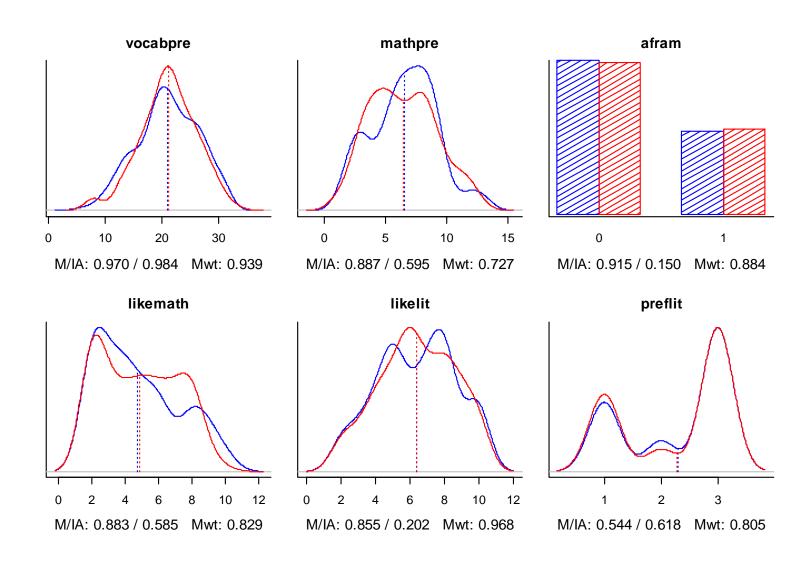
#### Balance

- Descriptive criteria that compare the covariate distribution of the treatment and control group before and after PS-adjustment:
  - □ *Visual inspection*: Comparison of histograms or kernel density estimates; QQ-plots

# Initial Imbalance before PS adjust. (Shadish et al., 2008)



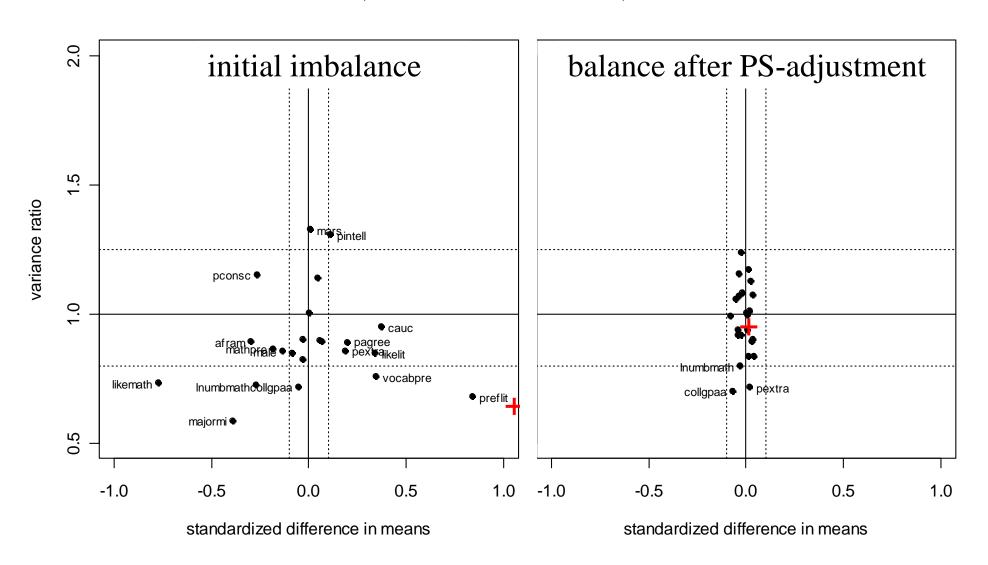
## Balance after PS-Adjustment



### Balance

- *Descriptive criteria* that compare the covariate distribution of the treatment and control group:
  - □ *Visual inspection*: Comparison of histograms or kernel density estimates; QQ-plots
  - □ Standardized mean difference & variance ratio (Rubin, 2001)
    - Standardized mean difference (Cohen's d)  $d = (\overline{x}_t - \overline{x}_c) / \sqrt{(s_t^2 + s_c^2)/2}$ d should be close to zero (|d| < 0.1)
    - Variance ratio  $v = s_t^2 / s_c^2$ v should be close to one (4/5 < v < 5/4)
    - Can also be applied to PS-strata separately for checking balance at different regions of the PS distribution

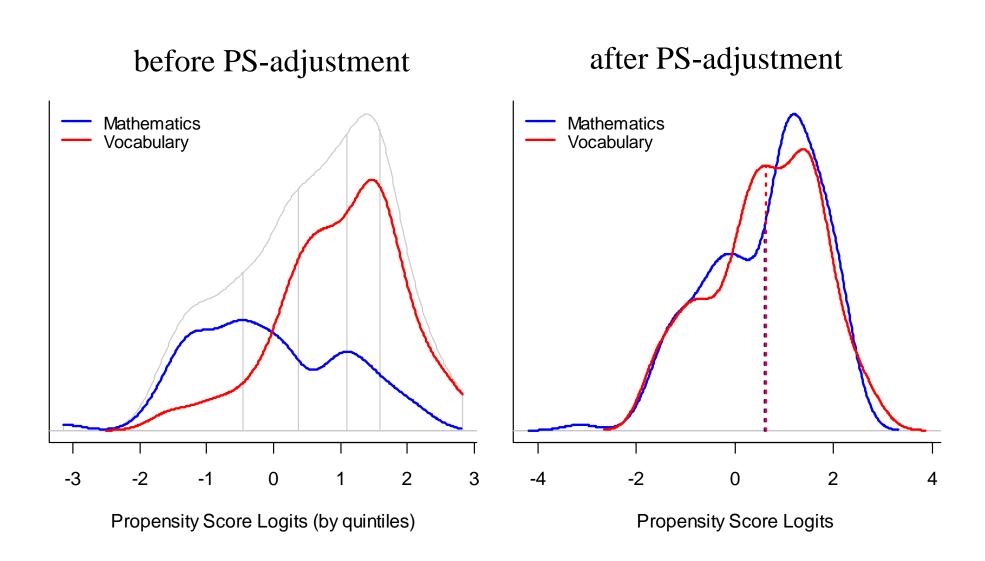
# (Im)balance in Covariates and PS-logit (23 covariates)



#### Balance

- *Inferential criteria* that compare the covariate distribution of the treatment and control group:
  - $\Box$  *t*-test comparing group means
  - $\square$  Hotelling's T (multiviariate version of t-test)
  - □ Kolmogorov-Smirnov test comparing distributions
  - □ Regression tests
- (Dis)advantage of inferential criteria:
  - □ Depend on sample size
    - No enough power for small samples
    - Hard to achieve balance for large samples

## Overlap & Balance (PS-logit)

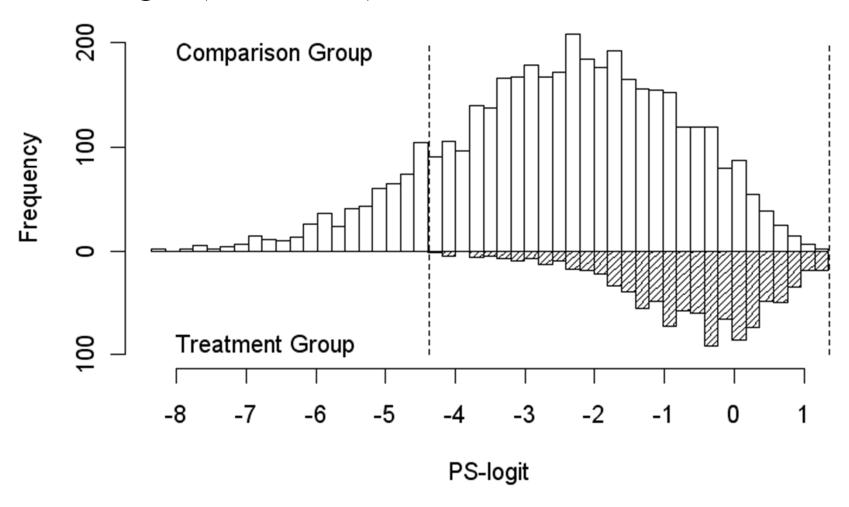


### Overlap

- Overlap of groups: the treatment and control group's common support region on the PS(-logit)
  - □ For *non-overlapping* cases the treatment effect cannot be estimated
  - □ Non-overlapping cases might be the reason why balance cannot be achieved
    - Balance should be checked after deletion of nonoverlapping cases
  - □ Discarding non-overlapping cases restricts the generalizability of results
    - Discarding on the basis of covariate values instead of PS retains the interpretability of the treatment effect

## Overlap

■ Example (simulation)



## How to Specify a PS Model

- 1. Estimate a good fitting logistic regression by
  - ☐ Including higher-order terms and interaction effects of covariates (also try transformed covariate, e.g., log)
  - □ Using usual fitting criteria, e.g. AIC or log-likelihood test
- 2. Delete non-overlapping cases
- 3. Check balance using the same PS techniques as is later used for estimating the treatment effect
- 4. If balance is not satisfying re-specify PS model (on all cases) by including further covariates, higher-order terms or interaction effects
- 5. Go to 2, repeat until satisfactory balance is achieved

## What if Balance/Overlap cannot be achieved?

- Indication that groups are too *heterogeneous* for estimating a causal treatment effect (particularly if groups only overlap on their tails)
- If imbalance is not too severe one could hope that the *additional covariance adjustment* removes the residual bias due to imperfect balance
- It is usually harder to achieve balance on a large set of covariates than on small set of covariates

## PS Analysis

#### Steps in conducting a PS analysis

- 1. Choice of the treatment effect of interest
- 2. Assessing strong ignorability
- 3. Estimation of the PS
- 4. Estimation of the treatment effect & standard errors
- 5. Sensitivity analysis

### Choice of PS technique

- Four groups of PS techniques (Schafer & Kang, 2008; Morgan & Winship, 2007:
  - □ Individual case matching on PS
  - □ PS-Stratification
  - □ Inverse PS-Weighting
  - □ Regression estimation (ANCOVA) using PS
- Combination with an additional covariance adjustment (within the regression framework)
  - ☐ Makes estimates "doubly robust", i.e., robust against the misspecification either of the PS model or outcome model
  - □ However, if both models are misspecified, bias can also increase (Kang & Schafer, 2007)

## Individual Case Matching

- General idea: for each treated subject find at least one control subject who has the same or a very similar PS
  - ☐ Matching estimators typically estimate ATT
- Types of matching strategies/algorithms
  - □ *Number of matches*: 1:1 matching, 1:k matching, full matching
  - □ *Replacement*: With or without replacement of previously matched subjects
  - □ *Similarity of matches*: Exact matching, caliper matching
  - □ *Algorithm*: Optimal matching, genetic matching, greedy matching, ...

## Individual Case Matching

- Standard matching procedures results in two groups:
  - □ *Matched treatment group* (might be smaller than the original treatment group if no adequate matches were found)
  - □ *Matched control group* (those cases of the original control group that where matched to one or more treatment cases)
- Estimation of the treatment effect:
  - □ Difference in the treatment and control means of the matched samples,  $\hat{\tau} = \overline{Y}_{m1} \overline{Y}_{m0}$  (one- or two sample *t*-test)
  - □ Alternatively, using regression without or with additional covariance adjustment:

$$\hat{Y}_m = \hat{\alpha} + \hat{\tau} Z_m \qquad \hat{Y}_m = \hat{\alpha} + \hat{\tau} Z_m + \mathbf{X}_m \hat{\beta}$$

## Individual Case Matching

■ Matching procedures are available in different statistical (programming) packages:

□ R: MatchIt (Ho, Imai, King & Stuart, in print)

Matching (Sekhon, in print)

optmatch (Hansen & Klopfer, 2006)

□ STATA: PSMATCH2 (Leuven & Sianesi, 2003)

MATCH (Abadie et al., 2004)

PSCORE (Becker & Ichino, 2002)

□ SAS: greedy matching (Parsons, 2001)

optimal matching (Kosanke & Bergstralh, 2004)

□ SPSS: greedy matching (Levesque R. Raynald's SPSS

Tools)

#### **PS-Stratification**

- Use the PS to stratify all observations into j = 1, ..., k strata
  - □ Most frequently quantiles are used
  - □ Quantiles: remove approx. 90% of the bias (given SI holds)
  - □ More strata remove more bias but the number of treated and control cases should not be too small (> 10 obs.)
- Estimate the treatment effect within each stratum
  - $\square$  mean difference within each stratum  $\hat{\tau}_j = \overline{Y_1}^j \overline{Y_0}^j$  or
  - $\square$  additional covariance adjustment removes residual bias: regression within each k strata:  $\hat{Y}_i = \hat{\alpha}_i + \hat{\tau}_i Z_i + \mathbf{X}_i \hat{\beta}_i$

#### **PS-Stratification**

■ Pool the stratum-specific treatment effects

$$\Box \hat{\tau} = \sum_{j=1}^k \hat{\tau}_j w_j$$

□ with weights

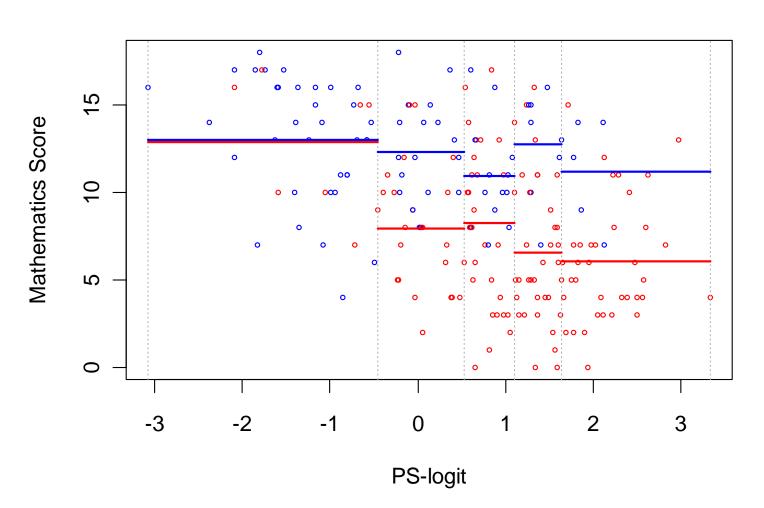
$$w_j = (n_{0j} + n_{1j})/n$$
 for ATE  
 $w_j = n_{1j}/n_{1.}$  for ATT

Stratum	j=1	j=2		j = k	sum	
Z = 0	$n_{01}$	$n_{02}$	•••	$n_{0k}$	$n_{0.}$	
Z = 1	$n_{11}$	$n_{12}$	•••	$n_{1k}$	$n_{1.}$	← ATT
sum	$n_{.1}$	$n_{.2}$	•••	$n_{.k}$	n	← ATE

□ Standard error: pooled stratum-specific standard errors

## PS-Stratification Example (Shadish et al.)

#### **Mathematics**



#### **PS-Stratification**

- Shadish et al. (2008): Mathematics outcome
  - □ Stratum-specific means and treatment effect & ATE

means	j=1	j=2	j=3	j=4	<i>j</i> = 5	mean	
Z = 0	12.9	8.0	8.2	6.5	6.0	8.3	
Z=1	13.0	12.3	10.9	12.8	11.2	12.0	
$\hat{ au}$	0.1	4.2	2.7	6.3	5.2	3.7	<b>←</b>

**ATE** 

□ Sample size and ATE-weights

Stratum	j=1	j=2		• • •	j = k	sum	
Z = 0	7	24	29	34	37	131	
Z = 1	35	18	13	8	5	79	
sum	42	42	42	42	42	210	← ATE
weights	.2	.2	.2	.2	.2	1	

#### **PS-Stratification**

□ Stratum-specific means and treatment effect & ATT

means	j=1	j=2	j=3	j=4	j=5	mean	
Z = 0	12.9	8.0	8.2	6.5	6.0	8.3	
Z = 1	13.0	12.3	10.9	12.8	11.2	12.0	
$\hat{ au}$	0.1	4.2	2.7	6.3	5.2	2.4	<b>←</b>

$$\hat{\tau} = .1 \cdot .44 + 4.2 \cdot .23 + 2.7 \cdot .16 + 6.3 \cdot .10 + 5.2 \cdot .06 = 2.4$$

**ATT** 

□ Sample size and ATT-weights

Stratum	j=1	j=2		•••	j = k	sum	
Z = 0	7	24	29	34	37	131	
Z = 1	35	18	13	8	5	79	← ATT
sum	42	42	42	42	42	210	
weights	.44	.23	.16	.10	.06	1	

## Inverse PS-Weighting

- Same idea as inverse probability weighting in survey research (Horvitz & Thompson, 1952)
- Weights for ATE:
  - $\square$   $w_i = 1/e(\mathbf{X})$  for the treated (Z = 1)
  - $\square$   $w_i = 1/(1 e(\mathbf{X}))$  for the untreated (Z = 0)
- Weights for ATT:
  - $\square$   $w_i = 1$  for the treated (Z = 1)
  - $\square$   $w_i = e(\mathbf{X})/(1-e(\mathbf{X}))$  for the untreated (Z=0)

## Inverse PS-Weighting

#### ■ Treatment effect:

- ☐ Treatment effect is the difference between the weighted treatment and control means, which can also be estimated using
- □ Weighted least squares regression (WLS) without or with covariates

#### ■ Disadvantage:

- ☐ Sensitivity to large weights
- □ Larger standard errors

### Regression Estimation using PS

- PS can be included as additional covariate in standard regression approaches. The PS can be included as
  - □ Quadratic or cubic *polynomial* of PS or *PS-logit* (the latter is more likely linearly related to the outcome)
  - □ *Dummy variables* for PS strata like deciles, for instance. Is less restrictive than the polynomial.
  - □ *Combination* of dummy variables and the linear PS-logit
- As simple regression estimate can be obtained by using the cubic polynomial of the PS logit, for instance,  $\hat{Y} = \hat{\alpha} + \hat{\tau}Z + \hat{\beta}_1\hat{l} + \hat{\beta}_2\hat{l}^2 + \hat{\beta}_3\hat{l}^3 + \mathbf{X}\hat{\gamma}$ 
  - □ strong functional form assumptions (same from for T & C)
  - □ ATE or ATT is not clearly defined

#### Regression Estimation using PS

- A better approach is as follows (regression estimation; Schafer & Kang, 2008):
  - □ Estimate the regression model for each group separately
  - ☐ Use the estimated treatment regression for predicting the missing treatment outcomes of the control group
  - ☐ Use the estimated control regression for predicting the missing control outcomes of the treatment group
  - □ Compute for each subject the difference between the observed (predicted) treatment and predicted (observed) control outcome
  - ☐ The average across these differences estimates ATE

## ATE & Bias Reduction: Vocabulary

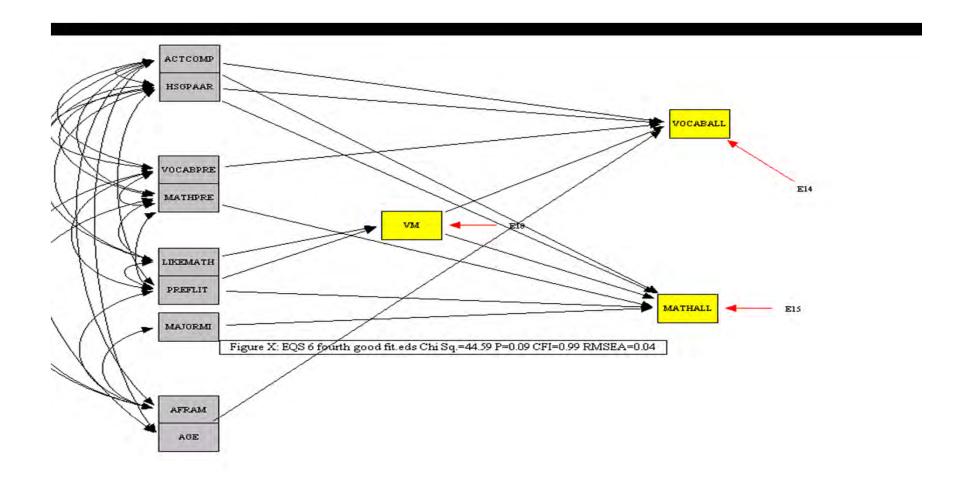
Vocabu	lary Outcome		
	Mean Diff. (s.e.) $R^2$		Percent Bias Reduction
Covariate-Adjusted Randomized Exp.	8.25 (.37)	.71	
Unadjusted Quasi-Experiment	9.00 (.51)	.60	
PS Stratification	8.11 (.52)	.76	80%
PS Linear ANCOVA	8.07 (.47)	.76	76%
PS Nonlinear ANCOVA	8.03 (.48)	.77	70%
PS Weighting	8.19 (.51)	.76	91%
ANCOVA Using Observed Covariates	8.21 (.43)	.76	94%

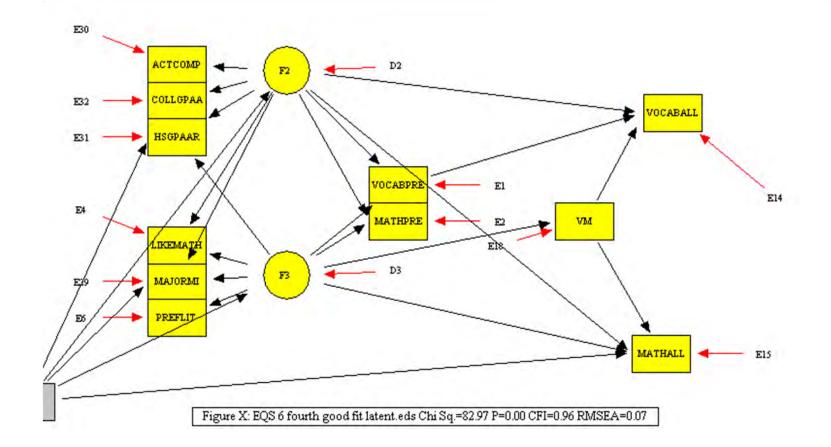
#### ATE & Bias Reduction: Mathematics

Mathem	atics Outcome		
	Mean Diff. (s.e.)	$R^2$	Percent Bias Reduction
Covariate-Adjusted Randomized Exp.	4.01 (.35)	.58	
Unadjusted Quasi-Experiment	5.01 (.55)	.28	
PS Stratification	3.74 (.42)	.66	73%
PS Linear ANCOVA	3.65 (.42)	.64	64%
PS Nonlinear ANCOVA	3.67 (.42)	.63	66%
PS Weighting	3.71 (.40)	.66	70%
ANCOVA Using Observed Covariates	3.85 (.44)	.63	84%

## Structural Equation Models as Adjustments

- If ordinary ANCOVA did well, perhaps SEM would do well too.
- After all, it can do more complex models than ordinary ANCOVA:





Randomized Results		Math Effect	Vocab Effect
		4.01	8.25
Observed Variable	Models		
Model	CFI	Math Effect	Vocab Effect
First good fit	.987	4.37 .95% bias	8.48
Second good fit	.971	3.83 /reduction	8.19
Third good fit	.980	3.96 /	8.38
Fourth good fit	.990	3.96	8.43
Fifth good fit	.978	3.91	8.32
Observed Mediatio	nal Models	76% bias reduction	
Fourth good fit	.983	3.96	8.43
Latent Variable Mo	dels		
Model	CFI	Math Effect	Vocab Effect
Fourth good fit	.961	3.69	8.49

#### Which Method Should be Chosen?

#### Bias

- □ Without additional covariance adjustment *matching* and *stratification* results in some residual bias (due to inexact matching)
- □ *Regression estimation* relies on functional form assumptions

#### Efficiency

- □ Some *matching* estimator are sometimes less efficient because not all available observations are used (except for optimal full matching)
- □ *Weighting* estimators are less efficient (due to extreme weights)

#### Which Method Should be Chosen?

- Sample Size Requirements
  - □ *Matching* does better with large samples and a considerably larger number of control cases than treatment cases
  - □ In general, PS methods are a large sample technique
- Treatment effect
  - □ *Matching* is better suited for estimating ATT
  - ☐ All other methods handle ATT and ATE equally well
- Ease of Implementation
  - □ Matching requires special software tools, but they are available in most major statistical software packages
  - □ Other methods are easy to implement (except s.e. estimation)

#### Which Method Should be Chosen?

- Additional covariance adjustment
  - □ Should be done, though it might sometimes introduce additional bias
  - □ With an additional covariate adjustment, the difference between PS methods gets smaller
- Does the choice of specific technique make a difference?
  - ☐ In general, differences are small
  - ☐ However, for specific data sets it might make a difference

## PS Analysis

#### Steps in conducting a PS analysis

- 1. Choice of the treatment effect of interest
- 2. Assessing strong ignorability
- 3. Estimation of the PS
- 4. Estimation of the treatment effect & standard errors
- 5. Sensitivity analysis

### Sensitivity Analysis

- How sensitive is the estimated treatment effect to *unobserved* confounders?
  - □ Need to assume how strongly such an unobserved covariate is correlated with treatment selection and potential outcomes
    - assume that the unobserved covariate is as strongly correlated with treatment and outcome as the (second) most important observed covariate
    - Sensitivity is determined using a regression approach
  - □ Alternatively one can ask how strongly a covariate has to be correlated with treatment and outcome such that the treatment effect is no longer different from zero

## Empirical Test of PS

- Relative Importance of
  - □ Covariate selection
  - □ Reliable measurement
  - ☐ Choice of analytic methods

## Relative Importance

- What are the most important factors for establishing SI and getting an unbiased causal estimate?
  - □ Selection of *constructs* related to both treatment selection and potential outcomes
  - □ *Reliable measurement* of constructs
  - □ *Balancing* pre-treatment baseline differences (i.e., correct specification of PS model)
  - □ Choice of the *analytic method* (PS methods or ANCOVA)

Ruling out hidden bias

Avoiding overt bias

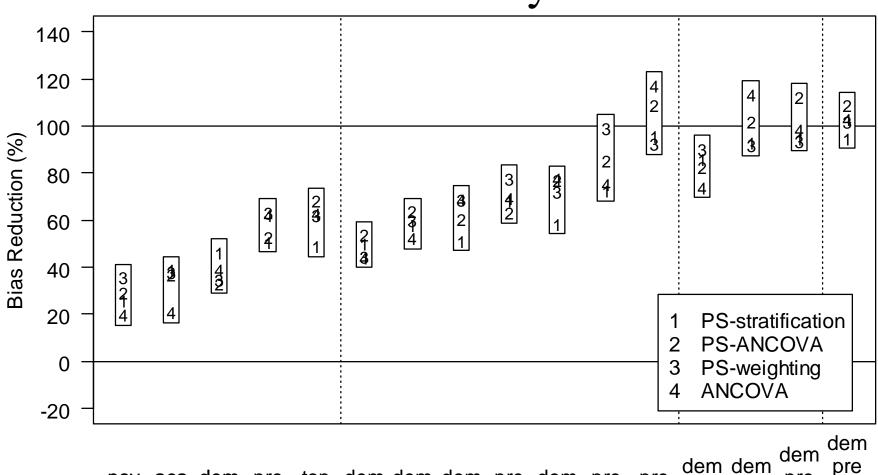
### Importance of Covariate Selection Steiner, Cook, Shadish & Clark (2011)

- Investigate the importance of
  - □ Construct domains
  - □ Single constructs

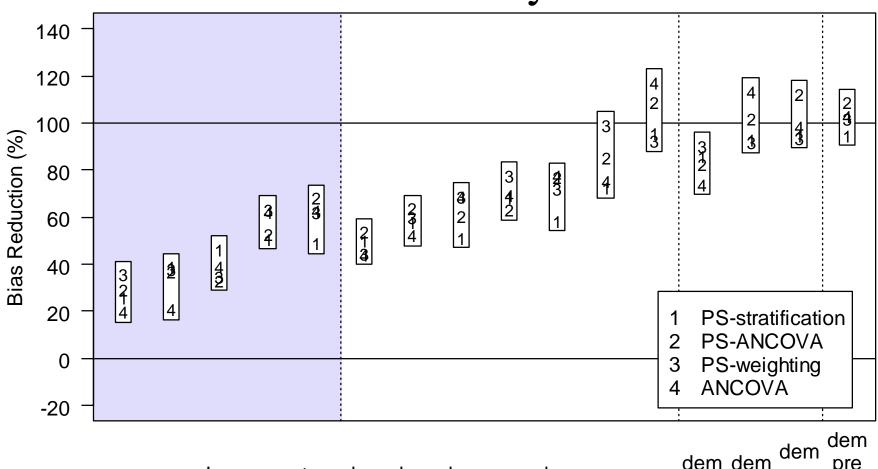
for establishing strong ignorability

Compare it to the importance of choosing a specific analytic method

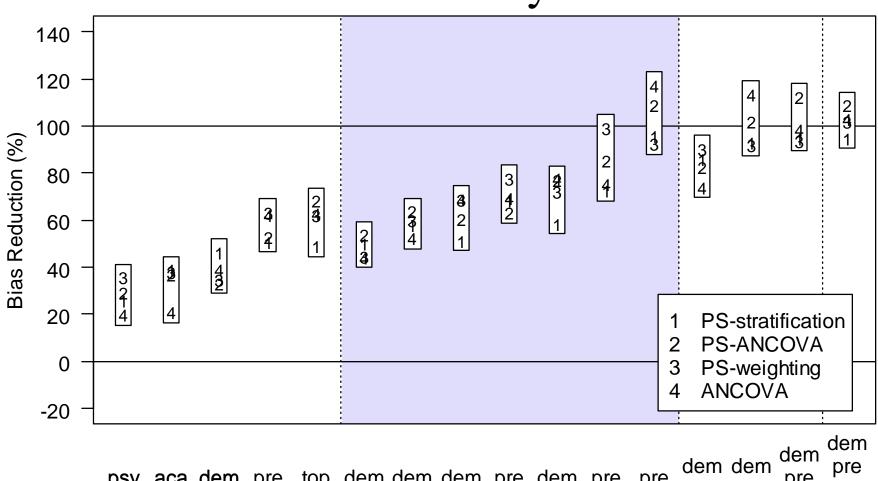
# Bias Reduction: Construct Domains Vocabulary



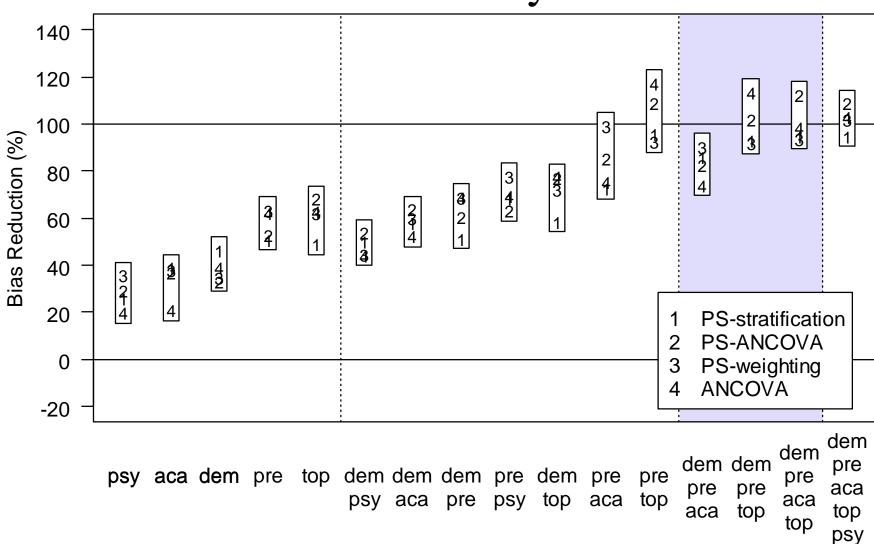
# Bias Reduction: Construct Domains Vocabulary



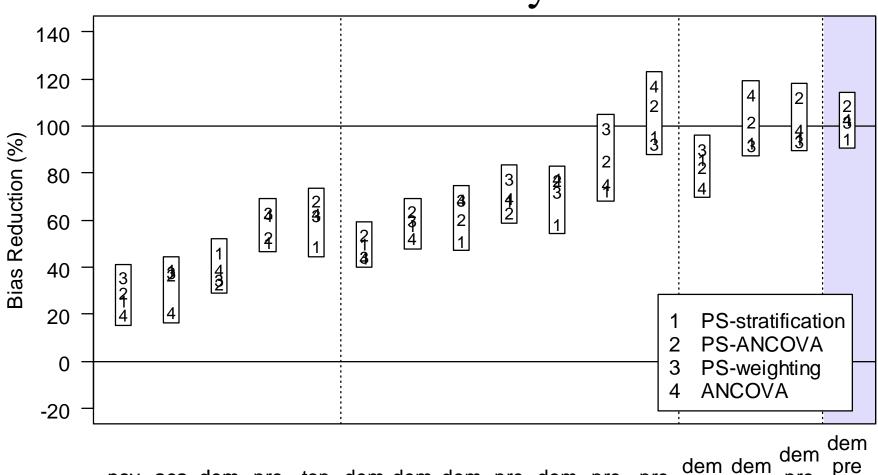
# Bias Reduction: Construct Domains Vocabulary

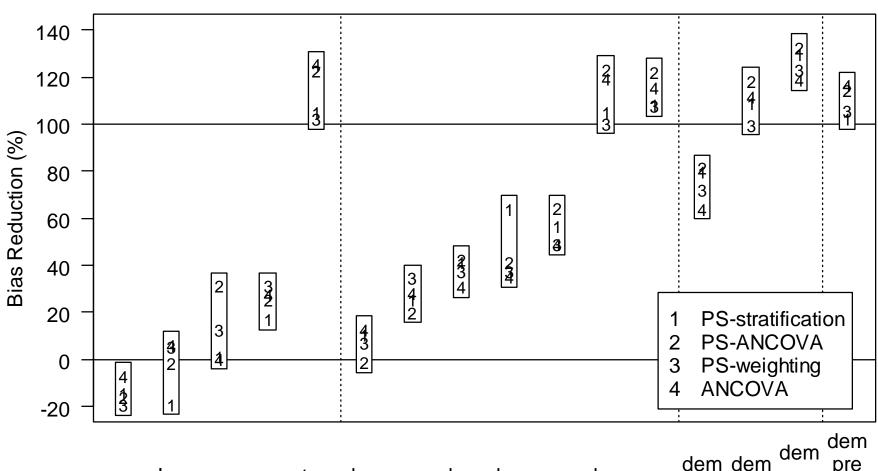


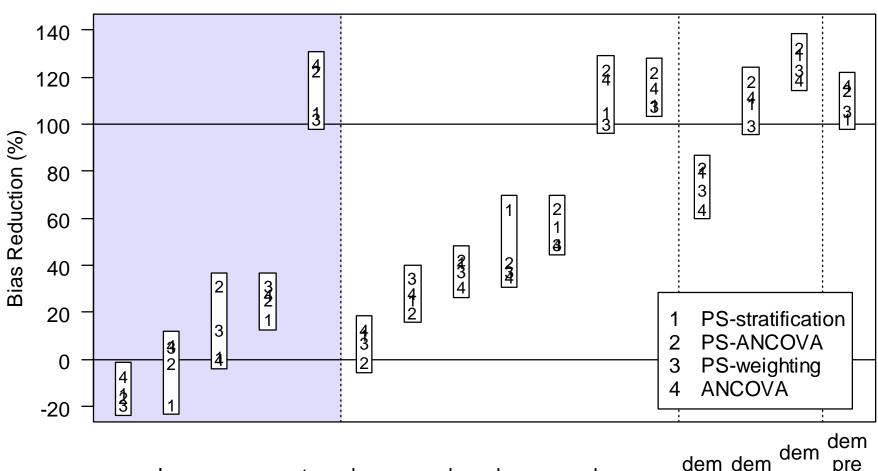
## Bias Reduction: Construct Domains Vocabulary

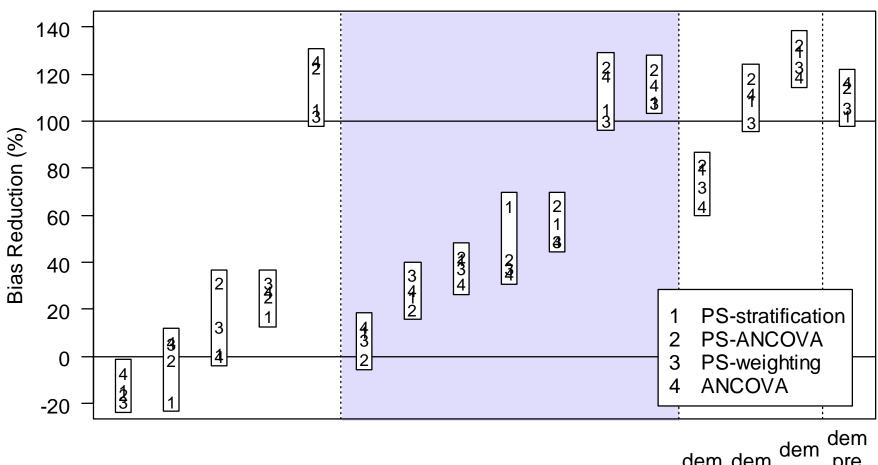


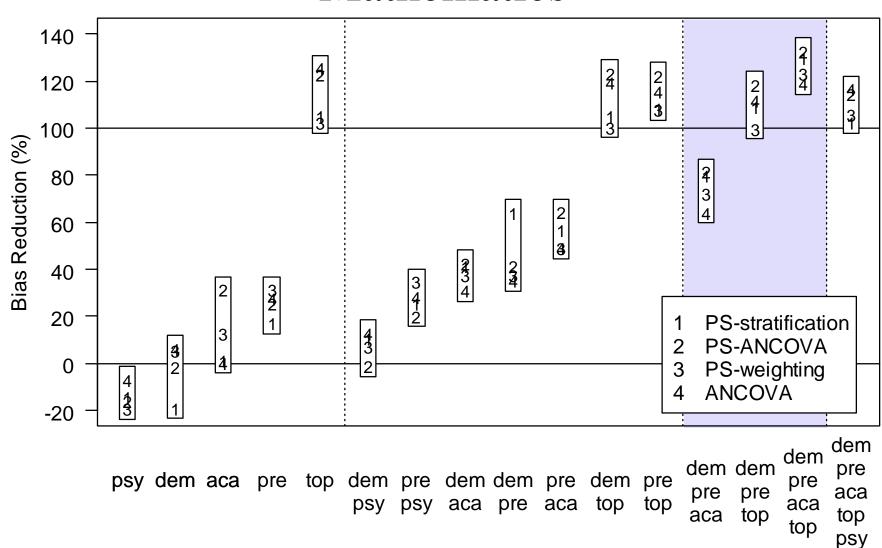
## Bias Reduction: Construct Domains Vocabulary

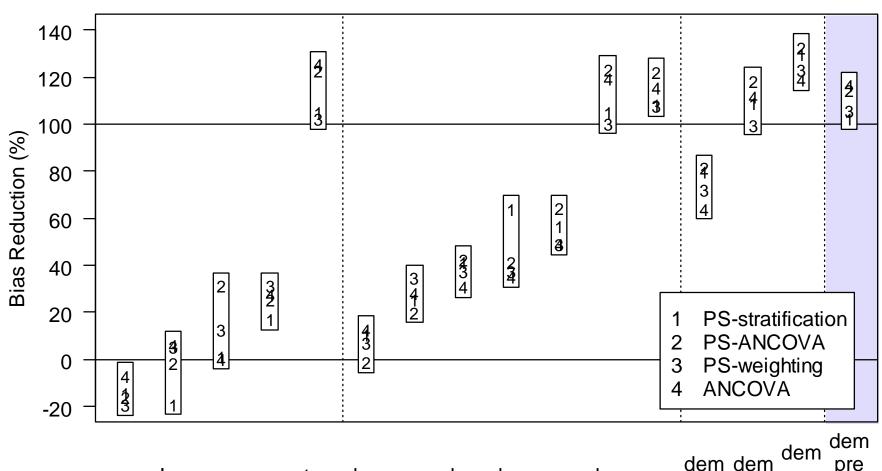




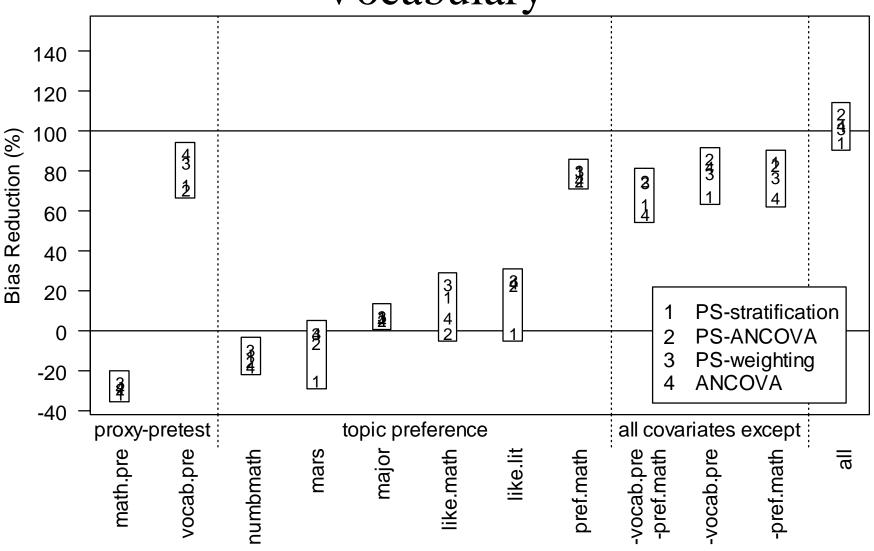




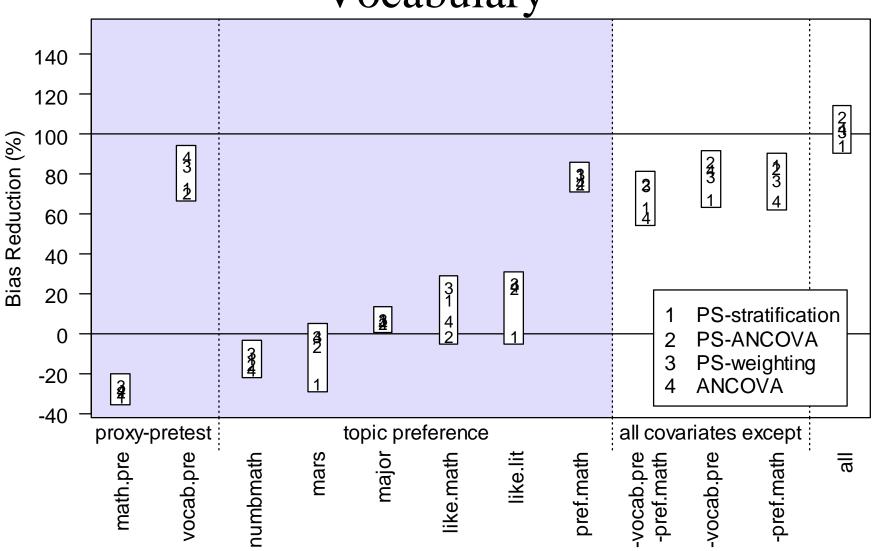




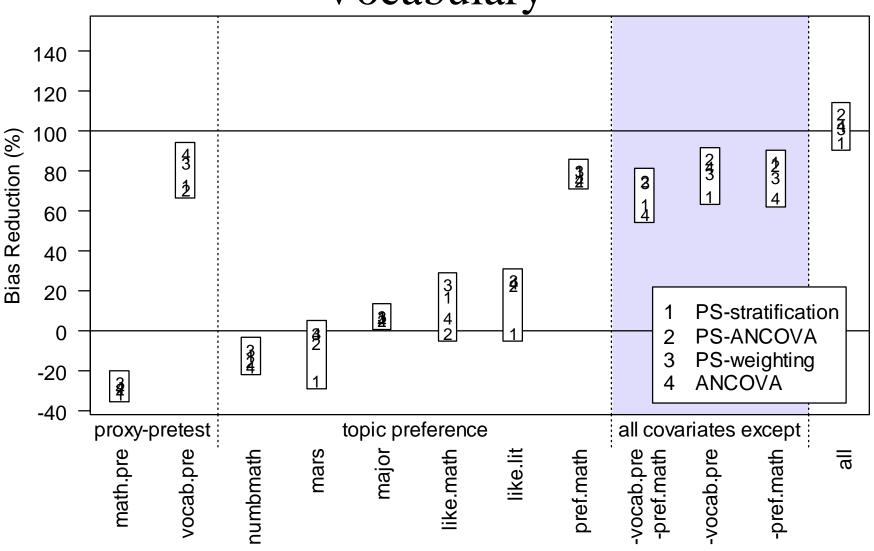
## Bias Reduction: Single Constructs Vocabulary



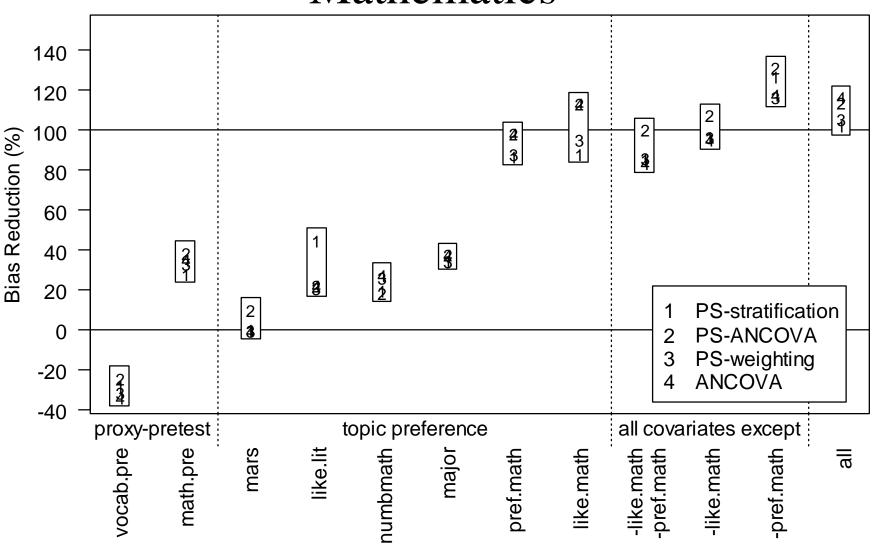
## Bias Reduction: Single Constructs Vocabulary



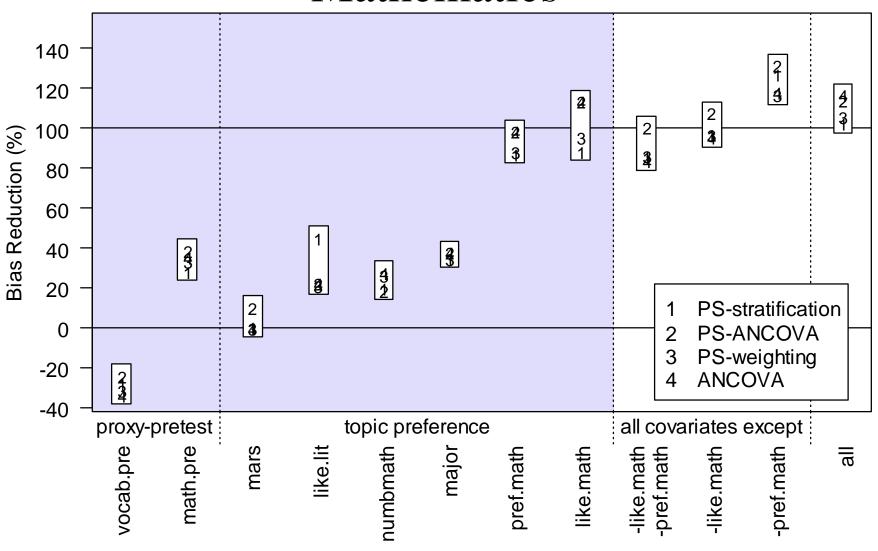
## Bias Reduction: Single Constructs Vocabulary



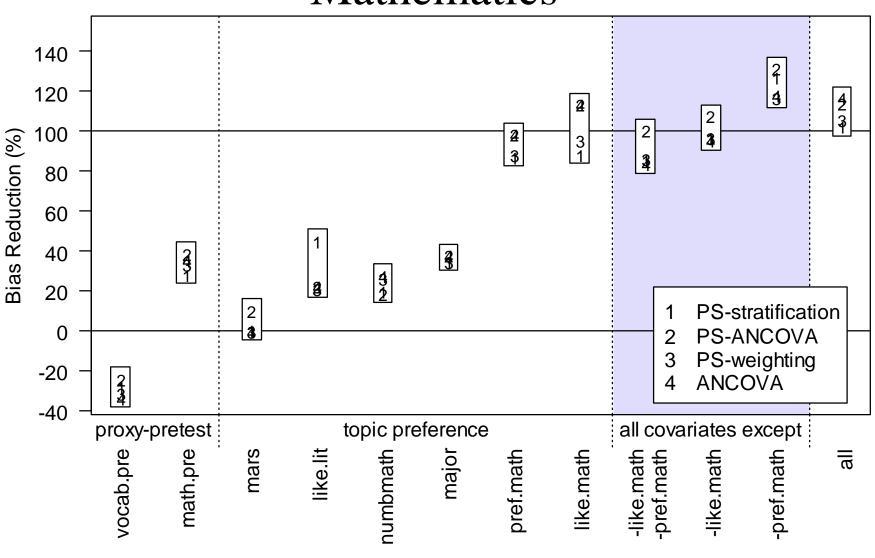
### Bias Reduction: Single Constructs Mathematics



### Bias Reduction: Single Constructs Mathematics



### Bias Reduction: Single Constructs Mathematics

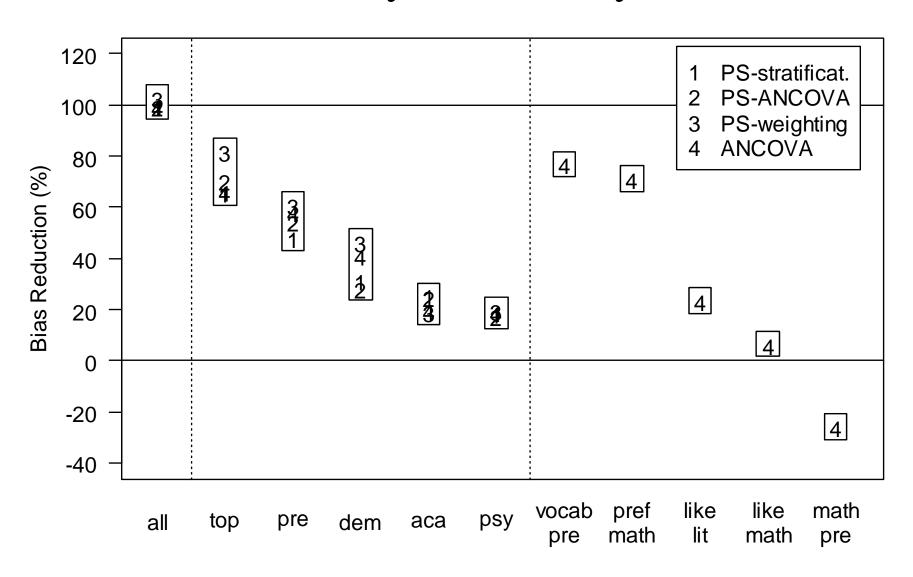


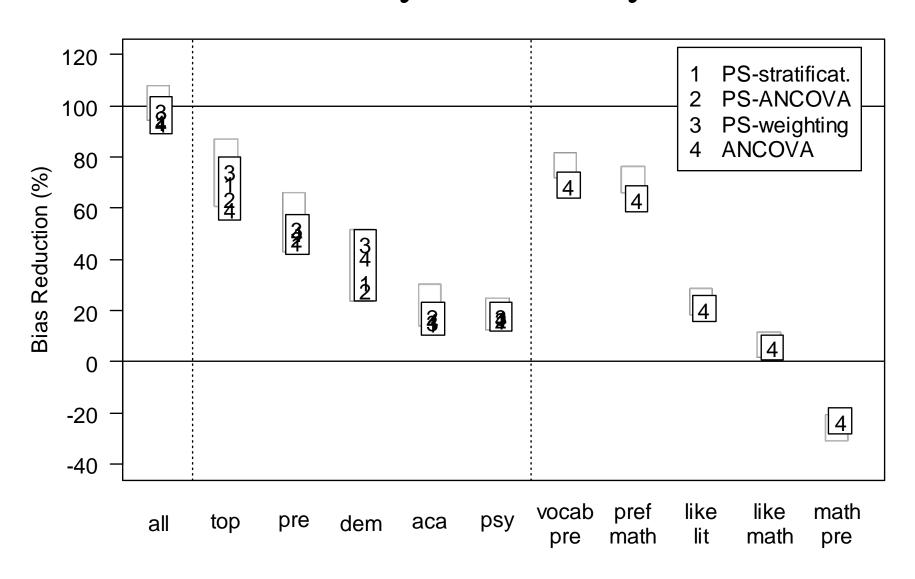
#### Constructs: Conclusion

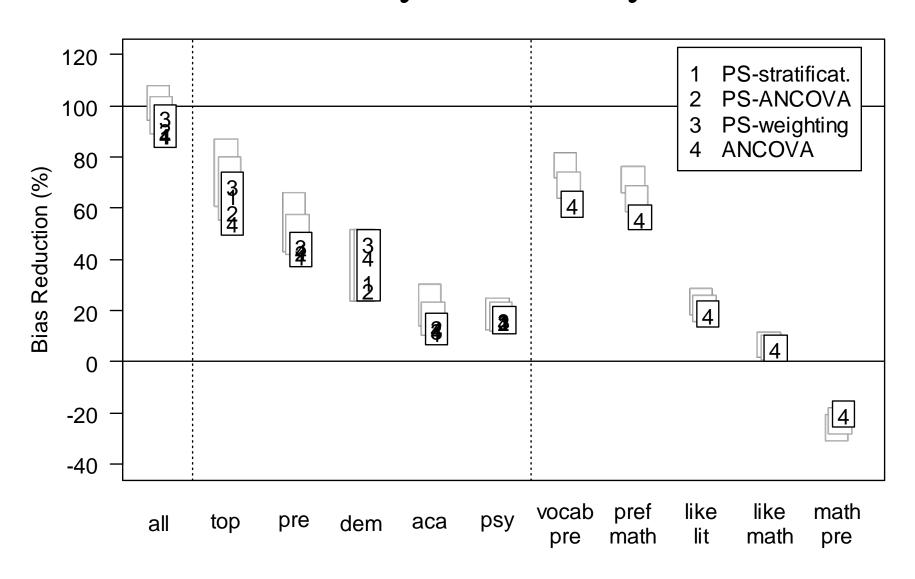
- In establishing SI selection of constructs matters
  - □ Need those *construct domains* that effectively reduce bias (those related to both treatment selection and outcome)
  - □ Need the right *single constructs* within domains because only a few covariates successfully reduce bias
- Choice of analytic method is less important (given its competent implementation)
  - □ No systematic difference between PS methods
  - □ ANCOVA did as well (at least in that case)

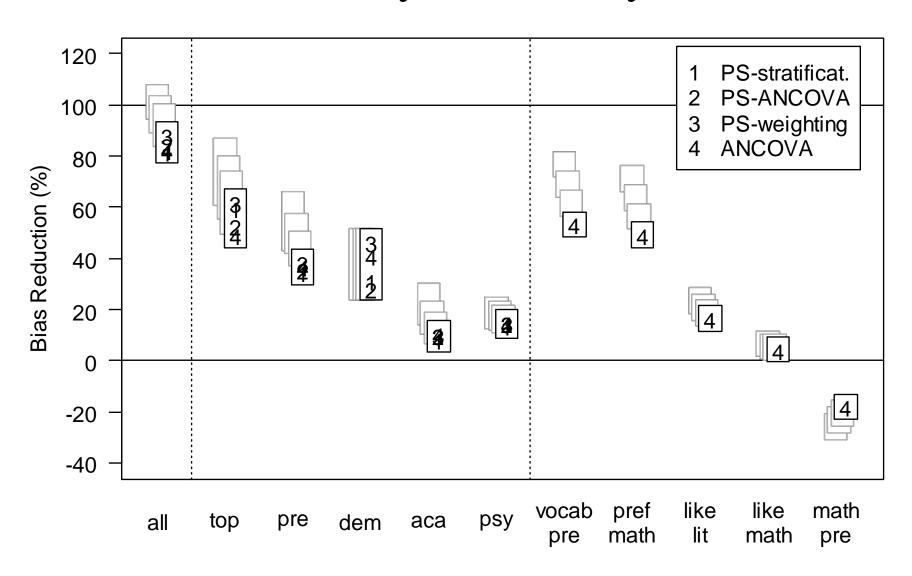
#### Reliability of Construct Measurement Steiner, Cook & Shadish (in press)

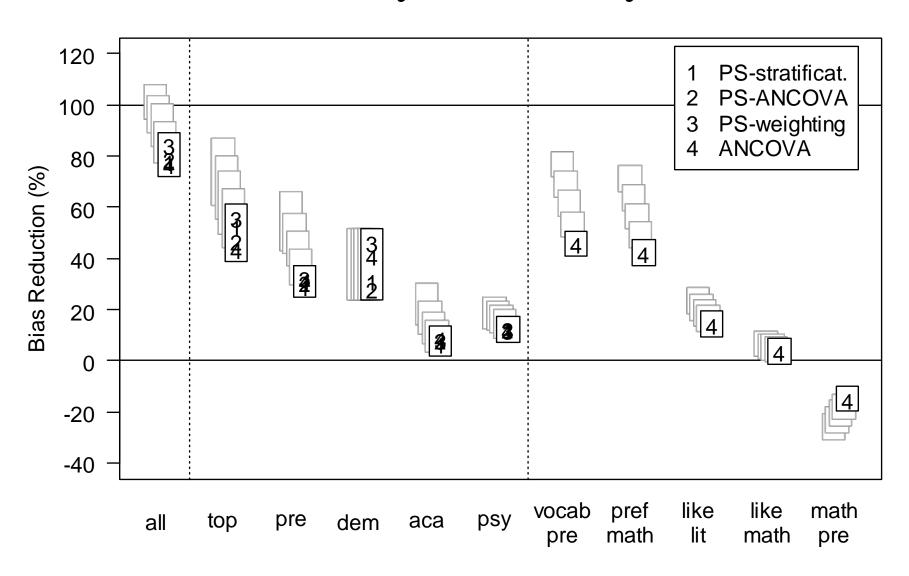
- How important is the *reliable measurement* of constructs (*given selection on latent constructs*)?
  - □ Does the inclusion of a large set of covariates in the PS model compensate for each covariate's unreliable measurement?
- Add measurement error to the observed covariates in a simulation study
  - ☐ Assume that original set of covariates is measured without error and removes 100% of selection bias
  - □ Systematically added measurement error such that the reliability of each covariate was  $\rho = .5, .6, .7, .8, .9, 1.0$

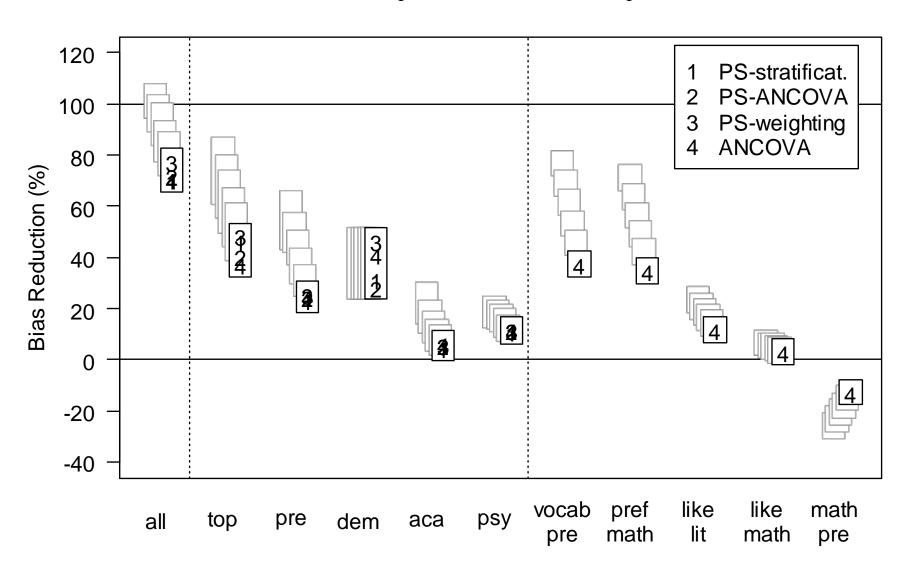


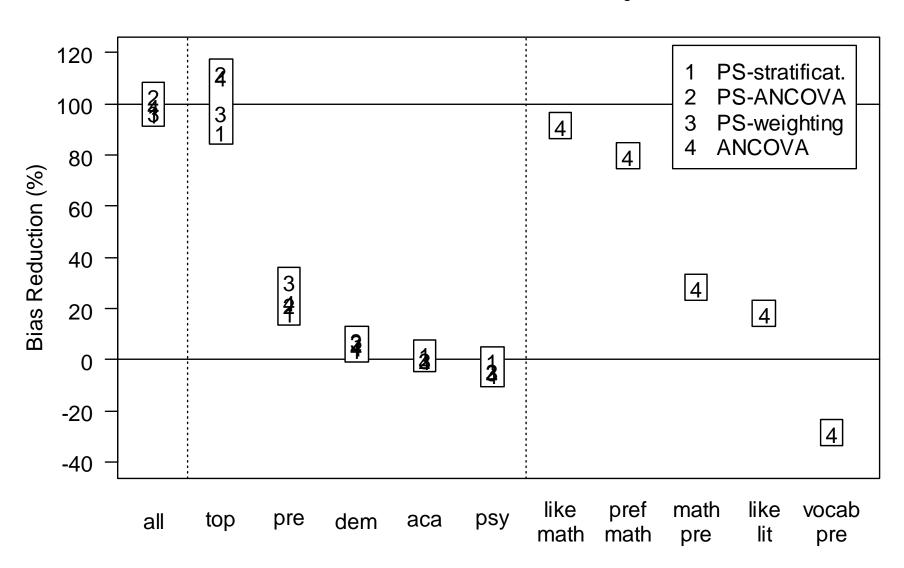


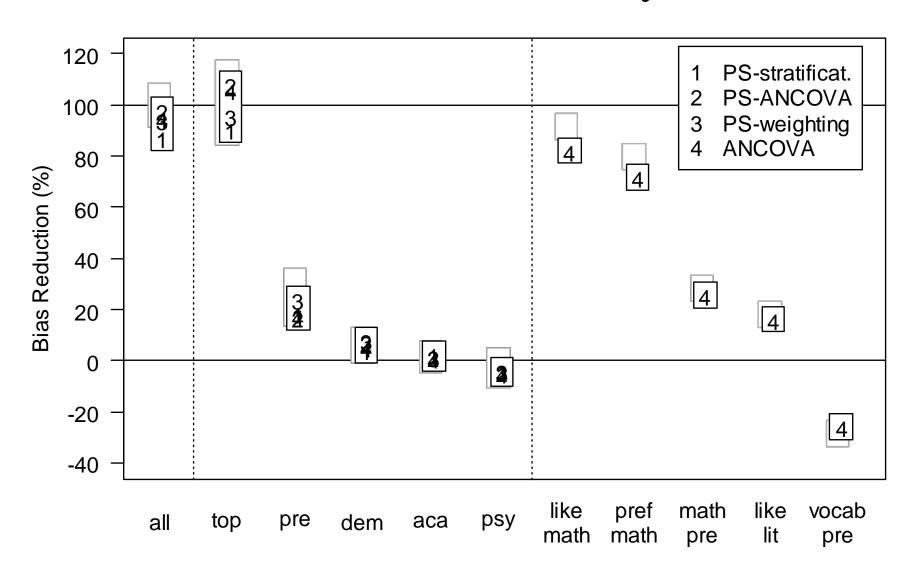


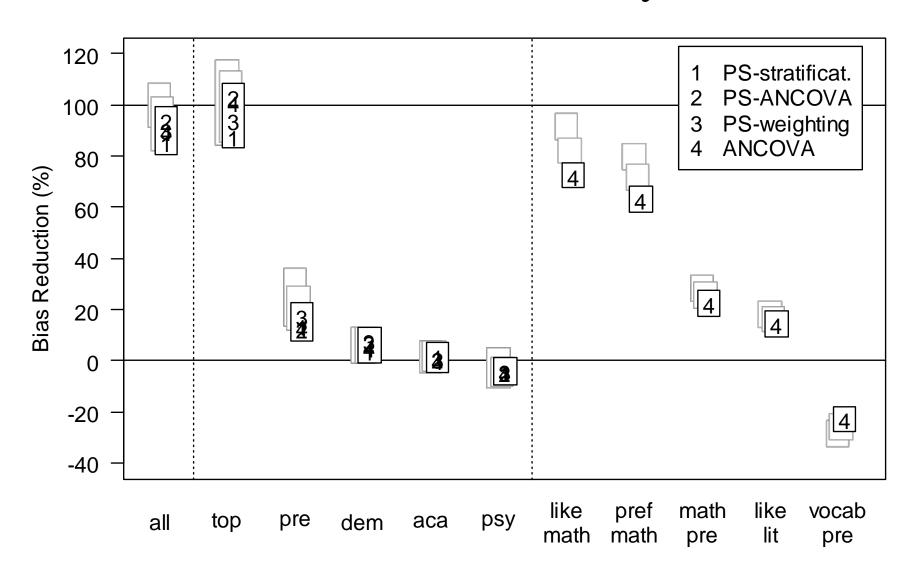


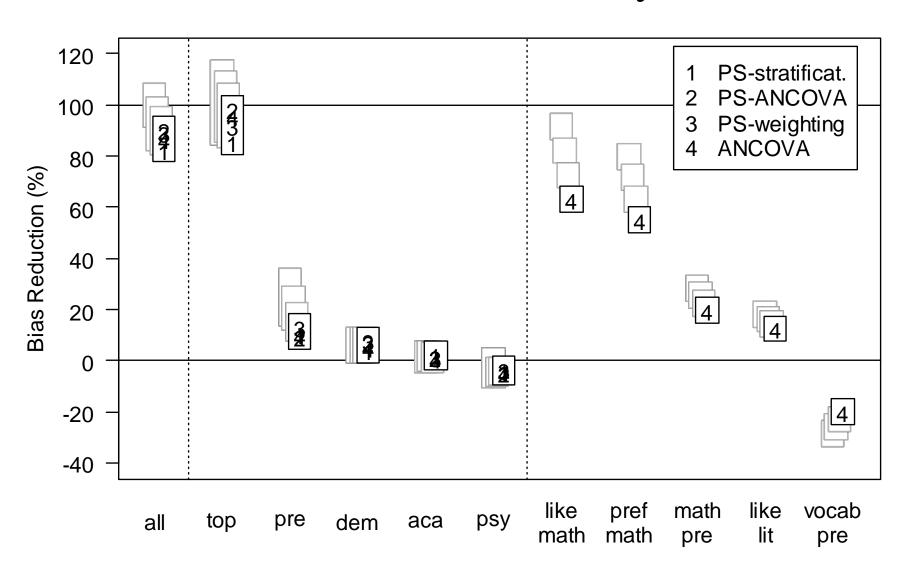


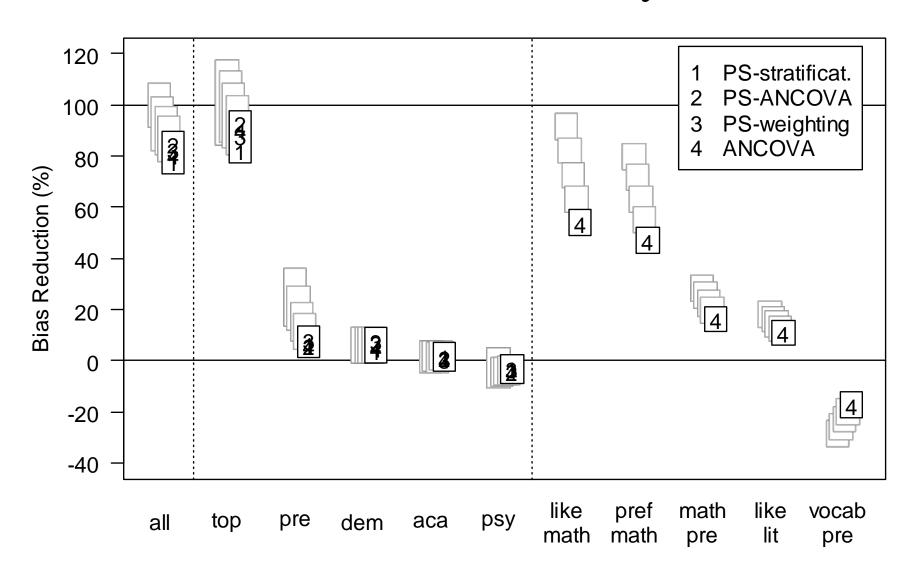


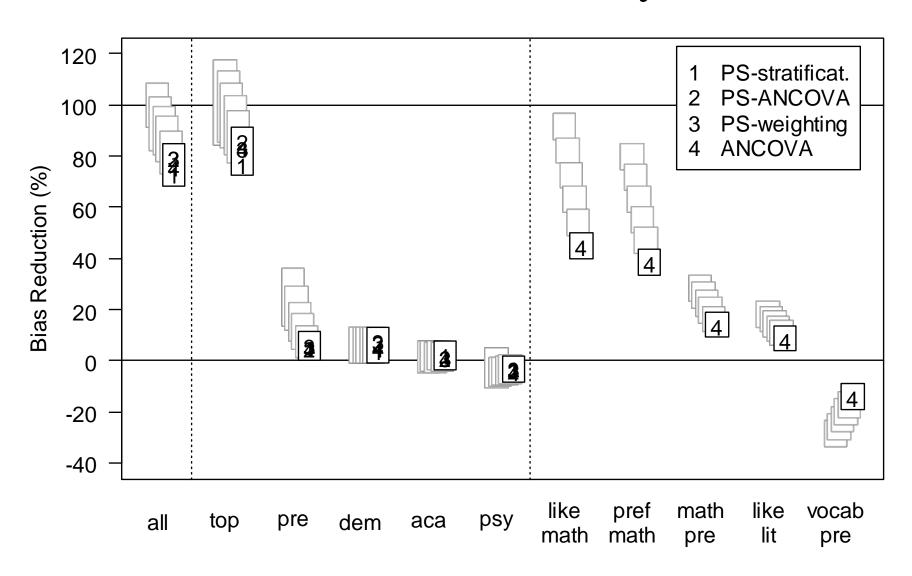












#### Reliability: Conclusions

- Measurement error *attenuates the potential* of covariates for reducing selection bias
- The measurement of a large set of interrelated covariates compensates for unreliability in each covariate—but does so only partially
- The reliability of *effective covariates* matters. Measurement error in ineffective covariates has almost no influence on bias reduction.
- Choice of *analytic method* is less important (no systematic difference between methods)

#### Conclusions

- The most important factors for establishing SI are:
  - 1. The *selection of constructs* is most important for establishing SI (Bloom et al. 2005, Cook et al. 2008, Glazerman et al. 2003)
  - 2. The next important factor is their *reliable measurement*
  - 3. PS has to *balance* observed pretreatment group differences in order to remove all overt bias
  - 4. Choosing a specific *analytic method*—PS techniques or ANCOVA—is of least importance given its competent implementation (as also demonstrated by reviews of within-study comparisons and meta-analyses in epidemiology)

#### PS vs. Regression: Summary

#### **PS** Methods

- Advantages
  - □ Blinding
  - □ Exclusion of nonoverlapping cases
  - □ No outcome modeling (type I error holds)
- Disadvantages
  - □ Balancing metrics
  - ☐ Standard errors
  - ☐ Generalizability

#### **Regression Approaches**

- Advantages
  - □ Generalizability
  - □ Standard errors
  - ☐ Criteria for model selection
- Disadvantages
  - □ Funct. form assumptions& extrapolation
  - □ No blinding
  - $\square$  Model specification ( $\alpha$ )

#### Implications for Practice

- Need strong *theories* on the selection process and outcome model for ruling out hidden bias and avoiding overt bias
- 1. Ruling out hidden bias
  - Cover *different construct domains* that are related to both treatment selection and outcome—administrative data or demographics alone will usually not suffice (e.g., Diaz & Handa 2006)
  - ☐ Measure *several constructs* within each construct domain
  - □ *Reliably* measure constructs—particularly the effective ones
- 2. Avoiding overt bias
  - □ *Balance* pretreatment group differences
  - □ Choose an *analytic method* (appropriate for the causal estimand, sample sizes, assumed functional form)

### Some Other Within-Study Comparisons

## Pohl, Steiner, Eisermann, Soellner & Cook (2009)

- Very close replication of Shadish et al. (2008) in Germany (Berlin)
  - ☐ Math and English Training/Outcome (English instead of Vocabulary)
  - □ Same extensive covariate measurement as in Shadish et al.
  - □ Different selection mechanism due to the English (instead of Vocabulary) alternative → no selection bias in mathematics outcome

#### Results:

- □ PS methods and ANCOVA removed all the selection bias for English outcome (and didn't introduce bias in mathematics)
- □ SEM did slightly better then standard ANCOVA

# Diaz & Handa: Other Non-Equivalent Sample

- Sample of non-equivalent and clearly richer villages
- Big differences between randomly formed eligible families and non-eligible ones, but some overlap
- Use of same means of material welfare as used for treatment assignment
- When used as covariates, all bias is non-experiment explained away

#### Diaz & Handa and Shadish et al.

- *Complete knowledge* of selection process in Diaz & Handa
- *Highly plausible knowledge* in Shadish et al.
- Shadish et al second route to bias reduction

### Earliest and Most Negative Evidence: Job Training Within Study Comparison in Job Training

- 12 studies
- No close correspondence of results
- Identification of some facilitating conditions
- Deep pessimism

# Within Study Comparison in Cook, Shadish & Wong

- Multiple domains
- 12 comparisons in 10 studies
- RD
- Intact Group Matching
- Selection process known
- 4 others
- More optimistic implications

#### Education: Wilde & Hollister

- The Experiment is Project Star in 11 sites
- The non-equivalent comparison group formed from other Tenn. sites via propensity scores
- No pretest, but proxy background variables and some school data
- Analysis of 11 exp vs non-exp comparisons
- Conclusions are: (1) no equivalence in individual site comparisons of exp and non-exp results
- (2) pooled across sites, each significant but they differ in magnitude (.69 vs. 1.07)

#### What's debatable here?

Design first: Who would design a quasi-experiment on this topic w/o pretest? with non-local and non-intact matches? Does this study compare a good experiment with a bad quasi-experiment?

Analysis: How good is a propensity score analysis with mostly demographic data?

- How valid is it to examine separate sites vs across sites?
- In any event, we have recovered the full cross-site treatment effect by first matching schools and then matching individual students within schools

### Agodini & M.Dinarski

- The experiment is on dropout at individual level at middle and high school
- The workhorse design uses propensity scores
- They are constructed separately from two sources-one another study at the middle school level and the other national data
- Findings: Few balanced matches are possible (29 of 128), given covariate data available and overlap achieved;
- Where balanced experiment and non-experiment do not produce same results

#### Commentary

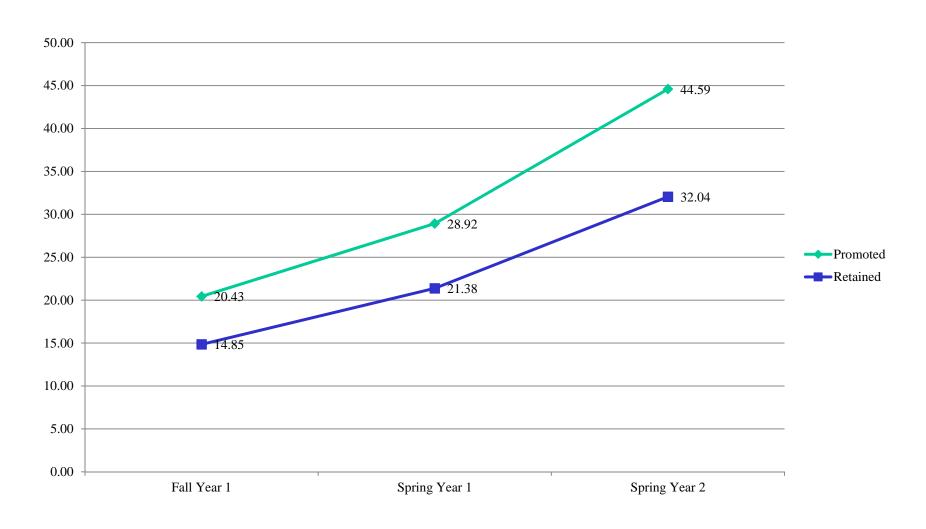
How good was the experiment? 2 of 5 sites

- Control cases not from high school
- Testing at different times
- Pretest measures mostly not available
- How rich was the covariate structure? No theory of dropping out used, merely what was available in archive
- Modest exp contrasted with poor non-experiment

# Role of Pretest: Reanalysis of Hong and Raudenbush (2005; 2006)

- Hong and Raudenbush used the rich covariates in the ECLS-K to estimate the effect of kindergarten retention on academic outcomes in math and reading
- Provided subset of data used in original analysis which included students who attended schools where at least some students were retained in kindergarten
  - □ 10,726 students in 1,080 schools
  - □ 144 pretreatment covariates

# Mean Unadjusted Math Scores for Retained and Promoted Students



## Purposes of Reanalysis

- This study tests whether
  - ☐ Two pretest time measures are superior to one
  - □ Proxy pretests can substitute for true pretest measures
  - ☐ Including a rich and large set of covariates that *excludes* the true and proxy pretests might be as effective in reducing bias as when pretest measures are included

### Analytic Approach

- Broke 144 possible covariates in three groups:
  - □ Pretest measures of the outcome
  - □ Proxy pretests (teacher ratings)
  - □ All other covariates
    - Further divided into domain specific categories in later analyses
- Created propensity scores with each set of covariates and estimated effects on reading and math, examined
  - □ Bias reduction compared to benchmark model
  - □ Whether estimates were statistically distinguishable from one another using bootstrap standard errors

#### Math Effect Estimates

	Mean	Standard Error
Unadjusted	-11.86	0.17
All Covariates	-5.29	0.85
Pretests:		
One Pretest	-7.21	0.77
Two Pretests	-5.76	0.74
First Pretest and Slopes	-5.89	0.76
Proxy Pretests:		
One Proxy Pretest	-9.56	0.77
Two Proxy Pretests	-5.65	0.73
Other Covariates &		
Combinations: Other Covariates	-7.58	0.99
One Pretest and Other Covariates	-6.06	1.05
One Proxy and Other Covariates	-5.37	1.01

#### Math Effect Estimates

	Math
All covariates	-5.29
	(0.88)
All other covariates	-7.58
without pretests	(0.99)
Child demographics	-10.77
	(0.90)
Child social skills	-8.76
	(0.80)
Classroom	-12.56
demographic	(0.75)
composition	
Classroom Learning	-11.88
Environment	(0.70)
Home environment	-11.78
	(0.74)
School structures	-12.61
and supports	(0.76)
School demographic	-12.56
composition	(0.74)
Teacher	-12.71
demographics	(0.75)

#### Conclusions

#### ■ In *this data* set:

- ☐ Two true pretests are superior to a single pretest and are not different from our benchmark causal estimate
- □ A single pretest is superior to a single proxy pretest
- ☐ Two proxy pretests are not different from two true pretests or our benchmark causal estimate
- □ A large and heterogeneous set of covariates without a true or proxy pretest reduces more bias than a homogeneous set of covariates, though remaining bias is still more than when two true or proxy pretests or the benchmark model is used

## Study Limitations

- No RCT benchmark
  - □ Analysis relies on the crucial assumption that the full set of observed covariates approximates meeting strong ignorability better than when fewer variables are used
- Ongoing work is examining whether these findings hold using data from several randomized experiments

# Summary about Validity of Workhorse Design

- Much discussion that workhorse design is empirically not validated
- True for low quality quasi-experiments that no-one trained in Campbell tradition would ever do
- Not universally true--e.g., local focal matching can often reproduce the results of experiments cos of strong common support overlap prior to case match
- Not true if a rich set of covariates is available that assess assignment process well--e.g. Shadish et al., and here pretest plays special role
- The mantra: We have a problem of partially unknown differential selection; not of selection per se