# COMP10002 Workshop Week 11

≤45	Representation of integers and floats
minutes	Ex. 13.1 and 13.2
LAB	<ul> <li>Try your hand at Exercise 11.3, time limit: 10 minutes (including reviewing file operations in lec08.pdf)</li> <li>Assignment 2: <ul> <li>ask questions during the workshop,</li> <li>finish by 11:59PM today or 2AM tonight ©</li> </ul> </li> </ul>
Require ments in Canvas	<ul> <li>Discuss Exercises 13.1 and 13.2. Try your hand at Exercise 11.3</li> <li>Then focus the rest of your energy on Assignment 2. Make sure that you make regular submissions right through the week.</li> <li>And browse through the discussion questions from time to time. Reading them might save you some marks.</li> </ul>

# A2 situation (deadline 6PM this Friday)

Please send me a letter on

how far you went with assignment 2?

- A. finished ass2, or just need to do some refinements or improvements along the marking rubric
- B. finished stage 1
- C. finished stage 0
- D. finished none, but have clear idea on how to do and can certainly finish stage 0 very soon
- E. still struggling with stage 0

And which deadline is occupying your mind:

- 1-6PM this Friday
- 2- oct 26 ©

### Numeral Systems

214.39	2	1	1	•	3	9
Position	2	1	0	Dot	-1	-2
Value	$2 \times 10^{2}$	$1 \times 10^{1}$	$4 \times 10^{0}$		3 x 10 <sup>-1</sup>	9 x 10 <sup>-2</sup>

$$\rightarrow$$
 base = 10 (decimal)

Other bases: binary (base= 2), octal (base= 8), hexadecimal (16)

$$21.3_{(10)} = 2 \times 10^{1} + 1 \times 10^{0} + 3 \times 10^{-1}$$

$$1001_{(2)} = 1 \times 2^{3} + 0 \times 2^{2} + 0 \times 2^{1} + 1 \times 2^{0} = 9_{(10)}$$

$$5B_{(16)} = 5 \times 16^{1} + 11 \times 16^{0} = 91_{(10)}$$

Note: hexadecimal uses 6 additional digits: A, B, C, D, E, F with the values 10-15

# Converting between bases 2 and 16 is easy!

16=2<sup>4</sup>: 1 hexadecimal digit is equivalent to 4 binary digits.

```
1 1010 1101 \rightarrow 1AD

111 1101 1011 \rightarrow
110.1100 0100 \rightarrow 6.C8

3CD \rightarrow 11 1100 1101

AF.B5 \rightarrow
```

# Converting Binary → Decimal

*Just expand using the base=2:* 

$$a_{1} b_{2} b_{1} b_{0} b_{1} b_{2} b_{1} b_{2} b_{2$$

Examples: 1101 
$$\rightarrow$$
 1.011  $\rightarrow$ 1 + 0 + 0.25 + 0.125 = 1.375

Practical advice: remember

# Decimal -> Binary: Convertion using definition

Changing integer x to binary: represent x as the sum of power of two.

```
Example: 23.375 = 16 + 0x8 + 4 + 2 + 1 + 0x0.5 + 0.25 + 0.125

So: 23.375 = 10111.011_{(2)}

129 \rightarrow 10000001

128 \rightarrow 10000000

127 \rightarrow 1111111
```

### Decimal -> Binary: General Method for the Fraction Part

Problem: Converting fraction could be complicated

$$0.1 = 0x0.5 + 0x0.25 + 0x0.125 + 0.0625 + 0.0375 (= ...)$$

**Easy method:** Multiply it, and subsequent fractions, by 2 until getting zero. Result= sequence of integer parts of results, in appearance order. Examples:

0.375			0.1		
operation	int	fraction	operation	int	fraction
.375 × 2	0	.75	.1 x 2	0	.2
.75 x 2	1	.5	.2 x 2	0	.4
.5 x 2	1	.0	.4 x 2	0	.8
			.8 x 2	1	.6
			.6 x 2	1	.2

So: 
$$0.375 = 0.011_{(2)}$$
  $0.1 = 0.0(0011)_{(2)}$ 

# Exercise: Converting Decimal->Binary

$$130_{(10)} =$$
 $6.625_{(10)} = 110.101 \quad 0.125$ 
 $9.23_{(10)} = 1001.001110...$ 

	x 2 =		
.23	0	.46	
.46	0	.92	
.92	1	.84	
.84	1	.68	
.68	1	.32	
.32	0	.64	

### Representation of integers (in computers) using w bits

- Note that we use a fixed amount of bits w
- Make difference between unsigned and signed integers (unsigned int and int in C)

#### unsigned integers:

```
Range: 0..2^{W}-1 (0..255 for w=8)
```

Representation: Just convert to binary, then add 0 to the front to have enough w bits.  $18 \rightarrow 10010 \rightarrow 00010010$ 

### Representation of signed integers using w bits

```
signed integer range: -2^{W-1} to 2^{W-1}-1
(-128 ... +127 for w=8)
(-2<sup>31</sup> ... +2<sup>31</sup>-1 for w=32)
```

- To represent signed integers x:
  - Positive numbers (x≥0):
    - the binary representation of x in w bits,
    - the first bit will always be 0.
  - Negative numbers (x<0):</li>
    - using *twos-complement* of  $|\mathbf{x}|$  in  $\mathbf{w}$  bit,
    - the first bit will always be 1.

### Finding twos-complement representation in w bits in 3 steps

Suppose that we need to find the twos-complement representation for -x, where x is positive, in w=16 bits. Do it in 3 steps:

- 1) Write binary representation of |x| in w bits
- Find the rightmost one-bit
- 3) Inverse (ie. flip 1 to 0, 0 to 1) all bits on the left of that rightmost one-bit

find the 2-comp repr of -40	Bit sequence			
1) bin repr of 40 in 16 bits	0000	0000	0010	1000
2) find the rightmost 1	0000	0000	0010	1000
3) inverse its left	1111	1111	1101	1000

**Why?** Think about finding (-x) so that x + (-x) = 0, where x is a bit pattern.

# Ex. 13.1 (grok's W12)

Suppose that a computer uses w = 6 bits to represent integers. Calculate the two-complement representations for 0, 4, 19, -1, -8, and -31; Verify that 19-8 = 11;

value	representation
0	000000
4	000400
19	010011
1	00000 <mark>1</mark>
-1	111111
	00 <mark>1</mark> 000
-8	111000
31	01111 <mark>1</mark>
-31	100001

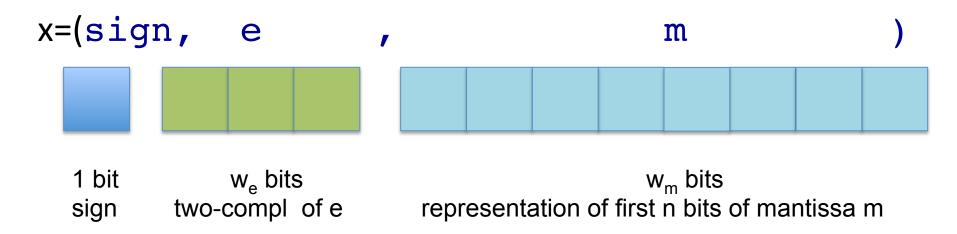
verify!					
	19	010011			
+	-8	111000			
=	11	001011			

# Representation of floats

#### We learnt 2 different formats:

- one as described in lec09.pdf and in the text book
- another is an IEEE standard, which is:
  - employed in most of modern computers,
  - demonstrated in the lecture, and
  - you can find/experiment with using the program
     floatbits.c (lec09.pdf, around page 26).
- How to represent a float x? (General approach):
  - replace x by 3 intergers: sign s, mantissa m, exponent e
  - s can be 0 (positive) or 1 (negative)
  - x is equivalent to  $0.m \times 2^e$  or  $1.m \times 2^e$

### Representation of floats (w=16, as described in lec09.pdf)



Convert |x| to binary form, and transform so that:

```
|x| = 0.b<sub>0</sub>b<sub>1</sub>b<sub>2</sub>... x 2<sup>e</sup> where b<sub>0</sub> = 1
e is called exponent, m= b<sub>0</sub>b<sub>1</sub>b<sub>2</sub>... is called mantissa
x is represented as the triple (sign, e, m) as shown in the
diagram. x= 1101.10= 1101.10 x 2^0 = 110.110x2^1=
0.1101100000x2^4
```

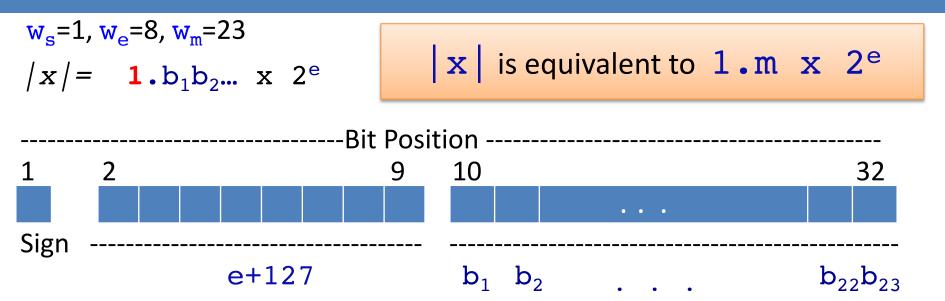
|x| is equivalent to  $0.m \times 2^e$ 

# Ex. 13.2 (grok's W12)

Suppose  $w_s=1$ ,  $w_e=3$ ,  $w_m=12$ , what's the representation of 2.0, -2.5, 7.875?

value	binary form	x  is equivalent to 0.m x 2e	representation
2.0	10.0	0.100 x 2^2, m= 100 e=2	0 010 1000 0000 0000
-0.25	-0.01	-0.1 x 2^(-2), m= 1 e=-2 2->010	1 110 1000 0000 0000
7.875			

### Representation of 32-bit float: (IEEE 754, as in floatbits.c)



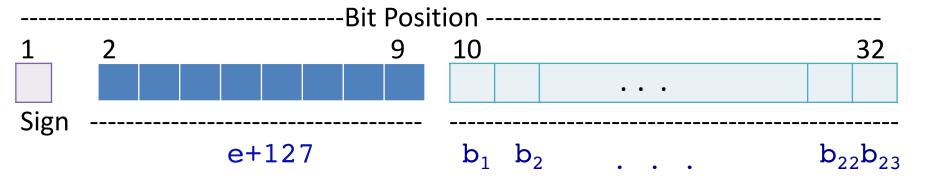
#### That is:

- The sign bit is 0 or 1 as in the previous case
- e is represented in the excess-127 format, which means e is represented as the unsigned value e+127 in  $w_e=8$  bits
- The first bit of the mantissa is omitted from the representation, and the mantissa is represented as just b<sub>1</sub>b<sub>2</sub>...b<sub>23</sub>

**Note:** There are 256 possible values for e. But valid e is -126  $\rightarrow$  +127, corresponding to bit patterns values 0000 0001  $\rightarrow$  1111 1110

### Representation of 32-bit float: (IEEE 754, as in floatbits.c)

$$w_e = 8$$



Un-important note on the exponent part:

- Valid e is -126 → +127, corresponding to bit patterns 0000 0001 → 1111 1110
- Pattern 0000 0000 used for representing 0.0,
- Pattern 1111 1111 used to represent infinity,
- And, 0.0 is all 32 zero-bit, and infinity is all 32 one-bit.

### Representation of 32-bit float: (IEEE 754, as in floatbits.c)

$$w_s = 1, w_e = 8, w_m = 23$$

• 
$$|x| = 1.m \times 2^e$$

Example: x = 3.5

In binary: x = 11.1 converted to  $|x| = 1.11 \times 2^1$ 

- → sign bit: 0
- $\rightarrow$  e=1 is represented as e+127= 128 in 8 bits ->
- → e is represented as 1000 0000
- → 23-bit mantissa: 110 0000 0000 0000 0000 0000
- → Final representation:

or 4 0 6 0 0 0 0 0 <sub>(16)</sub>

# Ex 11.3 (grok's W11)

The Unix tee command writes its stdin through to stdout in the same way that the cat command does. But it also creates an additional copy of the file into each of the filenames listed on the command-line when it is executed.

Implement a simple version of this command. Hint: you will need an array of files all opened for writing.

```
./program file1 file2
Hello world!
[^D]
Hello world!
```

[The program will also create two files named file1 and file2, both containing "Hello world!\n" as the file content.]

```
// here is an implementation of command cat
// change it to that for tee
int main(int argc, char *argv[]) {
  char c:
  while ( (c=getchar()) != EOF) {
    putchar(c);
  return 0;
```

## LAB TIME: Assignment 2 + exercise 11.3

- 2b|!2b: do exercise 11.3 in less than 10 minutes
- Work on assignment 2, ask questions

#### **PROTOCOL**

- You can just do your work, and ask questions when needed.
- Your choice between:
  - joining your break-out room, and
  - staying in the main room.
- You can discuss, but not show your code.
- Look at the big picture (depending on your progress) first, and ask questions.
- It's possible for Anh to have a look at your code, but you should:
  - ask him first, and he will put you in a "lock-down" room where you can show your code
  - prepare questions well so that you will probably need just <= 5 minutes to work with Anh

# Additional Slides

## Decimal $\rightarrow$ Binary: A procedure for converting Integer Part

Changing integer x to binary: Just divide x and the subsequent quotients by 2 until getting zero. The sequence of remainders, in reverse order of appearance, is the binary form of x. Example: 23

operation	quotoion	remainder
23 :2	11	1 1
11:2	5	1
5:2	2	1
2:2	1	0
1:2	0	1

So: 
$$23 = 10111_{(2)}$$
  $11 = _{(2)}$   $46 = _{(2)}$