

## Outlook:

1	Representation of integers and floats
2	Ex. 13.1 and 13.2
3	Working with text files Design & implement 11.3
4	Design 9.3 Implement 9.3 and/or 9.11
5	Q&A

# N numeral Systems

	214.39	2	1	1	.	3	9
	Position	2	1	0	Dot	-1	-2
2	Value	$2 \times 10^2$	$1 \times 10^1$	$4 \times 10^0$		$3 \times 10^{-1}$	$9 \times 10^{-2}$

→ *base* = 10 (decimal)

*Other bases: binary (base= 2), octal (base= 8), hexadecimal (16)*

$$21.3_{(10)} = 2 \times 10^1 + 1 \times 10^0 + 3 \times 10^{-1}$$

$$1001_{(2)} = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 9_{(10)}$$

$$5B_{(16)} = 5 \times 16^1 + 11 \times 16^0 = 91_{(10)}$$

# Converting between bases 2 and 16 is easy!

Because 1 hexadecimal digit is equivalent to 4 binary digits.

# Converting Binary → Decimal

*Just expand using the base=2:*

$$\begin{array}{c} \dots b_3 \quad b_2 \quad b_1 \quad b_0 . b_{-1} \quad b_{-2} \dots \quad (2) \\ \text{is} \quad \dots b_3 \times 2^3 + b_2 \times 2^2 + b_1 \times 2^1 + b_0 + b_{-1} \times 2^{-1} + b_{-2} \times 2^{-2} \dots \end{array}$$

*Examples: 1101 →*

*1.011 →*

*Practical advise: remember*

128	64	32	16	8	4	2	1	0.5	0.25	0.125
$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$	$2^{-1}$	$2^{-2}$	$2^{-3}$

# Converting Decimal → Binary: Method 1 (lectured)

*It's better to convert the integer and the fractional parts separately.  
Remember and apply the power-of-two sequence:*

	$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$	.	$2^{-1}$	$2^{-2}$	$2^{-3}$
	128	64	32	16	8	4	2	1		.5	.25	.125
$13.625_{10}$ → $1101.101_2$					8	4		1		.5		.125
$109_{10}$ →		64	32		8	4		1				
19												
1.3125												

# Decimal → Binary. Method 2A: Integer Part

*Changing integer  $x$  to binary: Just divide  $x$  and the subsequent quotients by 2 until getting zero. The sequence of remainders, in reverse order of appearance, is the binary form of  $x$ .*

*Example: 23*

operation	quotient	remainder
23 :2	11	1
11:2	5	1
5:2	2	1
2:2	1	0
1:2	0	1

So:  $23 = 10111_{(2)}$

## Decimal→Binary. Method 2B: Fraction Part

*Fraction: Multiply it, and subsequent fractions, by 2 until getting zero. Result= sequence of integer parts of results, in appearance order. Examples:*

0.375				0.1		
operation	int	fraction		operation	int	fraction
.375 x 2	0	.75		.1 x 2	0	.2
.75 x 2	1	.5		.2 x 2	0	.4
.5 x 2	1	.0		.4 x 2	0	.8
				.8 x 2	1	.6
				.6 x 2	1	.2

So:  $0.375 = 0.011_{(2)}$        $0.1 = 0.00011(0011)_{(2)}$

Now try convert: 6.875 to binary

# Converting Decimal->Binary

$$7_{(10)} = ?_{(2)}$$

$$130_{(10)} =$$

$$6.375_{(10)} =$$

$$9.2_{(10)} =$$



# Representation of integers (in computers) using $w$ bits

- Note that we use a fixed amount of bits  $w$
- Make difference between **unsigned** and **signed** integers
- To represent **unsigned** integers : Just convert to binary, then add **0** to the front until having  $w$  bits. Largest value:  $2^w - 1$
- To represent **signed** integers  $x$ :
  - Positive integers: a **0-bit**, followed by the binary representation of  $x$  in  $w-1$  bits. Largest value:  $2^{w-1} - 1$
  - Negative integers: using twos-complement of  $x$  in  $w$  bit. Smallest value:  $-2^{w-1}$ . The first bit will always be **1**.

# Finding twos-complement representation in $w$ bits for negative numbers in 3 step

*Suppose that we need to find the twos-complement representation of  $-x$ , where  $x$  is positive, in  $w=16$  bits. It can be done easily in 3 steps:*

- 1) Write binary representation of  $|x|$  in  $w$  bits*
- 2) Find the **rightmost one-bit***
- 3) Inverse (ie. flip 1 to 0, 0 to 1) all bits on the left of that **rightmost one-bit***

<i>find the 2-comp repr of -40</i>	Bit sequence			
1) bin repr of 40 in 16 bits	0000	0000	0010	1000
2) find the <b>rightmost 1</b>	0000	0000	0010	<b>1000</b>
3) inverse its left	<b>1111</b>	<b>1111</b>	<b>1101</b>	<b>1000</b>

# Examples

$w=8$ , then  $2^{w-1}=128$ , and only numbers from -128 to +127 can be represented.

Decimal		Representation	Hexa equivalent
2	→ 10	→ 0000 0010	= 02
93	→ 1011101	→ 0101 1101	= 5D
-93	→	→ 1010 0011	= A3
92	→		
-92	→		
130	→		
-128	→		

## Ex: 2-complement representation in $w=16$ bits

Q: What are 17,  $-17$ , 34, and  $-34$  as 16-bit two's-complement binary numbers, when written as (a) binary digits, and (b) hexadecimal digits?

## Ex. 13.1

*Suppose that a computer uses  $w=6$  bits to represent integers. Calculate the two-complement representations for 0, 4, 19, -1, -8, and -31; Verify that  $19-8 = 11$ ;*

# Quiz

What is the binary form of  $255_{(10)}$  ?

A. 1000 0000

B. 1111 1111

C. 1 0000 0000

D. 1 1111 1111

# Quiz

What is the binary form of  $13.625_{(10)}$  ?

A. 1101.101

B. 1011.101

C. 1101.11

D. 1011.11

# Quiz

What is your tutor's name?

A. Alistair

B. Anh

C. Artem

D. Ahn



## 5 min break for qoct : quality of casual teaching survey

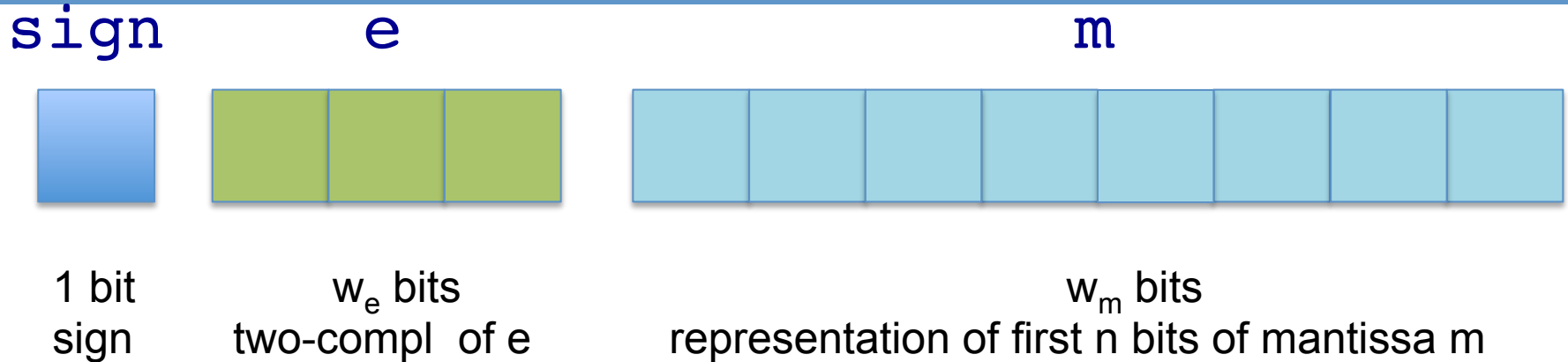
- Please do the survey right now
- To access the survey, just google:  
    qoct

# Representation of floats

We learnt 2 different formats:

- one as described in [lec09.pdf](#) and in the text book
- another is an [IEEE standard](#), which is:
  - employed in most of modern computers,
  - demonstrated in the lecture, and
  - you can find/experiment with using the program [floatbits.c](#).

# Representation of floats (as described in lec09.pdf)



Convert  $|x|$  to binary form, and transform so that:

$$|x| = 0.b_0b_1b_2... \times 2^e \quad \text{where } b_0 = 1$$

**e** is called exponent, **m** =  $b_0b_1b_2...$  is called mantissa

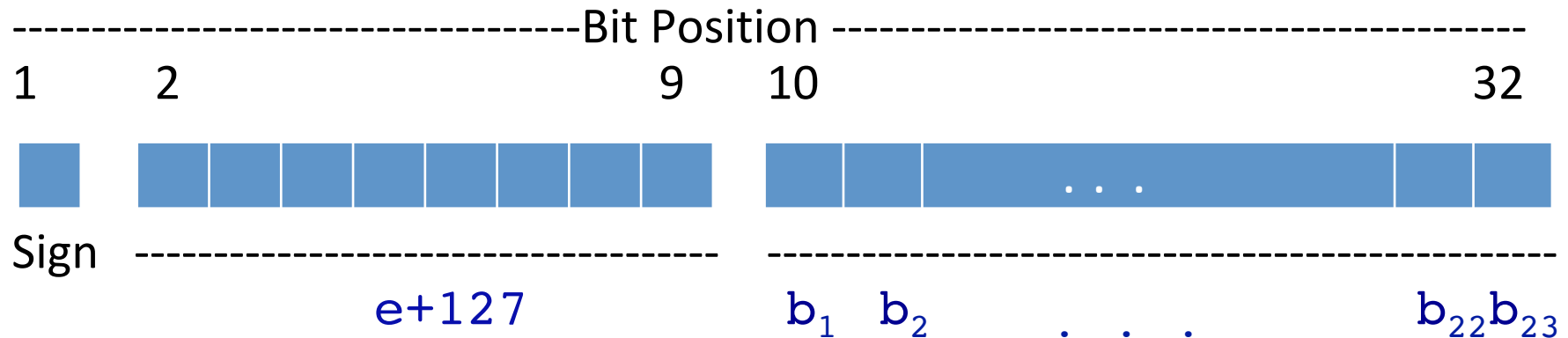
Three components: **sign**, **e**, **m** are represented as in the diagram.

## Ex. 13.2

Suppose  $w_s=1$ ,  $w_e=3$ ,  $w_m=12$ , what's the representation of 2.0, -2.5, 7.875 ?

# Representation of 32-bit float: (IEEE 754, as in floatbits.c )

$w_s=1, w_e=8, w_m=23$      $|x| = 1.b_1b_2... \times 2^e$  [Note the *different transformation* of  $|x|$ ]



That is:

- The sign bit is 0 or 1 as in the previous case
- $e$  is represented in excess-127 format, which means  $e$  is represented as the unsigned value  $e+127$  in  $w_e$  bits
- The first bit of the mantissa is omitted from the representation, and the mantissa is just  $b_1b_2...b_{23}$

**Note:** Valid **e** is  $-126 \rightarrow +127$ , corresponding to values  $1 \rightarrow 254$ . Value 0 used for representing **0.0**, value 255 used to represent **infinity**. And, zero is all 32 **zero-bit**, and infinity is all 32 **one-bit**.

# Representation of 32-bit float: (IEEE 754, as in floatbits.c )

$$w_s=1, w_e=8, w_m=23 \quad |x| = 1.b_1b_2... \times 2^e$$

Example:  $x = 3.5$

In binary:  $x = 11.1 = 1.11 \times 2^1$

→ sign bit: 0

→  $e=1$  is represented as  $e+127=128$  in 8 bits

→  $e$  is represented as 1000 0000

→ mantissa: 110 0000 0000 0000 0000 0000

→ Final representation:

0100 0000 0110 0000 0000 0000 0000 0000

or    4    0    6    0    0    0    0    0    0    (16)

# Working with text files. Ex 11.3

The Unix `tee` command writes its `stdin` through to `stdout` in the same way that the `cat` command does. But it also creates an additional copy of the file into each of the filenames listed in the command-line when it is executed.

Implement a simple version of this command.

Hint: you will need an array of files all opened for writing.





## Design 9.3

Write a program that deals four random five-card poker hands from a standard 52-card desk:

```
$ ./poker
```

```
player 1:    3-S, Ac-C, Qu-D, 4-H, Qu-H  
            . . . (all 4 players)
```

Then, modify your program to allow you to estimate the probability that a player obtains a simple pair (2 cards with the same face value) in their initial hand. Compute that probability using 40,000 hands dealt from 10,000 shuffled decks.

...

# Design 9.3

Implement: 11.2 , 9.3 OR 9.11

*Possible follow-up subjects next year:*

sem1: [comp20007](#) – Design of Algorithms

sem2: [comp20003](#) – Algorithms & Data Structures

*Good Luck!*