COMP20003 Workshop Week 6
Hashing + Assignment 2

Distribution Counting (aka. Counting Sort)

Hashing

2-3-4 Trees

LAB

- Implementation W6.5 (a small exercise)
- Assignment 2

Distribution Counting: An Unusual Sorting Algorithm

Strengths

Does not rely on key comparisons

Runs in linear time $\boldsymbol{\Theta}(\mathbf{n + k})$, where *k* is the key range ➔ $\Theta(n)$ time if $k \in O(n)$

Stable (preserves the order of equal keys)

Limitations

Requires $\Theta(n+k)$ extra memory

Inefficient when the key range *k* is much larger than *n* ➔ not a general-purpose sort

Constraints

Keys must be integers (or *mapped to integers*) in small range k

Operation

counts the frequency of each key

builds the cumulative array to determine positions

places elements into the output array in order

Example

Special Example: sort an array where the keys are non-negative integers, each ≤ 2:

```
input keys: {0,1,2,0,0,1,2,1,1,0,0,0}
```

freq(0) =**6**	freq(1)= **4**	freq(2)= **2**
keys 0 start from index **0**	keys 1 starts from index **6**	keys 2 starts from index **10**

Output Sorted array: `{ 0,0,0,0,0,0, 1,1,1,1, 2, 2 }`

Note: here k= 2

Array F[] = {**6, 4, 2**} is the frequency array

Array C[] = {**0, 6, 10**} is the cumulative array

Counting Sort for sorting array A[0..n-1]

Input: A[0..n-1], k (such that $0 \le A[i] \le k$)

Output: B[0..n-1] which is the sorted version of A[]

Step 1a: build the *count array* F[] such that

F[**x**] = frequency of **x** in A[]

Step 1b: transfer F[] to the *cumulative array* C[]:

C[**x**]= starting position of value **x**

in the sorted array B[].

Step 2: scan A[] from let to right and copy elements A to sorted array B[] **.** For each A[i] :

$$B[C[A[i]]{+}{+}] = A[i] \quad \Leftrightarrow \quad \left\{ \begin{array}{l} x = A[i]; \\ B[C[x]] = x; \\ C[x]{+}{+}; \end{array} \right.$$

A[0..11]= {2,0,1,0,3,0,1,2,1,1,0,0}

k= 3

F[]= count table

idx	0	1	2	3
	5	**4**	**2**	**1**

C[]= table of “next position for i”

idx	0	1	2	3
	0	**5**	**9**	**11**

B[]=

A[]= {2,0,1,0,3,1,2,1,1,0,0,0}

Peer Activity: Sorting Numbers

Suppose that we have a sequence of binary numbers (either 0 or 1).

Which sorting algorithm is best suited for sorting this sequence of numbers?

a. Quick sort
b. Selection sort
c. Insertion sort
d. Distribution counting

Hashing – Key Features

Concept:

Maps keys to indices in an array (hash table) using a hash function.

Aims for near-constant time insertion, deletion, and search.

Strengths:

Fast average-case operations: $O(1)$ for insert, search, delete.

Flexible: can store integers, strings, or complex objects via suitable hash functions.

But:

$O(n)$ worst case for insert, search, delete.

Typical Use Cases:

Implementing dictionaries, sets, symbol tables, caches.

Fast membership testing and lookups.

Example of Hashing

```
// hash function
int hash(int key) {
    return key % 13;
}
```

```
Want to insert:
  14, 20, 33, 46, 31, 72, 16

hash(14) == 1
hash(30) ==
hash(17) ==
```

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Key													

Example of Collision

```
// hash function
int h(int key) {
    return key % 13;
}
```

```
Want to insert:
  14, 20, 33, 46, 31, 72, 16
```

`h(14) == 1`

`h(20) == 7`

`h(33) == 7`

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Key		14						20					

Collision at index 7: where to put 33?

Collision Resolution: Method 1A – Open Addressing with Linear Probing

Open Addressing : store all elements in the table itself; on collision, probe for next empty slot.

- **1A - Linear Probing:** ***probe*** with step=1: $h(k,i) = (h(k) + i) \bmod m$

```
// hash function
int h(int key) {
    return key % 13;
}
```

```
Want to insert:
  14, 20, 33, 46, 31, 72, 16
```

h(14) == 1

h(20) == 7

h(33) == 7
index 7 is busy
found vacant
location 8 when i==1

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Key		14						20	**33**				

Collision Resolution: Method 1A – Open Addressing with Double Hashing

- **1B- Double Hashing: *probe*** with step= h2(x)

  h(k,i)= (h1(k)+ i*h2(k)) mod m

```
// hash function
int h(int key) {
    return key % 13;
}
```

```
// using 2nd hash function
int h2(int key) {
    return key % 5 + 1;
}
```

h(14) == 1

h(20) == 7

h(33) == 7
index 7 is busy
found vacant location
7+4= 11 when i==1

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Key		14						20				**33**	

Want to insert:**14, 20, 33,** 46, 31, 72, 16

Collision Resolution: Method 2 – Separate Chaining

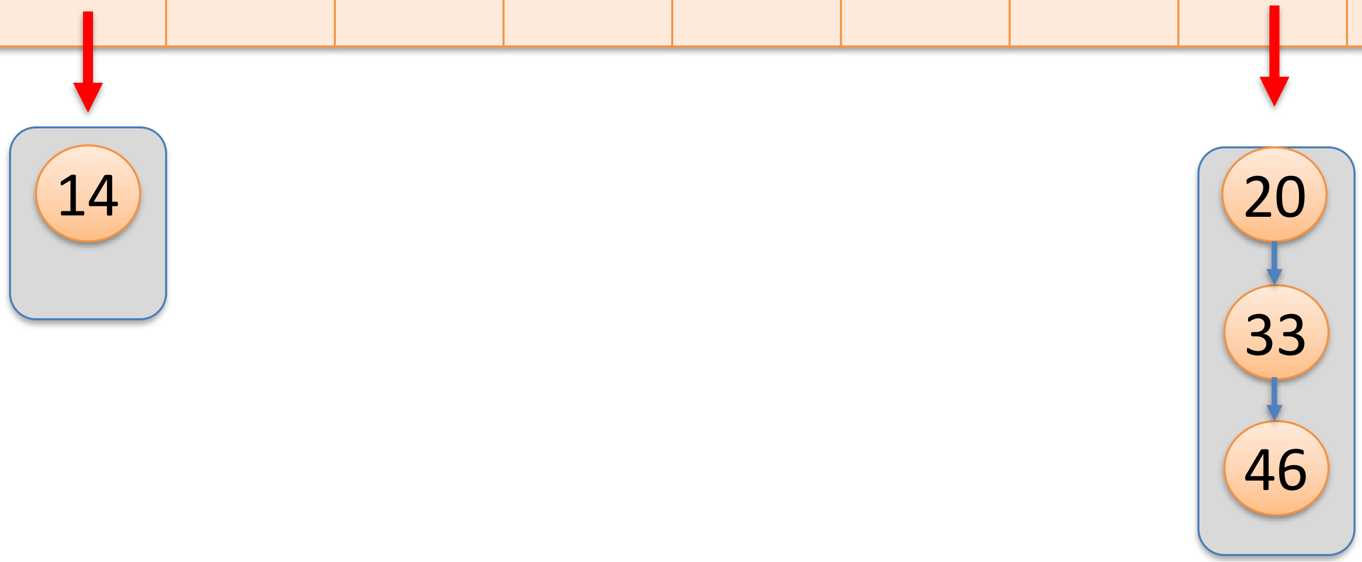
Separate Chaining : store a linked list (or other dynamic structure) at each hash table slot; all keys that hash to the same index are inserted into that list.

```
// hash function
int h(int key) {
    return key % 13;
}
```

```
Want to insert:
14, 20, 33, 46, 31, 72, 16
```

```
h(14) == 1
h(20) == 7
h(33) == 7
```

Index	0	1	2	3	4	5	6	7	8	9	10	11	12
Key													



Peer Activity: Overfilled Hash Table

Suppose that we have a **hash table** that:

- uses linear probing/double hashing for collision resolution
- is currently full

What should be done to insert another item into the hash table?

a. Give up; nothing can be inserted into an overfilled hash table.
b. `realloc()` the key array and insert the new item into this hash table.
c. `realloc()` the key array, rehash the existing keys, and insert the new item into this hash table.
d. Create another identical, empty hash table and insert the new item there.

Peer Activity: Overfilled Hash Table

What should be done to insert another item into the hash table?

c. `realloc()` the key array, rehash the existing keys, and insert the new item into this hash table.

Why?

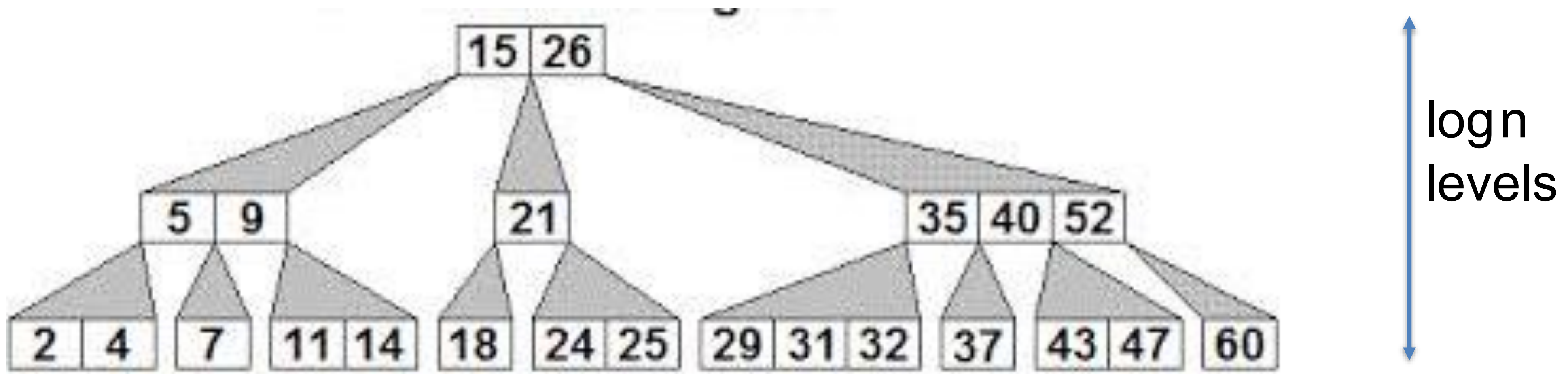
- `realloc()` the key array to get more space to store keys
- rehash existing keys to redistribute them over the enlarged key array

Suppose that we have a **hash table** that:

- uses linear probing/double hashing for collision resolution
- is currently full

2-3-4 Trees (B-trees of order 4)

What? It's a search tree, but not a binary tree!
Each node might have 1 to 3 keys, and hence 2 to 4 children.

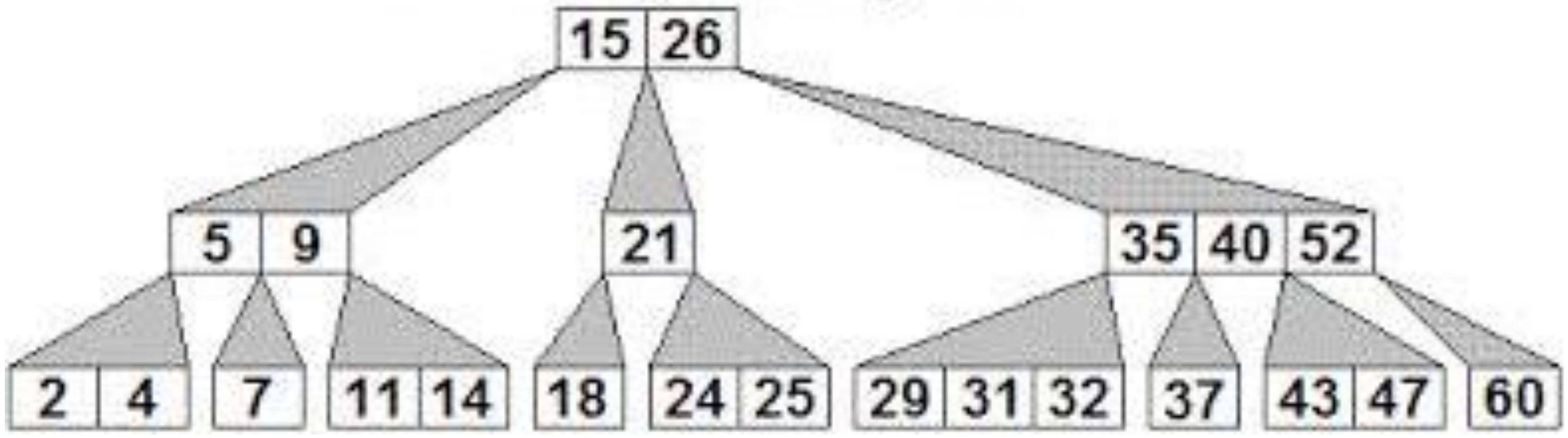


always balanced: all leaf nodes are in the same level
➔ the height of the tree is O(log n)
➔ search/insert/delete is O(log n)

2-3-4 Trees: Insertion

How to insert a new data ***key*** :

- start from root, use ***key*** to go down to a ***leaf node***, and *insert* ***key*** *to that* ***leaf node***
- if the ***leaf node*** has ≤ 2 data: insert to that node
- if the ***leaf node*** has 3 data: promote the median key to its parent *before insertion*
- the promoting might continue several levels upward until getting a parent with ≤ 3 data



insert 8 is easy: node [7] become [7,8]

insert 30 :

→ promote 31... → promote 40

→ insert 30 to node 29 → (29,30)

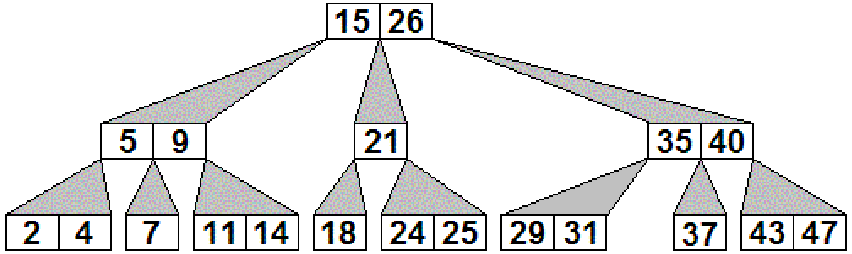
→ promote 40 to the root

Class example:

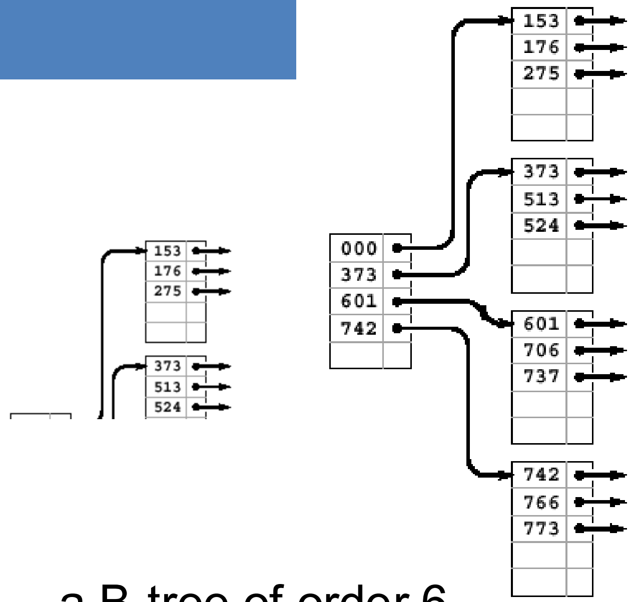
Insert the following keys into an initially-empty 2-3-4 Tree.

20 10 5 15 30

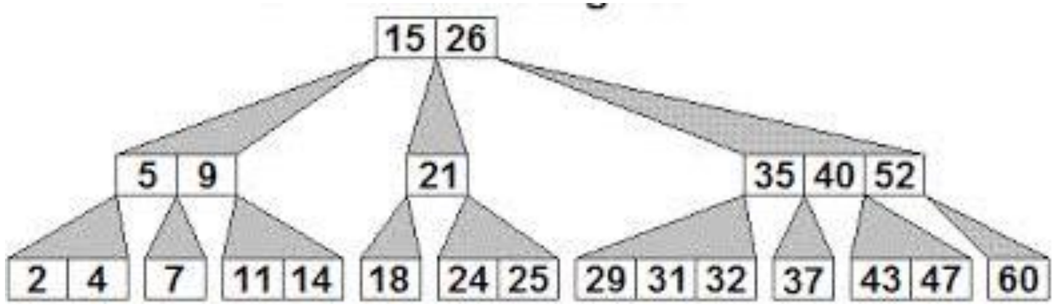
2-3 Trees, 2-3-4 Trees, B-Tree



2-3 trees= B-trees of order 3
(order= max number of children)



a B-tree of order 6



2-3-4 trees= B-trees of order 4

B-tree principles

- Always insert at leaves
- When a node full: promote the median data to the node's parent [and walk up further if needed]

Image sources: ?? and http://anh.cs.luc.edu/363/notes/06DynamicDataStructures.html

Practice: W6.2

- Understanding linear probing & double hashing with `W6.2`
- Programming with `W6.3`:
  - hash table framework already implemented
  - **just implement 3 functions in `hashT.c`**

Programming Notes:

- functions insertLP and insertDH5 are used in insert,
- the parameter key in insertLP , insertDH5, insertDH is actually the first mapping position, value is the step (ie. value of h2(x))

Tips:

- start with insertDH
- then, insertLP and insertDH5
- before implementing any function, read its comments in hashT.h

ASS2: Q&A

Requirements:
Code & Report

- Report: perhaps next week
- what to consider when writing the code?

Assignment 2 Strategies

- Choose a good and working version of A1 to start, for example:
  - your code (which is the best!)
  - Anh's solution (not the best, but in the workshop style)
  - Assignment 1 solution (needs more time to digest)
- If the A1 version is good, then it's
  - easy to add just a single module for Patricia trie
  - simple to adapt the main() for using in Assignment2
- Report: understand report's requirements by reading the specs, section "Assessment", then:
  - plan your report to address all 5 key points
  - build the hypothesis
  - plan the experiments to support/reject the hypothesis
  - think about having graphs to compare efficiency

Additional Slides

Distribution Counting summary

Unlike other sorting algorithms, *Distribution Counting does not use key comparison.*

Normally not applicable. Can only be useful when

- keys can be considered as integers in small range
- ie. when `min ≤ keys ≤ max` and `max-min ∈ O(n)`

Time complexity, supposing r= max-min+1*:*

- ***P***(n+r), or
- ***P***(n) if r ∈ O(n)

Special properties:

- *not in-place*, ie. requiring additional arrays for data records
- additional memory: ***P***(n+r), or ***P***(n) if r ∈ O(n)
- the sorting is *stable*, ie. it preserves the relative order of equal keys