COMP20003 Workshop Week 6
Hashing + Assignment 2

Distribution Counting (aka. Counting Sort)

Hashing

2-3-4 Trees

LAB

- Implementation W6.5 (a small exercise)
- Assignment 2

Distribution Counting: An Unusual Sorting Algorithm

Strengths

Does not rely on key comparisons

Runs in linear time **Θ(n + k)**, where *k* is the key range ➔ **Θ**(n) time if k ∈ O(n)

Stable (preserves the order of equal keys)

Limitations

Requires **Θ**(n+k) extra memory

Inefficient when the key range *k* is much larger than *n* ➔ not a general-purpose sort

Constraints

Keys must be integers (or mapped to integers) in small range k

Operation

counts the frequency of each key

builds the cumulative array to determine positions

places elements into the output array in order

Example

Special Example: sort an array where the keys are non-negative integers, each ≤ 2:

`input keys: {0,1,2,0,0,1,2,1,1,0,0,0}`

freq(0) = **6**   freq(1) = **4**   freq(2) = **2**

| keys 0 start from index **0** | keys 1 starts from index **6** | keys 2 starts from index **10** |

Output Sorted array: { 0,0,0,0,0,0,   1,1,1,1,   2, 2 }

Note: here k= 2

Array F[] = {**6, 4, 2**} is the frequency array

Array C[] = {**0, 6, 10**} is the cumulative array

Input: A[0..n-1], k (such that $0 \leq A[i] \leq k$)
Output: B[0..n-1] which is the sorted version of A[]

Step 1a: build the *count array* F[] such that
F[**x**] = frequency of **x** in A[]

Step 1b: transfer F[] to the *cumulative array* C[]:
C[**x**]= starting position of value **x**
in the sorted array B[].

Step 2: scan A[] from let to right and copy elements A to sorted array B[] . For each A[i] :

x= A[i];

B[C[A[i]]]++]= A[i]   ⇔   B[C[x]]= x;

C[x]++;

A[0..11]= {2,0,1,0,3,0,1,2,1,1,0,0}
k= 3

F[]= count table
idx   0   1   2   3

| 5 | 4 | 2 | 1 |

C[]= table of "next position for i"
idx   0   1   2   3

| 0 | 5 | 9 | 11 |

B[]=

A[]= {2,0,1,0,3,1,2,1,1,0,0,0}

Peer Activity: Sorting Numbers

Suppose that we have a sequence of binary numbers (either 0 or 1).

Which sorting algorithm is best suited for sorting this sequence of numbers?

  a. Quick sort
  b. Selection sort
  c. Insertion sort
  d. Distribution counting

Hashing – Key Features

Concept:

Maps keys to indices in an array (hash table) using a hash function.

Aims for near-constant time insertion, deletion, and search.

Strengths:

Fast average-case operations: O(1) for insert, search, delete.

Flexible: can store integers, strings, or complex objects via suitable hash functions.

But:

O(n) worst case for insert, search, delete.

Typical Use Cases:

Implementing dictionaries, sets, symbol tables, caches.

Fast membership testing and lookups.

```
// hash function
int hash(int key) {
    return key % 13;
}
```

Want to insert:
    14, 30, 17, 55, 31, 29, 16

hash(14) == 1
hash(30) ==
hash(17) ==

Index	0	1	2	3	4	5	6	7	8	9	10	11
Key												

Example of Collision

```
// hash function
int h(int key) {
    return key % 13;
}
```
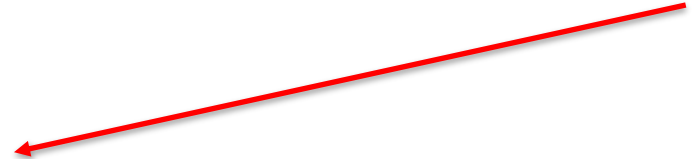
Want to insert:
14, 20, 33, 46, 31, 29, 16

h(14) == 1

h(20) == 7
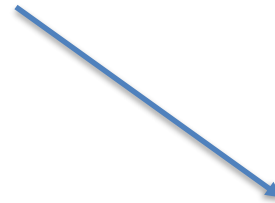
h(13) == 7

Index	0	1	2	3	4	5	6	7	8	9	10	11
Key		14						20				

where to put 7?

Open Addressing : store all elements in the table itself; on collision, probe for next empty slot.

- **1A - Linear Probing:** check slots sequentially: $h(k,i)= (h(k) + i) \bmod m$

```
// hash function
int h(int key) {
    return key % 13;
}
```
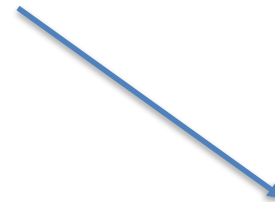
Want to insert:
14, 20, **33**, 46, 31, 29, 16

$h(\mathbf{33}) == \mathbf{7}$
index 7 is busy
found vacant
location 8 when i==1

$h(14) == 1$

$h(20) == 7$

Index	0	1	2	3	4	5	6	7	8	9	10	11
Key		14						20	**33**			

- **1B- Double Hashing:** use second hash for step:

$h(k,i)= (h1(k)+ i*h2(k))$ mod m

```
// hash function
int h(int key) {
    return key % 13;
}
```

```
// 2nd hash function
int h2(int key) {
    return key % 5 + 1;
}
```

h(33) == 7
index 7 is busy
found vacant location
7+4= 11 when i==1

h(14) == 1

h(20) == 7

Index	0	1	2	3	4	5	6	7	8	9	10	11
Key		14						20				**33**

Want to insert:14, 20, **33**, 46, 31, 29, 16

Separate Chaining : store a linked list (or other dynamic structure) at each hash table slot; all keys that hash to the same index are inserted into that list.

```
// hash function
int h(int key) {
    return key % 13;
}
```
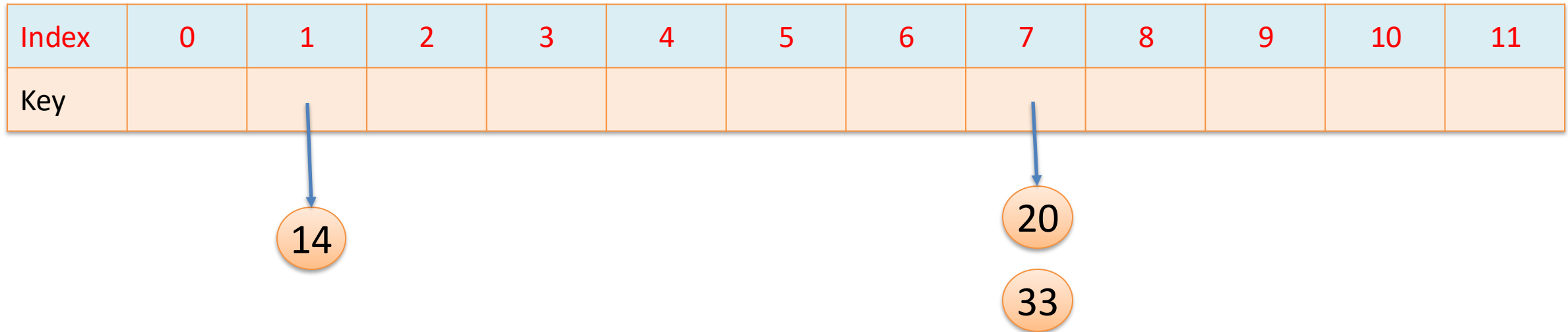
Want to insert:
14, 20, **33**, 46, 31, 29, 16

h(14) == 1
h(20) == 7
h(33) == 7

Index	0	1	2	3	4	5	6	7	8	9	10	11
Key												

14

20

33

Peer Activity: Overfilled Hash Table

Suppose that we have a **hash table** that:

- uses linear probing/double hashing for collision resolution
- is currently full

What should be done to insert another item into the hash table?

a. Give up; nothing can be inserted into an overfilled hash table.

b. `realloc()` the key array and insert the new item into this hash table.

c. `realloc()` the key array, rehash the existing keys, and insert the new item into this hash table.

d. Create another identical, empty hash table and insert the new item there.

Peer Activity: Overfilled Hash Table

What should be done to insert another item into the hash table?

    c. `realloc()` the key array, rehash the existing keys, and insert the new item into this hash table.

Why?

- `realloc()` the key array to get more space to store keys
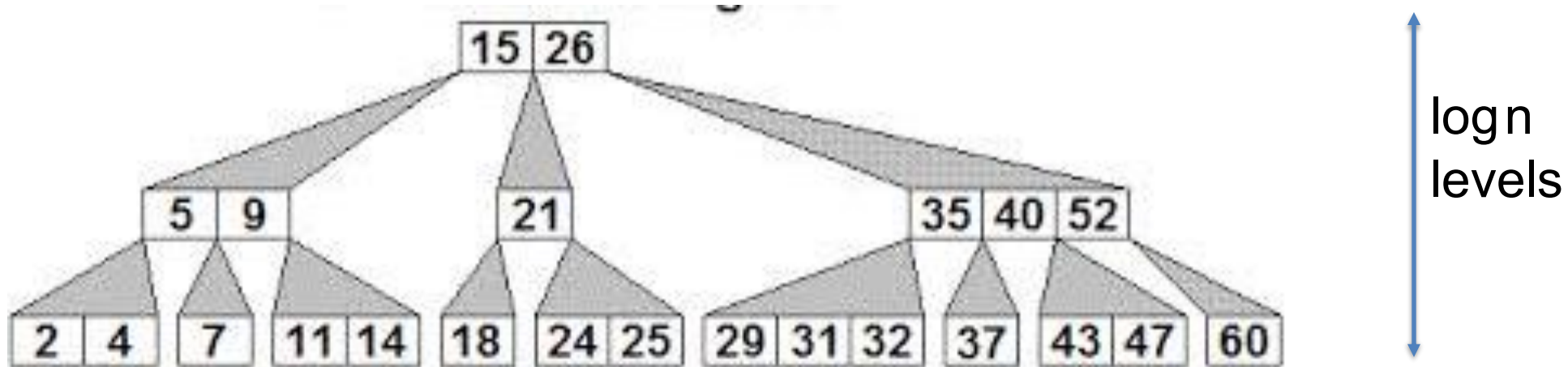- rehash existing keys to redistribute them over the enlarged key array

Suppose that we have a **hash table** that:

- uses linear probing/double hashing for collision resolution
- is currently full

What? It's a search tree, but not a binary tree!
Each node might have 1 to 3 keys, and hence 2 to 4 children.

log n
levels

always balanced: all leaf nodes are in the same level
➔ the height of the tree is O(log n)
➔ search/insert/delete is O(log n)

How to insert a new data **key** :

- start from root, use **key** to go down to a *leaf node*, and *insert **key** to that* *leaf node*
- if the *leaf node* has ≤ 2 data: insert to that node
- if the *leaf node* has 3 data: promote the median key to its parent *before insertion*
- the promoting might continue several levels upward until getting a parent with ≤ 3 data
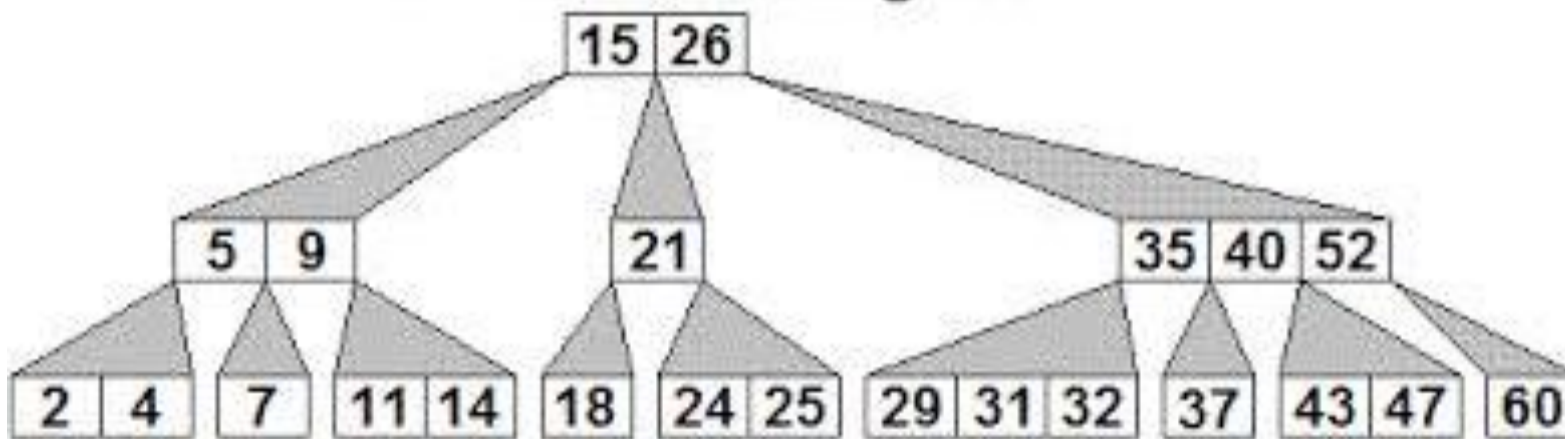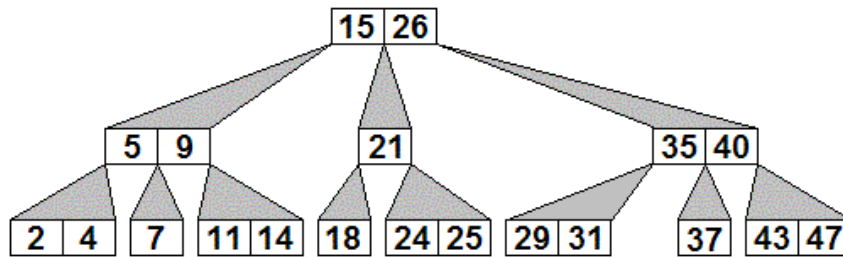

insert 8 is easy: node [7] become [7,8]

insert 30 :
→ promote 31... → promote 40
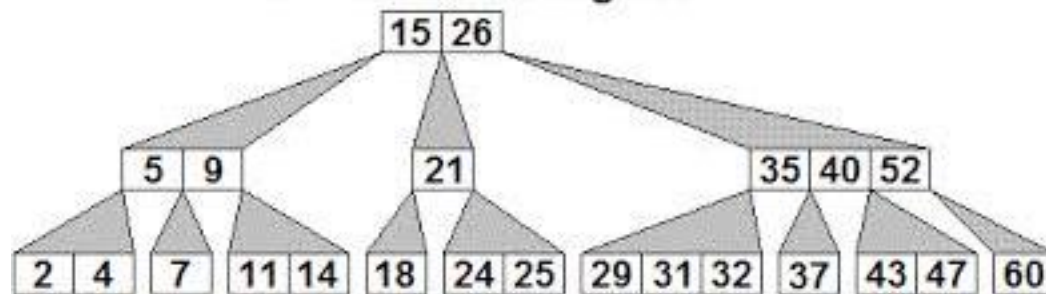→ insert 30 to node 29 → (29,30)
→ promote 40 to the root

Class example:

Insert the following keys into an initially-empty 2-3-4 Tree.
20 10 5 15 30
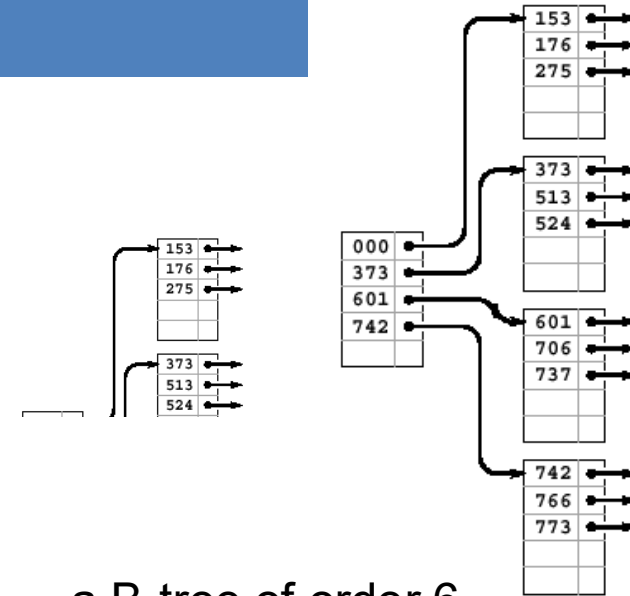
2-3 trees= B-trees of order 3
(order= max number of children)

a B-tree of order 6

2-3-4 trees= B-trees of order 4

B-tree principles
- Always insert at leaves
- When a node full: promote the median data to the node's parent [and walk up further if needed]

Image sources: ?? and http://anh.cs.luc.edu/363/notes/06DynamicDataStructures.html

- Understanding linear probing & double hashing with `W6.2`
- Programming with `W6.3`:
  - hash table framework already implemented
  - **just implement 3 functions in `hashT.c`**

Programming Notes:

- functions insertLP and insertDH5 are used in insert,
- the parameter key in insertLP , insertDH5,   insertDH is actually the first mapping position, value is the step (ie. value of h2(x))

Tips:

- start with insertDH
- then, insertLP and insertDH5
- before implementing any function, read its comments in hashT.h

Requirements:
Code & Report

- Report: perhaps next week
- what to consider when writing the code?

How to Start A2 Coding?

- Use any good and working version of Assignment 1 at the starting point, for example:
  - your code (which is the best!)
  - Assignment 1 solution (needs more time to digest)

- If the A1 version is good, it should be easy to add just one single module for working with Patricia trie (with 2 main functions for search and insert)

- cp driver.c driver2.c then change driver2.c to be the main file for dict2
- Add .c and .h files for the Patricia trie
- Build the trie node structure, basically following the spec

Additional Slides

Distribution Counting summary

Unlike other sorting algorithms, *Distribution Counting does not use key comparison.*

Normally not applicable. Can only be useful when

- keys can be considered as integers in small range

- ie. when `min ≤ keys ≤ max` and `max-min ∈ O(n)`

Time complexity, supposing r= max-min+1*:*

- *P*(n+r), or

- *P*(n) if   r ∈ O(n)

Special properties:

- *not in-place*, ie. requiring additional arrays for data records

- additional memory: *P*(n+r), or *P*(n) if   r ∈ O(n)

- the sorting is *stable*, ie. it preserves the relative order of equal keys