COMP20005 Workshop Week 9

1	Discussion 1: Representation of integer
2	Discussion 2: Representation of float
LAB	Your choice: • Q&A for quiz 2 • Array exercises: Consolidate your understanding of arrays, by completing Exercises 7.7, 7.8, and 7.15. *Arrays, and being able to manipulate data stored in them, are the absolute backbone of this subject.*

- Quiz 2: Monday
- Assignment 2: released this week, due Fri week 11

Numeral Systems

211.39	2	1	1	.	3	9
Position	2	1	0		–1	–2
Value	2×10^{2}	1×10^{1}	1×10^{0}		3×10^{-1}	9×10^{-2}

→ *base* = 10 (decimal)

Other bases: binary (base= 2), hexadecimal (base=16), …

$1001.01_{(2)} = 1x2^{3} + 0x2^{2} + 0x2^{1} + 1x2^{0} + 0x2^{-1} + 1x2^{-2} = 9.25_{(10)}$

$5B_{(16)} = 5x16^{1} + 11x16^{0} = 91_{(10)}$

Note: hexadecimal uses 6 additional digits: A, B, C, D, E, F with the values 10-15

Converting between bases 2 and 16 is easy!

$16=2^4$ ➔ 1 hexadecimal digit is equivalent to 4 binary digits.

For examples $5_{(16)} \Leftrightarrow 0101_{(2)}$, $C_{(16)} \Leftrightarrow 1100_{(2)}$

Class Exercises:

$111110101101_{(2)} \Leftrightarrow ?_{(16)}$

$110011011_{(2)} \Leftrightarrow ?_{(16)}$

$3D_{(16)} \Leftrightarrow ?_{(2)}$

$BF5_{(16)} \Leftrightarrow ?_{(2)}$

Converting Binary → Decimal

binary → decimal: do expansion

$1101_{(2)} = 2^3 + 2^2 + 1 = 8 + 4 + 1 = 13$

$1.011_{(2)} = 1 + 2^{-2} + 2^{-3} = 1 + 0.25 + 0.125 = 1.375$

Remember:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
128	64	32	16	8	4	2	1	0.5	0.25	0.125

Converting Decimal → Binary

decimal→binary: first express the decimal value as the sum of powers-of-two

$42 = 32(=2^5) + 8(=2^3) + 2(=2^1)$

$= 1\ 0\ 1\ 0\ 1\ 0_{(2)}$

$5.875 =$

$= \quad_{(2)}$

Notes:

- better to convert the integer and the fractional parts separately
- see additional slides for an algorithm to convert decimal → binary

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
128	64	32	16	8	4	2	1	0.5	0.25	0.125

Twos-compliment representation of integers using w bits

- Note that we use a fixed amount of bits w
- w bits gives 2^w combinations → possible range : -2^{w-1} .. $+(2^{w-1}-1)$
  - -128 to $+127$ if $w=8$
  - -2^{31} to $+2^{31}-1$ if $w=32$

Representation of a signed integers :

- Positive number x: as the binary representation of x in w bits. Note: the first bit is always 0
- Negative numbers x: as the *twos-complement* of (the representation) of $|x|$ in w bits. The first bit will always be 1 and has the value -2^{w-1}

A trick: Finding twos-complement representation in w bits

Suppose that we need to find the twos-complement representation of a negative x *in* w=16 *bits. Do it in 3 steps:*

1) *Write binary representation of* |x| *in* w *bits*
2) *Find the rightmost* one-bit
3) *Inverse (ie. flip* 1 *to* 0*,* 0 *to* 1*) all bits* ***on the left*** *of that rightmost* one-bit

find the 2-comp repr of **-36**	**Bit sequence**			
1) bin repr of 36 in 16 bits	0000	0000	0010	0100
2) find the **rightmost 1**-bit	0000	0000	0010	0100
3) **inverse** *the left* of that **1**-bit	1111	1111	1101	1100

→ -36 *is represented as the bit pattern* 1111 1111 1101 1100*,*
or FFDC *in the hexadecimal format*

Exercise 1: 2-complement representation in w=16 bits

Q: What are `17`, `-17`, `34`, and `-34` as 16-bit twos-complement binary numbers, when written as (a) binary digits, and (b) hexadecimal digits?

Decimal	twos-complement as binary digits	twos-complement as hexadecimal digits
17		
–17		
34		
–34		

Representation of `float`

There are several ways to represent floats. In this workshop we focus on an `IEEE standard`, which is:

- employed in most of modern computers,
- demonstrated in the lecture, and
- you can find/experiment with using the program `floatbits.c`

Representation of float x

Convert the absolute value $|x|$ to binary form, then transform it so that:

$|x| = 1.b_1b_2b_3\ldots \times 2^e$ *where the integer part is exactly* **1**

Example: 5 → 101 → 1.01 x 2^2

0.375 → 0.011 → 1.1 x 2^{-2}

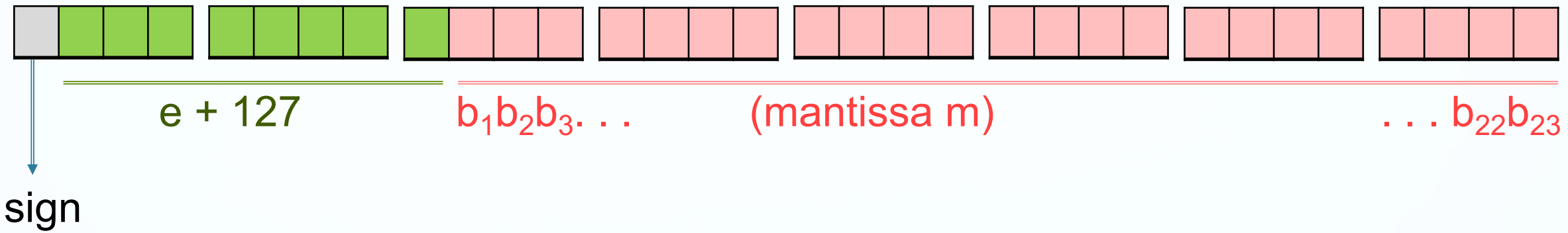
➔ $|x|$ can be represented by the pair:

- value e (called *exponent*)
- bit pattern $b_1b_2b_3\ldots$ (called *mantissa*)

Then, x is represented as triple (sign, exponent e, mantissa m), and:

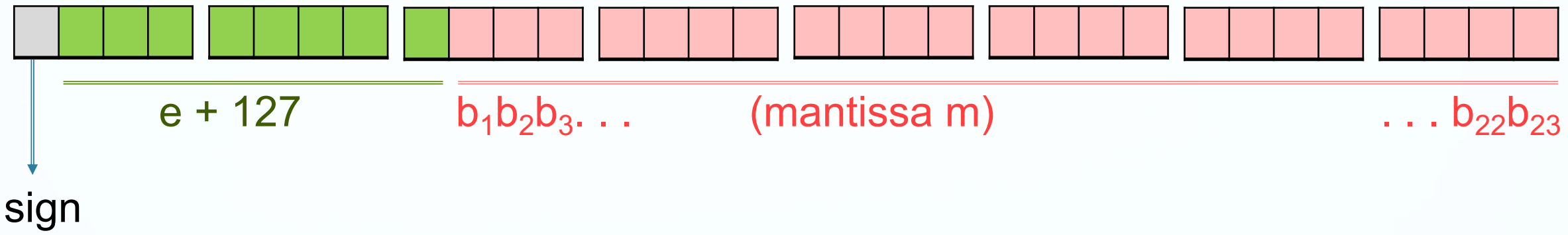
- using 1 bit for sign (0: positive, 1:negative)
- using w_e bit for e (note e can be negative/positive)
- using w_m bits for m (with possible cutting off right bits or adding 0-bits to the right of mantissa to fit into w_m bits)

Representation of 32-bit float: (IEEE 754, as in floatbits.c)



- using w_e = 8 bit for e:
  - e is represented as the (unsigned) bit pattern for the value 127+e
  - pattern 0111 1111 is for value e= 0
  - pattern 1000 0000 is for value e= 1
  - so:
    - e= 5 is represented as ???
    - e= -2 is represented as ???
- using w_m = 23 bits for m

Representation of 32-bit float: (IEEE 754, as in floatbits.c)



Important Note:

- remembering that w_e=8, and that the bit pattern for `e=0` is `0111 1111` and/or for `e=1` (`1000 0000`) could be helpful!

Un-important Notes, just for fun:

- we say that `e` is represented in `excess-127` format
- Valid `e` is -126 → +127, corresponding to the bit pattern `0000 0001` → `1111 1110` .
- zero is 32 `zero-bit`, and infinity is 32 `one-bit`.

Representation of 32-bit float: (IEEE 754, as in floatbits.c)

w_s=1, w_e=8, w_m=23 $\quad |x| = 1.b_1b_2\ldots \times 2^e$

Example: x= 6.5

In binary: x= 110.1= 1.101 x 2^2

- sign bit: 0
- e=2=1+1 is represented as 1000 0000 + 1
  - e is represented as 1000 0001
- mantissa: 110 10. . .0

- Final representation:

0100 0000 1110 1000 0000 0000 0000 0000

or 4 0 E 8 0 0 0 0 (16)

Exercise 2 (modified)

Class/Group Exercise: using representation in modern PC (an IEEE standard), what are the floating point representations of -3.125, 17.5, and 0.09375, when written as (a) binary digits, and (b) hexadecimal digits?

Note: this question was modified from the original question, which as for the 12-bit floating point representation described in the textbook.

Do it, then check your answer:

-3.125 →

17.5 →

0.09375 →

- transform binary number to **1.??? x 2^e**
- w=32, w_e=8, the bit pattern for e=0 is 0111 1111 , for e=1 is 1000 0000

Pointers & Variable Pointers

Each data type has a corresponding pointer type, for example

```
int a;  double b;
int *pa= &a;  double *pb= &b; int **ppa= &pa;
```

Different pointer types are NOT compatible, don't use:

```
pa= pb;     ×
pa == pb;   ×
ppa = pa;   ×
```

Pointer arithmetic: We can ***only*** add/subtract an integer to/from pointers:

```
pa =  pa + 5; ✓
pa = pa + pa; ×
```

Array Names are Constant Pointers

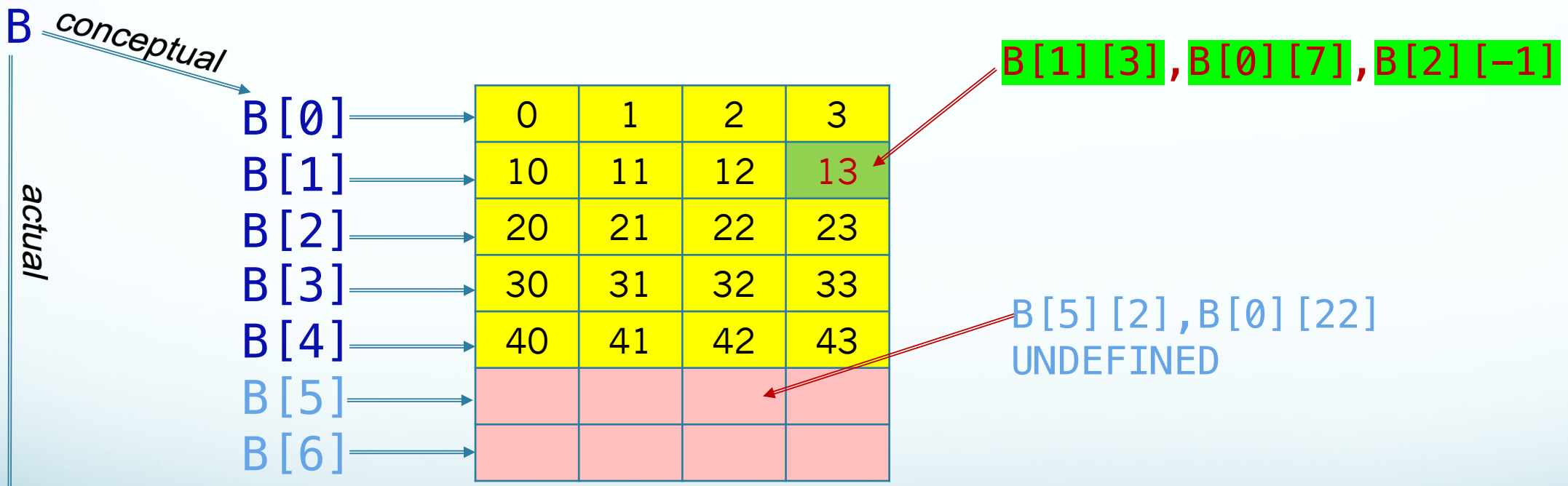
Each data type has a corresponding pointer type, for example

```
int A[5],  B[3][4];
//      B is an  array of       “array of 4 int”
//      B is a   pointer-to      pointer-to-int
int *pa, **pb;
//      pb is a ”pointer-to-pointer-to-int”
pa = A;          ✓
pa = A+3;        ✓
pb = B;          ✓
pb = &B[0];      ✓
pb = &B[0][0]; ×
A = A+3;         ×
```

if A is an 1D array, i is an int, A[i] is valid, but might be undefined!
if B is an 2D array, B[i][j] is valid, but might be undefined!

```
int B[5][4]={{0,1,2,3},{10,11,12,13},{20,21,22,23},{30,31,32,33},{40,41,42,43}};
```

- B is an array of arrays, each B[i] is an array (and a pointer)
- B[i] points to B[i][0], ie. B[i] has the value of &B[i][0]



0	1	2	3	10	11	12	13	20	21	22	23	30	31	32	33	40	41	42	43

example

```
int A[5]={4,3,5,2,1};
int B[3][5]
   ={{1,12,3,14,5},{15,4,6,2,13},{10,7,8,11,9}};
```

What's the value of

1. `B[A[4]][A[1]]`
2. `B[A[1]][A[2]]`
3. `B[A[4]][A[3]]`
4. `B[A[3]][A[2]]`

A Task: Searching in Arrays

General task: given an array of n elements A[n], find a specific element.

Example:

1. Find a special value, such as:
   a) being equal to x, ie. find i such that A[i]==x
   b) find the value of k-smallest, or k-largest element
2. Find some “optimized” value, such as:
   a) find the maximal/minimal element
   b) Find the most frequent element

Searching for a specific element

Task: search for a particular value in an array

Examples

- Return index i such that A[i]==x (or return -1 if NOTFOUND): *there is a chance for NOTFOUND*

- Exercise 7.07

```
#define NOTFOUND -1

// return the first index i
int search(int A[], int n) {
  ???
  for (i=0; i<n; i++) {
    // process A[i], test if A[i] is the solution
    //                return i or A[i] if YES
  }
  return NOTFOUND;
}
```

Searching for an "optimized" element

Optimization Task: Example

- search for the maximal (or minimal) value in an array.
- exercise 7.08

Note: *solution always found, ie. there is normally no* NOTFOUND *case*

```
// return a kind of min/max value in A
//   pay attention when there are multiple soln

int find(int A[], int i) {
  // soln= first estimation
  for (i=0; i<n; i++) {
    // process A[i]:
    //    test if A[i] is a better solution
    //    if YES, update soln
  }
  return soln;
}
```

LAB

- Questions on Quiz 2

- Review/Q&A for quiz 2, including:
  - Array exercises: completing Exercises 7.7, 7.8, 7.03, and 7.15.
    - 7.07: find most frequent value
    - 7.08: find k-smallest
    - 7.03: sort & remove duplicates
    - 7.15: anagrams
  - Other exercises in Chapter 7
  - Remember: *Arrays, and being able to manipulate data stored in them, are the absolute backbone of this subject.*
- [If you want a challenge] Write a program that outputs the bit-patterns associated with 64-bit double values.

Additional Material

- Procedures for converting between from decimal to binary representation

Procedure for Decimal→Binary: Integer Part

Changing integer x to binary (when the solution not obvious):

Just divide x and the subsequent quotients by 2 until getting zero. The sequence of remainders, in reverse order of appearance, is the binary form of x. 6567

Example: 23

operation	quotoion	remainder
23 :2	11	1
11:2	5	1
5:2	2	1
2:2	1	0
1:2	0	1

So: 23 = $10111_{(2)}$ 11= $_{(2)}$ 46= $_{(2)}$

Procedure for Decimal→Binary: Fraction Part

Fraction: Multiply it, and subsequent fractions, by 2 until getting zero. Result= sequence of integer parts of results, in appearance order. Examples: 0.1+01+… +0.1

0.375				0.1		
operation	int	fraction		operation	int	fraction
.375 x 2	0	.75		.1 x 2	0	.2
.75 x 2	1	.5		.2 x 2	0	.4
.5 x 2	1	.0		.4 x 2	0	.8
				.8 x 2	1	.6
				.6 x 2	1	.2

So: $0.375 = 0.011_{(2)}$ $0.1 = 0.00011(0011)_{(2)}$