COMP20005 Workshop Week 9

1	Representation of integers
2	Representation of floats
3	Numerical Computation & Root Finding
4	Small-group exercises
5	Implement: • Ex 9.11 if you think you remember physics • Output the bit-patterns of a double value

Numeral Systems

2 1 1 . 3 9

$2 \times 10^2 + 1 \times 10^1 + 1 \times 10^0 + 3 \times 10^{-1} + 9 \times 10^{-2}$

→ *base* = 10

Other bases: binary (base= 2), octal (base= 8) …

$21.3_{(8)} = 2 \times 8^1 + 1 \times 8^0 + 3 \times 8^{-1} = 17.375_{(10)}$

$1001_{(2)} = 1x2^3 + 0x2^2 + 0x2^1 + 1x2^0 = 9_{(10)}$

$1021_{(2)} = ?$

Changing Binary → Decimal

Just expand using the base=2:

$$\ldots b_3 \; b_2 \; b_1 \; b_0 \; . \; b_{-1} \; b_{-2} \ldots \; (2)$$

is $\ldots \; b_3 x 2^3 + b_2 x 2^2 + b_1 x \, 2^1 + b_0 + b_{-1} x 2^{-1} + b_{-2} x 2^{-2} \ldots$

so $1\;0\;1\;0\;0\;1$ *and* $1\;.\;1\;0\;1$

are $2^5 + 2^3 + 1 = 41$ *and* $1 + 2^{-1} + 2^{-3} = 1.625$

Practical advise: remember

128	64	32	16	8	4	2	1	0.5	0.25	0.125	0.0625
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}

Changing Decimal → Binary

Remember and apply the power-of-two sequence:

128	64	32	16	8	4	2	1	.	0.5	0.25	0.125
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}

So:

$13_{(10)} = 8 + 0 + 2 + 1 = 1101_{(2)}$

$2.625_{(10)} = 2 + 0 + 0.5 + 0 + 0.125 = 10.101_{(2)}$

Algorithm: Decimal→Binary, Integer Part

Changing integer x to binary: Just divide x and the subsequent quotients by 2 until getting zero. The sequence of remainders, in reverse order of appearance, is the binary form of x.

Example: 23

operation	quotoion	remainder
23 :2	11	1
11:2	5	1
5:2	2	1
2:2	1	0
1:2	0	1

So: $23 = 10111_{(2)}$

Algorithm: Decimal → Binary, Fraction Part

Fraction: Multiply it, and subsequent fractions, by 2 until getting zero. Result= sequence of integer parts of results, in appearance order. Examples:

0.375			0.1		
operation	int	fraction	operation	int	fraction
.375 x 2	0	.75	.1 x 2	0	.2
.75 x 2	1	.5	.2 x 2	0	.4
.5 x 2	1	.0	.4 x 2	0	.8
			.8 x 2	1	.6
			.6 x 2	1	.2

So: $0.375 = 0.011_{(2)}$ $0.1 = 0.00011(0011)_{(2)}$

Now try convert: 6.875 to binary

Converting Decimal->Binary

- $17_{(10)} = ?_{(2)}$
- $34_{(10)} =$
- $6.375_{(10)} =$

Representation of integers

representation of integers in w bits

CASE	w data bits for:
$x \geq 0$	binary form of x
$x < 0$	twos-compliment form of |x|

Examples using `w=4`*, representation:*

- *of* 2 *is* `0010`
- *of* 5 *is* `0101`
- *of* -5 *is* `1011`

Finding twos-complement representation in w bits for negative numbers in 3 step

Suppose that we need to find the twos-complement representation of `-x`, *where* `x` *is positive, in* `w=16` *bits. It can be done easily in 3 steps:*

1) *Find binary representation of* `|x|` *in w bits*
2) *Take the result above, inverse* `1` *to* `0` *and vice versa*
3) *Add* `1` *to the above to get the final twos-complement representation*

find the 2-comp repr of **`-40`**	**Bit sequence**			
1) bin repr of 40 in 16 bits	`0000`	`0000`	`0010`	`1000`
2) inverse	`1111`	`1111`	`1101`	`0111`
3) add 1	`1111`	`1111`	`1101`	`1000`

Note: Step 3 (adding `1`*) can be easily be done by:*

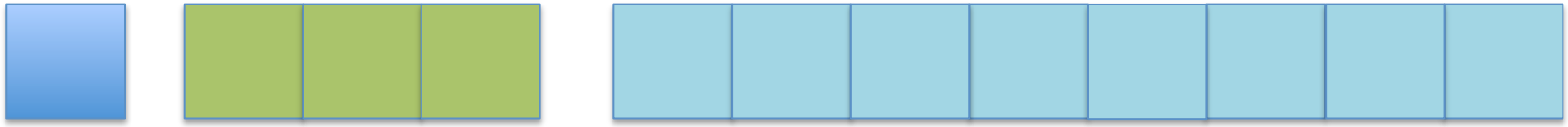
- *finding the right most* `zero-bit`*, then*
- *inversing all bits from this position to the right end.*

This note can be combined with steps 1-2 to make a shorter algorithm ☺

Ex: 2-complement representation in w=16 bits

Q: What are 17, -17, 34, and -34 as 16-bit twos-complement binary numbers, when written as (a) binary digits, and (b) hexadecimal digits?

Representation of floats



1 bit sign

w_e bits two-compl of e

w_m bits representation of first n bits of mantissa m

- Convert $/x/$ to binary form, and transform so that:

  $/x/ \;=\; 0.b_1b_2b_3\ldots \times 2^{\mathbf{e}}$ *with* $b_1 = 1$

- **e** is called exponent, **m**= $b_1b_2b_3\ldots$ is called mantissa
- Three components: sign, **e**, **m** are represented as in the diagram.

Ex: Representation of float

Q: Using the same representation as on page 27 ($w=16$, $w_s=1$, $w_e=3$, $w_m=12$) of the numericA.pdf slides, what are the floating point representations of `-3.125`, `17.5`, and `0.09375`, when written as (a) binary digits, and (b) hexadecimal digits?

Quiz

What is the binary form of $13.625_{(10)}$?

A. 1101.101

B. 1011.101

C. 1101.11

D. 1011.11

Quiz

What is the binary form of $65535_{(10)}$?

A. 1000 0000 0000 0001

B. 1000 0000 0000 0000

C. 111 1111 1111 1111

D. 1111 1111 1111 1111

Numerical Computations

`int`, `float`, `double` all have some range:

- `int` 32 bits: about -2×10^9 – 2×10^9
- `int` 64 bits:
- `float`
- `double`

computation on `float`/`double` is imprecise

- use `fabs(x) < EPS` instead of `x==0`
- adding a large value to a small value might have problems

Roof Finding for f(x)=0

Know about the methods:

- bracketing vs open
- bisection
- false position
- fixed-point iteration
- Newton-Raphson
- secant

Exercise

Re-examine the cube_root() function on page 77 of the textbook, croot.c. What method does it use? Explore what happens if: (a) very large numbers are provided as input; (b) very small (close to zero) numbers are provided; and (c) if ITERATIONS is made larger or smaller.

```
double cube_root(double v) {
    double x= 1.0;
    int i;
    …
    for (i=0; i<ITERATIONS; i++) {
        x= (2*x+v/(x*x))/3;
    }
    return x;
}
```

Lab

1. Ex 9.11 if you think you remember physics
2. Output the bit-patterns of a double value

Hints for 2): mimic Alistair's `floatbits.c`, or DIY by

a) using a "`unsigned long long`" pointer to access the "`double`" variable

b) using bitwise operations to get value of a bit. For example, if `n` is an "`unsigned long long`" then:

```
( n >> (63-i) ) & 1
```

would give the value of the `i`-*th* bit of `n` (from the left).

Decimal→Binary: Integer Part

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