

COMP20007 Workshop Week 9

Preparation:

- *have draft papers and pen ready*
- open [ws10.pptx/pdf](#) from github.com/anhvir/c207
- open [wokshop10.pdf](#) (from [LMS](#)), and
- download lab files from [LMS](#)

1 Hashing: Problems T1, T2

2 Huffman Coding: Problems T3, T4

3 Revision on demands:

Complexity (problem T5)

Solving Recurrences

and others

LAB

Lab: playing with hashing code

Hashing

- What? Why? How?
- Collision, separate chaining, open addressing.

T1: Separate chaining

Consider a hash table in which the elements inserted into each slot are stored in a linked list. The table has a fixed number of slots $L=2$. The hash function to be used is

$$h(k) = k \bmod L.$$

- a) Show the hash table after insertion of records with the keys
17 6 11 21 12 33 5 23 1 8 9
- b) Can you think of a better data structure to use for storing the records that overflow each slot?

T2: Open addressing

Consider a hash table in which each slot can hold one record and additional records are stored elsewhere in the table using linear probing with steps of size $i=1$. The table has a fixed number of slots $L=8$. The hash function to be used is $h(k) = k \bmod L$.

- a) Show the hash table after insertion of records with the keys
17 7 11 33 12 18 9
- b) Repeat using linear probing with steps of size $i = 2$. What problem arises, and what constraints can we place on i and L to prevent it?
- c) Can you think of a better way to find somewhere else in the table to store overflows?

T3,T4: Huffman Coding

Huffman Coding, encoding and decoding

Build Huffman code for [a:3, b:4, c:2, d:4, e:7]

T3,T4: Huffman Coding

Build Huffman code for [a:3, b:4, c:2, d:4, e:7]

Note: there are different versions of Huffman code, all we need to do is to choose a way and keep consistency. For instance:

- maintaining current weight/frequency table in increasing order, taking into account the alphabetic order
- always choosing the first 2 smallest weights to join
- when assigning code, always set 0 to the left, 1 to the right edge

T3: Huffman Code Generation

Huffman's Algorithm generates prefix-free code trees for a given set of symbol frequencies. Using these algorithms generate two code trees based on the frequencies in the following message:

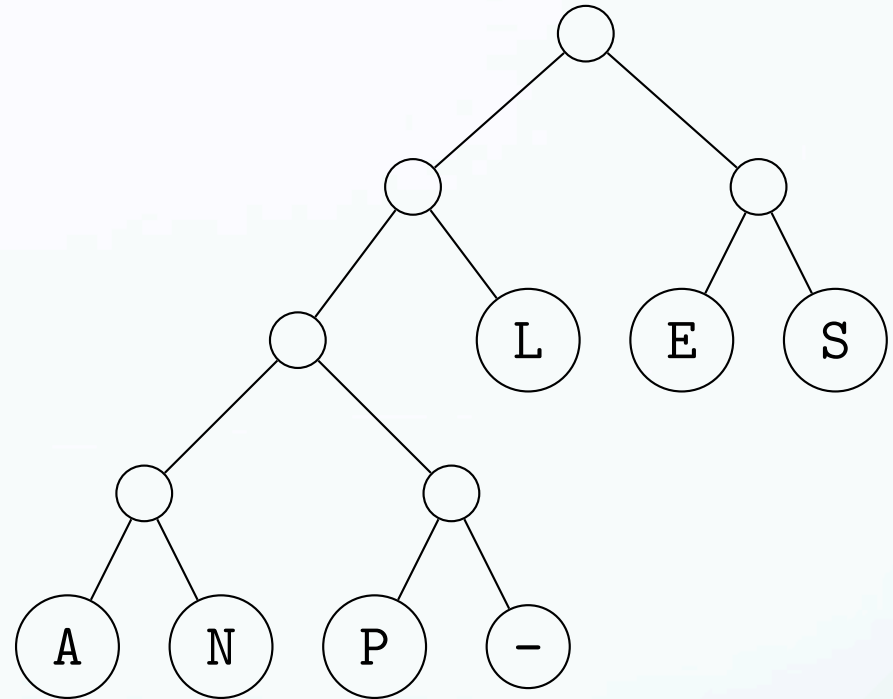
`losslesscodes`

What is the total length of the compressed message using the Huffman code?

T4: Canonical Huffman decoding

The code tree was generated using Huffman's algorithm, and converted into a Canonical Huffman code tree. Note: _ denotes space.

Assign codewords to the symbols in the tree, such that left branches are denoted 0 and right branches are denoted 1. Use the resulting code to decompress the following message:



00100110000011100011011011110011110110100010011011110001101111

Revision 1: Complexity Analysis – 03.pdf & 04.pdf

$$1 < \log n < n^\varepsilon < n^c < n^{\log n} < c^n < n^n \quad \text{where } 0 < \varepsilon < 1 < c$$

$$(\log n)^\alpha < (\log n)^\beta \quad \text{and} \quad n^\alpha < n^\beta \quad \text{where } 0 < \alpha < \beta$$

$$\begin{aligned} O(f(n) + g(n)) &= O(\max\{f(n), g(n)\}) \\ O(cf(n)) &= O(f(n)) \\ O(f(n) \times g(n)) &= O(f(n)) \times O(g(n)) \end{aligned} \quad \begin{array}{l} \text{note: these 3 also applied} \\ \text{to big-}\theta \end{array}$$

$$\begin{aligned} 1 + 2 + \dots + n &= n(n+1)/2 &= \theta(n^2) \\ 1^2 + 2^2 + \dots + n^2 &= n(n+1)(2n+1)/6 &= \theta(n^3) \\ 1 + x + x^2 + \dots + x^n &= (x^{n+1}-1)/(x-1) \quad (x \neq 1) \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \begin{cases} 0 & f(n) = O(g(n)) \\ c & f(n) = \theta(g(n)) \\ \infty & f(n) = \Omega(g(n)) \end{cases}$$

$$\lim_{n \rightarrow \infty} \frac{t(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{t'(n)}{g'(n)}$$

R1 exercises: Problem T5

For each of the following cases, indicate whether $f(n)$ is $O(g(n))$, or $\Omega(g(n))$, or both (that is, $\Theta(g(n))$)

(a) $f(n) = (n^3 + 1)^6$ and $g(n) = (n^6 + 1)^3$,

(b) $f(n) = 3^{3n}$ and $g(n) = 3^{2n}$,

(c) $f(n) = \sqrt{n}$ and $g(n) = 10n^{0.4}$,

(d) $f(n) = 2 \log_2 \{(n + 50)^5\}$ and $g(n) = (\log_e(n))^3$,

(e) $f(n) = (n^2 + 3)!$ and $g(n) = (2n + 3)!$,

(f) $f(n) = \sqrt{n^5}$ and $g(n) = n^3 + 20n^2$.

R2: Recurrences $T(n) = ? T(<n) + f(n)$ and $T(1)=c$

- Apply the Master Theorem ([10.pdf](#)) if possible (ie. if having a , b , and $\Theta(n^d)$ or $O(n^d)$)
- Otherwise, using substitution to expand until $T(1)$

Note that we can easily make mistakes with substitution. *Do it step by step using draft paper, don't rush.*

Exercises: Supposing $T(1)=1$, solve the following using substitution *and* using master theorem when possible:

a) $T(n) = 3T(n-1) + 1$

b) $T(n) = T(n/3) + 1$

R3: 05.pdf: exhaustive string search, knapsack

- exhaustive string search = naïve search
- exhaustive knapsack = find all subsets of a set

Exercises:

R4: graphs

- [06.pdf](#): graph concepts
- [07.pdf](#): DFS and BFS, topological sort
- [08.pdf](#) : Prim & Dijkstra

Exercises:

R5: Sorting algorithms

- [11.pdf](#): Sorting algorithm properties, *insertion sort* & *selection sort*
- [12.pdf](#): *top-down mergesort*, *quicksort with Lomuto partitioning*, *quicksort with Hoare partitioning*

Exercises: For each of the above 5 algorithms:

- is it input-sensitive, in-place, stable? What's the complexity? Best case and worst case?
- Show how it works on:

EXAMPLE

R6: Binary Heap (13.pdf)

- Binary Heap: complexity of insertion, removeMin, heapify
- Heap Sort and its complexity

Exercises:

- Show how to insert into an originally-empty min-heap:

EXAMPLE

R7: Search Trees (14.pdf)

- BST and AVL
- 2-3 Tree

Exercises: For each of the above 2 types of search trees:

- What is the complexity of insertion, of search?
- Perform the insertion into originally-empty tree:

TREBALNCD

R8: Hashing

- [15.pdf](#): Hashing.

Exercises:

R9: Huffman Coding

- [16.pdf](#): Coding and Huffman Coding

Exercises:

Anything missing in our list of revision?

Of course, there is no guarantee that the list is complete. You can fill in things like:

- stacks and queues, priority queues