COMP20007 Workshop Week 5

1	**Topic 1: Graph Traversal,** Q 5.4, 5.6, 5.7 Traversal Trees: Q 5.5
2	**Topic 2:** MST, Paths & Shortest Paths Dijkstra's (Greedy) Algorithm, Q5.9 ***Probably HomeWork:*** Prim's Algorithm Problems 5.8 and some others?
LAB	**MST and A1 Q&A**

Current Issues

- MST worth 10%, know
  - When, Where

Assignment 1:

- Q&A today
- due Tue Week 6: it might take heap of time to finish!

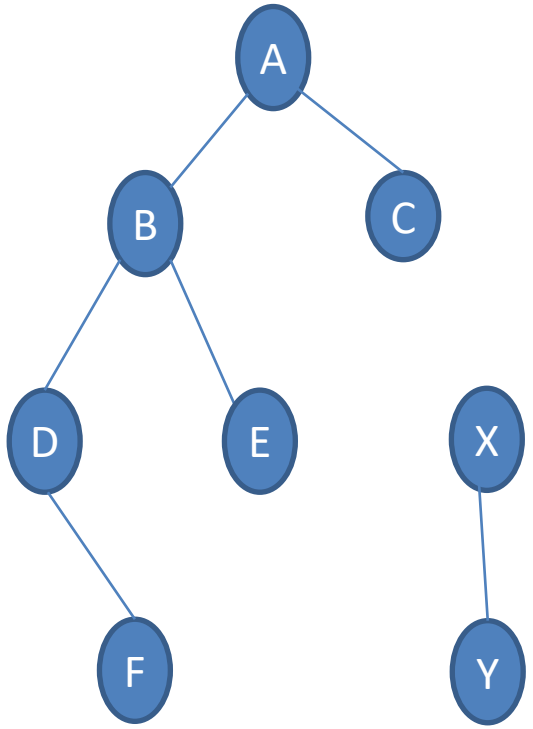
Graph Algorithms (in Lectures 7 and 8)

- Graph Traversal:
  - DFS
  - BFS

- Transitive Closure and Minimum Spanning Tree:
  - Prim's Algorithm for MST

- Shortest Path:
  - Dijkstra's Alfgorithm

Graph Traversal with DFS & BFS



function DFS(G= (V,E))	function BFS(G= (V,E))

// Traverses G in a systematic manner (from node to node using edges)

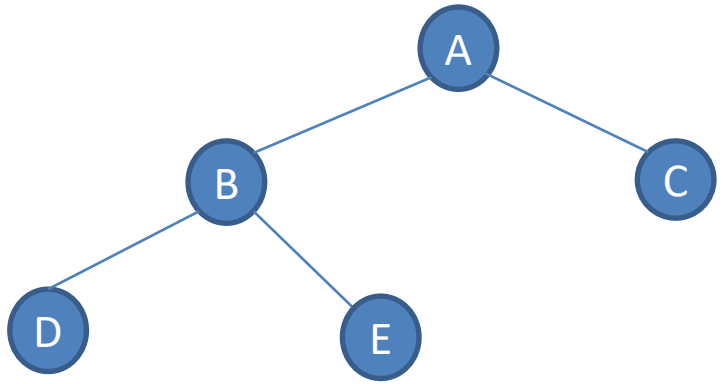
```
function DFS( G= (V,E))
  for each v ∈ V do
    visited[v] := false

  for each v ∈ V do
    if !visited(v) then
       DfsExplore(v)


function DfsExplore(v)
  // visit v and all
  //  connected-to-v nodes
  //
```

```
function BFS( G= (V,E))
  for each v ∈ V do
    visited[v] := false
  Q := empty queue
  for each v ∈ V do
    if !visited(v) then


      // visit v and all
      //  connected-to-v nodes
```

? From which node we start DFS or BFS?

? How to detect the number of connected components in G?

DfsExplore

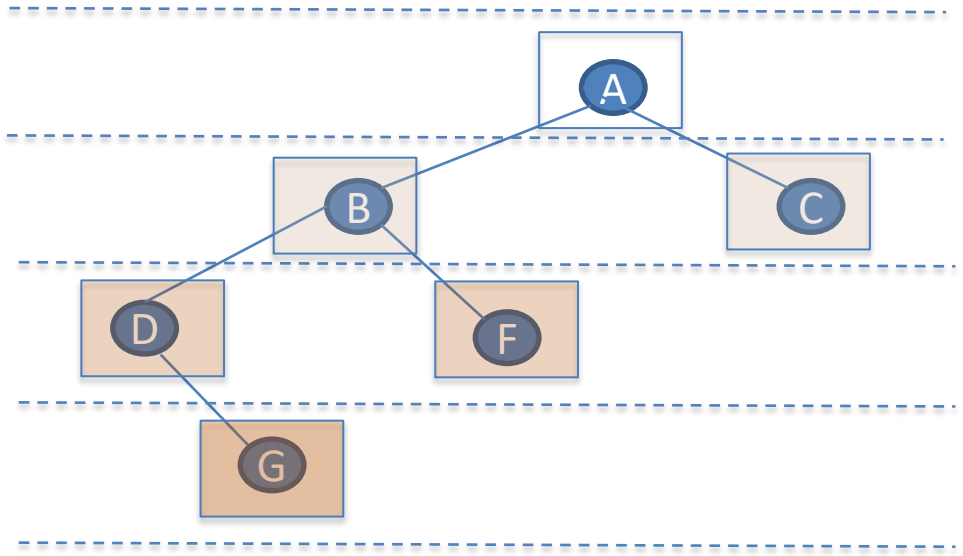


?: in DFS, how to:

- print out the nodes in the visited order (push-order)?
- print the nodes in the reverse of being visited order (pop-order)?

```
DfsExplore(A):
    # start visit A
    mark A as visited
    do pre-visit stuffs

    visit B
        visit D

        visit E

    visit C

    do post-visit stuffs
    # end visit A
```

```
DfsExplore(v):
    # start visit v
    mark u as visited
    do pre-visit stuffs

    for each neighbor w of v
      if w is unvisited
        DfsExplore(w)

    do post-visit stuffs
    # end visit v
```

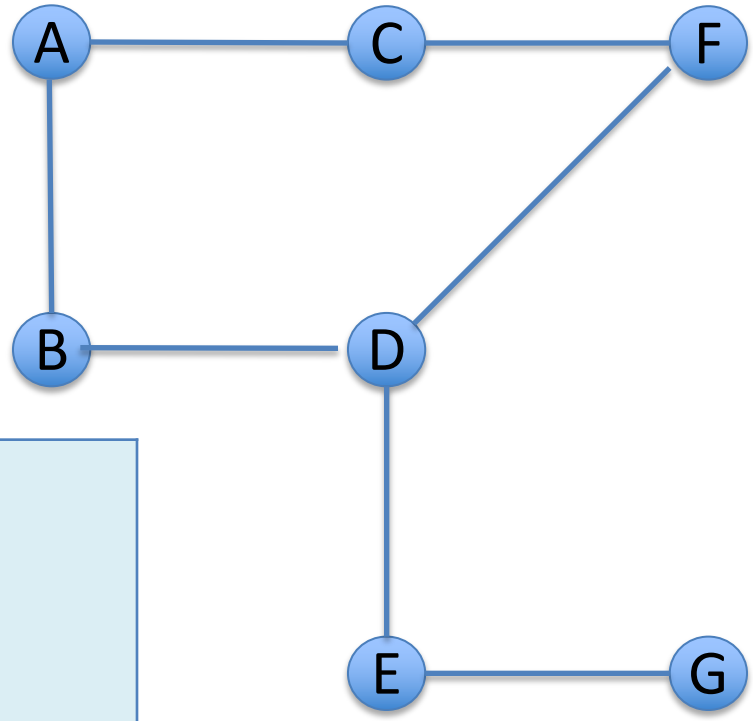


BFS(G=(V,E))

```
for each v ∈ V visited[v] := FALSE
for each v ∈ V
   // BFSExplore(v)
   visit v    #do all “visiting” stuffs
   visited[v] := TRUE
   Q := empty queue
   enqueue(Q,v)
   while Q is not empty
      v := dequeue(Q)
      for each neighbor w of v
        if !visited(w)
           visit w
           visited[v] := TRUE
           enqueue(Q,w)
```

Q5.4: DFS & BFS

- List the order of the nodes visited by the a) DFS and b) BFS algorithms



YOUR ANSWER:

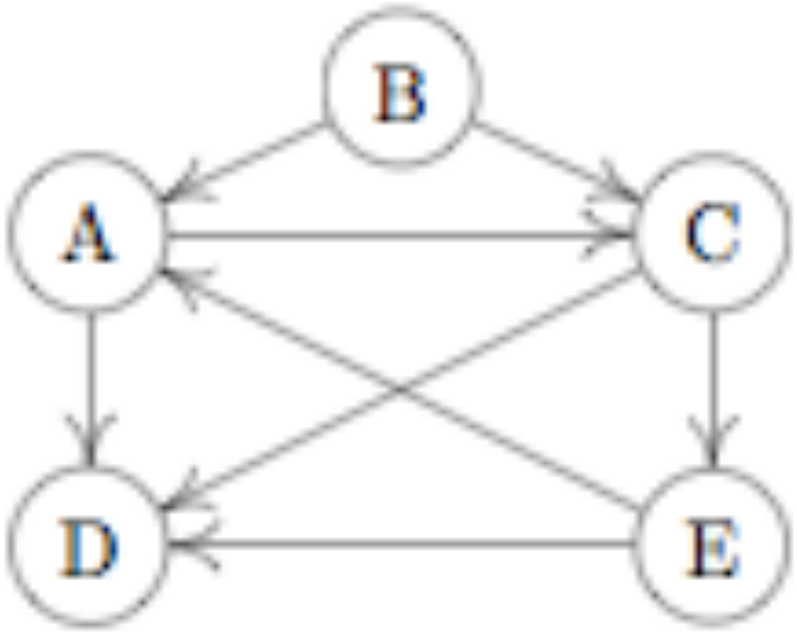
a) The order of the nodes visited by DFS is:

A

b) The order of the nodes visited by BFS is:

A

Q5.5: Tree, Back, Forward and Cross Edges



A DFS of a di-graph can be represented as a collection of trees. Each edge of the graph can then be classified as a *tree edge*, a *back edge*, a *forward edge*, or a *cross edge*. A tree edge is an edge to a previously un-visited node, a back edge is an edge from a node to an ancestor, a forward edge is an edge to a non-child descendent and a cross edge is an edge to a node in a different sub-tree (i.e., neither a descendent nor an ancestor)

Draw a DFS tree based on the following graph, and classify its edges into these categories.

In an undirected graph, you wont find any forward edges or cross edges. Why is this true? You might like to consider the graph above, with each of its edges replaced by undirected edges.

Q5.6: Finding Cycles – do in group of 2-3

a) Explain how one can use BFS to see whether an undirected graph is cyclic.

Transform function BFS to:

`isCyclic(G=(V,E))`

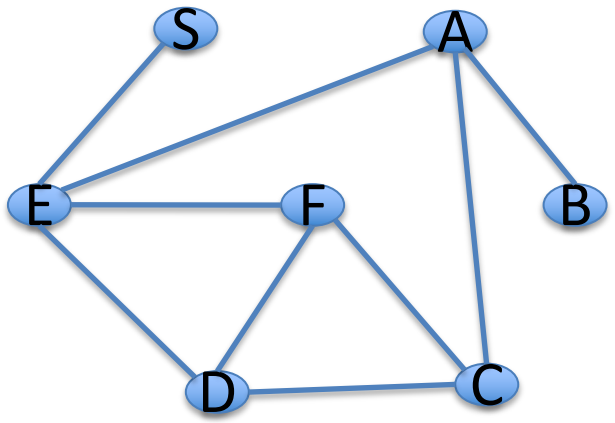
b) Can we use DFS for the task?

Which one is better: DFS or BFS?

BFS algorithm – as in lecture

```
function isCyclic(G=(V,E))
  mark each node in V with 0
  Q := empty queue
  for each v in V do
    if v is marked with 0 then
      mark v with 1
      INJECT(Q, v)
      while Q ≠ Ø  do
        u := EJECT(Q)
        for each edge (u,w) do
          if w is marked with 0 then
            mark w with 1
            INJECT(Q, w)
          else
            return YES
  return NO
```

5.7: 2-Colourability – possibly homework



Design an algorithm to check whether an undirected graph is 2-colourable, that is, whether its nodes can be coloured with just 2 colours in such a way that no edge connects two nodes of the same colour.

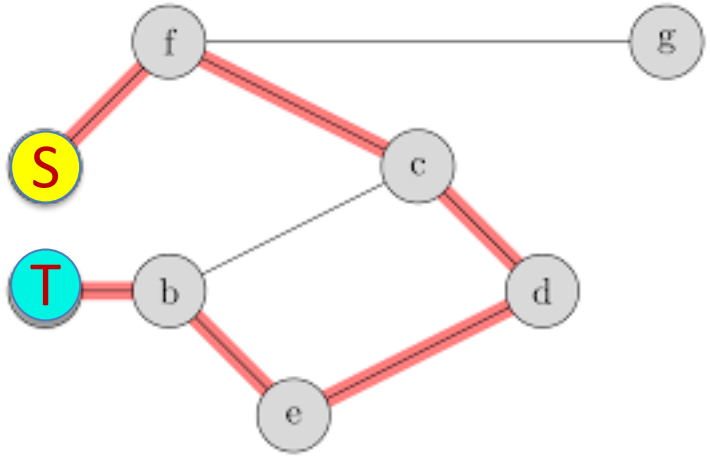
To get a feel for the problem, try to 2-colour the following graph (start from **S**).

Do you expect we could extend such an algorithm to check if a graph is 3-Colourable, or in general: k-Colourable?

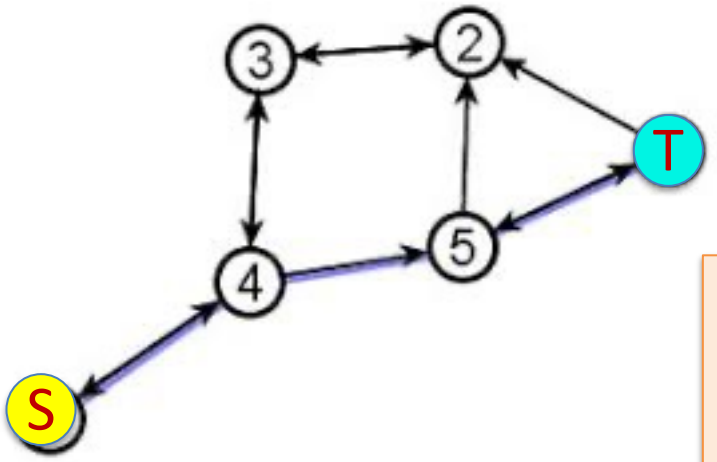
```
// Aim: transform this to is2Colorable
function DFS(G=(V,E))
  mark each node in V with 0
  for each v in V do
    if v is marked with 0 then
      DfsExplore(v)

function DFsExplore(v)
  mark v with 1
  for each edge (v,w) do
    if w is marked with 0 then
      DfsExplore(w)


// Can we use BFS?
```

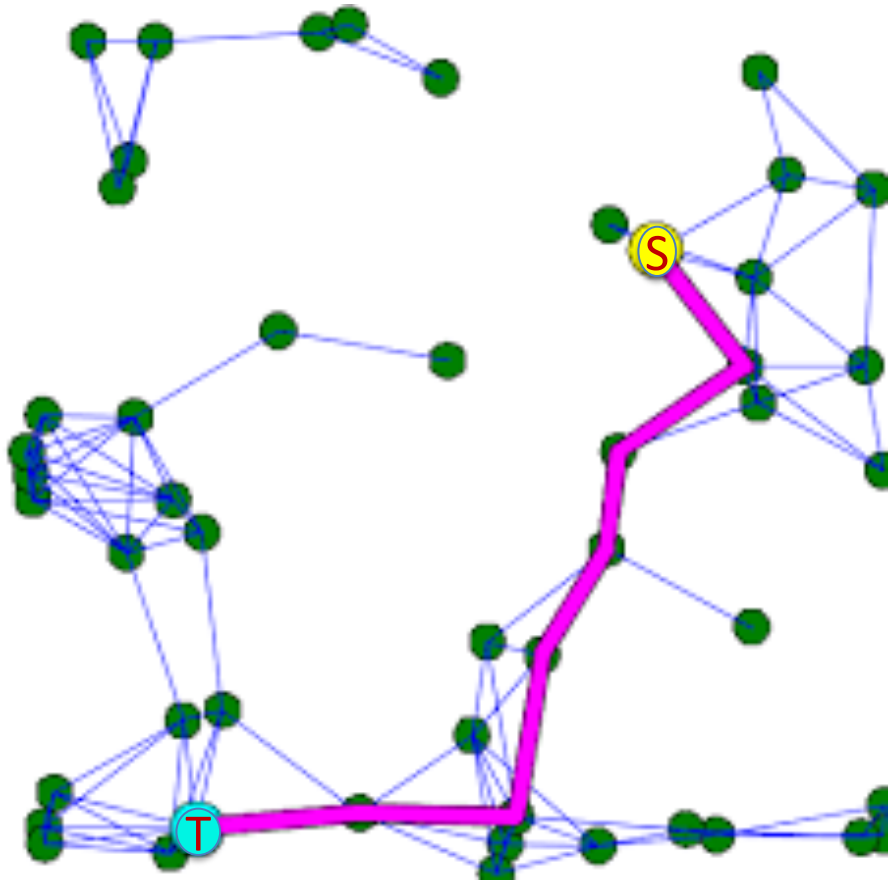
Paths in unweighted graphs: path length, shortest path

Path= S→f→c→d→e→b→T, length=6



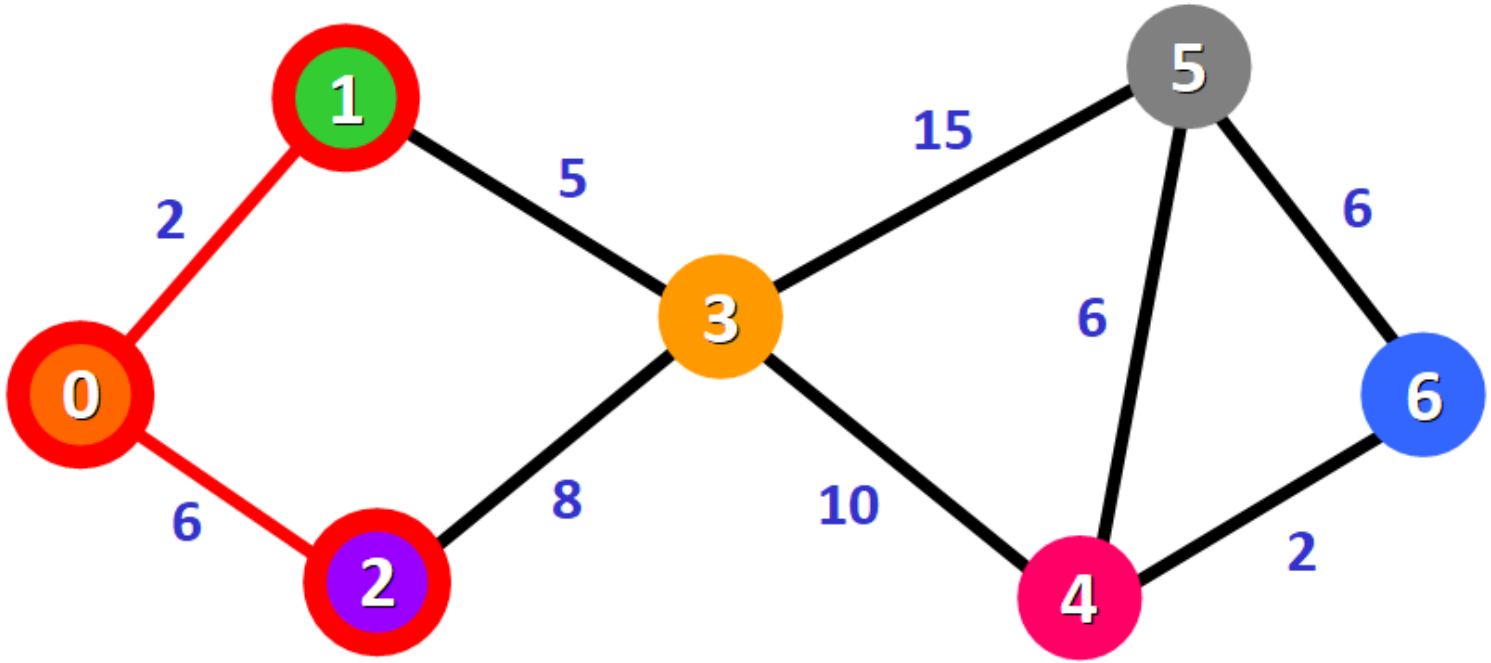
Path= S→4→5→T, length= 3





- For finding a *shortest path* from S to T can we use DFS? BFS?

How about shortest path on weighted graphs?



Path from 0 to 3:

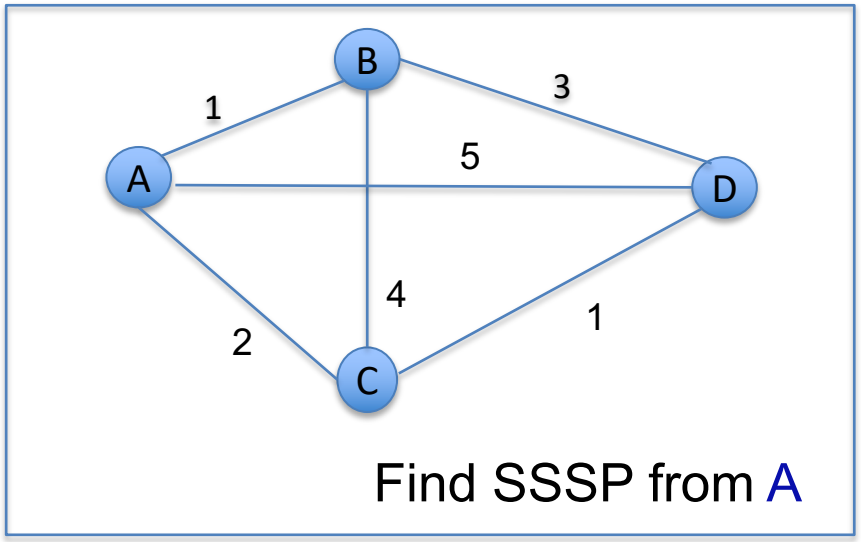
- possible paths:
  - 0→1→3,
  - 0→2→3
- shortest path: 0→1→3 with total weight= 7

Dijkstra's Algorithm: Single Source Shortest Path (SSSP)

The task:

- Given a weighted graph $G=(V,E,w(E))$, and $s \in V$, and supposing that *all weights are positive*.
- Find shortest path (path with min total weight / min distance) from s to all other vertices.

Input



Output

Destination	Shortest Path	Path Length
A	A→A	0
B	A→B	1
C	A→C	2
D	A→C→D	3

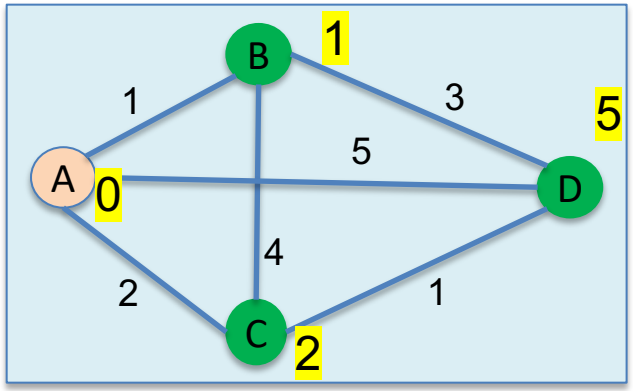
- Solution: using Dijkstra's Algorithm!
- Q: Can the Dijkstra's Algorithm be used for the SSSP in unweighted graphs?

We start from A, and shortest-distance-so-far to all other nodes is ∞

`dist[]= {0, ∞, ∞, ∞}`

We use the edges from A to update the shortest-distance-so-far

`dist[]= {0, 1, 2, 5}`



What's next?
explore C, B, or D?

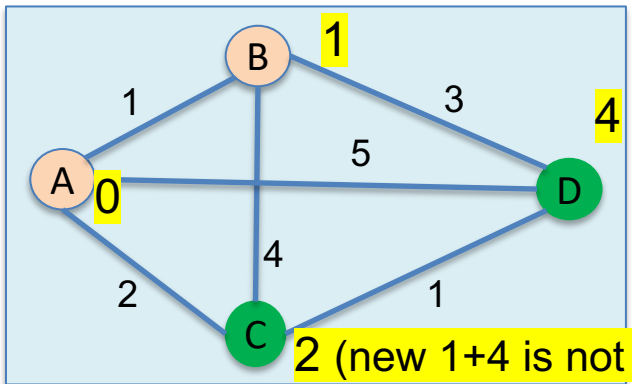
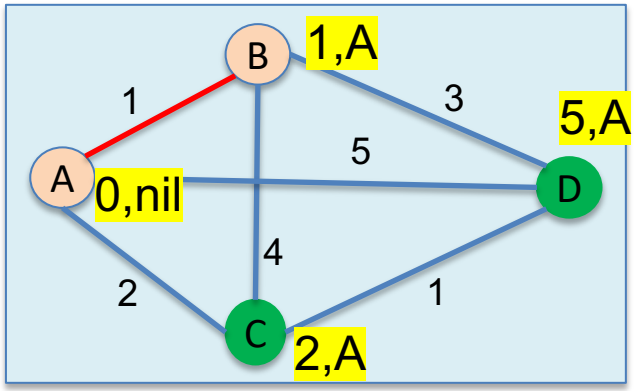


image: https://medium.com/analytics-vidhya/what-is-the-greedy-algorithm-5ed71f9a7b3a

- ➔ Using this greedy policy, shortest path from A to B found with `dist[B]= 1`.
- ➔ For retrieving the actual path, we also need to keep `prev[B]=A`.
- ➔ continue this way to update dist[] for other nodes

Keep track of our steps: for each v maintain

- dist[v]: shortest-distance-so-far from A to v
- prev[v]: node that precedes v in the path



this column: nodes with found shortest path

	A	B	C	D
	0, nil	∞,nil	∞,nil	∞,nil
A		**1,A**	2,A	5,A
B			**2,A**	4,B

prev[D]= A node that precedes D in the path A→D

dist[D]= 5 shortest-so-far distance from A

queue {B,C,D} includes not-yet-done elements at this stage.
From the queue we remove B - the one with lowest dist, so shortest path A→B found

From the queue, we always remove the element with min value ➔ for better efficiency, we use a priority queue instead of a normal FIFO queue.

Dijkstra's algorithm

```
set dist[u]:= ∞, prev[u]:=nil for all u
dist[s]:= 0
PQ:= makePQ(V) using dist[] as the weight
while (PQ not empty)
    u:= deleteMin(PQ)
    visit  u
    for all (u,v) in G:
      if (dist[u]+w(u,v)<dist[v]):
         update dist[v] and pred[v]
```

A	B	C	D
0,nil	∞,nil	∞,nil	∞,nil
	1,A	2,A	5,A
		2,A	4,B
			3,C

Programming note:

Here PQ is a priority queue.

"update dist[v] and pred[v]" means:

1. dist[v]:= dist[u]+w(u,v), pred[v]:= u
2. change the weight of v in PQ to the new dist[v].

Complexity:

Will see:

- makePQ of n elems: $O(n)$
- delemin(PQ): $O(\log n)$
- change a weight in PQ: $O(\log n)$

So:

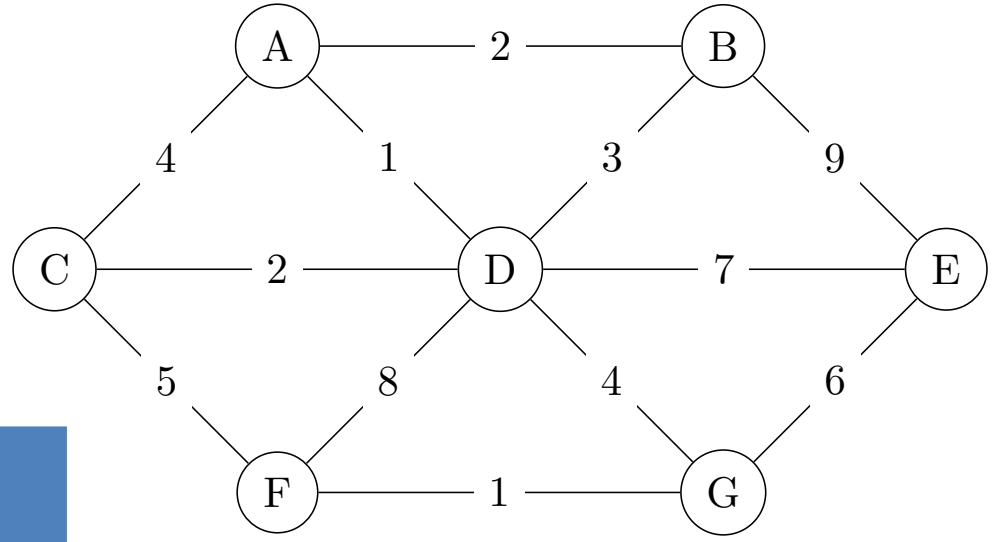
Complexity of DA is: $O(\ ?\ )$

Dijkstra's algorithm: a variation for finding shortest path from s to t

```
set dist[u]:= ∞, prev[u]:=nil, visit[u]:=0 for all u
dist[s]:= 0

~~set PQ= makePQ(V) using dist[] as the weight~~
PQ:= new empty PQ
enPQ(s, dist[s])

while (PQ not empty)
   u:= deleteMin(PQ)
   if visit[u]=1 continue
   visit[u]= 1
   if u=t break;   #path s→t found

   for all (u,v) in G:
     if (dist[u]+w(u,v)<dist[v]):
        update dist[v] and pred[v]
        PQ := enPQ(v, dist[v])
```

Q5.9 a): SSSP with Dijkstra's Algorithm

Trace Dijkstra's algorithm on the following graph, with node E as the source.
How long, and what, is the shortest path from E to A?



step››	node done	A	B	C	D	E	F	G
0								
1								
2								
3								
4								
5								
6								
7								

assignment 1: Q&A

- Do it early! Submit early, submit as many times as you want!
- Read & participate in discussion forum!
- Make sure that you follow well the specification.
- Make sure that the report part is presented clearly and concisely

- If finished, make sure you don't have memory leak:
  - check that every execution of `malloc` matches with an execution of `free`, and
  - use `valgrind` to test for memory leak.

Note:

- the assignment could be time-consuming!

How to use valgrind in Ed to check for possible bugs?

When opening a programming terminal. Now, suppose you want to run `problem2a` with input data `test_cases/2a-1.txt`, you normally do with:

```
make problem2a
./problem2a <test_cases/21-1.txt
```

Now, if you want to test with `valgrind`, just run:

```
valgrind –leak-check=full  ./problem2a <test_cases/21-1.txt
```

`valgrind` will give some output. If at the end of the output, you see lines:

```
==??== All heap blocks were freed -- no leaks are possible
==??== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
```

then your probably doesn't have bugs (well, you need to check for correctness yourself). If you don't see those 2 lines, then that means `valgrind` has detected some potential bugs. You need to scroll up and look at the start of the `valgrind` output, you will see `valgrind` reports some line numbers from some `.c` files that might cause problems. Inspect these lines to find the bugs.

LAB

-work on assignment 1

- or, if applicable, prepare for MST: **see sample MST test and solution in Ed**

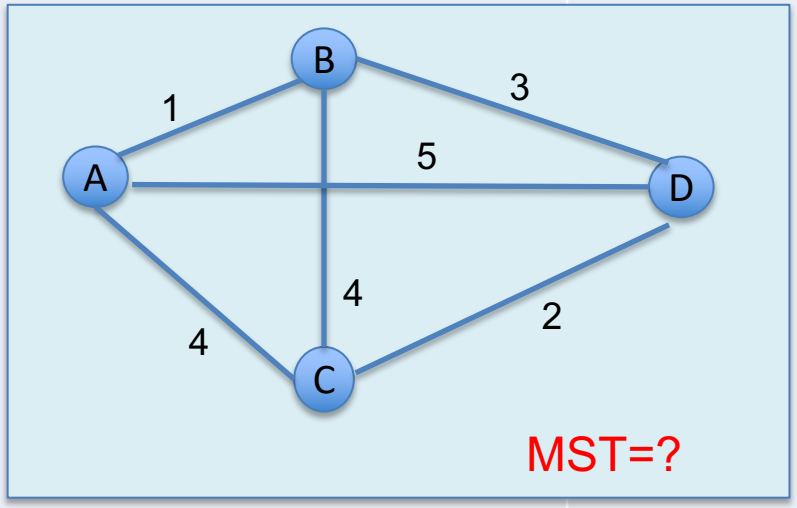
- or, work on not-yet-done this week's workshop problems: Questions 9, 8, 6, 7 (if not finished)

Prim's Algorithm vs Dijkstra's Algorithm. Discuss concepts

	Prim's	Dijkstra's
Aim	find a MST	find SSSP from a vertex s
Applied to	connected weighted graphs with weights ≥ 0	weighted graphs with weights ≥ 0
Works on directed graphs?	?	
Works on unweighted graph?	?	

Related concepts for Prim's

- spanning trees = ?
- MST = ?
- is MST unique?



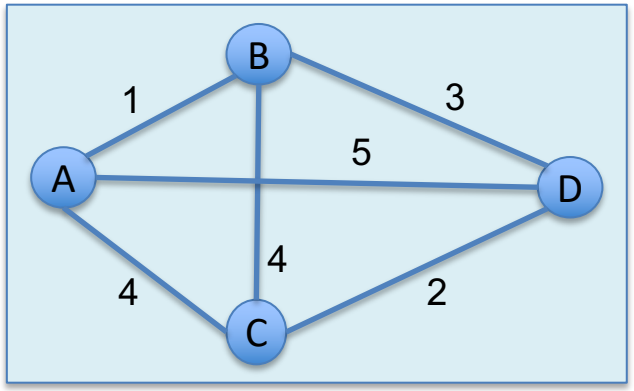
Dijkstra's and Prim's are similar

Dijkstra(G=(V,E),S)	Prim(G=(V,E))
Task: Find SSSP from S (that involves all nodes of a *connected* graph)	Task: MST (that involves all nodes of a *connected* graph)

`for each v ∈V do`
`  cost[v]:= ∞`
`  prev[v]:= nil`
 	
`cost[S]= 0`
`PQ:= create_priority_queue(V,cost) with`
`cost[v] as priority of v∈V`
	`for each v ∈V do`
	`  cost[v]:= ∞`
	`  prev[v]:= nil`
	pick initial s
	`cost[S]:=0`
	`PQ:= create_priority_queue(V, cost)`
	`with cost[v] as priority of v∈V`

`while (PQ is not empty) do` `  u := ejectMin(PQ)`	`while (PQ is not empty) do` `  u := ejectMin(PQ)`

`  for each neighbour v of u do`
 	
`  if  dist[u]+w(u,v) < cost[v] then`
`  cost[v]:= cost[u]+w(u,v)`
`  update (v, cost[v]) in PQ`
`  prev[v]:= u`
	`  for each neighbour v of u do`
	
	`  if  w(u,v) < cost[v] then`
	`  cost[v]:= w(u,v)`
	`  update (v, cost[v]) in PQ`
	`  prev[v]:= u`



Running Prim's Algorithm to find a MST

At each step, we add a node to MST.

We choose the node with **minimal edge cost**.

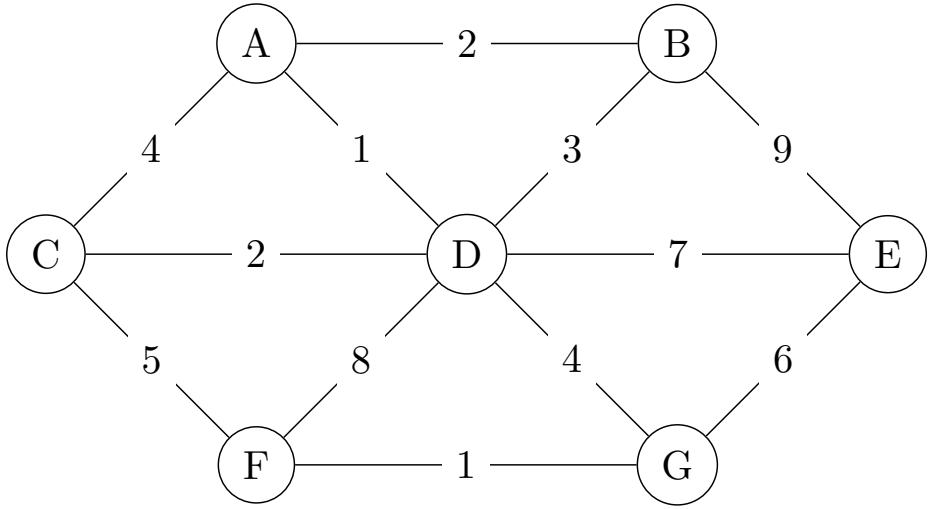
We start with A according to the alphabetical order.

step	node added to MST	A	B	C	D
0		0,nil	∞,nil	∞,nil	∞,nil
1					
2					
3					
4					

Question 5.8: Minimum Spanning Tree with Prim's Algorithm

Prim's algorithm finds a minimum spanning tree for a weighted graph. Discuss what is meant by the terms 'tree', 'spanning tree', and 'minimum spanning tree'.

Run Prim's algorithm on the graph below, using A as the starting node. What is the resulting minimum spanning tree for this graph? What is the cost of this minimum spanning tree?

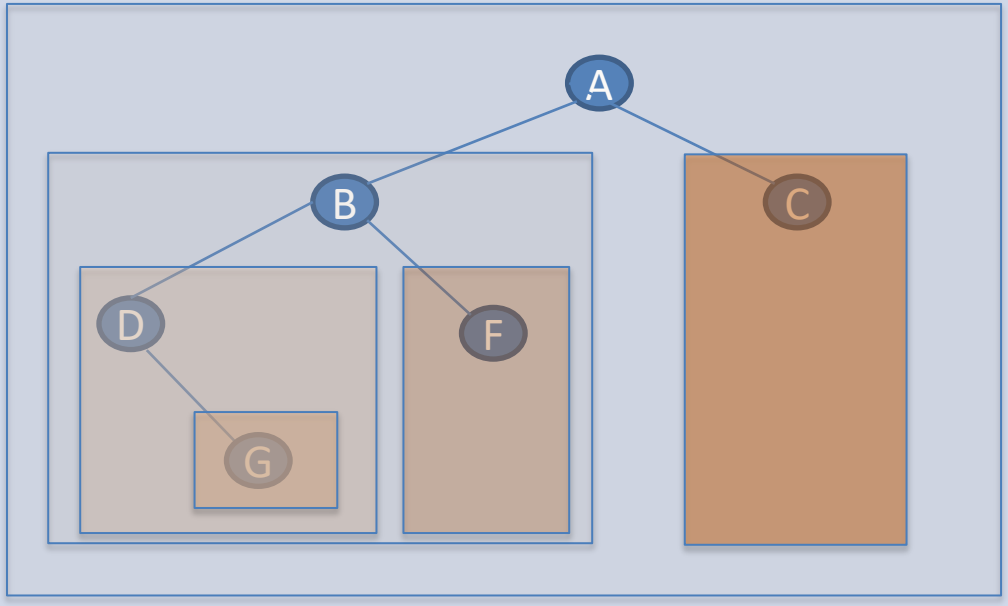


step	node done	A	B	C	D	E	F	G
0		0/nil	∞/nil	∞/nil	∞/nil	∞/nil	∞/nil	∞/nil
1								
2								
3								
4								
5								
6								
7								

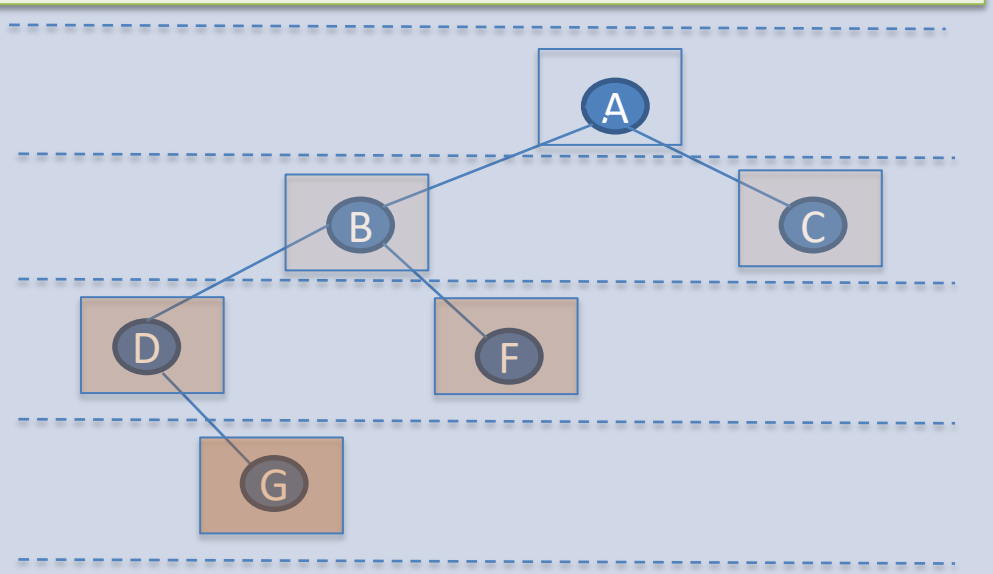
Additional Slides

DFS vs BFS	DfsExplore(v)	BfsExplore(v)

```
#start visit v
visit v
mark v as visited


for each neighbor w of v
  if !visited(w)
    DfsExplore(w)

#end visit v
```

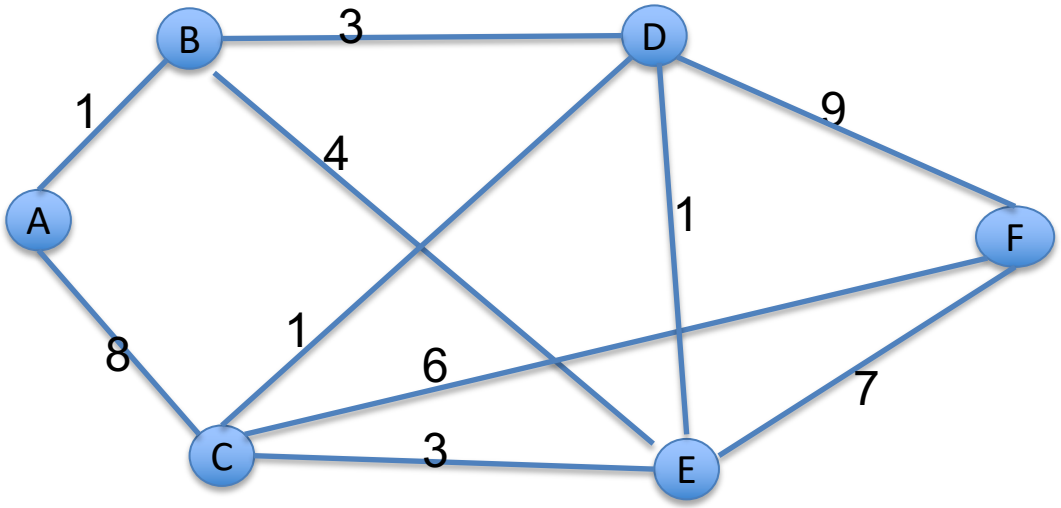



```
visit v #including start...end
mark v as visited
Q := empty queue
enqueue(Q,v)
while Q is not empty
   v := dequeue
   for each neighbor w of v
     if !visited(w)
       visit w
       mark w as visited
       enqueue(Q,w)
```



Each visit is represented by a box. In BFS, a visit ends before a next visit starts. In DFS, a visit includes other visits in itself.

Tracing Dijkstra's Algorithm



Still confused on how to trace the Dijkstra's Algorithm?

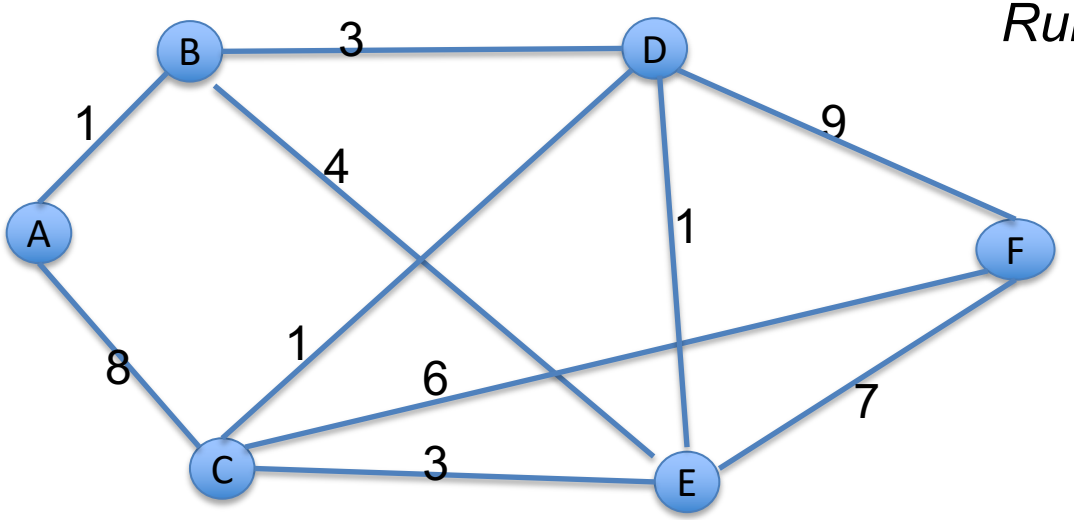
Try to do with the above graph then compare with the solution in the next 3 slides. The task:

Given the above graph. Find a shortest path:

- From A to B
- From A to C
- From A to F
- From A to any other node

by tracing the Dijkstra's Algorithm.

Run Dijkstra's Algorithm from A



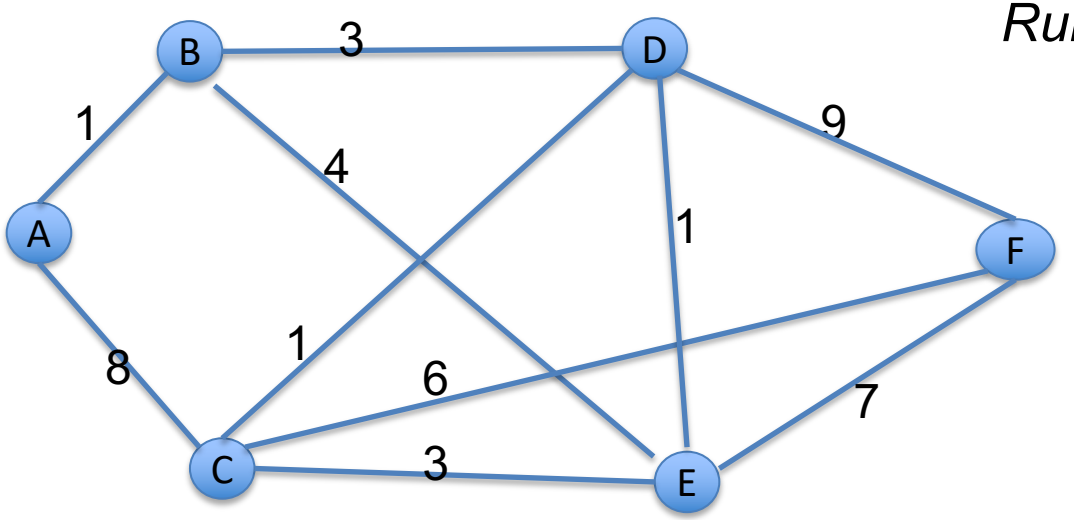
done	A	B	C	D	E	F
	0, nil	∞,nil	∞,nil	∞,nil	∞,nil	∞,nil
A						

this column: nodes with shortest path found

`dist[B]`: shortest-so-far distance from A

`pred[D]`: node that precedes D in the path A→D

Run Dijkstra's Algorithm from A

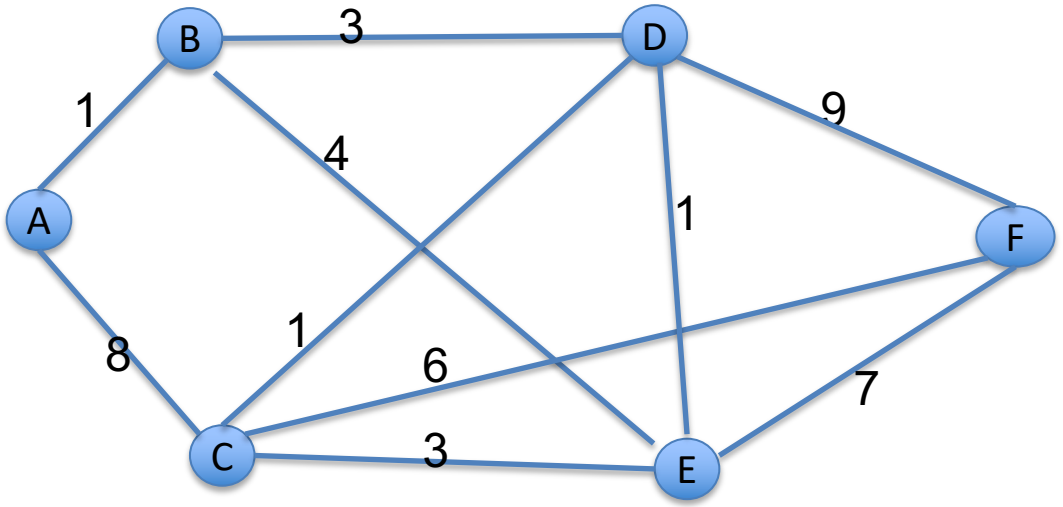


The dist at A is 0, there is an edge A->C with length 8, so we can reach C from A with distance 0+8, and 8 is better than previuosly-found distance of ∞

done	A	B	C	D	E	F
	0, nil	∞,nil	∞,nil	∞,nil	∞,nil	∞,nil
A		**1,A**	8,A	∞,nil	∞,nil	∞,nil
B			8,A	**4,B**	5,B	∞,nil
D			**5,D**		5,B	13,D
C					**5,B**	11,C
E						**11,C**
F						

Update this cell because now we can reach C from D with distance 4 (of D) + 1 (of edge D→C), and 5 is **better** than 8

At this pointy, we can reach E from D with distance 4 (of D) + 1 (of edge D→E), but new distance 5 is **not better** than the previously found 5, so no update!



Find a shortest path A→F:

- *the shortest path has weight 11 (last row of column F)*
- *the path is*

C→F
D→C
B→D
A→B
path= A→B→D→C→F

What's the found shortest path from A to F?
distance= 11, path=A→B→D→ C→F

done	A	B	C	D	E	F
	0, nil	∞,nil	∞,nil	∞,nil	∞,nil	∞,nil
A		**1,A**	8,A	-	-	-
B			8,A	**4,B**	5,B	-
D			**5,D**		5,B	13,D
C					**5,B**	11,C
E						**11,C**
C						

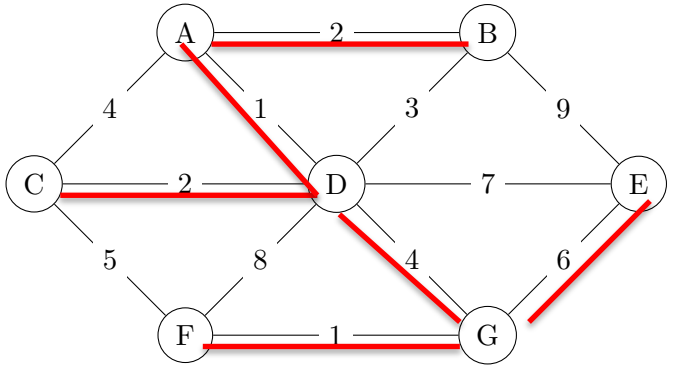
pred[B]= A:
A→B→D→ C→F

pred[D]= B:
B→D→ C→F

pred[C]= D:
D→ C→F

pred[F]= C, that is we came to F from C: C→F

the shortest distance from A to F is 11



Check: Soln to Q5.8 : Tracing Prim's Alg

What's the resulted MST: the red-linked
What's the cost of that MST?
cost= 0+2+2+1+6+1+4= 16

step	node ejected	A	B	C	D	E	F	G
0		**0/nil**	∞/nil	∞/nil	∞/nil	∞/nil	∞/nil	∞/nil
1	A		2,A	4,A	**1,A**	∞/nil	∞/nil	∞/nil
2	D		**2,A**	2,D		7,D	8,D	4,D
3	B			**2,D**		7,D	8,D	4,D
4	C					7,D	5,C	**4,D**
5	G					6,G	**1,G**	
6	F					**6,G**		
7	G							

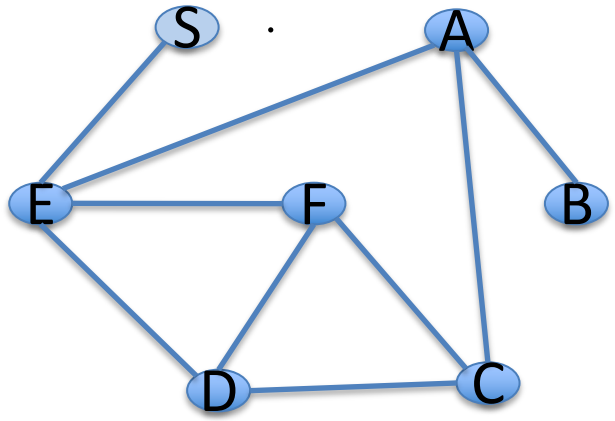
Q5.6 (Additional Homework) Finding Cycles with DFS

Explain how one can also use DFS to see whether an undirected graph is cyclic.

```
function DFS(G=(V,E))
  mark each node in V with 0
  for each v in V do
    if v is marked with 0 then
      DFSEXPLORE(v)

function DFSEXPLORE(v)
  mark v with 1
  for each edge (v,w) do
    if w is marked with 0 then
      DFSEXPLORE(w)
```

5.7: 2-Colourability (Additional Homework)

```
// adapt this to is2Colorable
//    iif you haven't done with DFS

function is2Colorable(G=(V,E))
  mark each node in V with 0
  Q := empty queue
  for each v in V do
    if v is marked with 0 then
      mark v with 1
      inject(Q, v)
      while Q ≠ ∅  do
        u := eject(Q)
        for each edge (u,w) do
        if w is marked with 0 then
          mark w with 1
          inject(Q, w)
        ?
```



b) Design an algorithm to check whether an undirected graph is 2-colourable, that is, whether its nodes can be coloured with just 2 colours in such a way that no edge connects two nodes of the same colour.

Perhaps by editing BFS or DFS. BFS attached here, DFS in the next page.

c)Do you expect we could extend such an algorithm to check if a graph is 3-Colourable, or in general: k-Colourable?