COMP20007 Workshop Week 11

	Preparation:
	- have draft papers and pen ready
1	Why BST, AVL, 2-3 Tree?
2	BST: Rotation, Balance factor: Q 11.1, 11.2
3	AVL Tree: Concepts, Insertion: Q 11.3, Deletion: Q11.4
4	2-3 Tree: Concepts, Insertion: Q 11.5, Deletion: Q11.6 (time permits)
5	B-tree?
LAB	Assignment 2, or
	Revision: Questions for previous week materials

Why AVL, 2-3 Trees?

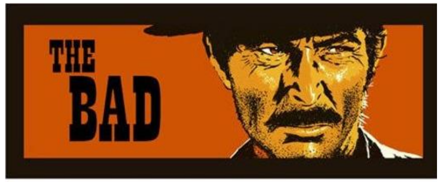
Peer Activity: Applying Data Structures

We have a set of **elements**, on which we want guaranteed log n search and insert complexity.

Which data structure is most suitable for storing these elements?

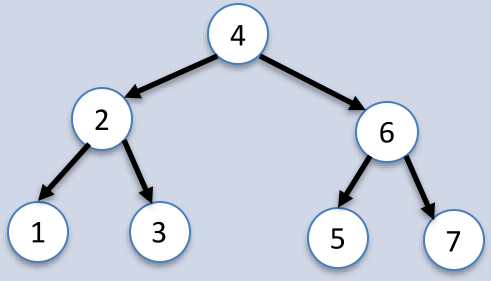
a. AVL tree
b. binary search tree
c. sorted array
d. sorted linked list

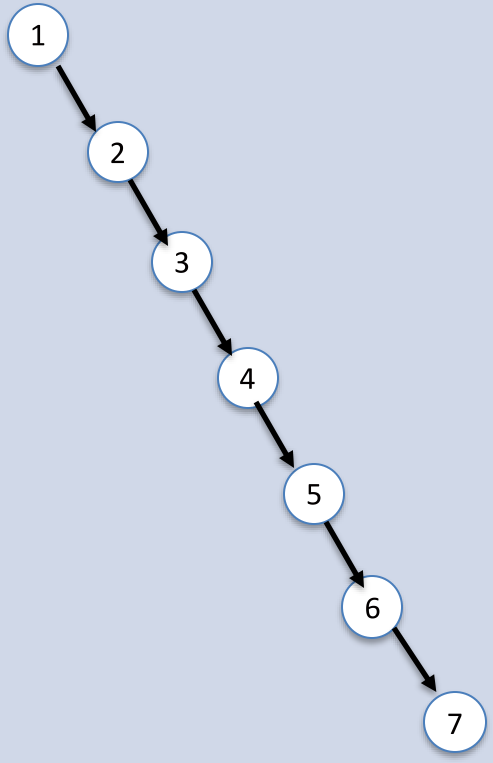
BST efficiency depends on the order of input data

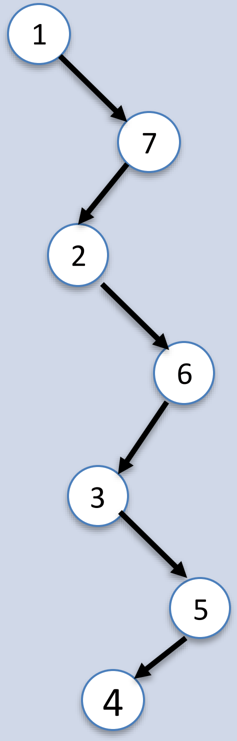


AND

4 6 2 1 3 7 5	1 2 3 4 5 6 7	1 7 2 6 3 5 4



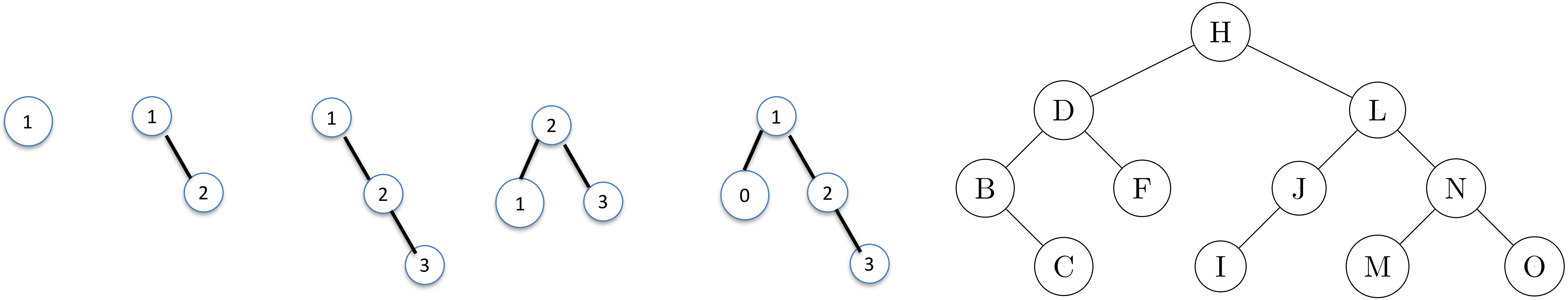




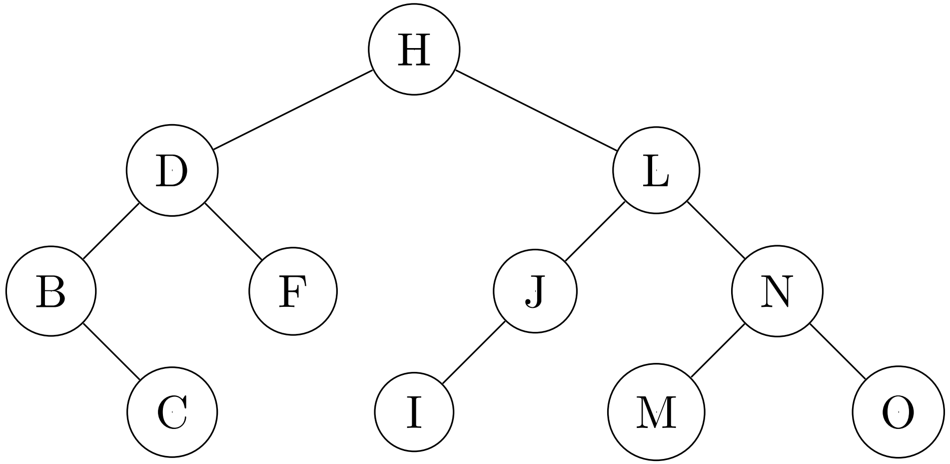
Want The Good, no matter what's the data input order? Use AVL (or …)!

Good-Bad_Ugly Picture Source: https://www.pinterest.com.au/pin/170573904624610413/

Are these BST balanced ?



[Class] Q 11.2: A node's '*balance factor*' is defined as the height of its right subtree minus the height of its left subtree. Calculate the balance factor of each node in the following binary search tree.

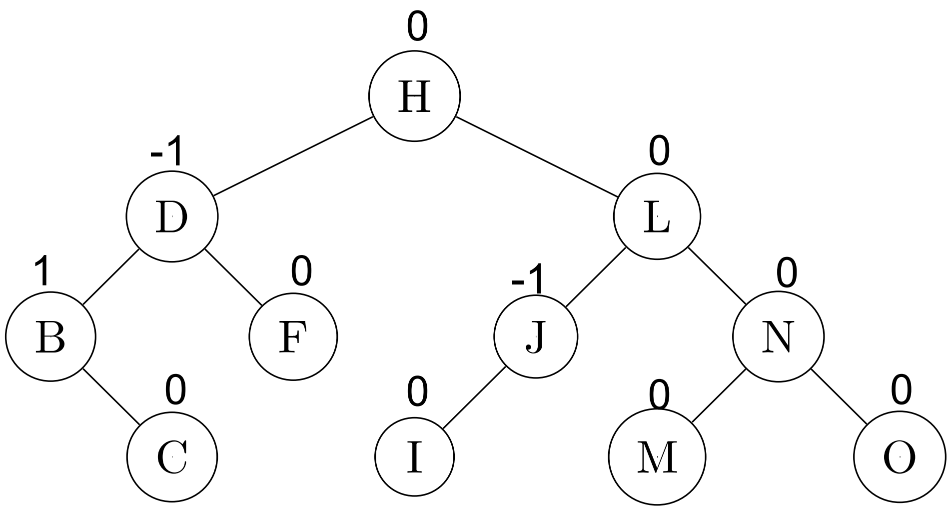


Note on balance factor (BF) definition

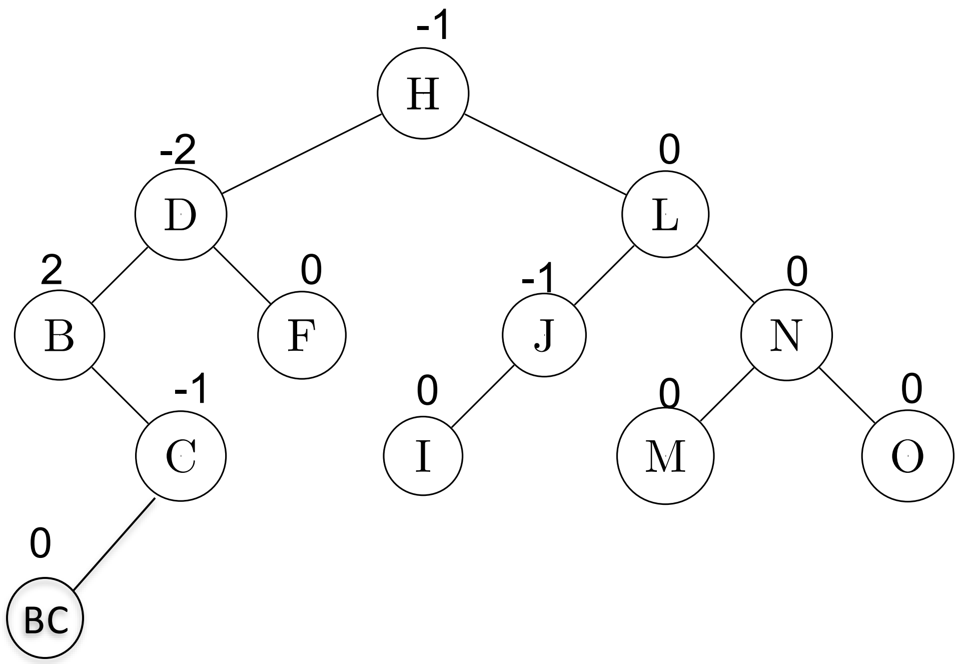
- Popular (as in lectures):
  BF= height(T.left) - height(T.right)
- Option 2 (here):
  height(T.right) - height(T.left)

Both are OK, but needs **consistency**!
To see if a tree is balanced, we just need absolute value of the height difference.

Balanced BST



a balanced BST



an unbalanced BST

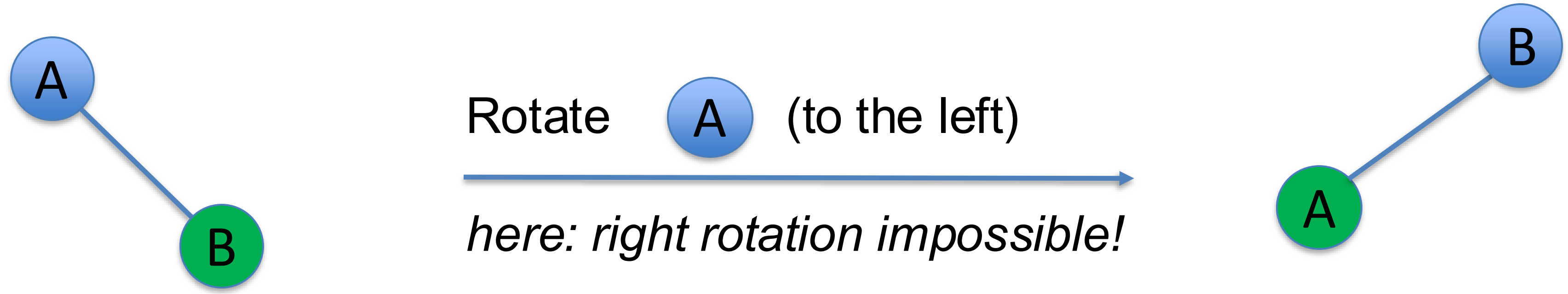
Balanced tree = when the balance factor of each node is 0, -1, or +1

= for each node, the difference of subtree heights is at most 1

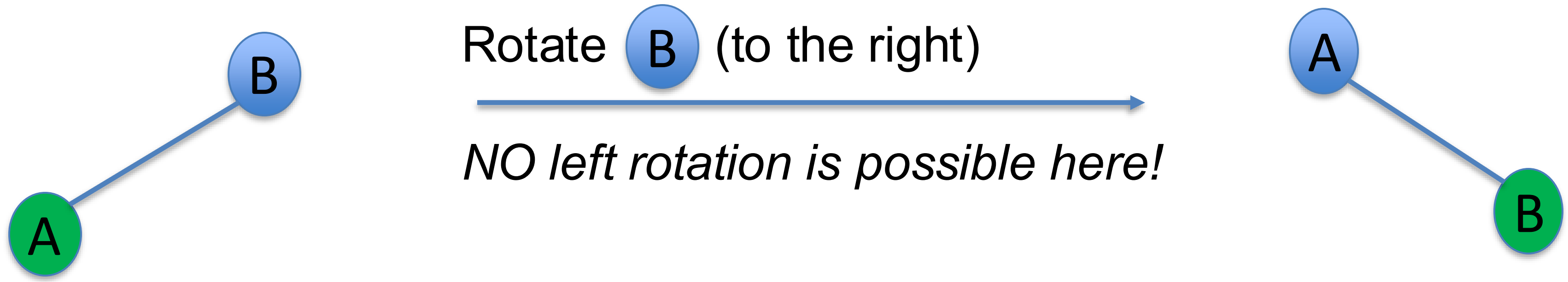
BST: what's a rotation

A *rotation reverses* the parent-child relationship of a parent and a child of it in a BST.

left rotation: rotate parent down to the left (parent becomes the left child)



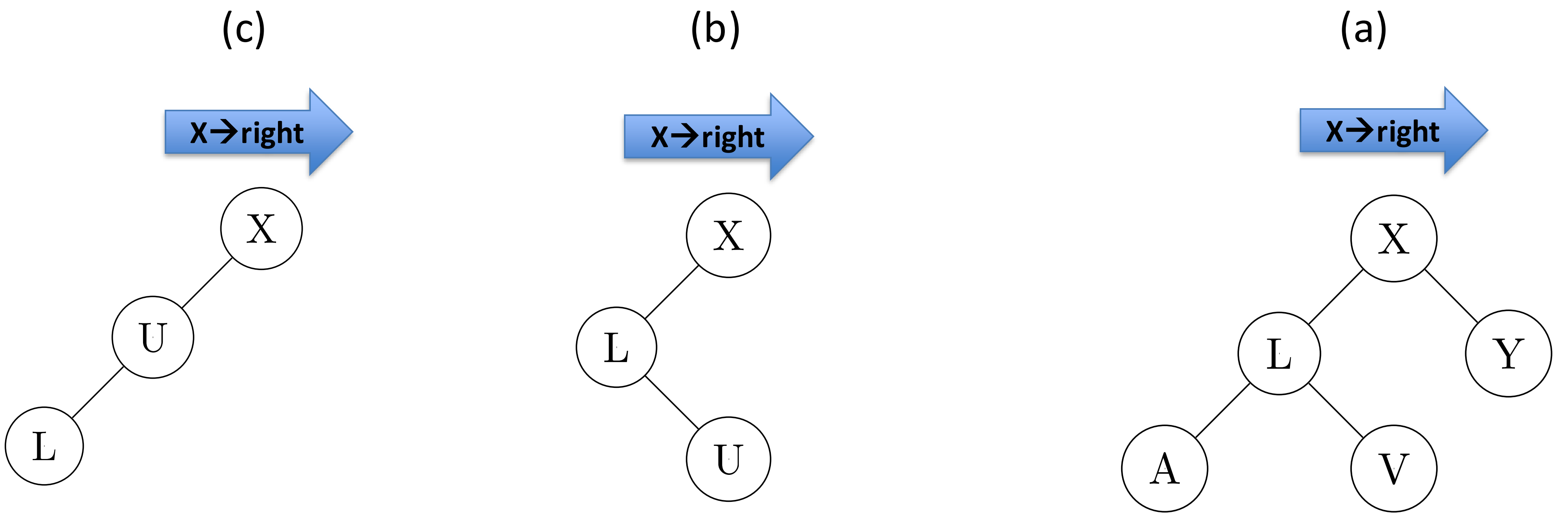
right rotation: rotate parent down to the right (to become the right child)



Rotation := use a child node as the *pivot*, and rotate the parent node down to the left or right.

[Individual] Q 11.1: Rotation

In the following binary search trees, rotate the ‘X’ node to the right (that is, rotate it and its left child). Do these rotations improve the overall balance of the tree?



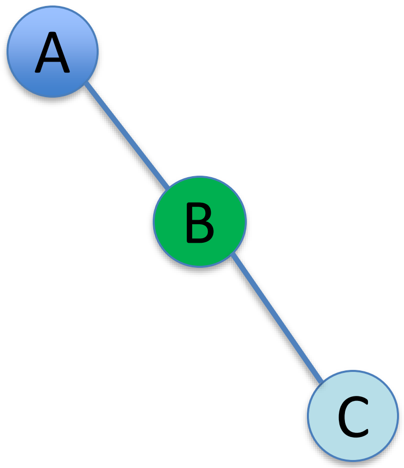
Recall: Only 2 types of rotations: *Right Rotation* (a node and its left child), and *Left Rotation* (a node and its right child)

AVL= self-balanced BST

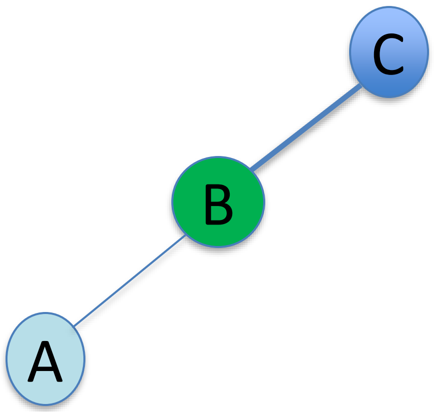
- AVL = balanced BST
- An empty BST is an AVL
- After each single insertion or deletion on an AVL, it might become unbalanced, and need to be re-balanced using rotations.

Two Basic Rotations for AVL: 1) Single Rotation

Applied when an unbalanced AVL subtree (or tree) is a ”stick”:



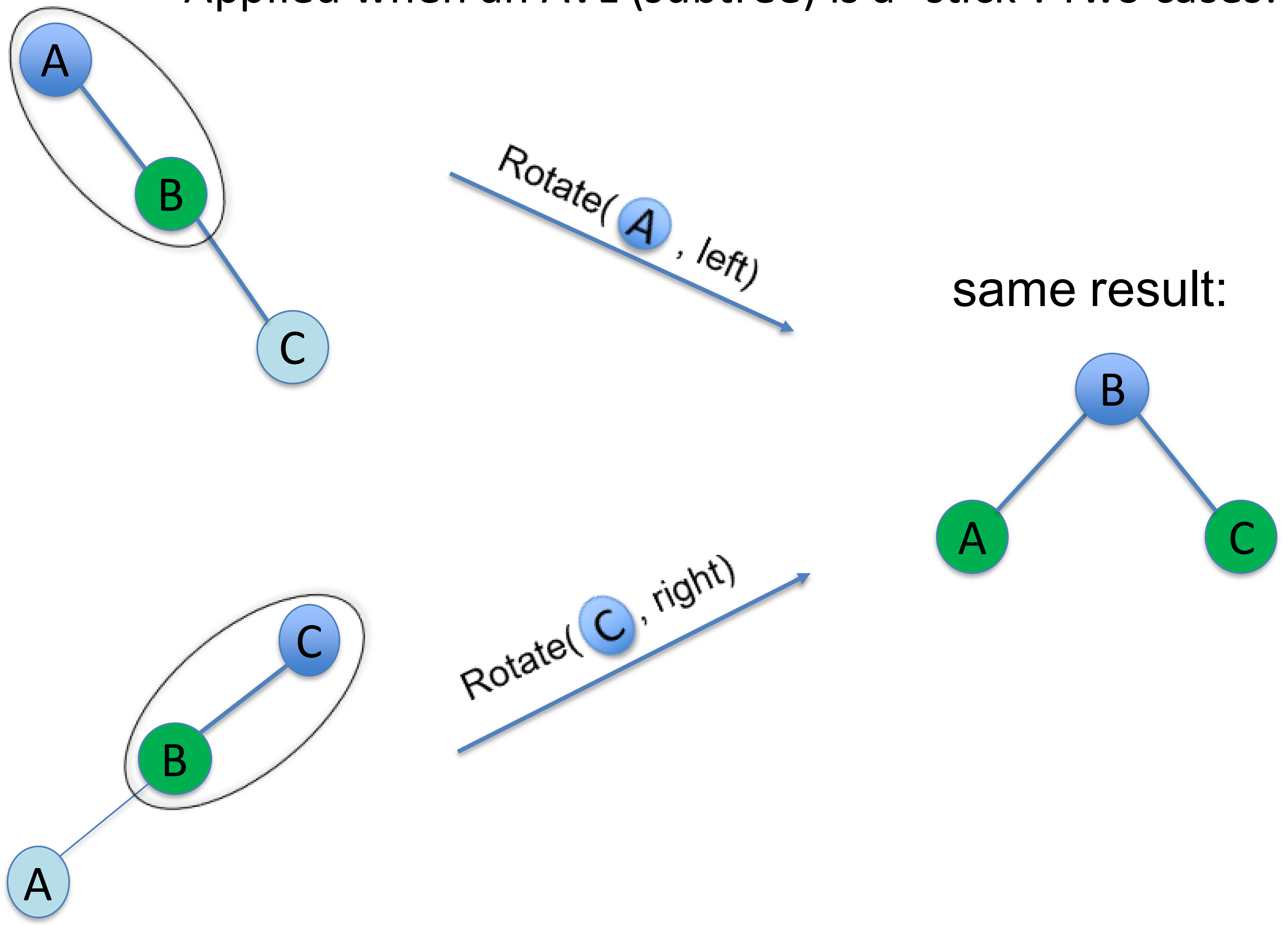
HERE: we have an un-balanced “stick”:
(unbalanced root)-child-grandchild



HOW:
→ Rotate the root down and hence balance the stick

AVL Rotations: 1) Single Rotation

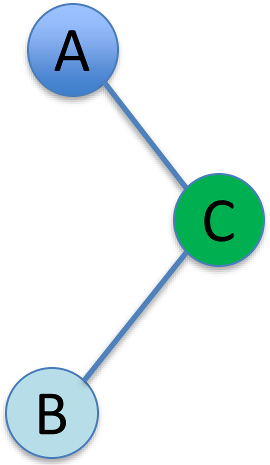
Applied when an AVL (subtree) is a ”stick”. Two cases:

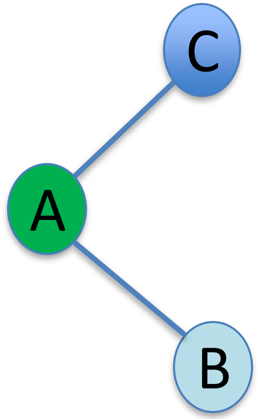


AVL: Two Basic Rotations: 2) Double Rotation

Applied when an unbalanced 3-node AVL subtree has a non-stick (that is, zig-zag) form.

Two cases: (a) (b)





We do 2 rotations to re-balance the non-stick unbalanced AVL.

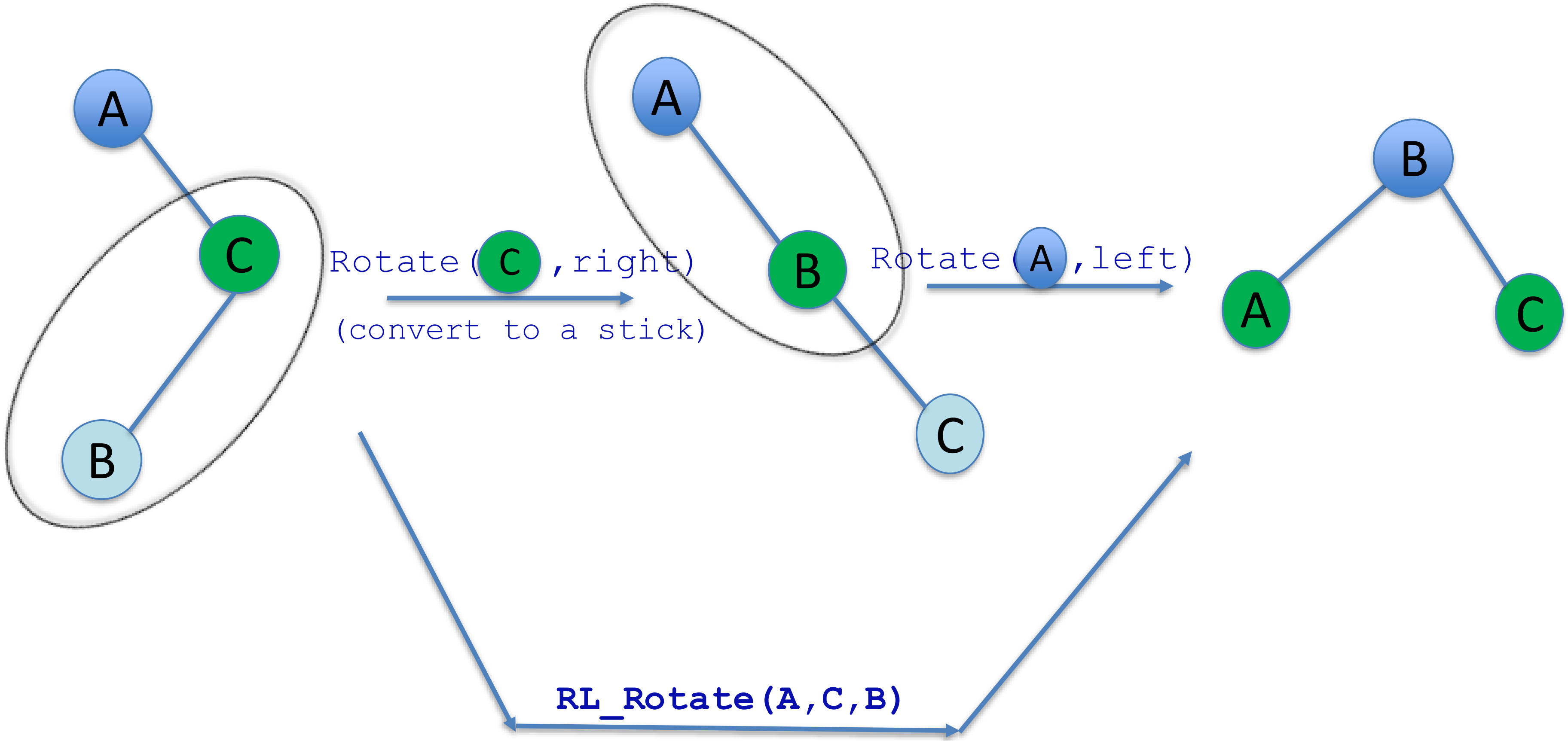
Rotation1:

- Rotate the **middle node** to turn the tree to a stick

Rotation2:

- Rotate the **unbalanced root** of the new stick.

Double Rotation Example: RL rotation



Do it Yourself: Perform `LR_Rotate(C,A,B)` for the other case of the previous page

AVL Tree: Summary of Rotations

AVL: Rebalance AVL after Insert, Q 11.3

Problem: When inserting a node, AVL might become unbalanced

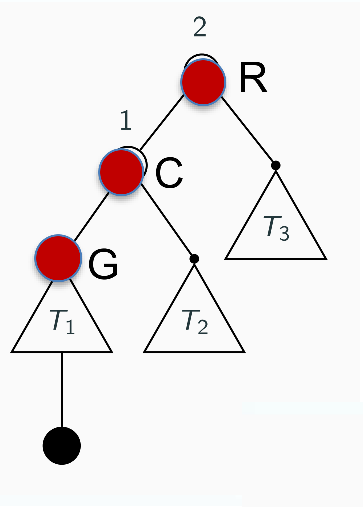
Approach: Rotations (to rebalance WHAT?, and HOW?)

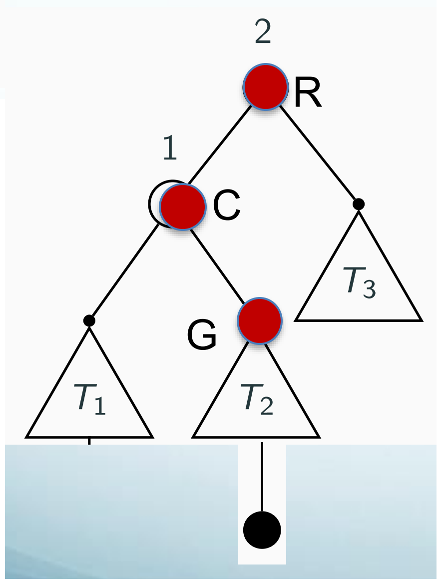
Rebalance WHAT?

- From the new node, walk up, find the *lowest* node R which is unbalanced. **Only** the tree rooted at R needs to be rebalanced.

HOW

- Consider *the 3 nodes* R, C, G on the path from R to the new node
- Apply a single rotations if R→C→G is a stick, double rotation otherwise





Insert the following keys into an initially-empty AVL Tree.

Class example:

20 10 5 15 30 17

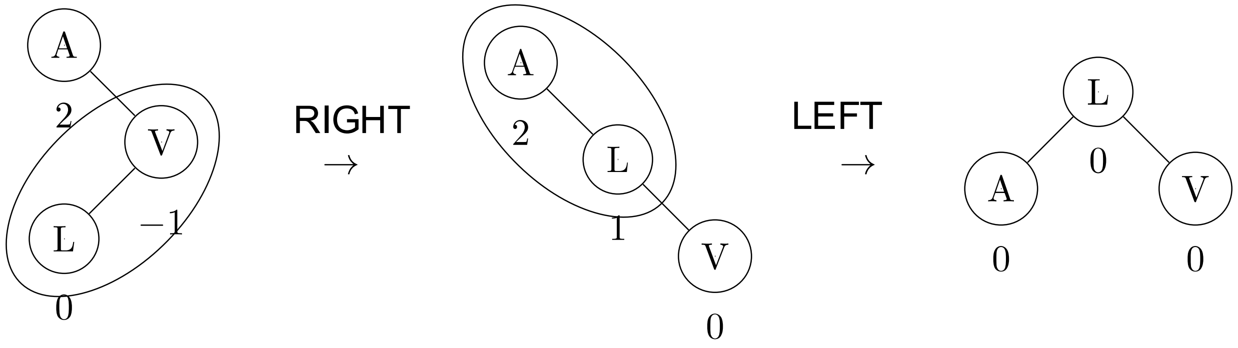
Q11.3 [*group/individual*]:

A V L T R E X M P

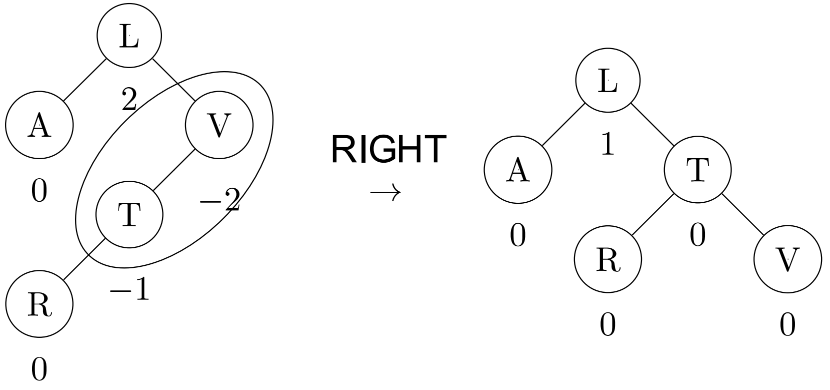
Q 11.3: Check your solution

Insert the following letters into an initially-empty AVL Tree: A V L T R E X M P

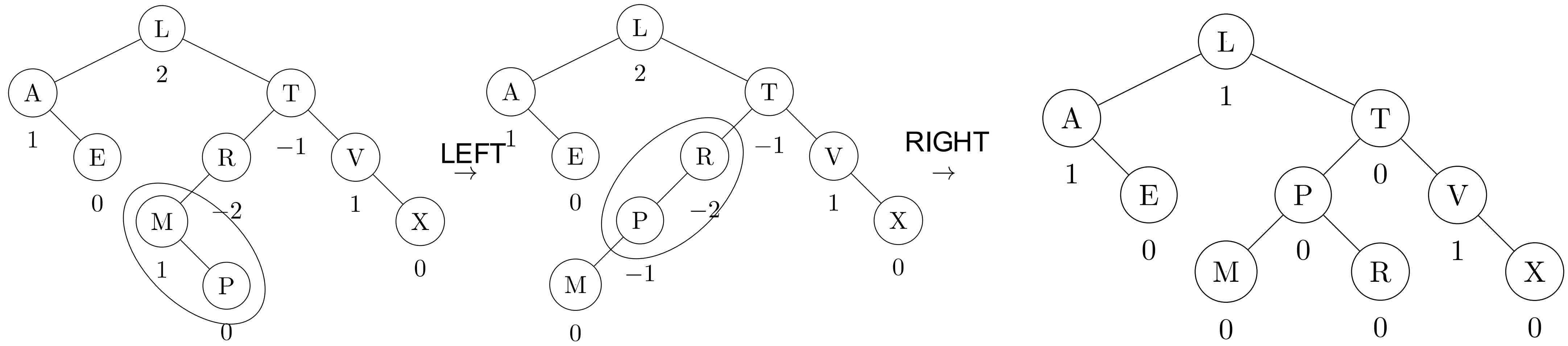
after A V L:



after T R:



after R E X M P

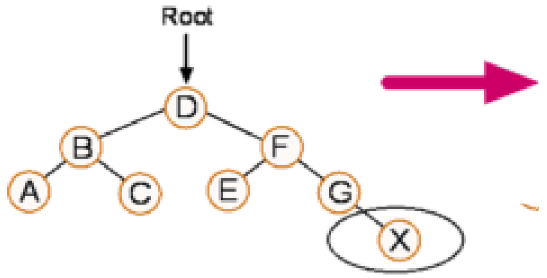


Deletion in BST

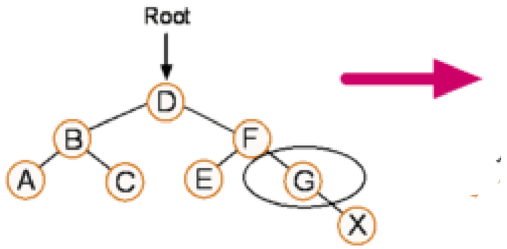
After deletion, do minimal work to keep the new tree valid, ie.:

- connected as a binary tree
- satisfying condition of a BST

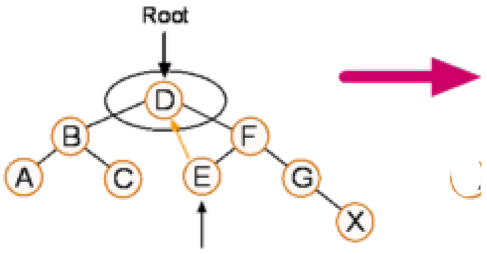
Case 1: delete a leaf node (X)



Case 2: delete node single-child node (G)

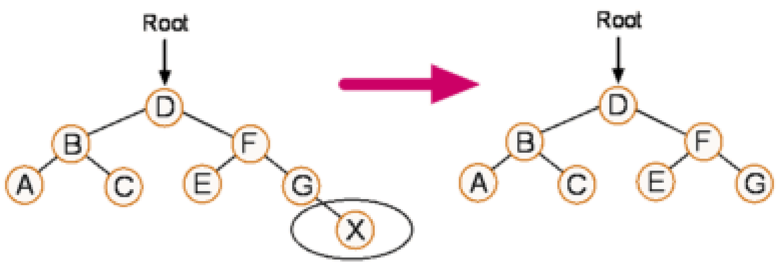


delete two-children node (D):

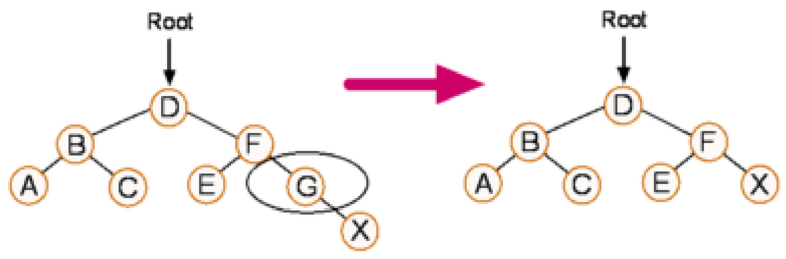


Check: Deletion in BST

Case 1: delete leaf node (X)
by removing it



Case 2: delete node single-child node (G)
by replacing it with its child, then delete the child

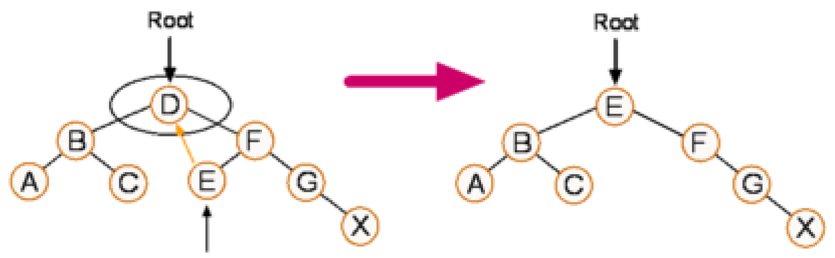


Case 3: To delete two-children node (D) : 2 options

- option 1: replace the node D with its in-order predecessor C
- option 2: replace the node D with its in-order successor E

Then, delete the original C (or E)

Note*: Case 1 and 2 are just special cases of case 3*

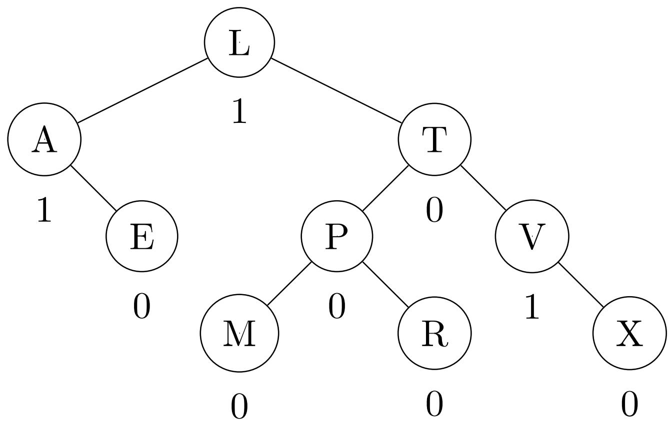


Deletion in AVL, Q11.4

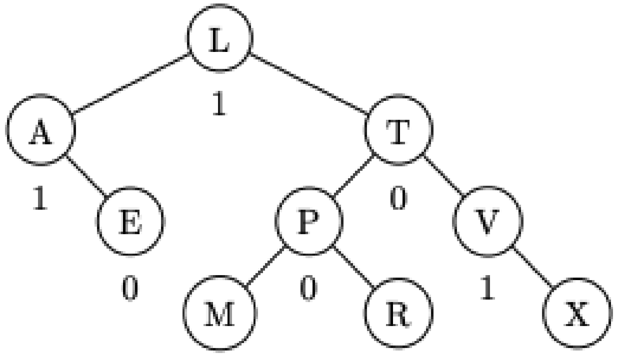
Same as deletion in BST, but two extra steps:

- Walk upwards starting from deleted node
- Rotate at each node if unbalanced.

Q11.4 (homework): Delete T, V, X, E in the previous AVL (for two-children node, swap with its in-order predecessor):

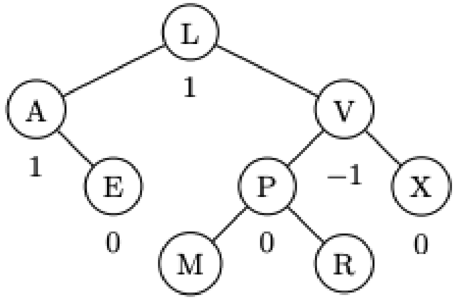


Q11.4: check your answer (delete T, V, X, E)



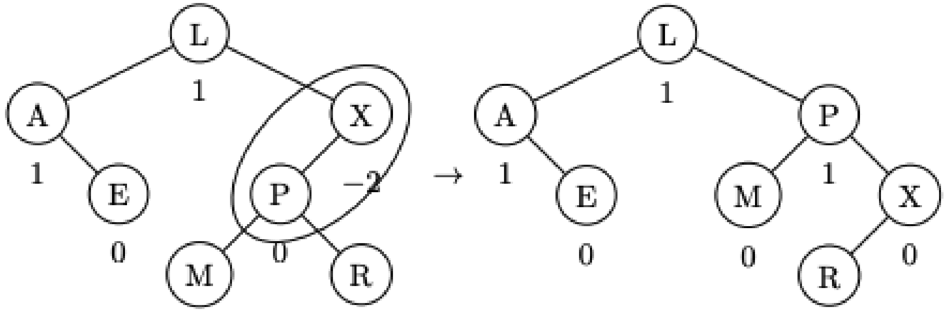
Delete T (having 2 children)

- swap T with V
- del renewed T (that has 1 child X) by replacing with X
- new tree is balanced!



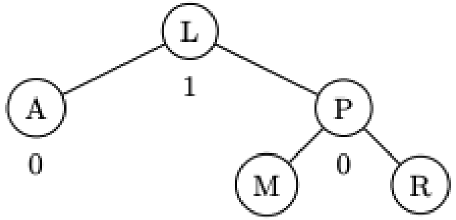
Delete V (having 2 children)

- swap V with X
- del renewed V which has no child
- need to re-balance X with a Left Rotation



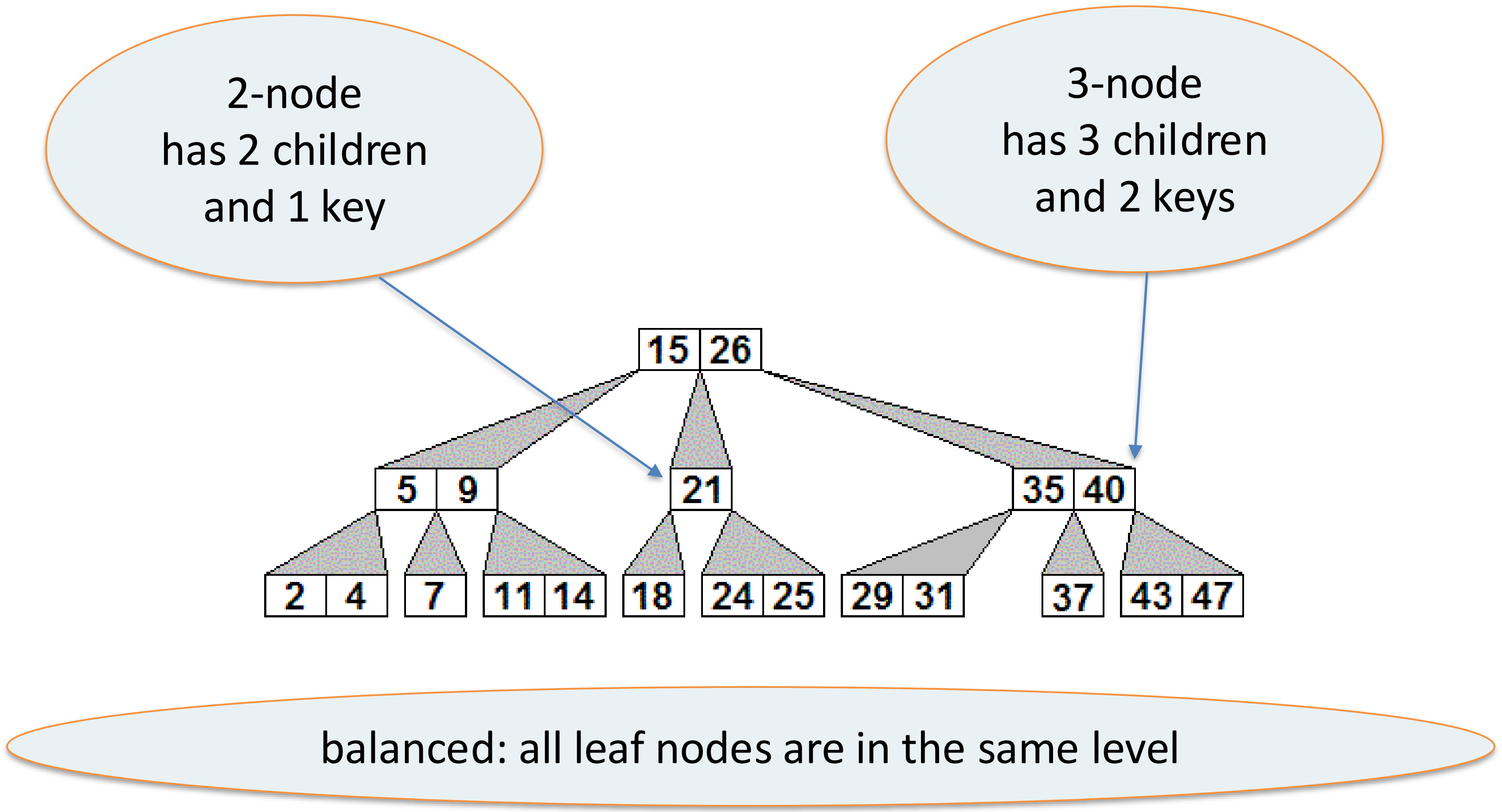
Delete X, and E (no child nodes)

- tree remains balanced after each deletion



2-3 Trees

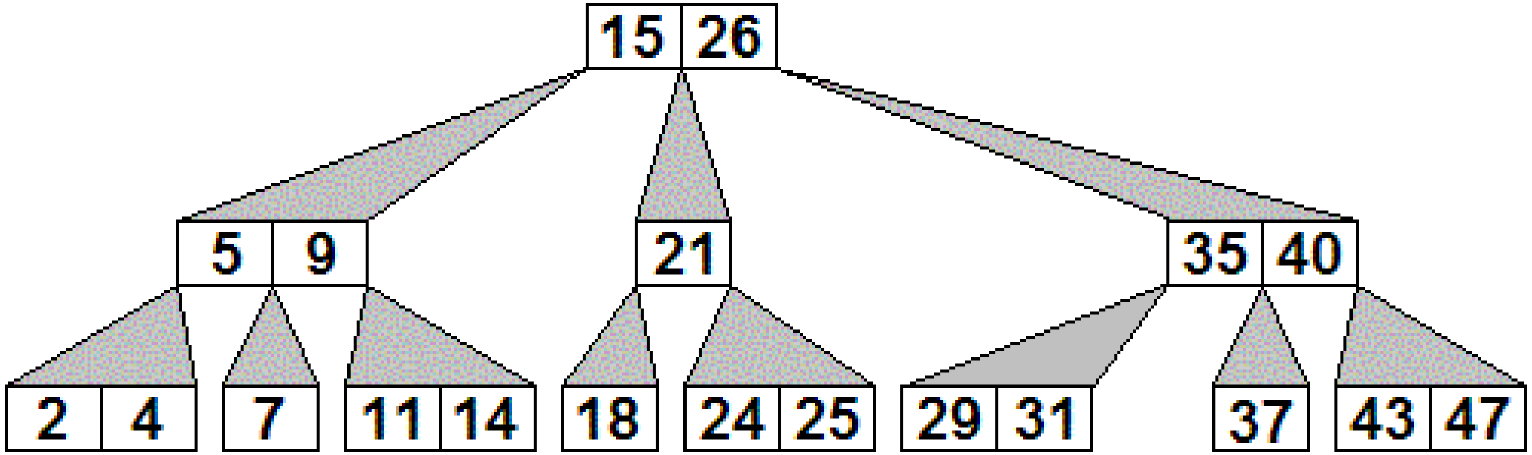
What? It's a search tree, but not a binary tree! Each node might have 1 or 2 keys/data, and hence 2 or 3 children.



2-3 Trees: Insertion

How to insert:

- start from root, go down and *insert new data to a* ***leaf node***
- if the new leaf has <=2 keys, it's good, done
- if the new leaf has 3 keys: promote the median key to the parent (the promoting might continue several levels upward)



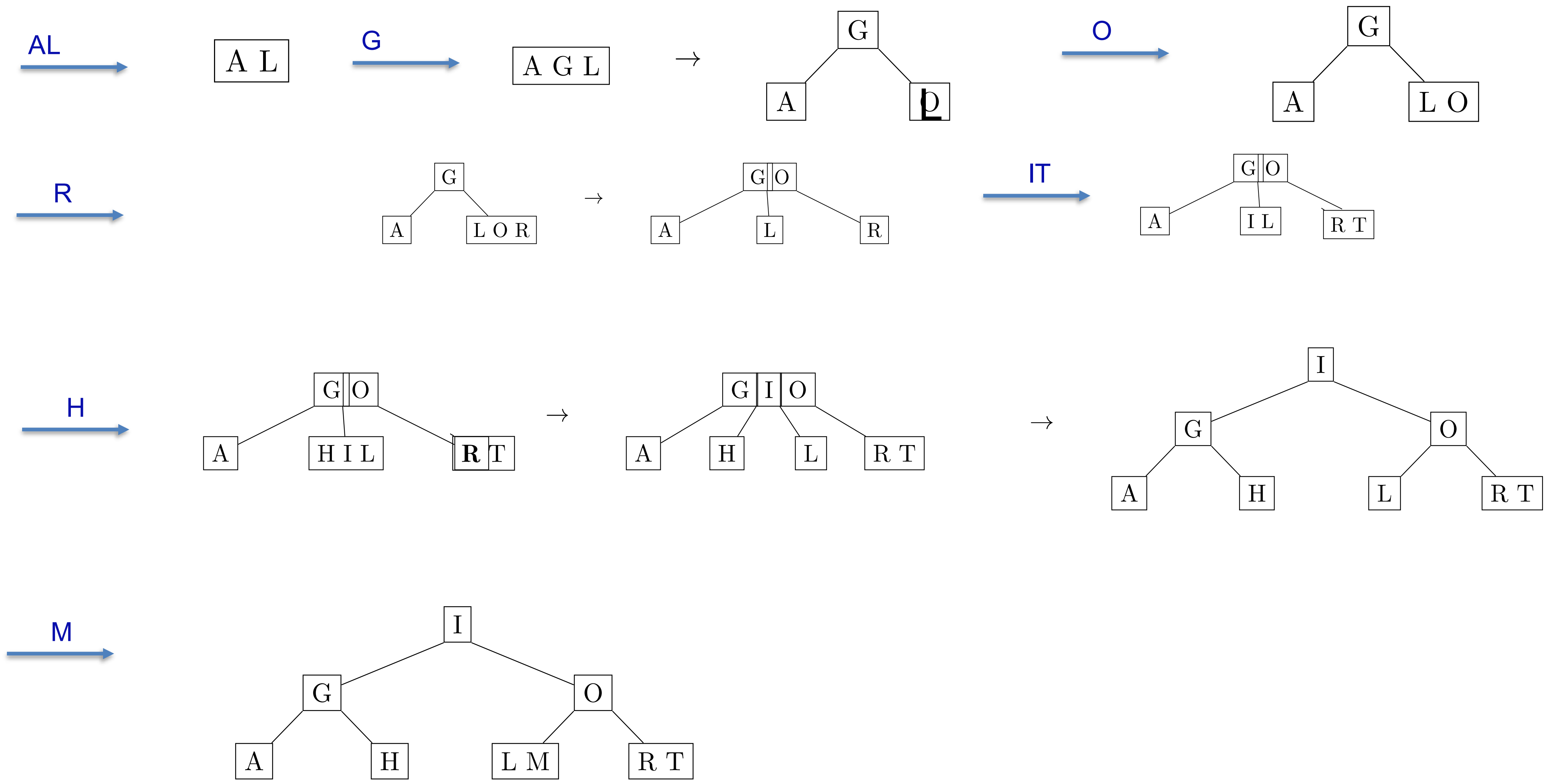
insert 8 is easy: node [7] become [7,8]
insert 45 make node [43,47] be [43,45,47]
→ promote 45, parent become [35,40,45]
→ promote 40, root become [15,26,40]
→ promote 26 to a new root

Insert the following keys into an initially-empty 2-3 Tree.
Class example:
20 10 5 15 30

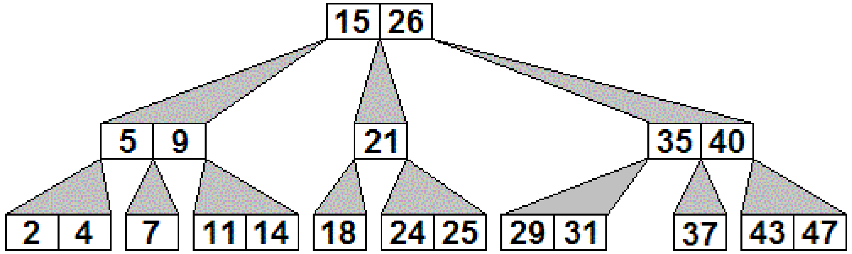
[group/individual] Q 11.5:
insert the keys into an initially empty 2-3 tree
A L G O R I T H M

Q 11.5: Check your solution

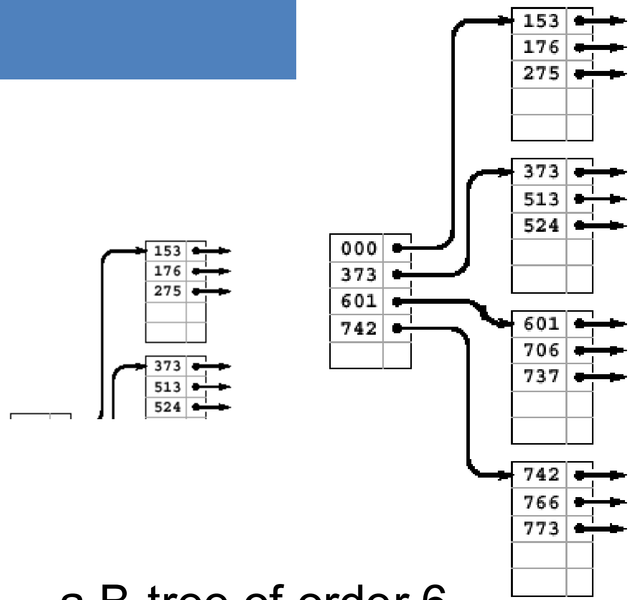
Insert the following keys into an initially-empty 2-3 Tree: A L G O R I T H M



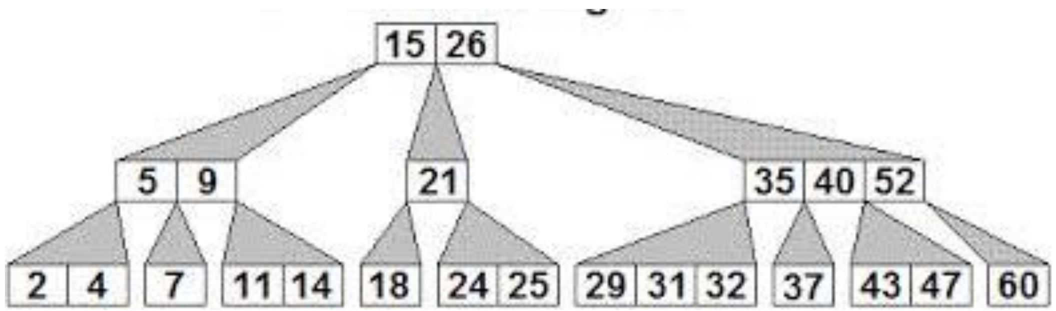
2-3 Trees, 2-3-4 Trees, B-Tree



2-3 trees= B-trees of order 3
(order= max number of children)



a B-tree of order 6



2-3-4 trees= B-trees of order 4

Image sources: ?? and http://anh.cs.luc.edu/363/notes/06DynamicDataStructures.html

B-tree principles

- Always insert at leaves
- When a node full: promote the median data to the node's parent [and walk up further if needed]

Additional Slides

Including:

- Deletion in 2-3 Trees
- Lab on BST insert

Deletion in 2-3 Trees

Keep the 2-3 property after deletion by

Step 1: If the deleted key not in a leaf node: swap it with the rightmost left key or the left most right key (similar to BST)

Step 2: Turn the deleted key into a "hole" and try to remove it. Stop if the removal is possible (=lucky). Otherwise repeat:

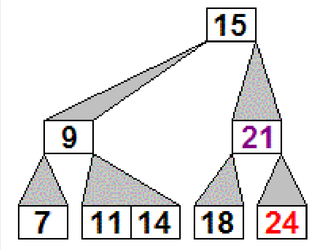
- "moving up" by swapping the hole with a valid parent key, merge the key's children into one node and promote the middle key to the hole if applicable

until:

- the new parent node is a valid 2-3 node: job done
- the new parent doesn't have any key, but is the root: remove the root

Note: Be cunning! If have more than 1 choices, choose the simpler one!

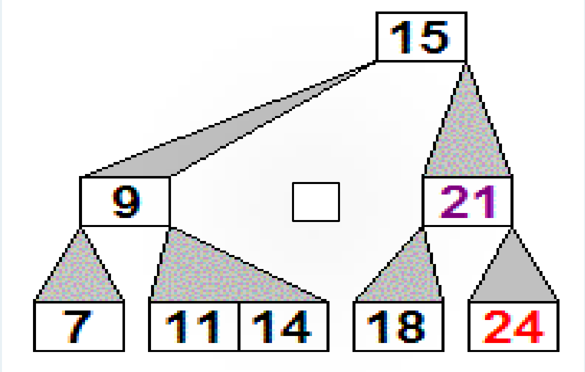
step 1:



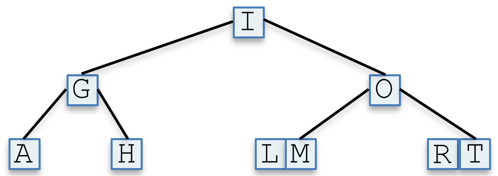
del 21: swap 21 with 18 or 24
del 15: swap 15 with 14 or 18

step 2:

HOLE at 11 or 14: lucky!
HOLE at 9: promote 11, lucky!
HOLE at 21: swap HOLE up to 15 …



Question 11.6: Delete `I`, then `L`, then `A` from the tree



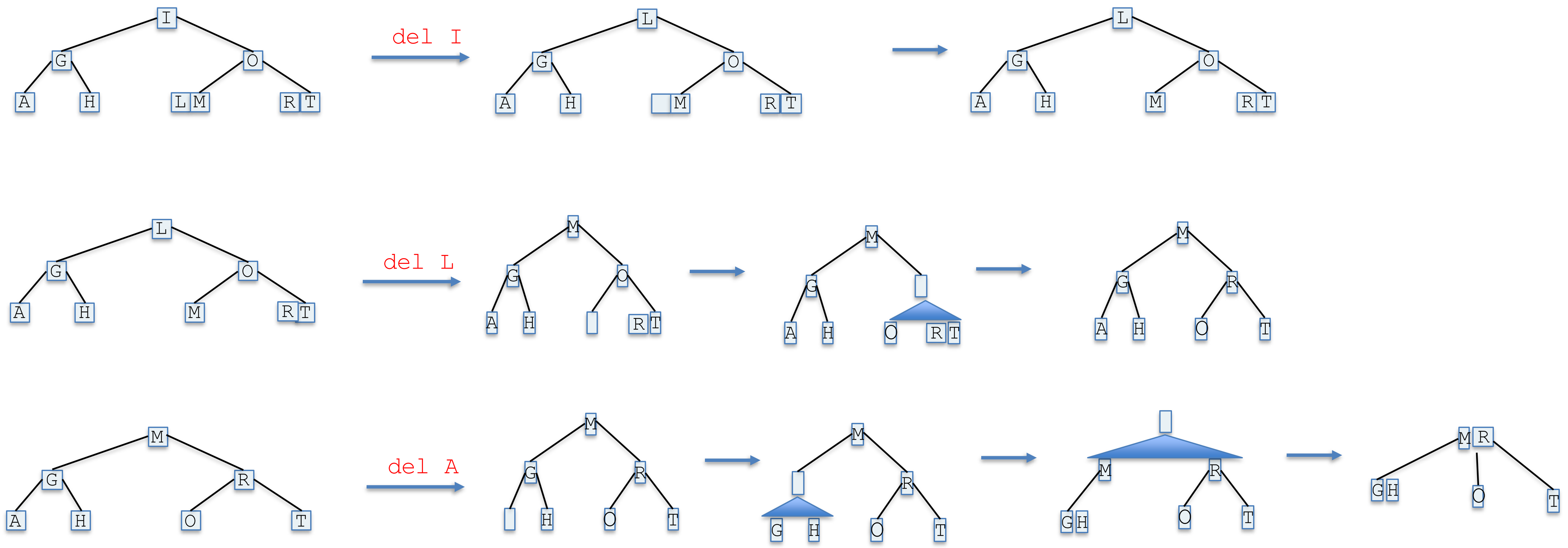
del I

del L

del A

Your notes:

Check Q 11.5: Delete I, then L, then A from the tree



Your notes:

Lab

Discuss: BST-insert

Assignment 2: Q&A

Q&A on previous week materials, revision, and/or A2

[Optional] Implement BST: insert, build tree from data, printing the tree. Note:

- Build your program from scratch
- But you can use the list and queue modules from previous weeks

Simple defs for binary trees

```
typedef struct treenode *tree_t;
struct treenode {
  int key;
  tree_t left, right;
} ;
```

```
tree_t insert(tree_t t, int key);
//OR
void insert(tree_t *t, int key);
```

Example of creating a tree

```
tree_t t= NULL;

//  insert 10 to tree t
t= insert(t, 10); //OR
insert(&t, 10);
//  depending on insert header
```