COMP20007 Workshop Week 11

LAB	**Preparation:** - *have draft papers and pen ready* - open `ws11.pptx`/pdf from `github.com/anhvir/c207` - open `a2_spec.pdf`, `wokshop11.pdf` (from `LMS`), and - download assignment 2 files from `LMS` Counting & Radix sort: Problems T1, T2, T3 Horspool's Algorithm: Problems T4, T5, T6 Assignment 2 Revision on demands: assignment 2

Counting Sort for array {5,2,3,2,1,0,3,1}

Counting Sort for sorting array A[0..n-1]: Review

Conditions:

small range of keys, for example, `A[i] <= k`, and `k= O(n)`

The algorithm:

- Build frequency array `F[0..k]` such that `F[i]`= frequency of key value `i`
- Convert `F[0..k]` so that `F[i]`= starting index of key value `i` in the *sorted* array
- Using another array `B[0..n-1]`, scan `A[]` again and copy to `B` using:

```
k= A[i];  // here C used for convenience
B[  F[k]++ ] = k;
```

Notes:

- Time complexity: $\Theta(n+k)$, or $\Theta(n)$ if k small
- Not in-place, additional memory: $\Theta(n+k)$, or $\Theta(n)$ if k small

Counting Sort & Radix Sort Exercises

1. Counting Sort Use counting sort to sort the following array of characters:

`[a, b, a, a, c, d, a, a, f, c, b]`

How much space is required if the array has n characters and our alphabet has k possible letters?

2. Radix Sort Use radix sort to sort the following strings:

`abc bab cba ccc bbb aac abb bac bcc cab aba`

As a reminder radix sort works on strings of length k by doing k passes of some other (stable) sorting algorithm, each pass sorting by the next most significant element in the string. For example in this case you would first sort by the 3rd character, then the 2nd character and then the 1st character.

3. Stable Counting Sort Which property is required to use counting sort to sort an array of tuples by only the first element, leaving the original order for tuples with the same first element. For example the input may be:

$(8, \texttt{campbell}),\ (6, \texttt{tal}),\ (3, \texttt{keir}),\ \ldots (6, \texttt{gus}),\ (0, \texttt{nick}),\ (8, \texttt{tom})$

Discuss how you would ensure that counting sort satisfies this property. Can you achieve this using only arrays? How about using auxiliarry linked data structures?

Counting Sort: Problem T1

Counting Sort Use counting sort to sort the following array of characters:

[a, b, a, a, c, d, a, a, f, c, b]

How much space is required if the array has n characters and our alphabet has k possible letters.

Radix Sort: Problem T2

Radix Sort: Use radix sort to sort the following strings:

```
abc bab cba ccc bbb aac abb bac bcc cab aba
```

As a reminder radix sort works on strings of length k by doing k passes of some other (stable) sorting algorithm, each pass sorting by the next most significant element in the string. For example in this case you would first sort by the 3rd character, then the 2nd character and then the 1st character.

Counting Sort: Problem T3

Stable Counting Sort: Which property is required to use counting sort to sort an array of tuples by only the first element, leaving the original order for tuples with the same first element. For example the input may be:

(8, campbell), (6, tal), (3, keir), . . . (6, gus), (0, nick), (8, tom)

Discuss how you would ensure that counting sort satisfies this property. Can you achieve this using only arrays? How about using auxiliarry linked data structures?

Horspool's Algorithm

The task: Seaching for a pattern `P` (such as "HELL" that has length m=5) in a text `T` (such as "SHE SELLS SEA SHELLS", having length n=20).

The Algorithm:

Stage 1: build `SHIFT[x]` for every possible character `x`, by:

```
P= HELL    → SHIFT(L)=  SHIFT(E)=  SHIFT(H) =
              SHIFT(any_other)=
```

Stage 2: searching

```
i=    0   4    8   12   16   20
      SHE SELLS SEA SHELLS
      SELL                    no match, c=' '
```

Horspool's Algorithm Review

The task: Seaching for a pattern `P` (such as "HELL" that has length m=5) in a text `T` (such as "SHE SELLS SEA SHELLS", having length n=20).

The Algorithm:

Stage 1: build `SHIFT[x]` for every possible character `x`, by:

1. first, set `SHIFT[x]= m` for all `x`, then
2. for each character `x` in `P`, except for the last one: `SHIFT(x)`= distance from the last character to last appearance of `x`

Stage 2: searching, by first set `i=m-1`, then

1. set `c= P[i]`, align `P` with `T` so that `P[m-1]` aligned with `T[i];`
2. compare character backwardly from the last character of `P` until the start or until finding the first mismatch:
3. if no mismatch found: return solution which is `i-m+1`
4. otherwise, set `i= i+ SHIFT[c]`, back to step 1 (note: use `c`)

Horspool's Algorithm: Problem T4

Use Horspool's algorithm to search for the pattern `GORE` in the string `ALGORITHM`.

Horspool's Algorithm: Problem T5

How many character comparisons will be made by Hor-spool's algorithm in searching for each of the following patterns it the binary text of one million zeros?

(a) 01001

(b) 00010

(c) 01111

Horspool's Algorithm: Problem T6

Horspool's Worst-Case Time Complextity: Using Horspool's method to search in a text of length n for a pattern of length m, what does a worst-case example look like?

Assignment 2
(for self-understanding)

- Due: 7PM Thursday 04/June
- Marks:
  - 14 for programming (6/a1 + 7/a2 +1)
  - 6 for writing (3/a3+3/a4)

Programming 2: How to read data

- You can use fgets as employed in lab files week 10 (ie. week 9 in LMS.Modules)
- Example of correct read data:

```
3   5
abc
cdefg
agh
```

```
scanf("%d %d ", &n, &x);
// the trailing space in the format
//    string is IMPORTANT
char s[258];
for (i=0; i<n; i++) {
  if (!fgets(s, 256, stdin)) break;
  int l= strlen(s);
  s[l-1]= '\0'; // get rid of last \n
  // copy s to store somewhere
}
n= i;
```

Programming 2.1

Note: Many things will be used in both question 1a) and 1b). Make sure to organise them into functions. Example:

- reading a set of n strings
- mapping a letter/digit to an int
- compute hash value for a string
- …?

Need to decide:

- what's the value of the hashtable's cells
- what's the SENTINEL for hashtable's cells

Programming 2.1: The Horner's Rule

if $S = c_0 c_1 \cdots c_{\ell-1}$ is a string of ℓ characters then the hash value

$$h_M(S) = \sum_{i=0}^{\ell-1} \text{chr}(c_i) \times 64^{\ell-1-i} \mod M$$

The Horner 's rule:

$h = 0$

$h = (h*64 + c_0) \ mod \ M$

$h = (h*64 + c_1) \ mod \ M$

$h = (h*64 + c_2) \ mod \ M$

...

$h = (h*64 + c_{l-1}) \ mod \ M$

2.1b) Notes

Collisions should be handled using linear probing with a step size of K.

How do we know that collision cannot be solved?

When a collision cannot be solved:

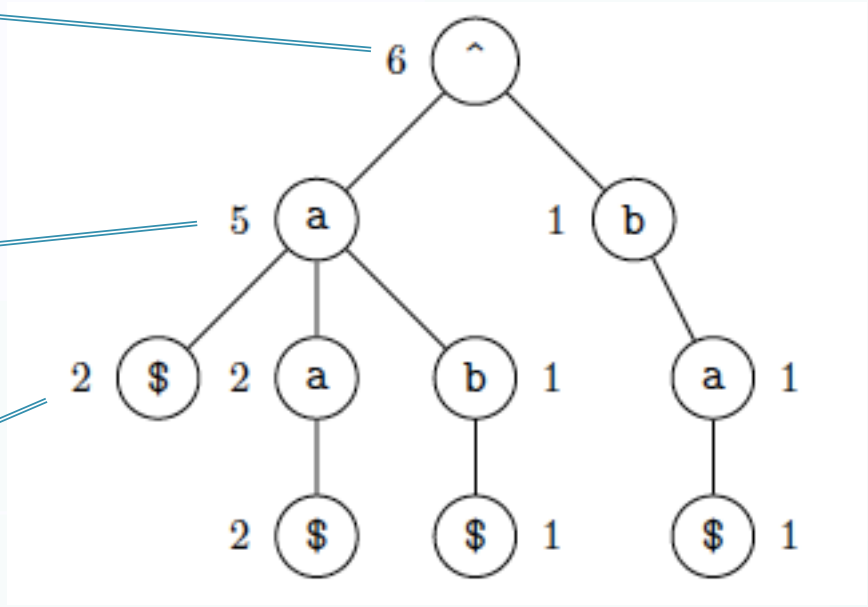
- double the size of the hash table, ie. M= 2M, then build new hash table with new size M
- rehash the strings already in the old hash table, these strings must be rehashed *in the order in which they appear in the old hash table*
- continue to hash the remaining strings

A2.2: The Described Trie

this trie contains 6 strings

there are 5 strings that share the prefix “a”

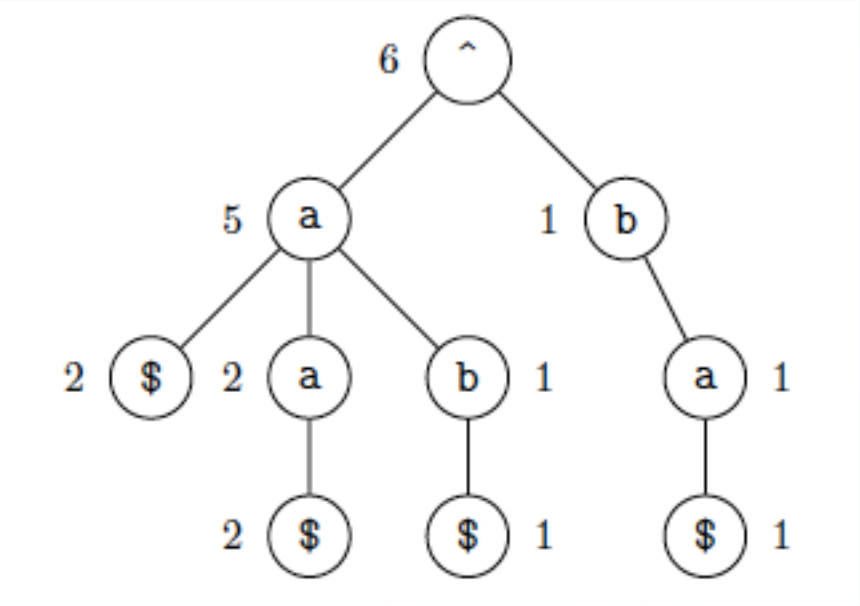
there are 2 strings “a”



Decide & Draw:

- *What does the **empty trie** look like?*
- *What the empty trie become after inserting just “a”?*
- *Can we insert an empty string into the trie?*

A2.2: How to represent?



Operations:

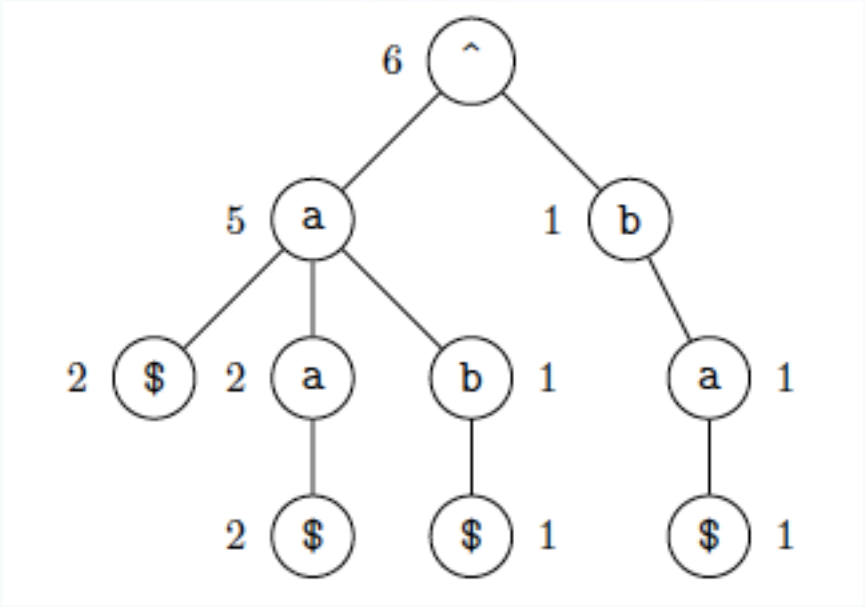
- *insert a string like "abc"*
- *search for string "abc" and get its frequency*
- *search for suffix "abc" and get its frequency*
- *…*

How to make difference between suffix and strings?

What a node should contains:

- a character itself?
- a frequency?
- pointers to children?
- …?

A2.2: How to insert?

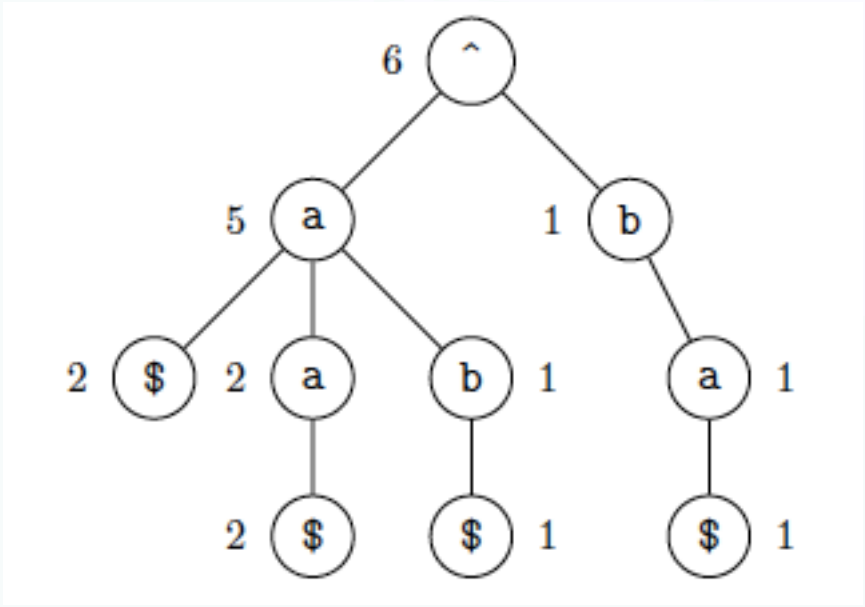


For the above trie, how to:

- insert "abc" ?

- insert "b"?

A2.2a: What to print?

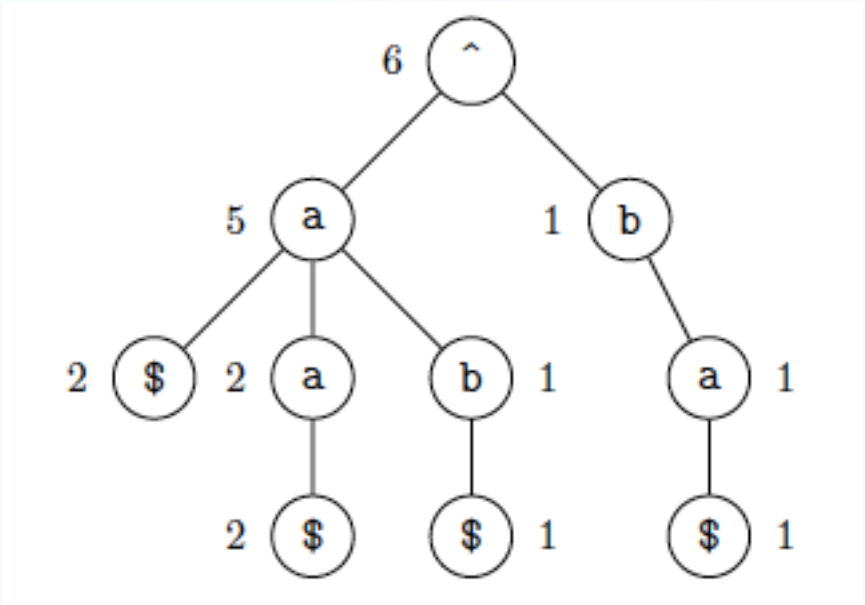


For the given trie:

```
^
a
$
a
$
b
$
b
a
$
```

- Check that the printout is the list of nodes in pre-order traversal
- How to implement that pre-order traversal?

A2.2b: How to print suffixes at level k?

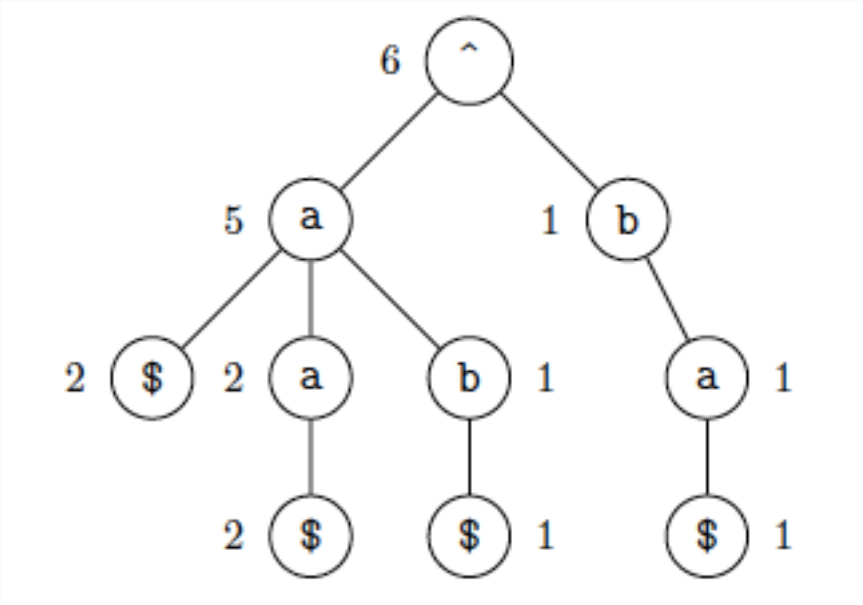


For the given trie and k=2:

```
aa 5
ab 1
ba 1
```

- Should we use (incomplete) BFS (level-order traversal)?
- Or, can we employ (incomplete) pre-order traversal?
- At the second ‘a’, how do we generate “aa”, then, how do we generate “ab” after that?

A2.2c: and the probability for autocompletion?



After typing (the suffix) `"a"` *the typist wants to finish the word as:*

`"a"` – with probability `2/5`

`"aa"` – with probability `2/5`

`"ab"` – with probability `1/5`

- How to compute the total frequencies?
- How to get all the related strings and their frequencies?
- How to print them in decreasing order of probability?

a2.3

The task is to *select a good sorting algorithm* for each case and *justify your choice*, in a maximum of 2 sentences.

Example case: an airplane factory wants to improve the embedded system in its airplanes. The system uses a range of sensors that constantly collect large amounts of data from wind currents outside the airplane. This data needs to be sorted as a preprocessing step to ease their visualisation by the pilot. The sorting algorithm should be completely in-place, as extra memory usage should be minimised in an embedded system. The algorithm should also have good performance even in the worst case, since it should be robust to hijacking. What algorithm would you use?

Solution: Heapsort, because it is in-place and has guaranteed $\Theta(n \log n)$ worst case performance, which makes it robust to adversarial attacks.

It's important to keep in mind that this is a subjective question and there might be more than one correct answer. Many real world scenarios do not have a single correct solution—this is why your argument about your chosen algorithm is important.

a2.3

The task is to *select a good sorting algorithm* for each case and *justify your choice*, in a maximum of 2 sentences.

Example case: an airplane factory wants to improve the embedded system in its airplanes. The system uses a range of sensors that *constantly collect large amounts of data* from wind currents outside the airplane. This data needs to be sorted as a preprocessing step to ease their visualisation by the pilot. The sorting algorithm should be completely *in-place*, as extra memory usage should be minimised in an embedded system. The algorithm should also have *good performance even in the worst case*, since it should be robust to hijacking. What algorithm would you use?

constantly collect large amounts of data: all data available right away

in-place: mergesort/distribution sorts not applicable

good performance: quicksort, heap sort?

good performance even in the worst case: no quicksort!

Solution: Heapsort, because it is in-place and has guaranteed Θ(n log n) worst case performance, which makes it robust to adversarial attacks.

a2.4

The task:

Revision Slides from Last Weeks (for individual review)

Review the topics in the remaining pages and ask questions if needed.

Revision 1: Complexity Analysis – 03.pdf & 04.pdf

$1 \prec \log n \prec n^{\varepsilon} \prec n^{c} \prec n^{\log n} \prec c^{n} \prec n^{n}$ where $0 < \varepsilon < 1 < c$

$(\log n)^{\alpha} \prec (\log n)^{\beta}$ and $n^{\alpha} \prec n^{\beta}$ where $0 < \alpha < \beta$

$O(f(n) + g(n)) = O(\max\{f(n), g(n)\})$
$O(c\, f(n)) = O(f(n))$
$O(f(n) \times g(n)) = O(f(n)) \times O(g(n))$

note: these 3 also applied to big-θ

$1+2+\ldots+n = n(n+1)/2 = \theta(n^2)$
$1^2 + 2^2 + \ldots + n^2 = n(n+1)(2n+1)/6 = \theta(n^3)$
$1 + x + x^2 + \ldots + x^n = (x^{n+1}-1)/(x-1) \quad (x \neq 1)$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \begin{cases} 0 & f(n) = O(g(n)) \\ c & f(n) = \theta(g(n)) \\ \infty & f(n) = \Omega(g(n)) \end{cases}$$

$$\lim_{n\to\infty} \frac{t(n)}{g(n)} = \lim_{n\to\infty} \frac{t'(n)}{g'(n)}$$

R1 exercises: Problem T5

For each of the following cases, indicate whether $f(n)$ is $O(g(n))$, or $\Omega(g(n))$, or both (that is, $\Theta(g(n))$)

(a) $f(n) = (n^3 + 1)^6$ and $g(n) = (n^6 + 1)^3$,

(b) $f(n) = 3^{3n}$ and $g(n) = 3^{2n}$,

(c) $f(n) = \sqrt{n}$ and $g(n) = 10n^{0.4}$,

(d) $f(n) = 2\log_2\{(n + 50)^5\}$ and $g(n) = (\log_e(n))^3$,

(e) $f(n) = (n^2 + 3)!$ and $g(n) = (2n + 3)!$,

(f) $f(n) = \sqrt{n^5}$ and $g(n) = n^3 + 20n^2$.

Other exercises: review exercises and solution for Workshop Week 3,

R2: Recurrences T(n)= ? T(<n) + $f(n)$ and T(1)=c

- Apply the Master Theorem (`10.pdf`) if possible (ie. if having a, b, and $\boldsymbol{\Theta}(n^d)$ or $O(n^d)$)
- Otherwise, using substitution to expand until T(1)

Note that we can easily make mistakes with substitution. *Do it step by step using draft paper, don't rush.*

Exercises: Solve the following recurrence relations. Give both a closed form expression in terms of n and a Big-Theta bound.

a) $T(n)= T(n/2)+1,\ T(1)= 1$

b) $T(n)= T(n-1) + n/5,\ T(0)= 0$

c) $T(n)= 3T(n-1) + 1,\ T(1)=1$

d) $T(n)= T(n/3) +1,\ T(1)= 1$

Other exercises: review exercises and solution for Workshop Week 3, Workshop Week 8 (master theorem)

R3: `05.pdf`: exhaustive string search, knapsack

- exhaustive string search = naïve search
- exhaustive knapsack = find all subsets of a set

Exercises:

Other exercises: review exercises and solution for Workshop Week 4.

R4: graphs

- `06.pdf`: graph concepts
- `07.pdf`: DFS and BFS, topological sort
- `08.pdf` : Prim & Dijkstra

Exercises:

Review exercises and solution in Workshops:

- graphs concepts: Workshop Week 4
- DFS, BFS: Workshop Week 5 [Week 5 according to the numbering in our subject's LMS.Modules, and is week 6 in uni's calendar)]
- Topological Sort: Workshop Week 6
- Prim & Dijkstra: Workshop Week 5, Week 6

R5: Sorting algorithms

- `11.pdf`: Sorting algorithm properties, *insertion sort* & *selection sort*
- `12.pdf`: *top-down mergesort*, *quicksort with Lomuto partitioning*, *quicksort with Hoare partitioning*

Exercises: For each of the above 5 algorithms:

- is it input-sensitive, in-place, stable? What's the complexity? Best case and worst case?
- Show how it works on:

`EXAMPLE`

Other exercises: review exercises and solution for Workshop Week 7.

R6: Binary Heap (13.pdf)

- Binary Heap: complexity of insertion, removeMin, heapify
- Heap Sort and its complexity

Exercises:

- Show how to insert into an originally-empty min-heap:

EXAMPLE

Other exercises: review exercises and solution for Workshop Week 8.

R7: Search Trees (14.pdf)

- BST and AVL
- 2-3 Tree

Exercises: For each of the above 2 types of search trees:

- What is the complexity of insertion, of search?
- Perform the insertion into originally-empty tree:

  TREBALNCD

Other exercises: review exercises and solution for Workshops:

Binary Trees & BST: Workshop Week 6

AVL & 2-3 Trees: Workshop Week 8

R8: Hashing

- `15.pdf`: Hashing.

Exercises: Workshop Week 9

R9: Huffman Coding

- `16.pdf`: Coding and Huffman Coding

Exercises: Workshop Week 9.

Any topics missing in our revision list?

Of course, there is no guarantee that the list is complete. You can fill in things like:

- stacks and queues, arrays and linked lists [Workshop week 2]
- priority queues?

Notes: The exercises provided in this file are similar to some workshop exercises, and their main purpose is to highlight the concepts provided in the lectures. The exercises do not necessarily represent all kinds of exam/test questions.