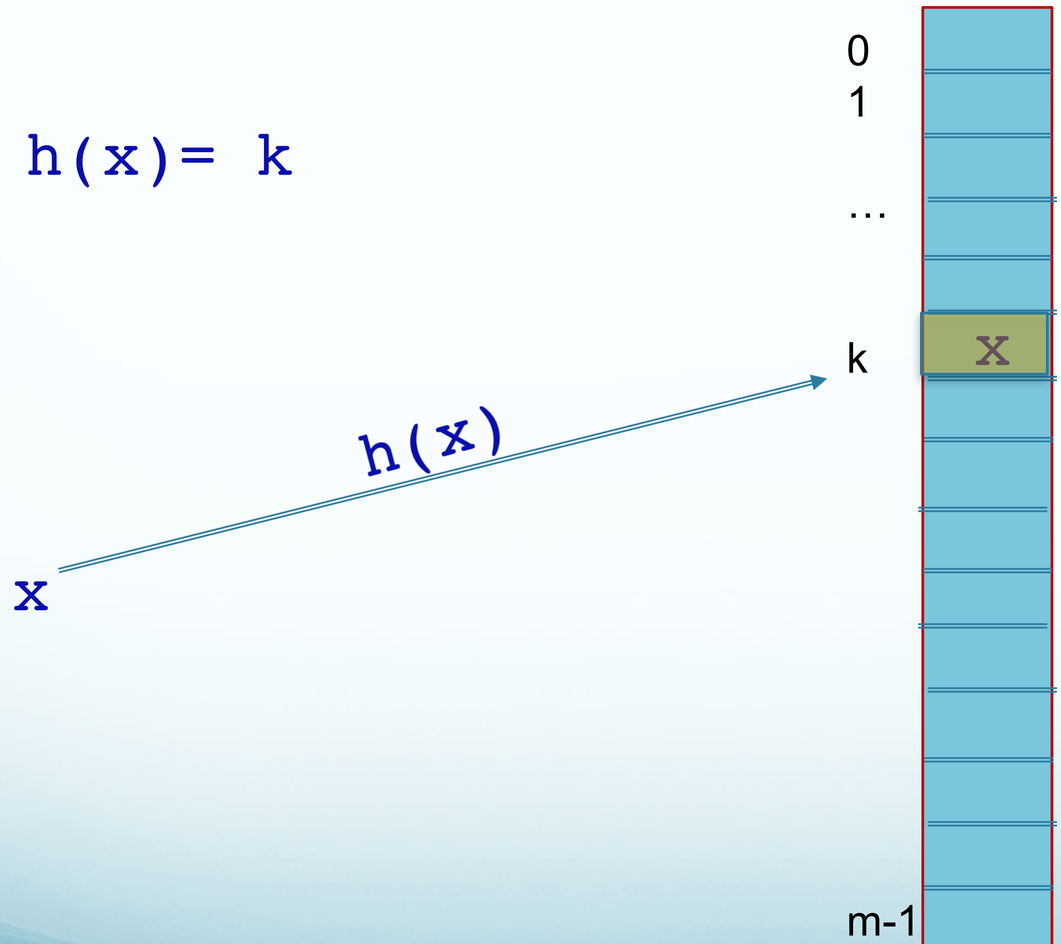
COMP20007 Workshop Week 9

	Preparation:
	- *have draft papers and pen ready, or ready to work on whiteboard*
	- open `wokshop10.pdf` (from `LMS`), and
	- download lab files from `LMS`
1	Hashing: Problems T1, T2
2	Huffman Coding: Problems T3, T4
3	Revision on demands:
	Complexity (problem T5)
	Solving Recurrences
	and others
LAB	Lab: playing with hashing code

Hashing: dictionary with O(1) search/insert

- *Hashing*= hash table `T[m]` + hash functions `h(x)`: store key `x` in `T`, at position `h(x)`



Example:
storing a list of 800 student records, each student has a unique student number in the range of:

1. 1..999
2. 7001..7999
3. 100..200000

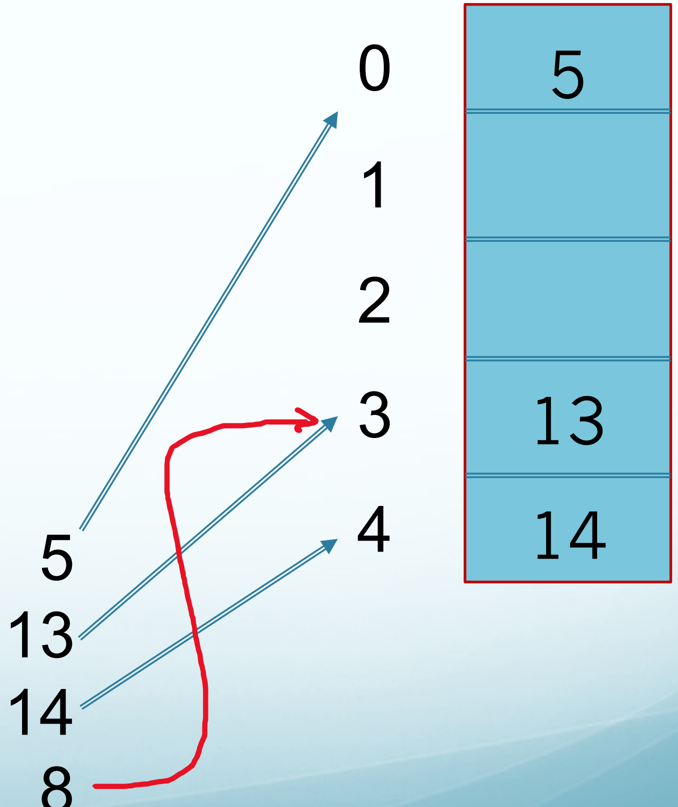
h(x)= ?, m=?

Collisions

- `h(x1) = h(x2)` for some `x1≠ x2`.
- Collisions are normally unavoidable.
- Example with student numbers: `m= 997` (a prime number >800) , `x1= 998, x2= 9971`

Collisions

Example:
`m=5`, `h(x)= x% m`
Here: `h(8)= h(5)`



One method *to reduce collisions* using *a prime number* for hash table size `m`.

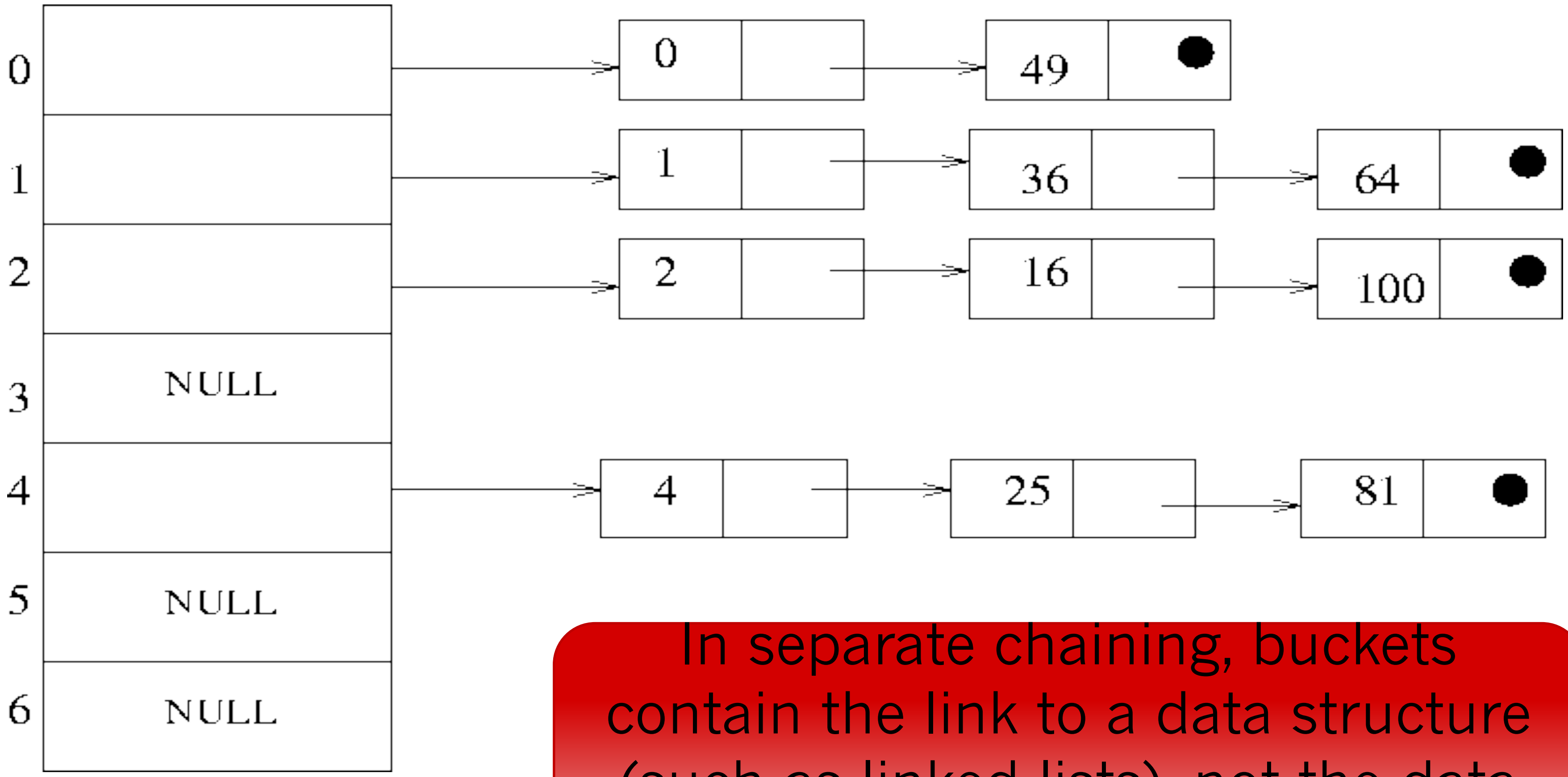
Another method is to make the table size `m` *big* enough (but that affects space efficiency).

Even though, collisions might still happen

Collision Solution 1: Separate Chaining

`h(x) = x % 7,` keys entered in decreasing order:

`100, 81, 64, 49, 36, 25, 16, 4,2,1,0`



In separate chaining, buckets contain the link to a data structure (such as linked lists), not the data themselves.

Solution 2: Linear Probing (here, data are in buckets)

When inserting we do some probes until getting a vacant slot. H(x,probe) can be summarized as:

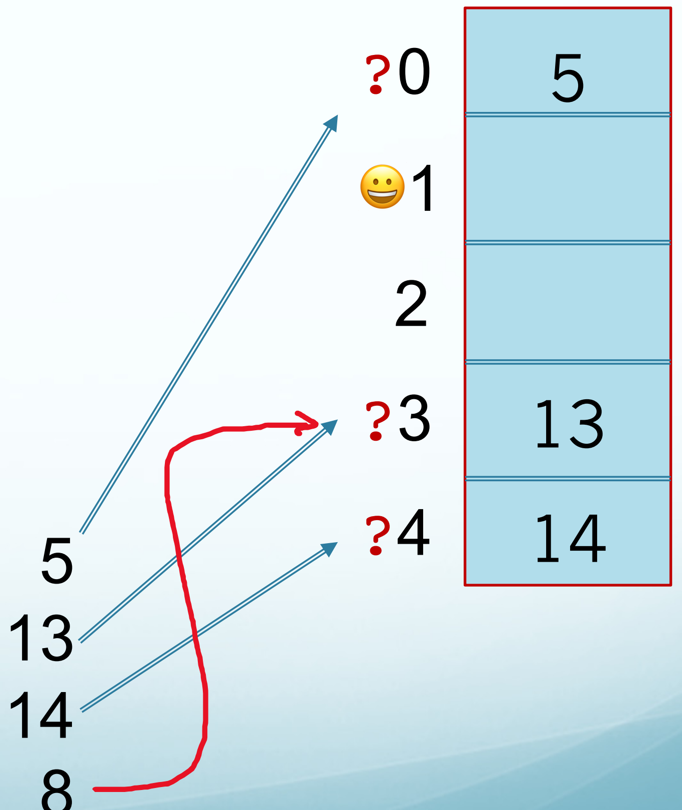
```
h(x) → H(x, probe) =
      ( h(x) + probe ) mod m
```

where m `probe` is `0, 1, 2 …` (until reaching a vacant slot).

Example: `m=5, h(x)= x mod m,`

Notes:

- The same procedure for search
- Deletion is problematic!



Double hashing

Double hashing is similar to *linear probing*, but employ a second hash function h2(x):

`H(x, probe) = ( h(x)+ probe*h2(x) ) mod m`

where `probe` is `0, 1, 2, …` (until reaching a vacant slot). Note that:

- `h2(x) ≠ 0 for all x`, (why?)
- to be good, `h2(x)` should be co-prime with `m`, (how?)
- *linear probing* is just a special case of *double hashing* when `h2(x)=1`.

Problem 1&2 [Group/Individual]: Separate chaining

Problem 1: Consider a hash table in which the elements inserted into each slot are stored in a linked list. The table has a fixed number of slots `L=2`. The hash function to be used is `h(k)=k mod L`.

a) Show the hash table after insertion of records with the keys
   `17 6 11 21 12 33 5 23 1 8 9`

b) Can you think of a better data structure to use for storing the records that overflow each slot?

Problem 2: Consider a hash table in which each slot can hold one record and additional records are stored elsewhere in the table using linear probing with steps of size `i=1`. The table has a fixed number of slots `L=8`. The hash function to be used is `h(k)=k mod L`.

a) Show the hash table after insertion of records with the keys
   `17 7 11 33 12 18 9`

b) Repeat using linear probing with steps of size `i = 2`. What problem arises, and what constraints can we place on `i` and `L` to prevent it?

c) Can you think of a better way to find somewhere else in the table to store overflows?

Problem 1 [Group/Individual]: Separate chaining

Consider a hash table in which the elements inserted into each slot are stored in a linked list. The table has a fixed number of slots `L=2`. The hash function to be used is `h(k)=k mod L`.

a) Show the hash table after insertion of records with the keys
`17 6 11 21 12 33 5 23 1 8 9`

b) Can you think of a better data structure to use for storing the records that overflow each slot?

Problem 2[Group/Individual]: Open addressing

Consider a hash table in which each slot can hold one record and additional records are stored elsewhere in the table using linear probing with steps of size `i=1`. The table has a fixed number of slots `L=8`. The hash function to be used is `h(k)=k mod L`.

a) Show the hash table after insertion of records with the keys
`17 7 11 33 12 18 9`

b) Repeat using linear probing with steps of size `i = 2`. What problem arises, and what constraints can we place on `i` and `L` to prevent it?

c) Can you think of a better way to find somewhere else in the table to store overflows?

Huffman's Coding & Data Compression

The task:

Input: a message T such as `that cat, that bat, that hat` over some alphabet

Output: an encoded message T' – an efficient storage of T with the guarantee that T can be reproduced from T'. For example

T'= 111000111101000101111…

Principle: Use less number of bits (shorter codeword) for symbol that appears more frequently.

Input message: `that cat, that bat, that hat`

alphabet= [`a` `b` `c` `h` `t` `,` ` `] (or perhaps all ASCII characters)

How to compress:

1.Modeling: making assumptions about the structure of messages

`that cat, that bat, that hat` (character model)

`that cat, that bat, that hat` (word model)

`that cat, that bat, that hat` (bi-character model)

2. Statistics: find symbol distribution

3. Coding: build the *code table* and do *encoding*

Input message: `that cat, that bat, that hat`

alphabet= [`a` `b` `c` `h` `t` `,` ` `] (or perhaps all ASCII characters)

1. Modeling: making assumptions about the structure of messages

`that cat, that bat, that hat` (character model)

2. Statistics: build table of frequencies, aka weight table. For the character model:

a	b	c	h	t	,	_
6/28	1/28	1/28	4/28	9/28	2/28	5/28

or just

a	b	c	h	t	,	_
6	1	1	4	9	2	5

3. Coding: build the *code table* and do *encoding*

a	b	c	h	t	,	_
01	0000	0001	100	11	001	101

→ 1110001111010001011...

Huffman Coding = a method for building code

Build Huffman code:

- make a node for each weight
- join 2 *smallest weights* and make a parent node (of binary tree), continue until having a single root
- for each node, assign 0- and 1-bit for 2 associated edges

Example:

a	b	c	h	t	,	_
6	1	1	4	9	2	5

Huffman Coding

Note: there are different versions of Huffman code for a same weight table (depending on dealing with ties, assigning 0- and 1-bits), all we need to do is to choose a way and keep consistency. For instance (*canonical Huffman's coding*):

- when joining 2 weights into one, always make the smaller weight be the left child (hence, need to always keeps current roots in weight ordering)
- choose a consistent way for breaking ties
- when assigning code, always set 0 to the left edge, 1 to the right edge

Additional notes
• When sending encoded messages, the sender also need to send the weight table (or something equivalent). • It's important that the receiver/decoder builds the code in the same way as the sender/coder does.

Problem 3: Huffman Code Generation

Huffman's Algorithm generates prefix-free code trees for a given set of symbol frequencies. Using these algorithms generate two code trees based on the frequencies in the following message:

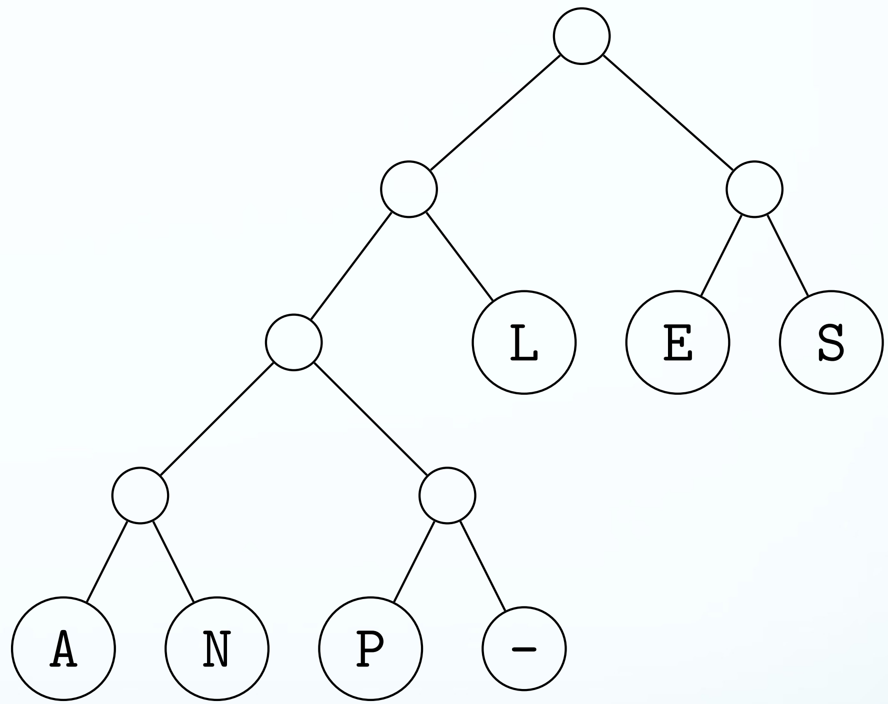
`losslesscodes`

What is the total length of the compressed message using the Huffman code?

Problem 4: Canonical Huffman decoding

The code tree was generated using Huffman's algorithm, and converted into a Canonical Huffman code tree. Note: _ denotes space.

Assign codewords to the symbols in the tree, such that left branches are denoted 0 and right branches are denoted 1.
Use the resulting code to decompress the following message:



001001100000011100011011011110011110110100010011011110001101111

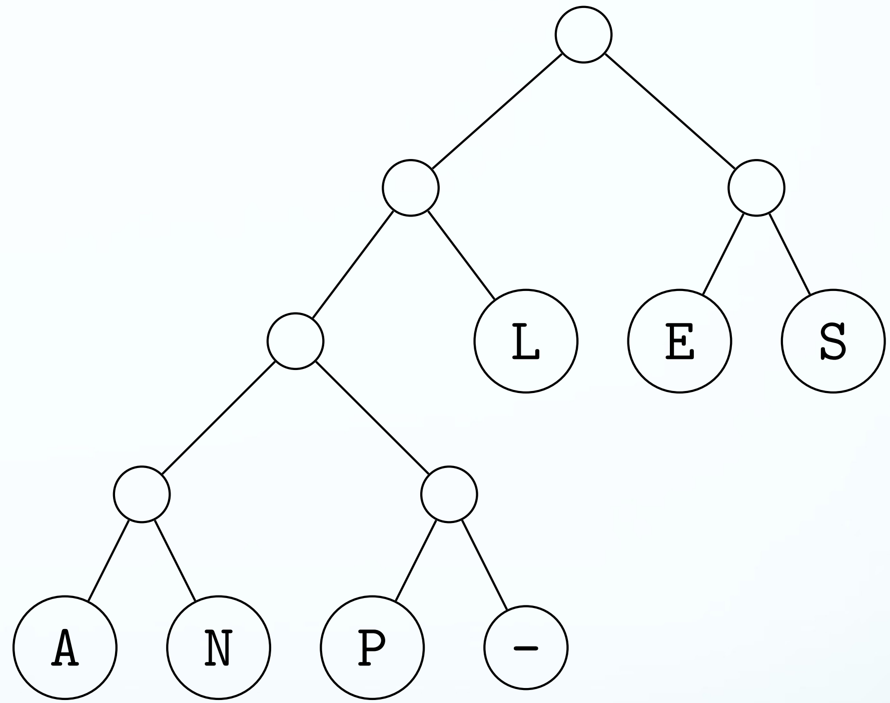
Problem 3&4: Huffman Code Generation

Problem 3: Huffman's Algorithm generates prefix-free code trees for a given set of symbol frequencies. Using these algorithms generate two code trees based on the frequencies in the following message:

`losslesscodes`

What is the total length of the compressed message using the Huffman code?

Problem 4: the code tree



Problem 4: Decode:

00100110000011100011011011110011110110100010011011110001101111

Revision 1: Complexity Analysis – `03.pdf` & `04.pdf`

$1 \prec \log n \prec n^{\varepsilon} \prec n^{c} \prec n^{\log n} \prec c^{n} \prec n^{n}$ where $0 < \varepsilon < 1 < c$

$O(f(n) + g(n)) = O(\max\{f(n), g(n)\})$
$O(c\, f(n)) = O(f(n))$
$O(f(n) \times g(n)) = O(f(n)) \times O(g(n))$

note: these 3 also applied to big-Θ

$1+2+ \ldots + n = n(n+1)/2 = \Theta(n^2)$
$1^2 + 2^2 + \ldots + n^2 = n(n+1)(2n+1)/6 = \Theta(n^3)$
$1 + x + x^2 + \ldots + x^n = (x^{n+1}-1)/(x-1) \quad (x \neq 1)$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \begin{cases} 0 & f(n) = O(g(n)) \\ c & f(n) = \Theta(g(n)) \\ \infty & f(n) = \Omega(g(n)) \end{cases}$$

$$\lim_{n\to\infty} \frac{t(n)}{g(n)} = \lim_{n\to\infty} \frac{t'(n)}{g'(n)}$$

Revision exercises: Problem 5

For each of the following cases, indicate whether $f(n)$ is $O(g(n))$, or $\Omega(g(n))$, or both (that is, $\Theta(g(n))$)

(a) $f(n) = (n^3+1)^6$ and $g(n) = (n^6+1)^3$,

(b) $f(n) = 3^{3n}$ and $g(n) = 3^{2n}$,

(c) $f(n) = \sqrt{n}$ and $g(n) = 10n^{0.4}$,

(d) $f(n) = 2\log_2\{(n+50)^5\}$ and $g(n) = (\log_e(n))^3$,

(e) $f(n) = (n^2+3)!$ and $g(n) = (2n+3)!$,

(f) $f(n) = \sqrt{n^5}$ and $g(n) = n^3 + 20n^2$.

Other exercises: review exercises and solution for Workshop Week 3,

Lab

Download `lab_files.zip`, unzip it, and “play” with the hashing code by following the instructions in the `workshop10.pdf` sheet.

and/or continue with reviewing.