Discrete Mathematics - MAD101

Assignment 2

SUMMER 2024 **Date: 6/17/2024**

Question 1: Devise an algorithm that finds the sum of all the integers in a finite list.

Question 2: List all the steps used to search for 9 in the list: $1, 2, 4, 5, 7, 8, 9, 16$ using a linear search and a binary search.

Question 3: Use the greedy algorithm to make change using quarters, dimes, nickels and pennies for 1016 cents.

Question 4: Find the least integer n such that $f(x)$ is $\mathcal{O}(x^n)$ for each of these functions:

$a) f(x) = \dfrac{x^4 + 3x^2 - 2x + 1}{x^2(\log x)^5 + 2x + 1}$ $b) f(x) = 2^x.$

Question 5: Find the number of operations (additions and multiplications) used in this segment of an algorithm:

$t := 0$
for $i := 1$ **to** 3
  for $j := 1$ **to** 4
    $t := t + ij$

Question 6: Find the largest n for which one can solve within one second a problem using an algorithm that requires $f(n)$ bit operations, where each bit operation is carried out in 10^{-9} seconds, with these functions $f(n)$?

$a) f(n) = \log n$ $b) f(n) = 2n^2$

Question 7: How much time does an algorithm using 2^{50} operations need if each operation takes these amounts of time?

$a) 10^{-6}$ $b) 10^{-12}$

Question 8: Find a **div** m and a **mod** m if $a = -116$ and $m = 6$.

Question 9: Find the integer a such that

$$a \equiv 43 (mod\ 23), \quad -22 \leq a \leq 0.$$

Question 10: Find x_2 if $x_0 = 3$ and

$$x_{n+1} = (4x_n + 1) \textbf{ mod } 7, \quad n \geq 0.$$

Question 11: Determine whether the following set of integers is pairwise relatively prime: $8, 9, 11, 13, 16$.

Question 12: Determine $gcd(1000, 625)$ and $lcm(1000, 625)$.

Question 13: Find the base 5 expansion of $(1289)_{10}$.

Question 14: How many divisions are required to find $gcd(22, 63)$ using the Euclidean algorithm.

Question 15*: Use mathematical induction to prove that

$$1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}, \quad n \geq 1.$$

Question 16: Let $P(n)$ be the statement that a postage of n cents can be formed using just 3-cent stamps and 5-cent stamps. Show that the statements $P(8)$, $P(9)$ and $P(10)$ are true.

Question 17: Find $f(1)$, $f(2)$ and $f(3)$ if $f(0) = 2$ and

$$f(n+1) = f(n)^2 - 2f(n) + 2, \quad n \geq 0.$$

Question 18: Let S be a set defined recursively by

Basis step: $0 \in S$

Recursive step: If $x \in S$, then $x + 5 \in S$ and $x - 5 \in S$.

Determine S?

Question 19: Consider the following algorithm for computing the Fibonacci numbers (see Section 5.4 in the student textbook). Find the amount of additions required to compute f_6?

ALGORITHM 7 A Recursive Algorithm for Fibonacci Numbers.

procedure *fibonacci*(n: nonnegative integer)
if $n = 0$ **then return** 0
else if $n = 1$ **then return** 1
else return *fibonacci*$(n-1)$ + *fibonacci*$(n-2)$
{output is *fibonacci*(n)}

Question 20: How many positive integers between 100 and 999 inclusive:

a) are divisible by 5 or 7?

b) are divisible by 4 but not 6.

Question 21: How many bit strings of length 10 either begin with two 0s or end with four 1s.

Question 22: A person deposits \$1000 in an account yields 10% interest compounded annually. How much money will the account contain after 50 years?

Question 23: Suppose that $f(n) = 2f(n/5) + n^2$ and $f(1) = 2$. Find $f(125)$?

Question 24: How many one-to-one functions are there from a set with 4 elements to a set with 8 elements.

Question 25: A young pair of rabbits (one of each sex) is placed on an island. A pair of rabbits does not breed until they are 2 month old. After they are 2 month old they will produce 4 pairs of rabbits each month. Find the number of pairs of rabbits after 5 months.