

MAE101 – PT2 QUESTIONS  
NOTE: REAL EXAM HAS ONLY 15 QUESTIONS!

**Câu 1.** Let  $A = \begin{bmatrix} 1 & 0 & 1 \\ k & 1 & k \\ 3 & k & 2 \end{bmatrix}$ . Find the (2,1)-entry of the inverse matrix  $A^{-1}$ .

- A. 0
- B. 1
- C. None of the other choices is correct
- D. -k
- E. -1

**Câu 2.** Given that  $\lambda = 9$  is an eigenvalue for the matrix

$$\begin{bmatrix} 5 & 2 & -2 \\ 2 & 5 & -2 \\ -2 & -2 & 5 \end{bmatrix}$$

Find basic eigenvectors corresponding to  $\lambda = 9$ .

- (i)  $[1 \ 1 \ -1]^T$                       (ii)  $[1 \ -1 \ -1]^T$   
(iii)  $[1 \ 1 \ 1]^T$  and  $[0 \ 1 \ 1]^T$                       (iv)  $[1 \ 1 \ -1]^T$  and  $[1 \ 0 \ 1]^T$

- A. (iii)
- B. (i)
- C. None of the other choices is correct
- D. (ii)
- E. (iv)

**Câu 3.** Find x so that the matrix  $\begin{bmatrix} -x & x+1 & x-1 \\ 0 & x^2 & x \end{bmatrix}$  is in reduced row-echelon form.

- A. 0
- B. -1
- C. None of the other choices is correct
- D. 0 or -1
- E. 1 or -1

**Câu 4.** Let  $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$ . Find AB.

- (i)  $\begin{pmatrix} 7 & 1 \\ 4 & 1 \end{pmatrix}$                       (ii)  $\begin{pmatrix} 1 & 7 \\ 1 & 4 \end{pmatrix}$                       (iii)  $\begin{pmatrix} 1 & 1 \\ 7 & 4 \end{pmatrix}$

- A. (ii)
- B. (iii)
- C. (i)
- D. None of the other choices is correct

**Câu 5.** Find all values of  $a$  such that the following system has nontrivial solutions

$$\begin{cases} x + 2y + z = 0 \\ x + 3y + 6z = 0 \\ 2x + 3y + az = 0 \end{cases}$$

- A. -4
- B. All of the other choices are incorrect
- C. All numbers but -3
- D. -3
- E. All numbers but -4

**Câu 6.** Rewrite the matrix equation as a system of linear equations

$$\begin{bmatrix} 14 & 5 \\ 5 & 17 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 7 \end{bmatrix}$$

- A. None of the other choices is correct
- B.  $14x + 5y = -11$ ,  $5x + 17y = -7$
- C.  $14x + 5y = 11$ ,  $17x + 5y = 7$
- D.  $14x + 5y = 11$ ,  $5x + 17y = 7$
- E.  $5x + 14y = 11$ ,  $5x + 17y = 7$

**Câu 7.** John and Joe earn a total of \$34 when John works 2 hours and Joe works 3 hours. If John works 3 hours and Joe works 2 hours, they get \$33,5. Find John's hourly rates (in dollars)

- A. 6,5
- B. 7
- C. 5,5
- D. 6
- E. 8

**Câu 8.** Find all values of  $m$  so that the system

$$\begin{cases} x - y - z = 1 \\ x + y - z = 2 \\ -x + 3y + z = m \end{cases}$$

has no solution.

- A. None of the other choices is correct
- B. 1
- C. Any number but 1
- D. Any real number
- E. Any number but 0
- F. 0

**Câu 9.** If  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is rotation through  $2\pi/3$ , then  $T \begin{bmatrix} 2 \\ -6 \end{bmatrix}$  is:

(i)  $\begin{bmatrix} 3\sqrt{3} - 1 \\ \sqrt{3} + 3 \end{bmatrix}$       (ii)  $\begin{bmatrix} 1 - 3\sqrt{3} \\ \sqrt{3} - 3 \end{bmatrix}$       (iii)  $\begin{bmatrix} \sqrt{3} \\ 2 \end{bmatrix}$       (iv)  $\begin{bmatrix} 3\sqrt{3} - 1 \\ 3 - \sqrt{3} \end{bmatrix}$

- A. (i)  
 B. None of the other choices is correct  
 C. (iv)  
 D. (iii)  
 E. (ii)

**Câu 10.** How many solutions would a HOMOGENOUS system of linear equations of 4 equations and in 4 variables have?

- A. No solution  
 B. Unique solution  
 C. Infinitely many solutions  
 D. There is not enough information

**Câu 11.** Find the solution of the linear system whose augmented matrix is

$$\left[ \begin{array}{cccc|c} 1 & 2 & 4 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right]$$

- A. The system is inconsistent  
 B.  $(-1, 0, 1, 1)$   
 C. None of the other choices is true  
 D.  $(-1-2t, t, 1, 1)$

**Câu 12.** Find the  $(1, 2)$ -entry of the inverse of the matrix

$$A = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- A. -2  
 B. None of the other choices is correct  
 C. 0  
 D. 2  
 E. This matrix is not invertible

**Câu 13.** If  $\det \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix} = 2$ , compute  $\det \begin{bmatrix} -p & -q & -r \\ 2p + a & 2q + b & 2r + c \\ p + 3x & q + 3y & r + 3z \end{bmatrix}$

- A. -3  
 B. 3  
 C. 6  
 D. None of the other choices is true  
 E. -6

**Câu 14.** Let A, B, C be 10 x 10 matrices with  $\det A = 3$ ,  $\det B = 2$  and  $\det C = -1$ . Find  $\det(A^{-1} B^T C^{-1})$ .

- A. All of the choices are incorrect
- B. -6
- C. 1/3
- D. 2/3
- E. -2/3

**Câu 15.** If  $6 \begin{bmatrix} 3 & 2 & -4 \\ 0 & z & x/6 \end{bmatrix} = \begin{bmatrix} x & 4z & -y \\ 0 & y-6 & 18 \end{bmatrix}$ , then:

- A. None of the other choices is correct
- B.  $x = 3, y = 4, z = 3$
- C.  $x = 6, y = 24, z = 6$
- D.  $x = 3, y = 4, z = \frac{1}{2}$
- E. There is no solution
- F.  $x = 18, y = 24, z = 3$

**Câu 16.** Find b such that the graph of  $y = a + bx + cx^2$  passes through (0, 5), (-1, -7), (2, 11)

- A. -3
- B. -9
- C. None of the choices is correct
- D. 3
- E. 9

**Câu 17.** Let A be an invertible matrix such that  $A^{-1} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}$ .

Find the (2, 3)-entry of the matrix  $(3A^T)^{-1}$ .

- A. 6
- B. 2/3
- C. -2/3
- D. 0
- E. None of the other choices is correct

**Câu 18.** Let  $A = (a_{ij})$  be the matrix of rotation in the plane through  $\pi/6$ . Find  $a_{21}$ .

- (i)  $\frac{1}{2}$                       (ii)  $-\frac{1}{2}$                       (iii)  $\frac{\sqrt{3}}{2}$                       (iv)  $-\frac{\sqrt{3}}{2}$

- A. None of the other choices is correct
- B. (iv)
- C. (i)
- D. (iii)
- E. (ii)

**Câu 19.** Consider the system  $AX = B$  where  $A$  is an invertible matrix with

$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 1 & 3 \\ 0 & 4 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \text{ Solve for } x_2.$$

- A. All of the other choices are incorrect
- B. -8
- C. 3
- D. 0

**Câu 20.** Find all numbers  $a$  such that the following matrix has rank 2.

$$\begin{bmatrix} -1 & 4 & 5 & 3 \\ 2 & 3 & -2 & 6 \\ 3 & 10 & a & 15 \end{bmatrix}$$

- A. None of the other choices is correct
- B. All numbers but -1
- C. All numbers but 1
- D. 1
- E. -1

**Câu 21.** Let  $A = (a_{ij})$  be the  $2 \times 2$  matrix of reflection in the  $x$ -axis followed by reflection in the line  $y = x$ . Find  $a_{22}$ .

- A. -1
- B. 2
- C. 1
- D. 0
- E. 2

**Câu 22.** Evaluate the determinant of the matrix  $\begin{bmatrix} 1+a & a & a \\ b & 1+b & b \\ c & c & 1+c \end{bmatrix}$

- A.  $1-abc$
- B.  $abc$
- C. None of the other choices is correct
- D.  $1+a+b+c$
- E.  $(a+b+c)/2$

**Câu 23.** Find all values of  $a$  such that the following system has nontrivial solutions

$$\begin{cases} x + 2y + z = 0 \\ x + 3y + 6z = 0 \\ 2x + 3y + az = 0 \end{cases}$$

- A. All numbers but -4
- B. All of the other choices are incorrect

- C. All numbers but -3
- D. -3
- E. -4

**Câu 24.** Let  $A$  be a  $2 \times 2$  matrix. Given that 1 and 2 are eigenvalues of  $A$  with corresponding eigenvectors  $[1 \ 3]^T$  and  $[0 \ 1]^T$ . Find the (1, 2)-entry of the matrix  $A$ .

- A. 0
- B. -3
- C. 2
- D. None of the other choices is true
- E. 1

**Câu 25.** Let  $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ . Which of the following matrices is  $A^{100}$ ?

(i)  $\begin{bmatrix} 100 & -100 \\ 0 & 100 \end{bmatrix}$       (ii)  $\begin{bmatrix} 1 & -100 \\ 0 & 1 \end{bmatrix}$       (iii)  $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

- A. (ii)
- B. (i)
- C. (iii)
- D. All of the other choices are incorrect

**Câu 26.** Find the (1,2)-entry of the inverse of the matrix  $A = \begin{bmatrix} 2 & 2 & 4 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

- A. 0
- B. -2
- C. None of the other choices is correct
- D. This matrix is not invertible
- E. 2

**Câu 27.** Solve for  $A$  if  $3A - 2A^T = \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}$

(i)  $\begin{bmatrix} 2 & 0 \\ 5 & 0 \end{bmatrix}$       (ii)  $\begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}$       (iii)  $\begin{bmatrix} 5 & 0 \\ 2 & 0 \end{bmatrix}$

- A. (ii)
- B. (i)
- C. None of the other choices is correct
- D. (iii)

**Câu 28.** Let  $A$  be a matrix and  $x$  a vector defined by

$$A = \begin{bmatrix} 0 & 8 & 3 & 6 \\ 9 & 0 & 8 & -4 \end{bmatrix} \quad x = \begin{bmatrix} 5 \\ 8 \\ -2 \\ 7 \end{bmatrix} \quad \text{Find the product } Ax.$$

- (i)  $\begin{bmatrix} 100 & 1 \end{bmatrix}$       (ii)  $\begin{bmatrix} 10 & 10 \end{bmatrix}^T$       (iii)  $\begin{bmatrix} 100 & 1 \end{bmatrix}^T$       (iv)  $\begin{bmatrix} 10 & 10 \end{bmatrix}$
- A. (ii)  
 B. (i)  
 C. None of the choices is correct  
 D. (iii)  
 E. (iv)

**Câu 29.** Find the number  $k$  for which the matrix  $A = \begin{bmatrix} 1 & 2 & k \\ 3 & -1 & 1 \\ 5 & 3 & -5 \end{bmatrix}$  has no inverse.

- A. 5  
 B. All numbers but 5  
 C. All numbers but -3  
 D. None of the other choices is correct  
 E. -3

**Câu 30.** Let  $A$  be a matrix of size  $m \times n$ . How many of the following statements are true?

- (i) If  $A$  has a zero entry, then  $A$  has a row of zeros  
 (ii) If  $Ax = 0$  where  $x \neq 0$ , then  $A = 0$ .  
 (iii) If  $Ax = 0$  has only the trivial solution  $x = 0$ , then  $Ax = b$  has a solution for every  $b$
- A. 2      B. 3      C. 0      D. 1

**Câu 31.** Let  $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 7 \\ 1 & 1 \end{bmatrix}$ , and  $X$  such that  $AX = B$ . The second row of matrix  $X$  is

- A.  $\begin{bmatrix} 1 & 1 \end{bmatrix}$     B.  $\begin{bmatrix} 1 & 0 \end{bmatrix}$     C.  $\begin{bmatrix} 0 & 1 \end{bmatrix}$     D.  $\begin{bmatrix} -1 & 1 \end{bmatrix}$     E. None of the other choices is correct

**Câu 32.** Determine how many solutions and how many parameters are possible for a homogenous system for 5 linear equation in 7 variables with augmented matrix  $A$  and  $\text{rank } A = 1$ .

- A. There are 5 parameters and infinitely many solutions  
 B. There are 5 parameters and a unique solution  
 C. There are 6 parameters and infinitely many solutions  
 D. There are 7 parameters and infinitely many solutions  
 E. None of the other choices is correct

**Câu 33.** Find the determinant of the matrix  $\begin{bmatrix} 1 & 1 & 1 \\ a & b+1 & c+2 \\ a+1 & b+2 & c+3 \end{bmatrix}$

- A. 0  
 B.  $a+b+c$   
 C.  $abc-a-2b-3c$

D. None of the other choices is correct

E.  $a(b+1)(c+2)$

**Câu 34.** Consider a homogenous system of 3 linear equations in 5 variables. Assume that the augmented matrix has rank 2. Choose correct statements.

A. There are exactly 2 parameters

B. The system has no solution

C. There are exactly 3 parameters

D. None of the other choices is correct

E. The system has only the trivial solution

**Câu 35.** Let  $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ . Find the (1, 1)-entry of  $A^{10}$ .

A. 1024

B. 512

C. 10

D. 20

**Câu 36.** Given that -3 is an eigenvalue for the matrix  $\begin{bmatrix} 3 & -6 \\ 8 & -11 \end{bmatrix}$ . Find all eigenvectors corresponding to this eigenvalue  $\lambda = -3$ .

A. None of the other choices is correct

B.  $t(1, 1)$ ,  $t$  is nonzero

C.  $t(1, -2)$ ,  $t$  is nonzero

D.  $t(1, -1)$ ,  $t$  is nonzero

E.  $t(2, 1)$ ,  $t$  is nonzero

**Câu 37.** Let  $A = (a_{ij})$  be the matrix of rotation in the plane through  $\pi/3$ . Find  $a_{12}$ .

(i)  $\frac{1}{2}$                       (ii)  $-\frac{1}{2}$                       (iii)  $\frac{\sqrt{3}}{2}$                       (iv)  $-\frac{\sqrt{3}}{2}$

A. None of the other choices is correct

B. (iv)

C. (iii)

D. (ii)

E. (i)

**Câu 38.** Find the rank of the matrix  $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 3 \\ 5 & 3 & 4 \end{bmatrix}$

A. 2

B. 0

C. 1

D. 3

**Câu 39.** Let  $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$ . Find  $B - 2A^T$ .

(i)  $\begin{pmatrix} -1 & 5 \\ 2 & -5 \\ -1 & 2 \end{pmatrix}$                       (ii)  $\begin{pmatrix} -1 & 5 \\ 2 & -5 \\ 1 & 2 \end{pmatrix}$                       (iii)  $\begin{pmatrix} -1 & -5 \\ 2 & -5 \\ -1 & 2 \end{pmatrix}$



- A. (ii)
- B. (iii)
- C. (i)
- D. None of the other choices is correct

**Câu 40.** Find the product  $AB$ , where:  $A = \begin{bmatrix} -5 & 7 & 6 \\ 9 & -3 & -1 \\ 2 & 4 & -8 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 \\ 8 \\ 5 \end{bmatrix}$

- (i)  $[76 \ -11 \ -4]^T$
- (ii)  $[75 \ -11 \ -4]^T$
- (iii)  $[76 \ -12 \ -4]^T$
- (iv)  $[76 \ -11 \ -3]^T$

- A. (iv)
- B. (ii)
- C. (iii)
- D. None of the other choices is correct
- E. (i)

**Câu 41.** Solve for the following system of linear equations.

$$\begin{cases} x + y + z = 2 \\ 2x + 3y - z = 8 \\ x - y - z = -8 \end{cases}$$

- A.  $(-3; 19/4; 1/4)$
- B. None of the other choices is correct
- C.  $(-3; 17/4; -1/4)$
- D. The system is inconsistent
- E.  $(3; 19/4; -1/4)$

**Câu 42.** Find the (2,3)-entry of the product

$$\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \begin{bmatrix} 0 & 5 & -4 \end{bmatrix}$$

- A. -14
- B. -22
- C. 15
- D. 8
- E. None of the other choices is correct