Chapter 5-7

Counting

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Counting Rules

- Product rule: task = {task 1 ∧ task 2 ∧ … ∧ task n} → product rule
- A = {a, b, c}, B = {1, 2}. How many ways to choose a character from A AND a number from B?

Choose (a character AND a number) = {task 1: choose a character ∧ task 2: choose a number}

→ product rule: there are 3x2 = 6 ways. //a1, a2, b1, b2, c1, c2

Ex: A→B : 5 ways
B→C : 7 ways
How many ways from A→C?

A B C

AB: 5 BC: 7 $5 \times 7 = 35$

Counting Rules

- Product rule: task = {task 1 $\wedge$ task 2 $\wedge$ … $\wedge$ task n} ➔ product rule
- How many positive integers are there with 3 distinct digits?

Choose a number (abc)= {task 1: choose a $\wedge$ task 2: choose b $\wedge$ task 3: choose c}

➔ product rule: 9x9x8 =648 numbers.

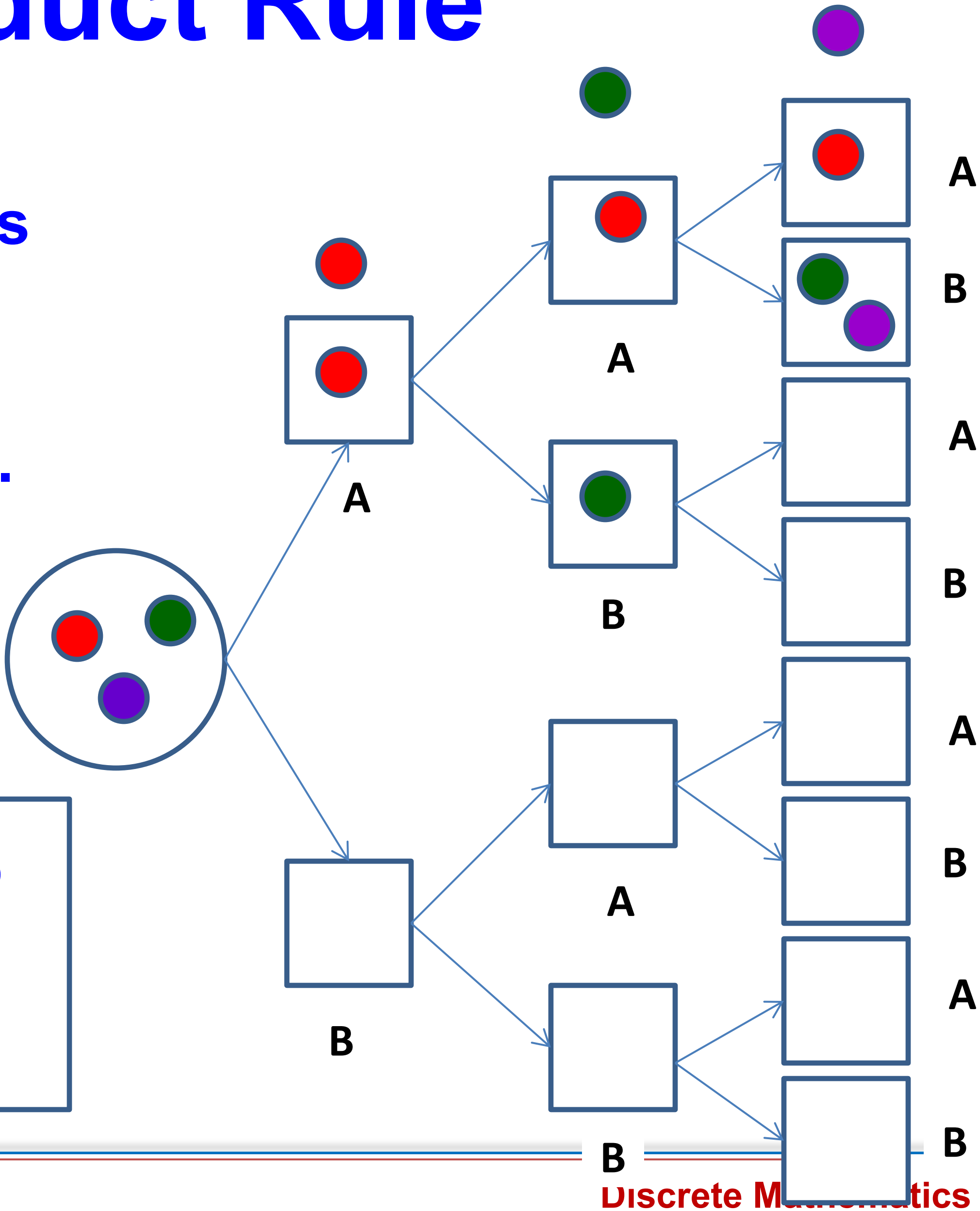
- How many functions are there from {a, b, c} to {1, 2, 3, 4}?

function = {(a, ?), (b, ?), (c, ?)}

➔ 4x4x4 = 4^3 functions.

Product Rule

- Ex: Distribute 3 distinct balls to 2 bags
- We must do 3 times, each time we have 2 ways to select a bags.
- $2.2.2 = 2^3$ ways



There are r^n ways to distribute n distinct objects to r bags

Product Rule

- 12 teams. How many ways to select 3 teams for the first, second and third teams

I

II

III

Select 1/ 12
→ 12 ways

After select 1 team for the first position, one of 11 teams can be select into the second team
→ 11 ways

After select 1 team for the second position, one of 10 remaining teams can be select into the third team
→ 10ways

12.11.10= 1320 ways

Counting rules

- Sum rule: task = {one of n_1 ways OR one of n_2 other ways}

→ n_1+n_2 ways in total.

- How many ways to choose one character in a..z OR one number in 1..9?

→task = {choose one character OR choose one number // number ≠ character}

→ 26 + 9 = 35 ways in total.

More Complex Counting Problems

- Example 14: Name of a variable in BASIC language, a case insensitive language, is a string of 1 or 2 characters in which the first must be a character, the second may be an alphanumeric characters, and must be different from 5 two-character keywords . How many different variable names are there?

26 characters (a..z), 10 digits (0..9)

Name with 1 character: 26

Name with 2 character: 26.36 -5 =931

→ There are 957 different names

Counting rules

- Inclusion-exclusion principle:

$|A \cup B| = |A| + |B| - |A \cap B|$

- How many positive integers not exceeding 100 are divisible by 3 or 5?

$A = \{x \mid x \text{ is divisible by } 3\}$, $B = \{x \mid x \text{ is divisible by } 5\}$

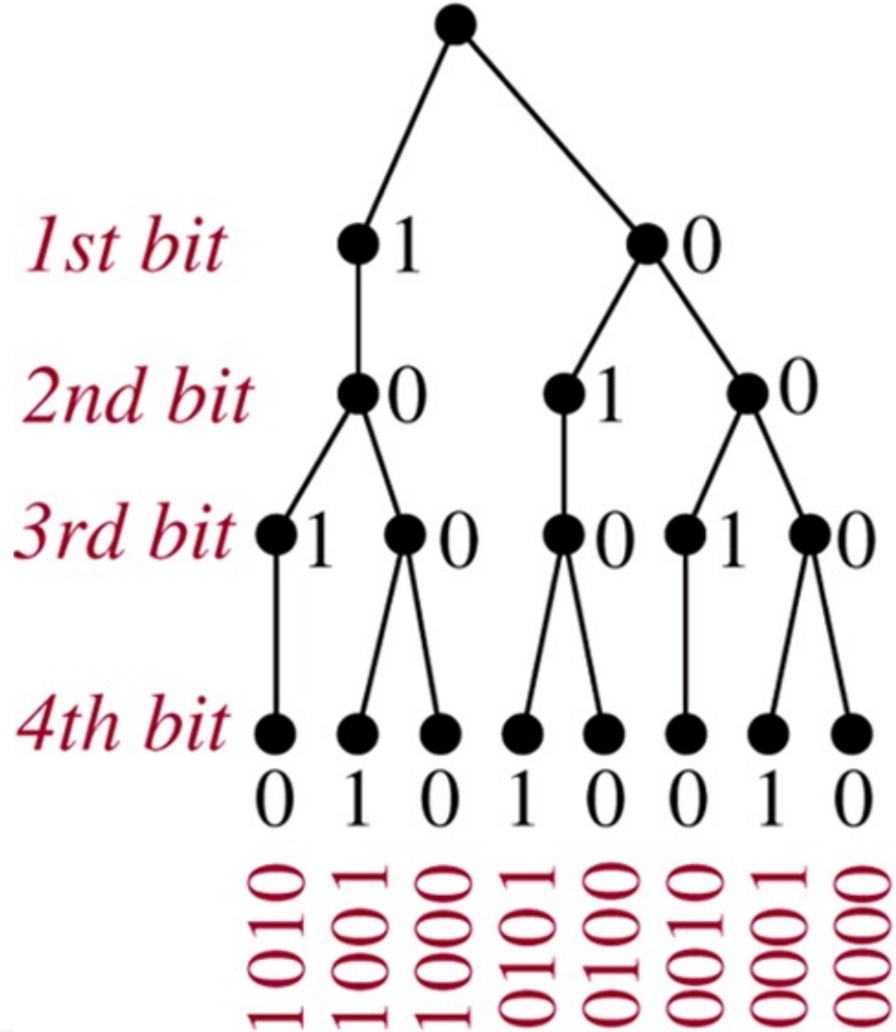
$\rightarrow$ $A \cup B = \{x \mid x \text{ is divisible by 3 OR 5}\}$, $A \cap B = \{x \mid x \text{ is divisible by 3 AND 5}\}$

$\rightarrow$ $|A \cup B| = |A| + |B| - |A \cap B| = 33 + 20 – 6 = 47$

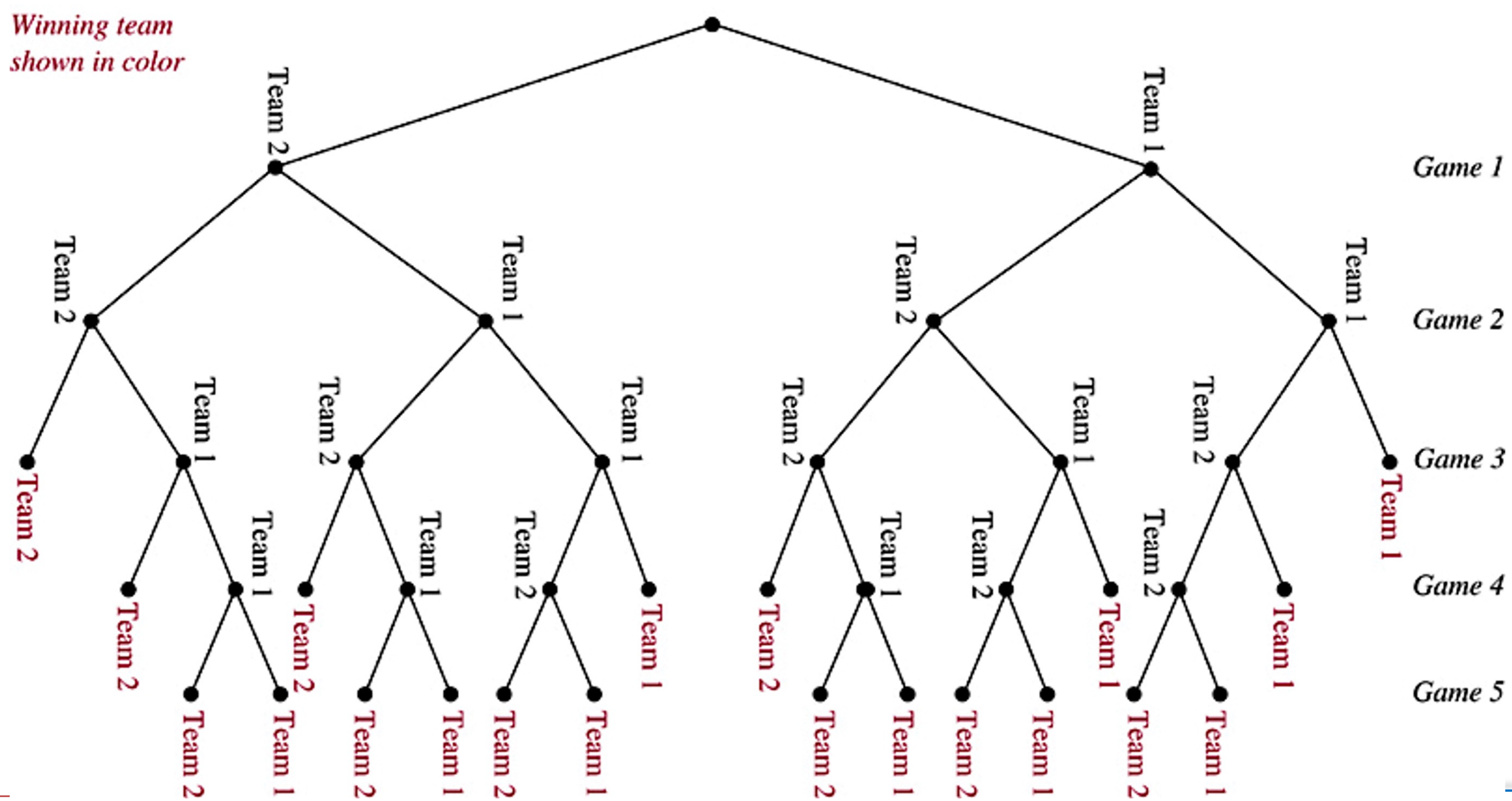
How many positive integers not exceeding 2021 are divisible by 7 or 5?

Tree Diagrams

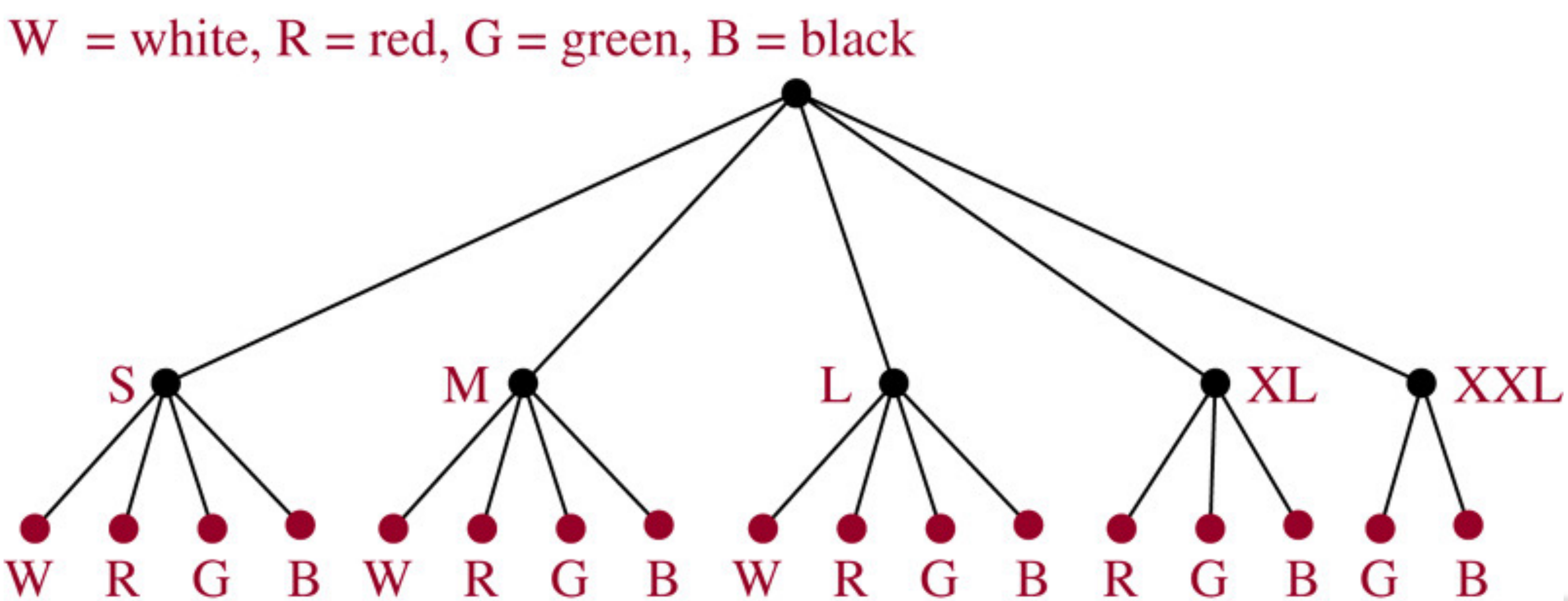
- Counting problem can be solved using **Tree Diagram.**
- **Example: How many 4-bit strings do not have two consecutive 1s?**



Example 20: A playoff between two teams consists of at most 5 games. The first team that wins 3 games wins the playoff. In how many different ways can the playoff occur?



- **Example 22**: T-Shirts come in 5 sizes: S, M, L, XL, XXL. Each size comes in 4 colors, W, R, G, B excepts for XL, which comes only in R, G, B, and XXL, which comes only in G and B.
- How many different shirts a souvenir shop have to stock to have at least one of each available size and color of the T_shirt?



7.1. Recurrence Relations

- Definition 1: Recurrence relation on the sequence { a_n }:

  a_n is expressed in terms of one or more of previous items for all n with $n \geq n_0$
- $\{a_n\}$ is called a **solution** of recurrence relation.
- Example 1: $a_0=3$, $a_1= 5$ $a_n=a_{n-1} - a_{n-2}$, $n>1$
- Example 2: Determine whether $\{a_n\} = 3n$, $n \geq 0$ is a solution of the recurrence relation $a_n= 2a_{n-1} - a_{n-2}$, $n \geq 2$?
- **Initial conditions**: Terms that **precede the first item where the recurrence takes effect.**

Modeling with Recurrence Relations

- Problem ← Modeling with recurrence relation
- Example 3 (page 451): Compound interest Problem – **Lãi gộp:**

P_0: Initial deposite

r: interest rate per year

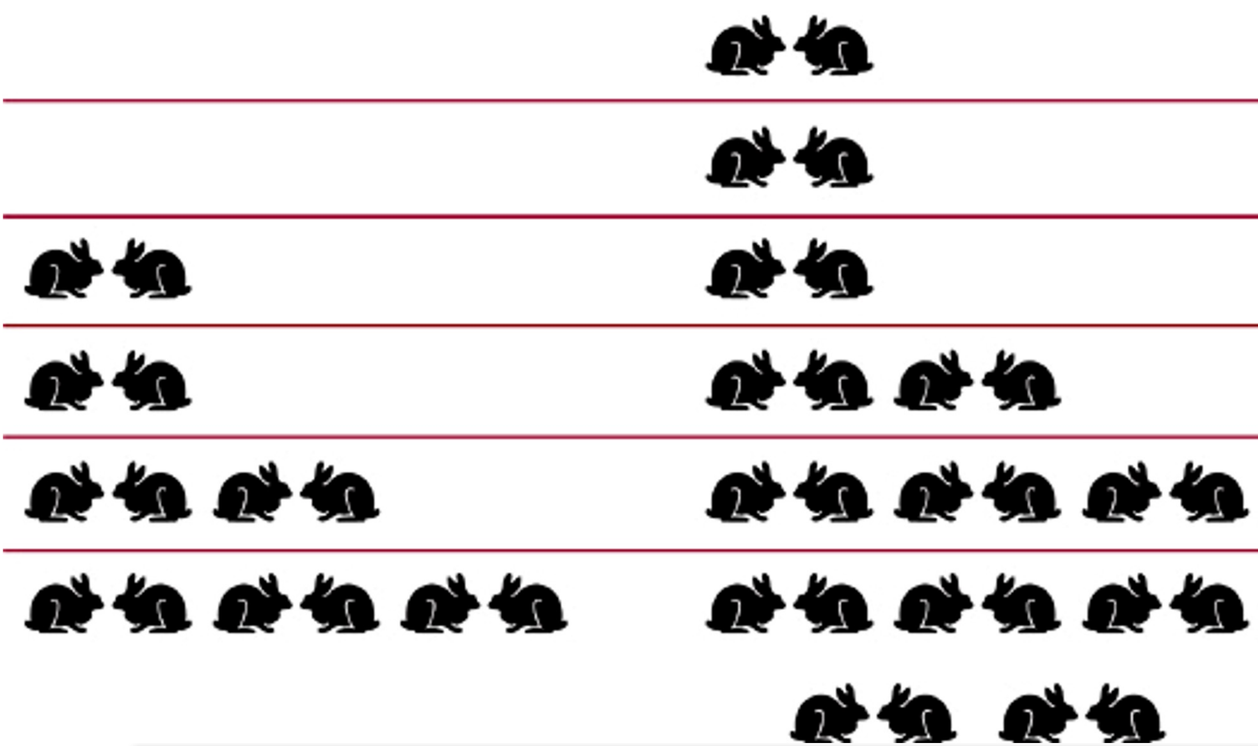
P_n: amount at the n^{th} year

$P_n = P_{n-1} + rP_{n-1}$

➔ $P_n = (1+r)^n P_0$

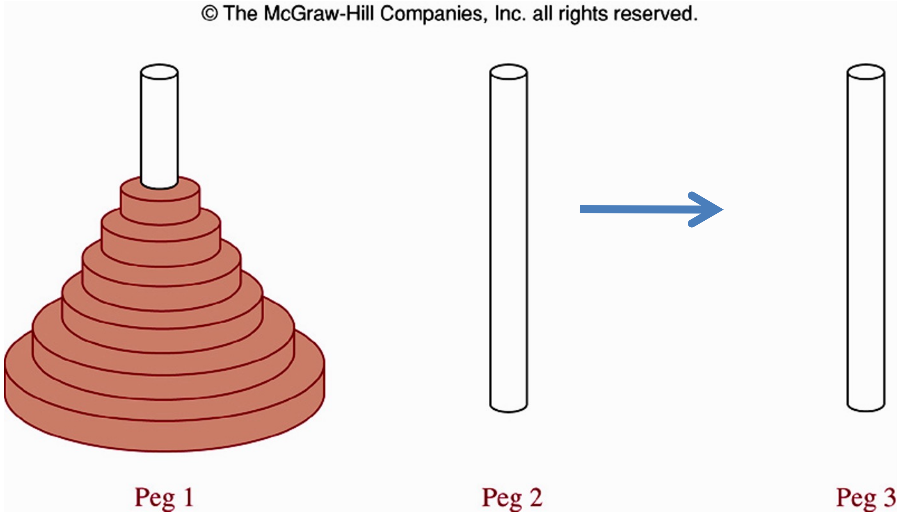
Fibonacci numbers of Leonardo Pisano

- Example 4: : $f_1=f_2=1;$ $f_n= f_{n-2} + f_{n-1}$



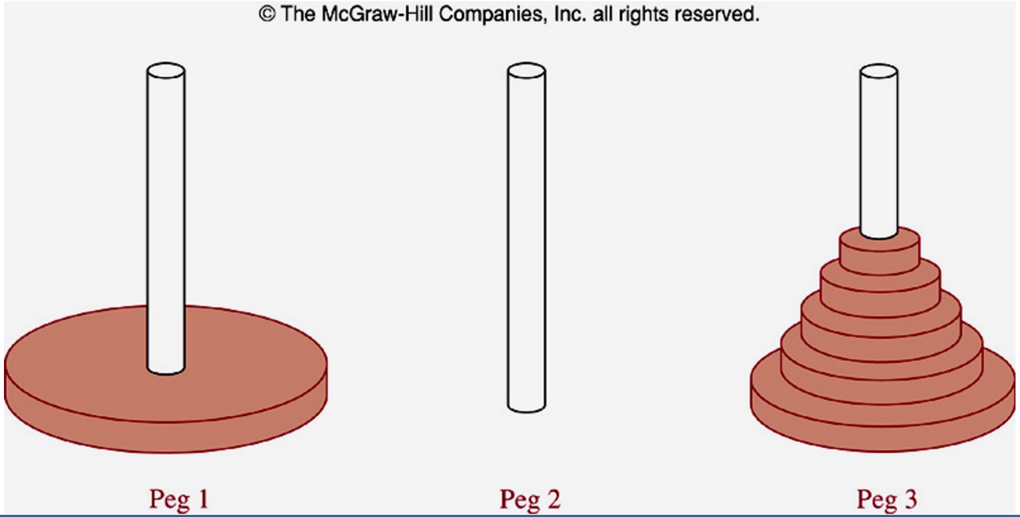
Reproducing pairs (at least two months old)	Young pairs (less than two months old)	Month	Reproducing pairs	Young pairs	Total pairs
		1	0	1	1
		2	0	1	1
		3	1	1	2
		4	1	2	3
		5	2	3	5
		6	3	5	8

The Tower of Hanoi Problem- Ex5, page 452



How many steps this problem is solved if there is n disks on the peg 1?

The Tower of Hanoi Problem- Ex5, page 452



Let H_n is the number of moves needed to solve the Tower of Hanoi problem. We will set up a recurrence relation for the sequence H_n.

-Begin with n disk on peg 1.

-We transfer the top n-1 disk from peg1 to peg3 → H_{n-1} moves .

-We transfer the largest disk from peg1 to peg 2 → 1 move

-We transfer n-1 disk from peg3 to peg 2 → H_{n-1} moves

→ $H_n = 2H_{n-1} + 1$

$H_1=1$ → $H_n = 2^n - 1$

n=64 → $2^{64}-1=18\ 446\ 744\ 073\ 709\ 551\ 615$. With 1 move/sec → 500 billion years.

Find recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0s. How many such bit strings are there of length five.

- a_n: number of bit string of length n that do not have two consecutive 0s.
- One bit string can terminate with bit 1 or 0
- Format of a bit string that **do not have two consecutive 0s**:

End with 1	Any bit string of length n-1 with no two consecutive 0s	1
End with 0	Any bit string of length n-2 with no two consecutive 0s	1 0

→ $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$ { Fibonacci sequence }

→ $a_1=2$ { two string "0" and "1" }

→ $a_2= 3$ { "10" , "01" , "11" }

With n=5

→ $a_3 = a_2+ a_1 =5$

→ $a_4= a_3 + a_2 = 5+3=8$

→ $a_5=a_4 + a_3 = 8+5=13$

Give a recursive definition of the sequence {*an*}, n=1, 2, 3, . . . if $a_n = 6n$.

(A) $a_{n+1} = a_n + 6 \ \mathit{for} \ n \geq 1 \ \mathit{and} \ a_1 = 6$

(B) $a_{n+1} = a_n - 6 \ \mathit{for} \ n \geq 1 \ \mathit{and} \ a_1 = 6$

(C) $a_{n+1} = 6a_n \ \mathit{for} \ n \geq 1 \ \mathit{and} \ a_1 = 6$

(D) $a_{n+1} = a_n + 6 \ \mathit{for} \ n \geq 1 \ \mathit{and} \ a_1 = 1$

7.3. Divide-and-Conquer Algorithms and recurrence Relations

- **Divide:** Dividing a problem into one or more instances of the same problem of smaller size
- **Conquer**: Using the solutions of the smaller problems to find a solution of the original problem, perhaps with some additional work.

Divide-and-Conquer Recurrence Relations

- n : size of the original problem
- a: number of sub-problem
- n/b : size of the sub-problem
- f(n) : number of operation required to solve the original problem.

→ f(n/b) : number of operation required to solve a sub-problem.

- g(n): number of extra operations are required in the **conquer** step of the algorithm.
- Recurrence relation: **$f(n) = a\, f(n/b) + g(n)$**

Recurrence Relations for Binary Search

```
procedure binary-search(x, i, j)
if i>j then location=0
m= ⌊(i+j)/2⌋
 if x= a_m then location =m
else if x< a_m then location= binary-search(x, i, m-1)
else location= binary-search(x, m+1, j)
```

$$f(n) = f(n/2) + 2$$

Recurrence Relations for Finding Maximum of a sequence

procedure max(i,j: integer ,a_i,a_{2+1},…,a_j: integers)
if i=j **then**
begin
max:= a_i
end
else
begin
m= $\lfloor (i+j)/2 \rfloor$
max1= max (i,m,a_i,a_{i+1},…,a_m)
max2= max (m+1,j,a_{m+1},a_{m+2},…,a_j)
if max1>max2 **then** max:= max1
else max:=max2

$$f(n) = 2f(n/2) + 1$$

Theorem 1

- Let f be an **increasing function** that satisfies the recurrence relation f(n)= af(n/b) + c

whenever n is divisible by b, where $a \geq 1$, b>1 is an integer, and c >0. **Then**

$$f(n)\ is \begin{cases} O(n^{\log_b a}) & \text{if } a>1 \\ O(\log n) & \text{if } a=1 \end{cases}$$

Furthermore, when $n=b^k$, where k is a positive integer,

$$f(n) = C_1 n^{\log_b a} + C_2$$

Where C_1=f(1) + c/(a-1) and C_2= -c/(a-1)

Example

- Estimate the number of comparisons used by a binary search

  $f(n) = f(n/2) + 2$, $a=1 \rightarrow f(n) = O(\log n)$ // theorem 1

- Estimate the number of comparisons to locate the maximum element in a sequence

  $f(n) = 2f(n/2) + 1$, $a=2 \rightarrow f(n) = O(n^{\log a}) = O(n)$ // theorem 1

Example 6: $f(n)= 5f(n/2) + 3$, $f(1)=7$. Find $f(2^k)$, k is a positive integer. Estimate f(n) if f is increasing function.

Using theorem 1: $a= 5$, $b=2$, $c=3$, $n=2^k$

$C_1=f(1) + c/(a-1) = 7+3/(5-1)= 7+3/4=31/4$

$C_2= -c/(a-1) = -3/(5-1)= -3/4$

$$n^{\log_b a} = 2^{k\log_2 a} = a^k$$

$f(n)= f(2^k) = C_1a^k + C_2 = a^k.31/4 -3/4$

f is increasing, $a>1 \rightarrow f(n) = O(n^{\log a}) = O(n^{\log 5})$

Theorem 2: Master Theorem

Let f be an **increasing function that satisfies the recurrence relation f(n) = af(n/b) + cn^d**

If n= b^k, where k is a positive integer, a $\geq$ 1, $b \geq 1$ is an integer, c >0 and $d \geq 0$. Then

$$f(n) \quad is \quad \begin{cases} O(n^d)\, if\ a < b^d \\ O(n^d \log n)\, if\ a = b^d \\ O(n^{\log_b a})\, if\ a > b^d \end{cases}$$

Summary

- **Product rule**: Suppose that a procedure can be broken down into a sequence of two tasks. If there is n1 ways to do the first task, there is n2 ways to do the second task, then there are n1n2 ways to do the procedure.
- **Sum rule**: If a task can be done either in one of n1 ways or in one of n2 ways, where none of the set of n1 ways is the same as any of the set of n2 ways, then there are n1 + n2 ways to do this task.

Summary

- **Inclusion-Exclusion Principle**: Suppose that a task can be done in n1 or n2 ways, but that some ways in the set of n1 ways are the same as some ways in the set of n2 ways.

Summary

- **Recurrence relation** on the sequence { a_n } is an expression in which a_n is expressed in terms of one or more of previous items for all n with n ≥ n_0 . {an} is called a **solution** of recurrence relation.
- **Initial conditions**: Terms that **precede the first item where the recurrence takes effect.**
- **Modeling with recurrence relation:** Finding out an recurrence relation for input of a problem.

Summary

- **Divide:** Dividing a problem into one or more instances of the same problem of smaller size
- **Conquer**: Using the solutions of the smaller problems to find a solution of the original problem, perhaps with some additional work
- Recurrence relation: $\mathbf{f(n) = af(n/b) + g(n)}$

Summary – Theorem 1

- Let f be an **increasing function** that satisfies the recurrence relation f(n)= af(n/b) + c

whenever n is divisible by b, where a ≥ 1 , b is an integer and greater than 1, and c is a positive real number. **Then**

$$f(n)\ is \begin{cases} O(n^{\log_b a}) & \text{if } a>1 \\ O(\log n) & \text{if } a=1 \end{cases}$$

Furthermore, when $n=b^k$, where k is a positive integer,

$$f(n) = C_1 n^{\log_b a} + C_2$$

Where C_1=f(1) + c/(a-1) and C_2= -c/(a-1)

Proof: page 477

Summary – Theorem 2- Master Theorem

Let f be an **increasing function that satisfies the recurrence relation $f(n) = af(n/b) + cn^d$**

Whenever $n = b^k$, where k is a positive integer, a $\geq$ 1, b is an integer greater than 1, and c and d are real numbers with c positive and d nonnegative. Then

$$f(n) \quad is \quad \begin{cases} O(n^d) \; if \; a < b^d \\ O(n^d \log n) \; if \; a = b^d \\ O(n^{\log_b a}) \; if \; a > b^d \end{cases}$$

Q1. A certain type of push-button door lock requires you to enter a code before the lock will open. The lock has five buttons, numbered 1,2,3,4,5.

a. If you must choose an entry code that consists of a sequence of four digits, with repeated numbers allowed, how many entry codes are possible?

b. If you must choose an entry code that consists of a sequence of four digits, with no repeated digits allowed, how many entry codes are possible?

- Answer:

a. We need to fill in the four blanks in _ _ _ _ , where each blank can be filled in with any of the five digits 1,2,3,4,5. By the generalized product rule this can be done in $5^4 = 625$ ways.

b. We need to fill in the four blanks in _ _ _ _, but each blank must be filled in with a distinct integer from 1 to 5.

By the generalized product rule that can be done in $5 \cdot 4 \cdot 3 \cdot 2 = 120$ ways.

Q2. Count the number of print statements in this algorithm:

- For i:= 1 to n

```
begin
    For j:= 1 to n
        Print “hello”
    For k:= 1 to n
        Print “hello”
end
```

Answer.

For each value of i, both the j-loop and k-loop are executed. Thus for each i, the number of print statements executed is n + n, or 2n. Therefore the total number of print statements executed is $n \cdot 2n = 2n^2$.

Q3. Count the number of print statements in this algorithm:

```
For i:= 1 to n
begin
        For j:= 1 to i
                Print "hello"
        For k:= i+1 to n
                print "hello"
End
```

Answer. For each value of i, both the j-loop and k-loop are executed. Thus for each i, the number of print statements executed is i in the first loop plus n-i in the second loop. Therefore, for each i, the number of print statements is $i+(n-i) = n$.

Therefore the total number of print statements executed is $n \cdot n = n^2$.

Q4. Suppose that a "word" is any string of seven letters of the alphabet, with repeated letters allowed.

1. How many words are there?

Answer. Word = letter1-letter2-letter3-letter4-letter5-letter6-letter7

➔ Product rule ➔ 26^7

2. How many words begin with R and end with T?

Answer. Word = R-letter2-letter3-letter4-letter5-letter6-T

➔ Product rule ➔ 26^5

Q4. Suppose that a "word" is any string of seven letters of the alphabet, with repeated letters allowed.

3. How many words begin with A or B?

Answer. Word = A****** ➔ product rule: 26^6 OR

Word = B******* ➔ 26^6

sum rule ➔ $26^6 + 26^6$

4. How many words begin with A or end with B?

Answer. Word = A****** OR ******B (including words in form A*****B)

➔ Inclusion-Exclusion Principle $|A \cup B| = |A| + |B| - |A \cap B| = 26^6 + 26^6 - 26^5$

Q5. Suppose that a “word” is any string of seven letters of the alphabet, with repeated letters allowed.

1. How many words begin with A or B and end with A or B?

Answer. Word = ‘A OR B’*****’A OR B’ ➔ $2.26^5.2$

2. How many words begin with a vowel and end with a vowel?

Answer. Word = ‘a vowel’*****’a vowel’ ➔ $5.26^5.5$

3. How many words begin with a vowel or end with a vowel?

Answer. Word = ‘a vowel’****** OR word =*****’a vowel’ // containing cases ‘a vowel’*****’a vowel” //

➔ $5.26^6 + 26^6.5 – 5.26^5.5$

4. How many words have no vowels?

Answer. 21^7

5. How many words have exactly one vowel?

Answer. Word = vowel in any position AND 6 letter, not vowel

➔ $5.7.21^6$

6. How many words have at least one vowel?

Answer. number of words with at least one vowel = number of all possible words – number of words with no vowel ➔ $26^7 - 21^7$

Q6. Find the number of words of length 8 that have exactly one B.

- Answer.

Word = ******** // 8 letters and contains exactly one B in any *-position. For example, a word may be KG**B**NFUNN.

Task = {task 1 = choose position for B: 8 ways AND task 2 = choose letter 2: 25 ways AND … AND task 8 = choose letter 8: 25 ways}

➔ Product rule: 8.25^7

- Similar questions:

1. Find the number of words of length 8 that have at least one C.

Answer. $26^8 - 25^8$

2. Find the number of words of length 8 that begin with L or end with R.

Answer. $26^7 + 26^7 - 26^6$

Q7. Find the number of subsets of S = {1,2,3,…,10} that contain the number 5.

Answer. Subsets contain 5 ➔ {5, others} ➔ 2 choices for 1 (belongs or does not belong to subset) AND 2 choices for 2 AND … AND 2 choice for 10 ➔ product rule 2.2….2 = 2^9.

1. Find the number of subsets of S = {1,2,3,…,10} that contain neither 5 nor 6.

Answer. 2^8

2. Find the number of subsets of S = {1,2,3,…,10} that contain both 5 and 6.

Answer. 2^8

Q8. How many function are there form A to B if A = {a, b, c} and B={1, 2, 3, 4}?

Answer.

- A function from A to B = {(a, ?), (b, ?), (c, ?)} // "?" can be one of {1, 2, 3, 4}.
- Task = construct a function = {task1: 4 ways for (a, ?) AND task2: 4 ways for (b, ?) AND task 3: 4 ways for (c, ?)}
- ➔ product rule ➔ we have 4.4.4 = 4^3 different functions from A to B in total.

Questions:

1. Suppose $|A| = 4$ and $|B| = 10$. Find the number of functions $f : A \to B$.

   Answer. 10^4

2. Suppose $|A| = 4$ and $|B| = 10$. Find the number of 1-1 functions $f : A \to B$.

   Answer. 10.9.8.7

3. *How many functions are there from {x, {x, ∅}, {x}} to {z, {z, y, x}, {z}, z, x}?*

   Answer. 4^3

Q9.

Question. Suppose a restaurant serves a “special dinner” consisting of soup, salad, entree, dessert, and beverage. The restaurant has five kinds of soup, three kinds of salad, ten entrees, five desserts, and four beverages. How many different special dinners are possible? (Two special dinners are different if they differ in at least one selection.)

Answer.

Special dinner = (soup: 5 ways, salad: 3 ways, entrée: 10 ways, dessert: 5 ways, beverage: 4 ways)

➔ Product rule: $5 \times 3 \times 10 \times 5 \times 4$.

Q10. Describe the sequence

$a_n = 1 + 2 + 3 + \ldots + n$ *recursively*.

Include initial conditions and assume that the sequences begin with a_1.

- Answer. $a_n = 1+2+3\ldots+(n-1) + n \Rightarrow a_{n-1} = 1+2+3+\ldots+(n-1)$

$\Rightarrow a_n = a_{n-1} + n$ and initial condition $a_1 = 1$.

Similar questions: Describe each sequence recursively and give initial condition a_1.

1. $a_n = 5^n$ // $a_n = 5a_{n-1}$ and $a_1 = 5$
2. $a_n = 5$ for all positive integers n // $a_n = a_{n-1}$ and $a_1 = 5$
3. $a_n = 3n - 1$ // $a_n = a_{n-1} + 3$ and $a_1 = 2$
4. ½, 1/3, ¼, 1/5, 1/6, …
5. 0.1, 0.11, 0.111, 0.1111, …
6. $1^2, 2^2, 3^2, 4^2, \ldots$

Q11. Determine whether the sequence $a_n = 2^n$ is a solution of the recurrence relation

$$a_n = 3a_{n-1} - 2a_{n-2}$$

Answer.

- $a_n = 2^n \Rightarrow a_{n-1} = 2^{n-1}, a_{n-2} = 2^{n-2}$

$\Rightarrow$ right hand side $= 3a_{n-1} - 2a_{n-2} = 3(2^{n-1}) - 2(2^{n-2})$
$= 3.2(2^{n-2}) - 2.2^{n-2} = (6-2)2^{n-2} = 4.2^{n-2} = 2^n = a_n =$ left hand side

$\Rightarrow$ solution

1. $a_n = 0$ for all n // solution
2. $a_n = 7$ // solution
3. $a_n = n$ // not a solution
4. $a_n = 3^n$ // not a solution

Q11. Suppose $f(n) = 3f(n/2) + 1$, $f(1) = 1$. Find $f(8)$.

- Answer. 40
- $f(8) = 3f(4) + 1 = 40 \Leftarrow f(4) = 3f(2) + 1 = 13 \Leftarrow f(2) = 3f(1) + 1 = 4$

Similar questions:

1. Suppose $f(n) = f(n/3) + 2n$, $f(1) = 1$. Find $f(27)$.

   Answer. 79

2. Suppose $f(n) = 2f(n/2)$, $f(8) = 2$. Find $f(1)$.

   Answer. 1/4

3. Suppose $f(n) = 2f(n/2) + 3$, $f(16) = 51$. Find $f(2)$.

   Answer. 15/4

4. Suppose $f\ n) = 4f(n/2) + n + 2$, $f(1) = 2$. Find $f(8)$.

   Answer. 226

More exercises

1. 12. How many one-to-one functions from a set of 5 elements to a set of 4 elements?
2. How many 8-digits numbers that are divisible by 11?
3.

More exercises

A young pair of rabbits (one of each sex) is placed on an island. A pair of rabbits does not breed until they are 3 month old. After they are 3 month old they will produce 1 pair of rabbits each month. Let f_n be the number of pairs of rabbits after n months.

Find a recursive relation for f_n.

(i) $f_n = f_{n-1} + f_{n-3}$

(ii) $f_n = f_{n-1} + f_{n-2}$

(iii) $f_n = f_{n-1} + f_{n-2} + f_{n-3}$

More exercises

Find f(16) given that

$$f(1) = 2 \text{ and } f(n) = \left(f\left(\frac{n}{4}\right)\right)^2$$

1. A car model comes with the following choices: 9 colors, with or without air conditioning, with or without sunroof, with or without automatic transmission, with or without a spoiler, and with or without antilock brakes. In how many ways can the car be ordered?

2. You are about to take a 8 question multiple choice test. Each of these questions have 4 answers (A, B, C, or D). How many ways can you answer the test if you leave an answer for each question?

3. A social security number contains nine digits, such as 000-00-0000. How many different social security numbers can be formed using any numerals from 0 to 9?

4. How many different four digit alarm codes can be formed for a house alarm. The first digit must be a 2, 4 or 9 and all the other digits can be any number and numbers can be repeated.

5. How many 5 character license plates can be made if the first 3 characters are letters and last 2 characters are numbers. Repetition of characters is allowed but the first letter must be a P, W, Q, E, L, or K?

6. Telephone numbers is the United States begin with 3 digit area codes followed by 7 digit local telephone numbers. **Area codes and local telephone numbers** cannon begin with 0 or 1. How many telephone numbers are possible?

8. There are 8 horses in race, how many ways can they finish first, second and third?

9. A menu has 6 different sandwiches, with 3 choices of potato, 3 types of salad, and 5 different beverages. How many different lunches can be ordered consisting of a sandwich, potato, salad and beverage?

10. Assume a postal coded consists of 6 characters. Each character can be any letter from A to Z or any numeral from 0 to 9. How many postal codes are possible in this situation?

11. A lock uses the letters A through H on the first dial and the digits 0 through 9 on the second and third dials.

a) How many possible codes are there for this lock?

b) How many possible codes are there if the same number is not used twice?

12. In how many ways can a teacher seat 5 girls and 3 boys in a row of 8 seats if a boy must be seated in the first seat and a girl in the last seat?

13. Draw a tree diagram of all the possible outcomes there are when a coin is flipped in the air and dice is rolled at the same time.

How many license plate numbers

are available if a license plate number consists of three letters followed by three numbers

(0 through 9).?

A restaurant menu has a price-fixed complete dinner that consists of an appetizer, an entrée, a

beverage, and a dessert. You have a choice of five appetizers, ten entrées, three beverages, and six desserts. Determine the total number of possible dinners.

If there are 10 multiple-choice questions on an exam, each having three possible answers, how many different sequences of answers are there?

- You need to travel in between city A and B. You can either fly, take a train, or a bus. There are 12 different flights in between A and B, 5 different trains and 10 buses. How many options do you have to get from A to B?

- The minimum password length is 6 and the maximum is 8. The password can consist of either an uppercase letter or a digit. There must be at least one digit in the password. • How many different passwords are there?

There are 5 roads leading from Bluffton to Hardeeville, 9 roads leading from Hardeeville to Savannah, and 4 roads leading from Savannah to Macon. How many ways are there to get from Bluffton to Macon?

1) There are 45 multiple-choice questions on an exam. If each question has 4 possible answers, how many different ways are there to complete the test?

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2) If a student has a choice of 6 different CPUs, 5 different monitors and 3 different printers, how many ways can she select a computer system containing one CPU, one monitor, and one printer?

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3) A password consists of 3 randomly selected letters, which must be in the correct order. There are 26 letters in the alphabet. How many passwords are there if:

a) there are no restrictions on the letters used

b) no letter may be repeated

Find $f(125)$ if $f(1)=3$ and $f(n)=f(n/5)+\frac{n^2}{3}$.

$$f(125) = f\left(\frac{125}{5}\right) + \frac{125^2}{3} = f(25) + \frac{125^2}{3}$$
$$f(25) = f(5) + \frac{25^2}{3}$$
$$f(5) = f(1) + \frac{5^2}{3} = 3 + \frac{25}{3}$$

$f(1) = 1, f(2) = 3, f(n) = 2f(n-1) + f(n-2)$. Find $f(10)$?

- thanks