

# Computational Modeling of Memory Consolidation

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## 1 Introduction

Memory consolidation – the neural process by which transient experiences transform into enduring memories – serves as a cornerstone of human cognition. This phenomenon underpins learning, adaptive decision-making, and the continuity of personal identity, making it a critical focus of cognitive neuroscience [1, 2, 3]. While traditional views framed consolidation as a passive stabilization mechanism [4], contemporary research emphasizes its active, integrative nature [5, 6]. Memories are not merely stored but dynamically reorganized, with new information selectively merged into existing networks based on relevance, shared features, and inhibitory interactions between competing traces [7, 8, 9]. Despite advances, the mathematical principles governing these multiscale neural processes remain elusive, particularly how distinct brain systems with varying stability and timescales interact to shape long-term retention [10, 11, 12].

Computational modeling offers a powerful lens to dissect these mechanisms. By translating biological hypotheses into testable mathematical frameworks, we can simulate memory formation, predict retention outcomes, and uncover design principles invisible to experimental methods alone [13, 14]. This project proposes a dynamical systems model to investigate how short-term memories evolve into stable long-term representations. The model will incorporate four key factors: (i) contextual relevance (modulating attention-driven encoding) [15], (ii) inhibitory competition (modeling interference between overlapping memories) [16], (iii) temporal delays (between initial encoding and consolidation phases) [2], and (iv) timescale separation (distinguishing transient vs. stable memory subsystems) [10, 12].

In this project, I set out to understand the dynamical relationship between short-term and long-term memory in both single-trace and multi-trace settings. The first theme will be explored is how individual memory traces evolve over time, focusing on how encoding strength, consolidation delay, and intrinsic decay shape the transition from short-term activation to long-term retention. The model will be then extended to include multiple interacting memory traces, allowing me to study how competition and self-reinforcement influence the outcome of consolidation when memories coexist or overlap. A key question throughout was whether the model could reproduce certain qualitative behaviors of memory consolidation observed in humans, such as graded retention of multiple items or winner-take-all selection of dominant traces, which is based purely on

its underlying dynamics. In the end, this approach allowed me to test whether a relatively simple mathematical framework can capture both the quantitative retention of individual memories and the qualitative selection processes that emerge in more complex memory systems. The source code for the project is available at this URL.

## 2 Methods

### 2.1 Modeling Single Memory Dynamics

I first describe the evolution of a single memory trace by separating its short-term and long-term components.

#### Short-Term Memory (STM)

The instantaneous dynamics of the STM trace  $S(t)$  are governed by

$$\dot{S}(t) = c I(t) - (\alpha + \beta) S(t). \quad (1)$$

Here:

- $S(t)$  is the strength of the short-term memory at time  $t$ .
- $c I(t)$  represents external input encoding, with  $c$  quantifying contextual encoding level.
- $\alpha$  is the intrinsic decay rate of STM, which is often relatively large to model its rapid fading.
- $\beta$  is the consolidation rate from STM into LTM.

The equation (1) is a first-order linear ordinary differential equation with constant coefficients. Its general solution, given the initial condition  $S(0) = S_0$ , can be written in closed form using the integrating factor or variation of constants method as follows

$$S(t) = S_0 e^{-(\alpha+\beta)t} + c \int_0^t e^{-(\alpha+\beta)(t-s)} I(s) ds. \quad (2)$$

#### Long-Term Memory (LTM)

Once consolidated, the long-term trace  $L(t)$  evolves according to

$$\dot{L}(t) = \beta S(t - \tau) - \gamma L(t), \quad (3)$$

where:

- $L(t)$  is the strength of the long-term memory at time  $t$ .
- $\beta S(t - \tau)$  captures delayed consolidation from STM into LTM after a latency  $\tau$ .

- $\gamma$  is the decay rate of LTM, reflecting gradual forgetting, which is usually very small to reflect minimal decay, resulting in persistent long-term memory over the duration of the simulation.

The equation (3) is a first-order linear delay ordinary differential equation whose general solution, assuming an initial condition  $L(0) = L_0$  and a known history for  $S(t)$ , can be written:

$$L(t) = L_0 e^{-\gamma t} + \beta \int_0^t e^{-\gamma(t-s)} S(s - \tau) ds. \quad (4)$$

Together, these equations capture the core processes of encoding, consolidation, and decay for a single memory trace for both STM and LTM.

## 2.2 Extending to Multiple Memory Traces

For each trace  $i = 1, \dots, N$ , the separate differential equations for its short-term and long-term strengths can be presented as follows:

### Short-Term Memory (STM)

$$\dot{S}_i(t) = c_i I(t) - (\alpha + \beta) S_i(t) - \kappa S_i(t) \sum_{j \neq i} S_j(t) \quad (5)$$

- $c_i I(t)$ : external input scaled by trace sensitivity.
- $(\alpha + \beta) S_i$ : STM decay ( $\alpha$ ) plus consolidation into LTM ( $\beta$ ).
- $\kappa S_i \sum_{j \neq i} S_j$ : multiplicative inhibition among STM traces.

The term  $\kappa S_i \sum_{j \neq i} S_j$  mathematically implements inhibition that is proportional to both the strength of trace  $i$  (i.e.,  $S_i$ ) and the summed strength of its competitors (i.e.,  $\sum_{j \neq i} S_j$ ), ensuring zero competition if either the target trace or competitors are inactive and increasing competition as both become more active.

The set of equations (5) defines a system of first-order, nonlinear ordinary differential equations due to the multiplicative coupling term  $\kappa S_i(t) \sum_{j \neq i} S_j(t)$ . Because of its nonlinearity, it does not admit a general closed-form solution except in the special case  $\kappa = 0$ , where each trace reduces to a linear ODE solvable via standard integrating-factor techniques. In the general case ( $\kappa > 0$ ), analytic solutions are unavailable and the system must be solved numerically.

### Long-Term Memory (LTM)

$$\dot{L}_i(t) = \beta S_i(t - \tau) + \rho_i \sigma(L_i(t)) - \gamma L_i(t) - \lambda L_i(t) \sum_{j \neq i} L_j(t) \quad (6)$$

where  $\sigma(x) = 1/(1 + \exp[-a(x - \theta)])$

- $\beta S_i(t - \tau)$ : delayed consolidation from STM to LTM.

- $\rho_i \sigma(L_i)$ : self-recurrent excitation of LTM trace  $i$ .
- $\gamma L_i$ : intrinsic decay of long-term memory.
- $\lambda L_i \sum_{j \neq i} L_j$ : multiplicative competition among LTM traces with a similar argument from the STM part.

The sigmoid term  $\sigma(x) = 1/(1 + \exp[-a(x - \theta)])$  is added because it creates a simple threshold and saturation effect. It means that recurrent self-excitation only becomes significant when the long-term trace  $L_i$  exceeds the threshold  $\theta$  and that feedback is limited at high activation levels to prevent runaway growth. Long-term memory formation relies on recurrent feedback to stabilize synaptic changes. Therefore, the term  $\rho_i \sigma(L_i)$  appears only in the LTM equation, as short-term memory is assumed not to use recurrent reinforcement.

The set of equations (6) defines a system of first-order, nonlinear delay differential equations. The presence of a fixed time delay ( $\tau$ ), a sigmoid nonlinearity ( $\sigma(L_i)$ ), and multiplicative competition among traces makes it analytically intractable except in trivial linear limits (e.g.,  $\rho_i = \lambda = 0$ ). Consequently, no general closed-form solution exists, and the dynamics must be obtained through numerical simulation.

## 2.3 Simulation Protocol

In general, the model is a system of nonlinear delay differential equations with multiplicative coupling and sigmoid activation, for which no closed-form solution exists. Therefore, numerical simulations are necessary.

$N = 3$  is selected for all simulations because a two-trace network cannot simultaneously demonstrate robust competitive suppression and graded long-term retention, whereas a four-trace system introduces unnecessary complexity without yielding new qualitative insights. A three-trace configuration is the minimal model that captures the core memory phenomena – namely persistent retention of dominant traces and winner-take-all competition – and allows these behaviors to systematically vary with contextual weights of each trace input ( $c_i$ ) and the self-recurrent strengths within each trace population ( $\rho_i$ ).

### 2.3.1 Single-trace settings

All single-trace simulations were performed using a forward Euler integration scheme with a time step of  $\Delta t = 0.01$  and a total duration of  $T = 100$  time units. Unless otherwise noted, a fixed set of baseline parameters will be used across all runs (Table 1). The encoding strength was set to  $c = 1.0$ , representing a moderately salient input. The short-term memory decay rate was  $\alpha = 0.5$ , while the consolidation rate into long-term memory was set to  $\beta = 0.1$ , allowing gradual transfer from transient to persistent representations. The long-term memory decay rate was set to  $\gamma = 10^{-5}$ , which ensures the stability of consolidated traces or persistent LTM across the simulation period. A consolidation delay of  $\tau = 5.0$  time units was used to model a biologically plausible latency between short-term encoding and long-term integration.

Table 1: Default parameter values for the single-trace memory model

Parameter	Description	Default Value
$c$	Encoding strength (input salience)	1.0
$\alpha$	Short-term memory decay rate	0.5
$\beta$	Consolidation rate (STM→LTM)	0.1
$\gamma$	Long-term memory decay rate	$10^{-5}$
$\tau$	Consolidation delay (time units)	5.0

### Experiment 1: Encoding Strength Variation

To quantify the influence of input salience on long-term retention, encoding strength  $c$  was varied from 0.0 to 2.0 in increments of 0.1 (Table 2). A single square-pulse input ( $I(t) = 1$  for  $t \in [0, 5]$ ) was applied, and the system was simulated until  $t = 100$ . For each value of  $c$ , I recorded the final LTM value  $L(100)$ .

### Experiment 2: Consolidation Delay Variation

To examine the effect of consolidation timing, the delay parameter  $\tau$  was varied from 0 to 15 (step = 1) while holding all other parameters at their baseline values (Table 2). Using the same square-pulse input and simulation protocol as experiment 1, I measured the final LTM value  $L(100)$ .

### Experiment 3: Long-Term Decay Rate Variation

To characterize forgetting dynamics, the long-term decay rate  $\gamma$  was systematically varied from 0.0 to 0.5 in increments of 0.05 (Table 2). Similarly, using the same square-pulse input and simulation protocol as experiments 1 and 2, I recorded the final LTM value  $L(100)$ .

Table 2: Parameter ranges and hypotheses for single-trace simulations

Parameter	Hypothesis	Range (step)
$c$	Increasing contextual encoding strength will proportionally raise the final long-term memory strength	0.0–2.0 (0.1)
$\tau$	Increasing consolidation delay will postpone transfer from STM to LTM, modestly reducing final LTM strength when the short-term trace cannot be sustained.	0–15 (1)
$\gamma$	Increasing long-term decay rate will accelerate forgetting, resulting in lower final LTM strength and a steeper decay slope	0–0.5 (0.05)

## 2.3.2 Multiple-trace settings

In the multi-trace simulations, a network of  $N = 3$  interacting memory traces was implemented, with all base parameters held constant unless explicitly varied (Table 3). Each trace was assigned an identical encoding strength ( $c_i = 1.0$ ), a short-term decay rate of  $\alpha = 0.5$ , and a consolidation

rate of  $\beta = 0.1$ . The long-term decay rate was fixed at  $\gamma = 10^{-5}$  to ensure minimal forgetting over the simulation interval, and a uniform consolidation delay of  $\tau = 5.0$  time units was imposed to model delayed transfer from STM to LTM. Competitive interactions were introduced via multiplicative inhibitory coupling in both STM ( $\kappa = 0.5$ ) and LTM ( $\lambda = 0.5$ ), while self-recurrent excitation was set to zero ( $\rho_i = 0$ ) by default (no competition). These parameter settings established a controlled baseline against which targeted manipulations could be performed in the latter experiments.

Table 3: Default parameter values for the multi-trace memory model

Parameter	Description	Default Value
$c_i$	Encoding strength (input salience)	1.0
$\alpha$	Short-term memory decay rate	0.5
$\beta$	Consolidation rate (STM→LTM)	0.1
$\gamma$	Long-term memory decay rate	$1 \times 10^{-5}$
$\tau$	Consolidation delay (time units)	5.0
$\kappa$	STM inhibition strength	0.5
$\lambda$	LTM competition strength	0.5
$\rho_i$	Self-recurrent excitation strength	0.0

Each following experiment was simulated for 100 time units using a forward Euler integrator ( $\Delta t = 0.01$ ), with 30 replicates per experiment to estimate mean final LTM strength and variability.

#### Experiment 4: Competition vs. Encoding Strength

This tested whether competitive interactions in long-term memory amplify differences in contextual encoding strength when multiple items are learned together. I used a 2×2 design that crossed encoding heterogeneity (all three traces equally salient:  $c_i = [1, 1, 1]$  vs. varied salience:  $c_i = [1, 0.5, 0.1]$ ) with competition (disabled:  $\lambda = 0$  vs. enabled:  $\lambda = 0.5$ ), as detailed in Table 4. All other parameters, including recurrent excitation, were held at their baseline values.

I anticipated that when all traces have equal salience input levels, competition would have minimal impact on final LTM strength. In contrast, when encoding strengths differ, enabling competition was expected to exaggerate these differences – resulting in a winner-take-all outcome favoring the trace with the highest input salience.

Table 4: Experiment 4: Competition vs. Encoding Strength

Condition	LTM Competition ( $\lambda$ )	Encoding Strength $c_i$
No Competition, Homogeneous Encoding	0.0	[1, 1, 1]
No Competition, Heterogeneous Encoding	0.0	[1, 0.5, 0.1]
With Competition, Homogeneous Encoding	0.5	[1, 1, 1]
With Competition, Heterogeneous Encoding	0.5	[1, 0.5, 0.1]

#### Experiment 5: Recurrent Strength under Low vs. High Decay

This experiment evaluated whether self-recurrent excitation can protect memories from accelerated forgetting. Encoding heterogeneity ( $c_i = [1.0, 0.5, 0.1]$ ) and active competition ( $\lambda = 0.5$ ) were held constant while long-term decay ( $\gamma$ ) and recurrent strength ( $\rho_i$ ) were varied factorially (Table 5). I hypothesized that when the decay rate is low, the changes in recurrent excitation would have little effect on final LTM strength, but under high decay, increasing recurrent strength would substantially rescue consolidation and improve retention.

Table 5: Experiment 5: Recurrent Strength under Low vs. High Decay

Condition	Long-Term Decay ( $\gamma$ )	Recurrent Strength ( $\rho_i$ )
Low Decay	$1 \times 10^{-5}$	[0.0, 0.5, 1.0]
High Decay	0.05	[0.0, 0.5, 1.0]

### Experiment 6: Recurrent Compensation for Low Encoding Strength

This experiment tested whether self-recurrent excitation can rescue memory traces that are disadvantaged due to weak initial encoding. Encoding strengths were deliberately chosen so that trace 2 had substantially lower salience ( $c_2 = 0.1$ ) than traces 1 and 3 ( $c_1 = 1.0$ ,  $c_3 = 0.3$ ), thereby biasing the consolidation process against trace 2. To assess whether recurrent dynamics could compensate for this disadvantage, I varied the recurrence profile  $\rho = (\rho_1, \rho_2, \rho_3)$  across conditions, all under active LTM competition ( $\lambda = 0.5$ ).

Four recurrent configurations were compared (Table 6): a baseline with no recurrence; a selective profile enhancing only trace 2; a moderate differential profile in which all traces received recurrent excitation, but trace 2 had a slight advantage; and a high differential profile in which trace 2 received much stronger recurrence than the others. I hypothesized that Trace 2, although initially weak, would benefit from recurrent feedback – specifically in the selective and differential conditions – and that this could reverse its disadvantage in the final LTM outcome.

Table 6: Experiment 6: Recurrent Compensation for Low Encoding Strength

Condition	Recurrent Strength ( $\rho_1, \rho_2, \rho_3$ )	Encoding Strength $c_i$
Baseline Recurrence	[0.0, 0.0, 0.0]	[1.0, 0.1, 0.3]
Selective Recurrence	[0.0, 1.0, 0.0]	[1.0, 0.1, 0.3]
Moderate Differential	[0.5, 0.55, 0.5]	[1.0, 0.1, 0.3]
High Differential	[0.5, 1.0, 0.5]	[1.0, 0.1, 0.3]

## 3 Results

### 3.1 Single-trace Simulations

In this simulation (Figure 1), the short-term memory trace rises quickly during the 5-time unit input interval and decays shortly after, reflecting fast encoding and intrinsic fading. Due to the long consolidation delay ( $\tau = 5$ ), the long-term memory trace begins to accumulate only after the

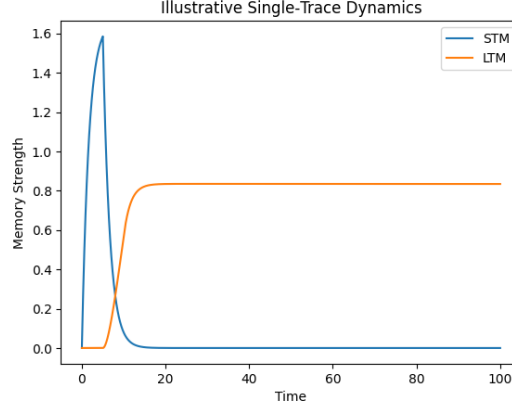


Figure 1: Dynamics of STM and LTM in a single memory trace

input ends, resulting in a delayed but sustained rise. The near-zero LTM decay rate ( $\gamma = 10^{-5}$ ) allows the consolidated memory to persist almost indefinitely, demonstrating effective long-term retention despite the delayed onset.

To investigate how different model parameters shape memory retention, I systematically varied one parameter at a time while holding all others fixed at their baseline values. For each condition, I recorded the final LTM strength at  $t = 100$  time units following a 5-unit input pulse. The observed effects are summarized in Experiments 1–3 below.

### 3.1.1 Experiment 1: Effect of Encoding Strength ( $c$ )

Contextual encoding strength significantly modulated final LTM strength. As predicted in Table 2, final memory increased approximately linearly with  $c$ , rising from 0 at  $c = 0$  to approximately 1.67 at  $c = 2.0$  (Figure 2). The relationship exhibited a near-perfect linear fit (slope  $\approx 0.834$ ,  $R^2 = 1.00$ ), confirming that greater input salience produces proportionally stronger consolidated LTM strength.

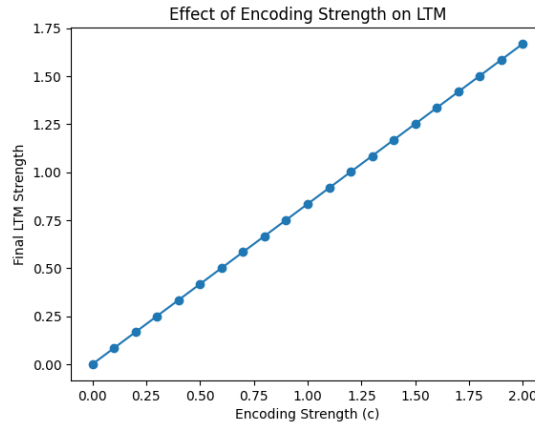


Figure 2: Effect of Encoding Strength ( $c$ ) on Final LTM Strength



### 3.1.2 Experiment 2: Effect of Consolidation Delay ( $\tau$ )

Contrary to the hypothesis that longer delays reduce consolidation efficacy, final LTM strength remained essentially constant (mean  $\approx 0.8343$ ; range 0.83420–0.83433) across  $\tau = 0$  to 15 (Figure 3). This invariance suggests that delay has a negligible impact on long-term retention as long as short-term memory activation does not fully diminish during the delay period.

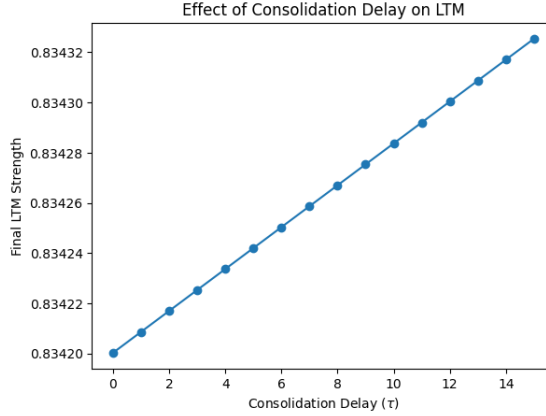


Figure 3: Effect of Consolidation Delay ( $\tau$ ) on Final LTM Strength

### 3.1.3 Experiment 3: Effect of LTM Decay Rate ( $\gamma$ )

In agreement with hypothetical expectations, final LTM strength declined rapidly as the LTM decay rate  $\gamma$  increased. Memory strength dropped from approximately 0.835 at  $\gamma = 0$  to nearly zero for  $\gamma \geq 0.05$  (Figure 4). Even small increases in the decay rate led to substantial forgetting by  $t = 100$ , demonstrating that decay is a dominant factor in determining long-term retention in the absence of reactivation or maintenance mechanisms.

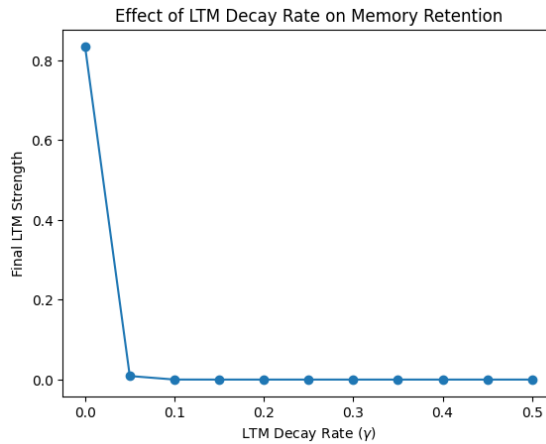


Figure 4: Effect of LTM Decay Rate ( $\gamma$ ) on Final LTM Strength

## 3.2 Multiple-trace Simulations

### 3.2.1 Experiment 4: Competitive Interactions in LTM

Experiment 4 investigated whether competitive interactions in long-term memory amplify differences in encoding strength when multiple items are learned simultaneously.

- Homogeneous encoding (Figure 5a): When all traces were equally salient, disabling competition produced modest and equivalent final LTM strength across traces. Enabling competition uniformly suppressed consolidation, resulting in minimal retention for all items.
- Heterogeneous encoding (Figure 5b): When encoding strengths varied, disabling competition yielded graded retention proportional to salience (trace 1 > trace 2 > trace 3). Enabling competition produced a winner-take-all outcome, with the most salient trace (trace 1) selectively preserved while lower-salience traces (traces 2 and 3) were nearly eliminated.

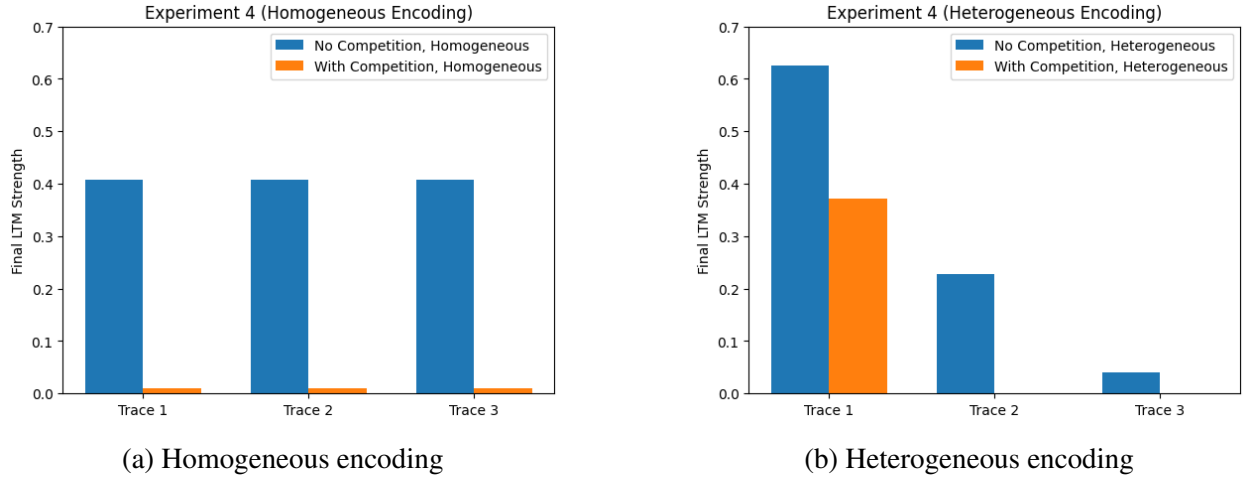


Figure 5: Final LTM strength for each trace under homogeneous and heterogeneous encoding.

### 3.2.2 Experiment 5: Effects of Decay and Recurrent Strength on LTM Consolidation

The purpose of experiment 5 was to examine how varying decay rates and recurrent excitation influence long-term memory consolidation. In particular, the experiment sought to determine how recurrence compensates for rapid decay and equalizes retention across traces.

- Under low decay (Figure 6a), minimal recurrence produced a clear winner-take-all effect – only the highest-salience trace achieved substantial final LTM strength while all others were effectively suppressed – whereas progressively increasing recurrence reduced led to convergence of retention across traces, demonstrating that even modest recurrent feedback counteracts competitive suppression and promotes uniform long-term memory consolidation.

- Under high decay (Figure 6b), final LTM strength was essentially zero without recurrence, reflecting rapid forgetting. Introducing any nonzero recurrence immediately rescued consolidation and produced uniformly high retention across all traces, identical to the low-decay condition, demonstrating that high decay has no effect in the presence of recurrence.

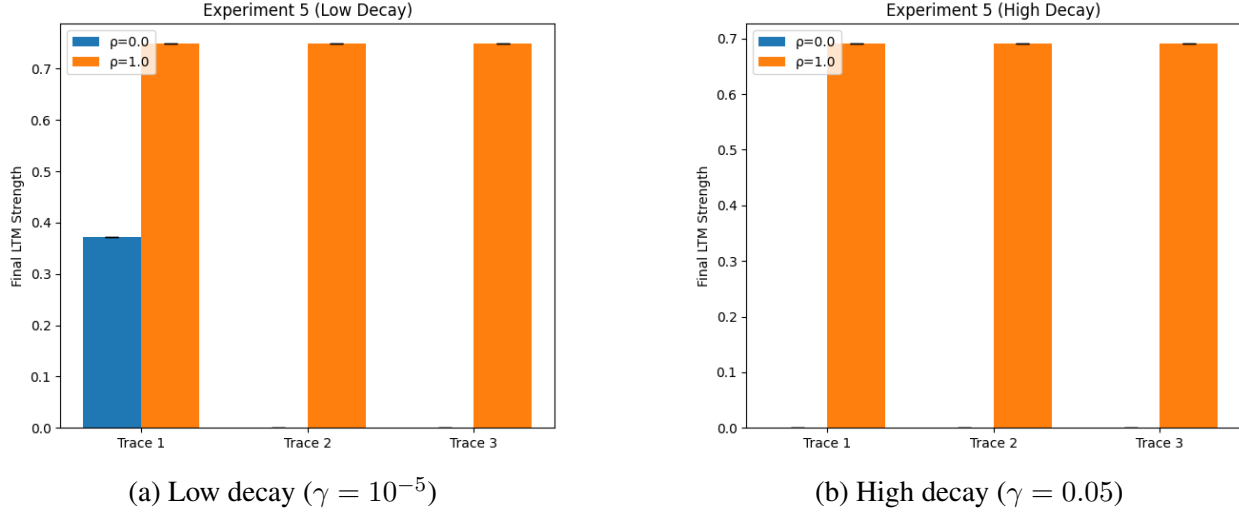


Figure 6: Final LTM strength for each trace under low and high decay

### 3.2.3 Experiment 6: Effects of Differential Recurrent Excitation on Time Course of Memory Consolidation

The purpose of Experiment 6 was to examine how varying patterns of recurrent excitation affect the time evolution of LTM consolidation across three distinct memory traces. Each trace was initialized with a different encoding strength, and the recurrent excitation parameter  $\rho$  was varied across four qualitatively distinct conditions.

- Baseline ( $\rho = [0, 0, 0]$ ): No recurrent excitation was applied to any trace. As shown in Figure 7, only trace 1 (with the highest encoding strength) survived consolidation. Traces 2 and 3 were nearly eliminated by competition. This outcome reflects a passive one-winner-take-all regime, in which stronger initial encoding is sufficient to dominate in the absence of recurrent reinforcement.

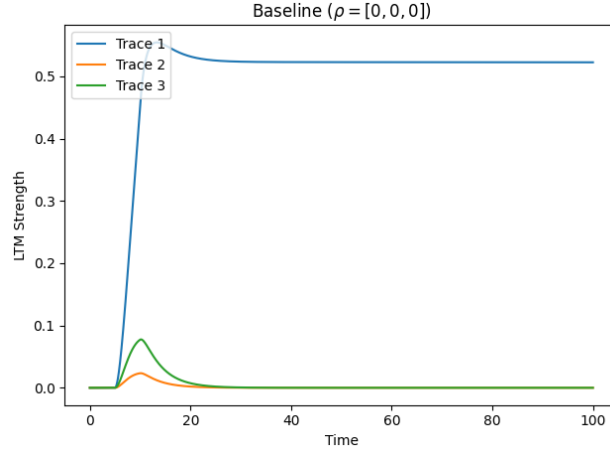


Figure 7: Time course for LTM in the Baseline condition. No recurrent excitation is applied to any trace ( $\rho = [0, 0, 0]$ ).

- Selective Recurrent ( $\rho = [0, 1, 0]$ ): Recurrent excitation was applied only to trace 2. In figure 8, trace 2, despite having the weakest contextual encoding, outperformed traces 1 and 3. This is a one-winner-take-all outcome in which recurrence selectively rescues and amplifies a low-salience trace.

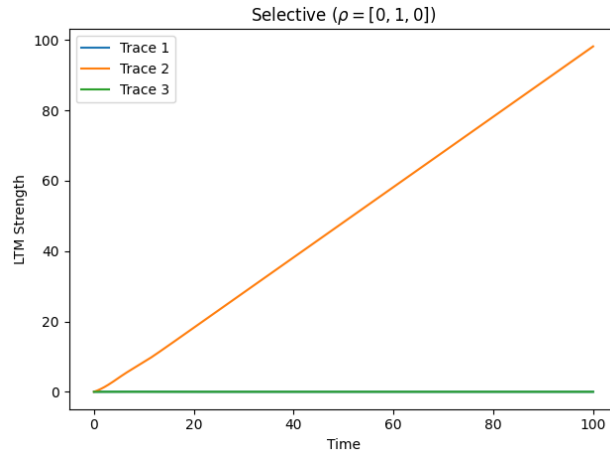


Figure 8: Time course for LTM in the Selective condition. Recurrent excitation is applied only to Trace 2 ( $\rho = [0, 1, 0]$ ).

- Moderate Differential Recurrent ( $\rho = [0.5, 0.55, 0.5]$ ): All traces received recurrent excitation but trace 2 has a slightly higher salience. As seen in figure 9, all traces consolidated well and persisted over time, and trace 2 was slightly favored. This reflects a graded retention regime where multiple memory traces persist together, and small differences in recurrence produce only modest asymmetries in the retention of LTMs.

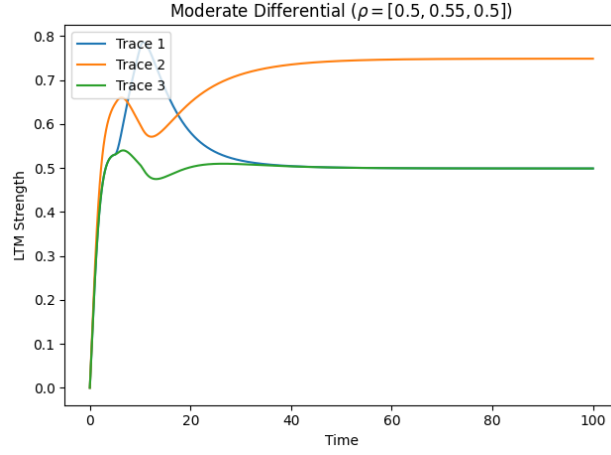


Figure 9: Time course for LTM in the Moderate Differential condition. Trace 2 receives slightly more excitation ( $\rho = [0.5, 0.55, 0.5]$ ).

- High Differential Recurrent ( $\rho = [0.5, 1.0, 0.5]$ ): All traces received recurrent excitation, but trace 2 got a much larger recurrent boost. Figure 10 shows that despite its weak initial encoding, trace 2 outcompeted and suppressed the other two traces, producing a clear one-winner-take-all outcome in which strong recurrent feedback overrides the initial disadvantage in salience.

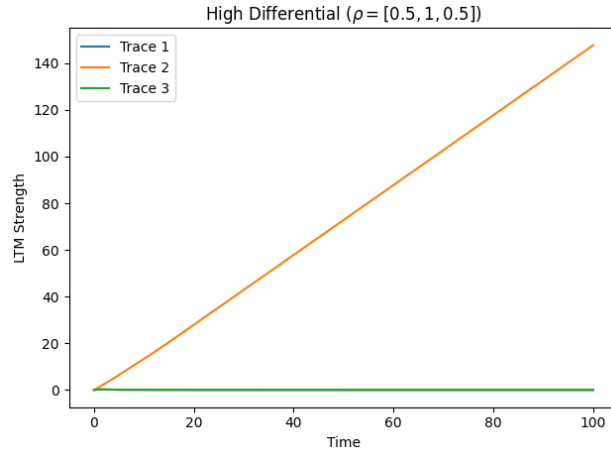


Figure 10: Time course for LTM in the High Differential condition. Trace 2 receives substantially stronger excitation ( $\rho = [0.5, 1, 0.5]$ ).

## 4 Discussion

This project examined whether a relatively simple mathematical model could reproduce some basic behaviors of memory consolidation. When considering a single memory trace, the model illustrated that stronger inputs led to stronger and longer-lasting memories, and forgetting was driven mainly by how quickly long-term memory faded rather than by when consolidation occurred. When multiple memories were considered together, the model showed that stronger memories can suppress weaker ones, while feedback mechanisms can protect weaker memories from being lost – and even allow them to become dominant if the recurrent feedback is strong enough. These results demonstrate that a relatively simple model is able to capture some qualitative behaviors of memory consolidation, including short-term and long-term memory dynamics, graded retention, and competitive selection of memories or winner-takes-all.

However, not all outcomes matched the initial hypotheses. For instance, varying the consolidation delay  $\tau$  produced almost no change in the final long-term memory strength, which was contrary to our initial expectation that longer delays would reduce consolidation efficacy. This suggests that the model’s handling of delay is overly simplistic and may not reflect the true timing-dependent processes observed in biological memory systems.

There are several key features missing from the current model. First, the equations assume fixed parameters for decay, consolidation, and inhibition. In real neural systems, these parameters can change with time or depend on other conditions such as attention and neuromodulation. Second, the deterministic nature of the model ignores the role of randomness or noise, which can be important in explaining variability in memory recall and phenomena like spontaneous recovery. On top of that, the competition and recurrent excitation are implemented with simple multiplicative terms and a basic sigmoid function. This oversimplifies the complex, nonlinear interactions that occur in the brain. Moreover, the model also omits other crucial factors – such as dynamic synaptic plasticity, heterogeneous network interactions, neuromodulatory influences, and many other factors – that are essential for capturing the full complexity and variability of biological memory consolidation.

In conclusion, this model can reproduce some key features of memory consolidation, such as strength-dependent retention, competitive dynamics, and recurrent-based retention, but it is still a simple version of a much more complex biological process and only partly answers the original research question. Future work should include adaptive, time-varying parameters, stochastic components, and more sophisticated representations of neural network interactions to better capture the full complexity of memory consolidation dynamics. To fully understand memory consolidation, we will need to compare these models with brain recordings, controlled behavior studies, and reports of personal experience, since real memory depends on ongoing brain changes, chemical signals, and a person’s attention, emotions, and context.

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