# MAT1856/APM466 Assignment 1

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## Fundamental Questions - 25 points

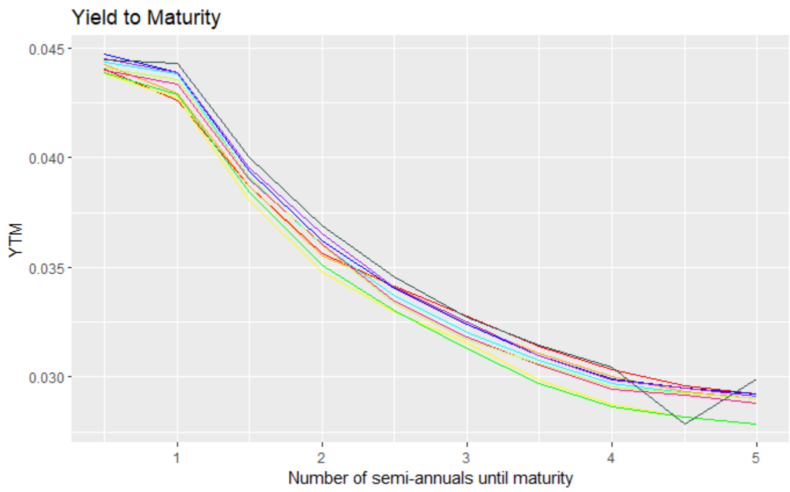
1.
   (a) Simply printing money causes its devaluation, therefore inflation; bonds are basically loans, so issuing them does not cause such troubles.
   (b) The yield curve flattens when long-term interest rates fall more than short-term interest rates and vice versa. Flat yield curve implies an uncertain economic situation: this could happen at the end of a period of high economic growth (that eventually leads to inflation) or it may appear from time to time when the central bank is expected to raise interest rates.
   (c) Quantitative easing is, in fact, increasing the money supply, while also reducing interest rates. The Fed employed this method by purchasing a big amount of assets to help the economic activity at the beginning of the COVID-19 pandemic.

2. *CAN 0.25 Feb 23, CAN 0.25 Aug 23, CAN 0.75 Feb 24, CAN 2.75 Aug 24, CAN 3.75 Feb 25, CAN 0.5 Sep 25, CAN 0.25 Mar 26, CAN 1 Sep 26, CAN 1.25 Mar 27, CAN 2.75 Sep 27*
   This exact data is chosen for a couple of reasons:
   - Their coupon rates are close. There are some relative outliers like *CAN 3.75 Feb 25*, but there were definitely higher values like 8 or 9.
   - The gap between maturing dates is around 6 months in all of them (there was a one month shift once, but it's okay). It is reasonable because usually coupon payments happen once in 6 months, so it would be the most precise case.

3. In this case eigenvalues and eigenvectors are our instruments to handle bigger data. Basically, we are able to create a "summary" for larger datasets with certain parameters that are much easier to analyze. The eigenvectors - principal components - will be the directions of the new space, while the eigenvalue is the magnitude.

## Empirical Questions - 75 points

4.
   (a) To calculate Yield to Maturity, I gathered all the needed data and used the bond.yield() R function to calculate it, sorting out all the needed arguments.
   YTM curve:




(b) To calculate spot rates for the first bond (which matures in less than 0.5 year - there are no more coupon payments) I used:

$$SpotRate = -\frac{log\frac{P}{N}}{T}$$

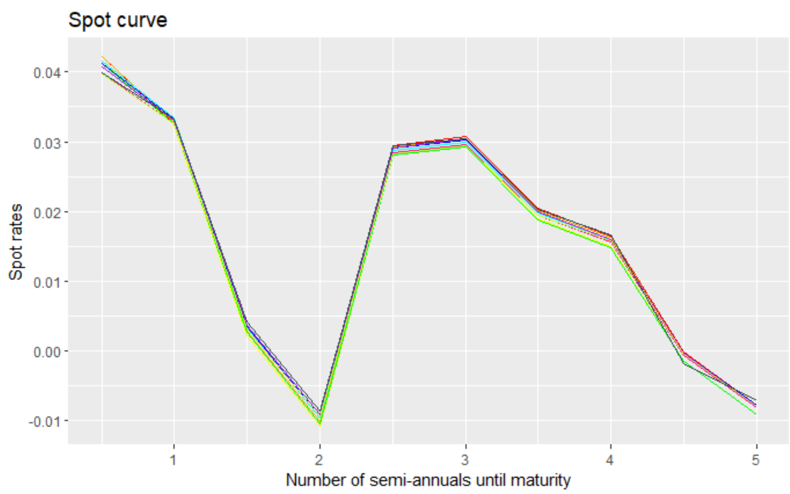
(this is for zero coupon bonds, so that is why we needed a bond that has its maturity date). Or we also just could have used the fact that for the bonds that mature in less than a year, their YTMs and spot rates are the same.

To calculate every bond that matures next, we will use bootstrapping: calculating the spot rates of the next bond, using the spot rates of the previous one that we already know.

$$SpotRate = -1 * \frac{log\frac{DP-PV}{p^n}}{tn},$$

where PV is calculated using previous spot rates

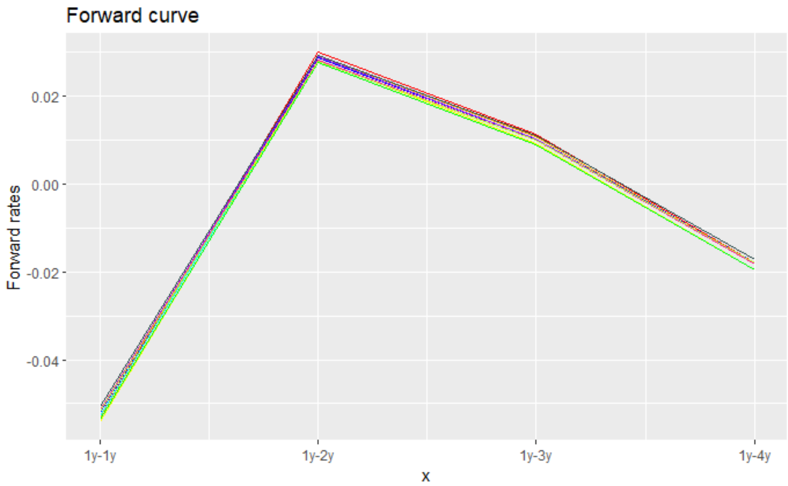
Spot curve:




(c) Using the simplified formula from the lecture and 4(b) since we need spot rates, we get the forward rates for the given time periods (1-1 yrs, 1-2 yrs, 1-3 yrs, 1-4 yrs).

$$ForwardRate(t1, t2) = \frac{r2 * t2 - r1 * t1}{t2 - t1}$$

Forward curve:




5. YTM:

```
> covM
             [,1]         [,2]         [,3]         [,4]         [,5]
[1,] 0.0001029264 0.0001323244 1.614435e-04 6.696290e-05 0.0001977409
[2,] 0.0001323244 0.0002352368 2.787576e-04 1.256705e-04 0.0003340396
[3,] 0.0001614435 0.0002787576 3.812997e-04 8.473866e-05 0.0004109140
[4,] 0.0000669629 0.0001256705 8.473866e-05 6.884545e-04 0.0002606619
[5,] 0.0001977409 0.0003340396 4.109140e-04 2.606619e-04 0.0005515904
```

Forward rates:

```
> covFM
              [,1]          [,2]          [,3]          [,4]          [,5]
[1,]  0.0001827765 -0.0003309745 -0.0009374043  0.0004288071  0.0001827765
[2,] -0.0003309745  0.0007640568  0.0019503924 -0.0008848184 -0.0003309745
[3,] -0.0009374043  0.0019503924  0.0064406071 -0.0029482324 -0.0009374043
[4,]  0.0004288071 -0.0008848184 -0.0029482324  0.0013667141  0.0004288071
[5,]  0.0001827765 -0.0003309745 -0.0009374043  0.0004288071  0.0001827765
```

6. The largest eigenvalue and its eigenvector are important to understand the variability and variation of the data. Eigenvalue explains how variable the data(its biggest variation) is in the eigenvector's direction.

YTM:

```
> evalM <- eigen(covM)$values # eigenvalue
> evalM
[1] 1.318054e-03 5.681202e-04 3.125276e-05 2.754514e-05 1.453576e-05
> evecM <- eigen(covM)$vectors # eigenvector
> evecM
           [,1]       [,2]       [,3]        [,4]        [,5]
[1,] -0.2303863 -0.1230822  0.1324811  0.89937187 -0.32457959
[2,] -0.3916005 -0.1869448 -0.2280841  0.21274113  0.84523344
[3,] -0.4694747 -0.3598479 -0.6528249 -0.22381323 -0.41692967
[4,] -0.4251926  0.8893197 -0.1601018  0.01895049 -0.04827088
[5,] -0.6263992 -0.1718210  0.6918190 -0.30890123 -0.06378163
```

Forward rates:

```
> evalFM <- eigen(covFM)$values
> evalFM
[1]  8.676013e-03  1.921049e-04  5.524745e-05  1.356568e-05 -5.707481e-20
> evecFM <- eigen(covFM)$vectors
> evecFM
           [,1]       [,2]        [,3]        [,4]          [,5]
[1,]  0.1280407 -0.3182025 -0.61763206  0.02972166  7.071068e-01
[2,] -0.2669352  0.8321364 -0.48051722  0.07346944  1.026956e-15
[3,] -0.8604644 -0.2722265 -0.05866400 -0.42667586  4.579670e-16
[4,]  0.3944082  0.1758862 -0.05218247 -0.90043501  1.450229e-15
[5,]  0.1280407 -0.3182025 -0.61763206  0.02972166 -7.071068e-01
```

## References and GitHub Link to Code

GitHub:
https://github.com/anhxlina/APM466A1

References:
https://www.rdocumentation.org/packages/jrvFinance/versions/1.4.3/topics/bonds
https://www.investopedia.com/terms/f/flatyieldcurve.asp
https://seco.risklab.ca/apm466-mat1856-library/