Notes for CPSC 4148: Theory of Computation

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Contents

0 Mathematical Review

0.1 Sets

Sets are **unordered collections** of **elements**, or **members**, commonly denoted using curly brackets. A set of two elements is called an **unordered pair**.

$$\begin{aligned}
&\textbf{Let:}\\
&U = \{-1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}\\
&A = \{1, 2, 4, 5, 6\}\\
&B = \{1, 3, 5, 7, 9\}\\
&A \subset U\\
&B \subset U
\end{aligned}$$

Sets do **not** count for the multiplicity of elements; each appears only once in the result of set operations. Below are some characteristics of an example set A, the set of **integers** $\mathbb{Z}$, the set of **natural numbers** $\mathbb{N}$, the **empty** set $\emptyset$, and basic set **operations** on sets A and B.

$$\begin{aligned}
2 \in A &\iff 2 \in \{1, 2, 4, 5, 6\}\\
7 \notin A &\iff 7 \notin \{1, 2, 4, 5, 6\}\\
\mathbb{N} &= \{1, 2, 3, \ldots\}\\
\mathbb{Z} &= \{\ldots, -1, 0, 1, \ldots\}\\
\emptyset &= \{\}\\
A \cup B &= \{1, 2, 3, 4, 5, 6, 7, 9\}\\
A \cap B &= \{1, 5\}\\
\overline{A} = \{-1, 0, 3, 6, 7, 8, 9\} &\iff \overline{A} = U - A\\
\overline{B} = \{-1, 0, 2, 4, 6, 8\} &\iff \overline{B} = U - B\\
A \times B &= \{(1, 1), (1, 3), (1, 5), \ldots, (6, 5), (6, 7), (6, 9)\}
\end{aligned}$$

Note: $A \times B$ is the **cross**, or **Cartesian**, **product** of two sets. Each element of each set in the set of sets is multiplied together. The set $\mathbb{N}^2 = \mathbb{N} \times \mathbb{N}$ consists of all ordered pairs of natural numbers. It can also be written as $\{(i, j) | i, j \geq 1\}$

0.2 Sequences and Tuples

Sequences are **ordered collections** of **elements**, or **members**, commonly denoted using parentheses. Finite sequences are often called **tuples**; the **2-tuple** is referred to as an **ordered pair**.

$$\begin{aligned}
&\textbf{Let:}\\
&A = (1, 2, 3)\\
&A \neq (2, 3, 1)
\end{aligned}$$

0.3 Funtions and Relations

n	f(n)
0	1
1	2
2	3
3	4
4	0

Table 1: $f(n) = n + 1 \mod 5$

Note: $n+1 \mod 5$ is of the form $dividend \mod divisor$. Subtract the divisor from the dividend over and again until the remainder is less than the divisor. That remainder is your answer.

n	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Table 2: $g(i, j) = i + j \mod 4$

0.4 Graphs

0.5 Strings and Languages

0.6 Boolean Logic

a	b	$(\neg a \vee b)$	$(b \Rightarrow a)$	$((a \Rightarrow a) \wedge (b \Rightarrow a))$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	1
1	1	1	1	1

Table 3:

0.7 Proofs

Proof. □

1 Regular Languages

1.1 Finite Automata

References

[1] Michael Sipser. *Introduction to the Theory of Computation.* Course Technology, Boston, MA, third edition, 2013.