Role of Two-qubit Entangling Operators in Preventing the Onset of Chaos in Duopoly Game

Contributed Talk

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Classically, game theory is the study of the decision-making process involving two or more competing players. It has a wide range of applications, including economics, biology, and the social sciences, among others. The advancement of quantum mechanics in the 20th century led to the incorporation of quantum mechanical tools into game theory. David Meyer pioneered the study of quantum game theory by showing the significance of quantum strategies over classical strategies. Since then, quantization schemes such as the Eisert- Wilkens-Lewenstein (EWL) scheme and the Marinatto-Weber (MW) scheme have been proposed to understand the role of entanglement in quantum game theory. In recent years, researchers have focused on incorporating quantum mechanical concepts into economics. Notably, in 2002, Li, Du, and Massar proposed a quantization scheme based on a continuous variable system to analyze duopoly markets. In economics, a duopoly represents a market structure in which two competing manufacturers strategically interact by setting prices or determining quantities of commodities. While duopoly games have been widely explored in continuous variable systems, we aim to study these games using a modified Eisert-Wilkens- Lewenstein scheme. The modified EWL scheme admits a wider range of two-qubit entangling operators compared to the well-known EWL scheme, which is restricted to controlled unitaries, with CNOT being the known two-qubit entangling operator. Moreover, using this scheme allows for an understanding of game theory from the perspective of quantum operators. Next, the strategies of the players are considered dynamic, wherein the players undergo strategic adjustments in each period to attain the Nash equilibrium. By studying the Cournot duopoly game in dynamic settings following key-findings emerge: To begin, monopoly in a duopoly game can be avoided with the use of special perfect entanglers. Secondly, the numerical analysis highlights the two-qubit entangling operators which can stabilize a chaotic system or at least delay chaos. Finally, there exists an appropriate choice of initial state, speed of adjustments, and entangling operators that can decrease the sensitivity of the system. In short, while we know the importance of entangling operators in quantum game theory, the paper indicates the significance of operators in the context of a chaotic system [4].

References

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