Three-rotor Problem: Classical and Quantum Aspects

Invited Talk 14

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The quantum three-rotor problem concerns the dynamics of three equally massive particles moving on a circle subject to pairwise attractive cosine potentials and can model coupled Josephson junctions. Classically, it displays order-chaos-order behavior with increasing energy as well as a band of energies where the dynamics appears to be ergodic, mixing and possibly globally chaotic. We also find a geometric cascade of isochronous and period-doubling bifurcations in a family of periodic orbits, that accumulates at the libration to rotation threshold energy around which widespread chaos sets in. The quantum system admits a dimensionless coupling with semiclassical behavior at strong coupling. Single valuedness of stationary state wavefunctions allow for nontrivial boundary conditions in the quantum theory. Restricting to periodic 'relative' wave functions, perturbative and harmonic approximations capture the spectrum at weak coupling and that of low-lying states at strong coupling. More generally, the cumulative distribution of energy levels obtained by numerical diagonalization is well-described by a Weyl-like semiclassical estimate. However, the system has an $S3 \times Z2$ symmetry that is obscured when working with relative angles. By exploiting a basis for invariant states, we obtain the spectrum restricted to the identity representation. To uncover universal quantum hallmarks of chaos, we partition the spectrum into energy windows where the classical motion is regular, mixed or chaotic and unfold each separately. At strong coupling, we find striking signatures of transitions between regularity and chaos in spacing distributions and the number variance. Some nonuniversal features in the number variance and spectral form factor are also uncovered. Deviations from Poisson spacings at asymptotically low and high energies are explained by quantum harmonic and free-rotor spectra projected to the identity representation at strong and weak coupling. Interestingly, the degeneracy of free-rotor levels admits an elegant formula that we deduce using properties of Eisenstein primes.