

Study of Propagation Dynamics of Optical Beam in an Inhomogeneous Nonlocal Nonlinear Medium

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This study investigates the propagation of optical beams in nonlocal nonlinear media, with a focus on the effects of inhomogeneous diffraction profiles. The Lagrangian Variation Method (LVM) is employed to derive a set of coupled ordinary differential equations governing the evolution of key beam parameters. These equations are numerically solved using the Runge-Kutta (RK4) method and the split-step Fourier transform technique. Our results indicate that in homogeneous media, the beam characteristics remain invariant, whereas in inhomogeneous media, the phase-front curvature and beam width increase as the beam propagates. These findings provide insights into pulse propagation in inhomogeneous optical fibers.

Discussion: The propagation of the optical beams in the nonlocal nonlinear medium is described by the nonlocal nonlinear Schrodinger equation (NNLSE)

$$i\frac{\partial\psi}{\partial z} + \mu(z)\frac{\partial^2\psi}{\partial x^2} + \rho\psi \int_{-\infty}^{\infty} R(x-\xi)I(\xi, z)d\xi = 0 \quad (4)$$

Here σ is the width of the response function. In real fibers, the core medium is often non-uniform, with potential deformities arising from manufacturing defects or variations in the lattice parameters of the fiber material. To better understand pulse propagation in such fibers, we are considering various diffraction-decreasing profiles in this study as given below [1,2,3], Constant diffraction profile:

$$\mu(z) = \mu_0 \quad (5)$$

Gaussian diffraction profile:

$$\mu(z) = \mu_0 \exp\left(-\beta \frac{z^2}{L^2}\right) \quad (6)$$

Hyperbolic diffraction profile:

$$\mu(z) = \mu_0 \left(\frac{L}{(\beta - 1)z + L} \right) \quad (7)$$

Solving the NNLSE Eq. (4) directly is challenging, and due to the lack of an exact solution to Eq. (4), But in the case of the strongly nonlocal nonlinear medium the nonlocal NLSE given Eq.(4) can be reduced to the linear form by using Snyder – Mitchel model as follows,

$$i\frac{\partial\psi}{\partial z} + \mu(z)\frac{\partial^2\psi}{\partial x^2} + \rho\psi \frac{P_0}{\sqrt{\pi}\sigma} \left(1 - \frac{x^2}{\sigma^2}\right) = 0. \quad (8)$$

we employ the Lagrangian Variation Method (LVM), to solve the above equation, by considering the solution of the Eq. 4 in the form of a Gaussian profile.

$$\psi(x, z) = A(z) \exp\left(-\frac{x^2}{2\omega^2(z)}\right) \exp[ic(z)x^2 + i\theta(z)] \quad (9)$$

Where, $A(z)$ is amplitude, $\omega(z)$ is beam width, $c(z)$ is the phase front curvature, and $\theta(z)$ is the phase of the beam. By using the Lagrangian density corresponding to the Eq. (8), we will find the Lagrangian of the system by using Rayleigh-Ritz optimization procedure. By using Euler-Lagrange equation, we will find the fol-

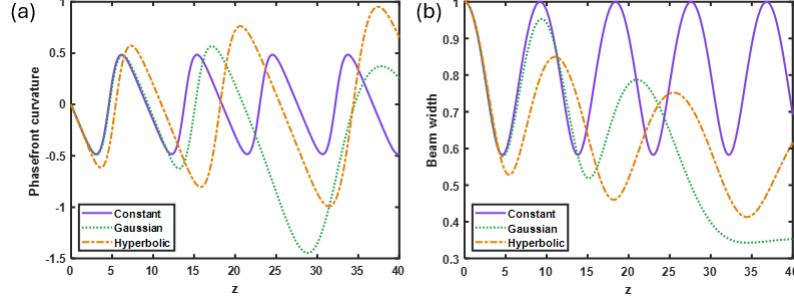


Figure 19: comparison of (a) phase front curvature and (b) beam width for constant Gaussian and Hyperbolic diffraction profile.

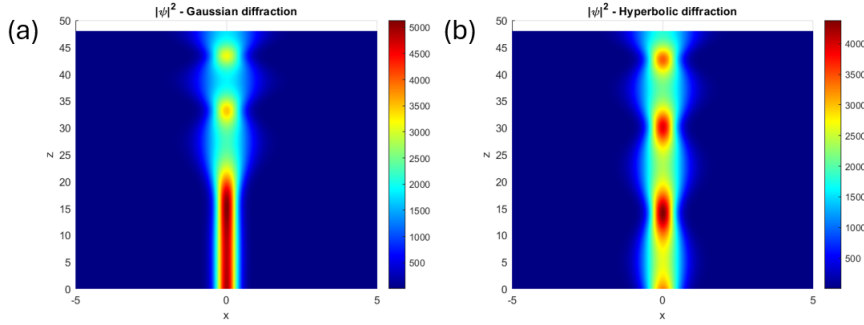


Figure 20: Comparison of (a) phase front curvature and (b) beam width for constant Gaussian and Hyperbolic diffraction profile

lowing three coupled ordinary differential equation, which describes the evolution characteristics of the various beam parameters,

$$\frac{\partial c}{\partial z} = \mu(z) \left(-4c^2 + \frac{1}{\omega^4} \right) - \frac{2\rho P_0}{\sqrt{\pi} (\sigma^2 + 2\omega^2)^{3/2}} \quad (10)$$

$$\frac{\partial \theta}{\partial z} = \frac{\rho P_0 (5\omega^2 + 2\sigma^2)}{2\sqrt{\pi} (\sigma^2 + 2\omega^2)^{3/2}} - \frac{\mu(z)}{\omega^2} \quad (11)$$

$$\frac{\partial \omega}{\partial z} = 4c\omega\mu(z) \quad (12)$$

To better understand pulse evolution in inhomogeneous media, we numerically solved the coupled ordinary differential equations (ODEs) using the Runge-Kutta (RK4) method. As shown in Figure 19, we observe that the phase-front curvature and beam width remain constant in the case of homogeneous diffraction. However, in the inhomogeneous diffraction medium, the phase-front curvature and beam width broaden as the beam propagates along the z -direction. Additionally, the nonlocal nonlinear Schrödinger equation (4) is numerically solved using the split-step Fourier

transform (SSFT) method, as illustrated in Figure 20.

Conclusion: We explored the influence of inhomogeneous diffraction profiles on the propagation characteristics of optical beams in a nonlocal nonlinear medium. Our analysis focused on changes in beam width and phase-front curvature. These findings are relevant for real-life applications of pulse propagation in optical fibers.

References

- [1] Mishra, Manoj, et al. “Energy optimization of diffraction managed accessible solitons.” *JOSA B* 39.10 (2022): 2804-2812.
 - [2] Xiong, Gang, et al. “Analytical light bullet solutions in diffraction-decreasing media with inhomogeneous parameters.” *Results in Physics* 43 (2022): 106111.
 - [3] Mishra, Manoj, et al. “Generation, dynamics and bifurcation of high-power soliton beams in cubic-quintic nonlocal nonlinear media.” *Journal of Optics* 24.5 (2022): 055504.
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