

# Determination of Dynamical Exponents of Complex Systems using Machine Learning

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Despite the large size, complexity, and dynamics of non-linear interactions in real-world systems, they follow self-organizing principles and maintain their stability. The networked system's stability depends on the Jacobian matrix's principal eigenvalue for the steady states, and entries of the Jacobian weights depend on dynamics and networks Ref. [1]. For a large and complex system, determining the entries of complex systems is a challenging task. This work Ref. [1] show systematic analytical derivation to extract dynamical exponents using functions that are associated with capturing intrinsic mechanisms of complex systems  $M = (M_0, M_1, M_2)$  and the system are modeling using the differential equation define as:

$$\frac{dx_i}{dt} = M_0(x_i(t), f_{0i}) + g \sum_{j=1}^N A_{ij} M_1(x_i(t), f_{1i}) \times G_{ij} M_2(x_j(t), f_{2i}). \quad (1)$$

Here, you can see that the interaction function has a multiplicative form ( $M_1 M_2$ ). However, if the system does not follow this dynamical framework, extracting dynamical exponents analytically is challenging. Therefore, in our work, we use a

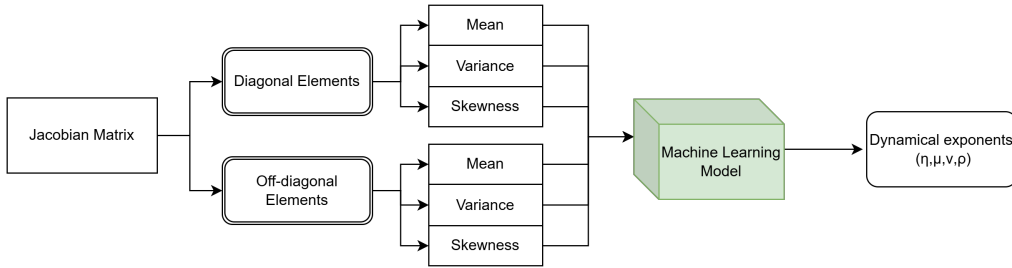


Figure 13: *Architecture of ML model to predict dynamical exponents of complex systems.*

machine learning algorithm to predict dynamical exponents for a general dynamical framework that defines the complex system. From Fig. 13, you can see that the input features are the statistical characteristics extracted from the numerically obtained Jacobian matrices. We generate a dataset consisting of several Jacobian matrices used to train the ML model. We use statistical characteristics as input for the ML models, such as SVM, Random Forest, ANN, and XGBoost algorithms, to predict the dynamical exponents for the family of equations, which capture more complex interaction mechanisms than Eq. (1). Our findings indicate that XGBoost (refer to Fig. 14) is the best ML model for determining the dynamical exponents

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Model	MSE	R2 Score
Random Forest	0.1015	0.9614
SVM	0.9408	0.4501
XGBoost	0.0137	0.9920 (Best)
ANN	0.6081	0.6432

Figure 14: *ML models evaluations to determine dynamical exponents.*

of complex systems based on statistical measures of dynamical Jacobian ensembles. We also find that using ML, we can determine the dynamical exponents of systems that are not within the framework of Eq. (1), and further, we can use the extracted exponents to determine the stability of such systems.

## References

- [1] Chandrakala Meena. Simcha Haber. Chittaranjan Hens. Stefano Boccaletti Suman Acharyya and Baruch Barzel, (2023), Emergent stability in complex network dynamics, Nature Physics.
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