Contributed Talk

Multi-sphere Model in Higher Dimensional Space

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Kuramoto oscillator [1] is a paradigmatic one-dimensional phase oscillator that has been extensively used to understand a plethora of collective phenomena including synchronization. Recent studies have extended the Kuramoto oscillator to the d-dimensional Kuramoto model to represent higher-dimensional systems [2]. In particular, two-dimensional Kuramoto oscillators have been employed to investigate the collective dynamics of swarmalator [3], and various other extended versions [4]. These models primarily focus on systems where the coupling is limited to one or two state variables. For instance, the Kuramoto oscillator typically involves a single state variable, where the variable represents location or phase, while swarmalators incorporate two variables, one for location, and the other representing orientation. These state variables may vary according to the application. If there are three or more state variables that are coupled with each other, how can we explain the collective dynamics of the system? In this work, we propose a generalized model capable of accommodating any number (let P) of state variables, where for each oscillator, the dynamics of all state variables are coupled to each other. In our approach, each state variable is associated with a hyper-sphere. So, with this generalization, we have no restriction on choosing the dimension of the oscillators, as well as on the choice of the number of state variables for the system. This multi-sphere framework captures collective dynamics with potential applications in diverse domains, including sociology, flocking patterns, neural networks, and beyond.

References

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