

Elliptic waves in coupled nonlinear Schrödinger system with four-wave mixing effects

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The formation of nonlinear coherent structures due to the interplay of intensity-dependent and phase-dependent nonlinearities, coupled with dispersion/diffraction effects, is a key topic in nonlinear wave research. Nonlinear optical systems and atomic condensates are two primary platforms for studying such phenomena. In nonlinear optics, short pulses propagating through a birefringent medium experience self-phase and cross-phase modulation due to Kerr nonlinearity, leading to four-wave mixing (FWM) effects. These effects, along with phase-dependent nonlinearities, are responsible for energy exchange between co-propagating pulses. In multi-mode fibers, dispersion engineering and tailored core diameters can enhance these effects. In atomic condensates, particularly Bose-Einstein condensates (BECs), matter wave mixing through atom-atom interactions can also give rise to nonlinear coherent effects. Multi-component BECs, including spinor condensates, exhibit phase-sensitive nonlinearities that allow for particle exchange among components, influencing the dynamics of FWM. In this context, we explore PT -invariant elliptic waves in coherently coupled nonlinear Schrödinger systems arising in nonlinear optics and atomic condensates. These include rational and standard forms of elliptic solitons, with their hyperbolic counterparts also derived. The four-wave mixing term plays a crucial role in shaping these PT-symmetric elliptic waves, which exhibit periodic soliton trains and localized coherent structures. Notably, we report PT-invariant coupled anti-dark-gray and gray-gray solitons, as well as intriguing W-shape and flat-top profile solitons. Numerical experiments confirm the stable propagation of these waves, with potential applications in pulse shaping in nonlinear optics.

References

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