Unveiling Noise-Induced Extreme Events: From Single Neuron to Network Dynamics

Contributed Talk

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Extreme events (EE) are rare and sudden occurrences characterized by significantly large or small amplitudes that deviate drastically from a system's typical behavior. This study explores the relatively under-researched phenomenon of noise-induced EE in nonlinear dynamical systems, with a particular focus on single neuronal models. This approach is vital because neurons, being highly sensitive to random stimuli, offer a critical platform for understanding how such phenomena arise. By isolating the dynamics of a single neuron, the study provides valuable insights into the local mechanisms within neuronal populations that may lead to seizures or extreme bursting events [1]. Using the FitzHugh-Nagumo (FHN) model [2], a well-established simplification of neuronal dynamics, we investigate the behavior of a scalar FHN oscillator under the influence of Gaussian white noise. Unlike previous studies that identified EE in coupled FHN systems, including globally coupled and small-world networks [3, 4], our findings reveal that noise alone can induce EE in a single FHN oscillator. The governing equations for the system are:

$$\dot{x} = x(a-x)(x-1) - y + \sqrt{2D}\eta(t),$$
$$\dot{y} = bx - cy,$$

where x and y represent the membrane potential and recovery variable, respectively, D denotes the noise intensity, $\eta(t)$is Gaussian white noise, and a, b, and c are system parameters. Under weak noise intensities, the deterministic system perturbed at its steady state exhibits small-amplitude oscillations (SAO) that sporadically transition into large-amplitude oscillations (LAO), classified as EE based on statistical criteria. The emergence of EE is confirmed through bifurcation analysis and probabilistic calculations of trajectory behavior in phase space. Furthermore, inter-spike interval (ISI) statistics reveal noise-induced bursting dynamics, while higher noise intensities result in frequent spiking, attributed to self-induced stochastic resonance. The large fluctuations approach [5] is employed to estimate trajectory escape rates, providing analytical and numerical explanations for EE emergence. Expanding the analysis to a globally coupled network of heterogeneous FHN oscillators subjected to non-identical noise sources highlights the impact of heterogeneity, a key feature of biological neural systems. Our results demonstrate that EE can emerge within the network across varying noise intensities and coupling strengths, even when individual oscillators are influenced by distinct noise sources. This work underscores the relevance of noise-induced EE in both single and networked neuronal systems, offering a foundation for further research into the mechanisms of EE and their biological implications.

References

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