Dynamic Stability of Complex Systems using Gershgorin Disc Theorem

Poster

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Prediction of the stability of large complex networks ranging from ecological, biological, chemical and mechanical systems to electrical grids and financial markets is a challenging task. In our research work, to determine the stability of complex dynamical networks, we have developed a comprehensive framework that integrates the Dynamical Jacobian Ensemble [1] and the Gershgorin Disc Theorem [2]. We extracted the Jacobian matrices for a complex dynamical network using the Dynamical Jacobian Ensemble [1] approach and corresponding to the ensemble, we generated Gershgorin Discs. As we know from the previous study, the Dynamical

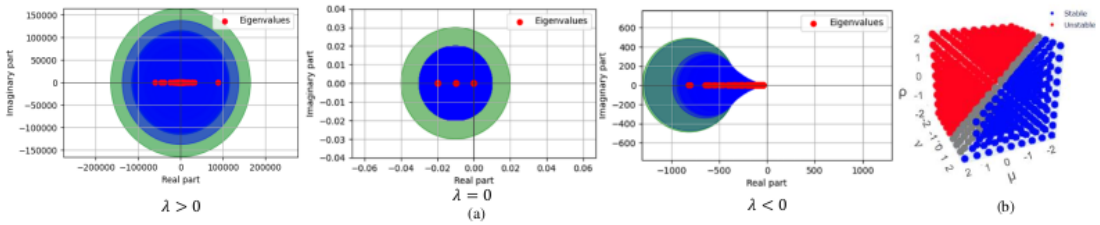

Figure 11: (a) Gershgorin Discs for three stability regimes. (b) Stability regimes: stable (blue), unstable (red), and grey (less effective analysis) in the parameter space of dynamical exponents (μ, ν and ρ).

Jacobian Ensembles for the various networks are fundamentally different from each other and depend on the dynamics through dynamical exponents η, μ, ν and ρ and on the network topology through the degrees and average nearest neighbour degree as $J_{ii} = -Cd_{nn}^{\eta}d_i^{u}$ and $J_{ii} = d_i^{\nu}A_{ij}G_{ij}d_j^{\rho}$ are the diagonal and off-diagonal entries of the Jacobian respectively. These fundamental changes in the Jacobian Ensembles, we can also observe through the Gershgorin Discs as well (Fig.11.a). Further, using the Gershgorin Disc Theorem, we establish a criterion, $S = \beta\eta - \eta\rho + \psi(\mu - \nu)$ that helps to classify the dynamical stability of the system into three distinct regimes: stable, unstable, and sensitively stable. We can clearly see the three distinct stability regimes in the parameter space (η, μ and ν) plotted in 3D (Fig.11b). Thus, the integration of the Dynamical Jacobian Ensemble with the Gershgorin Disc Theorem presents a novel and effective theoretical method for predicting the stability of complex dynamical systems. This framework not only advances theoretical knowledge but also has practical implications for improving the stability and control of systems across various fields.

Keywords: Gershgorin Discs, Dynamical Jacobian Ensemble, Dynamic Stability.

References

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