Firm Entry and Exit:

A Replication of Clementi-Palazzo (2016) and Khan-Thomas(2008)*

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Abstract: We analyze a simple version of Clementi and Palazzo (2016)'s **?** model of the firm lifecycle (which itself builds on Hopenhyan (1992) and Hopenhayn and Rogerson (1993)). The setup of it replicates my code for Khan-Thomas. Further, we study the steady state of the model without aggregate shocks. Further, we graph the results of our simulations.

PDF: `http://econ.jhu.edu/people/ccarroll/papers/BufferStockTheory.pdf`

Slides: `http://econ.jhu.edu/people/ccarroll/papers/BufferStockTheory-Slides.pdf`

Web: `http://econ.jhu.edu/people/ccarroll/papers/BufferStockTheory/`

GitHub: `http://github.com/llorracc/BufferStockTheory`

(In GitHub repo, see `/Code` *for tools for solving and simulating the model)*

CLICK HERE for an interactive Jupyter Notebook that uses the Econ-ARK/HARK toolkit to produce all of the paper's figures (warning: it may take several minutes to launch)

*I am grateful to Prof. Chris Carroll for his encouragement and advice for replicating these papers. All remaining errors are my own.

1 Introduction

Over the last 25 years, empiricists have pointed out a tremendous amount of between–firm and between–plant heterogeneity, even within narrowly defined sectors. A key issue in the macroeconomics literature is to gauge the importance of such heterogeneity for the evolution of aggregate magnitudes. The objective of this paper is to assess the role that entry and exit dynamics play in the propagation of aggregate shocks. The current exercise aims to complete the idiosyncratic version by modeling in aggregate shocks. We analyse a model of lumpy investment wherein firms face persistent shocks to common and plant-specific productivity, and convex adjustment costs. The model extends the real business cycle model to include firm heterogeneity and fixed capital adjustment costs. The aggregate state vector of the model contains the distribution of firms over idiosyncratic productivity and capital, which evolves over time in response to aggregate productivity shocks. The dynamics of the distribution must satisfy a complicated fixed point problem: each firm's investment decision depends on its expectations of the dynamics of the distribution, and the dynamics of the distribution depend on firms' investment decisions. This infinite-dimensional fixed point problem is at the heart of the computational challenges faced by the heterogeneous agent literature.

2 Related Literature

Clementi-Palazzo(2016) are not the first to find that entry and exit enhance the effects of aggregate shocks. Devereux, Head, and Lapham (1996), Chatterjee and Cooper (2014), Bilbiie, Ghironi, and Melitz (2012), and Jaimovic and Floetotto (2008) model entry and exit in general equilibrium models with monopolistic competition. In the former three, a surge in entry leads to greater diversity of the product space. Because of increasing returns, this encourages agents to work harder and accumulate more capital. In Jaimovic and Floetotto (2008), more entry means more competition and lower markups.

Khan-Thomas(2008)'s setup is utmost useful for our project.

3 Model

3.1 Firms

We have two groups of firms. The first group of *incumbent firms* behave similarly to Khan and Thomas (2008) model except that they have convex costs of capital adjustment rather than fixed costs. To be specific, each of these incumbent firms has a decreasing returns to scale production fucnction

$$y_{jt} = e^{\epsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu}$$

, where y_{jt} is output, ϵ_{jt} is idiosyncratic productivity, k_{jt} is the firm's capital stock, n_{jt} is the firm's labor input, and $\theta + \nu < 1$.

The idiosyncratic productivity ϵ_{jt} follows a Markov Chain process. Firms accumulate capital according to the accumulation equation

$$k_{jt+1} = (1-\delta)k_{jt} + i_{jt}$$

.

Capital accumulation incurs the convex adjustment cost

$$-\frac{\varphi}{2}(\frac{i_{jt}}{k_{jt}})^2 k_{jt}$$

, in units of output. At the beginning of each period, incumbent firms must pay a fixed cost c_f units of output to remain in operation. A firm that does not pay this fixed cost does not produce, sells its entire capital stock with value $(1-\delta)k$ and permanently exits the economy.

There is a continuum of the second group of firms, the *potential entrants*. These firms are ex-ante identical. At the beginning ofeach period, each firms decides whether to pay a fixed cost c_e and enter the economy. If a potential entrant enters the economy, it draws a value for idiosyncratic productivity ϵ_{jt} from some distribution ν and begins as an incumbent firm with k_{jt} =0. We also assume that there are no adjustment costs at $k_{jt} = 0$.

Further, we assume there is free entry among potential entrants, which implies that the exoected value from entering is less than or equal to the entry cost c_e, with equality if entry actually takes place. In equations, this condition is $c_e \leq \int v(\epsilon, 0)\nu(d\epsilon)$, with equality if $m^* > 0$ (where $v(\epsilon, k)$ is the value function of an incumbent firm and m^* is the mass of entrants in equilibrium.)

3.2 Households

Finally, there is a representative household with preferences over consumption C_t and labor N_t represented by the expected utility function

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t (logC_t - aN_t)$$

where β and a are parameters. Output here is used for consumption, investment, capital adjustment costs, entry costs and operating costs. A steady state recursive competitive equilibrium of this economy is a set of incumbent value functions $\nu(\epsilon, k)$. policy rules $k'(\epsilon, k)$ and $n(\epsilon, k)$, a mass of entrants per period m^*, a measure of active firms at the beginning of period $g^*(\epsilon, k)$ and real wage w^* such that

- incumbent firms maximize their firm value;
- the free entry condition holds;
- the labor market clears;
- the measure of active firms $g^*(\epsilon, k)$ is stationary.

More formally, let me define the recursive equilibirum now.

4 Recursive Equilibrium

At the beginning of each period, incumbent firms must pay a fixed cost c_f units of output to remain in operation. A firm that does not pay this fixed cost permanently exits the economy immediately and sells its entire capital stock with value $(1-\delta)k$, i.e. $V_x(k) = (1-\delta)k$.

Then, the start-of-period value of an incumbent firm is dictated by the function $V(\lambda, k, s)$ which solves the following functional equation:

$$V(\varepsilon, k) = \max \left\{ V_x(k), \tilde{V}(\varepsilon, k) - c_f \right\}$$

Given that firms accumulate capital according to the accumulation equation $k_{jt+1} = (1-\delta)k_{jt} + i_{jt}$ and capital accumulation incurs the adjustment cost $-\frac{\varphi}{2}\left(\frac{i_{jt}}{k_{jt}}\right)^2 k_{jt}$, in units of output. The prospective value of an entrant is

$$V(\varepsilon, 0) = \max \left\{ 0, \tilde{V}(\varepsilon, 0) - c_e \right\}$$

hence, she will invest and start operating if and only if $c_e \leq \int V(\varepsilon, 0) d\varepsilon$ For a given Markov process, a recursive competitive equilibrium consists of (i) value functions $V(\varepsilon, k), \tilde{V}(\varepsilon, k)$ and $V_e(\varepsilon, 0)$,; (ii) policy functions $n(\varepsilon, k), k'(\varepsilon, k)$,; (iii) bounded sequences of wages $\{w_t\}_{t=0}^{\infty}$, incumbents' measures $\{g_t\}_{t=1}^{\infty}$, and entrants' measures $\{m_t\}_{t=0}^{\infty}$ such that, for all $t \geq 0$.

1. $V(\varepsilon, k), \tilde{V}(\varepsilon, k)$, and $n(\varepsilon, k)$ solve the incumbent's optimization problem;
2. $V_e(\varepsilon, 0)$ and $k'(\lambda, 0)$ solve the entrant's optimization problem;
3. The representative houschold chooses consumption and labour such that $\frac{w(g)}{C(g)} = \frac{a}{N(g)}$;
4. The labour market clears: $N(w_t) = \int n(\varepsilon_t, k) g_t(\varepsilon, k) d\varepsilon dk \forall t \geq 0$;
5. The goods market clears: $C(g_t) = \int [y(\varepsilon_t, k) - i(\varepsilon_t, k)] g_t(\varepsilon, k) d\varepsilon dk : \forall t \geq 0$.

Parameter	Description	Value
β	Discount Factor	.961
φ	Convex Cost adjusment	.5
σ	Utility curvature	1
α	Inverse Frisch	$\lim \alpha \to 0$
χ	Labor Disutility	N*$= \frac{1}{3}$
ν	Labor Share	0.64
θ	Capital Share	0.21
δ	Capital Depreciation	0.1
ρ_z	Idiosyncratic TFP AR(1)	0.859
σ_z	Idiosyncratic TFP AR(1)	0.02
n_{ss}	Steady State Labor	0.6
c_f	Incumbent's cost	0.01
c_e	Entrant's cost	0.02
m	Tauchen Grid initializer	3

4.1 Representative Agent Steady State

We begin by analysing the steady state equilibrium of the model in which there is a representative firm and productivity is equal to the mean value of ε. In this scenario, the steady state recursive competitive equilibrium is characterized by a set $V^*(\bar{\varepsilon}, k), C^*, N^*, w^*$ and $g(\bar{\varepsilon}, k)^*$ such that 1. $V^*(\bar{\varepsilon}, k)$ solves the representative firm's optimization problem (i.e. Bellman eq.); 2. Taking N^* as given, the representative household's optimization is satisfied by $\frac{w^*}{C^*} = \frac{a}{N^*}$ 3. Labour market clearing follows from $N^*(w_t) = \int n(\bar{\varepsilon}, k) g(\bar{\varepsilon}, k) dk \forall t \geq 0$ 4. The goods market satisfies $C^* = \int [y(\bar{\varepsilon}, k) - i(\bar{\varepsilon}, k)] g(\bar{\varepsilon}, k) dk \forall t \geq 0$

Now, assume that steady state labour supply is $N^*_{rep} = 0.6$. Then, we can use the following system of equations to solve for K^*_{rep} and $w^*_{rep} = 0.6$. Then, we can use the

$$\bar{n} = 0.6$$

$\bar{r} = \frac{1}{\beta} - (1 - \delta)$ $\bar{k} = \frac{1}{\beta} - (1 - \delta)$ $\bar{k} = \delta \bar{k}$ $\bar{i} = \nu \bar{k}$ $\bar{w} = \nu \bar{k}^{\theta} \bar{n}^{\nu - 1}$ $\bar{c} = \bar{k}^{\theta} \bar{n}^{\nu} - \bar{i}$ $\bar{a} = \frac{w^* n^*}{c^*}$

In particular, $K^*_{rep} = 1.09$ and $w^*_{rep} = 0.78$.

5 Some plots

Figure 1: Firm Distribution

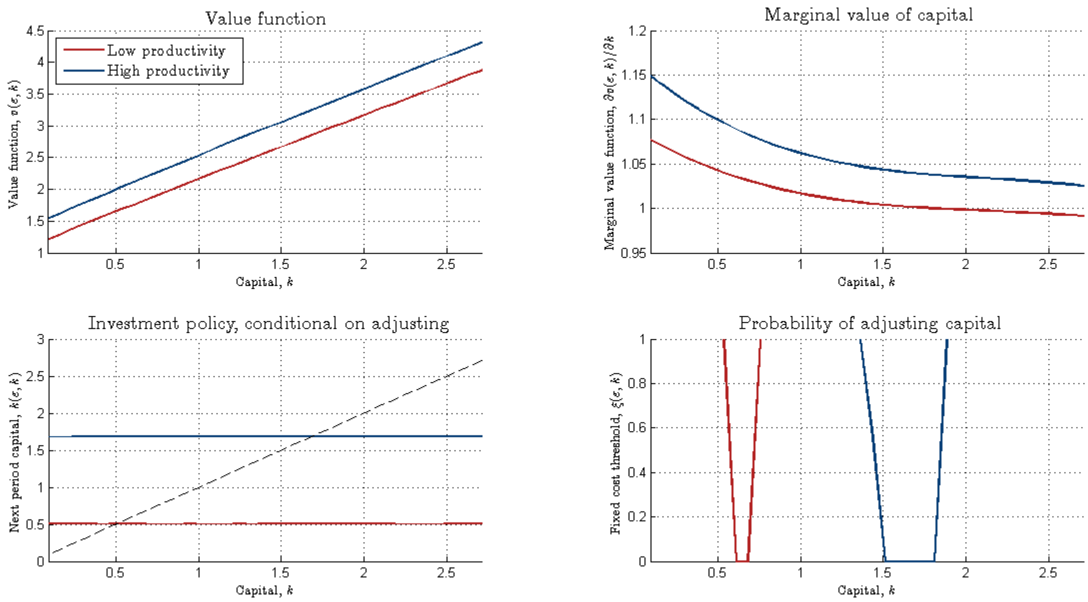
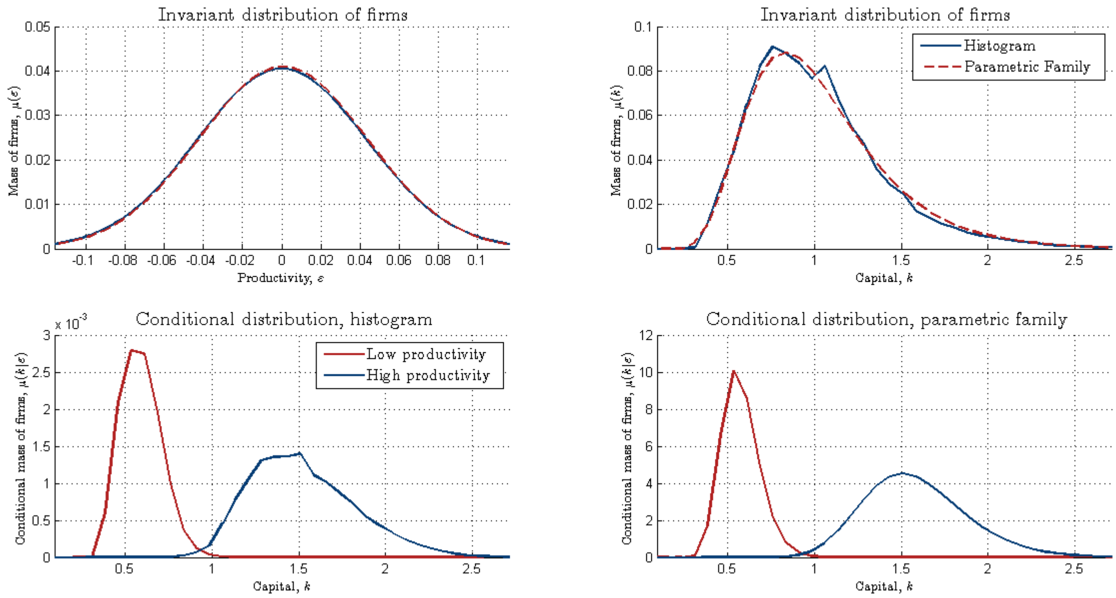
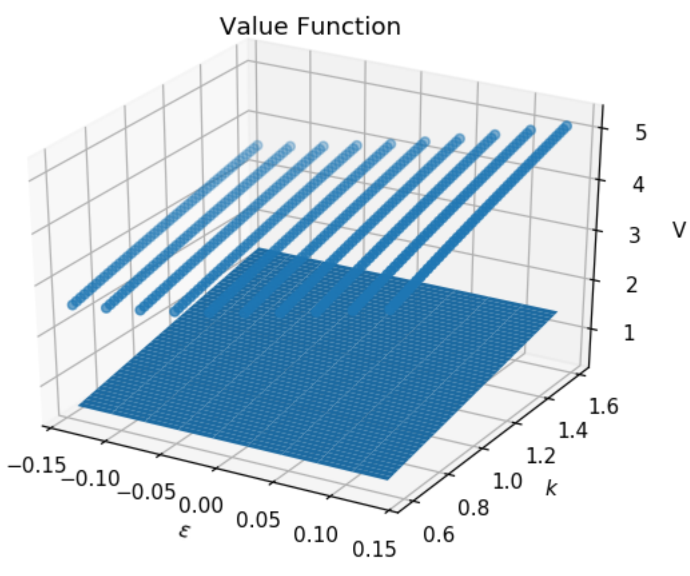
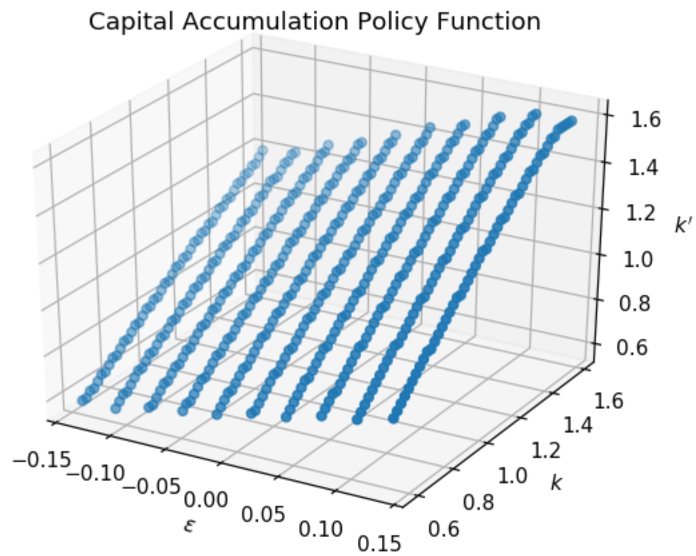


Figure 2: Marginal Product Distribution



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k
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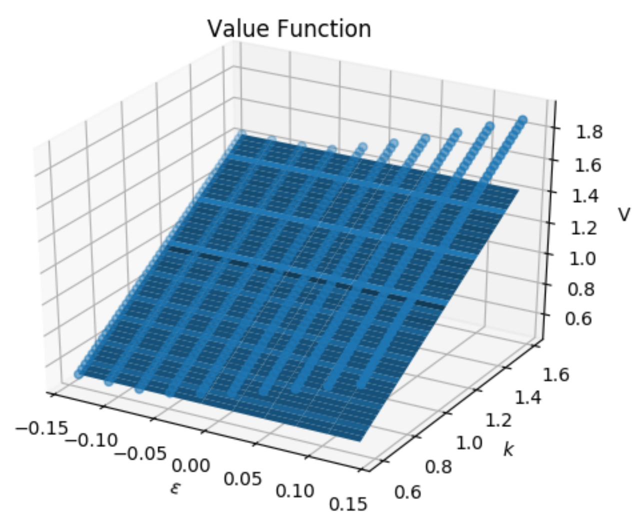
k′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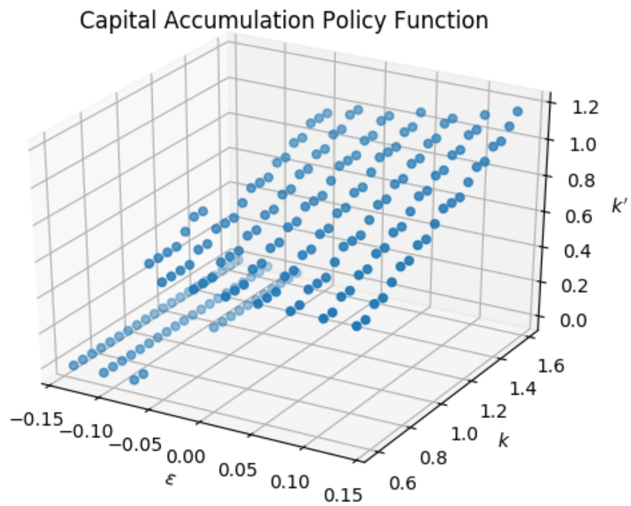
Capital Accumulation Policy Function
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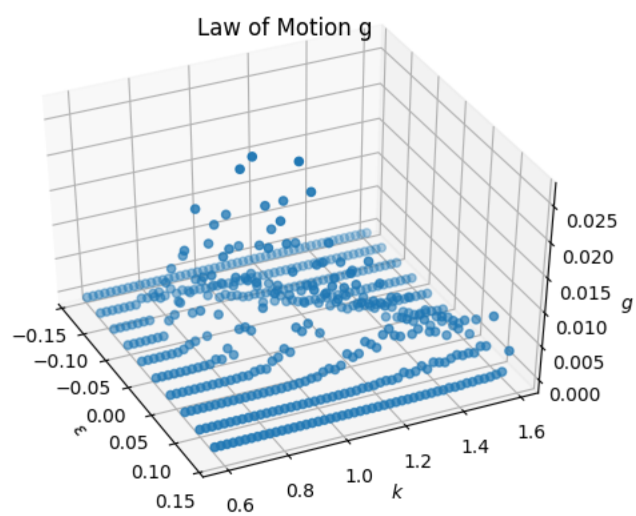
Law of Motion g
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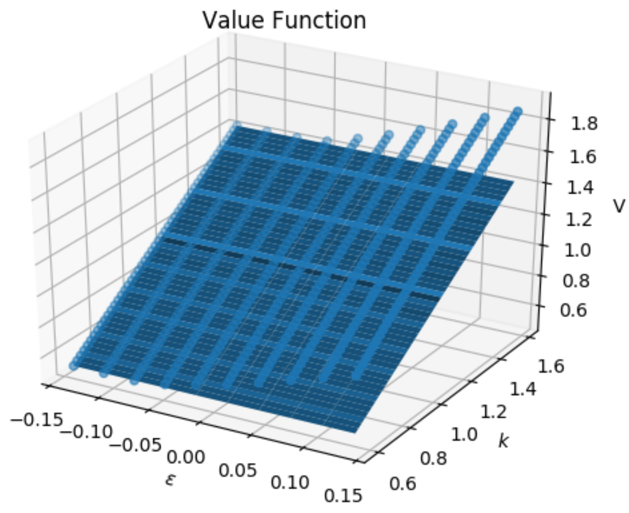
g


Value Function
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6 Appendix

Firm Entry and Exit: Clemento-Palazzo(2016) Aniruddha Ghosh

We analyze a simple version of Clementi and Palazzo (2016)'s model of the firm lifecycle (which itself builds on Hopenhyan (1992) and Hopenhayn and Rogerson (1993)). The setup of it replicates my code for Khan-Thomas. Further, we studied the steady state of the model without aggregate shocks.

The following graphs some simulations using my Matlab codes for the model.

Figure 3: Tracing the value function from Khan-Thomas replication

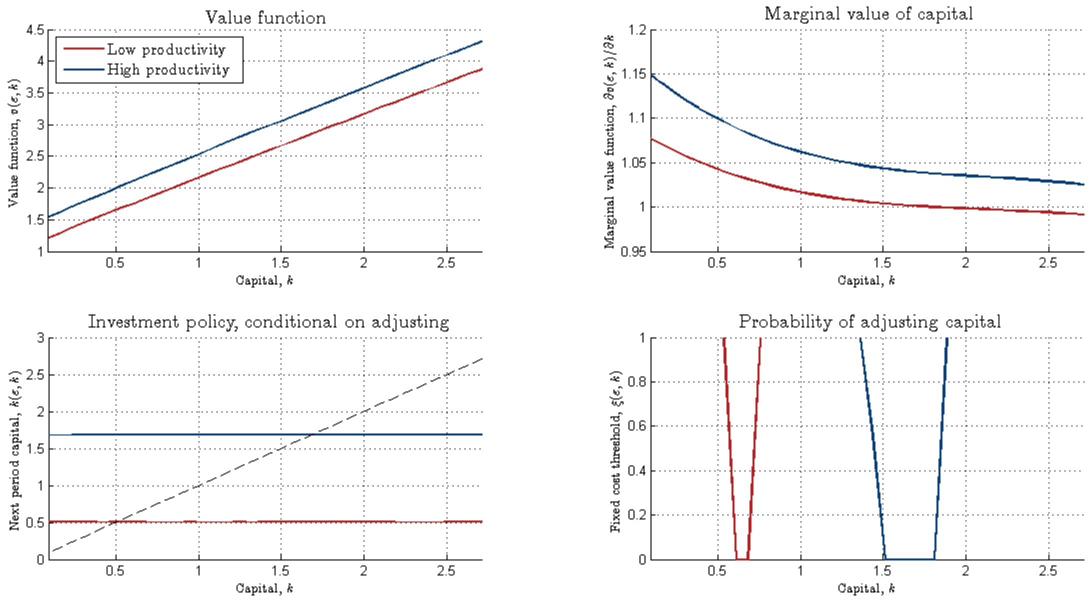

Figure 4: Tracing investment implications