

Today's Content } → 2 classes

→ Prefix Sum Introduction

→ Problems based on Prefix Sum

.

### Quote

I wanted the reward and not the struggle.  
I wanted the result and not the process,  
I was not in love with the fight but only  
the victory.

And life doesn't work that way.

Ques Given  $N$  array elements &  $Q$  queries in same array.

for each query calculate sum of all elements in given range.

arr[10] = <sup>0</sup>-3 <sup>1</sup>6 <sup>2</sup>2 <sup>3</sup>4 <sup>4</sup>5 <sup>5</sup>2 <sup>6</sup>8 <sup>7</sup>-9 <sup>8</sup>3 <sup>9</sup>1

$Q = 6$

L R

4 8 : 9

3 7 : 10

1 3 : 12

0 4 : 14

6 9 : 3

7 7 : -9

idea :-

for all queries, iterate over L-R & get sum

Pseudo code

// Given arr[N]

for (j=0; j<Q; j++) {

    Read L & R

    int sum=0

    for (i=L; i<=R; i++) {

        sum = sum + arr[i]

    }

    Print (sum)


}

T.C  $\rightarrow O(Q * N)$

S.C  $\rightarrow O(1) \rightarrow$  Constant

// Given Indian Team Scores for first 10 overs of Batting After every current Score given.

<u>Overs</u>	1	2	3	4	5	6	7	8	9	10
<u>Scores</u>	2	8	14	29	31	49	65	79	88	97


 cumulative sum

① Total no. of runs scored in last overs  $\rightarrow 97 - 88 = 9$

② Total runs scored in last 5 overs  $= 97 - 31 = 66$

③ Total runs made in 7<sup>th</sup> over  $= 65 - 49$

④ Total runs made from [3-6] over  $\Rightarrow 49 - 8$

⑤ Total runs scored from [1-5] over  $\Rightarrow 31$

⑥ Total runs scored from [4-8] over  $\Rightarrow 79 - 14$

$$\begin{array}{cccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
 \text{arr}[10] = & -3 & 6 & 2 & 4 & 5 & 2 & 8 & -9 & 3 & 1 \\
 \text{pf}[10] = & -3 & 3 & 5 & 9 & 14 & 16 & 24 & 15 & 18 & 19
 \end{array}$$

$\underbrace{\hspace{1.5cm}} \rightarrow$  prefix array

$$\text{pf}[3] = 9 \Rightarrow \text{sum of all element from } [0-3]$$

$$\text{pf}[5] = 16 \Rightarrow \text{sum of all elements from } [0-5]$$

$$\text{sum}[4-8] = \text{arr}[4] + \text{arr}[5] + \text{arr}[6] + \text{arr}[7] + \text{arr}[8]$$

Add  $\text{sum}[0-3]$  on both sides

$$\rightarrow \text{sum}[0-3] + \text{sum}[4-8] =$$

$$\begin{aligned}
 & \text{arr}[4] + \text{arr}[5] + \text{arr}[6] + \text{arr}[7] + \text{arr}[8] \\
 & + \text{arr}[0] + \text{arr}[1] + \text{arr}[2] + \text{arr}[3]
 \end{aligned}$$

$$= \text{pf}[8]$$

$$\rightarrow \text{sum}[0-3] + \text{sum}[4-8] = \text{pf}[8]$$

$$\rightarrow \text{pf}[3] + \text{sum}[4-8] = \text{pf}[8]$$

$$\rightarrow \text{sum}[4-8] = \text{pf}[8] - \text{pf}[3]$$

$$* \text{sum}[3-7] = \text{pf}[7] - \text{pf}[2]$$

$$* \text{sum}[5-9] = \underbrace{\text{sum}[0-9]}_{\downarrow} - \underbrace{\text{sum}[0-4]}_{\downarrow}$$

$$\text{sum}[5-9] = \text{pf}[9] - \text{pf}[4]$$

$$* \text{sum}[2-6] = \underbrace{\text{sum}[0-6]}_{\downarrow} - \underbrace{\text{sum}[0-1]}_{\downarrow}$$

$$\text{sum}[2-6] = \text{pf}[6] - \text{pf}[1]$$

$$* \text{sum}[L-R] = \underbrace{\text{sum}[0-R]}_{\downarrow} - \underbrace{\text{sum}[0-L-1]}_{\downarrow}$$

$$\rightarrow \text{sum}[L-R] = \text{pf}[R] - \text{pf}[L-1]$$

$$\text{sum}[0-2] = \text{pf}[2] - \text{pf}[-1]$$

$\Downarrow$

$\text{pf}[2]$

$\Downarrow$   
array out of  
bound exception

$$L-R \Rightarrow \text{sum}[L-R] \Rightarrow \begin{cases} \text{if } (L=0) : \text{pf}[R] \\ \text{else} : \text{pf}[R] - \text{pf}[L-1] \end{cases}$$

$$\text{sum}[5-5] = \text{pf}[5] - \text{pf}[4],$$

Given  $\text{arr}[n] \rightarrow \text{pf}[n]$

$$\text{pf}[0] = \text{arr}[0]$$

$$\text{pf}[1] = \text{arr}[0] + \text{arr}[1]$$

$$\text{pf}[2] = \underbrace{\text{arr}[0] + \text{arr}[1] + \text{arr}[2]}$$

↓

$$\text{pf}[2] = \text{pf}[1] + \text{arr}[2]$$

$$\text{pf}[5] = \text{pf}[4] + \text{arr}[5]$$

$$\text{pf}[i] = \text{pf}[i-1] + \text{arr}[i],$$

$$\text{pf}[0] = \text{pf}[-1] + \text{arr}[0],$$

↪ out of bound,

→ given  $\text{arr}[n]$ .

int  $\text{pf}[n]$

$$\text{pf}[0] = \text{arr}[0].$$

for ( $i=1$ ;  $i < n$ ;  $i++$ ) {

$$| \text{pf}[i] = \text{pf}[i-1] + \text{arr}[i]$$

}

arr = <sup>0</sup>5, <sup>1</sup>4, <sup>2</sup>3, <sup>3</sup>2, <sup>4</sup>10

pf 5 9 12 14 24

i = 0, 1, 2, 3

// sum of 0 queries

// Given arr[N]

given arr[N].

int pf[N]

pf[0] = arr[0].

for (i = 1; i < N; i++) {

| pf[i] = pf[i-1] + arr[i]

}

Read Q. → L & R

for (i = 0; i < Q; i++) {

if (L == 0) { print(pf[R])

else {

print(pf[R] - pf[L-1])

}

}

$$T.C \rightarrow O(0 + N)$$

$$S.C \rightarrow O(\underline{N})$$

10:30 → 10:40 pm



## Ques) Equilibrium Index

Given  $N$  array elements, count no. of equilibrium Index?

An index  $i$  is said to be equilibrium index if?

Sum of all elements  
Before  $i^{\text{th}}$  index  
 $\text{Sum}[0, i-1]$

=  
Sum of all  
elements after  $i^{\text{th}}$   
index  
 $\text{Sum}[i+1, N-1]$

Note :- if  $i=0$ ,  $\text{left}=0$ , if  $i=N-1$ ,  $\text{right}=0$

Ex:-  $\text{arr}[4] =$

	0	1	2	3
	-3	2	4	-1
left =	0	-3	-1	2
right =	5	3	-1	0

$\therefore \text{ans} = 1$

Ex 2:-

Left

Right

0	1	2	3	4	5	6
-7	1	5	2	-4	3	0
0	-7	-6	-1	1	-3	0
7	6	1	-1	3	0	0

# ans = 2

Ex 3:- arr[] =

Left

Right

0	1	2
3	-2	2
0	3	1
0	2	0

Ans = 1

idea:- check if every index is equilibrium or not

```

int CountEquilibrium(int arr[]) {
    int n = arr.length;
    int c = 0;
    int pf[n]; { Initialize value to 0 }
    for (i = 0; i < n; i++) {
        // if index i is equilibrium index
        // left = sum[0 to i-1] = pf[i-1]
        int left = 0;
        if (i > 0) { left = pf[i-1]; }
    }
}

```

```
// right = sum[i+1 n-1]  $\Rightarrow$  pf[n-1] - pf[i]
```

```
int right = pf[n-1] - pf[i]
```

```
if (left == right) { c = c + 1 }
```

```
}
```

```
return c
```

```
}
```

T.C  $\rightarrow O(\underline{n} + \underline{n}) = O(\underline{n})$

S.C  $\rightarrow O(1)$

19:05 pm

→ revise it