DSA using C/C++

DSA Notes based on Abdul Bari's Udemy Course

Thanks to following github author (sakshamgarg6500) for preparing such good notes:

https://github.com/sakshamgarg6500/Data-Structure-using-C-Notes

For examples another valuable resource worthy to take note of https://github.com/parasguglani1/DSA_Practice

Data Structure Using C

- Way of organising data
- Data should be efficiently used by the program

Various Data Structures

- ARRAY
- MATRIX
- LINKED LIST

Physical Data Structure
(How data is arranged)

- STACK
- QUEUE

Linear

- TREE
- GRAPH

Non Linear

- HASHING

Tabular

Logical Data Structure
(How data is utilized)

- RECURSION
- SORTING TECHNIQUE

Logical Data structures are implemented using physical Data structures

Basics of C/C++

BASICS

1. Arrays
2. Structure
3. Pointer
4. Reference
5. Parameter Passing
6. Classes
7. Constructor
8. Templates

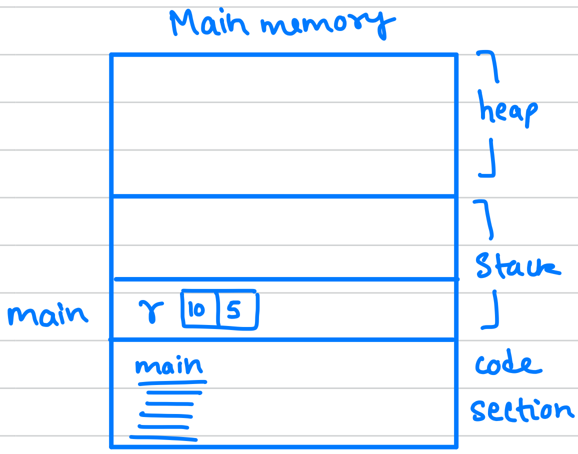
ARRAYS → Collection of similar datatypes

STRUCTURES → Collection of different data types

```
struct rectangle
{
     int length;   — 2
     int bredth;   — 2
                   4 bytes
};
```

but till here, it does not occupy any space in memory

```
int main()
{
   struct rectangle r = {10, 5}   → here it occupies memory
   r.length = 10 ;
   r.bredth = 5 ;
}
```



```
struct card
{
    int face;
    int shape;
    int colour;
};

int main()
{
    struct card deck[52] = {....}
    printf("%d", deck[0].face);
    printf("%d", deck[0].shape);
}
```

POINTERS

↳ address variables

1. Why pointers
2. Decleration
3. Initialization
4. Dereferencing
5. Dynamic allocation

```
int a=10;
int *p;            → decleration
p=&a;              → initialization
printf("%d", a);
printf("%d", *p);  → dereferencing
```

USES OF POINTER

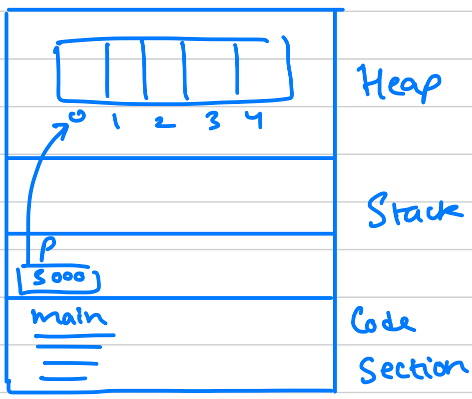
1. Accessing heap
2. Accessing resources like keyboard or mouse.
3. Parameter passing.

HOW TO USE POINTER FOR ALLOCATING HEAP

```
#include<stdlib.h>   → header file for malloc()

int main()
{
    int *p;
    p=(int *)malloc(5*sizeof(int));
}
```

↳ for returning as malloc function returns void pointer (type casting)

↓ Creating space for 5 integer values in heap.



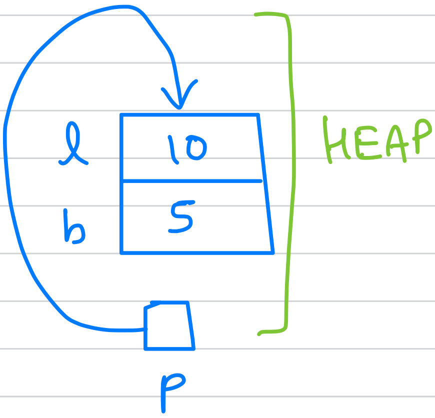
REFERENCE IN C++

```
int main()
{
    int a=10;
    int &r=a;
    cout<<a;
    cout<<r;
}
```

POINTER TO STRUCTURE

```
Struct rectangle
{
      int length;
      int bredth;
};

int main()
{
      struct rectangle r = {10, 5};
      struct rectangle *p = &r;

      r.length = 15;
   ✓  (*p).length = 20;
   ✓  p -> length = 20;
}
```

doesn't occupy 4 bytes of memory
(2 bytes)




```
int main()
{
      struct rectangle *p;

      p = (struct rectangle *)malloc( sizeof (struct rectangle))

      p -> length = 10;
      p -> bredth = 5;
}
```

FUNCTIONS

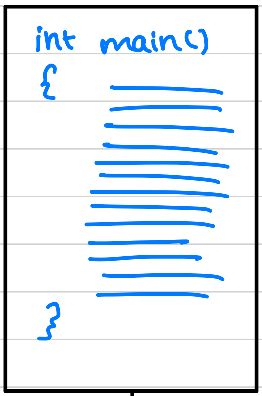
What are functions → Performs a specific task

Parameters of passing functions
- → Pass by value
- → Pass by address

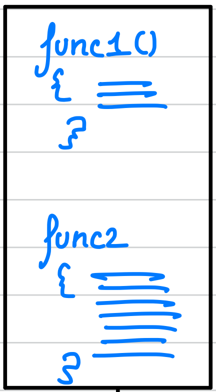
Only in C (pass by value, pass by address)

- → Pass by reference

In C++ (all three)



MONOLITHIC PROGRAMING



MODULAR PROGRAMING

FUNCTION EXAMPLE

prototype

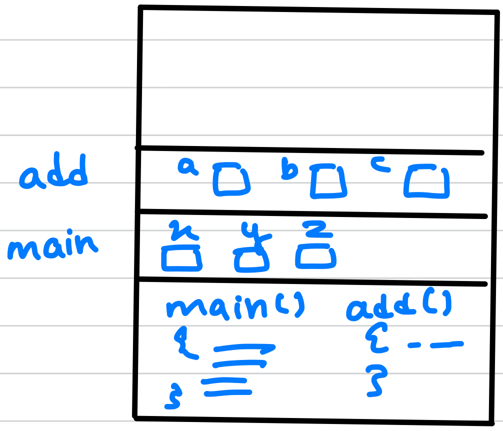
```
int add (int a ,int b)
{
      int c;
      c = a+b;
      return c;
 }
```

Formal Parameter

```
int main()
{
      int x,y,z;
      x = 10;
      y = 5;
      z = add (x,y);
      printf(" %d", z);
}
```

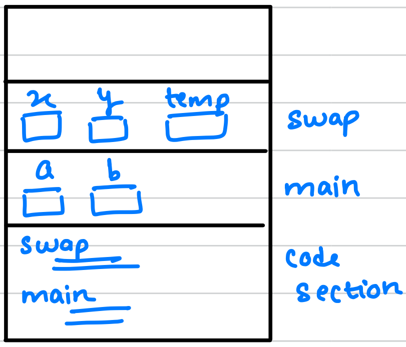
Actual Parameter

15



CALL BY VALUE

```
Void swap( int x , int y)
{
    int temp; temp = x;
    x = y;
    y = temp;
}
```

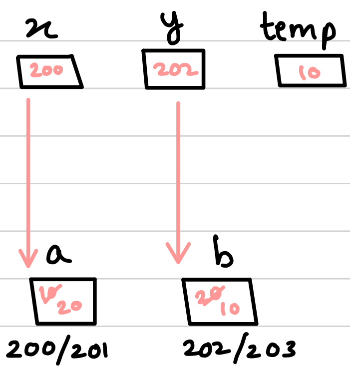



```
Int main()
{
    int a, b;
    a = 10;
    b = 20;
    swap (a,b);
    printf (" %d %d", a, b);
}
```

10 20

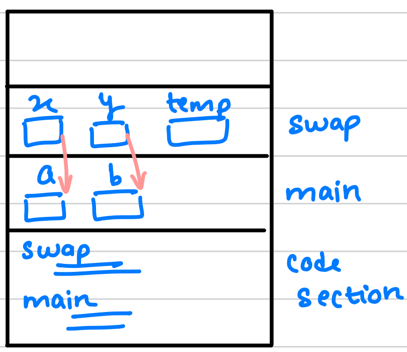
CALL BY ADDRESS

```
Void swap( int *x , int *y)
{
    int temp; temp = *x;
    *x = *y;
    *y = temp;
}
```



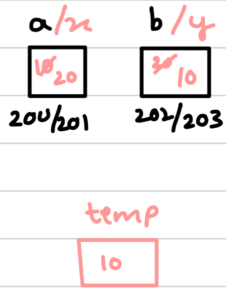

```
Int main()
{
    int a, b;
    a = 10;
    b = 20;
    swap (&a, &b);
    printf (" %d %d", a, b);
}
```

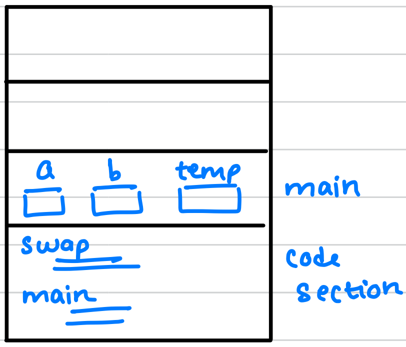
20 10



CALL BY REFERENCE (Only in C++)

```
Void swap( int &x ,int &y)
{
    int temp; temp = x;
    x = y;
    y = temp;
}

Int main()
{
    int a, b;
    a = 10;
    b = 20;
    swap (a,b);
    printf ("%d %d",a,b);
}                    20  10
```





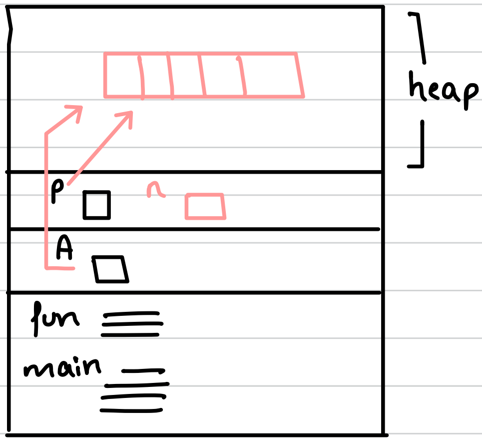
Only 2 bytes of memory is used as in reference, only another name is given to the pre-existing variable

ARRAY AS PARAMETER

```
void fun (int A[], int n)
{             └→ or int *A
    int i;
    for (i=0 ; i<n; i++)
        printf (" %d", A[i]);
}

int main()
{
    int a[5] = {2,4,6,8,10};
    fun ( A, 5);
}
```

as ARRAYS can only be passed as call by address.

```
        or  int * fun(int n)
int [] fun(int n)
{       it returns an array
    int *p;
    p = (int *)malloc(n * sizeof(int));
    return p;
}

int main()
{
    int *A;
    A = fun(5);
}
```



STRUCTURE AS PARAMETER

CALL BY VALUE

```
int area(struct rectangle r1)
{
    r1.length++;
    return r1.length * r1.bredth;
}

struct rectangle
{
    int length;
    int bredth;
};

int main()
{
    struct rectangle r = {10, 5};
    printf("%d", area(r));
```

CALL BY REFERENCE

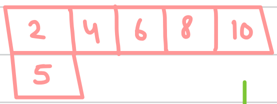
```
int area(struct rectangle &r1)
```

CALL BY ADDRESS

```
void changel(struct rectangle *p, int l)
{
    p -> length = l;
}

int main()
{
    struct rectangle r = {10, 5};
    changel(&r, 20);
}
```

PASSING ARRAYS AS CALL BY VALUE USING STRUCTURES

```
Void fun(struct test t1)
{
    t1.A[0] = 10;
}

int main()
{
    struct test t={{2,4,6,8,10}, 5};
    fun(t);
}
```

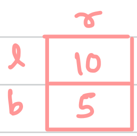



```
struct test
{
    int A[5];
    int n;
}
```

Changes made in the array in the function will not be reflected as it is call by value

STRUCTURES AND FUNCTIONS

```
Struct rectangle
{
    int length;
    int bredth;
}

void initialize(struct rectangle *r, int l, int b)
{
    r -> length = l ;
    r -> bredth = b;
}

int area(struct rectangle r)
{
    return r.length * r.bredth;
}
```




```
void changel(struct rectangle *r, int l)
{
    r -> length = l;
}
```

```
int main()
{
    Struct rectangle r;

    initialize(&r, 10, 5);
    area(r);
    changel(&r, 20);
}
```

because values need to be changed, so call by address.

CLASS AND CONSTRUCTOR

```
class rectangle
{
    private:
            int length;
            int bredth;
    public:

            rectangle (int l, int b)
            {
                length= l;
                bredth= b;
            }

            int area ()
            {
                return length*bredth;
            }

            void changelength (int l)
            {
                length = l;
            }
};
```

```
int main()
{
    rectangle r (10, 5);
    r. area()
    r. changelength (20);
}
```

```
class rectangle
{
    private:
            int length;
            int bredth;
    public:
            rectangle()
            {
                length= bredth= 1;
            }
            rectangle (int l, int b);
            int area();
            int perimeter();

            int getlength
            {
                return length;
            }
            void setlength (int l)
            {
                length = l;
            }
            ~ Rectangle ();
};
```

Constructors (overloaded): `rectangle()`, `rectangle (int l, int b);`

Facilitators (They do operations on data members): `int area();`, `int perimeter();`

Accessor: `int getlength`

Mutator: `void setlength (int l)`

Destructor → `~ Rectangle ();`

```
rectangle :: rectangle (int l, int b)
{
        length= l;
        bredth= b;
}

int rectangle :: area()
{
        return length* bredth;
}

int rectangle :: perimeter ()
{
        return 2* (length +bredth);
}

rectangle :: ~ rectangle ()
{
}


int main()
{
        rectangle r (10, 5);
        cout<< r. area;
        cout << r. perimeter;
        r. setlength (20);
        cout << r. getlength();
}
```

TEMPLATE CLASS

for using various datatypes. Using the same class for different datatypes.

template < class T >
class arithmetic
{
    private:
        T ~~int~~ a;
        T ~~int~~ b;

    public :
        arithmetic (T ~~int~~ a, T ~~int~~ b)
        T ~~int~~ add ();
        T ~~int~~ sub ();
};

template < class T > → because effect of previous template has ended
arithmetic <T> :: arithmetic (T ~~int~~ a, T ~~int~~ b)
{
    this.a = a;
    this.b = b;
}

template <class T>
T ~~int~~ arithmetic <T> :: add ()
{
    T ~~int~~ c;
    c = a + b;
    return c;
}

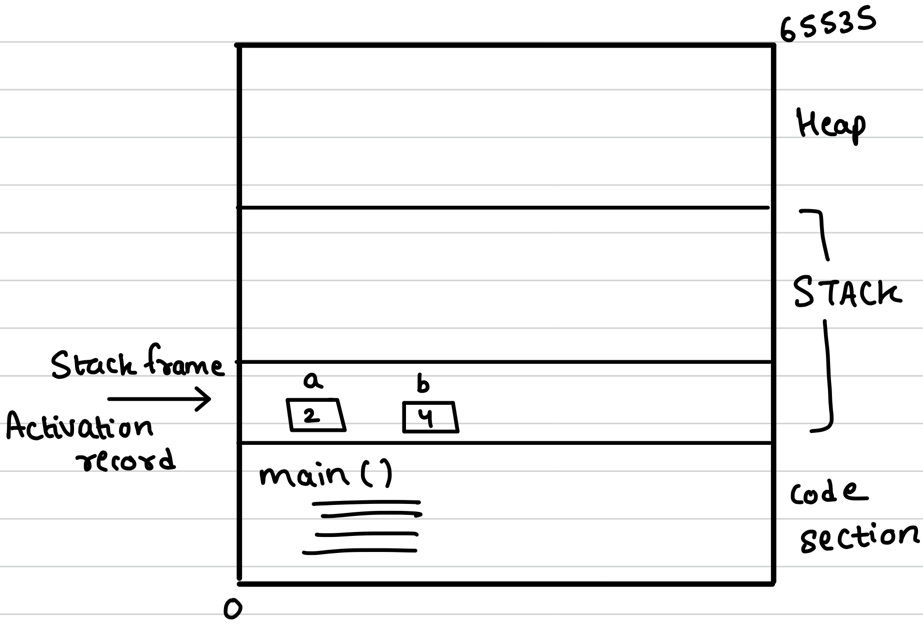
template < class T >
int arithmetic <T> :: sub()
{
    T ~~int~~ c;
    c = a - b;
    return c;
}

```
int main()
{
    arithmetic <int> ar (10, 5);
    cout << ar.add();
    arithmetic <float> ar1 (1.5, 1.2);
    cout << ar1.add();
}
```

INTRODUCTION

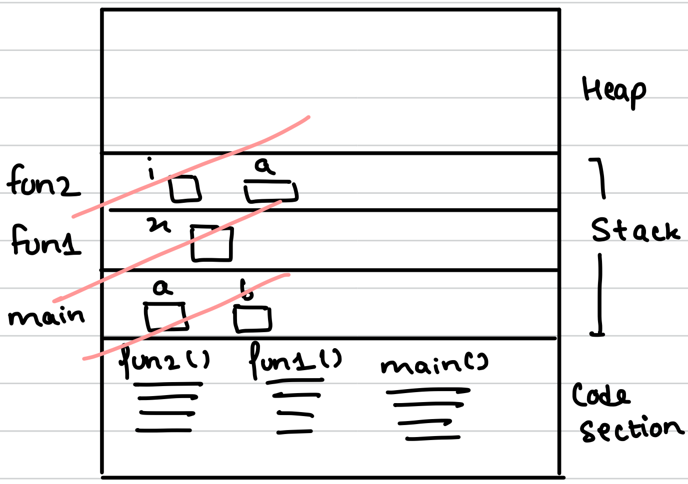
STATIC VS DYNAMIC MEMORY ALLOCATION

```
void main()
{
        int a;
        float b;
}
```



HOW DOES STACK WORK ?

```
void fun2(int i)
{
      int a;
}

void fun1 ()
{
      int x;
      fun2(x);
}

void main()
{
        int a;
        float b;
        fun1();
}
```

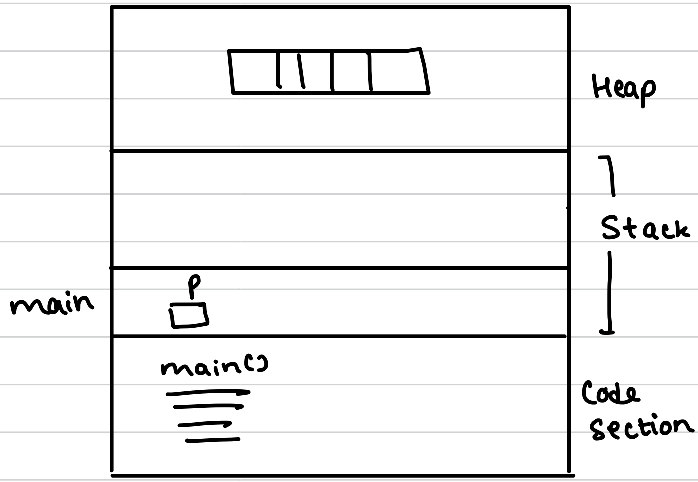


1. Stack is always organised memory.

HOW DOES HEAP MEMORY WORK ?

1. Heap may be organised memory or unorganised memory.
2. Heap must be treated as a resource i.e when it is needed, we must use it and when not needed, free it so that it can be used by other applications.
   For eg – printer is a resource.

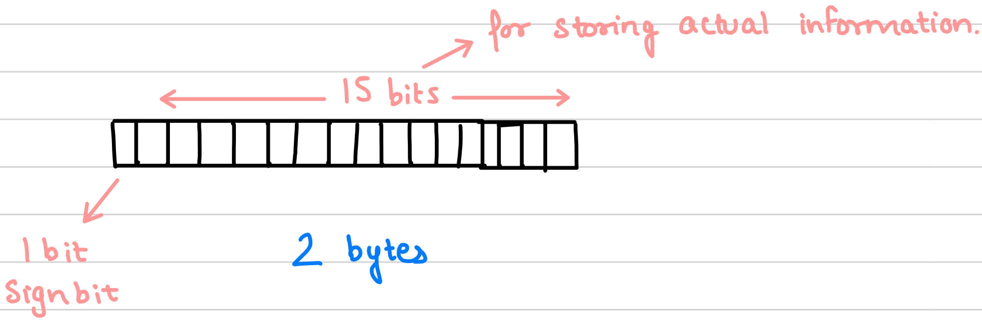
```
void main()
{
        int *p;

for C++   p= new int [5];
 for C    p= (int *) malloc (2*5);
              → for array
DEALLOCATION  delete []p;
              p= NULL;
}
```

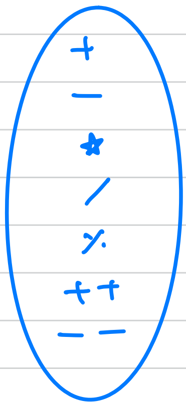


DATATYPE ?

↳ 1. Representation of data.
   2. Operation on data.

int x;





ABSTRACT DATATYPE

↳ hiding internal details

(part of object oriented programming)

What does log mean?

$\log_2 n$ ⟶ This gets divided by this until it reaches 1.

Time and Space complexity

```
void swap(x,y)
{
        int t;
        t=x;        ———— 1
        x=y;        ———— 1
        y=t;        ———— 1
}                        ———
                         3

            O(1)
```

```
int sum(int A[], int n)
{
        int s,i;
        s=0;                  ———— 1
        for(i=0;i<n;i++)      ———— n+1
        {       1     n
               s = s + A[i];  ———— n
        }
        return s;             ———— 1
}                             ————————
                              2n+3

                              O(n)
```

because i will be initialized (+1) and i will be incremented for n no of times and 1 time it will fail too (+n)

// because one time it will fail too.

```
Void  Add (int n)
{
    int i, j;
    for (i=0; i<n; i++)              ———— n+1
    {
        for (j=0; j<n; j++)          ———— n*(n+1)
        {
            c[i][j] = A[i][j] + B[i][j]; — n*n
        }
    }
}
```

because the nested loop will run again and again for (n+1) no of times.

$f(n) = 2n^2 + 2n + 1$

$O(n^2)$

Recursion

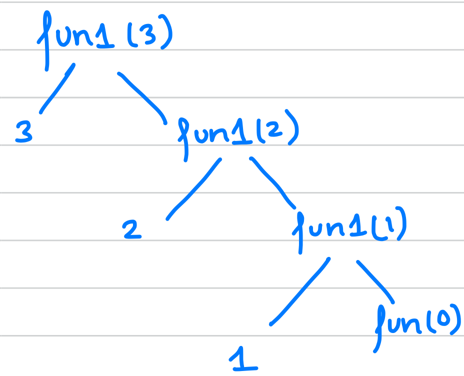
RECURSION

↳ When a function calls itself

EXAMPLE #1

```c
void fun1 (int n)
{
    if (n>0)
    {
        printf ("%d", n);
        fun1 (n-1);
    }
}

void main()
{
    int x = 3;
    fun1 (x);
}
```

TRACING TREE



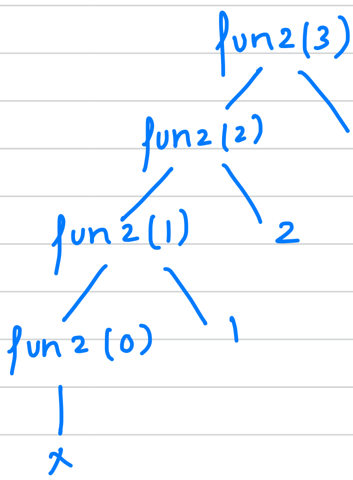
OUTPUT

3 2 1

EXAMPLE #2

```c
void fun2 (int n)
{
    if (n>0)
    {
        fun2 (n-1);
        printf ("%d", n);
    }
}

void main()
{
    int x = 3;
    fun2 (x);
}
```

TRACING TREE



OUTPUT

1 2 3

```
void fun (int n)
{
        if (n>0)
        {
ASCENDING    1. calling
             2. fun(n-1)
DESCENDING   3. returning
        }
}
```

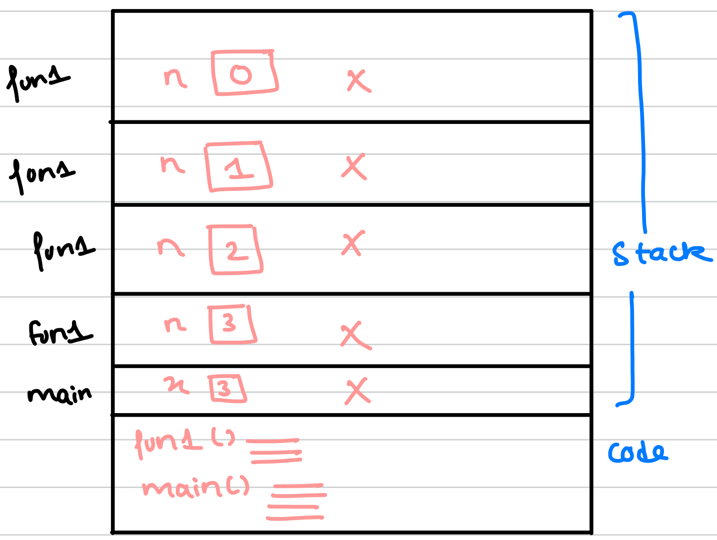
DIFFERENCE BETWEEN LOOP AND RECURSION

The main difference between loop and recursion is that recursion allows two phases i.e ascending and descending while loop allows only ascending

HOW STACK IS UTILISED IN RECURSIVE FUNCTIONS ?

EXAMPLE #1

```
void fun1 (int n)
{
        if (n>0)
        {
                printf ("%d", n);
                fun1 (n-1);
        }
}
```




```
void main()
{
        int x = 3;
        fun1 (x);
}
```

Here, value of n was 3, so there were 4 calls (Tracing tree).

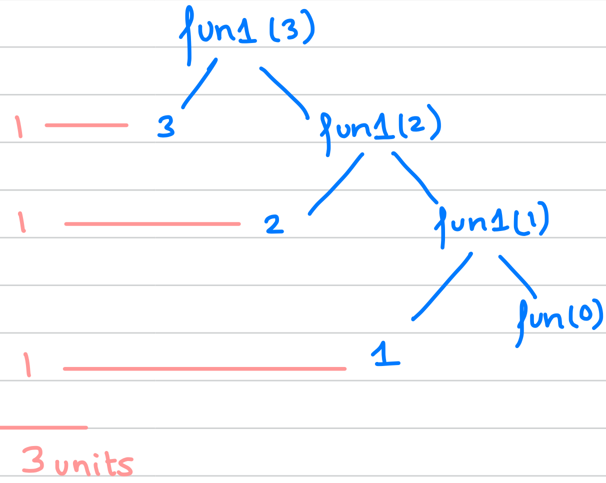
So for value of n, there will be n+1 calls

TIME COMPLEXITY (using Tree)

EXAMPLE #1

```
void fun1(int n)
{
    if (n>0)
    {
        printf("%d", n);
        fun1(n-1);
    }
}

void main()
{
    int x = 3;
    fun1(x);
}
```

TRACING TREE



Thus, for n calls $\longrightarrow$ n units of time.

$$\boxed{O(n)}$$

TIME COMPLEXITY (using recurrence relation)

$$T(n) = \begin{cases} 1 & n=0 \\ T(n-1) + \cancel{2}\ 1 & n>0 \end{cases}$$

Assume it as 1 for constant

$T(n)$ — This function takes n time.

```
        void fun1(int n)
        {
1   ——      if (n>0)
            {
1   ——          printf("%d", n);
T(n-1) ——       fun1(n-1);
            }
        }
```

As it is similar to $T(n)$

$$\underline{T(n) = T(n-1) + 2}$$

$T(n) = T(n-1) + 1$

$T(n) = T(n-1) + 1$

$T(n-1) = T(n-2) + 1$

$T(n) = \underbrace{T(n-2)}_{T(n-3)+1} + 1 + 1$

$T(n) = T(n-3) + 1 + 2$

$\vdots$

$T(n) = T(n-k) + k$

Assume $n-k=0 \ \therefore n=k$

$T(n) = T(n-n) + n$

$T(n) = T(0) + n$

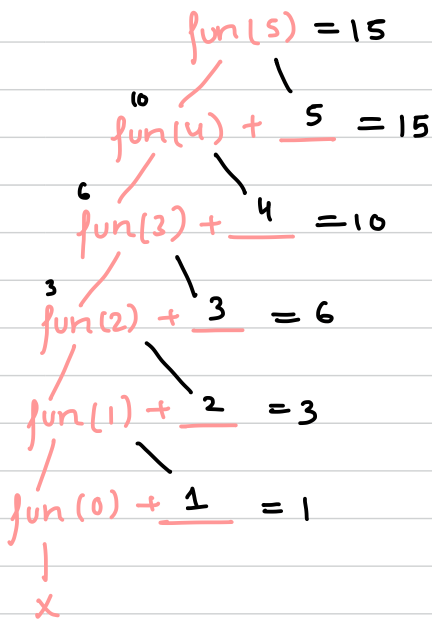
$T(n) = 1 + n$

$$\boxed{O(n)}$$

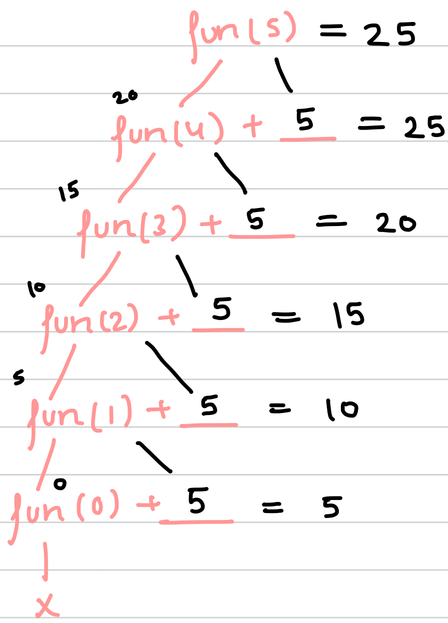
STATIC VARIABLES IN RECURSION

TRACING TREE

```c
int fun (int n)
{
        if (n>0)
        {
            return fun (n-1) +n;
        }
        return 0;
}

main()
{
        int a= 5;
        printf (" %d", fun (a));
}
```




```c
int fun (int n)
{       static int x=0;
        if (n>0)
        {   x++;
            return fun (n-1) + x;
        }
        return 0;
}

main()
{
        int a= 5;
        printf (" %d", fun (a));
}
```



TYPES OF RECURSION

1. Tail Recursion
2. Head Recursion
3. Tree Recursion
4. Indirect Recursion
5. Nested Recursion

1. TAIL RECURSION

When the function is calling itself and that call is the last call in the function.

```
fun (n)
{
    if (n > 0)
    {
        ______
        ______
        ______
        ______
        ______
        fun (n-1);
    }
}
```

1. Every operation is performed at calling time.
2. Tail recursions can easily converted into loops.

TAIL RECURSION AND LOOPS

Some compilers convert your program to loop if you have used tail recursion as they are more effecient

```
void fun (int n)
{
    while (n>0)
    {
        printf (" %d", n);
        n --;
    }
}

fun(3);
```

space O(1)

```
void fun ( int n)
{
    if (n>0)
    {
        printf (" %d", n);
        fun (n-1);
    }
}

fun (3);
```

O(n)

2. HEAD RECURSION

It means that the the function does not need to perform any operation at the time of calling. It has to do all operations at returning time.

```
void fun (int n)
{
    int i= 1;
    while (i<=n)
    {
        printf ("%d", i);
        i++;
    }
}
    fun (3);
```

```
void fun(int n)
{
    if (n>0)
    {
        fun (n-1);
        printf ("%d",n);
    }
}
    fun(3);
```

Head recursions cannot be so easily converted into loops.

3. TREE RECURSION

LINEAR RECURSION

```
fun(n)
{
    if (n>0)
    {
        fun(n-1);
    }
}
```

When the function calls itself only once.

TREE RECURSION

```
fun(n)
{
    if (n>0)
    {
        fun(n-1);
        fun(n-1);
    }
}
```

When the function calls itself more than one time.

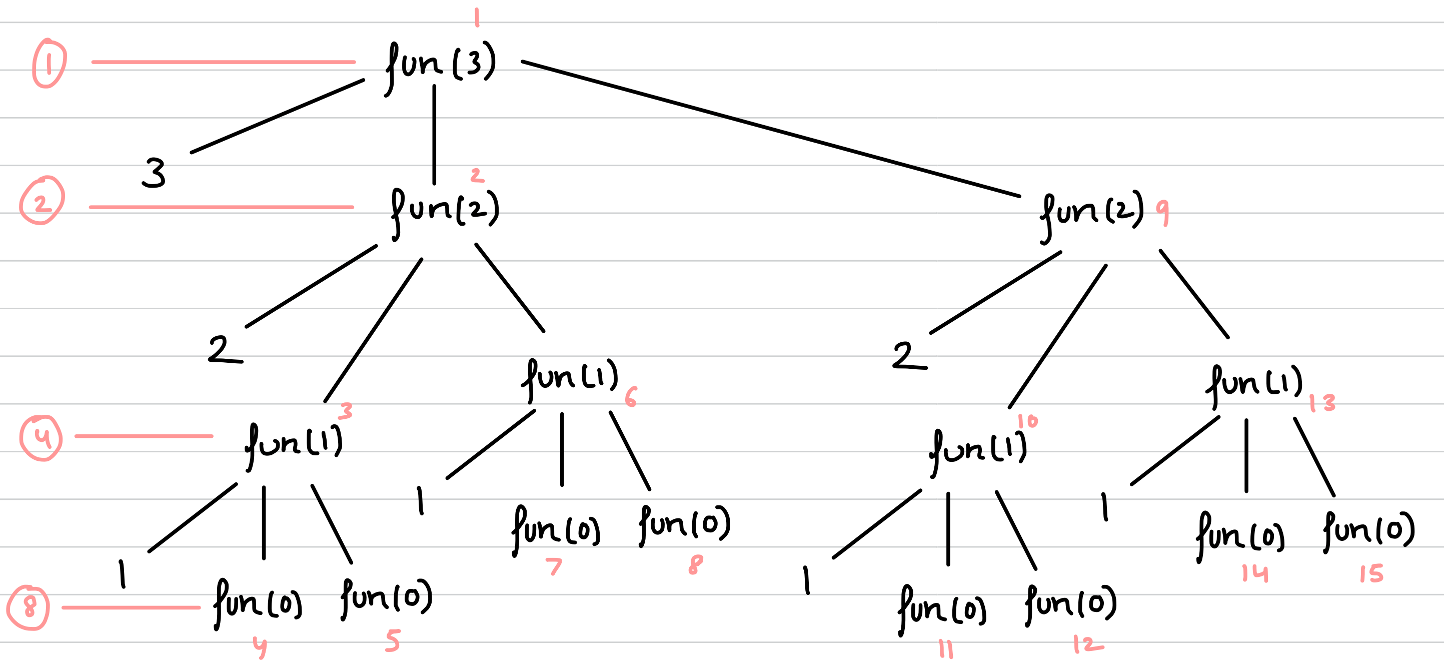
EXAMPLE OF TREE RECURSION

```
void fun (int n)
{
    if (n>0)
    {
        printf(" %d ", n);
        fun (n-1);
        fun (n-1);
    }
}
```

---

fun(3);

• ACTIVATION RECORDS



$1+2+4+8 = 15$

$2^0+2^1+2^2+2^3 = 2^{3+1}-1$ $\quad 2^{n+1}-1$

GP Series

Time $= O(2^n)$

Space $= O(n)$

[No of levels of activation records $\quad n+1$]

3 – parameter

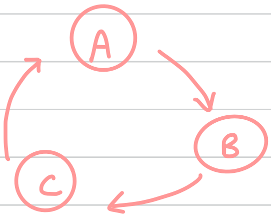
4 – levels

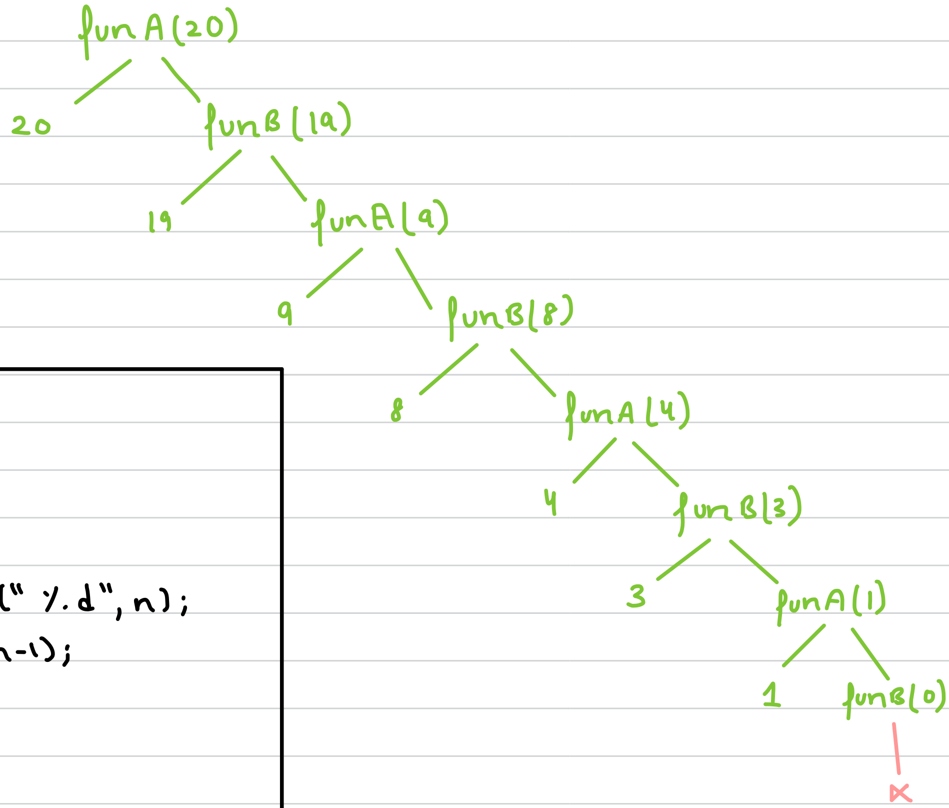
OUTPUT

3 2 1 1 2 1 1

4. INDIRECT RECURSION

In indirect recursion, there are more than one function and they are calling one another in a circular manner.






```
void funA(int n)
{
        if (n>0)
        {
                printf(" %d", n);
                funB(n-1);
        }
}

void funB(int n)
{
        if (n>1)
        {
                printf(" %d", n);
                funA(n/2);
        }
}
```

5. NESTED RECURSION

In a nested recursion, the recursive function will pass parameter as a recursive call.

```
int fun(int n)
{
    if (n>100)
        return n-10;
    else
        return fun(fun(n+11));
}

fun(95);
```

fun(95) = 91

fun(fun(95+11))   96 = fun(106)

106

fun(96)

fun(fun(96+11))   97 = fun(107)

107

fun(97)

fun(fun(97+11))   98 = fun(108)

108

fun(98)

fun(fun(98+11))   99 = fun(109)

109

fun(99)

fun(fun(99+11))   100 = fun(110)

110

fun(100)

fun(fun(100+11))   101 = fun(111)

111

fun(101)

91

SUM OF FIRST N NATURAL NUMBERS

$1+2+3+4 \ldots\ldots + n$

$sum(n) = 1+2+3+4+\ldots\ldots+(n-1)+n$

$sum(n) = sum(n-1)+n$

$$sum(n) = \begin{cases} 0 & n=0 \\ sum(n-1)+n & n>0 \end{cases}$$

```
int sum (int n)
{
      if (n = =0)
            return 0;
      else
            return sum(n-1) +n;
}
```

Time = O(n)
Space = O(n)

sum(5)
sum(4) + 5 = 15
sum(3) + 4 = 10
sum(2) + 3 = 6
sum(1) + 2 = 3
sum(0) + 1 = 1

```
int sum (int n)
{
      return n* (n+1) /2;
}
```

O(n)

```
Int sum (int n)
{
      int i, s=0;                 ——— 1
      for (i=1; i<= n; i++)       ——— n+1
            s= s+i;               ——— n
      return s;                   ——— 1
}
```

2n+3

O(n)

FACTORIAL USING RECURSION

$fact(n) = 1 * 2 * 3 * \ldots\ldots (n-1) * (n)$
$fact(n) = fact(n-1) * n$

$$fact(n) = \begin{cases} 1 & n = 0 \\ fact(n-1) * n & n > 0 \end{cases}$$

```
int fact (int n)
{
    if (n==0)
        return 1;
    else
        return fact(n-1)*n;
}
```

POWER USING RECURSION

$m^n = m * m * m\ m \ldots\ldots$ for n times
$pow(m,n) = m * m * m \ldots\ldots * (n-1)$ times $* m$
$pow(m,n) = pow(m,n-1) * m$

$$pow(m,n) = \begin{cases} 1 & m = 0 \\ pow(m,n-1) * m & m > 0 \end{cases}$$

```
int pow (int m, int n)
{
    if (n==0)
        return 1;
    return pow (m,n-1);
}
```

Here 10 calls are being performed

But this way is taking longer

$2^8 = (2^2)^4$
$= (2*2)^4$

$2^9 = 2 * (2^2)^4$

pow (2,9)
|
pow (2,8) * 2
|
pow (2,7) * 2
|
pow (2,6) * 2
|
pow (2,5) * 2
|
pow (2,4) * 2
|
pow (2,3) * 2
|
pow (2,2) * 2 = 6
|
pow (2,1) * 2 = 4
|
pow (2,0) * 2 = 2
|
1

REWRITING POWER FUNCTION

FASTER METHOD

```
int pow(m,n)
{
    if (n==0)
        return 1;

    else
        if (n%2==0)
            return pow(m*m, n/2);
        else
            return m*pow(m*m, (n-1)/2);
}
```

$pow(2,9)$ $= 2^9.$

$2 * pow(2^2, 4)$ $\quad 2 * 2^8 = 2^9$

$pow(2^4, 2)$

$pow(2^8, 1)$

$2^8 * pow(2^{16}, 0) = 2^8 \times 1 = 2^8$

1

TAYLOR'S SERIES

$$e^x = 1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots \frac{x^n}{n!}$$

$$e(x,4) = 1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

$$e(x,3) = 1 + \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3!}$$

$$e(x,2) = 1 + \frac{x}{1} + \frac{x^2}{2}$$

$$e(x,1) = 1 + \frac{x}{1}$$

$p = p * x \qquad f = f * 1 \qquad 1 + p/f$

$$e(x,0) = 1$$

```
int  e(int x, int n)
{
      static  int  p = 1 , f = 1;
      int r;

      if ( n == 0)
            return 1;

      else
      {
            r = e (x,n-1);
            p = p* x;
            f = f* n;
            return r + p/f;
      }
}
```

p: $\boxed{1}$ — $x*x*x*x$

f: $\boxed{1}$ — $1 \times 2 \times 3 \times 4$

TAYLOR'S SERIES USING HORNER'S RULE

$$e^x = 1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \dots \frac{x^n}{n!}$$

$\frac{x \times x}{2 \times 1}$ $\frac{x \times x \times x}{3 \times 2 \times 1}$

No of multiplications $\downarrow$

0 0 1+1 2+2

2 4 6 8 10

$2(1+2+3+\dots n)$

$2\frac{(n)(n+1)}{2}$

$O(n^2)$ Quadratic

$$1 + \frac{x}{1} + \frac{x^2}{1 \times 2} + \frac{x^3}{1 \times 2 \times 3} + \frac{x^4}{1 \times 2 \times 3 \times 4}$$

$$1 + \frac{x}{1}\left[\frac{x}{2} + \frac{x^2}{2 \times 3} + \frac{x^3}{2 \times 3 \times 4}\right]$$

$$1 + \frac{x}{1}\left[\frac{x}{2}\left[\frac{x}{3} + \frac{x^2}{3 \times 4}\right]\right]$$

$$1 + \frac{x}{1}\left[\frac{x}{2}\left[\frac{x}{3}\left[1 + \frac{x}{4}\right]\right]\right]$$

$O(n)$ Linear

```
int e(int x, int n)
{
        int s = 1;

        for ( ;n>0 ;n--)
                s = 1+ x/n*s ;

        return s;
}
```

USING FOR LOOP

```
int e(int x, int n)
{
        static int s = 1;

        if (n==0)
                return s;
        else
                s = 1+ x/n*s;

        return e(x, n-1);
}
```

USING RECURSION

FIBONACCI SERIES

fib(n)	0	1	1	2	3	5	8	13
n	0	1	2	3	4	5	6	7

$$fib(n) = \begin{cases} 0 & n=0 \\ 1 & n=1 \\ fib(n-2)+fib(n-1) & n>1 \end{cases}$$

PROGRAM USING ITERATION

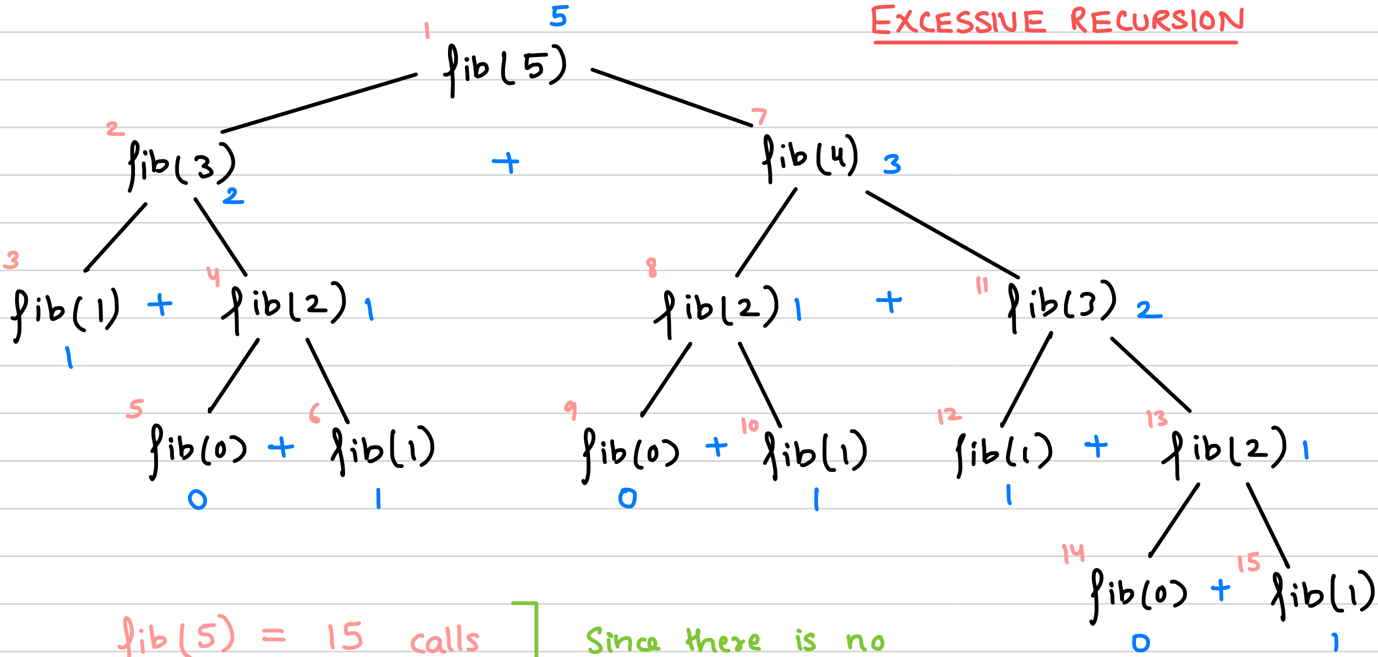
```
int fib (int n)
{
    int t0=0, t1=1, s, i;  ——— 1
    if (n<=1)
        return n;  ——— 1
    for (i=2; i<=n; i++)  ——— n
    {
        s = t0 + t1;  ——— n-1
        t0 = t1;  ——— n-1
        t1 = s;  ——— n-1
    }

    return s;  ——— 1
}                       4n
```

$O(n)$

PROGRAM USING RECURSION

```
int fib (int n)
{
        if (n<=1)
            return n;
        else
            return fib(n-2) + fib(n-1);
}
```

EXCESSIVE RECURSION



fib (5) = 15 calls
fib (4) = 9 calls
fib (3) = 5 calls

Since there is no definite pattern, then we have to assume that the function fib(n-2) + fib(n-1); is calling itself

2 fib (n-1)

↓

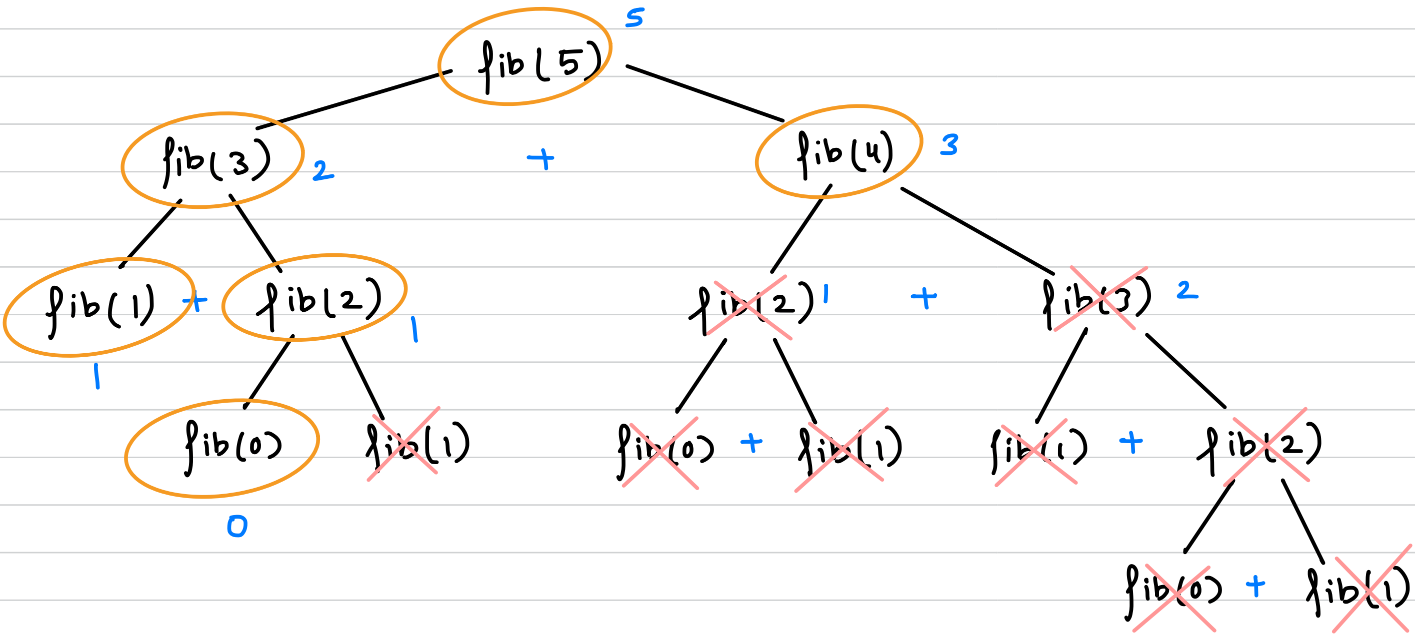
order of (2^n)

This tree lies in the category of excessive recursion because it calls itself multiple times for the same parameters.

To reduce the order of this function, we will write another program using static array and initialize all its values with -1.

F

0	1	2	3	4	5	6
~~-1~~ 0	~~-1~~ 1	~~-1~~ 1	~~-1~~ 2	~~-1~~ 3	~~-1~~ 5	-1



So, for 5 as n, 6 calls are made

∴ for n, n+1 calls are made

O(n)

This approach of storing result in an array is called MEMOIZATION.

↳ Storing the result of function calls, so they can be utilized again for avoiding excessive calls

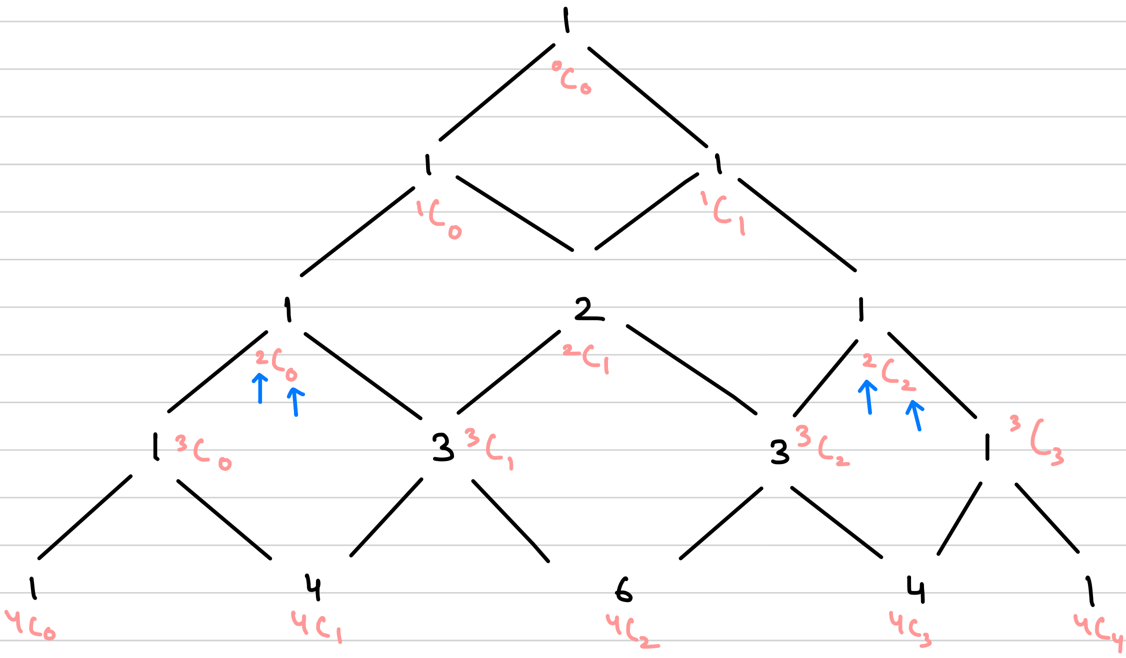
```
int F[10];

int fib(int n)
{
    if (n<=1)
    {
        F[n]=n;
        return n;
    }
    else
    {
        if(F[n-2]==-1)
            F[n-2]=fib(n-2);

        if (F[n-1]==-1)
            F[n-1]=fib(n-1);

        return F[n-2]+F[n-1];
    }
}
```

COMBINATION FORMULA

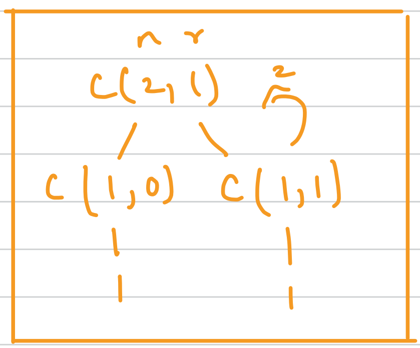
$${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$$

```
int C (int n, int r)
{
        int t1, t2, t3;
        t1 = fact (n);          ———— n
        t2 = fact (r);          ———— n
        t3 = fact (n-r);        ———— n

        return t1/(t2*t3);      ———— 1
}                                   ——
                                    3n
                                    O(n)
```

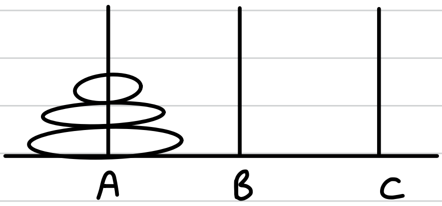
PASCAL'S TRIANGLE




```
int C (int n, int r)
{
        if (r == 0 || n == r)
                return 1;
        else
                return C(n-1, r-1) + C(n-1, r);
}
```

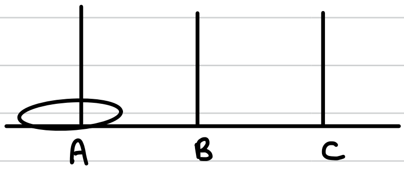


TOWER OF HANOI

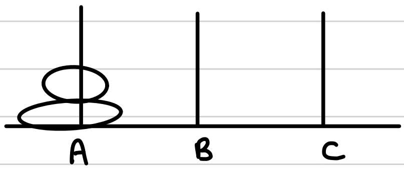


1. Move one disk at a time.
2. No bigger disk can be there above a smaller one.

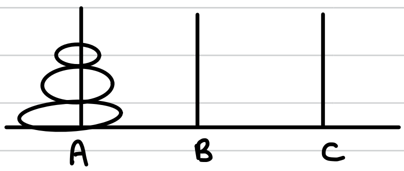
TOH (1, A, B, C)
Move disk A to C using B.



TOH (2, A, B, C)
1. TOH (1, A, C, B)
2. Move disk A to C using B.
3. TOH (1, B, A, C)



TOH (3, A, B, C)
1. TOH (2, A, C, B)
2. Move disk from A to C using B
3. TOH (2, B, A, C).



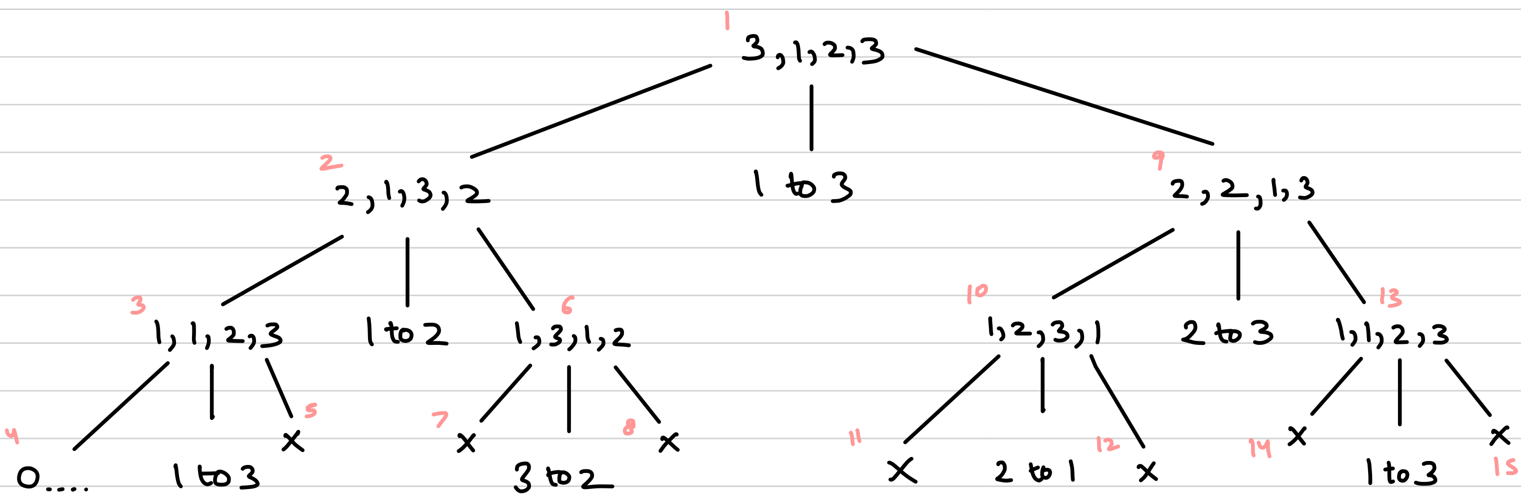
FOR n number of disk.

TOH ($\cancel{3}$ n, A, B, C)
1. TOH ($\cancel{2}$ n-1, A, C, B)
2. Move disk from A to C using B
3. TOH ($\cancel{2}$ n-1, B, A, C).

```
void TOH(int n, int A, int B, int c)
{
    if(n>0)
    {
        TOH(n-1, A, C, B);
        printf(" from %d to %d", A, C);
        TOH(n-1), B, A, C);
    }
}
```

TOH(3,1,2,3)

$A \rightarrow 1$
$B \rightarrow 2$
$C \rightarrow 3$



OUTPUT

(1 to 3), (1 to 2), (3,2), (1,3), (2,1), (2,3), (1,3)

	Calls	
$n = 3$	15	$1+2+2^2+2^3 = 2^4-1$
$n = 2$	7	$1+2+2^2 = 2^3-1$

$\boxed{2^{n+1}-1}$

$O(2^n)$

ARRAYS

Scalar

int x = 10;

x
10

100/101 2 bytes

int A[5];
A[2] = 15;

vector

	0	1	2	3	4
A			15		
	200/1	202/3	203/5	204/7	205/9

10 bytes

DECLARATION OF ARRAYS

1. int A[5];

	0	1	2	3	4
A	?	?	?	?	?

garbage

2. int A[5] = {2,4,6,8,10};

	0	1	2	3	4
A	2	4	6	8	10

3. int A[5] = {2,4};

	0	1	2	3	4
A	2	4	0	0	0

→ Rest of the elements get automatically initialized by 0.

4. int A[5] = {0};

	0	1	2	3	4
A	0	0	0	0	0

5. int A[] = {2,4,6,8,10,12}

	0	1	2	3	4	5
A	2	4	6	8	10	12

→ Depending upon the number of elements, size of the array is automatically allocated.

---

int A[5] = {2,5,4,9,8};

	0	1	2	3	4
A	2	5	4	9	8
	200/1	2/3	4/5	6/7	208/9

→ Array addresses are contagious.

```
printf("%d", A[2]);
printf("%d", 2[A]);
printf("%d", *(A+2));
```

→ OUTPUT
4

```
for (i = 0; i < 5; i++)
    printf("%u", &A[i]);
```

for printing address.

STATIC VS DYNAMIC ARRAY

Size of the array is static

size of the array is dynamic

→ Once an array is created, it's size cannot be modified.

In C

→ size of the array is decided at compile time

In C++

→ The size of the array can be decided at run time also.

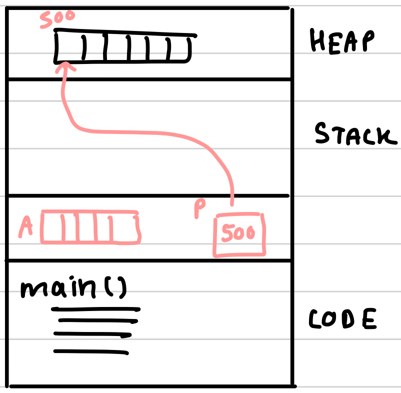
```
int n;
cin>>n;
int A[n];
```

ACCESSING HEAP

```
void main()
{
        int A[5];
        int *p;                         p[0] = 5

C++     p = new int[5];

C       p = (int *)malloc(5 * sizeof(int));
                    <stdlib.h>
        :
        :

C++     delete []p;
C       free(p);
}
```

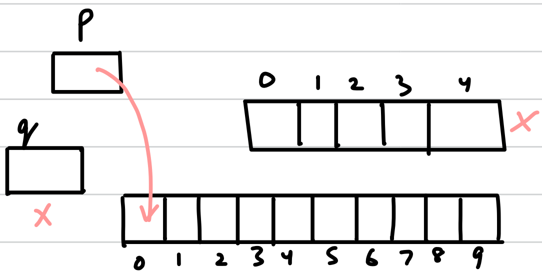
Otherwise MEMORY LEAK

shortage of memory.



ONE WAY OF INCREASING SIZE OF ARRAY

```
int *p = new int[5];
int *q = new int[10];

for(i=0; i<5; i++)
        q[i] = p[i]

delete []p;
p = q;
q = NULL;
```



2D - ARRAY

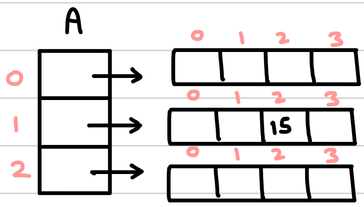
① int A[3][4] = { {1,2,3,4}, {2,4,6,8}, {3,5,7,9} };

Array will be stored inside stack.

	0	1	2	3
0				
1			15	
2				

A[1][2] = 15

② int *A[3]; → Array of pointers



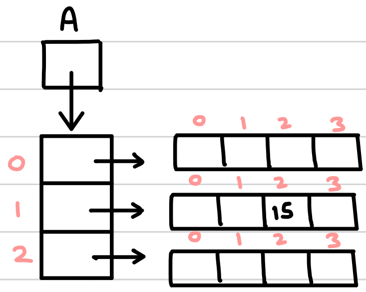
A[0] = new int[4];
A[1] = new int[4];
A[2] = new int[4];

Memory is created inside heap.

A[1][2] = 15;

We can access array of pointers in the same way.

③



int **A;
A = new int*[3];
A[0] = new int[4];
A[1] = new int[4];
A[2] = new int[4];

All the memory is allocated in heap.

for column
```
for (i=0; i<3; i++)
{
    for (j=0; j<4; j++)
    {
        A[i][j] = __;
    }
}
```
for row

	0	1	2	3
0	1	2	3	4
1	5	6	7	8
2	9	10	11	12

- Elements accessed in order

HOW COMPILER GENERATES FORMULA FOR ADDRESS OF AN ARRAY

int A[5] = { 3, 5, 8, 4, 2 };

3	5	8	4	2
200/1	2/3	4/5	6/7	208/209

L_0

ADD (A[3]) = 200 + 3 * 2 = 206

ADD (A[3]) = L_0 + 3 * 2

ADD (A[i]) = L_0 + i * ω → FORMULA

No of operations = 2

Base Address (L_0), Index (i), Size of datatype. (ω)

ADD (A[i]) = L_0 + (i-1) * ω → Formula, when indices are starting from one onwards

No of operations = 3

More time consuming.

That's why c and c++ do not start array index with 1. Only for one extra operation, time taken by the program will increase and this will make the program slower.

ROW MAJOR

int A[3][4] ;
m n

A

0	1	2	3	4	5	6	7	8	9	10	11
a_{00}	a_{01}	a_{02}	a_{04}	a_{05}	a_{06}	a_{07}	a_{08}	a_{09}	a_{10}	a_{11}	a_{12}
200/1	2/3	4	6	8	10	12	14	16	18	20	22
row_0				row_1				row_2			

Add (A[1][2]) = 200 + [4 + 2] * 2 = 212

Add (A[2][3]) = 200 + [2 * 4 + 3] * 2 = 222

Add (A[i][j]) = L_0 + [i * n + j] * ω
4 operations

Add([i][j]) = L_0 + [(i-1) * n + (j-1)] * ω
6 operations

COLUMN MAJOR

	0	1	2	3	4	5	6	7	8	9	10	11
A	a_{00}	a_{10}	a_{20}	a_{01}	a_{11}	a_{21}	a_{02}	a_{12}	a_{22}	a_{03}	a_{13}	a_{23}
	col 0			col 1			col 2			col 3		

$$ADD(A[1][2]) = 200 + [2 * 3 + 1] * 2 = 214$$

$$ADD(A[1][3]) = 200 + [3 * 3 + 1] * 2 = 220$$

$$ADD(A[i][j]) = L_0 + (j * m + i) * \omega$$

FORMULAS FOR nD ARRAYS

A[d1][d2][d3][d4]; 4 D ARRAY

Row Major

$$ADD(A[i_1][i_2][i_3][i_4]) = L_0 + [i_1 * d_2 * d_3 * d_4 + i_2 * d_3 * d_4 + i_3 * d_4 + i_4] * \omega$$

Column Major

$$ADD(A[i_1][i_2][i_3][i_4]) = L_0 + [i_4 * d_3 * d_2 * d_1 + i_3 * d_2 * d_1 + i_2 * d_1 + i_1] * \omega$$

ROW MAJOR FOR nD

$$L_0 + \sum_{p=1}^{n} \left[i_p * \prod_{q=p+1}^{n} d_q \right] * \omega$$

COLUMN MAJOR for nD

$$L_0 + \sum_{p=n}^{1} \left[i_p * \prod_{q=p-1}^{1} d_q \right] * \omega$$

(self tried)
(please check)

A[d1][d2][d3][d4]; 4D ARRAY HORNER'S RULE

Row Major

$$Add(A[i_1][i_2][i_3][i_4]) = L_0 + [\underbrace{i_1 * d_2 * d_3 * d_4}_{3} + \underbrace{i_2 * d_3 * d_4}_{2} + \underbrace{i_3 * d_4}_{1} + i_4] * w$$

$4D \rightarrow 3 + 2 + 1$

$5D \rightarrow 4 + 3 + 2 + 1$

$nD \rightarrow n-1 + n-2 + n-3 \ldots + 1 = \frac{n(n-1)}{2}$

$O(n^2)$

$$i_4 + i_3 * d_4 + i_2 * d_3 * d_4 + i_1 * d_2 * d_3 * d_4$$

$$i_4 + d_4 [i_3 + i_2 * d_3 + i_1 * d_2 * d_3]$$

$$i_4 + d_4 [i_3 + d_3 [i_2 + i_1 * d_2]]$$

$O(n)$

FORMULA FOR 3D ARRAYS

int A[l][m][n];

ROW MAJOR

$$Add(A[i][j][k]) = L_0 + [i * m * n + j * n + k] * w$$

COLUMN MAJOR

$$Add(A[i][j][k]) = L_0 + [k * m * l + j * l + i] * w$$

ARRAY ADT

→ Abstract Datatype

1. Representation of data
2. Operations on data

DATA

1. Array Space
2. Size
3. Length (No of elements)

① int A[10];

② int *A;

A = new int[size];

OPERATIONS

1. Display()
2. Add(x) / Append(x)
3. Insert (index, x)
4. Delete (index)
5. Search (x)
6. Get (index)
7. Set (index, x)
8. Max(), Min()
9. Reverse()
10. Shift() / Rotate()

1. DISPLAY

pseudo code

```
for (i = 0 ; i < length ; i++)
{
        print (A[i])
}
```

Array size = 10
Length = 6

8	3	7	12	6	9				
0	1	2	3	4	5	6	7	8	9

2. Add (x) / Append (x)

```
A[Length] = x;  —— 1
length ++;      —— 1
```

$f(x) = 2$

$f(n) = 2n^0$

$O(n^0) = O(1)$

Array size = 10
Length = 7

8	3	7	12	6	9	10			
0	1	2	3	4	5	6	7	8	9

3. Insert (4, 15)

index x

Array size = 10
Length = 8

8	3	7	12	6	9	10			
0	1	2	3	4	5	6	7	8	9

```
for (i = length ; i > index ; i--)
{
    A[i] = A[i-1];   — 0-n
}
```

Inserting at index 8 — Inserting at 0

```
A[index] = x;   ——— 1
Length++;
```

8	3	7	12	15	6	9	10		
0	1	2	3	4	5	6	7	8	9

O(1) min — O(n) max

4. Delete (3)

index

Array size = 10
Length = 7

8	3	7	12	15	6	9	10		
0	1	2	3	4	5	6	7	8	9

```
x = A[index];   ——— 1

for ( i = index ; i <= Length-1 ; i++)
    A[i] = A[i+1];   ——— 0-n

Length--;   ——— 1
```

min = 2
max = n+2

8	3	7	15	6	9	10			
0	1	2	3	4	5	6	7	8	9

Best O(1) Worst O(n)

Index should be in range of length
we cannot leave empty space between two elements in an array.

LINEAR SEARCH → Searching each element and incrementing

Array size = 10
Length = 10

8	9	4	7	6	3	10	5	14	2
0	1	2	3	4	5	6	7	8	9

```
for (i=0 ; i < length ; i++)
        if ( key == A[i])
                return i;
return -1;
```

Key = 5 ← Succesful → Best — O(1)

Key = 12 ← Unsuccesful → Worst — O(n)

Average — O(n)

Average

found at location 1
found at 2

$$\frac{1+2+3+\dots n}{n} = \frac{\frac{(n+1)\cancel{n}}{2}}{\cancel{n}} = \frac{n+1}{2}$$

FASTER WAY OF LINEAR SEARCH

1. Transposition : Moving the searched element one step back

8	9	4	7	6	3	10	5	14	2
0	1	2	3	4	5	6	7	8	9

key

```
for (i=0; i< length; i++)
 {
        if ( key == A[i])
        {
              swap( A[i], A[i-1]);
              return i-1;
        }
 }
```

2. Move to front / head : The searched element is brought to first position in array.

8	9	4	7	6	3	10	5	14	2
0	1	2	3	4	5	6	7	8	9

key

```
for (i=0; i< length; i++)
 {
        if ( key == A[i])
        {
              swap( A[i], A[0 ]);
              return 0;
        }
 }
```

BINARY SEARCH

Size = 15
Length = 15

A

4	8	10	15	18	21	24	27	29	33	34	37	41	43
1	2	3	4	5	6	7	8	9	10	11	12	13	14

Sorted Array

ITERATIVE VERSION

```
Algorithm BinSearch(l, h, key)
{
    while (l <= h)
    {
        mid = [(l+h)/2];

        if (key == A[mid])
            return mid;

        else if (key < A[mid])
            h = mid-1;
        else
            l = mid+1;
    }
    return -1;      # key not found
}
```

Binary search is faster than linear search

RECURSIVE VERSION

Tail Recursion

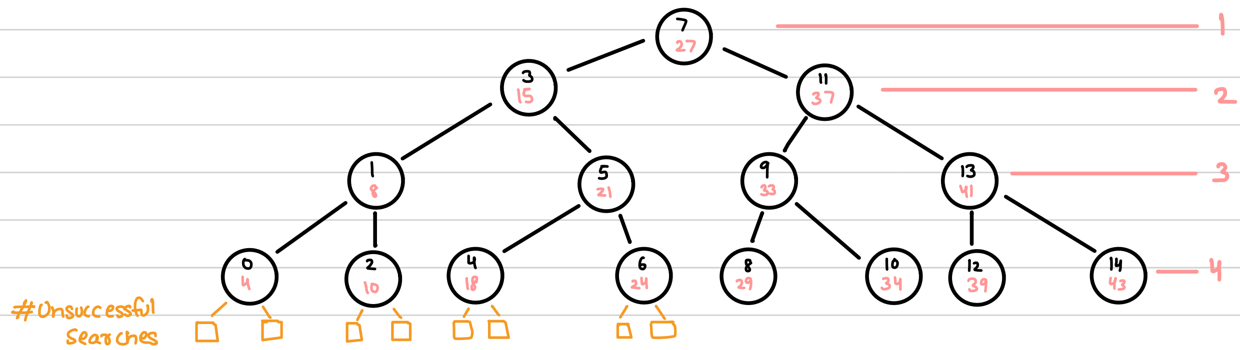
```
Algorithm RBinSearch(l, h, key)
{
    if (l <= h)
    {
        mid = [(l+h)/2];

        if (key == A[mid])
            return mid;

        else if (key < A[mid])
            return RBinsearch(l, mid-1, key);
        else
            return RBinsearch(mid+1, h, key);
    }
    return -1;      # key not found
}
```

ANALYSIS OF BINARY SEARCH

Size =15
Length =15

A

4	8	10	15	18	21	24	27	29	33	34	37	41	43
1	2	3	4	5	6	7	8	9	10	11	12	13	14



Best min – $O(1)$
Worst max – $O(\log n)$
Unsuccesful max – $O(\log n)$

Why $\log n$?

$\log n$

Let size of array be 16

$$\frac{16}{2}, \frac{}{2}, \frac{}{2}, \frac{}{2}$$

$2^4 = 16$
$4 = \log_2 16$

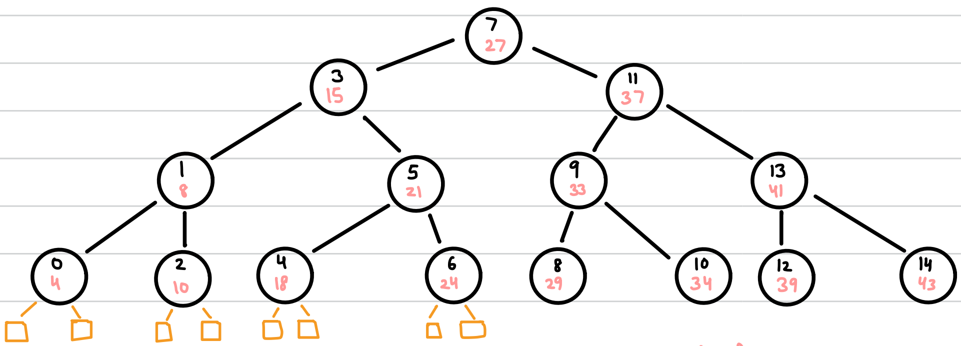
$\log_2 n$

Inverse of power is log.

16 is divided by 2 multiple times

AVERAGE CASE ANALYSIS OF BINARY SEARCH*

$$= \frac{\text{Total time taken in all possible cases}}{\text{Number of cases}}$$



$1 + 1\times 2 + 2\times 4 + 3\times 8$

No of comparisons ↲ ↳ No of elements

$1 + 1\times 2^1 + 2\times 2^2 + 3\times 2^3$

$\log n \rightarrow$ Level of tree (Here 4)

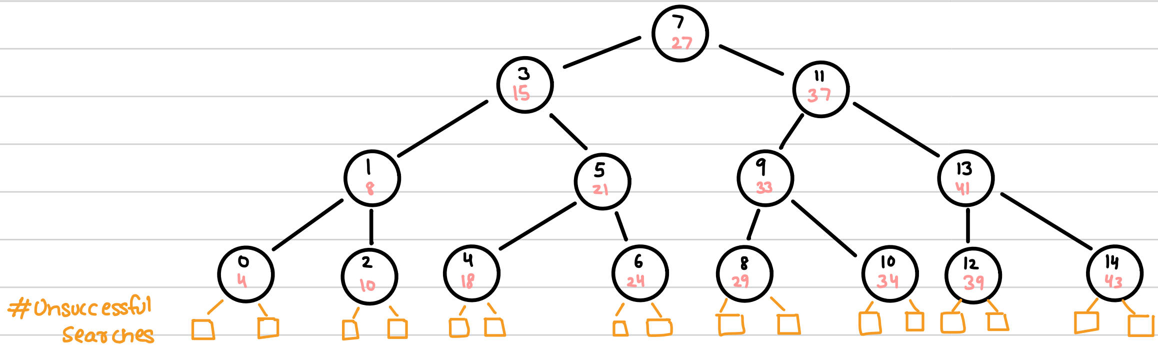
$$\frac{\sum_{i=1}^{\log n} i \times 2^i}{n} = \frac{\log n \times 2^{\log n}}{n}$$

i is replaced with logn

Same as formula*

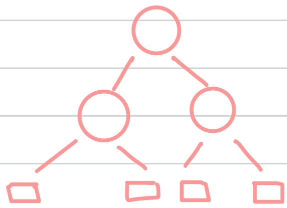
$$= \frac{\log n \times n^{\cancel{\log_2 2}}}{n}$$

$$= \log n$$



I = sum of all internal nodes
E = sum of all external nodes

$$E = I + 2n$$



$n = 3$
$I = 2$
$E = 2 \times 4 = 8$

$E = I + 2n$

i = num of internal nodes
e = num of external nodes

n = num of nodes

$$e = i + 1$$

<u>Average successful time for n elements</u>

$$A_s(n) = 1 + \frac{I}{n} \Rightarrow 1 + \frac{E - 2n}{n} \Rightarrow 1 + \frac{n\log n}{n} - 2$$

$$\Rightarrow 1 + \frac{E}{n} - 2 \Rightarrow \log n$$

<u>Average unsuccessful time</u>

$$A_U(n) = \frac{E}{n+1} = \frac{n\log n}{n+1} = \log n$$

$E = n\log n$
$E = I + 2n$
$I = E - 2n$

6. <u>Get(index)</u>:

```
if (index >= 0 && index < length)
    return A[index];
```

$O(1)$

7. <u>Set(index, x)</u>

```
if (index >= 0 && index < length)
    A[index] = x;
```

$O(1)$

8. Max()

```
max = A[0];                      ———— 1
for (i = 1; i < length; i++)     ———— n
{
      if (A[i] > max)            ———— n-1
            max = A[i];
}
return max;                      ———— 1
                                      ————
                                      2n+1   O(n)
```

9. Min()

```
min = A[0];
for (i = 1; i < length; i++)
{
      if (A[i] < min )
            min = A[i];
}
return min;
```

10. Sum()

```
Total = 0;                        ———— 1
for (i = 0; i < length; i++)      ———— n+1
          Total += A[i]           ———— n
return total                      ———— 1
                                       ————
                                       2n+3
                                       O(n)
```

$$sum(A, n) = \begin{cases} 0 & n < 0 \\ sum(A, (n-1)) + A[n] & n >= 0 \end{cases}$$

11. Avg()

```
Total = 0;
for (i = 0; i < length; i++)
          Total += A[i]
return total/n;
```

RECURSIVE CALL

```
int sum (A, n)
{
      if (n < 0)
            return 0;
      else
            return sum (A, n-1) + A[n];
}
```

sum (A, length-1) — call

REVERSE AND SHIFT AN ARRAY

1. Reverse
2. Left Shift
3. Left Rotate
4. Right Shift
5. Right Rotate

1. Reversing an array

Reversing using auxillary array B.

A

8	9	4	7	6	3	10	5	14	2
0	1	2	3	4	5	6	7	8	9

B

2	14	5	10	3	6	7	4	9	8
0	1	2	3	4	5	6	7	8	9

$A \rightarrow B$ $\quad n$

A

2	14	5	10	3	6	7	4	9	8
0	1	2	3	4	5	6	7	8	9

$B \rightarrow A$ $\quad n$

$2n$

$O(n)$

```
for (i = length-1; j=0; i>=0 ; i-- , j++)
        B[j] = A[i];
```

Reverse copying array

```
for (i=0 ; i < length ; i++)
        A[i] = B[i];
```

```
for ( i=0; j = length-1 ; i<j ; i++ ; j--)
{
        temp = A[i];
        A[i] = A[j];
        A[j] = temp;
}
```

$O(n)$

ANOTHER METHOD OF REVERSING ARRAY

A

8	9	4	7	6	3	10	5	14	2
0	1	2	3	4	5	6	7	8	9

2. Left Shift

6	3	8	5	9

3	8	5	9	0

3. Left Rotate

6	3	8	5	9

3	8	5	9	6

CHECK IF ARRAY IS SORTED

1. Inserting in an sorted array
2. Checking if array is sorted
3. Arranging -ve elements on left side and +ve on right side.

1. Inserting in an sorted array

A

4	8	13	16	20	25	28	33		
0	1	2	3	4	5	6	7	8	9

Insert - 18

```
x = 18 ;
i = length-1 ;
while (A[i] > x)
{
        A[i+1] = A[i];
        i-- ;
}
A[i+1] = x ;
```

Starting from last element, loop will run until the element is greater than the element to be inserted

2. Checking if array is sorted

A

4	8	13	16	20	25	28	33		
0	1	2	3	4	5	6	7	8	9

```
Algorithm isSorted (A, n)
{
        for (i=0 , i<n-1; i++)
        {
                if (A[i] > A[i+1])
                        return false;
        }
        return true;
}
```

We will check false condition first by checking if any element is greater than its next element.

O(n) - Max   Worst
O(1) - Min   Best

3. Arranging -ve elements on left side and +ve on right side.

```
i=0;
j=length-1;

while (i<j)
{
        while (A[i] < 0)
              i++;
        while (A[j] ≥ 0)
              j--

        if (i<j)
              swap(A[i], A[j])
}
```

A

-6	3	-8	10	5	-7	-9	12	-4	2
0	1	2	3	4	5	6	7	8	9
i									j

A

-6	-4	-8	-9	-7	5	10	12	3	2
0	1	2	3	4	5	6	7	8	9
				j	i				

O(n)   n+2 comparisons are made

MERGING ARRAYS

1. Append / Concat
2. Compare
3. Copy

```
i=0;
j=0;
k=0;
while (i<m && j<n)    ──→ # When either of i or j have reached end of array.
{
     if (A[i] < B[j])
     {
          c[k] = A[i];
          k++;
          i++;
     }

     else
     {
          c[k] = B[j];
          k++;
          j++;
     }
     for ( ; i<m; i++)  ──→ # When some elements in either of array are remaining to be copied.
          c[k++] = A[i];
     for ( ; j<n; j++)
          c[k++] = B[j];
}
```

Can also be written as c[k++] = A[i++];

c[k++] = B[j++]

A (m), i

3	8	16	20	25
0	1	2	3	4

B (n), j

4	10	12	22	23
0	1	2	3	4

C, k

3	4	8	10	12	16	20	22	23	25
0	1	2	3	4	5	6	7	8	9

$\Theta = (m+n)$

↳ Time is known

SET OPERATIONS

1. UNION

A (m)

3	5	10	4	6
0	1	2	3	4

B (n)

12	4	7	2	5
0	1	2	3	4

C

3	5	10	4	6	12	7	2		
0	1	2	3	4	5	6	7	8	9

- Copy all elements of A in C
- Then copy elements of B which are not there in C.

$m + m * n$

$n + n * n$

$n + n^2$

$O(n^2)$

A (m)

3	4	5	6	10
0	1	2	3	4

B (n)

2	4	5	7	12
0	1	2	3	4

C

2	3	4	5	6	7	10	12		
0	1	2	3	4	5	6	7	8	9

- Use merge procedure
- Copy the smaller element to C
- If same element, copy once
- Use i, j, k method

$\Theta(m+n)$

$\Theta(n+n)$

$\Theta(n)$

2. INTERSECTION

A (m)

3	5	10	4	6
0	1	2	3	4

B (n)

12	4	7	2	5
0	1	2	3	4

C

5	4								
0	1	2	3	4	5	6	7	8	9

Copy the common elements of A and B to C

- One by one, take each element of A. If it is present in B, copy it to C otherwise move on to next element

$n * m$

$n * n$

$O(n^2)$

A (m)

3	4	5	6	10
0	1	2	3	4

B (n)

2	4	5	7	12
0	1	2	3	4

C

4	5								
0	1	2	3	4	5	6	7	8	9

- Use merge method
- If element is smaller, don't copy.
- If element is same, copy
- Not merging but similar to it.

$\Theta(m+n)$

$\Theta(n)$

3. DIFFERENCE

A

3	5	10	4	6
0	1	2	3	4

m

B

12	4	7	2	5
0	1	2	3	4

n

C

3	10	6							
0	1	2	3	4	5	6	7	8	9

A-B

- we want elements of A which are not there in B.
- One by one, take each element of A If it is not present in B, copy it to C, otherwise move to next element

$m*n$

m^2

$O(n^2)$

A

3	4	5	6	10
0	1	2	3	4

m i

B

2	4	5	7	12
0	1	2	3	4

n j

C

3	6	10							
0	1	2	3	4	5	6	7	8	9

- Use merge procedure.
- Compare A[i] and B[j], if A[i] is small, copy it to C otherwise increment j.
- If same, don't copy.

$\Theta(m+n)$

$\Theta(n)$

FIND MISSING ELEMENT

1. Single missing element in an sorted array.
2. Multiple missing element in an sorted array.
3. Missing element in unsorted array.

1. Single missing element in an sorted array.

METHOD 1

ITERATIVE METHOD

A

1	2	3	4	5	6	8	9	10	11	12
0	1	2	3	4	5	6	7	8	9	10

```
for (i = 0 ; i < 11; i++)
        sum += A[i];
s = n* (n+1) /2 ;        # Formula for sum of
                           n natural numbers
s - sum
78 - 71 = 7  → Missing num
```

METHOD 2

A

6	7	8	9	10	11	13	14	15	16	17
0	1	2	3	4	5	6	7	8	9	10

6-0 = 6   7-1 = 6   8-2 = 6

$l = 6$
$h = 17$
$n = 11$
To be known

```
diff = l-0;
for (i=0; i<n; i++)
{
    if (A[i] - i != diff)
    {
        printf(" missing element : %d", i+diff);
        break;
    }
}
```

O(n)

2. Multiple missing element in an sorted array.

METHOD 1

A

6	7	8	9	11	12	15	16	17	18	19
0	1	2	3	4	5	6	7	8	9	10
6	6	6	6	7	~~7~~	9	9	9	9	9

~~8~~ 9

```
diff = 6-0;
for (i=0; i<n; i++)
{
    if (A[i] - i != diff)
    {
        while (diff < A[i]-i)
        {
            printf("%d", i + diff);
            diff++;
        }
    }
}
```

negligible (few elements)

O(n)

METHOD 2

A

0	1	2	3	4	5	6	7	8	9	10
6	7	8	9	11	12	15	16	17	18	19

0	1	2	3	4	5	6	7	8	9	10	11	12
~~0~~	~~0~~ 1	~~0~~ 1	~~0~~ 1	~~0~~ 1	0	~~0~~ 1	~~0~~ 1	~~0~~ 1	~~0~~ 1	~~0~~ 1	~~0~~ 1	~~0~~ 1

Hash Table / Bit set

A table is created of size equal to the largest element of array A. The array is initialized with 0 One by one, each element present in array A, the index of hash array is initialized by 1 corresponding to it.

```
for (i=0; i<n; i++)              ______  n
        H[A[i]]++;

for (i=l; i<=h; i++)             ______  n
      if (H[i]==0)                      ____
           printf("%d", i);             2n
                                        O(n)
```

FINDING DUPLICATE IN A SORTED ARRAY

0	1	2	3	4	5	6	7	8	9
3	6	8	8	10	12	15	15	15	20

```
lastDuplicate = 0;

for (i=0; i<n; i++)
{
      if (A[i] = A[i+1] && A[i]!= LastDuplicate)   # So that if there are
      {                                              more than 1 duplicate,
            printf("%d\n", A[i]);                    it will print only one
            last Duplicate = A[i];                   time.
      }
}
```

COUNTING NO OF TIMES OF DUPLICATE ELEMENT

```
for (i=0; i<n-1; i++)
{
        if (A[i] == A[i+1])
        {
                j= i+1;

                while (A[j]== A[i])
                        j++;

                printf ("%d is appearing %d times, A[i], j-1);
                i=j-1;
        }
}
```

3	6	8	8	10	12	15	15	15	20
0	1	2	3	4	5	6	7	8	9

O(n)

FINDING DUPLICATES IN SORTED ARRAY USING HASHING

3	6	8	8	10	12	15	15	15	20
0	1	2	3	4	5	6	7	8	9

0	0	0	~~0~~ 1	0	0	~~0~~ 1	0	~~0~~ 2	0	~~0~~ 1	0	~~0~~ 1	0	0	~~0~~ 3	0	0	0	0	~~0~~ 1
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

```
for (i=0; i<n; i++)
        H[A[i]]++;

for (i=0; i<=max; i++)
        if (H[i]>1)
                printf ("%d %d", i, H[i]);
```

$n+n = 2n$

O(n)

FINDING DUPLICATES IN AN UNSORTED ARRAY

METHOD 1

				-1		-1	-1		
8	3	6	4	~~6~~	5	~~6~~	~~8~~	2	7
0	1	2	3	4	5	6	7	8	9

```
for (i=0; i<n-1; i++)
{
        count = 1;

        if( A[i] != -1)
        {
                for (j = i+1; i<n; j++)
                {
                        if (A[i] == A[j])
                        {
                                count ++;
                                A[j] = -1;
                        }
                }

                if (count > 1)
                        printf("%d %d", A[i], count);
        }
}
```

$O(n^2)$

METHOD 2

USING HASH TABLE

8	3	6	4	6	5	6	8	2	7
0	1	2	3	4	5	6	7	8	9

0	0	~~0~~ 1	~~0~~ 1	~~0~~ 1	~~0~~ 1	~~0~~ 3	~~0~~ 1	~~0~~ 2
0	1	2	3	4	5	6	7	8

$$\frac{n + n}{O(n)}$$

FIND PAIR WITH SUM K (a+b=k)

SORTED

1	3	4	5	6	8	9	10	12	14
0	1	2	3	4	5	6	7	8	9
i									j

Let a + b = 10 (to be searched)

i + j

If i + j > 10
decrement j

i + j < 10
increment i

i + j = 10
increment i
decrement j

```
i = 0, j = n-1
while (i<j)
{
    if (A[i] + A[j] == k)
    {
        printf ("%d+ %d = %d", A[i], A[j], k);
        i++;
        j--;
    }
    else if ( A[i] + A[j] < k)
        i++;
    else
        j--;
}
```

O(n)

FIND PAIR WITH SUM K (a+b=k)

UNSORTED

1	3	4	5	6	8	9	10	12	14
0	1	2	3	4	5	6	7	8	9

```
for (i=0; i<n-1; i++)
{
    for (j= i+1; j<n; j++)
    {
        if (A[i]+A[j]==k)
            printf (" %d+%d = %d", A[i], A[j], k);
    }
}
```

$O(n^2)$

FIND PAIR WITH SUM K (a+b=k)

USING HASHING

1	3	4	5	6	8	9	10	12	14
0	1	2	3	4	5	6	7	8	9

```
for (i=0; i<n; i++)
{
    if (H[k-A[i]] != 0)
        printf ("%d + %d = %d", A[i], k-A[i], k);
    H[A[i]]++;
}
```

FIND MAX AND MIN IN SINGLE SCAN

1	3	4	5	6	8	9	10	12	14
0	1	2	3	4	5	6	7	8	9

```
min = A[0];
max = A[0];
```

n=10

O(n)

Best = 10, 9, 8, 7, 2, 1

comp = n-1

Worst = 1, 2, 3, 5, 8, 9

comp = 2(n-1)

```
for (i=1; i<n; i++)
{
    if (A[i] < min)
        min = A[i];
    else if (A[i] > max)
        max = A[i];
}
```

STRINGS

1. Character Sets / ASCII codes
2. Character Array
3. String
4. Creating a string

→ For English language

1. Character Sets / ASCII codes

American Standard Code for Information Interchange

A − 65　　a − 97　　0 − 48
B − 66　　b − 98　　1 − 49
⋮　　⋮　　⋮
Z − 90　　z − 122　　9 − 57

UNICODES
2 byte
16 bits　　Represented in form of
4×4 bits　　hexadecimal

↲ enter − 10
Space − 13
esc − 27

$\boxed{0-127}$ Total 128
$2^7 = 128$
7 bits

1 byte

2. Character Array

```
char temp;
temp = 'A';
```

65 (Stored as 65)

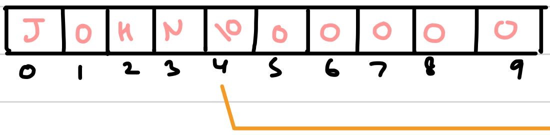
$\boxed{A}$

```
printf("%c", temp);
```

Displays 'A' and not 65

```
char x[5];
char x[5] = {'A','B','C','D','E'};

char x[5] = {65,66,67,68,69};
```

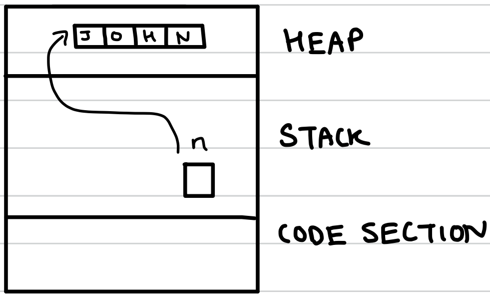
```
char name[10] = {'J','O','H','N','\0'};
```



String delimeter
End of string char
NULL char

CREATING A STRING

```
(1) char name[10] = {'J','O','H','N','\0'};

(2) char name[] = {'J','O','H','N','\0'};      size of array = 5

(3) char name[] = "John";      compiler takes null character on its own

(4) char *n = "John";
```



DISPLAYING A STRING

```
char name[10] = "David";
printf("%s", name);
scanf("%s", name);  → cannot read after space
gets (name)  → can read until you hit enter
```

FINDING LENGTH OF STRING


```
int main()
{
        char *s = "welcome";   // No size is required
        int i;
        for (i=0; s[i]'=0; i++)
        {
                   // This will remain empty
        }
}       printf("Length is %d", i);
```

CHANGING CASE OF A STRING

```
int main()
{
    char A[] = "WELCOME";
    int i;
    for (i=0; A[i] != '\0'; i++)
        A[i] = A[i] + 32;
    printf ("%s", A);
}
```

TOGGLE CASES

S	w	E	l	C	o	m	e	\0
	0	1	2	3	4	5	6	7

```
int main()
{
    char A[] = "wElCome";
    int i;

    for (i=0; A[i] != '\0'; i++)
    {
        if ( A[i] >= 65 && A[i] <= 90)
                A[i] += 32;
        else if ( A[i] >= 'a' && A[i] <= 122)
                A[i] -= 32;
    }
}
```

COUNTING NO OF VOWELS AND CONSONANTS

```
for (i=0; A[i] != '\0'; i++)
{
    if ( A[i] == 'a' || A[i] == 'e'..... )
            vcount ++;

    else if ((A[i] >= 65 && A[i] <= 90) || (A[i] >= 97 && A[i] <= 122))
            ccount ++;
}
```

COUNTING NO OF WORDS

```
int main()
{
    char A[] = "How are    you";
    int i, word = 1;
                        white space
                          ↳ when there are more than one space
    for (i=0; A[i] != '\0'; i++)
        if (A[i] == ' ' && A[i-1] != ' ')
            word ++;
}
```

VALIDATING A STRING

```
int valid ( char * name)
{
    int i;
    for (i=0; name[i] != '\0'; i++)
    {
        if ( ! ( name[i] >= 65 && name[i] <= 90 ) &&
             ! ( name[i] >= 97 && name[i] <= 122) &&
             ! ( name[i] >= 48 && name[i] <= 57))
                return 0;
        else
                return 1;
    }
}
```

```
int main()          → This type of string is not modifiable
{
    char * name = "Anil 321";
    if ( validate (name))
        printf (" Valid String");
    else
        printf (" Invalid String");
}
```

FINDING DUPLICATES IN A STRING

```
int main()
{
    char A[] = "finding";
    int H[26], i;

    for (i=0 ; A[i] != '\0' ; i++)
            H[A[i]-97] += 1;

    for (i=0 ; i<26 ; i++)
        if ( H[i] > 1)
            printf("%c", i+97);
}
```

FINDING DUPLICATES IN A STRING USING BITWISE OPERATIONS

Manipulating bits of a byte

1. Left Shift <<
2. Bits ORing (Merging)
3. Bits ANDing (Masking)

most significant bit

(Represents how 8 is stored)

7	6	5	4	3	2	1	0
0	0	0	0	1	0	0	0
128	64	32	16	8	4	2	1

1 BYTE = 8 BITS

least significant bit

char H=8;

(1) LEFT SHIFT

H = H << 2

7	6	5	4	3	2	1	0
0	0	1	0	0	0	0	0
128	64	32	16	8	4	2	1

(3) BITS ANDing

a = 10 → 1010
b = 6 → 0110
0010 → 2

(2) BITS ORing

a = 10 → 1010
b = 6 → 0110
1110 → 14

MASKING

	7	6	5	4	3	2	1	0
H	0	0	0	1	0	0	0	0
	128	64	32	16	8	4	2	1

Q We want to know whether 4th bit in H is on or off.

	7	6	5	4	3	2	1	0
a	0	0	0	0	0	0	0	0
	128	64	32	16	8	4	2	1

Sol

a = 1

a = a<<4

a & H (perform ANDing)

	7	6	5	4	3	2	1	0
a	0	0	0	0	0	0	0	1
	128	64	32	16	8	4	2	1

MERGING

	7	6	5	4	3	2	1	0
H	0	0	1	0	0	0	0	0
	128	64	32	16	8	4	2	1

	7	6	5	4	3	2	1	0
a	0	0	1	0	0	0	0	0
	128	64	32	16	8	4	2	1

Q we want to set 3rd bit in H as on.

Sol

a = 1

a = a<<2

H = a | H

FINDING DUPLICATES

ASCII Codes

	102	105	110	100	105	110	103	
A	f	i	n	d	i	n	g	\0
	0	1	2	3	4	5	6	7

H

0	0	0	0	0	0	0	0
31	30	29	28	27	26	25	24
0	0	0	0	0	0	0	0
23	22	21	20	19	18	17	16
0	0	1	0	0	0	0	1
15	14	13	12	11	10	9	8
0	1	1	0	1	0	0	0
7	6	5	4	3	2	1	0

32 bits created for 26 alphabets

0	0	0	0	0	0	0	1	X

X also consists of 32 bits

```
int main()
{
    char A[] = "finding";
    long int H=0 , x=0;
    for(i=0; A[i]!='\0'; i++)
    {
        x=1;
        x = x<< A[i]-97;
        if(x & H >0)
            printf("%c is duplicate", A[i]);
        else
            H = x | H;
    }
}
```

CHECK FOR ANAGRAM

When two strings are made up of same alphabets

A

100	101	99	105	109	97	108	
d	e	c	i	m	a	l	\0
0	1	2	3	4	5	6	7

B

m	e	d	i	c	a	l	\0
0	1	2	3	4	5	6	7

H

1		1	1	1				1			1	1													
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

H

0		0	0	0				0			0	0													
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

First check if the strings are of same length.

```
int main
{
    char A[] = " decimal ";
    char B[] = " medical ";
    int i , H[26] = {0};

    for ( i=0 ; A[i] != ' \0' ; i++ )          // Making it 1
                H[ A[i]-97] += 1;

    for ( i=0 ; B[i] != '\0'; i++ )            // Making it 0
                if ( H[A[i] -97] < 0 )
                {
                        printf(" Not Anagram");
                        break;
                }

    if ( B[i] == '\0')
                printf(" Anagram");
```

★★

PERMUTATION OF A STRING

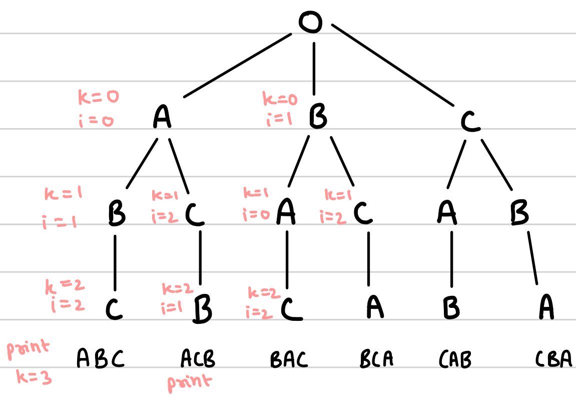
1st Method

A	B	C	\0
0	1	2	3

3!

n!

State Space Tree



Brute Force: Finding all possible permutations

Back Tracking

- Implemented using recursion
- To achieve brute force.

```
void perm(char s[], int k)
{
    static int A[10] = {0};
    static char Res[10];
    int i;

    if (s[k] == '\0')
    {
        Res[k] = '\0';
        printf("%s", Res);
    }
    else
    {
        for (i=0; s[i] != '\0'; i++)
        {
            if (A[i]==0)
            {
                Res[k] = S[i];
                A[i] = 1;
                perm(s, k+1);
                A[i] = 0;
            }
        }
    }
}
```

S

A	B	C	\0
0	1	2	3

A

0	0	0	0
0	1	2	3

Res

A	B	C	\0
0	1	2	3

```
main()
{
    char s[] = "ABC";
    perm(S, 0);
}
```

2nd Method

```
void perm(char s[], int l, int h)
{
    int i;
    if (l==h)
        printf("%s", S);
    else
    {
        for (i = l; i<=h; i++)
        {
            swap(s[l], s[i]);
            perm(s, l+1, h);
            swap(s[l], s[i]);
        }
    }
}
```

MATRICES

SPECIAL MATRICES

Only square matrix
n x n

1. Diagonal Matrix
2. Lower Triangular
3. Upper Triangular
4. Symmetric Matrix
5. Tridiagonal Matrix
6. Band Matrix
7. Toeplitz Matrix
8. Sparse Matrix

1. DIAGONAL MATRIX

$$
\begin{array}{c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
3 & 0 & 0 & 0 & 0 \\
0 & 7 & 0 & 0 & 0 \\
0 & 0 & 4 & 0 & 0 \\
0 & 0 & 0 & 9 & 0 \\
0 & 0 & 0 & 0 & 6
\end{bmatrix}
\end{array}
$$

A

3	7	4	9	6
0	1	2	3	4

ROWS COLUMNS ELEMENT TO BE INSERTED

```
int A[5];
void set ( int A[], int i, int j, int x )
{
        if (i==j)
                A[i-1] = x;
}
void get ( int A[], int i, int j)
{
        if ( i==j)
              return A[i-1];
        else
              return 0;
}
```

C++ Class For Diagonal Matrix

```
class Diagonal()
{
    private :
            int n;
            int *A;

    public :
            Diagonal (int n)
            {
                This -> n = n;
                A = new int(n);
            }

            void set (int i, int j, int k);
            void get ( int i, int j);
            void Display();
            ~ Diagonal()
            {
                delete []A;
            }
}
```

```
void Diagonal :: set (int i, int j, int x );
{
        if (i == j)
            A[i-1] = x;
}
```

```
int Diagonal :: get (int i, int j)
{
        if (i == j)
            return A[i-1];
        else
            return 0;
}
```

```
void Diagonal :: Display()
{
        for (i=0 ; i<n; i++)
        {
            for (j= 0; j<n; j++)
            {
                if ( i == j)
                    cout << A[i-1];
                else
                    cout << " 0 ";
            }
            cout << endl;
        }
}
```

LOWER TRIANGULAR MATRIX

$$M = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}$$

$M[i,j] = 0 \quad \text{if} \quad i < j$

$M[i,j] = \text{Non Zero} \quad \text{if} \quad i >= j$

$$\text{Non Zero} = 1+2+3+4+5$$
$$= 1+2+3+4 \ldots\ldots n$$
$$= \frac{n(n+1)}{2}$$

$$\text{Zero} = n^2 - \frac{n(n+1)}{2} = \frac{n(n-1)}{2}$$

ROW MAJOR

a_{11}	a_{21}	a_{22}	a_{31}	a_{32}	a_{33}	a_{41}	a_{42}	a_{43}	a_{44}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
row1	row2		row3			row 4				row 5				

$\text{Index}(A[4][3]) = [1+2+3]+2 = 8$
$\text{Index}(A[5][4]) = [1+2+3+4]+3 = 13$

$$\text{Index}(A[i][j]) = \left[\frac{i(i-1)}{2}\right] + j-1$$

COLUMN MAJOR

a_{11}	a_{21}	a_{31}	a_{41}	a_{51}	a_{22}	a_{32}	a_{42}	a_{52}	a_{33}	a_{43}	a_{53}	a_{44}	a_{54}	a_{55}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		col 1				col2				col3		col 4		col5

$\text{Index}(A[4][4]) = [5+4+3]+0 = 12$
$\text{Index}(A[5][4]) = [5+4+3]+1 = 13$
$\text{Index}(A[5][3]) = [5+4]+2 = 11$

$$\text{Index}(A[i][j]) = [n + n-1 + n-2 + \ldots. n-(j-2)] + (i-j)$$
$$= [n(j-1) - [1+2+3+\ldots\ldots j-2]] + (i-j)$$
$$= \left[n(j-1) - \frac{(j-2)(j-1)}{2}\right] + (i-j)$$

UPPER TRIANGULAR MATRIX

$$M = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & a_{22} & a_{23} & a_{24} & a_{25} \\ 0 & 0 & a_{33} & a_{34} & a_{35} \\ 0 & 0 & 0 & a_{44} & a_{45} \\ 0 & 0 & 0 & 0 & a_{55} \end{bmatrix}$$

$M[i,j] = 0 \quad \text{if} \quad i > j$

$M[i,j] = \text{Non Zero} \quad \text{if} \quad i <= j$

$$\text{Non Zero} = 5 + 4 + 3 + 2 + 1 = \frac{n(n+1)}{2}$$

$$\text{Zero} = n^2 - \frac{n(n+1)}{2} = \frac{n(n-1)}{2}$$

ROW MAJOR

a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{22}	a_{23}	a_{24}	a_{25}	a_{33}	a_{34}	a_{35}	a_{44}	a_{45}	a_{55}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		row1					row2			row3		row4		row5

$\text{INDEX}(A[4][5]) = [5+4+3]+1 = 13$

$\text{INDEX}(A[i][j]) = [n + n-1 + n-2 + \dots + n-(i-2)] + (j-i)$

$$= \left[(i-1)n - \frac{(i-2)(i-1)}{2}\right] + (j-i)$$

COLUMN MAJOR

a_{11}	a_{12}	a_{22}	a_{13}	a_{23}	a_{33}	a_{14}	a_{24}	a_{34}	a_{44}	a_{15}	a_{25}	a_{35}	a_{45}	a_{55}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

$\text{INDEX}(A[4][5]) = [1+2+3+4]+3 = 13$

$\text{INDEX}(A[i][j]) = [1+2+3+\dots+j-1] + i-1 = \left[\frac{j(j-1)}{2}\right] + i-1$

SYMMETRIC MATRIX

$$M = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 3 & 3 & 3 & 3 \\ 2 & 3 & 4 & 4 & 4 \\ 2 & 3 & 4 & 5 & 5 \\ 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

if $M[i,j] = M[j,i]$

Either we can store lower triangular matrix, or we can store upper triangular matrix

TRI DIAGONAL MATRIX

$$M = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & a_{54} & a_{55} \end{bmatrix}$$

a_{21}	a_{32}	a_{43}	a_{54}	a_{11}	a_{22}	a_{33}	a_{44}	a_{55}	a_{12}	a_{23}	a_{34}	a_{45}
0	1	2	3	4	5	6	7	8	9	10	11	12

lower diagonal | main diagonal | upper diagonal

Main diagonal $i - j = 0$

Lower diagonal $i - j = 1$

Upper diagonal $i - j = -1$

$|i-j| \le 1$

$$\boxed{\begin{array}{ll} M[i,j] = \text{Non Zero} & \text{if } |i-j| \le 1 \\ M[i,j] = \text{Zero} & \text{if } |i-j| > 1 \end{array}}$$

$$\boxed{\begin{array}{c} 5+4+4 \\ n + n-1 + n-1 \\ 3n-2 \end{array}}$$

Index $(A[i][j])$

case 1 : if $i-j=1$ index $= i-1$

case 2 : if $i-j=0$ index $= n-1+i-1$

case 3 : if $i-j=-1$ index $= 2n-1+i-1$

SQUARE BAND MATRIX (Same as TRI DIAGONAL matrix)

When there are more than one diagonals below the main diagonal and the number of lower and upper diagonal is equal.

TOEPLITZ MATRIX

* n

$$M = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 7 & 2 & 3 & 4 & 5 \\ 3 & 8 & 7 & 2 & 3 & 4 \\ 4 & 9 & 8 & 7 & 2 & 3 \\ 5 & 10 & 9 & 8 & 7 & 2 \end{array}$$

* n-1

2	3	4	5	6	7	8	9	10
0	1	2	3	4	5	6	7	8

row | column

$M[i,j] = M[i-1, j-1]$

No of elements we want to store : $n + n-1$ *

Index $A[i][j]$

case 1 : if $i <= j$ Index $= j-1$

case 2 : if $i > j$ Index $= n + i - j - 1$

CREATING A DYNAMICALLY ALLOCATED ARRAY

```
int *A, n;
printf(" Enter dimension");
scanf(" %d", &n);
A = (int*)malloc(n* sizeof(int));
A = new int[n]; C++
```

SPARSE MATRIX

Having many number of non - zero elements

Methods for storing sparse matrix

1. Coordinate list / Three column representation
2. Compressed sparse row

1. Coordinate list / Three column representation

$$
\begin{array}{c|ccccccccc|}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\
2 & 0 & 0 & 8 & 0 & 0 & 10 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 9 & 0 & 0 & 5 & 0 & 0 & 0 & 0 \\
\hline
\end{array}
$$

8 x 9 72 elements

72 x 2 = 144 bytes

row	column	element
8	9	8
1	8	3
2	3	8
2	6	10
4	1	4
6	3	2
7	4	6
8	2	9
8	5	5

2. Compressed sparse row

A[3, 8, 10, 4, 2, 6, 9, 5] → Non Zero elements

IA[0, 1, 3, 3, 4, 4, 5, 6, 8] → Cummulative sum of number of non zero elements in each row

0 1 2 3 4 5 6 7 8 → Row Number

JA[8, 3, 6, 1, 3, 4, 2, 5]

No of column in which each non zero element is located

8 + 9 + 8 = 25 x 2 = 50 bytes

1. <u>Coordinate list / Three column representation</u> (ADDITION)

$$A = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 6 & 0 & 0 \\ 2 & 0 & 7 & 0 & 0 & 0 & 0 \\ 3 & 0 & 2 & 0 & 5 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 4 & 0 & 0 & 0 & 0 & 0 \end{array}$$

0	1	2	3	4	5
5	1	2	3	3	5
6	4	2	2	4	1
5	6	7	2	5	4

$$B = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 3 & 0 & 0 & 5 & 0 \\ 3 & 0 & 0 & 2 & 0 & 0 & 7 \\ 4 & 0 & 0 & 0 & 9 & 0 & 0 \\ 5 & 8 & 0 & 0 & 0 & 0 & 0 \end{array}$$

0	1	2	3	4	5	6
5	2	2	3	3	4	5
6	2	5	3	6	4	1
6	3	5	2	7	9	8

$$C = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 6 & 0 & 0 \\ 2 & 0 & 10 & 0 & 0 & 5 & 0 \\ 3 & 0 & 2 & 2 & 5 & 0 & 7 \\ 4 & 0 & 0 & 0 & 9 & 0 & 0 \\ 5 & 12 & 0 & 0 & 0 & 0 & 0 \end{array}$$

0	1	2	3	4	5	6	7	8	9
5	1	2	2	3	3	3	3	4	5
6	4	2	5	2	3	4	6	4	1
	6	10	5	2	2	5	7	9	12

CREATING SPARSE MATRIX PROGRAM

Q

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 0 & 7 & 0 & 0 \\
2 & 2 & 0 & 0 & 5 & 0 \\
3 & 9 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 4
\end{array}
$$

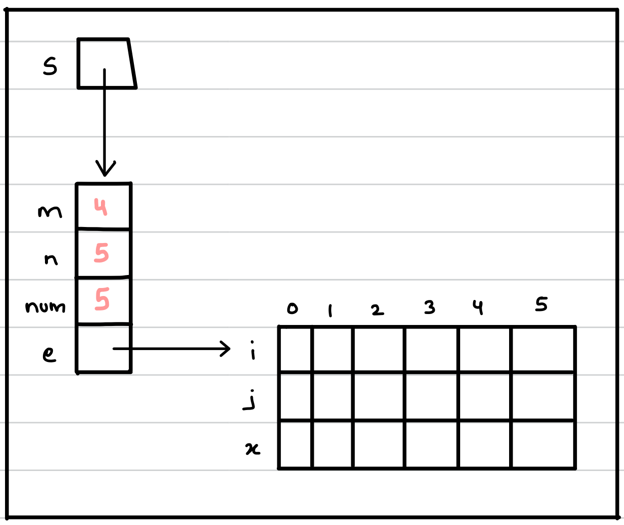
4x5 mxn

	0	1	2	3	4	5
i	4	1	2	2	3	4
j	5	3	1	4	1	5
k	5	7	2	5	9	4

```
Struct Element
{
        int i;
        int j;
        int x;
}
Struct sparse
{
        int m;
        int n;
        int num;
        Struct Element *e;
}
void main()
{
        Struct sparse s;
        create (&s);
}
```

```
void create ( struct sparse *s)
{
    int i;
    printf(" Enter dimensions");
    scanf (" %d %d", &s→m, &s→n);
    printf(" Enter number of non-zero elements");
    scanf (" %d ", &s→num);
    s→e = new Elements [s→num];
    printf(" Enter all elements ");

    for (i=0; i< s→num; i++)
          scanf(" %d %d %d", &s→e[i].i,
                             &s→e[i].j,
                             &s→e[i].x);
}
```

s→e = (Struct Element *) malloc
(s→num * sizeof(struct Element))



ADDING SPARSE MATRIX PROGRAM

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 0 & 3 & 0 & 0 \\
2 & 4 & 0 & 0 & 0 & 7 \\
3 & 0 & 0 & 5 & 0 & 8 \\
4 & 0 & 6 & 0 & 0 & 0
\end{array}
\qquad
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 0 & 0 & 0 & 2 \\
2 & 0 & 5 & 0 & 0 & 6 \\
3 & 4 & 0 & 8 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 9
\end{array}
$$

m	4
n	5
num	6
e	→

	0	1	2	3	4	5
i	1	2	2	3	3	4
j	3	1	5	3	5	2
x	3	4	7	5	8	6

i

m	4
n	5
num	6
e	→

	0	1	2	3	4	5
i	1	2	2	3	3	4
j	5	2	5	1	3	5
x	2	5	6	4	8	9

j

sum

m	4
n	5
num	
e	→

	0	1	2	3	4	5
i						
j						
x						

k

```
add ( struct sparse *s1 , struct sparse s2)
{
        struct Sparse *Sum;

        if ( s1 -> m! = s2-> m || s1->n! = s2->n)
                    return 0 ;
        sum= new sparse;
        sum -> m = s1 ->m;
        sum -> n = s1 ->n;
        sum -> e = new Element[s1->num + s2-> num];

        while (i< s1 ->num && j< s2->num)
        {
                if ( s1. e[i].i < s2. e [j].i )
                        sum ->e[k++] = s1->e [i++];

                else if (s1 ->e[i] .i > s2 ->e[j] . i)
                        sum -> e[k++] = s2-> e[j++];

                else
                {
                        if ( s1-> e[i] . j < s2 -> e[j] . j)
                                sum -> e [k++] = s1 -> e[i++];
                        else if ( s1-> e[i] . j > s2 -> e[j] . j )
                                sum -> e[k++] = s2 -> e[j++];
                        else
                        {
                                sum -> e[k] = s1 -> e[i++] ;
                                sum -> e [k++] .x += s2 -> e [j++] .x ;
                        }
                }
```

Rows are Compared

Column compared

POLYNOMIAL REPRESENTATION

1. Polynomial Representation
2. Evaluation of Polynomial
3. Addition of two Polynomials

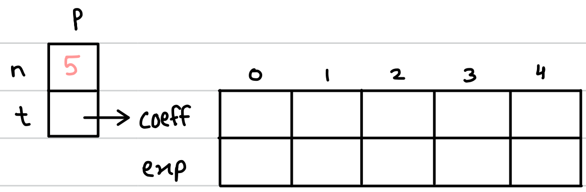
$p(x) = 3x^5 + 2x^4 + 5x^2 + 2x + 7$

coeff	3	2	5	2	7
exp	5	4	2	1	0

$n = 5$

```
Struct Term
{
        int coeff;
        int Exp;
}

Struct Poly
{
        int n;
        Struct Term *t;
}
```

```
Struct Poly P;
printf(" No of non-zero terms");
scanf(" %d", &p.n);
p.t = new Term[p.n];
printf(" Enter polynomial terms ");
for(i=0; i<p.n; i++)
{
        printf(" Term No : %d ", i+1);
        scanf("%d %d", & p.t[i].coeff, & p.t[i].Exp);
}
```



EVALUATION

```
Struct Poly P;
x = 5; sum = 0;

for( i=0; i<p.n; i++)
        sum += p.t[i].coeff * pow (x, p.t[i].Exp);
```

POLYNOMIAL ADDITION

$p1(x) = 5x^4 + 2x^2 + 5$

$p2(x) = 6x^4 + 5x^3 + 9x^2 + 2x + 3$

p: n = 3, t →

	0	1	2	3	4
coeff	5	2	5		
exp	4	2	0		

i

p: n = 5, t →

	0	1	2	3	4
coeff	6	5	9	2	3
exp	4	3	2	1	0

j

```
while ( i< p1.n && j< p2.n)
{
        if ( p1.t[i].Exp > p2.t[j].Exp)
                p3.t[k++] = p1.t[i++];

        else if ( p2.t[j].Exp > p1.t[i].Exp)
                p3.t[k++] = p2.t[j++];

        else
        {
                p3.t[k].Exp = p1.t[i].Exp;
                p3 t[k++].coeff = p1.t[i++].coeff + p2.t[j++].coeff;
        }
}
```

p: n = 5, t →

	0	1	2	3	4
coeff	11	5	11	2	8
exp	4	3	2	1	0

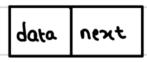
k

LINKED LIST

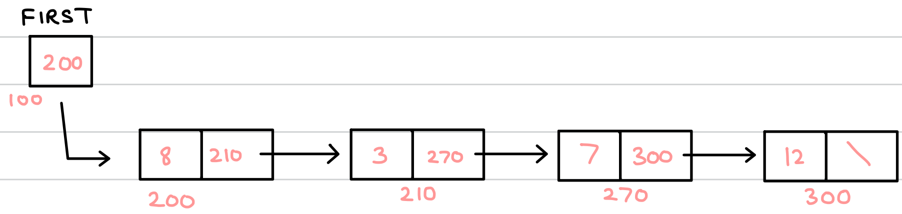
1. **Problem with arrays** : Fixed Size

Q What is a linked list?

Ans Linked list is a collection of nodes where each node contain data and pointer to next node.



NODE

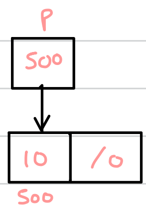



```
struct Node
{
        int data;                 ———— 2
        struct Node * next;       ———— 2
};                                     ————
                                       4 bytes
```

SELF REFERENTIAL STRUCTURE

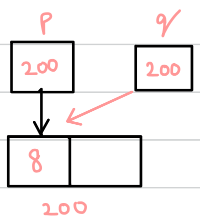
(same as datatype)

```
struct Node *p;
p= (struct Node *)malloc (sizeof(struct Node));
p = new Node;        C++

p → data = 10;
p → next = 0;
```

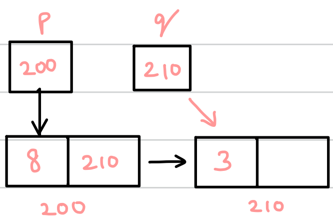


```
struct Node *p, *
```

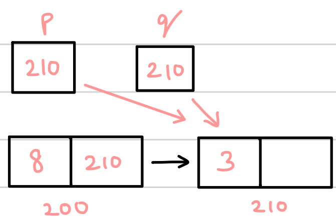
1. q = p



2. q = p → next

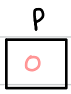


3. p = p → next



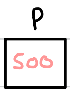
```
struct Node *p = NULL;
```

```
if (p == NULL)
if (p == 0)
if (!p)
if (p->next == NULL)
```



To check if pointer is not pointing anywhere

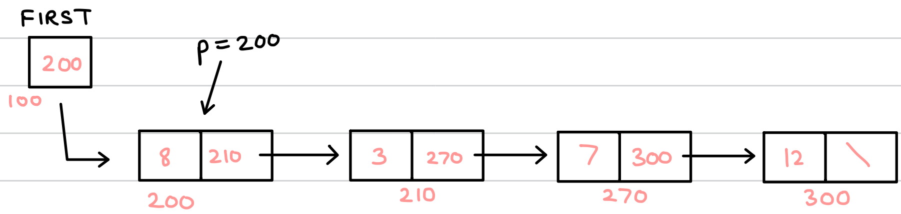
```
if (p != NULL)
if (p != 0)
if (p)
if (p -> next != NULL)
```



To check if pointer is not NULL.

TRAVERSING THROUGH LINKED LIST

```
struct Node * p = first;
while (p != 0)
{
        p = p->next;
}
```



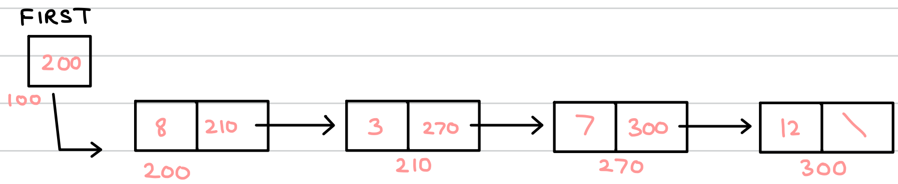

```
#include<stdio.h>
#include <stlib.h>

struct Node
{
      int data;
      struct Node *next;
} * first = NULL;
```

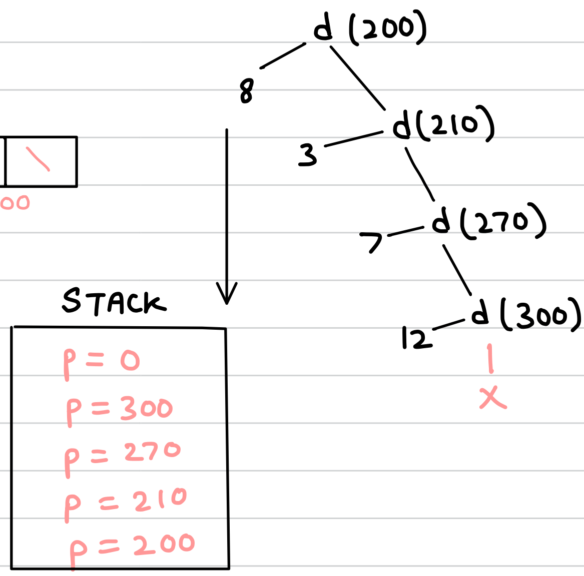
Check struct Node * first= NULL in main

```
void create (int A[], int n)
{
      int i;
      struct Node *t, *last;
      first = (struct Node *)malloc(sizeof(struct Node));
      first -> data= A[0]
      first -> next = NULL;
      last = first;
```

```
    for( i=1; i<n; i++)
    {
        t = (struct node *)malloc( sizeof( struct Node));
        t -> data = A[i];
        t -> next = NULL;
        last -> next = t;
        last = t;
    }
}

void Display( struct Node *p)
{
    while (p != NULL)
    {
        printf(" %d ", p -> data);
        p = p -> next;
    }
}

void main()
{
    struct Node * temp;
    int A[] = { 3, 5, 7, 10, 25, 8, 32, 2};
    create (A, 8);
    Display (first);
}
```

RECURSIVE DISPLAY OF LINKED LIST

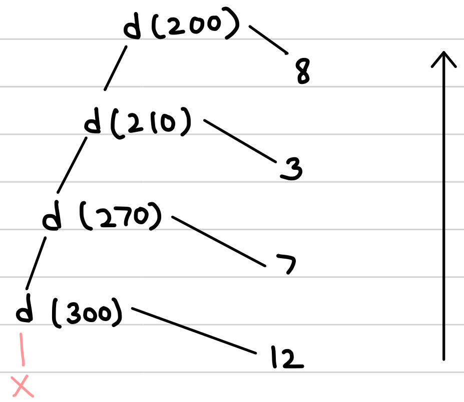





```
void Display (struct Node *p)
{
    if (p != NULL)
    {
        printf(" %d", p -> data);
        Display ( p -> next);
    }
}        Display ( first);
```

```
void Display (struct Node *p)
{
    if (p != NULL)
    {
        Display (p -> next);
        printf(" %d", p -> data);
    }
}
```

O(n)



COUNTING NODES IN LINKED LIST

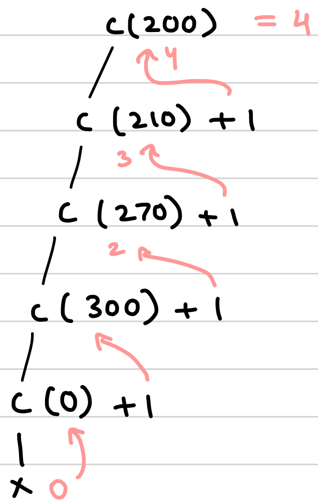
```
int count ( struct Node *p)
{
    int c=0;
    while (p!=0)
    {
        c++;
        p =p->next;
    }
    return(c);
}
```

O(n)

RECURSIVE FUNCTION FOR COUNTING NUMBER OF NODES

```
int count( struct Node *p)
{
    if (p==0)
        return 0;
    else
        return count(p->next)+1;
}
```

O(n)



SUM OF ALL ELEMENTS IN A LINKED LIST

```
int add ( Struct Node *)
{
    int sum = 0;
    while (p)
    {
        sum = sum + p->data;
        p = p->next;
    }
    return sum;
}
```

USING RECURSION

```
int Add( struct Node *p)
{
    if (p==0)
        return 0;
    else
        return Add(p->next)+ p->data;
}
```

O(n)

MAXIMUM ELEMENT IN A LINKED LIST

```
int max(struct Node *p)
{
    int m = -32768;
    while (p)
    {
        if (p->data > m)
            m = p->data;
        p = p->next;
    }
    return (m);
}
```

RECURSION

```
int max(Node *p)
{
    int x = 0;
    if (p == 0)
        return MIN_INT;
    else
    {
        x = max(p->next);
        if (x > p->data)
            return x;
        else
            return p->data;
    }
}
```

SEARCHING IN A LINKED LIST

Binary search is not suitable for linked list as we cannot go directly in the middle of the list

LINEAR SEARCH

```
Node* Search(struct Node *p, int key)
{
    while (p != NULL)
    {
        if (key = p->data)
            return (p);

        p = p->next;
    }
    return NULL;
}
```

RECURSIVE

```
Node* Search(Node *p, int key)
{
    if (p == NULL)
        return NULL;
    if (key == p->data)
        return (p);
    return Search(p->next, key);
}
```

IMPROVING SEARCHING

```
Node *search (Node *p, int key)
{
    Node *q = NULL;

    while ( p! = NULL)
    {
        if (key == p→data)
        {
            q→next = p→next;
            p→next = first;
            first = p;
        }
        q = p;
        p = p→next;
    }
}
```

INSERTING IN A LINKED LIST

Before first Node

```
Node *t = new Node;
t → data = x;
t → next = first;
first = t
```

Pos == 4

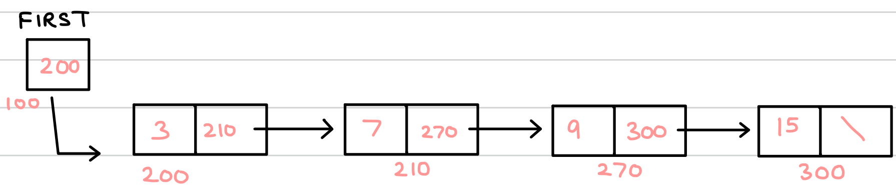
```
Node *t = new Node;
t→data = x;
p = first;
for (i=0 ; i < pos-1 ; i++)
        p = p→next;
t→next = p→next;
p→ next = t;
```

```
void Insert (int pos, int x)
{
    Node *t, *p;
    if (pos == 0)
    {
        t = new Node;
        t→data = x;
        t→next = first;
        first = t;
    }

    else if (pos >0)
    {
        p = first;
        for (i=0; i< pos-1 && p ;i++)
            p = p→next;
        if (p)
        {
            t = new Node;
            t→data = x;
            t→next = p→next;
            p→next = t;
        }
    }
}
```

INSERTING AT LAST

```
void Insertlast (int x)
{
    Node *t = new Node;
    t -> data = x;
    t -> next = NULL;

    if ( first == NULL)
    {
        first = last = t;
    }
    else
    {
        last -> next = t;
        last = t;
    }
}
```

INSERTING IN A SORTED LIST




```
p = first;
q = NULL        ( Tailing Pointer)

while ( p && p -> data < x)
{
    q = p;
    p = p -> next;
}
t = new Node;
t -> data = x;
t -> next = q -> next;
q -> next = t ;
```

DELETION FROM LINKED LIST

(1) Deletion of first Node

```
Node *p = first;          O(1)
first = first → next;
x = p → data;
free (p);
```

(2) Deletion from a given position

```
Node *p = first;          min O(1)
Node *q = NULL;           max O(n)

for (i = 0 ; i < pos -1 ; i++)
{
        q = p;
        p = p → next;
}

q → next = p → next;
x = p → data;
free (p);
```

CHECK IF LIST IS SORTED

```
int x = -32768;           O(n)
Node *p = first;
while ( p != NULL)
{
        if (p → data < x)
                return -1;
        x = p → data;
        p = p → next;
}
return 0;
```

REMOVE DUPLICATES FROM LIST

O(n)

```
Node *p = first;
Node *q = first->next;

while (q != NULL)
{
    if (p->data != q->data)
    {
        p = q;
        q = q->next;
    }
    else
    {
        p->next = q->next;
        free(q);
        q = p->next;
    }
}
```

REVERSING A LINKED LIST

(1) Reversing Elements
(2) Reversing Links

Reversing links is preferred over reversing elements because maybe there are lots of values in a single node.

(1) Reversing Elements

First create an array A equal to size of length of linked list.

O(n)

```
p = first;
i = 0;

while (p != NULL)
{
    A[i] = p->data;
    p = p->next;
    i++;
}
```

```
p = first; i--;

while (p != NULL)
{
    p->data = A[i--];
    p = p->next;
}
```

(2) *Reversing Links

TRACE IT!

```
p = first;
q = NULL;
r = NULL;
while (p != NULL)
{
    r = q;
    q = p;
    p = p -> next;
    q -> next = r;
}
first = q;
```

sliding pointers

REVERSING LINKED LIST USING RECURSION

```
void Reverse (Node *q, Node *p)
{
    if (p != NULL)
    {
        Reverse (p, p -> next)
        p -> next = q;
    }
    else
        first = q;
}
```

CONCATENATING TWO LINKED LIST

```
p = first;
while (p -> next != NULL)
    p = p -> next
p -> next = second;
second = NULL;
```

O(n)

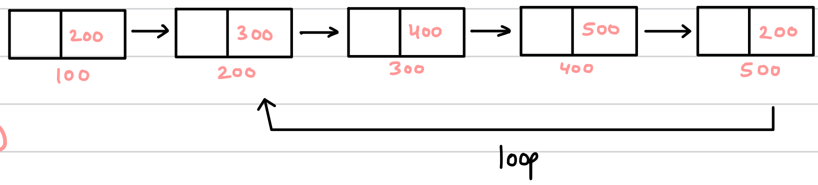
MERGING TWO LINKED LIST

We have two sorted linked list and we want to combine it into a single list.

$\theta(m+n)$

```
if ( first -> data < second data)
{
        third = last = first;
        first = first -> next;
        last -> next = NULL;
}
else
{
        third = last = second;
        second = second -> next;
        last -> next = NULL;
}

while ( first != NULL && second != NULL)
{
        if ( first -> data < second -> data)
        {
                last -> next = first;
                last = first;
                first = first -> next;
                last -> next = NULL;
        }
        else
        {
                last -> next = second;
                last = second;
                second = second -> next;
                last -> next = NULL;
        }
}
if ( first != NULL )
        last -> next = first;
else
        last -> next = second;
```

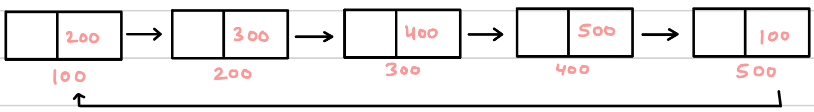
CHECK FOR LOOP IN LINKED LIST

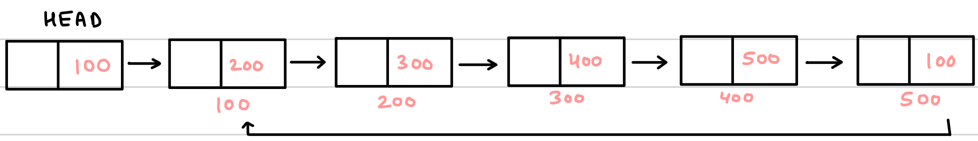


O(n)

```c
int isloop ( Struct Node *f)
{
    Struct Node *p, *q;
    p=q= f;

    do
    {
            p = p->next;
            q = q ->next;
            q = q ? q ->next : q;
    } while ( p && q && p!=q);

    if (p==q)
            return 1;
    else
            return 0;
}
```

CIRCULAR LINKED LIST

1 

2 

DISPLAY CIRCULAR LINKED LIST

```c
void Display ( Node *p )
{
     do
     {
           printf (" %d", p -> data);
           p = p -> next ;
     } while ( p != Head);
}
```

Display (Head);

DISPLAY CIRCULAR LINKED LIST USING RECURSION

```
void Display (Node *p)
{
        static int flag = 0 ;
        if ( p! = Hea  || flag = 0)
        {
                flag = 1;
                printf(" %d", p -> data);
                Display ( p -> next);
        }
}
```

CREATION OF CIRCULAR LINKED LIST

```
void create (int A[], int n)
{
        int i;
        struct Node *t, * last ;
        Head = (struct Node *) malloc (sizeof (struct Node));
        Head -> data = A[0];
        Head -> next = Head;
        last = Head;

        for (i= 1; i <n; i++)
        {
                t= (struct Node *)malloc (sizeof (struct Node));
                t -> data = A[i];
                t -> next = last -> next;
                last -> next = t;
                last = t;
        }
}

int main()
{
        int A[] = { 2,3,4,5,6};
        create (A , 5);
}
```

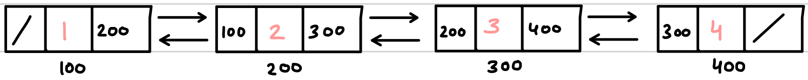
INSERTING IN A CIRCULAR LINKED LIST

```
void Insert( struct Node *p, int index , int x)
{
    struct Node *t;
    int i;

    if (index <0 || index > length())
        return;

    if (index ==0)
    {
        t= (struct Node *)malloc ( sizeof (struct Node));
        t -> data = x;

        if (Head = = NULL)
        {
            Head = t;
            Head -> next = Head;
        }
        else
        {
            while( p -> next != Head )
                p = p -> next;
            p -> next = t;
            Head = t;
            t -> next= Head;
        }
    }

    else
    {
        for (i =0; i< index-1 ; i++)
            p= p -> next;
        t= (struct Node *)malloc ( sizeof (struct Node));
        t -> data = x;
        t -> next = p -> next;
        p -> next = t;
    }
}
```

DELETION IN CIRCULAR LINKED LIST

```
int Delete(struct Node *p, int index)
{
    struct Node *q;
    int i, x;

    if (index<0 || index > length(Head))
        return -1;

    if (index==1)
    {
        while (p->next != Head)
            p=p->next;
        x= Head->data;

        if (Head == p)      // If there is only one node
        {
            free(Head);
            Head = NULL;
        }

        else
        {
            p->next = Head->next;
            free(Head);
            Head = p->next;
        }
    }

    else
    {
        for (i=0 ; i< index-2; i++)
            p = p->next;
        q = p->next;
        p->next = q->next;
        x = q->data;
        free(q);
    }
}
```

INSERT IN A DOUBLY LINKED LIST DOUBLY LINKED LIST




```
struct Node
{
        struct Node * prev;
        int data;
        struct Node * next;
} * first = NULL;


void  create (int A[], int n)
{
        struct Node *t, * last;
        int i;

        first = (struct Node *) malloc ( sizeof ( struct Node));
        first -> data = A[0];
        first -> prev = first -> next = NULL;
        last = first;

        for (i=1 ; i<n ; i++)
        {
                t = (struct Node *) malloc ( sizeof ( struct Node));
                t -> data = A[i];
                t -> next = last -> next
                t -> prev= last ;
                last -> next = t;
                last = t
        }
}
```

INSERT IN A DOUBLY LINKED LIST

(1) Insert at first node

```
Node *t = new Node;
t -> data = x;
t -> prev = NULL;
t -> next = first;
first -> prev = t;
first = t;
```

(2) Insert at any given position

```
Node *t = new Node
t -> data = x;

for (i = 0; i < pos-1; i++)
        p = p -> next;
t -> next = p -> next;
t -> prev = p;
if (p -> next)
        p -> next -> prev = t
p -> next = t;
```

// To check if a node is available after p or not

DELETE FROM A DOUBLY LINKED LIST

(1) Deleting first node

```
p = first;
first = first -> next;
x = p -> data;
delete p;
if (first)          // If first is not NULL
    first -> prev = NULL;
```

(2) Deleting Node from given index

```
p = first;
for (i=0; i<pos-1; i++)
        p = p -> next;
p -> prev -> next = p -> next;

if (p -> next)
        p -> next -> prev = p -> prev;

x = p -> data;
delete p;
```

if p -> next is not NULL

REVERSING A DOUBLY LINKED LIST

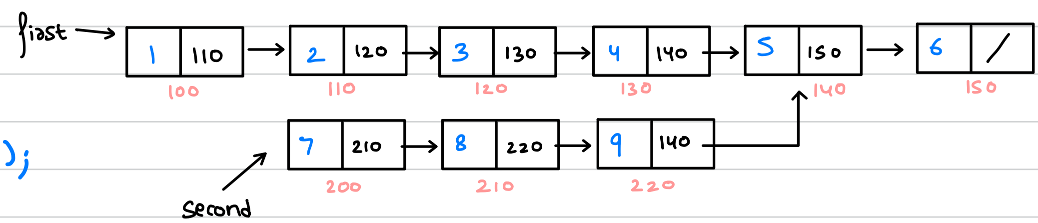
```
p = first;

while (p)
{
        temp = p→next;
        p→next = p→prev;
        p→prev = temp;
        p = p→prev

        if ( p != NULL && p→next = NULL)
                first = p;
}
```

FINDING MIDDLE NODE IN A LINKED LIST

```
p = q = first;

while (q)
{
        q = q→next;
        if (q)
            q = q→next;
        if (p)
            p = p→next;
}
```

Two pointers p and q will move at the same time.
q will move 2 nodes and p will move 1 node.

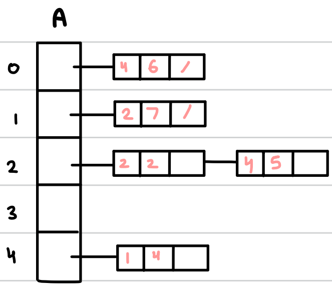
FINDING INTERSECTION POINT OF TWO LINKED LIST




```
p = first
while ( p != NULL)
        push (&stk1 , p);
p = second;
while ( p ! NULL)
        push (&stk2 , p);
while ( stacktop (stk1) == stacktop (stk2))
{
        p = pop(&stk1);
        pop (&stk2);
}
```

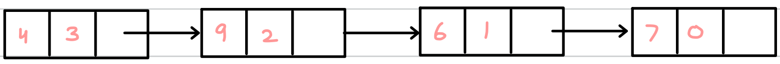
SPARSE MATRIX USING LINKED LIST

$$\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 0 & 6 & 0 & 0 \\ 2 & 0 & 7 & 0 & 0 & 0 & 0 \\ 3 & 0 & 2 & 0 & 5 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 4 & 0 & 0 & 0 & 0 & 0 \end{array}$$



POLYNOMIAL REPRESENTATION USING LINKED LIST

$p(x) = 4x^3 + 9x^2 + 6x + 7$



Recursion uses stack **STACK**

→ **LIFO** : Last In First Out

It is a collection of elements that follow LIFO for insertion and deletion.

ADT Stack (Abstract data type)

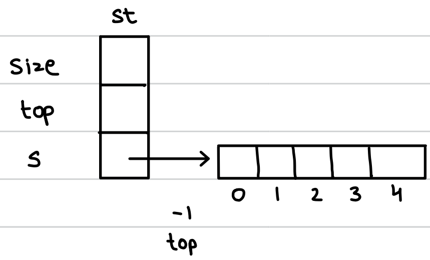
Data : 1. Space for storing elements
2. Top pointer

Operations:
1. push(x)
2. pop()
3. peek(index)
4. StackTop()
5. isEmpty()
6. isFull()

Implementation of stack using
1. Array
2. Linked List

IMPLEMENTATION OF STACK USING ARRAY

```
Struct stack
{
      int size;
      int top;
      int *s;
};
```




```
int main()
{
      Struct stack st;
      printf(" Enter size of stack ");
      scanf (" %d ", &st.size);
      st.s = new int [st.size];      for heap memory
      st.top = -1;
}
```

Stack empty — if (top == -1)
Stack full - if (top == size-1)

push() O(1)

```
void push (stack *st, int x)
{
        if ( st -> top == st -> size -1)
                printf (" Stack Overflow");
        else
        {
                st -> top ++;
                st -> s [st -> top] = x;
        }
}
```

pop() O(1)

```
int pop (stack *st)
{
        int x = -1;

        if ( st -> top == -1)
                printf (" Stack Underflow");
        else
        {
                x = st -> s [st -> top];
                st -> top --;
        }
        return x;
}
```

peek()

```
int peek( stack st, int pos)
{
        int x = -1;

        if ( st top - pos +1 < 0)
                printf (" Invalid Position");
        else
                x = st. s [st. top - pos+1];

        return x;
}
```

pos	Index = Top- pos +1
1	3
2	2
3	1
4	0

isEmpty()

```
int isEmpty (stack st)
{
        if (st. top == -1)
                return 1;
        else
                return 0;
}
```

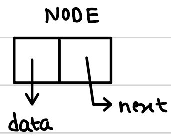
Stacktop()

```
int Stacktop (Stack st)
{
        if (st. top == -1)
                return -1;
        else
                return st. s [st. top];
}
```

isFull()

```
int isFull ( stack st)
{
        if (st. top == -1)
                return 1;
        else
                return 0;
}
```

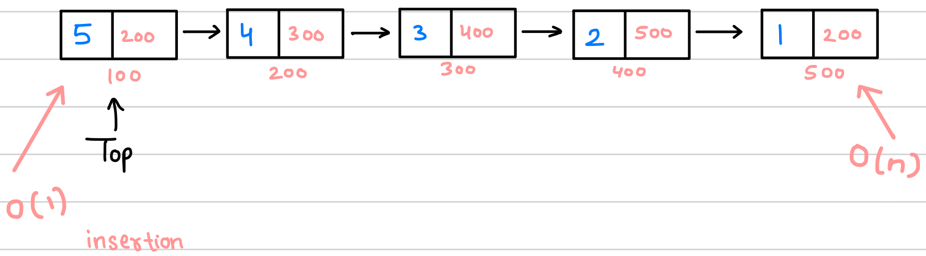
STACK USING LINKED LIST



Empty : if (top == NULL)

Full : Node *t = new Node;
if (t==NULL)

```
Struct Node
{
    int data;
    Struct Node *next;
}
```



push()

```
void push( int x)
{
    Node * t = new Node;

    if (t==NULL)
        printf("Stack Overflow");
    else
    {
        t → data = x;
        t → next = top;
        top = t;
    }
}
```

pop()

```
int pop()
{
    Node *p;
    int x=-1;

    if ( top == NULL)
        printf("Stack is empty");
    else
    {
        p = top;
        top = top→next;
        x = p →data;
        free(p);
    }
    return x;
}
```

peek()

```
int Peek ( int pos)
{
    int i;
    Node *p = top;

    for (i=0 ; p != NULL && i<pos-1 ; i++)
        p = p→next;

    if ( p!= NULL)
        return p→data;
    else
        return -1;
}
```

Stacktop()

```
int  stacktop()
{
      if(top)
          return top→data;
      return -1;
}
```

isFull

```
int  isFull()
{
      Node *t = new Node;
      int r = t ? 1 : 0;
      free(t);
      return r;
}
```

isEmpty

```
int  isEmpty()
{
      return Top ? 0 : 1;
}
```

False

True

Paranthesis Matching

exp

(	(	a	+	b	)	*	(	(	−	d	)	)	\0
0	1	2	3	4	5	6	7	8	9	10	11	12	13

```
int  isBalance(char *exp)
{
      Struct stack st;
      st.size = strlen(exp);
      St.top = -1;
      St.s = new char[st size];

      for(i=0; exp[i]! = '\0'; i++)
      {
            if(exp[i] == '(')
                  push(&st, exp[i]);
            else if(exp[i] == ')')
                  if(isEmpty(st))
                        return false;
                  else
                        pop(&st);
      }
      return isEmpty(st)? true : False;
}
```

initializing of stack

ASCII

(	40
)	41
[	91
]	93
{	123
}	125

Infix to Postfix Conversion

1. What is postfix
2. Why postfix
3. Precedence
4. Manual Conversion

1. **Infix:** Operand Operator Operand
   a + b

2. **Prefix:** Operator Operand Operand
   +ab

3. **Postfix:** Operand Operand Operator
   ab +

SYMBOL	PRECEDENCE	ASSOCIATIVITY
+, −	1	L-R
*, /	2	L-R
^	3	R-L
—	4	R-L
()	5	L-R

Unary minus

Eg a + b*c

(a + (b*c)) First fully paranthecise it.

prefix

(a + [*bc])
[+a*bc]

postfix

(a + [bc*])
[abc*+]

ASSOCIATIVITY

Left to Right

$a+b+c-d$

$(((a+b)+c)-d)$

Right to Left

$a=b=c=5$

$(a=(b=(c=5)))$

Power operator Example

a ^ b ^ c

(a ^ (b ^ c))

Postfix: (a ^ [bc ^])
abc ^ ^

Unary Operators Example

(1) − a negation of a
pre : − a
post : a −

(−(− a))

(2) * p
pre : * p
post : p *

(*(* p))

(3) n !
pre : ! n
post : n !

(4) log x
pre : log x
post : x log

Example: − a + b * log n !
− a + b * log [n !]
− a + b * [n ! log]
[a −] + b * [n ! log]
[a −] + [b n ! log *]
a − b n ! log * +

INFIX TO POSTFIX CONVERSION

a + b * c − d / e

SYMBOL	PRECEDENCE	ASSOCIATIVITY
+, −	1	L-R
*, /	2	L-R

Symbol	Stack	Postfix
a		a
+	+	a
b	+	ab
*	*, +	ab
c	*, +	abc
−	−	abc * +
d	−	abc * + d
/	/, −	abc * + d
e	/, −	abc * + de

abc * + de / − Ans

PROGRAM

infix a + b * c - d / e \0
0 1 2 3 4 5 6 7 8 9

```
char *convert( char *infix)
{
    struct stack st;        // Initialized
    char *postfix = new char[strlen(infix +1)];
                            └→ for null string
    int i=0 ; j=0 ;

    while( infix[i] != '\0')
    {
        if ( isOperand (infix[i]))
            postfix[j++] = infix[i++];

        else
        {
            if ( pre (infix [i]) > pre (stacktop(st))
                push (&st , infix [i++]);
            else
                postfix[j++] = pop (&t);
        }
    }

    while ( ! isEmpty (st))
        postfix [j++] = pop (&st);

    postfix [j] = '\0';
    return postfix ;
}
```

```
int pre (char x)
{
    if ( x == '+' || x == '-')
        return 1;
    else if (x == '*' || x == '/')
        return 2 ;
    return 0;
}
```

```
int isOperand (char x)
{
    if (x == '+' || x == '-' || x == '*' || x == '/')
        return 0;
    else
        return 1;
}
```

Q ((a+b)*c) − d^e^f
([ab+]*c) − d^e^f
[ab+c*] − d^e^f
[ab+c*] − d^[ef^]
[ab+c*] − [def^^]
ab+c*def^^−

SYMBOL	OUT STACK PRE	IN STACK PRE
+, −	1	2
*, /	3	4
^	6	5
(	7	0
)	0	?

because of R-L associativity

closing bracket cannot be pushed into Stack

EVALUATION OF POSTFIX

35*62/+4−

SYMBOL	STACK	OPERATION
3	3	
5	5, 3	
*		5*3
	15	
6	6, 15	
2	2, 6, 15	
/		6/2
	3, 15	
+		15+3
	18	
4	4, 18	
−		18−4
	14	

$x = 6+5+3*4$
$x = 65+34*+$

* Here + gets executed first instead of * because presedence and associativity are meant for parenthecisation, they don't decide which operator gets executed first.

PROGRAM FOR EVALUATION OF POSTFIX

postfix	3	5	*	6	2	/	+	4	−	\0
	0	1	2	3	4	5	6	7	8	9

```c
int Eval( char *postfix)
{
        struct stack st;
        int i, x1, x2, r;

        for ( i=0; postfix[i] != '\0'; i++)
        {
                if ( isOperand (postfix[i]))
                        push(&st, postfix[i]-'0');
                else
                {
                        x2 = pop(&st);
                        x1 = pop (&st);

                        switch (postfix[i])
                        {
                                case '+' : r = x1+x2 ; push(&st,r ; break;
                                case '-' : r = x1 - x2 ; push(&st,r ; break;
                                case '*' : r = x1 * x2 ; push(&st,r ; break;
                                case '/' : r = x1 / x2; push(&st,r ; break;
                        }
                }

                return pop(&st);
}
```

because operand will be pushed into the stack in its ASCII value because postfix expression is in char.

For eg : 3

'51' − '48' = 3

QUEUE

↳ FIFO
(First In First Out)

Queue ADT

Data : (1) Space for storing elements
(2) Front : for deletion
(3) Rear : for insertion

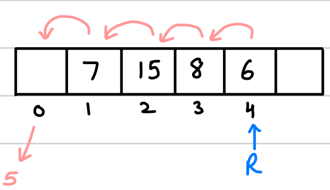
Operations : (1) enqueue (x)
(2) dequeue ()
(3) isEmpty ()
(4) isFull ()
(5) first ()
(6) last ()

(1) Array
(2) Linked List

QUEUE USING ARRAY

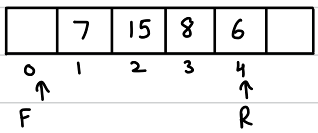
1. Queue using single pointer
2. Queue using front and rear.
3. Drawbacks of queue using array.

1. Queue using single pointer



Insert — O(1)
Delete — O(n)

2. Queue using front and rear



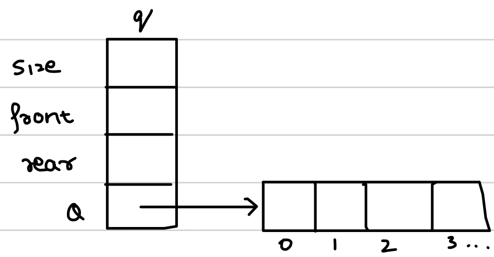
initially front = rear = -1

Insertion : increment rear and insert O(1)
Deletion : increment front and delete O(1)

Empty : if (front == rear)
Full : if (rear = size -1)

PROGRAM FOR QUEUE USING ARRAY

```
struct Queue
{
    int size;
    int front;
    int rear;
    int * Q;
}
```




```
int main ()
{
    struct Queue q;
    printf(" Enter Size : ");
    scanf (" %d ", &q. size);
    q. Q = new int[q size];
    q . front = q.rear = -1;
}
```

```
void enqueue (Queue *q , int x)
{
    if ( q ->rear == q ->size-1)
        printf(" Queue is full ");
    else
    {
        q ->rear++;
        q -> Q[q ->rear] =x;
    }
}
```

```
void dequeue ( Queue *q)
{
    int x = -1;

    if (q -> front == q->rear )
        printf(" Queue is empty");
    else
    {
        q-> front ++;
        x= q ->Q[q -> front];
    }
    return x;
}
```

DRAWBACK OF QUEUE USING ARRAY

1. We cannot utilize space of deleted element.

	7	15	8	6	9
0	1	2	3	4	5
F ↑					↑ R

2. Every location can be used only once

3. A situation where queue is empty and full

0	1	2	3	4	5
					↑↑ F R

USING SPACE AGAIN SOLUTION

(1) Resetting Pointers

Whenever Front and Rear are pointing at same place, initialize them as -1

(2) Circular Queue

In circular queue, front and rear initialize with array's first position.

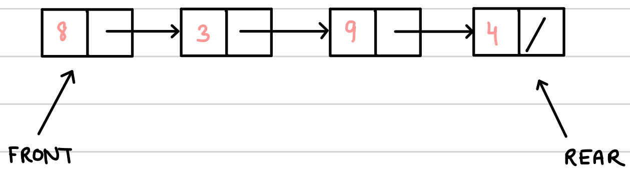
```
void enqueue( struct Queue *q , int x)
{
    if((q → Rear+1)% q → size == q→front)
            printf(" Queue is Full ");
    else
    {
            q→ Rear = (q →rear +1) % q→size;
            q → Q [q → rear] = x;
    }
}
```

To check if rear's next is front

rear =	(rear +1) % size
0	(0 +1) % 7 = 1
1	(1+1) % 7 = 2
2	(2+1) % 7 = 3
3	(3 +1) % 7 = 4
4	(4+1) % 7 = 5
5	(5+1)% 7 = 6
6	(6 +1)% 7 = 0

```
void dequeue ( struct Queue *q)
{
    int x=-1
    if ( q → front == q → rear)
            printf(" Queue is Empty");
    else
    {
            q→ front = (q → front+1) % q → size;
            x = q→ Q [ q → front];
    }
    return x;
}
```

QUEUE USING LINKED LIST



Empty : if (front == NULL)

Full : Node *t = new Node;
if (t == NULL)
// No more nodes can be created
i.e heap is full

```
void enqueue (int x)        O (1)
{
    Node *t = new Node;
    if (t == NULL)
        printf(" Queue is Full");
    else
    {
        t -> data = x;
        t -> next = NULL;
        if ( front == NULL)
            front = rear = t;
        else
        {
            rear -> next = t
            rear = t;
        }
    }
}
```

```
int  dequeue()        O (1)
{
    int x = -1;
    Node *p;
    if ( front == NULL)
        printf(" Queue is Empty");
    else
    {
        p = front;
        front = front -> next;
        x = p -> data;
        free (p);
    }
    return x;
}
```

DEQueue

Double Ended Queue

QUEUE

	Insert	Delete
front	✗	✓
rear	✓	✗

DEQUEUE

	Insert	Delete
front	✓	✓
rear	✓	✓

INPUT RESTRICTED DEQUEUE

	Insert	Delete
front	✗	✓
rear	✓	✓

OUTPUT RESTRICTED DEQUEUE

	Insert	Delete
front	✓	✓
rear	✓	✗

PRIORITY QUEUES

1. Limited Set of priorities
2. Element Priority

1. Limited Set of priorities

Priorities = 3

Element →	A	B	C	D	E	F	G	H	I	J
Priority →	1	1	2	3	2	1	2	3	2	2

Priority Queues

Q_1	A	B	F			
Q_2	C	E	G	I	J	
Q_3	D	H				

INSERTION

DELETION

For deletion, elements of Q_1 will be deleted first, then Q_2 and then Q_3

Elements will be deleted in FIFO

2. Element Priority

Elements → 6, 8, 3, 10, 15, 2, 9, 17, 5, 8

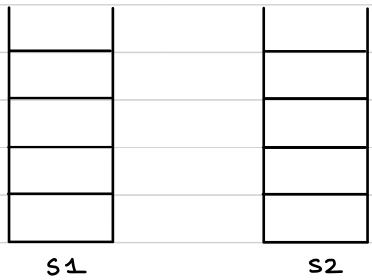
- Element is itself a priority
- Smaller number higher priority

1. Insert in same order O(1)
   Delete max priority by searching it O(n)

2. Insert in increasing order of priority O(n)
   Delete the last element of array O(1)

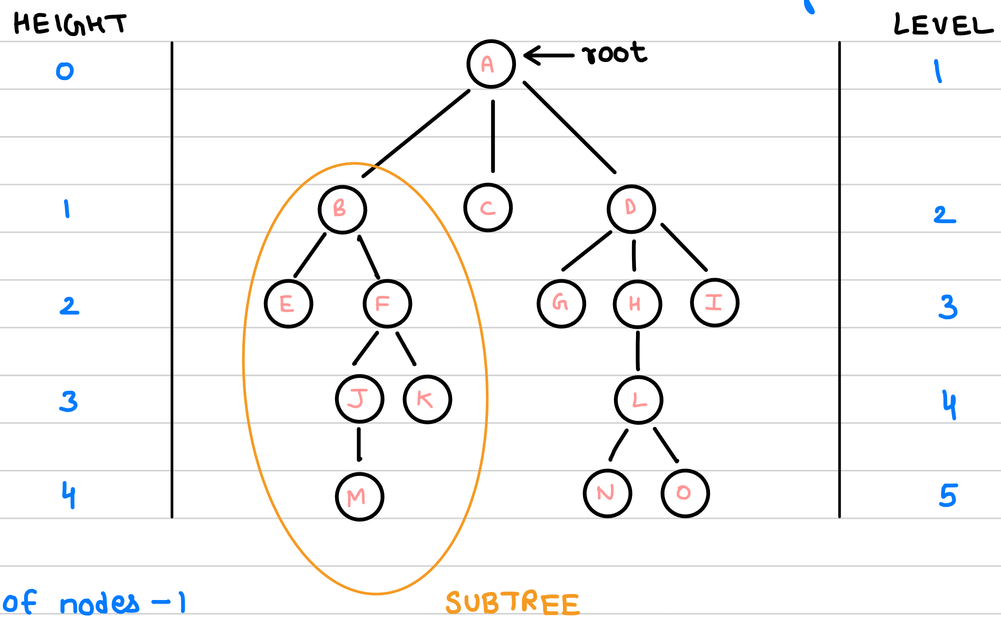
QUEUE using 2 STACKS

assume stacks to be implemented using linked list




```
enqueue (int x)
{
        push (&s1, x);
}

int dequeue()
{
        int x = -1;
        if( isEmpty (s2))
        {
                if ( isEmpty s1)
                {
                        printf(" Queue Empty");
                        return x;
                }
                else
                {
                        while (! isEmpty (s1))
                                push (&s2, pop (&s1));
                }
        }
        return pop (&s2);
}
```

TREES → Tree is a collection of nodes and edges.



No of edges = No of nodes − 1

TERMINOLOGY

(1) **Root** : The first node on the top

(2) **Parent** : A node is a parent to its very next descendents.

(3) **Child**

(4) **Sibling** : Children of same parent G, H, I

(5) **Descendents** : Set of all those nodes which can be reached from a particular node.
E, F, J, K, M are descendents of B.

(6) **Ancestors** : For any node, all the nodes along the path from that node to root node
For M, J, F, B, A are ancestors.

(7) **Degree of a node** : No of children it is having L−2

(8) **Internal Nodes / Non-Leaf Nodes / Non-Terminal Nodes** : Nodes with degree > 0

External Nodes / Leaf-Nodes / Terminal Nodes : Nodes with degree 0

(9) **Levels** (10) **Height**

(10) **Forest** : Collection of trees.

BINARY TREE

degree {2}

children – 0, 1, 2

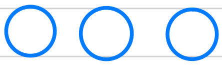
X BINARY TREE ✓

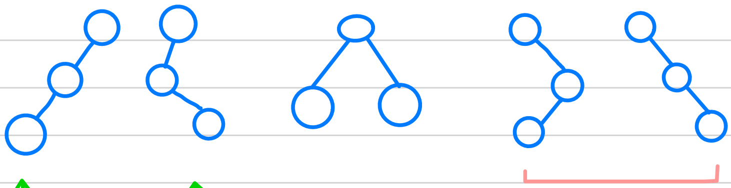
NUMBER OF BINARY TREES USING N NODES

(1) Unlabelled Nodes
(2) Labelled Nodes

No of trees with maximum height $= 4 = 2^2$
(2)

$n = 3$





height = 2

mirror images

$T(3) = 5$

$n = 4$

No of trees with maximum height $= 8 = 2^3$

$T(4) = 14$

+ other 7 mirror images

$$T(n) = \frac{^{2n}C_n}{n+1}$$

Catalan Number

No of trees with maximum height $= 2^{n-1}$

2^{nd} Method for finding Catalan Number

n	0	1	2	3	4	5	6
$T(n) = \frac{{}^{2n}C_n}{n+1}$	1	1	2	5	14	42	

$$T(6) = 1 \times 42 + 1 \times 14 + 2 \times 5 + 5 \times 2 + 14 \times 1 + 42 \times 1$$

$$T(6) = T(0) \times T(5) + T(1) \times T(4) + T(2) \times T(3) + T(3) \times T(2) + T(4) \times T(1) + T(5) \times T(0)$$

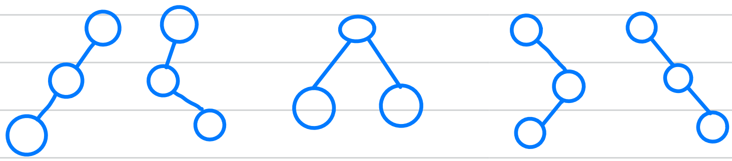
$$= 132$$

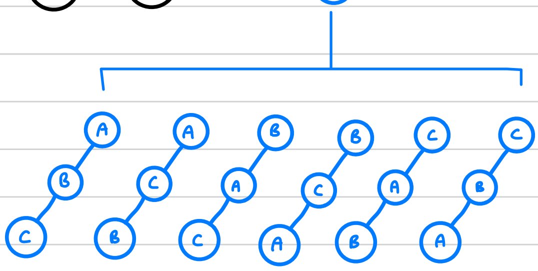
$$T(n) = \sum_{i=1}^{n} T(i-1) * T(n-i)$$

(2) Labelled Nodes

$n = 3$

A B C





$6 = 3!$

$$T(n) = \frac{{}^{2n}C_n}{n+1} * n!$$

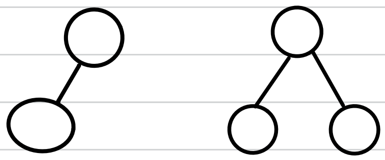
Shapes

Filling

Height vs Node

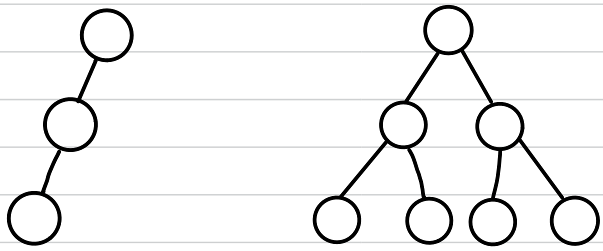
Height
h = 1

min max



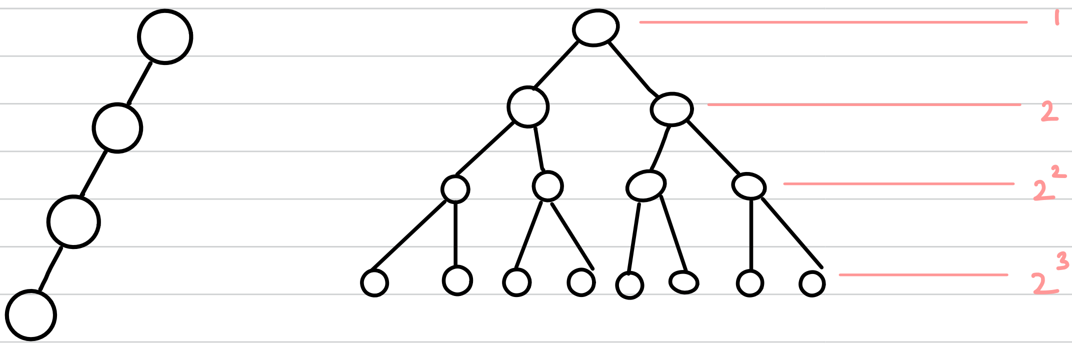
Height
h = 2

min max



Height h = 3

min max



$1 + 2 + 2^2 + 2^3 = 15$

Minimum Number of Nodes

min nodes = h+1

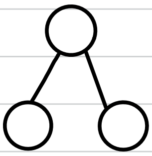
Maximum Number of Nodes

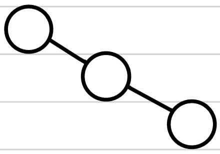
$$a + ar + ar^2 + ar^3 + ar^k = \frac{a(r^{k+1} - 1)}{r-1}$$

$$1 + 2 + 2^2 + 2^3 + \dots\dots 2^h = \frac{1 \cdot (2^{h+1} - 1)}{2-1} = 2^{h+1} - 1$$

Nodes $n = 3$

<u>min</u> <u>max</u>





$h = 1$ $h = 2$

<u>If nodes are given</u>

(1) Max height $h = n-1$

Can be obtained by previous formula

(2) For minimum height

$\Rightarrow \quad n = 2^{h+1} - 1$

$\Rightarrow \quad 2^{h+1} = n+1$

$\Rightarrow \quad h+1 = \log_2(n+1)$

$\Rightarrow \quad h = \log_2(n+1) - 1$

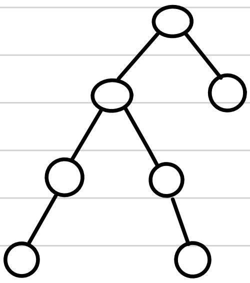
<u>Height of a binary tree</u>

$$\underbrace{\log_2(n+1) - 1}_{O(\log n)} \le h \le \underbrace{n-1}_{O(n)}$$

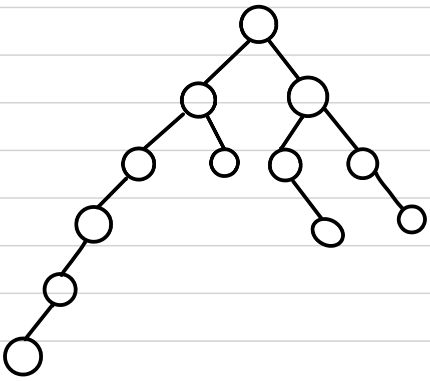
<u>Number of Nodes in a binary tree</u>

$$h+1 \le n \le 2^{h+1} - 1$$

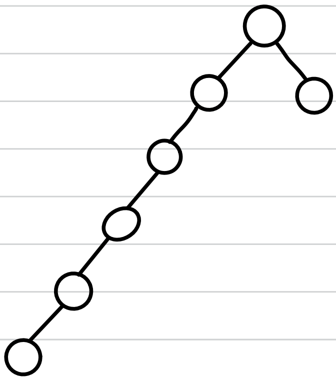
INTERNAL NODES VS EXTERNAL NODES IN A BINARY TREE



deg (2) = 2
deg (1) = 2
deg (0) = 3



deg (2) = 3
deg (1) = 5
deg (0) = 4

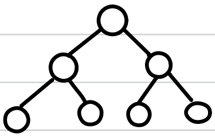


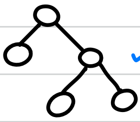
deg (2) = 1
deg (1) = 4
deg (0) = 2

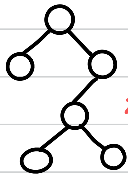
$\boxed{\deg(0) = \deg(2) + 1}$ True in binary tree

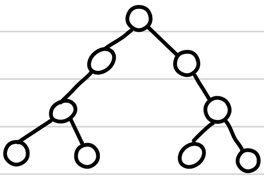
STRICT / PROPER / COMPLETE BINARY TREES

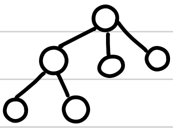
$\deg = \{\overset{\checkmark}{0}, \overset{\times}{1}, \overset{\checkmark}{2}\}$

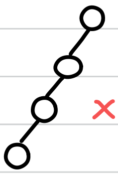
 ✓

 ✓

 ×

 ×

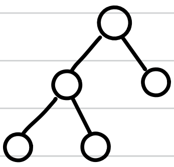
 ×

 ×

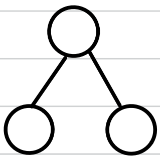
Height vs Node (Strict Binary Tree)

Height
$h = 1$

min

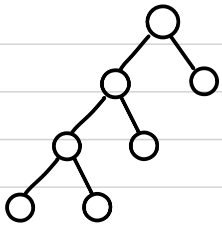


max

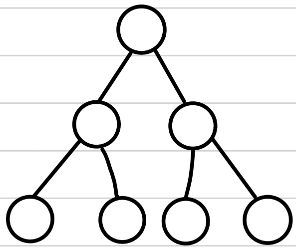


Height
$h = 2$

min

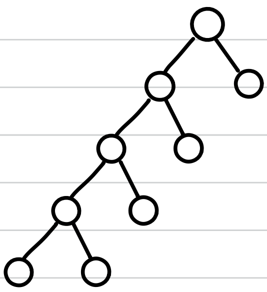


max

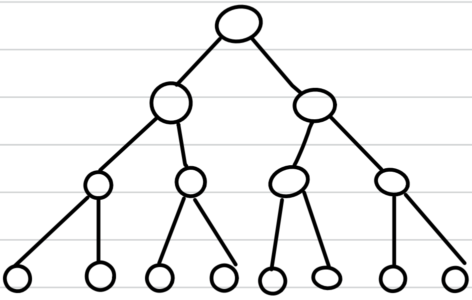


Height $h = 3$

min



max



Only minimum number changed

If height 'h' is given

Min Nodes $n = 2h + 1$

Max Nodes $n = 2^{h+1} - 1$

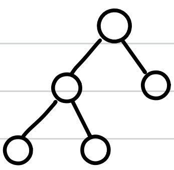
If 'n' nodes are given

Minimum height $h = \log_2(n+1) - 1$

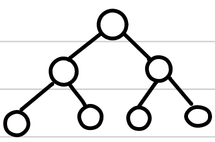
Max height $h = \frac{n-1}{2}$

$$\log_2(n+1) - 1 \leq h \leq \frac{n-1}{2}$$

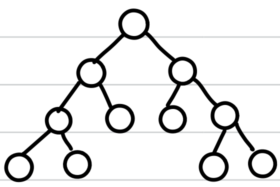
INTERNAL NODES VS EXTERNAL NODES IN A BINARY TREE (STRICT)



$i = 2$
$e = 3$



$i = 3$
$e = 4$

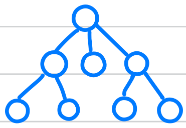


$i = 5$
$e = 6$

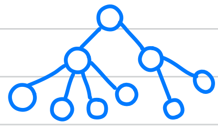
$\boxed{e = i + 1}$

n-ary Trees

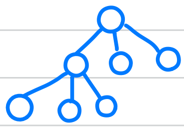
3-ary Tree $\{0,1,2,3\}$



4-ary Tree $\{0,1,2,3,4\}$



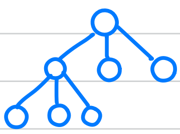
Strict 3-ary Tree $\{0,3\}$



Strict n-ary trees

h = 2

min



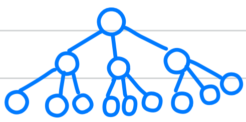
$n = 3+3+1$

$n = 2\times 3 + 1$

$i = 2$

$e = 5$

max



$i = 5$

$e = 9$

If height 'h' is given

Min Nodes $n = nh+1$

Max Nodes $n = \frac{m^{h+1}+1}{m-1}$

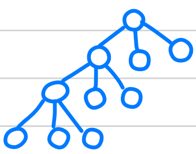
If 'n' nodes are given

Minimum height $h = \log_m[n(m-1)+1]-1$

Max height $h = \frac{n-1}{m}$

h = 3

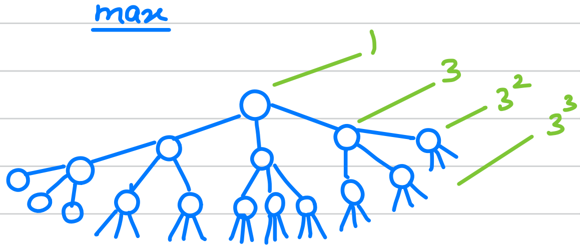
min



$i = 3$

$e = 7$

max



$\Rightarrow 1 + 3 + 3^2 + 3^3$

$\Rightarrow 1 + n + n^2 + n^3$

$\Rightarrow \frac{(n^{h+1}-1)}{n-1}$

$i = 13$

$e = 27$

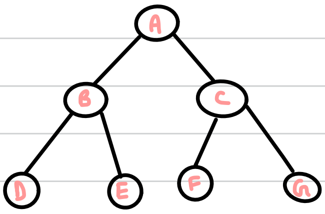
$e = 2i+1$

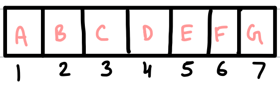
$\boxed{e = (n-1)i+1}$

REPRESENTATION OF BINARY TREE

1. Array Representation
2. Linked Representation

1. Array Representation





Element	Index	L. Child	R. Child
A	1	2	3
B	2	4	5
C	3	6	7
	i	$2*i$	$2*i+1$

Element $\longrightarrow$ i

L Child $\longrightarrow$ $2*i$

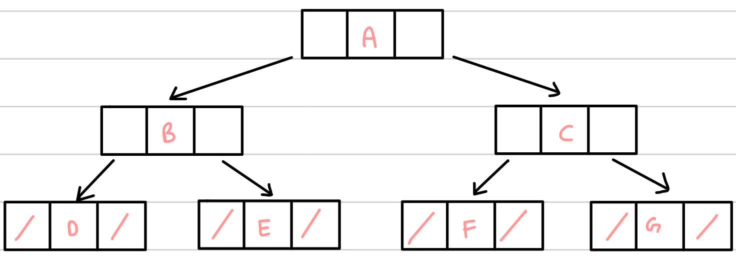
R Child $\longrightarrow$ $2*i+1$

Parent $\longrightarrow$ $\left\lfloor \frac{i}{2} \right\rfloor$

2 Linked Representation

NODE


```
Struct Node
{
    Struct Node *lchild;
    int data;
    Struct Node *rchild;
};
```

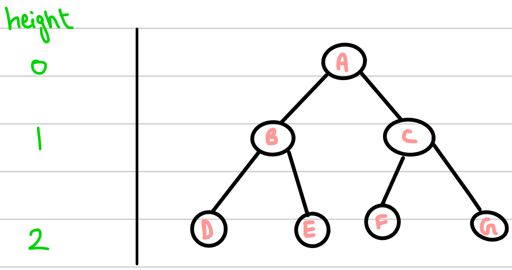


$n = 7$

nulls $= n+1$

Full Binary Tree: A binary tree of height h having maximum number of nodes.

Eg

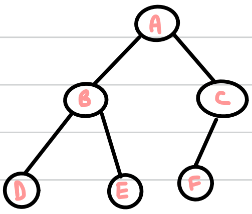


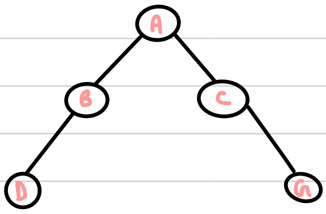
A	B	C	D	E	F	G
1	2	3	4	5	6	7

Complete Binary Tree: A complete binary tree of height h will be a full binary tree upto height h-1

A full binary tree is always a complete binary tree.

Eg





NOT A COMPLETE BINARY TREE

A	B	C	D	E	F	
1	2	3	4	5	6	7

NO BLANK SPACES BETWEEN ELEMENTS

A	B	C	D			G
1	2	3	4	5	6	7

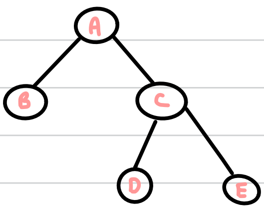
↑↑ BLANK SPACES

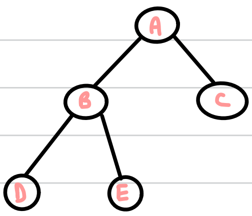
degree
{0, 2}

STRICT VS COMPLETE

STRICT → complete

COMPLETE → almost complete





A	B	C			D	E
1	2	3	4	5	6	7

strict ✓
complete ✗

A	B	C	D	E		
1	2	3	4	5	6	7

strict ✓
complete ✓

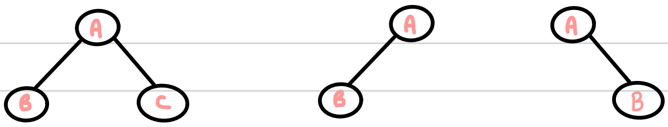
TREE TRAVERSALS

Preorder : NLR
Node , Left , Right

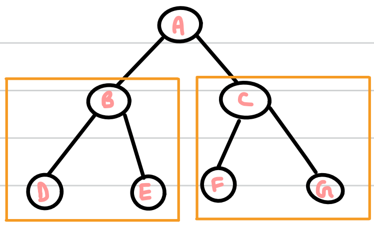
Inorder : LNR
Left , Node , Right

Postorder : LRN
Left , Right , Node

Level Order : level by level



Preorder	ABC	AB	AB
Inorder	BAC	BA	AB
Postorder	BCA	BA	BA
Level	ABC	AB	AB



Preorder A , (B, D, E) , (C, F, G)
A, B, D, E , C , F, G

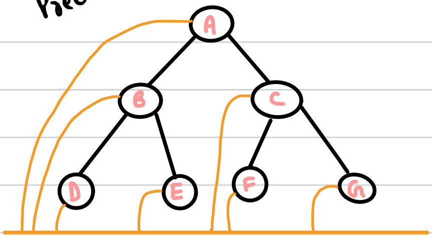
Inorder (D, B, E), A, (F , C , G)
D, B, E , A , F , C, G

Postorder (D, E , B) , (F, G , C) , A
D, E , B, F, G, C, A

Level A, B, C, D, E, F, G

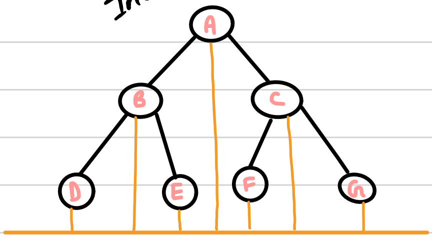
BINARY TREE TRAVERSAL EASY METHOD 1

Preorder



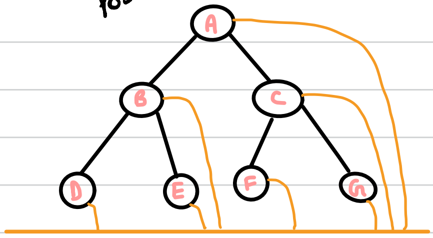
A, B, D, E, C, F, G

Inorder



D, B, E, A, F, C, G

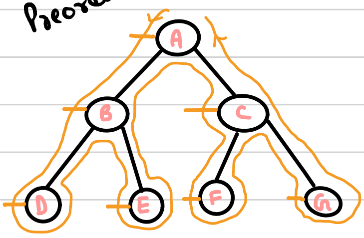
Postorder



D, E, B, F, G, C, A

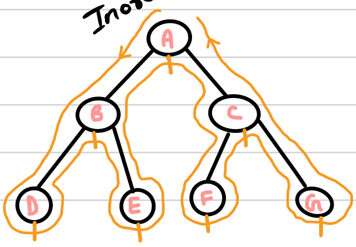
BINARY TREE TRAVERSAL EASY METHOD 2

Preorder



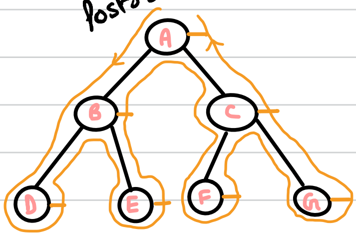
A, B, D, E, C, F, G

Inorder

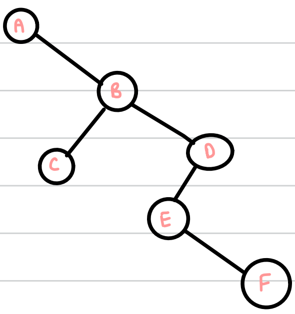


D, B, E, A, F, C, G

Postorder



D, E, B, F, G, C, A



Preorder : A, B, C, D, E, F

Inorder : A, C, B, E, F, D

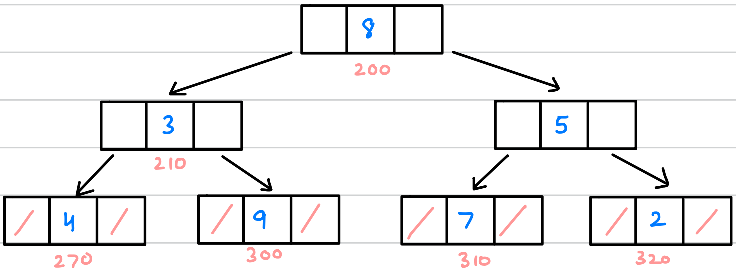
Postorder : C, F, E, D, B, A

PROGRAM TO CREATE BINARY TREE

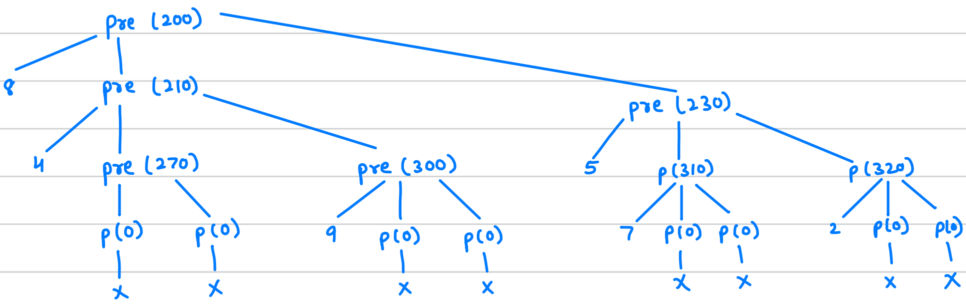
```
void create()
{
    Node *p, *t;
    int x;
    Queue q;          initializing a queue
    printf("Enter root value");
    scanf("%d", &x);
    root = malloc(.....);        initializing root
    root -> data = x;
    root -> lchild = root -> rchild = 0;
    enqueue(root);    Storing address of root in queue

    while(!isEmpty(q))
    {
        p = dequeue(&q);   take out a value from queue into p
        printf("Enter left child");
        scanf("%d", &x);

        if(x != -1)
        {
            t = malloc(....);
            t -> data = x; t -> lchild = t -> rchild = 0;
            p -> lchild = t;      p -> rchild = t    for right child everything
            enqueue(t);                              else will be same
        }
    }
}
```

RECURSIVE PREORDER TREE TRAVERSAL

```
void Preorder (Node *t)
{
    if ( t != NULL)
    {
        printf (" %d", t -> data);
        Preorder ( t -> lchild);
        Preorder ( t -> rchild);
    }
}
```



TRACING TREE



ACTIVATION RECORDS ARE CREATED USING STACK

RECURSIVE INORDER TREE TRAVERSAL

```
void Inorder ( Node *t)
{
    if ( t ! = NULL)
    {
        Inorder ( t -> lchild);
        printf ( " %d", t -> data);
        Inorder ( t -> rchild);
    }
}
```

RECURSIVE POSTORDER TREE TRAVERSAL

```
void Postorder ( Node *t )
{
    if (t ! = NULL)
    {
        Postorder ( t -> lchild) ;
        Post order ( t -> rchild ) ;
        printf (" %d", t -> data) ;
    }
}
```

ITERATIVE PREORDER TRAVERSAL

```
void Preorder ( Node *t)
{
    struct stack st;
                                    to check for empty stack
    while ( t != NULL || isEmpty (st) )
    {
        if (t != NULL)
        {
            printf(" %d ", t -> data);
            push ( &st , t); push address of t in stack
            t = t -> lchild;
        }

        else
        {
            t = pop ( &st); pop out address
            t = t -> rchild; from stack
        }
    }
}
```

ITERATIVE INORDER TRAVERSAL

```
Inorder ( Node *t)

push ( &st , t);
t = t -> lchild;

t = pop ( &st);
printf(" %d ", t -> data);
t = t -> rchild;
```

ITERATIVE POSTORDER TRAVERSAL*

```
void Inorder ( Node *t)
{
    struct stack st;
    long int temp;
    while ( t != NULL || isEmpty (st))
    {
        if ( t != NULL)
        {
            push (&st, t);
            t = t -> lchild;
        }
        else
        {
            temp = pop (&st);
            if ( temp > 0)
            {
                push (&st, -temp);
                t = ((Node *) temp) -> rchild;
            }
            else
            {
                printf("%d",((Node *) temp)
                                    -> data)
                t = NULL;
            }
        }
    }
}
```

LEVEL ORDER TRAVERSAL

```
void  Levelorder (Node *p)
{
        Queue  q;
        printf(" %d", p -> data);
        enqueue (&q, p);

        while( ! isEmpty (q))
        {
                p = dequeue (&q);
                if (p -> lchild)
                {
                        printf("%d", p -> lchild -> data);
                        enqueue( &q, p -> lchild);
                }

                if (p -> rchild)
                {
                        printf("%d", p -> rchild -> data);
                        enqueue( &q, p -> rchild);
                }
        }
}
```

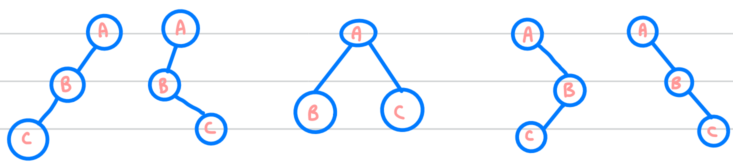
CAN WE GENERATE TREE FROM TRAVERSALS ?

$n = 3$

A B C

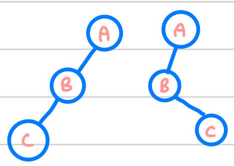
① Given preorder – A, B, C

$$\frac{{}^{2n}C_n}{n+1}$$



② Given preorder A, B, C
posorder C, B, A

more than one



CONCLUSION

If we are given either preorder, inorder, postorder we cannot generate unique tree.

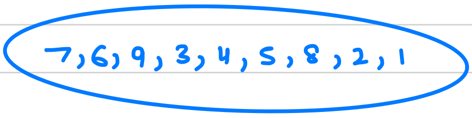
If we are given both preorder and postorder, then also unique tree cannot be generated

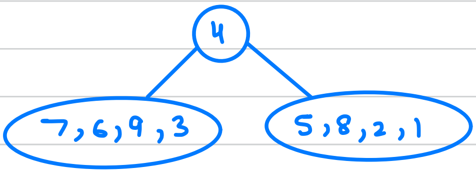
① preorder + inorder
② postorder + inorder

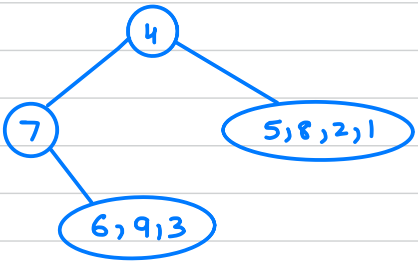
for generating unique tree

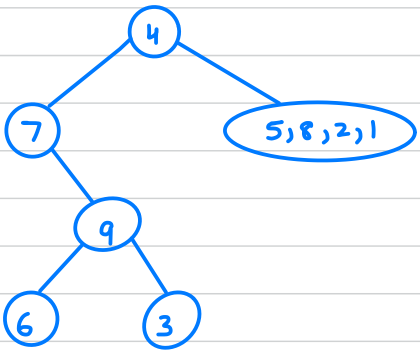
Q Preorder – 4, 7, 9, 6, 3, 2, 5, 8, 1
Inorder – 7, 6, 9, 3, 4, 5, 8, 2, 1

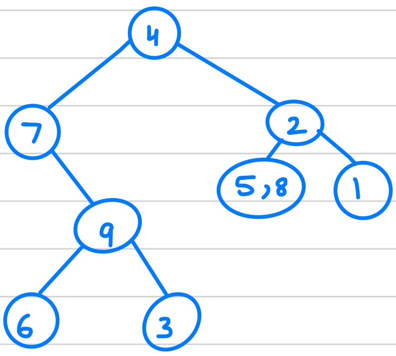
Sol



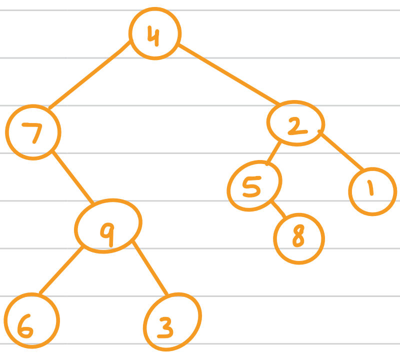






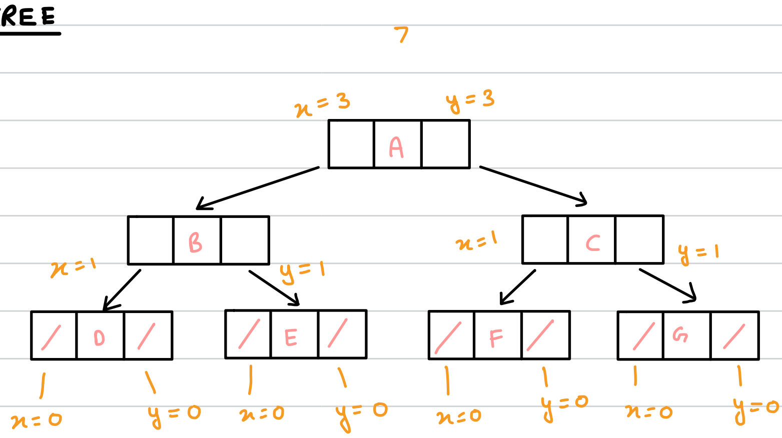


Unique Tree



COUNTING NO OF NODES IN A BINARY TREE

```
int count (Node *p)
{
    int x, y;

    if (p != NULL)
    {
        x = count (p -> lchild)
        y = count (p -> rchild)
        return x + y + 1;
    }
    return 0;
}
```



COUNTING LEAF NODES

```
int count (Node *p)
{
    int x, y;

    if (p != NULL)
    {
        x = count (p -> lchild);
        y = count (p -> rchild);

        if (p -> lchild == NULL && p -> rchild == NULL)
            return x + y + 1;
        else
            return x + y;
    }
    else
        return 0;
}
```

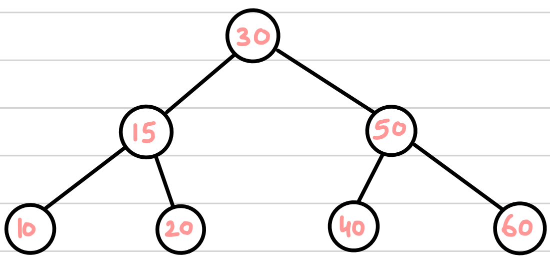
COUNTING NODES WITH DEGREE 2

COUNTING NODES WITH DEGREE 2 OR 1

COUNTING NODES WITH DEGREE 1

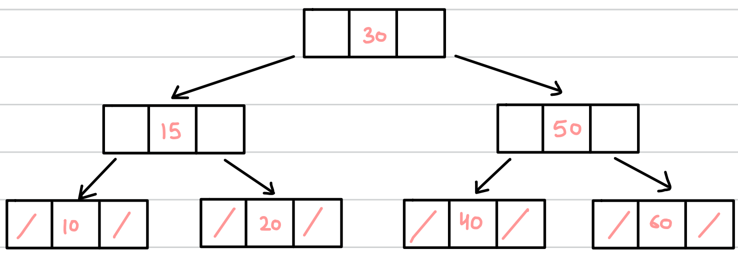
```
if (p -> lchild != NULL && p -> rchild != NULL)

if (p -> lchild != NULL || p -> rchild != NULL)

if ((p -> lchild != NULL && p -> rchild == NULL) ||
    (p -> lchild == NULL && p -> rchild != NULL))
        XOR
if (p -> lchild != NULL ^ p -> rchild != NULL)
```

BINARY SEARCH TREE



- Left subtree is smaller
- Right subtree is greater
- Useful for searching (in less number of comparisons)
- No duplicates
- Inorder gives sorted order
- For n number of nodes, catalan number of trees can be generated

$$T(n) = \frac{{}^{2n}C_n}{n+1}$$



- BST is represented using linked representation.
- Can also be represented using arrays.

SEARCHING IN A BINARY SEARCH TREE

(searching takes maximum time same as height of tree) O(logn)

(1) RECURSIVE SEARCH (tail recursion)

O(h)

$\log n \leq h \leq n$

```
Node* Rsearch(Node *t, int key)
{
    if (t == NULL)            // If element is not
        return NULL;          found
    if (key == t→data)
        return t;
    else if (key < t→data)
        return Rsearch(t→lchild, key);
    else
        return Rsearch(t→rchild, key);
}
```

(2) ITERATIVE VERSION O(log n)

```
Node * Search(Node *t, int key)
{
    while (t != NULL)
    {
        if (key == t→data)
            return t;
        else if (key < t→data)
            t = t→lchild;
        else
            t = t→rchild;
    }
    return NULL;
}
```

INSERTING IN BINARY SEARCH TREE

```
void Insert(Node *t, int key)
{
    Node *r = NULL, *p;

    while(t! = NULL)
    {
        if(key == t->data)
            return;        // To avoid duplication
        else if(key < t->data)
            t= t->lchild;
        else
            t= t->rchild;
    }
    p = new Node;
    p->data = key;
    p->lchild = p->rchild = NULL;

    if(p->data < r->data)
        r->lchild = p;
    else
        r->rchild = p;
}
```

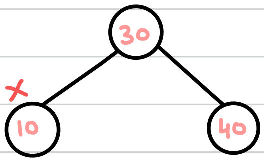
RECURSIVE INSERT IN BST

```
Node* Insert(Node *p, int key)
{
    if(p==NULL)
    {
        t = new Node;
        t-> data = key;
        t-> lchild = t->rchild = NULL;
    }
    if(key<p->data)
        p->lchild = insert(p->lchild, key);
    else if(key>p->data)
        p->rchild = insert(p->rchild, key);
    return p;
}
```

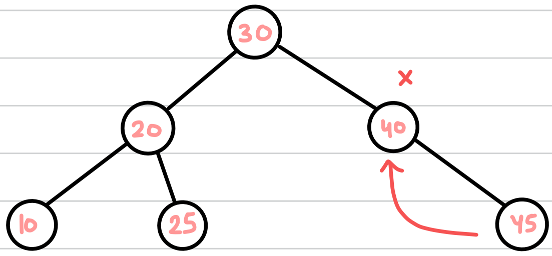
```
void main()
{
    Node *root = NULL;
    root = insert(root, 30);
    insert(root, 20);
    insert(root, 25);
}
```

DELETING FROM BST

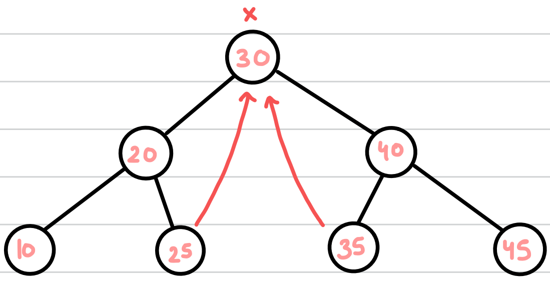
CASE 1 :



CASE 2 :



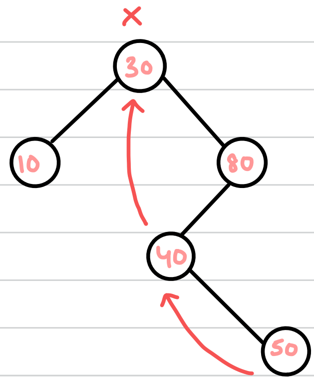
CASE 3 :



Inorder predecessor

Inorder successor

CASE 4 :



// Multiple changes to be made

```
int Height(struct Node *p)
{
        int x, y;
        if(p == NULL)
              return 0;
        x = Height(p->lchild);
        y = Height(p->rchild);
        return x>y ? x+1 : y+1;
}
```

```
struct Node *InPre(struct Node *p)
{
        while(p && p->rchild != NULL)
              p = p->rchild;
        return p;
}
```

```
struct Node *InSucc(struct Node *p)
{
        while(p && p->lchild != NULL)
              p = p->lchild;
        return p;
}
```

```c
Struct Node * Delete( struct Node *p , int key)
{
        struct Node *q;

        if (p == NULL)
                return NULL;

        if (p->lchild = NULL && p->rchild = NULL)   // Leaf Node
        {
                if (p == root)
                        root = NULL;
                free (p);
                return NULL;
        }

        if ( key < p->data)
                p->lchild = Delete (p->lchild, key);
        else if ( key > p->data)
                p->rchild = Delete (p->rchild, key);
        else
        {
                if( Height ( p->lchild) > Height (p->rchild))
                {
                        q = InPre (p->lchild);
                        p->data = q->data;
                        p->lchild = Delete (p->lchild, q->data);
                }

                else
                {
                        q = InSucc (p->rchild);
                        p->data = q->data;
                        p->rchild = Delete (p->rchild, q->data);
                }
        }
        return p;
}
```

GENERATING BINARY SEARCH TREE

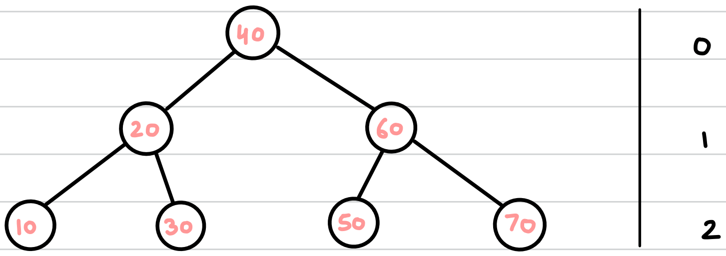
```
void Createpre( int pre[], int n)    O(n)
{
    Stack st;
    Node *t;
    int i=0;

    root = new Node;
    root -> data = pre[i++];
    root -> lchild = root -> rchild = NULL;
    p = root;

    while (i<n)
    {
        if (pre[i] < p -> data)
        {
            t= new Node;
            t -> data = pre[i++];
            t -> lchild = t -> rchild = NULL;
            p -> lchild = t;
            push (&stk, p);
            p = t;
        }
        else
        {
            if( pre[i] > p -> data && pre[i] < stacktop (stk) -> data)
            {
                t = new Node;
                t -> data = pre[i++];
                t -> lchild = t -> rchild = NULL;
                p -> rchild = t;
                p = t;
            }
            else
            {
                p = pop(&stk);
            }
        }
    }
}
```

BST can be generated using
- Preorder + Inorder
- Postorder + Inorder

Sorted preorder of BST gives its Inorder /postorder

DRAWBACK OF BINARY SEARCH TREE

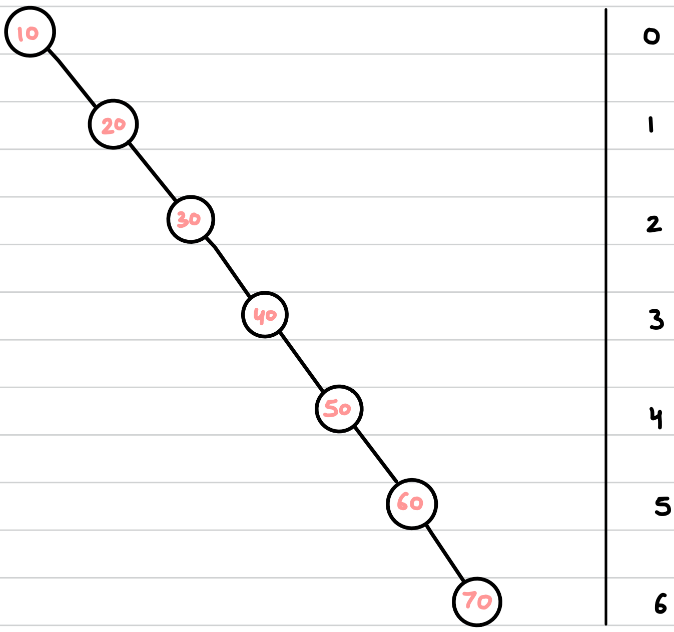
KEYS : 40, 20, 30, 60, 50, 10, 70



$h = \log_2(n+1) - 1$

$O(\log n)$

KEYS : 10, 20, 30, 40, 50, 60, 70



$h = n-1$

$O(n)$

There is no control over height of binary search tree.

It only depends on user's input or sequence of insertion

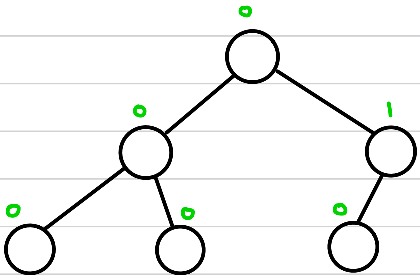
AVL TREES

→ Height Balanced Binary Search Trees

No of edges

Balance factor = Height of left subtree − Height of right subtree

{-1, 0, 1} ⟶ Balanced Tree



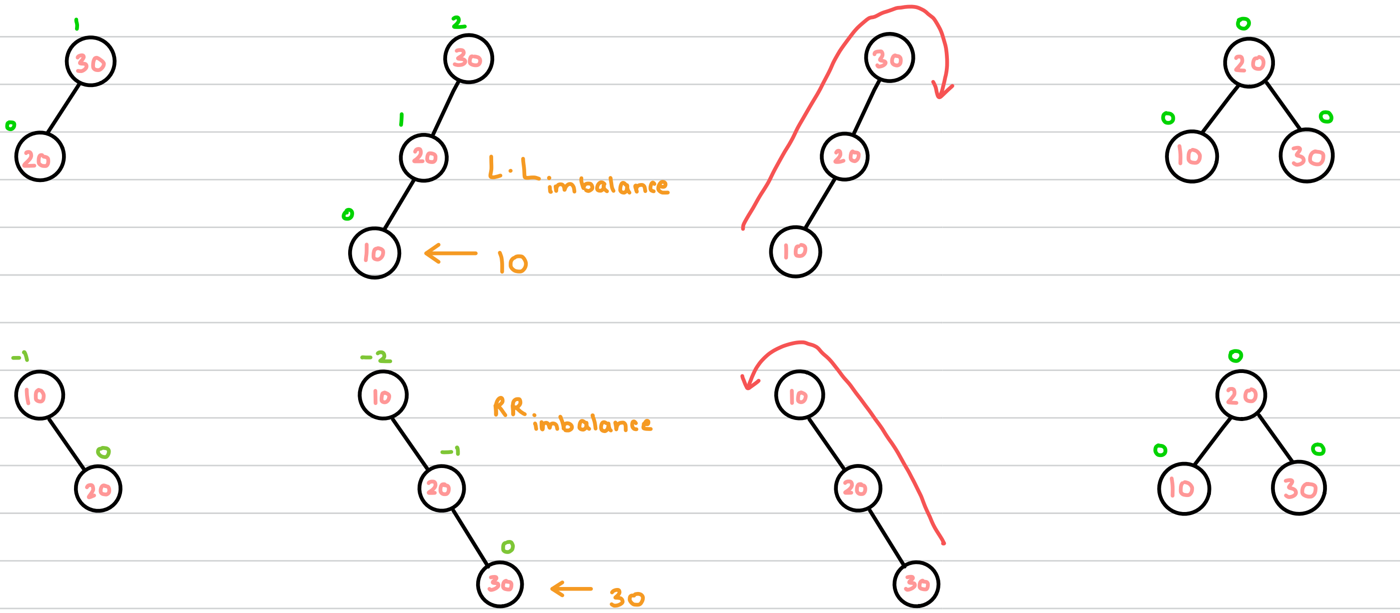
Balanced Tree

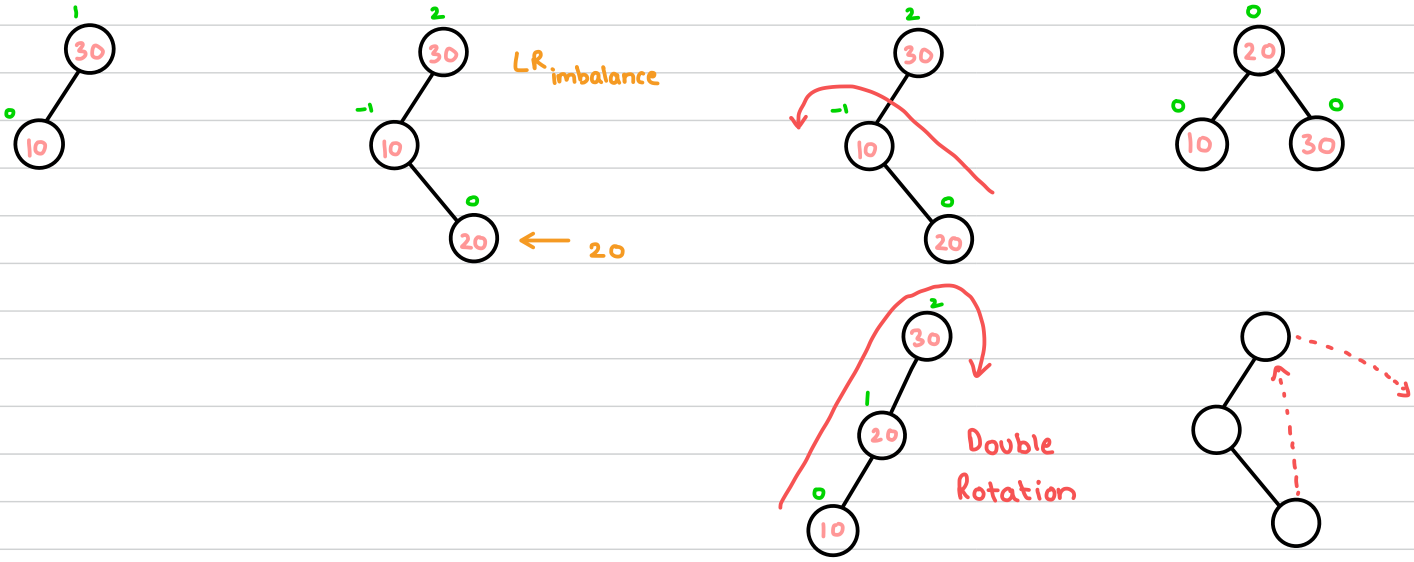
Rotations for insertion in AVL trees

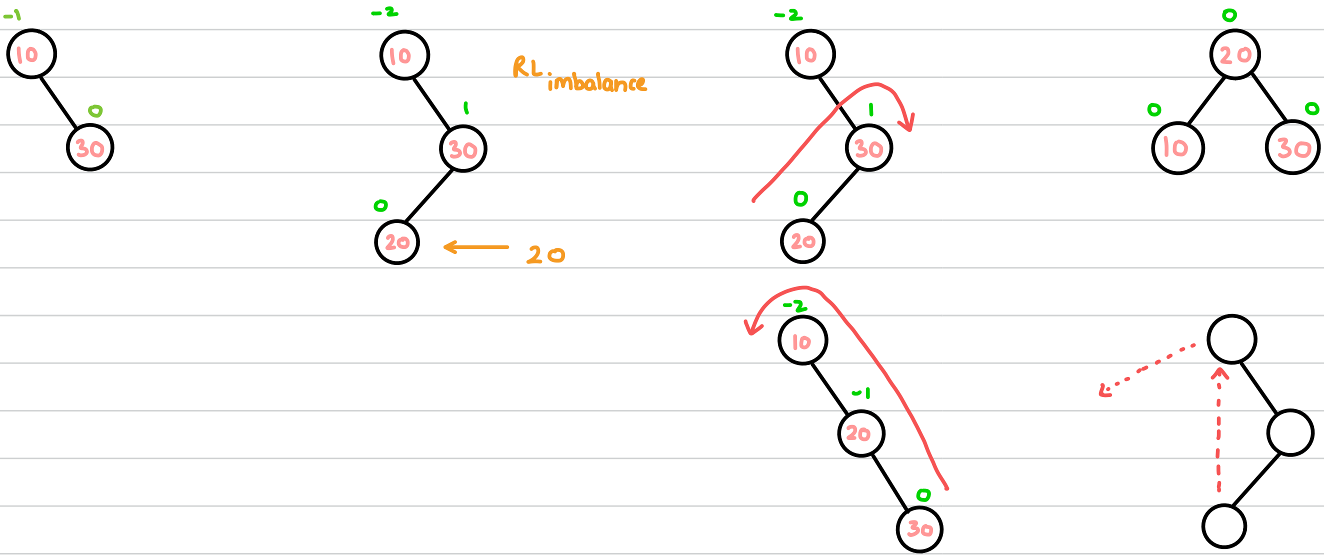
(1) LL Rotation
(2) RR Rotation] Single Rotation

(3) LR Rotation
(4) RL Rotation] Double Rotation

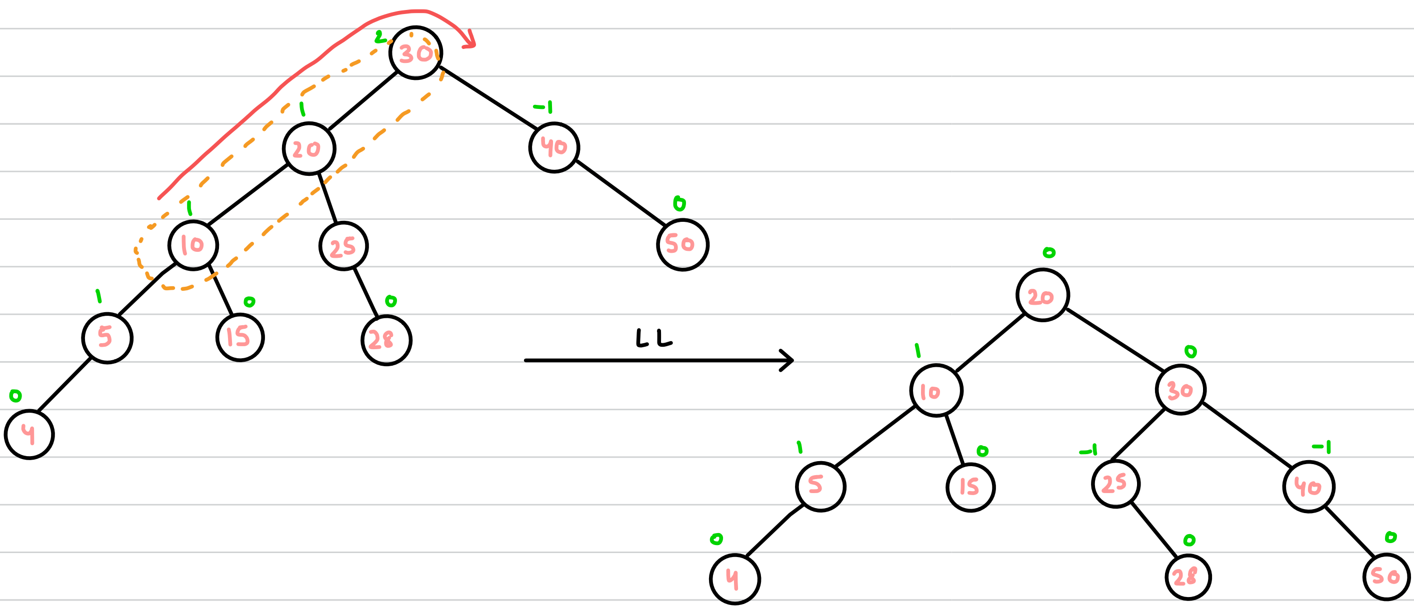
INITIAL | AFTER INSERTION | PERFORM ROTATION | AFTER ROTATION

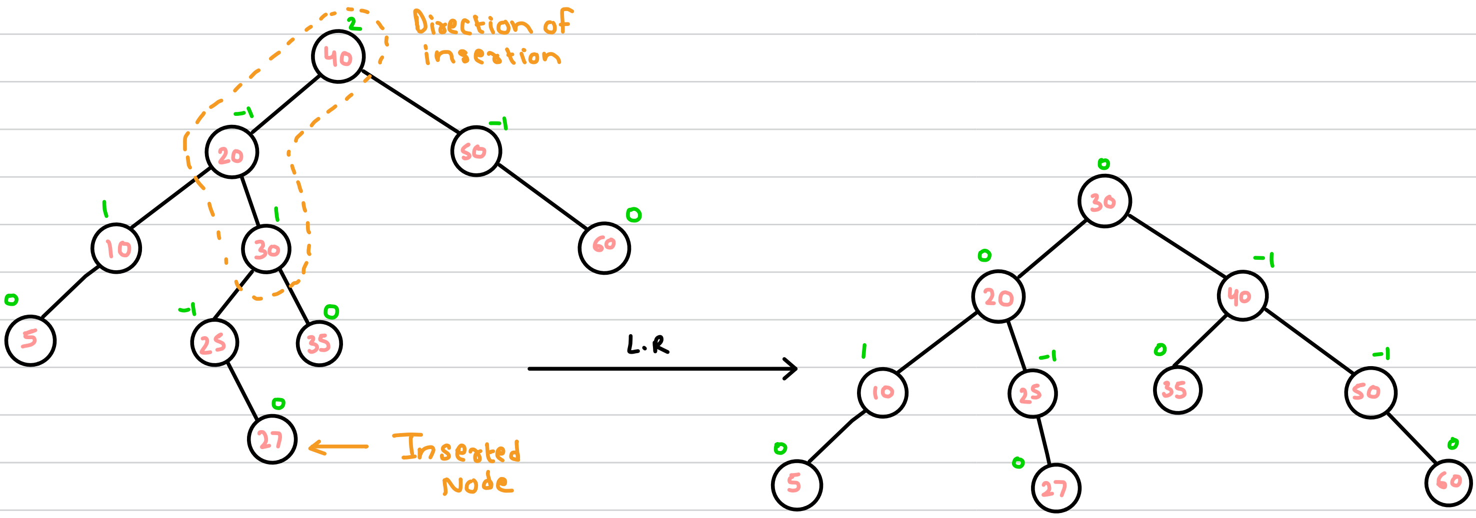






FORMULA OF ROTATION FOR INSERTION

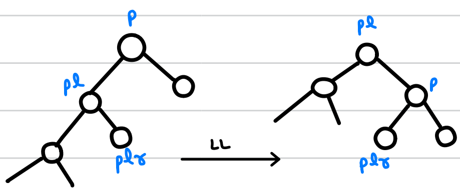




PROGRAM FOR LL ROTATION

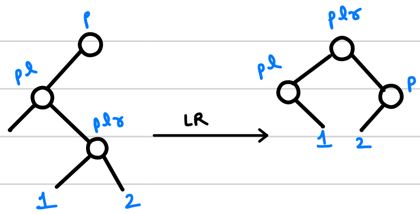
```
struct Node
{
    Struct Node *lchild;
    int data;
    int height;     // we will set height for
    Struct Node *rchild; each and every Node
}* root = NULL;

Struct Node* Insert(Node *p, int key)
{
    if(p==NULL)
    {
        t = new Node;
        t → data = key;
        t → height = 1;
        t → lchild = t → rchild = NULL;
        return t;
    }
    if ( key < p → data)
        p → lchild = insert (p → lchild, key);
    else if ( key > p → data)
        p → rchild = insert (p → rchild, key);
    p → height = NodeHeight(p);   // update height of each
                                     node at returning time
```

```
    if (BalanceFactor(p)==2 && BalanceFactor(p->lchild)==1)
          return LLRotation(p);
    else if (BalanceFactor(p)==2 && BalanceFactor(p->rchild)==-1)
          return LRRotation(p);
    else if (BalanceFactor(p)==-2 && BalanceFactor(r->child)==-1)
          return RRRotation(p);
    else if (BalanceFactor(p)==-2 && BalanceFactor(r->child)==1)
          return RLRotation(p);

    return p;
}

int NodeHeight(struct Node *p)
{
    int hl, hr;   // Height of left subtree (HL), height of right subtree (HR)
    hl = p && p->lchild ? p->lchild->height : 0;
    hr = p && p->rchild ? p->rchild->height : 0;
      NOT NULL
    return hl > hr ? hl+1 : hr+1;
}

int BalanceFactor(struct Node *p)
{
    int hl, hr;
    hl = p && p->lchild ? p->lchild->height : 0;
    hr = p && p->rchild ? p->rchild->height : 0;

    return hl - hr;
}

struct Node *LLRotation(struct Node *p)
{
    struct Node *pl = p->lchild;
    struct Node *plr = pl->lchild;

    pl->rchild = p;
    p->lchild = plr;
    p->height = NodeHeight(p);
    pl->height = NodeHeight(pl);
```



```
    if(root == p)        // If rotation was performed on
         root = pl;        root node, root needs to be
                           updated.
    return pl;
}
```

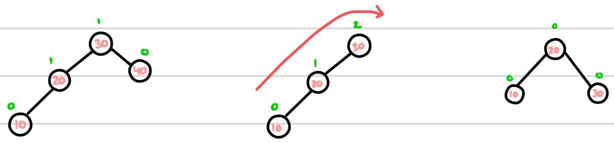
```
struct Node* LRRotation(struct Node *p)
{
    struct Node *pl = p->lchild;
    struct Node *plr = pl->rchild;

    pl->rchild = plr->lchild;
    p->lchild = plr->rchild;

    plr->lchild = pl;
    plr->rchild = p;

    pl->height = NodeHeight(pl);
    p->height = NodeHeight(p);
    plr->height = NodeHeight(plr);

    if(root == p)
        root = plr;
    return plr; // New root;
}
```



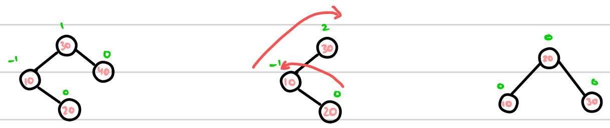
DELETION FROM AVL TREES WITH ROTATION

1. L 1 Rotation
2. L -1 Rotation
3. L 0 Rotation
4. R 1 Rotation
5. R -1 Rotation
6. R-0 Rotation

L1

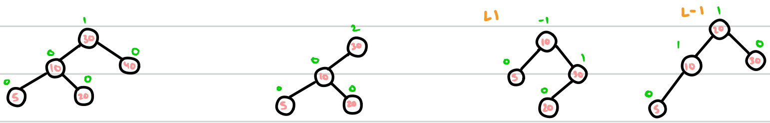


L-1



// Other three will be mirror images

L-0

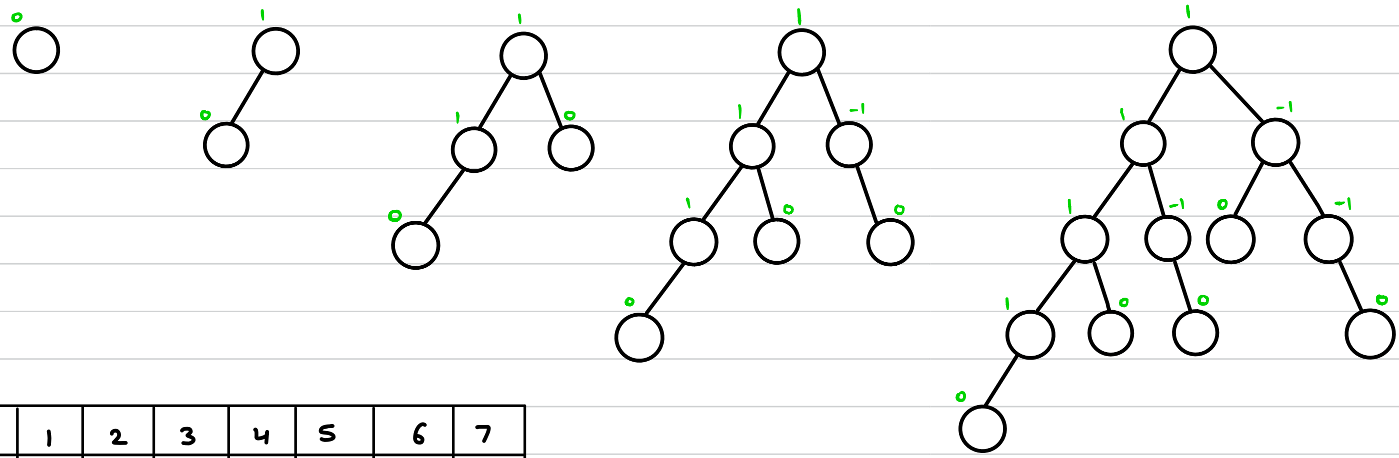


HEIGHT VS NODES OF AVL TREES

If height is given :

- Max nodes $n = 2^h - 1$ // Not $2^{h+1} - 1$ because height is starting from 1.
- Min nodes n = Look in table

$h=1$, $n=1$ | $h=2$, $n=2$ | $h=3$, $n=4$ | $h=4$, $n=7$ | $h=5$, $n=12$



h	1	2	3	4	5	6	7
n	1	2	4	7	12	20	33

+ +1

$$N(h) = \begin{cases} 0 & 0 \\ 1 & 1 \\ N(h-2) + N(h-1) + 1 & \text{otherwise} \end{cases}$$

// formula same as Fibonacci series

↳ balanced series

If 'N' Nodes are given find:

- Min height = $\log_2 (n+1)$
- Max height = Look in table

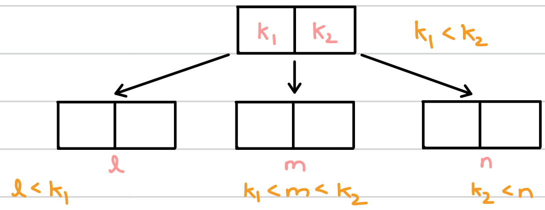
Foreg for 13 nodes $h = 5$ // Look from node towards height

19 nodes $h = 5$

SEARCH TREES

2 - 3 TREES

- Multiway Search Tree (M Way Search Tree) — M → degree
- Degree 3 (2-3 trees are Multiway Search Trees with degree 3)
- B Trees (These are height Balanced Search Trees)
- Rules
  - All leaf nodes at same level
  - Every node must have $\left\lceil \frac{n}{2} \right\rceil$ children

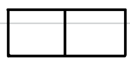


$$\left\lceil \frac{3}{2} \right\rceil = 2$$

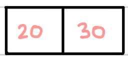
  - Cannot have duplicates

CREATION OF 2-3 TREE

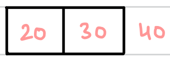
KEYS : 20 , 30 , 40 , 50 , 60 , 10 , 15 , 70 , 80 , 90

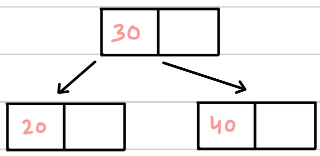


20 , 30

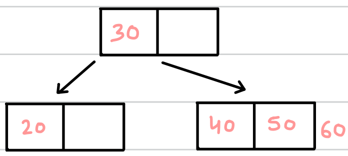


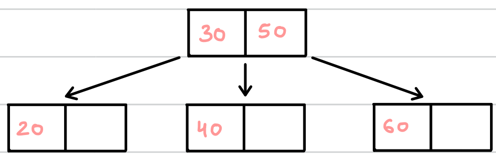
40



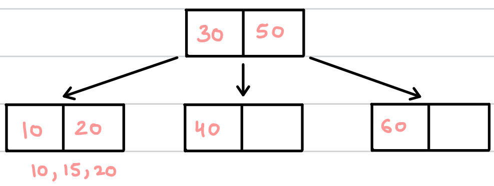


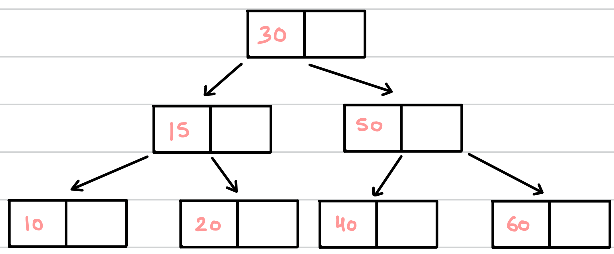
50,60



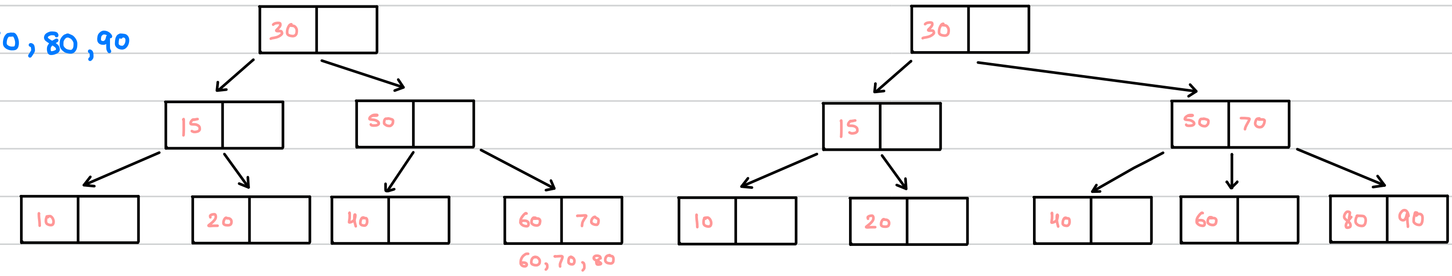


10 , 15



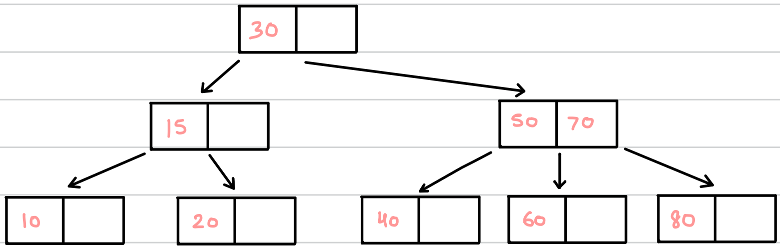


70, 80, 90

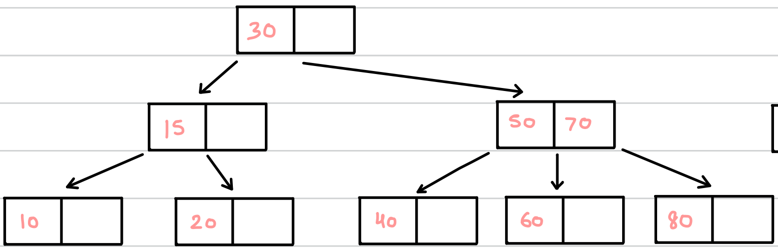


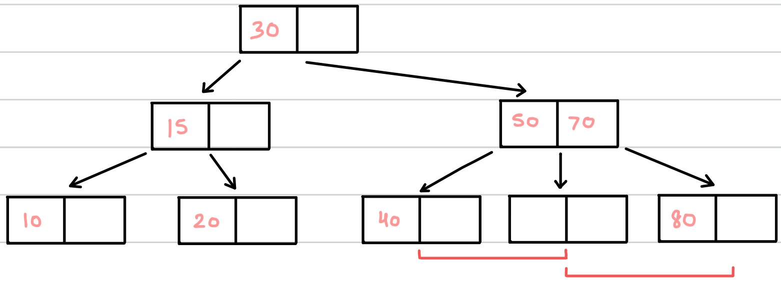
DELETING FROM 2-3 TREE

CASE 1 : Simply Delete
90

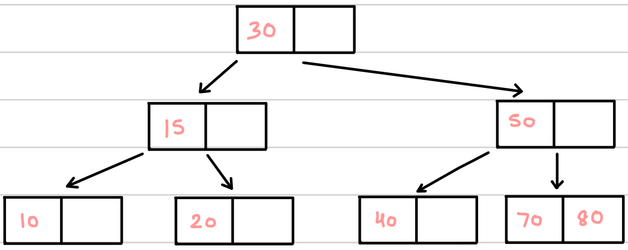


CASE 2 : Delete and merge
60



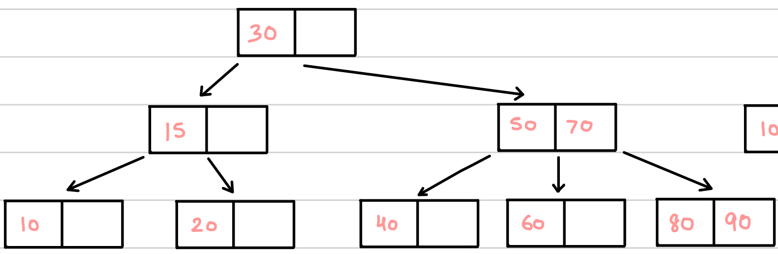


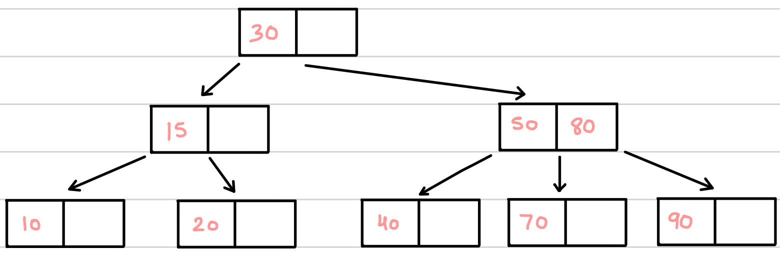
Join with either left sibling or right sibling



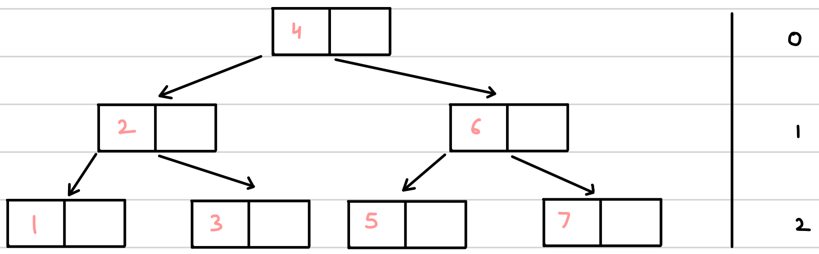
Merging with right sibling

CASE 3 : Borrow
60





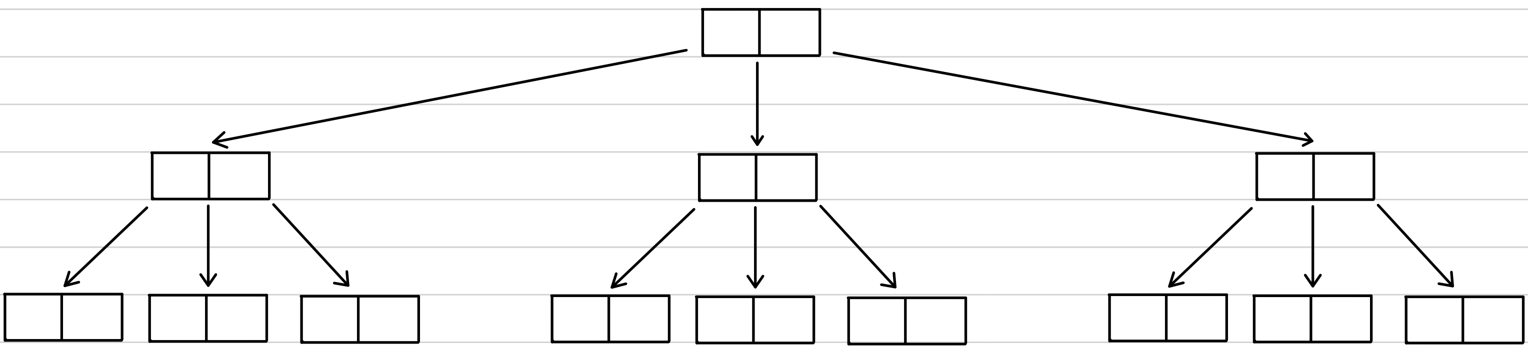
ANALYSIS



Tree with min nodes for given height

Minimum $n = 1 + 2 + 2^2 \ldots$
$= 2^{h+1} - 1$

Max $h = \log_2 (n-1) - 1$
$= O(\log_2 n)$



Max $n = 1 + 3 + 3^2 \ldots\ldots$

$$= \frac{3^{h+1} - 1}{3-1}$$

Min $h = \log_3 [n(3-1) + 1] - 1$

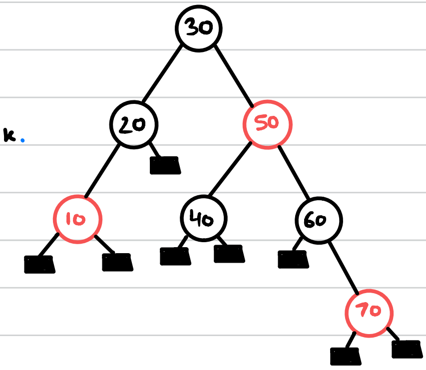
$O(\log_3 n)$

Minimum as well as maximum height is $\log n$

These trees are used for DBMS softwares because a node can have more than one value

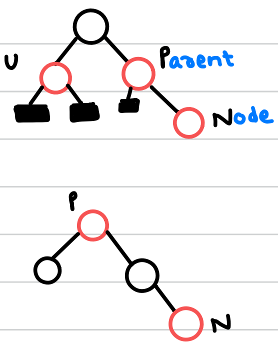
RED BLACK TREE

- It is a height balanced Binary Search Tree, similar to 2-3-4 tree.
- Every node is either Red or Black.
- Root of a Tree is Black
- NULL is also Black.
- Number of Blacks on paths from Root to leaf are same.
- No 2 consecutive Red, Parent and Children of red are Black.
- New inserted Node is Red.
- Height in $\log n \le h \le 2\log n$



CREATION OF RED BLACK TREE

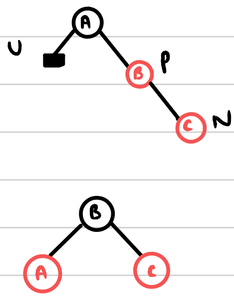
Uncle is Red (for Node)



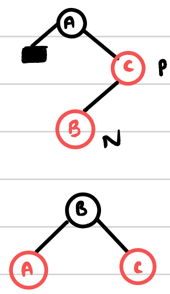
RECOLOURING

Uncle is Black

Zig-Zig (LL/RR)



Zig-Zag (RL/LR)



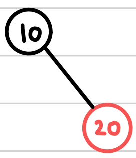
ROTATION

KEYS: 10, 20, 30, 50, 40, 60, 70, 80, 4, 8

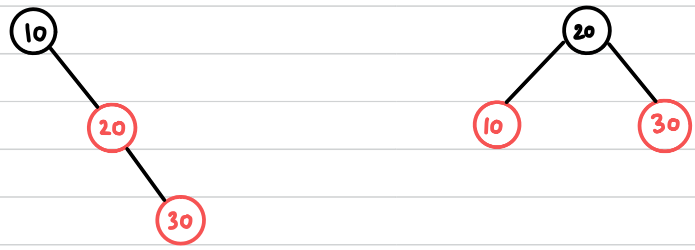
INSERT

10

20



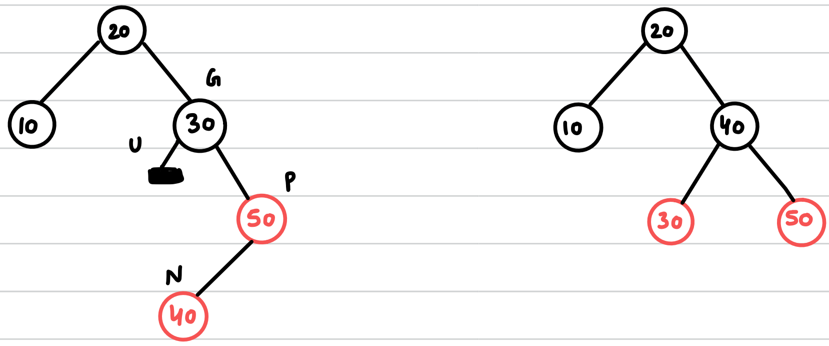
30



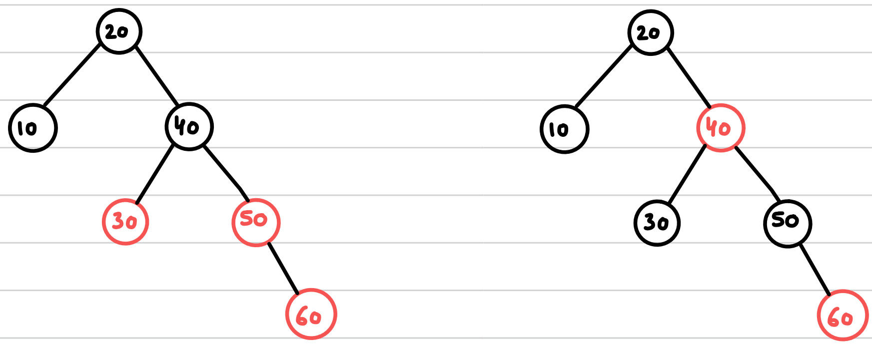
50

Root must be black

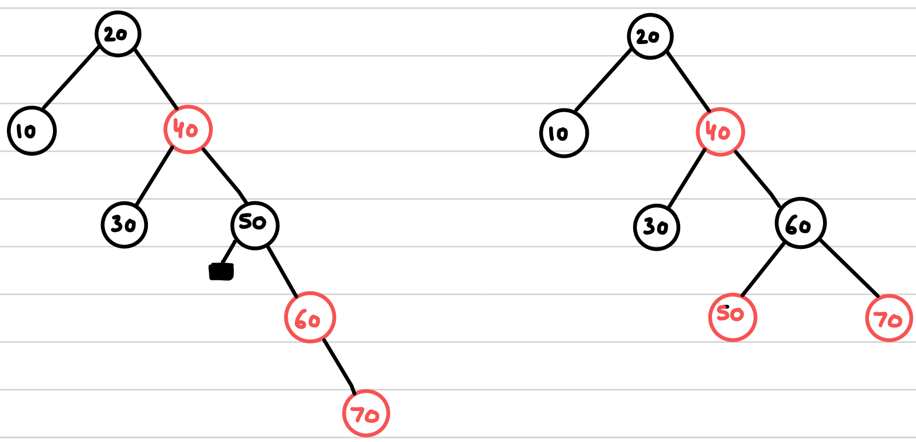
40



60

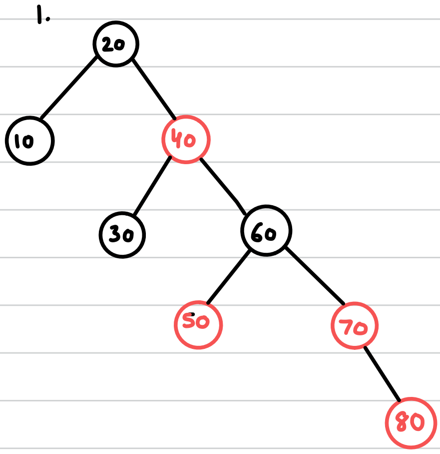


70

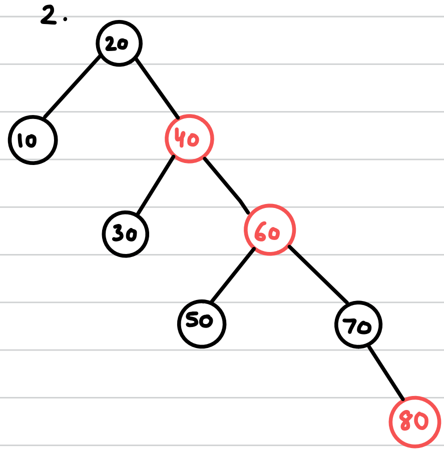


80

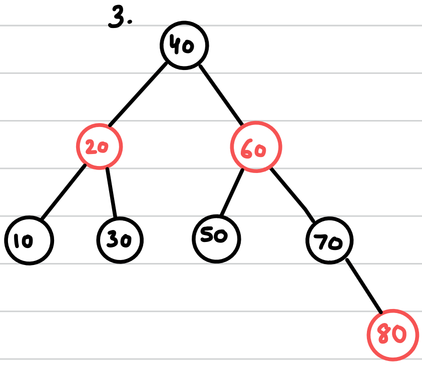
1.



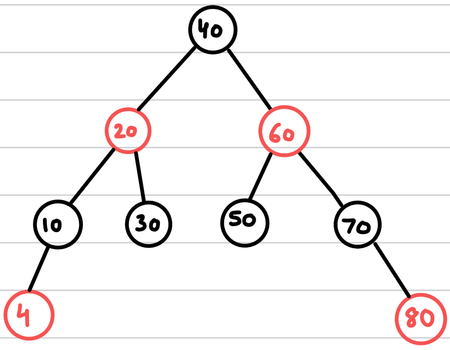
2.



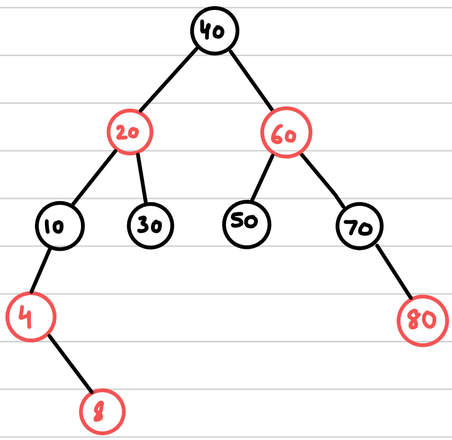
3.

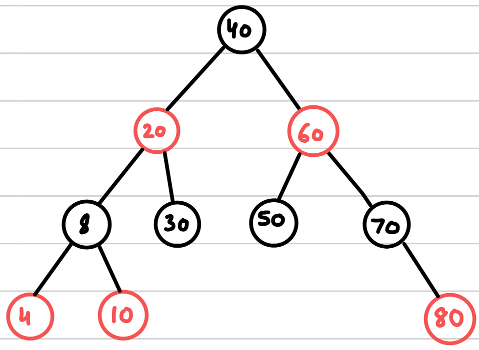


4



8

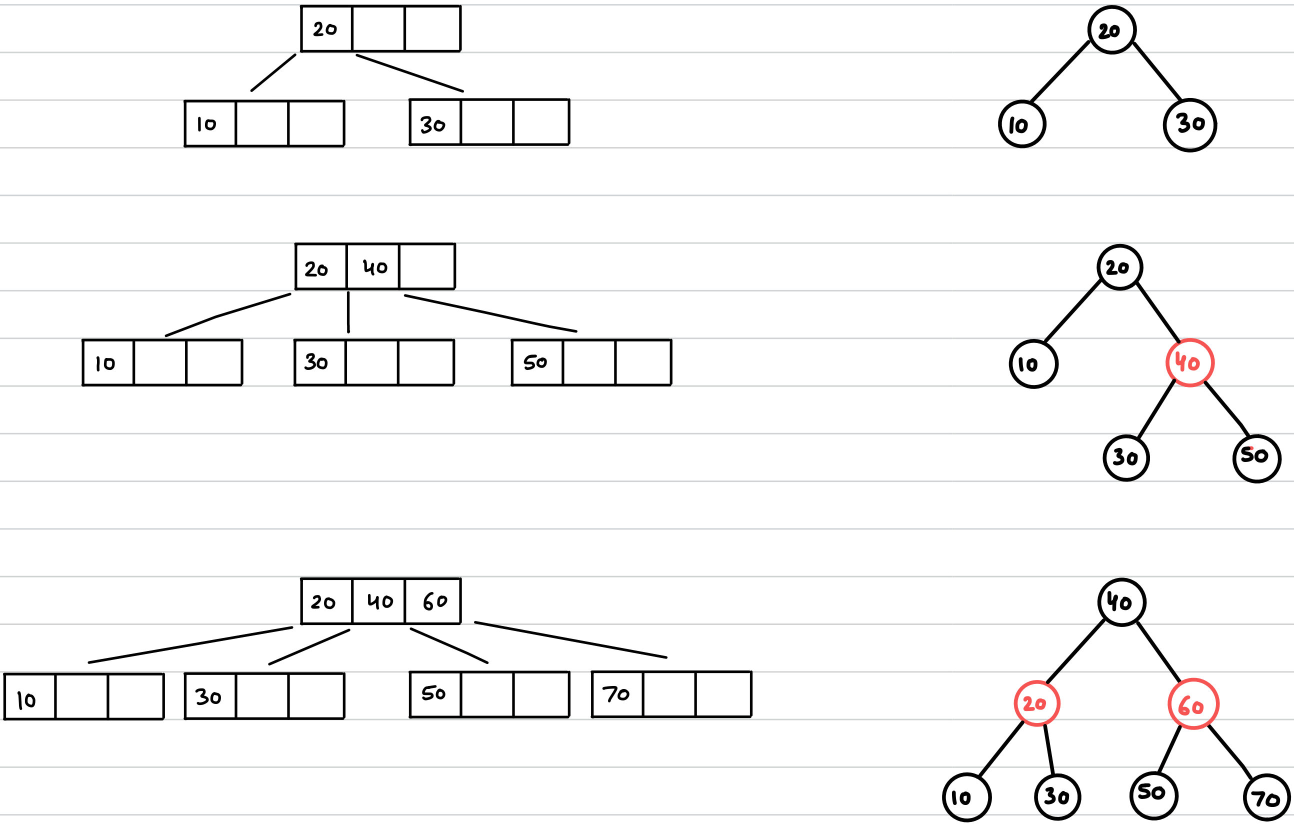




height only red / black = log n

height red and black = 2 log n

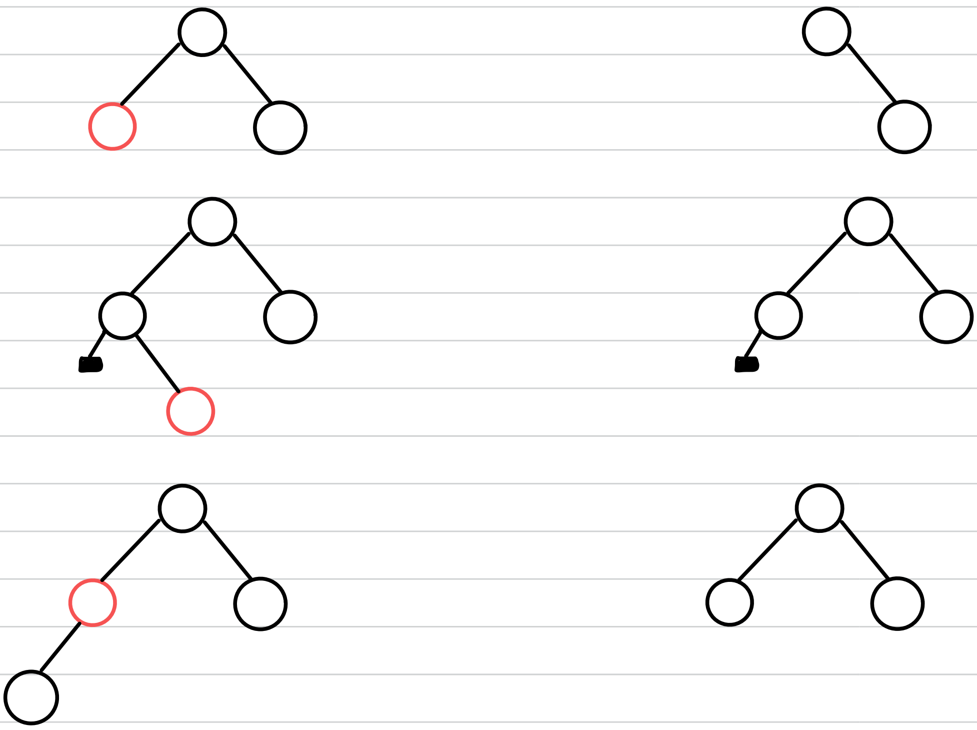
2-3-4 TREES VS RED BLACK TREES



RED BLACK TREE DELETION CASES

CASE 1 : Deleted Node is Red Node

Before Deletion | After Deletion

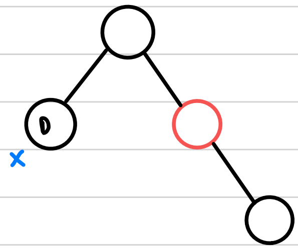


Simply delete as it is a leaf node and red

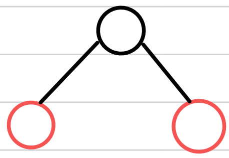
Simply delete because if red node is deleted, the path of black nodes remains unchanged

CASE 2 : Node is black and sibling is red

Before Deletion



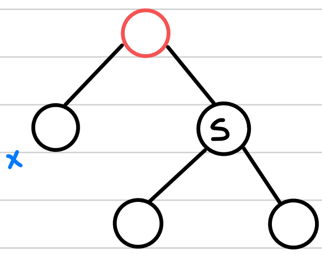
After Deletion



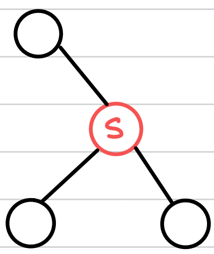
Perform rotation

CASE 3 : Node is black and sibling is also black

Before Deletion

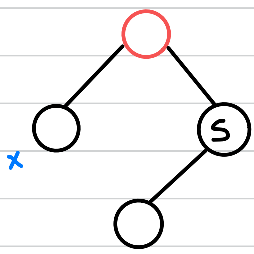


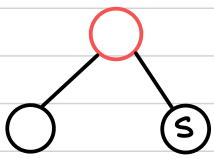
After Deletion



Change sibling to red and parent to black

Recolour





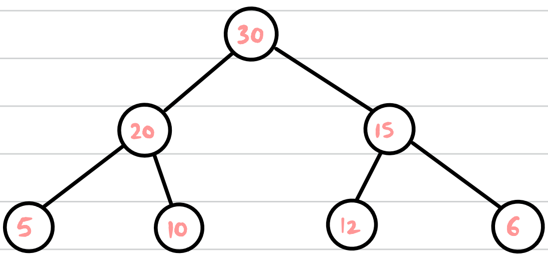
Perform rotation

HEAP

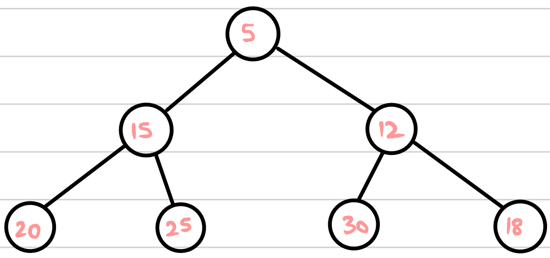
↳ It is a complete binary tree

- what is a heap?
- Insert in a heap
- Deleting from heap
- Heap Sort
- Heapify
- Priority Queues

Max Heap



Min Heap



30	20	15	5	10	12	6			
1	2	3	4	5	6	7	8	9	10

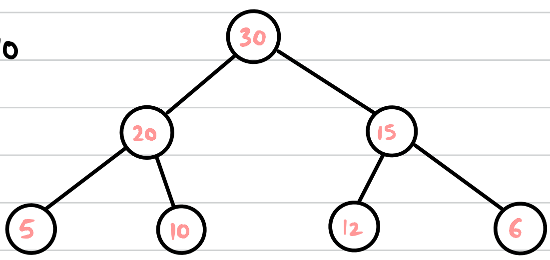
- For complete binary tree, there should not be any free space between the elements of array

Node at index i
Left child at index $2*i$
Right child at index $2*i+1$

- Every node should have an element greater than or equal to all its decendants (duplicates can be there)

INSERTING IN HEAP

Element to insert = 40



30	20	15	5	10	12	6			
1	2	3	4	5	6	7	8	9	10

Step 1 : Insert element at next free index in array

30	20	15	5	10	12	6	40		
1	2	3	4	5	6	7	8	9	10

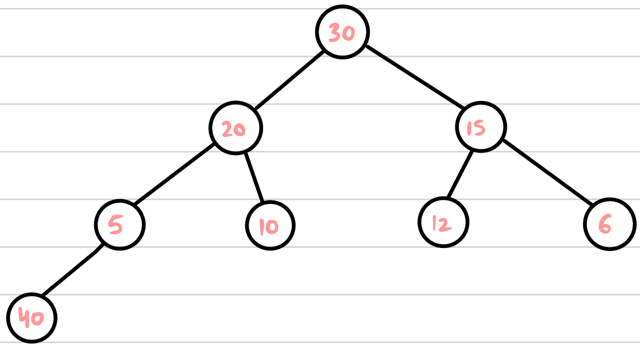
↑

Step 2 : Insert the element in the tree

Parent of 40 $= \frac{8}{2} = 4$

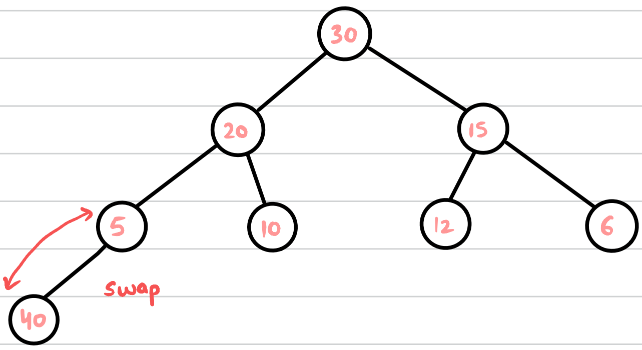
30	20	15	5	10	12	6	40		
1	2	3	4	5	6	7	8	9	10

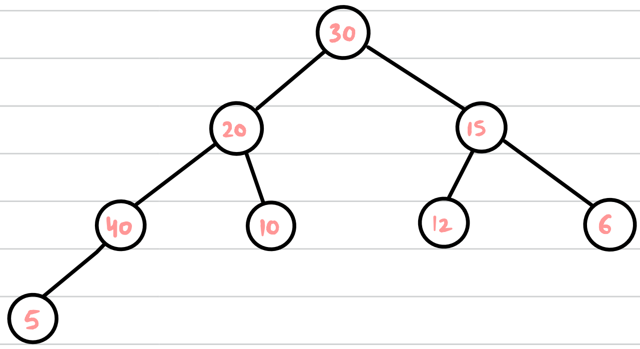
↑



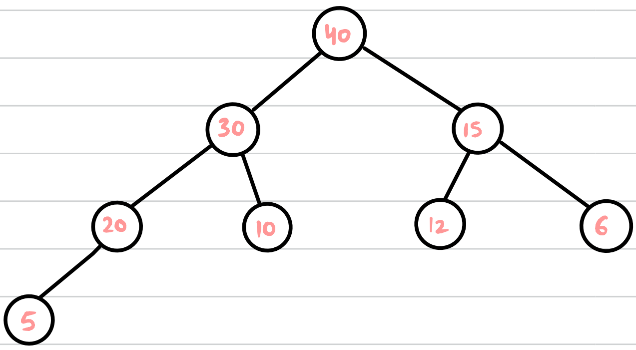
Step 3 : Now tree does not satisfy max heap condition

Compare 40 with its parent / ancestor, if node is found greater than ancestor , swap.





Repeat until condition is satisfied



Check this condition in array also. Compare node with parent $\left(\frac{i}{2}\right)$, if found greater than parent , replace

40	30	15	20	10	12	6	5		
1	2	3	4	5	6	7	8	9	10

PROGRAM

```
void Insert (int A[], int n)        O(logn)
{
    int temp, i=n;
    temp = A[n];

    while( i>1 && temp > A[i/2] )
    {
        A[i] = A[i/2]);
        i= i/2;
    }
    A[i] = temp;
}

void createheap()   O(nlogn)        1 element - logn
{                                   n elements - nlogn
    int A[] = {0,10,20,30,25,5,40,35};
    int i;
                 because first element is the heap and next element onwards,
    for (i=2; i<=7; i++)    insertion in heap is done.
        Insert (A,i);
}
```

Heapify : Faster method to implement creation of binary heap ← Separate Topics

Element Priority

Elements → 6, 8, 3, 10, 15, 2, 9, 17, 5, 8

- Element is itself a priority
- Smaller number higher priority

1. Insert in same order O(1) O(logn)
   Delete max priority by searching it O(n) O(logn)

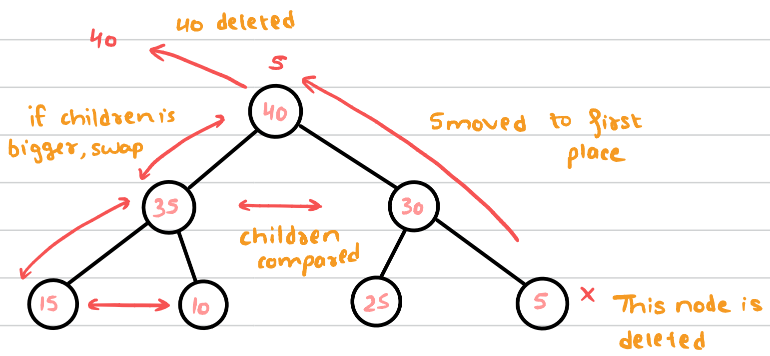
] After implementing using heap

Heap is used to implement priority queues

DELETION FROM HEAP

Only the root node can be deleted from heap i.e the first node

```
void Delete (int A[], int n)
{
    int x, i, j;
    x = A[n];
    A[1] = A[n]          // Smallest or last node is copied at first place
    i = 1; j = 2*i;      // i is at index 1 and j is at its left child

    while (j<n-1)
    {
        if (A[j+1] > A[j])   // Right child is greater than left child
            j = j+1;

        if (A[i] < A[j])   // Parent is smaller than child
        {
            swap (A[i], A[j]);
            i = j;
            j = 2*j;
        }
        else
            break;
    }
    A[n] = x;
}
```



HEAP SORT

While deleting, store the deleted element at vacant position

Through this, you will get sorted array in ascending order after deleting all the elements

1	2	3	4	5	6	7	8	9	10
40	35	30	15	10	25	5			

1	2	3	4	5	6	7	8	9	10
5	35	30	15	10	25				

$x = 40$

1	2	3	4	5	6	7	8	9	10
35	15	30	5	10	25	40			

STEPS

- $n \log n$ • Create heap of 'n' elements
- $n \log n$ • Delete 'n' elements 1 by 1

$O(n \log n)$

SORTING TECHNIQUES

1. Bubble Sort
2. Insertion Sort
3. Selection Sort

$O(n^2)$

4. Heap Sort
5. Merge Sort
6. Quick Sort
7. Tree Sort

$O(n \log n)$

8. Shell Sort

$O(n^{3/2})$

Comparison based Sorts

9. Count Sort
10. Bucket / Bin Sort
11. Radix Sort

$O(n)$

Index Based Sort
// Faster but memory consumption is high

There is no best sorting technique.

You have to chose the technique which matches your requirements

CRITERIA FOR ANALYSIS

1. Number of comparisons
2. Number of swaps
3. Adaptive // If already the array is sorted, it should make minimum comparisons
4. Stable
5 Extra Memory required

1. List sorted on basis of name

Name :	A	B	C	D	E	F	G
Marks :	5	8	6	4	6	7	10

Duplicate elements

2. List sorted on basis of marks

Name :	D	A	C	E	F	B	G
Marks :	4	5	6	6	7	8	10

Here also 'C' should come before 'E' as order must be preserved

This type of algorithms are useful in databases

If the sorting algorithm is preserving the order of duplicate elements in the sorted list then that algorithm is called 'Stable'.

1. BUBBLE SORT

A

8	5	7	3	2
0	1	2	3	4

n = 5

1st Pass

8	5	5	5	5
5	8	7	7	7
7	7	8	3	3
3	3	3	8	2
2	2	2	2	8

Largest element is sorted in first pass

4 comp
4 swap

2nd Pass

5	5	5	5
7	7	3	3
3	3	7	2
2	2	2	7
8	8	8	8

3 comp
3 swap

3rd Pass

5	3	3
3	5	2
2	2	5
7	7	7
8	8	8

2 comp
2 swap

4th Pass

3	2
2	3
5	5
7	7
8	8

1 comp
1 swap

→ Not actual number of swaps but maximum number of swaps

No of passes : 4
: (n-1)

No of comparison : 1+2+3+4
: 1+2+3+4 ...... (n-1)
: $\frac{n(n-1)}{2}$ $O(n^2)$

No of swaps : 1+2+3+4
1+2+3+4 ...... (n-1)
$\frac{n(n-1)}{2}$ $O(n^2)$

```
void BubbleSort(int A[], int n)
{
    int flag;
    for(i=0; i<n-1; i++)
    {
        flag=0;
        for(j=0; j<n-1-i; j++)
        {
            if (A[j] > A[j+1])
            {
                swap(A[j], A[j+1]);
                flag=1;
            }
        }
        if(flag==0)
            break;
    }
}
```

- Called as bubble sort as lighter or smaller elements come up same as bubbles
- Performing selected number of passes can give same number of largest elements as number of passes performed
- Bubble Sort is adaptive
- It is stable
- 'k' passes give 'k' number of largest element

2. INSERTION SORT

Inserting element in a sorted array at a sorted position

2	6	10	15	20	

ele = 12

// Start comparison from last element

EXAMPLE : A

8	5	7	3	2
0	1	2	3	4

// Assume only first element to be sorted

n = 5

1st Pass

8		7	3	2
0	1	2	3	4

5

1 comp

1 swap

Take out one element and compare and insert

5	8	7	3	2
0	1	2	3	4

2nd Pass

5	8		3	2
0	1	2	3	4

7

2 comp

2 swap

5	7	8	3	2
0	1	2	3	4

3rd Pass

5	7	8		2
0	1	2	3	4

3

3 comp

3 swap

3	5	7	8	2
0	1	2	3	4

4th Pass

3	5	7	8	
0	1	2	3	4

2

4 comp

4 swap

2	3	5	7	8
0	1	2	3	4

No of passes : $(n-1)$

No of comparison : $\frac{n(n-1)}{2}$ $O(n^2)$

No of swaps : $\frac{n(n-1)}{2}$ $O(n^2)$

- Insertion sort is adaptive as no swapping is done if array is sorted. Only 'n' comparisons are done which take minimum time
  It is adaptive by nature (no flag used)
- Insertion sort is stable
- Used in linked list.

```
void InsertionSort (int A[], int n)
{
    for(i=1 ; i<n ; i++)
    {
        j = i-1 ;
        x = A[i];
        while ( j > -1 && A[j] > x )
        {
            A[j+1] = A[j];
            j-- ;
        }
        A[j+1] = x ;
    }
}
```

3. SELECTION SORT

↳ Selecting a position in the array and finding the smallest element suitable for that.

A

8	6	3	2	5	4
0	1	2	3	4	5

$n = 6$

	1st Pass	2nd Pass	3rd Pass	4th Pass	5th Pass	
0	8 ← i	2	2	2	2	2
1	6	6 ← i	3	3	3	3
2	3	3 ← k	6 ← i	4	4	4
3	2 ← k	8	8	8 ← i	5	5
4	5	5	5	5 ← k	8 ← i	6
5	4	4	4 ← k	6	6 ← k	8

No of comparison : $\frac{n(n-1)}{2}$ $O(n^2)$

No of swaps : $n-1$ $O(n)$

- k passes give k number of smallest elements
- Selection Sort is not adaptive
- Selection Sort is not stable
- Selection Sort performs minimum number of swaps

```
void SelectionSort( int A[] ,int n)
{
    int i,j,k;

    for (i=0 ; i< n-1; i++)
    {
        for ( j=k=i ; j<n ; j++)
        {
            if ( A[j] < A[k])
                k = j ;
        }
        swap ( A[i] , A[k]);
    }
}
```

5. MERGING

- Merging two lists in third array
- Merging two lists in single array
- Merging multiple list

• MERGING TWO LISTS IN THIRD ARRAY

O(m+n)

index	A(m)		index	B(n)		index	C	
0	2	i	0	4	j	0	2	k
1	10		1	9		1	4	
2	18		2	19		2	9	
3	20		3	25		3	10	
4	23					4	18	
						5	19	
						6	20	
						7	23	
						8	25	

```
void Merge (int A[], int B[], int m, int n)
{
    int i, j, k;
    i = j = k = 0;

    while(i<m && j<n)
    {
        if (A[i] < B[j])
            c[k++] = A[i++];
        else
            c[k++] = B[j++];
    }

    for (; i<m; i++)            // for remaining elements
        c[k++] = A[i];          in either of list

                                // Only one of these loops
    for (; j<n ; j++)           will work
        c[k++] = B[j];
}
```

• MERGING TWO LISTS IN SINGLE ARRAY

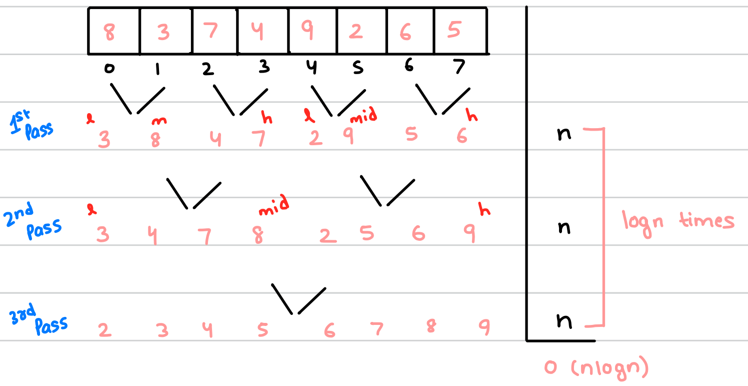
	l			mid				h
A	2	5	8	12	3	6	7	10
	0	1	2	3	4	5	6	7
	i				j			

B	2	3	5	6	7	8	10	12
	0	1	2	3	4	5	6	7
	k							

```
void Merge (int A[], int l, int mid, int h)
{
    int i, j, k;
    int B[h+1];
    i=l; j = mid+1; k=l;

    while( i<=mid && j<=h)
    {
        if (A[i] < A[j])
            B[k++] = A[i++];
        else
            B[k++] = A[j++];
    }

    for (; i<=mid; i++)
        B[k++] = A[i];

    for (; j<=h; j++)
        B[k++] = A[j];
}
```

ITERATIVE MERGESORT

We consider each of these elements to be a sorted list



RECURSIVE MERGE SORT O(nlogn)

```
void RMergeSort(int A[], int l, int h)
{
    if(l<h)
    {
        mid = ⌊(l+h)/2⌋;
        RMergeSort(A, l, mid);
        RMergeSort(A, mid+1, h);
        Merge(A, l, mid, h);
    }
}
```

```
void IMergeSort(int A[], int h)
{
    int p, i, l, mid, h;

    for(p=2; p<=n; p=p*2)
    {
        for(i=0; i+p-1<n; i=i+p)
        {
            l=i;
            h=i+p-1;
            mid = ⌊(l+h)/2⌋;

            Merge(A, l, mid, h);
        }
    }

    if(p/2<n)
        Merge(A, 0, p/2, n-1);
        // If number of elements are odd
}
```

6. QUICK SORT

→ Selecting a element and then finding its position

Pivot

50 70 60 90 40 80 10 20 30

i ... j

i ↳ Starts searching for an element greater than pivot — STEP 1

j ↳ Starts searching for element smaller or equal than pivot — STEP 2

When larger and smaller elements found, exchange them ← STEP 3

PARTITIONING PROCEDURE

50 30 60 90 40 80 10 20 70 (i at 30, j at 70)

50 30 20 90 40 80 10 60 70 (i at 20, j at 60)

50 30 20 10 40 80 90 60 70 (i at 10, j at 90)

50 30 20 10 40 80 90 60 70 (j at 40, i at 80)

If i becomes greater than j, we have checked entire list Now j^{th} index element and pivot should be swapped STEP 4

(40 30 20 10) 50 (80 90 60 70)

j

perform quick sort left and right side

partitioning position

```
int Partition(int A[], int l, int h)
{
    int Pivot = A[l];
    int i=l, j=h;

    do
    {
        do{i++} while (A[i] ≤ pivot);
        do{j--} while (A[i] > pivot);

        if (i<j)
            swap(A[i], A[j]);
    }while(i<j);
    swap(A[l], A[j]);
    return j;
}
```

lowest index (0) — highest index (8) in this example

```
void QuickSort(int A[], int l, int h)
{   int j;
    if( l<h)
    {       j= Partition(A, l, h);
            QuickSort(A, l, j);
            QuickSort (A, l+1, h);
    }
}
```

Best Case : If partitioning is in middle
O(nlogn)

Worst Case : If partioning is on any end
O(n^2)

Avg case : O(nlogn)

- best case : Sorted list
  (first make middle element as pivot)
- worst case : partitioning on any end
  O(n^2)

8. **SHELL SORT**

Used for large arrays

```
void ShellSort( int A[],int n)
{
    int gap, i, j, temp;

    for( gap = n /2 ; gap >= 1 ; gap /= 2)
    {
        for (i= gap ; i<n ; i++)
        {
            temp = A[i];
            j= i - gap;

            while( j >= 0 && A[j] > temp)
            {
                A[j + gap] = A[j];
                j = j - gap;
            }

            A[j+gap] = temp;
        }
    }
}

void main()
{
    int A[] = { 11, 13 , 7, 12, 16, 9 , 24 , 5 , 10 , 3 }, n=10 ;

    ShellSort (A , n);
}
```

9. COUNT SORT

A

3	6	8	8	10	12	15	15	15	20
0	1	2	3	4	5	6	7	8	9

C

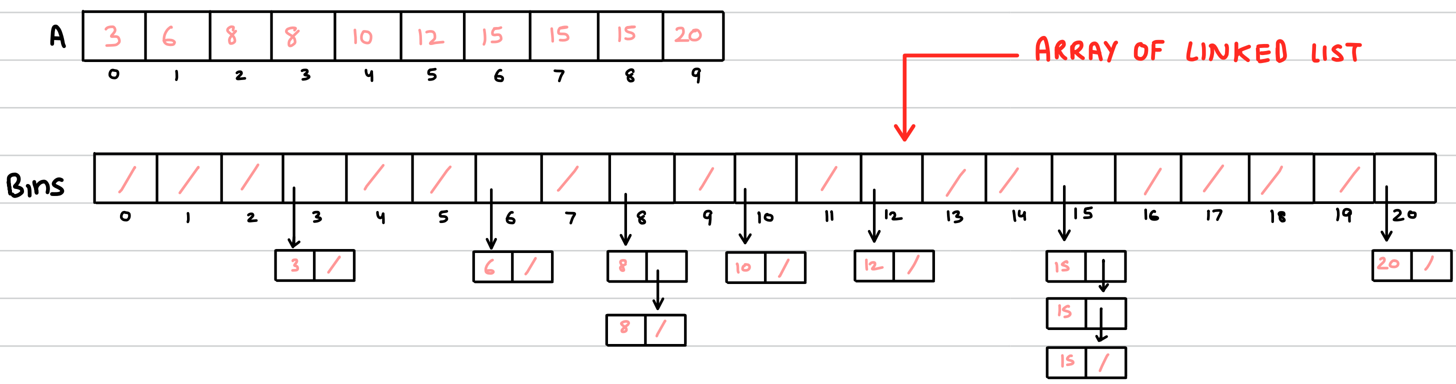
0	0	0	$\cancel{0}$ 1	0	0	$\cancel{0}$ 1	0	$\cancel{0}$ 2	0	$\cancel{0}$ 1	0	$\cancel{0}$ 1	0	0	$\cancel{0}$ 3	0	0	0	0	$\cancel{0}$ 1
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Fastest sorting method but high memory consumption

```
void CountSort(int A[], int n)
{
    int max, i, j;
    int *c;

    max = findMax(A, n);
    C = new int[max+1];              c = (int *) malloc (sizeof(int) * (max+1));

    for(i=0; i<max+1; i++)           n
        c[i] = 0;

    for(i=0; i<n; i++)               n
        C[A[i]]++;

    i=0, j=0;

    while(i<max+1)                   n
    {
        if(c[i]>0)
        {
            A[j++] = i;
            c[i]--;
        }

        else
            i++;
    }
}
```

O(n)

10. BUCKET / BIN SORT




```
void BinSort(int A[], int n)
{
    int max, i, j;
    max = findMax(A, n);
    Bins = new Node*[(max+1)];

    for (i=0; i<max+1; i++)
        Bins[i] = NULL;

    for (i=0; i<n; i++)
        Insert(Bins[A[i]], A[i]);      // Insert at end of linked list

    i=0, j=0;

    while (i < max+1)
    {
        while (Bins[i] != NULL)
        {
            A[j++] = Delete(Bin[i]);     O(n)
        }
        i++;
    }
}
```

11. RADIX SORT

1st Pass

Elements arranged on basis of one's digit

A

0	1	2	3	4	5	6	7	8	9
237	146	259	348	152	163	235	48	36	62

Bin ← ARRAY OF LINKED LIST

0	1	2	3	4	5	6	7	8	9
/	/	152 62	163	/	235	146 36	237	348 48	259

A

0	1	2	3	4	5	6	7	8	9
152	62	163	235	146	36	237	348	48	259

// Take out the elements in FIFO way and store them

2nd Pass

two's digit

Bin

0	1	2	3	4	5	6	7	8	9
/	/	/	235 36 237	146 348 48	152 259	62 163	/	/	/

A

0	1	2	3	4	5	6	7	8	9
235	36	237	146	348	48	152	259	62	163

3rd Pass

three's digit

Bin

0	1	2	3	4	5	6	7	8	9
36 48 62	146 152 163	235 237 259	348	/	/	/	/	/	/

A

0	1	2	3	4	5	6	7	8	9
36	48	62	146	152	163	235	237	259	348

two digit number's third digit considered as 0

Decimal numbers require 10 sized bin (0 - 9)

Binary numbers require 2 sized bin (0, 1)

Octal numbers require 8 sized bin (0-7)

1. [A[i] / 1] % 10
2. [A[i] / 10] % 10
3. [A[i] / 100] % 10

O(n) and minimum storage required

Storage depends on maximum number of digits of largest element

HASHING TECHNIQUE

• **Why hashing?**

There are three methods of searching

1. Linear Search – O(n)
2. Binary Search – O(logn) // But in binary search, elements must be arranged in sorted order, so we have to do some extra work
3. Hashing – O(1)

→ It is the fastest method, but requires very large space as it depends on maximum element

DRAWBACK

A

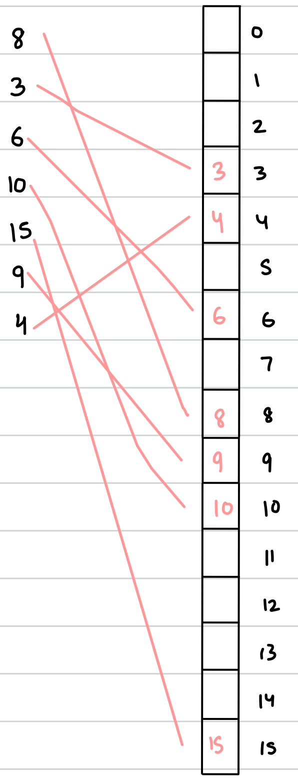
3	6	8	8	10	12	15	15	15	20
0	1	2	3	4	5	6	7	8	9

C

0	0	0	3	0	0	6	0	8	0	10	0	12	0	0	15	0	0	0	0	20
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

• **Ideal Hashing**

$h(x) = x$ hash function



Mapping

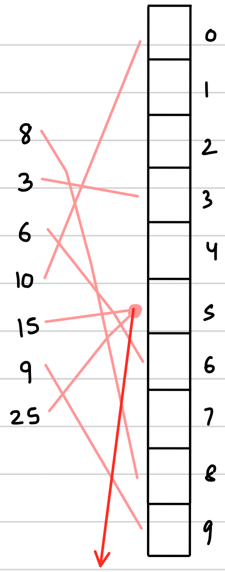
✓ One to one func

one to many

✓ many to one func

many to many

$h(x) = x \% 10$ gives remainder many-one



COLLISION

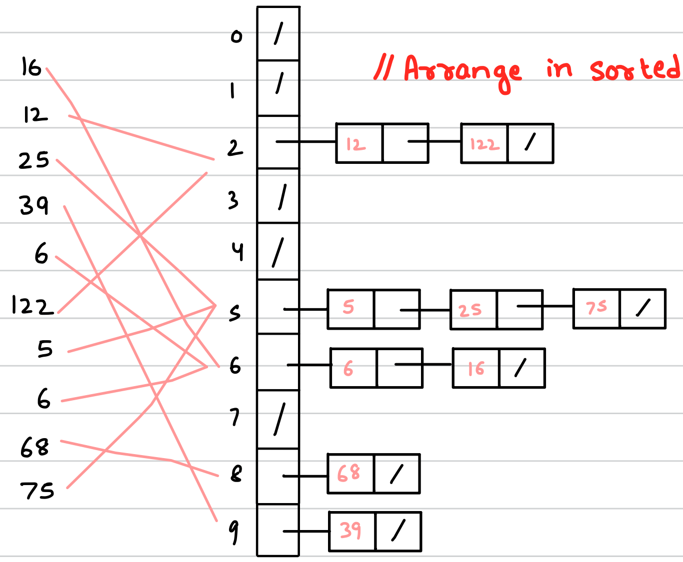
- **How to avoid COLLISION?**

Open Hashing // No space restriction
- Chaining

Closed Hashing // Limited Space
- Open Addressing // If collision, store element elsewhere in free space
  1. Linear Probing
  2. Quadratic Probing
  3. Double Hashing

  Methods for storing in free space

- **CHAINING**



Arrange on basis of first digit

$h(x) = x \% 10$ ← Hash Function

$n = 100$	No of keys
$size = 10$	No of index
$\lambda = \frac{n}{size}$	LOADING FACTOR

Average Successful Search

$$t = 1 + \frac{\lambda}{2}$$

Average Unsuccessful Search

$$t = 1 + \lambda$$

If keys are: 5, 35, 95, 145, 175, 265, 845

Then modify your hash function

PROGRAM

```c
void Insert(struct Node *H[], int key)
{
        int index = hash(key);
        SortedInsert(&H[index], key);
}

int hash(int key)
{
        return key % 10;
}

void SortedInsert(struct Node **H, int x)
{
      struct Node *t, *q = NULL, *p = *H;
      t = (struct Node *)malloc(sizeof(struct Node));
      t -> data = x;
      t -> next = NULL;

      if(*H == NULL)
            *H = t;
      else
      {
            while(p && p -> data < x)
            {
                  q = p;
                  p = p -> next;
            }

            if(p == *H)
            {
                  t -> next = *H;
                  *H = t;
            }
            else
            {
                  t -> next = q -> next;
                  q -> next = t;
            }
      }
}
```

```c
struct Node
{
      int data;
      struct Node *next;
};
```

```
void main()
{
    struct Node *HashTable[10], *temp;
    int i;

    for(i=0; i<10; i++)
        HashTable[i]= NULL;

    Insert (HashTable, 12);
    Insert (HashTable, 22);
    Insert (HashTable, 42);

    temp = Search (HashTable[hash(21)], 21);
}


struct Node *Search (struct Node *p, int key)
{
    while(p != NULL)
    {
        if (key == p->data)
            return p;
        else
            p = p->next;
    }
    return NULL;
}
```

LINEAR PROBING

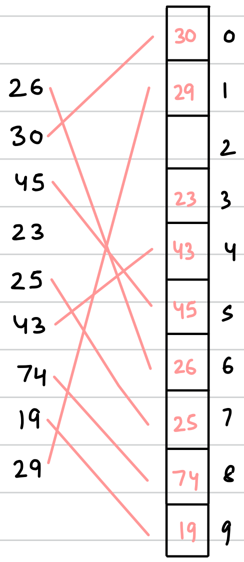
$h(x) = x \% 10$

$h'(x) = (h(x) + f(i)) \% 10$ // For probing

$f(i) = i$

where $i = 0, 1, 2 \ldots$

In case of collision, insert at next free space



$h'(25) = (h(25) + f(0)) \% 10$
$= (5 + 0) \% 10 = 5$

$h'(25) = (h(25) + f(1)) \% 10$
$= (5 + 1) \% 10 = 6$

$h'(25) = (h(25) + f(2)) \% 10$
$= (5 + 2) \% 10 = 7$

$h'(29) = (h(29) + f(0)) \% 10$
$= (9 + 0) \% 10 = 9$
$= (9 + 1) \% 10 = 0$
$= (9 + 2) \% 10 = 1$

Cyclic behaviour

For searching :

Key : 45 ✓ — $45 \% 10 = 5$ — found at index 5

Key : 74 ✓ — $74 \% 10 = 4$
- Not found at index 4
- Search Next index until 74 is found or blank space is found
- Found at index 8

// Search takes more than constant time

Key : 40 ✗ — $40 \% 10 = 0$
- Not found at index 0
- At index 2, blank space found
- Element does not exist

ANALYSIS

$\lambda = \frac{n}{size} = \frac{9}{10} = 0.9$

DRAWBACK

** $\lambda \leq 0.5$ // Table should be atmost half filled so, there are blank spaces and searching can be made faster

Average Successful Search

$t = \frac{1}{\lambda} \ln \left(\frac{1}{1-\lambda} \right)$

Average Unsuccessful Search

$t = \frac{1}{1-\lambda}$

DELETION IS NOT EASY IN LINEAR PROBING

PROGRAM

```
int hash( int key)
{
    return key % 10 ;
}

int probe ( int H[], int key)
{
    int index = hash (key) ;
    int i= 0 ;

    while ( H[ (index +i ) % 10] ! = 0)
        i++ ;
    return (index +i) % 10 ;
}

void Insert ( int H[] , int key)
{
    int index = hash (key);
    if ( H[ index] ! = 0)
        index = probe (H , key);
    H[index] = key ;
}

int Search (int H[], int key)
{
    int index = hash (key);
    int i=0 ;

    while ( H[(index +i) % 10] ! = key)
        i++ ;
    return (index + i) % 10 ;
}

void main()
{
    struct Node * HashTable [10], *temp ;
    int i;

    for (i = 0 ; i < 10 ; i++)
        HashTable [i] = NULL;

    Insert (HashTable , 12) ;
    Insert (HashTable , 22) ;
    Insert (HashTable , 42) ;

    temp = Search ( HashTable , 35) ;
}
```

Quadratic Probing

The drawback of linear probing is that elements cluster together and form a group. To resolve this issue, quadratic probing is introduced.

$h'(x) = (h(x) + f(i)) \% 10$ where $f(i) = i^2$

$i = 0, 1, 2 \ldots$

$h'(23) = (h(13) + f(0)) \% 10$

$= (3 + 0) \% 10 = 3$

$h'(43) = (h(43) + f(0)) \% 10$

$= (3 + 0) \% 10 = 3$

$= (3 + 1) \% 10 = 4$

$h'(13) = (h(13) + f(2)) \% 10$

$= (3 + 4) \% 10 = 7$

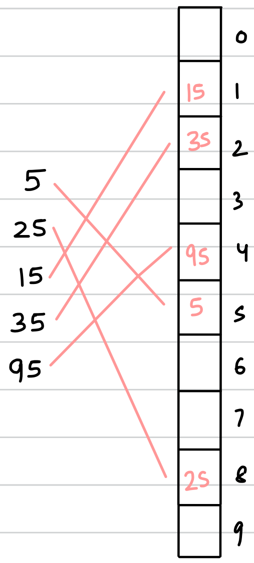
Average Successful Search

$$\frac{-\log_e(1-\lambda)}{\lambda}$$

Average Unsuccessful Search

$$\frac{1}{1-\lambda}$$

Double Hashing



$h1(x) = x \% 10$

$h2(x) = R - (x \% R)$

$h'(x) = (h1(x) + i * h2(x)) \% 10$

where R is prime num smaller than Size of Hash Table

where $i = 0, 1, 2 ..$

$h'(25) = (5 + 1 * 3) \% 10 = 8$

$7 - (25 \% 7)$

$7 - 4 = 3$

$h'(15) = (5 + 1 * 6) \% 10 = 1$

$7 - (15 \% 7)$

$7 - 1 = 6$

$h'(35) = (5 + 1 * 7) \% 10 = 2$

$7 - (35 \% 7)$

$7 - 0 = 7$

$h'(95) = (5 + 3 * 3) \% 10 = 4$

$7 - (95 \% 7)$

$7 - 4 = 3$

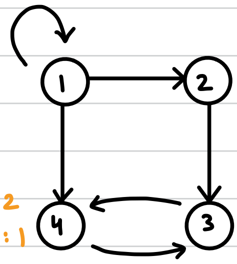
GRAPHS

$G = (V, E)$

$V =$ Set of vertices

$E =$ Set of edges

Self loop

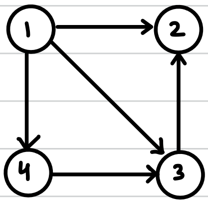


Indegree : 2
Outdegree : 1

Parallel Edges

Directed Graph

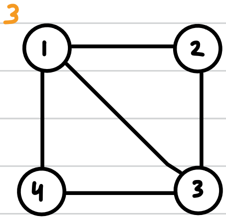
- Edges are having directions



Simple Diagraph

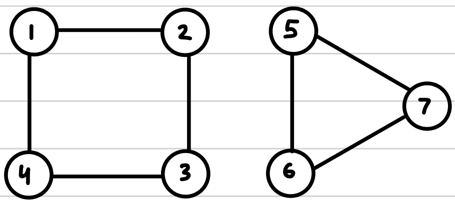
- Without self loop
- Without parallel edges

degree 3



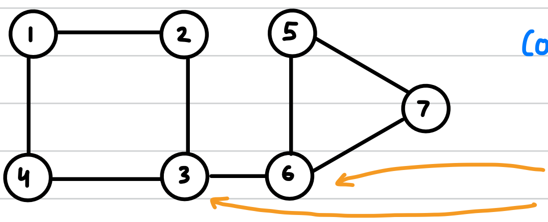
Graph / Non - directed Graph

- Undirected Edges



2 components

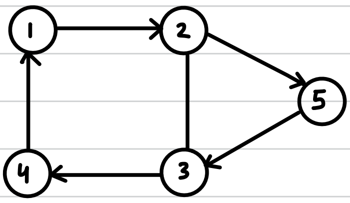
Non - connected Graph



Connected components

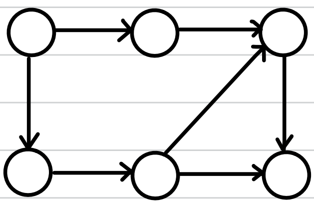
Articulation Points

- Those vertices whose removal will divide the graph into multiple components



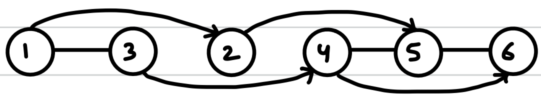
Strongly Connected Graph (Directed Graph)

- All vertices can be reached from any vertex
- There is a path between every pair of vertices
- **Path** is set of vertices which are connecting pair of vertices
- **Cycle** is a circular path that is starting from same vertex and ending at same vertex



Directed Acyclic Graph (DAG)

- Directed Graph
- with no cycles
- These can be arranged linearly such that edges are going in only forward direction

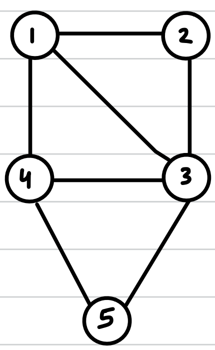


Topological Ordering

REPRESENTATION OF UNDIRECTED GRAPH

(1) Adjacency Matrix
(2) Adjacency List
(3) Compact List

ADJACENCY MATRIX



	1	2	3	4	5
1	0	1	1	1	0
2	1	0	1	0	0
3	1	1	0	1	1
4	1	0	1	0	1
5	0	0	1	1	0

5 x 5

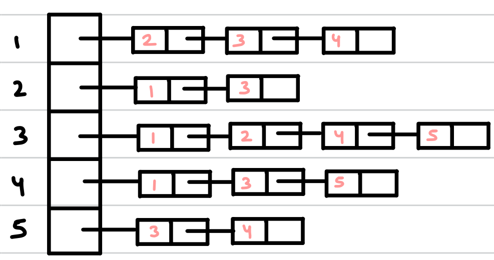
$|V| = n = 5$

$|E| = e = 7$

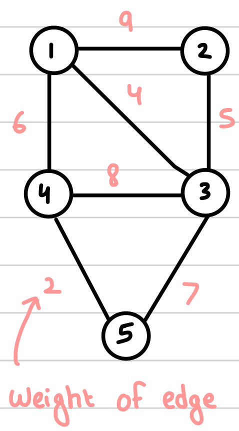
(i, j)

$A[i][j] = 1$

ADJACENCY LIST



COST ADJACENCY MATRIX (Representation of weight)



	1	2	3	4	5
1	0	9	4	6	0
2	9	0	5	0	0
3	4	5	0	8	7
4	6	0	8	0	2
5	0	0	7	2	0

5 x 5

COST ADJACENCY LIST

Add another area to node for weight of edge

COMPACT LIST

Ending point of last vertex

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
~~ ~~	7	10	12	16	19	21	2	3	4	1	3	1	2	3	4	1	3	5	3	4
	Vertices						1			2		3				4			5	

Starting indexes

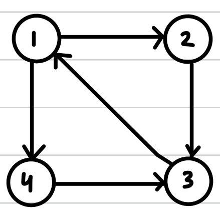
$|V| + 2|E| + 1$

$5 + 2 \times 7 + 1 = 20$

$20 + 1 = 21$

REPRESENTATION OF DIRECTED GRAPH

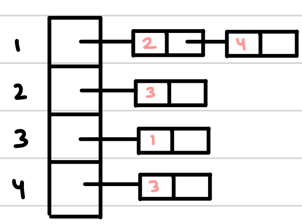
ADJACENCY MATRIX



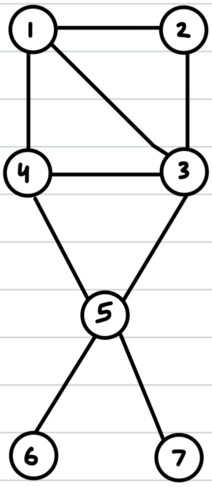
	1	2	3	4
1	0	1	0	1
2	0	0	1	0
3	1	0	0	0
4	0	0	1	0

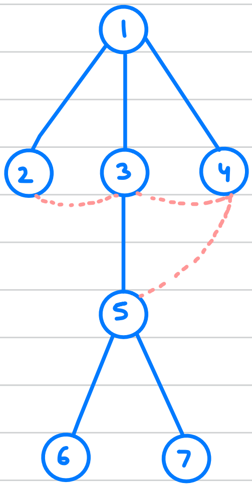
4 x 4

ADJACENCY LIST

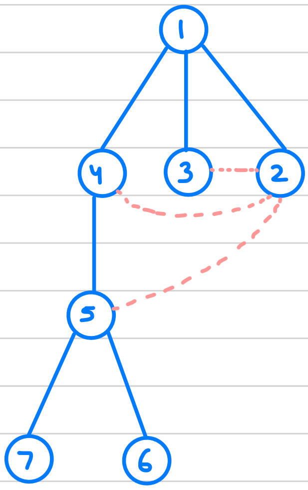


BREADTH FIRST SEARCH (BFS)





TREE 1



TREE 2

Multiple trees can be generated from the same graph

- We can start from any vertex
- Each vertex should be explored fully
- Dotted pink lines represent edges that make a complete cycle called cross edges
- These trees are called BFS Spanning Trees

BFS TREE 1 : 1, 2, 3, 4, 5, 6, 7 // Same as Level order Traversal in trees

BFS TREE 2 : 1, 4, 3, 2, 5, 7, 6

Starting vertex

$O(n^2)$

```
void BFS (int i)
{
    int u, v;
    printf("%d", i);
    visited[i] = 1;
    enqueue(q, i);

    while (!isEmpty(q))
    {
        u = dequeue(q);
```

Take out vertex from queue

For scanning row of matrix

Should not be visited

```
        for (v = 1; v <= n; v++)
        {
            if (A[u][v] == 1 && visited[v] == 0)
            {
                printf("%d", v);
                visited[v] = 1;
                enqueue(q, v);
            }
        }
    }
}
```

A	0	1	2	3	4	5	6	7
0								
1		0	1	1	1	0	0	0
2		1	0	1	0	0	0	0
3		1	1	0	1	1	0	0
4		1	0	1	0	1	0	0
5		0	0	1	1	0	1	1
6		0	0	0	0	1	0	0
7		0	0	0	0	1	0	0

QUEUE q

visited

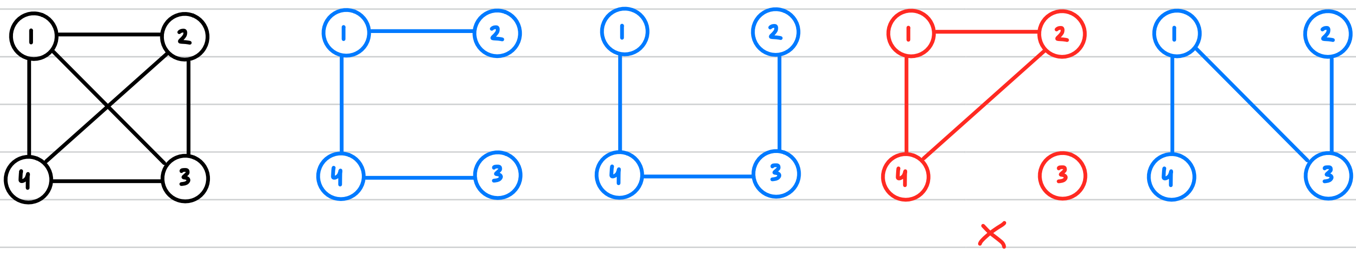
0	1	2	3	4	5	6	7
0	1	1	1	1	0	0	0

DEPTH FIRST SEARCH (DFS)

```
void  DFS (int u)
{
      int  v;

      if ( visited [u] == 0)
      {
            printf (" %d", u);
            visited [u] = 1;

            for ( v = 1 ; v <= n; v++)
            {
                  if ( A [u] [v] == 1 && visited [v] ! = 0)
                        DFS (v);
            }
      }
}
```

SPANNING TREES

Spanning tree is a sub graph of a graph having all vertices of a graph and |V|-1 edges and there should not be any cycle.



$|V| = 4$

$|E| = 6$

$^{|E|}C_{|V|-1} = {}^{6}C_{3}$ ways or number of spanning trees

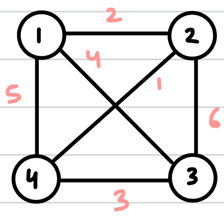
but not include trees forming cycles

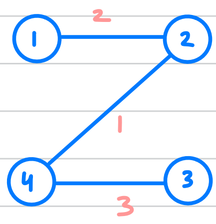
$^{|E|}C_{|V|-1} - \text{cycles} = {}^{6}C_3 - 4$ // In this example

$= 16$

MINIMUM COST SPANNING TREE

If weights are added to edges of graph, then the spanning tree with minimum cost of weights is called minimum cost spanning tree.





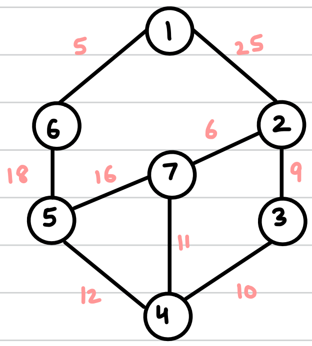
= 6

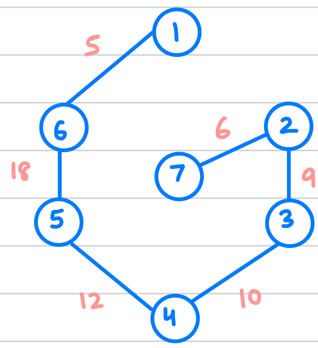
PRIM'S MINIMUM COST SPANNING TREE

(|V| − 1) |E|

ne = n x n

O (n²)



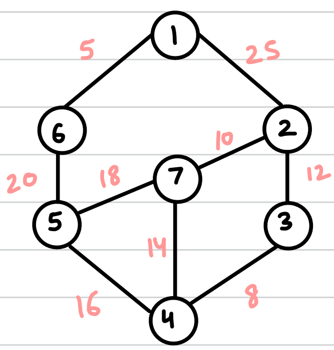


STEP 1: Select the edge with least weight along with its vertices

STEP 2: Then compare the edges joined with the vertices and select the one with least weight along with its vertex

STEP 3: Repeat step 2 until spanning tree is obtained

PROGRAM



COST	0	1	2	3	4	5	6	7
0	-	-	-	-	-	-	-	-
1	-	-	25	-	-	-	5	-
2	-	25	-	12	-	-	-	10
3	-	-	12	-	8	-	-	-
4	-	-	-	8	-	16	-	14
5	-	-	-	-	16	-	20	18
6	-	5	-	-	-	20	-	-
7	-	-	10	-	14	18	-	-

~~-~~	-	-	-	-	-	-	-
0	1	2	3	4	5	6	7

near

t						
0						
1						
	0	1	2	3	4	5

// Only either of lower or upper triangular matrix will be sufficient

Largest possible integer value

Initialize as {I}

Initialize as the given matrix (Replace '–' with 'I')

INITIAL PROCEDURE

For accessing upper triangular matrix only

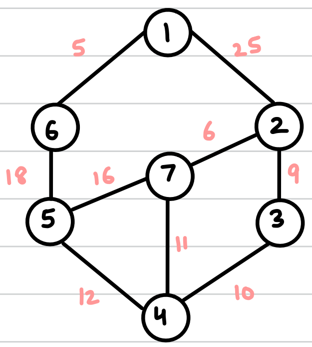
```
#define  I   32767

int  cost[8][8], near[8], t[2][6];

void  main()
{
    int  i, j, k, u, v, n=7, min = I;

    for (i=1; i<=n; i++)
    {
        for (j=i; j<=n; j++)
        {
            if (cost[i][j] < min)
            {
                min = cost[i][j];
                u=i, v=j ;
            }
        }
    }

    t[0][0] = u;
    t[1][0] = v;

    near[u] = near[v] = 0;

    for (i=1; i<=n ; i++)
    {
        if (near[i]!=0 && cost[i][u] < cost[i][v])
                near[i]=u;
        else
                near[i]=v;
    }
```

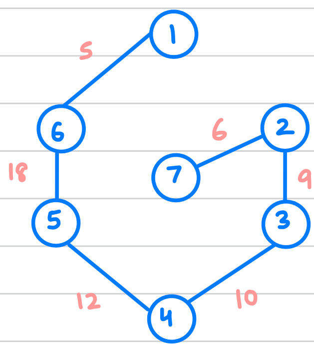
RECURSIVE PROCEDURE

```
    for (i= 1; i<n-1; i++)
    {
        min = I ;

        for (j=1; j<=n ; j++)
        {
            if (near[j]!=0 && cost[j][near[j]] < min)
            {
```

```
                min = cost[j][near[j]],
                k = j;
            }
     }

     t[0][i] = k;
     t[1][i] = near[k];
     near[k] = 0;

     for (j= 1; j<=n; j++)
     {
          if ( near[j]! = 0 && cost[j][k] < cost[j][new[j]])
                near[j] = k;
     }
  }

  Print t;
}
```

KRUSKAL'S METHOD





STEP 1 : Select the edge with least weight along with its vertices

STEP 2 : Select the next minimum edge if it is not forming a cycle

DISJOINT SUBSET

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

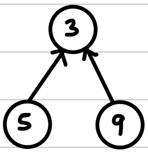
Consider its two subsets

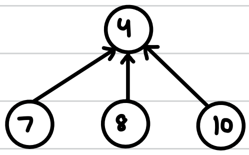
$A = \{3, 5, 9\}$

$B = \{4, 7, 8, 10\}$

$A \cap B = \phi$ // Disjoint subset

$A = \{3, 5, 9\}$ $B = \{4, 7, 8, 10\}$





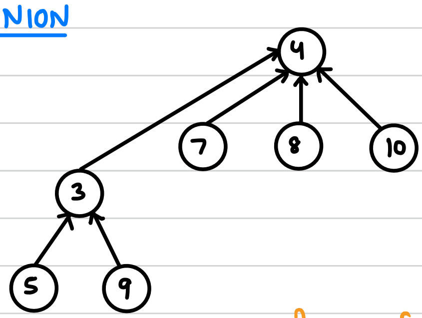
			-3	-4	3		4	4	3	4
0	1	2	3	4	5	6	7	8	9	10

No of nodes in negative
(4 is having 4 nodes)

Rest are representing parent

Representation of set

(1) UNION



The parent with more number of nodes

			4	-7	3		4	4	3	4
0	1	2	3	4	5	6	7	8	9	10

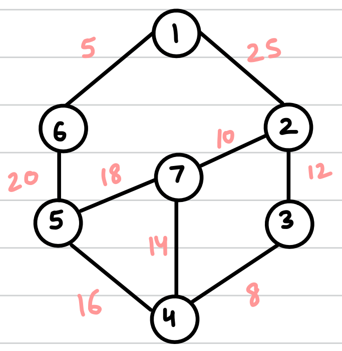
Parent of nodes

```
void Union (int u, int v)
{
    if ( s[u] < s[v] )
    {
        s[u] = s[u] + s[v];
        s[v] = u;
    }
    else
    {
        s[v] = s[u] + s[v];
        s[u] = v;
    }
}
```

(2) FIND

```
int Find (int u)
{
    int x = u;

    while ( s[x] > 0)
        x = s[x];

    while ( u != x)
    {
        v = s[u];
        s[u] = x;
        u = v;
    }
}
```

Connecting vertices directly to parent



	0	1	2	3	4	5	6	7	8
0	1	1	2	2	3	4	4	5	5
1	2	6	3	7	4	5	7	6	7
2	25	5	12	10	8	16	14	20	18

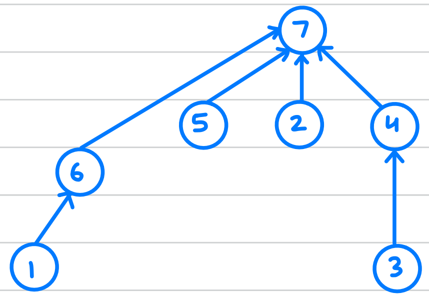
edges

x	6	7	4	7	7	-2	-5
0	1	2	3	4	5	6	7

set

0	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8

included



t

	0	1	2	3	4	5
0	1	3	2	2	4	5
1	6	4	7	3	5	6

```
#define   I   32767

int  edges[3][9] = { { 1,1, 2,2 ,3,4,4, 5,5},
                     { 2,6, 3,7 ,4,5,7,6, 7},
                     { 25,5,12,10,18,16,14,20,18}};

int   set[8] = {-1};
int   included[9] = {0} , t[2][6];


void main()
{
      int  i=0,j,k,n=7,e=9,min,u,v;
                         <--  No of vertices -1
      while (i < n-1)
      {
            min = I;
                              For finding minimum cost edge
            for(j=0;j<e;j++)  <-
            {
                  if ( included[j]==0   && edges[2][j] <min)
                  {
                        min = edges[2][j];
                        k = j ;
                        u = edges[0][j];
                        v = edges[1][j];
                  }
            }
```

```
if ( find (u) != find (v))
{
        t[0][i] = u ;
        t[1][i] = v ;
        union ( find (u) , find (v) );
        i++ ;
}

included [k] = 1;
    }
}
```

To check if cycle is forming or not

ASYMPTOTIC NOTATION

$f(n) = \sum_{i=1}^{n} i \;-\; 1+2+3 \ldots\ldots +n = \frac{n(n+1)}{2} = O(n^2)$ ← We can get exact value

$f(n) = \sum_{i=1}^{n} i \times 2^i$ Exact value is not possible that is simplified

So, we use asymptotic notations

Lower Bound	Ω	Omega	less than or equal to
Upper Bound	O	Big O	Greater than or equal to
Tight Bound	θ	Theta	equal