Faculty of Engineering Science

Denoising and inpainting with wavelets

Wavelets with application in signal and Image processing

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1 Wavelet-based denoising

1.1 A univariate functions with noise

1.1.1 Question 2.1

Function being sampled, $N = 1000$, between $[-2, 2]$

$$f(x) = (2 + \cos(x))|x| \operatorname{sign}(x - 1)$$

Tested for wavelet transform of 4 levels deep, using the Daubechies 2

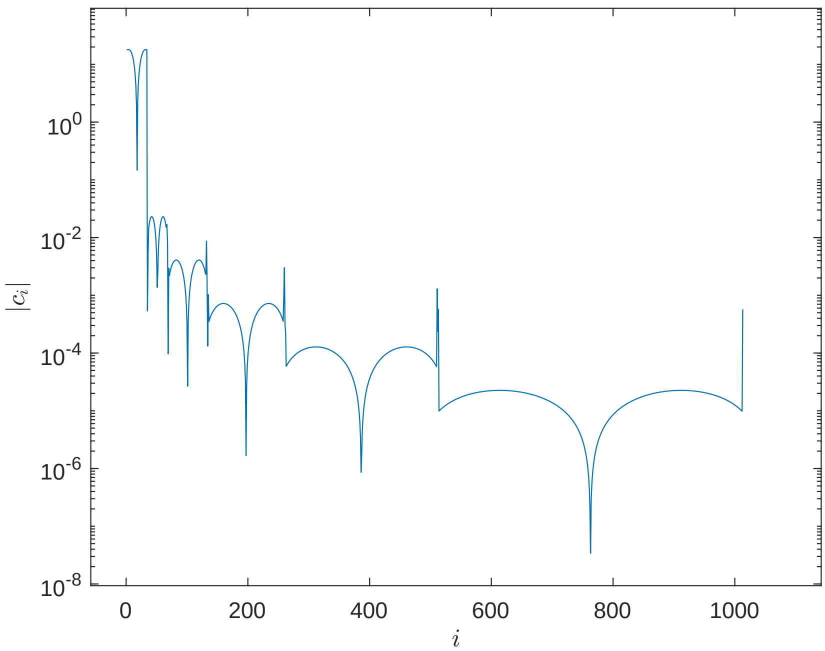


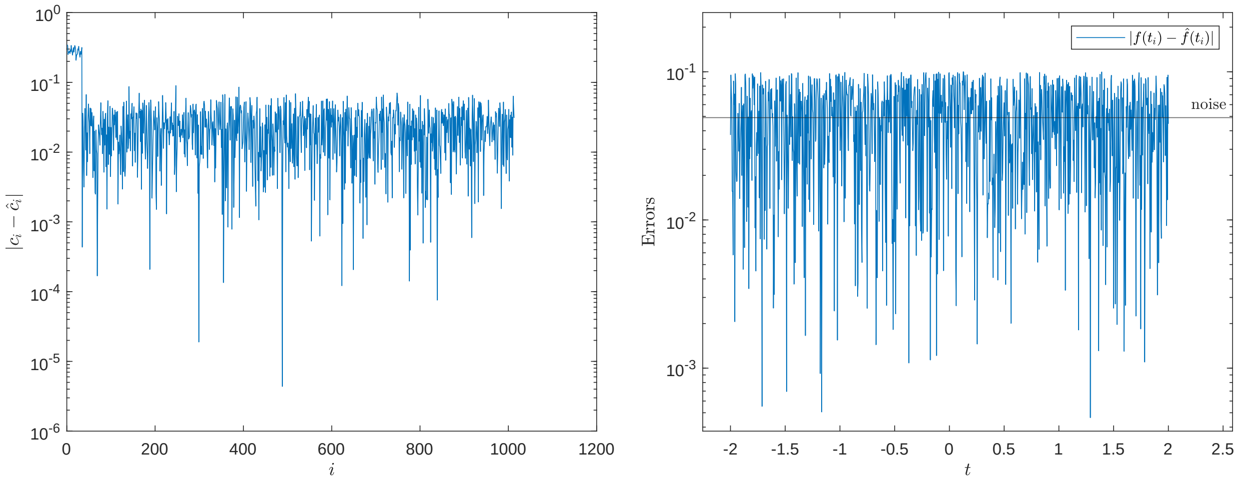
Figure 1: Coefficients of Wavelet transform (4 levels deep, using the Daubechies 2, $N = 1000$ sample points, between $[-2, 2]$

We can see in fig. 1, that the size of the coefficients decreases as i increases. The meaning of this is that the coefficients of small size are of less importance to the reconstruction of the signal, and can be more easily effected by added noise.

1.1.2 Task 2.2

Noise is added to the signal $\tilde{f}_i = f_i + \epsilon\mathcal{U}(0, 1)$ ($\hat{f}$ is the reconstructed $\hat{f}$). We chose $\epsilon =$ `1e-1`. First let us choose $\delta = 0$, the results are in fig. 2. We can see in fig. 2a, that the highest error is in the first few coefficients. Which makes sense since we are in some sense moving the mean of the function f by adding a value sampled from the uniform distribution, $\rightarrow$ lower frequencies should be more effected. The total error $E = \|f - \hat{f}\|_2 = 1.805$.

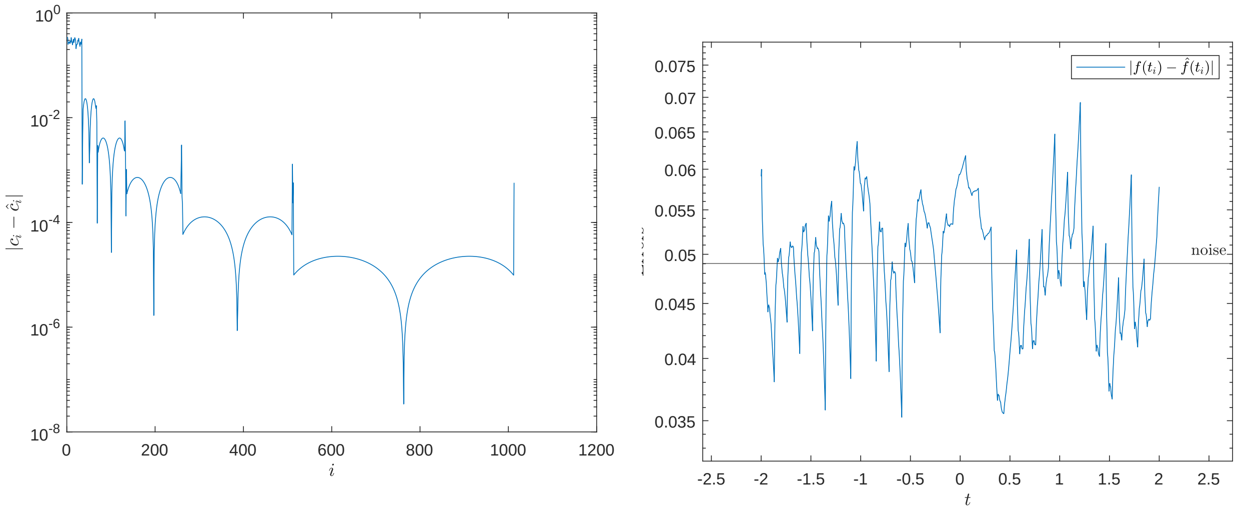
Now let us set $\delta =$ `1e-1` (notice it is the order of the error), the resulting figures are fig. 3. $E = 1.5643$, which is indeed lower then the previous one. This seems manly due to the fact that $\hat{f}$ is smoother then $\tilde{f}$, see fig. 3b. If knowledge of the mean of the noise is available, we can come put a better reconstruction of f by $\hat{f} + mean(noise)$, the resulting $E = 0.1982$, see fig. 4, which has the same order as ϵ.



(a) The error between the real coefficients and noisy ones

(b) The error between real signal and noisy one

Figure 2: Plots for $\delta = 0$, black line is mean of noise



(a) The error between the real coefficients and reconstructed ones

(b) The error between real signal and reconstructed one

Figure 3: Plots for $\delta = 0.1$, black line is mean of noise

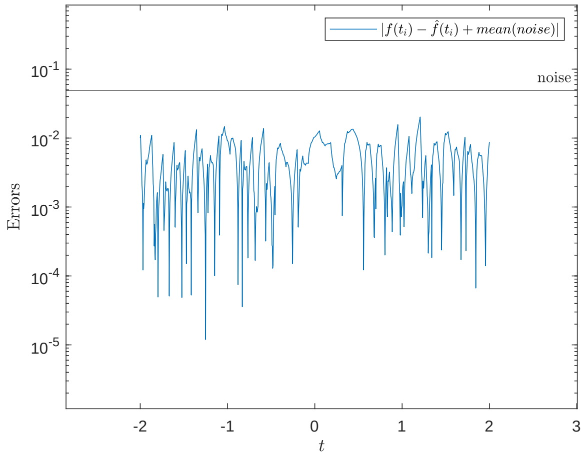


Figure 4: The error between real signal and reconstructed one, black line is mean of noise

1.1.3 Question 2.3

Now let us try to find the best parameter δ in order to minimize the noise. This will be done by simply checking MSE between the real and filtered coefficients, for different δ's. The results are shown in fig. 5, the best delta according to MSE is $\delta = 0.501$. The reconstructed is function does have less error then the noisy one, however it is still near the order of the noise. The error is especially noticeable for $x = 0$, probably due to the fact that f is not differenctiable there $|x|$. The same should be said for $x = 1$ because of the sign but it is not the case.

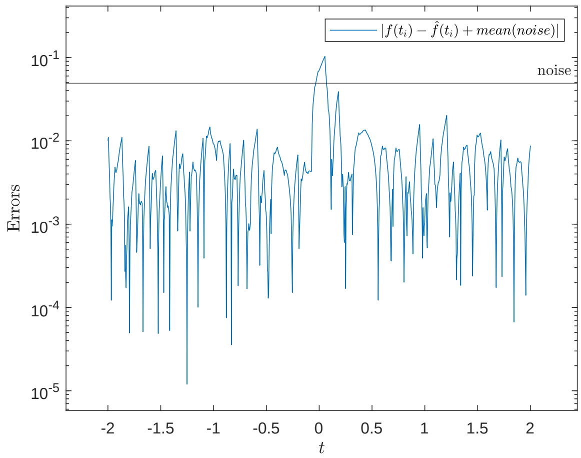


Figure 5: The error between real signal and reconstructed one, black line is mean of noise

1.2 Images with noise

1.2.1 Task 2.4

1.2.2 Question 2.5

1.3 Using a redundant wavelet transform

1.3.1 Task 2.6

1.3.2 Question 2.7

1.3.3 Question 2.8

1.3.4 Task 2.9

2 Wavelet-based inpainting

2.1 An iterative algorithm

2.1.1 Task 3.1

2.1.2 Question 3.2

2.1.3 Question 3.3

2.1.4 Question 3.4