DS 288 (AUG) 3:0 Numerical Methods
Assignment–1 [1]

Due date: Sep 16, 2018
Instructor: Prof. Sashikumaar Ganesan

1. The modes of a system are described by the Bessel functions $J_i(x)$ for i = 1,2,..., n. As a numerical methods expert, your job is to compute these Bessel functions using the recurrence relation
$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x) \tag{1}$$
   (a) Compute the recursion in the forward direction, i.e., compute $J_2(x)$ from $J_1(x)$ and $J_0(x)$ with starting values taken from the table-1. Use only the first 6 digits given in the table for each quantity ($e.g.$, $J_0(1) = 7.65198e-01$) when supplying the starting values to your program. For x = 1,5, and 50, how accurate is $J_{10}(x)$? Compute both the absolute and relative errors of these values taking the tabulated values (table-1) as truth. [1 point]

   (b) Compute the recursion backward, i.e. start with $J_{10}(x)$ from $J_9(x)$ compute $J_8(x)$. Again use only first 6 digits and for x = 1,5, and 50, how accurate is $J_0(x)$ in this backward approach? Compute both the absolute and relative errors of these values taking the tabulated values (table-1) as truth. Is the last value computed by the recurrence relation is having less or more error compared to the forward approach? [1 point]

n	$J_n(1)$	$J_n(5)$	$J_n(50)$
0	7.6519768656e-01	-1.7759677131e-01	5.5812327669e-02
1	4.4005058574e-01	-3.2757913759e-01	-9.7511828125e-02
2	1.1490348493e-01	4.6565116278e-02	-5.9712800794e-02
3	1.9563353983e-02	3.6483123061e-01	9.2734804062e-02
4	2.4766389641e-03	3.9123236046e-01	7.0840977282e-02
5	2.4975773021e-04	2.6114054612e-01	-8.1400247697e-02
6	2.0938338002e-05	1.3104873178e-01	-8.7121026821e-02
7	1.5023258174e-06	5.3376410156-02	6.0491201260e-02
8	9.4223441726e-08	1.8405216655e-02	1.0405856317e-01
9	5.2492501799e-09	5.5202831385e-03	-2.7192461044e-02
10	2.6306151237e-10	1.4678026473e-03	-1.1384784915e-01

Table 1: Bessel functions of integer order (n = 0-10) for x = 1, 5, and 15.

2. Using Newton's method, Secant method, and Modified Newton's method, find the solution of $f(x) = 0$ for the functions listed below. Iterate until you reach a relative tolerance of 10^{-6} between successive iterates. Report the root found and the number of iterations needed for each method.
(a) $f(x) = x \sin x + 3 \cos x - x$, find root(s) in the interval $(-6, 6)$.
(b) $f(x) = \sin x - 0.1x$, find all positive, nonzero roots.
Comment on the observed convergence rates in these cases. Does your results agree with the analysis did in the class? [2 points]

[1] Posted on: September 10, 2018.

3. Develop the functional form for a cubicly convergent fixed point iteration function $g(p_n)$ to solve the problem $f(x) = 0$ by writing

$$g(x) = x - \phi(x)f(x) - \psi(x)f^2(x)$$

and determining $\phi(x)$ and $\psi(x)$. Specify the asymptotic order of convergence (α) and write the asymptotic error constant (λ). Write all expressions in terms of $f(p)$ and its derivatives and *simplify* your answers. You are allowed to scan the hand-written derivation for this part alone.
Hint: Extend the approach we used in class to derive Newton's method. The scheme you will produce is often referred to as "Cubic Newton's Method". [`2 points`]

4. The equations

$$\begin{aligned} \sin x + 3\cos y &= 2 \\ \cos x - \sin y &= -0.2 \end{aligned}$$

have a solution in the vicinity of the point $(1, 1)$. Refine the solution using Newton's method for the system. Compute to a relative tolerance of 10^{-8} and report the number of iterations required to reach this level of convergence. [`2 points`]