

CS 663: Assignment 5

Question 1.

d) Given g_1 and g_2 and assuming h_1 and h_2 are known, derive a formula to determine f_1 and f_2 assuming that there was no relative motion between the camera and the scene outside while the two pictures are being acquired and there is no change in the scene.

$$\begin{aligned} A) \quad g_1(x, y) &= f_1(x, y) + h_2 * f_2(x, y) \quad \dots (1) \\ g_2(x, y) &= h_1 * f_1(x, y) + f_2(x, y) \quad \dots (2) \end{aligned}$$

Taking Fourier Transforms of (1) and (2) we get and using convolution th^m we get,

$$G_1(u, v) = F_1(u, v) + H_2(u, v) \cdot F_2(u, v) \quad \dots (3)$$

$$G_2(u, v) = H_1(u, v) F_1(u, v) + F_2(u, v) \quad \dots (4)$$

Multiplying (3) by $H_1(u, v)$ and subtracting (4) from it, we get

$$H_1(u, v) G_1(u, v) - G_2(u, v) = H_1(u, v) H_2(u, v) F_2(u, v) - F_2(u, v)$$

$$\therefore \hat{F}_2(u, v) = \frac{G_2(u, v) - H_1(u, v) G_1(u, v)}{1 - H_1(u, v) H_2(u, v)}$$

[Here while dividing by $(1 - H_1(u, v) H_2(u, v))$ we need to check if it is not zero; will be used later] ;

Now taking the inverse fourier transfer of $\hat{F}_2(u, v)$, we get

$$f_2(x, y) = F^{-1}(\hat{F}_2(u, v))$$

Similarly we can get f_1 .

$$\hat{F}_1(u, v) = \frac{G_1(u, v) - H_2(u, v) G_2(u, v)}{1 - H_2(u, v) H_1(u, v)}$$

Taking IFT, we get

$$f_1(x, y) = F^{-1}(\hat{F}_1(u, v))$$

Note:- Even with all these assumptions; there is something inherently problematic.

We know that h_1 and h_2 are blur ~~filter~~ kernels and hence H_1 and H_2 are low pass filters; \therefore at low frequencies both H_1 and H_2 will tend to 1. Hence $H_1(u, v) \cdot H_2(u, v)$ will tend to 1; and $1 - H_1(u, v) \cdot H_2(u, v)$ will tend to 0. Hence, we won't be able to extract the low frequency components ^{accurately} of $H_1(u, v)$ and $H_2(u, v)$ since this will blow up. And the low frequency components calculated inaccurately affects the overall perception of natural images, since low frequency components are perceived by the humans ~~as the base of~~ prominently.