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Machine Learning assignment

) a) Performance function:

$$f(K_x) = \lambda e^{-\lambda x_n}$$

Likelihood function:

$$\Rightarrow p(x) = \prod_{i=1}^{N} \lambda e^{-\lambda x_i}$$

Optimization function:

$$\log(p(x)) = \log\left(\prod_{i=1}^{N} \lambda e^{-\lambda x_i}\right)$$

$$\Rightarrow \sum_{i=1}^{N} \log(\lambda e^{-\lambda x_i})$$

$$\Rightarrow \sum_{i=1}^{N} \log(\lambda) + \sum_{i=1}^{N} \log(e^{-\lambda x_i})$$

$$\Rightarrow N \log \lambda + \sum_{i=1}^{N} -\lambda x_i$$

$$\Rightarrow N \log \lambda - \lambda \sum_{i=1}^{N} x_i$$

Differentiate w.r.t. λ

$$\frac{\partial}{\partial \lambda} = 0 \qquad \frac{N}{\lambda} - \sum_{i=1}^{N} x_i = 0$$

$$\Rightarrow \frac{N}{\lambda} - \sum_{i=1}^{N} x_i = 0$$

$$\Rightarrow \frac{N}{\lambda} = \sum_{i=1}^{N} x_i$$

$$\Rightarrow \frac{1}{\lambda} = \frac{\sum_{i=1}^{N} x_i}{N}$$

1b) $D = \{1.5, 3, 2.5, 2.75, 2.9, 3\}$

$$= 1.5+3+2.5+2.75+2.9+3 = 2.6083$$

$$\frac{1}{\lambda} = \frac{\sum_{i=1}^{N} x_i}{N} = 2.6083$$

$$\boxed{\lambda = \frac{1}{2.6083} = 0.383}$$

1c) MAP approach using Conjugate prior

$$P_{\alpha,\beta}(\lambda) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$$

$$\text{~~Posterior~~} = \frac{\text{Likelihood} \times \text{Prior}}{\text{Evidence}}$$

Posterior

Likelihood from 1ⓐ is $\lambda^{n} e^{(-\lambda \sum_{i=1}^{n} x_i)}$

$$P_{\alpha,\beta}(\lambda) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$$

Taking out the constant we get

$$P(\lambda) = \lambda^{\alpha-1} e^{-\beta\lambda}$$

$$P(\lambda|x) = \alpha \lambda^{n} e^{(-\lambda \sum_{i=1}^{n} x_i)} \cdot \lambda^{(\alpha-1)} e^{(-\beta\lambda)}$$

So, $P(\lambda|x) \alpha \text{ Gamma}\left[n+\alpha-1, \sum_{i=1}^{n} x_i + \beta\right]$

Maximum a Posterior Estimate (MAP):

Taking Log on both sides

$$\log(P(\lambda|x)) \alpha \quad -\lambda\left(\sum_{i=0}^{n} x_i + \beta\right) + (n+\alpha-1)\log\lambda$$

Differen ciate → set to zero →

$$0 = -\sum_{i=0}^{n} x_i - \beta + \frac{n+\alpha-1}{\lambda}$$

$$n+\alpha-1 = \lambda\left(\sum_{i=0}^{n} x_i + \beta\right)$$

$$\boxed{\lambda = \frac{n+\alpha-1}{\sum_i x_i + \beta}}$$

3 1 c) b. $\alpha = 5$ $\quad D = \{1.5, 3, 2.5, 2.75, 2.9, 3\}$

$\beta = 10$

$$\lambda = \frac{6+5-1}{15.65+10} = \frac{10}{25.65} = \underline{\underline{0.389}}$$

Machine Learning Assignment

2a) Given points: (155, 40, 35), (170, 70, 32), (175, 70, 35)
(180, 90, 20)

Case 1 (155, 40, 35)

$((170,57,32), W) = \sqrt{523} = 22.86$ |
$((192,95,28), M) = \sqrt{4443} = 66.65$ * *
$((150,45,30), W) = \sqrt{75} = 8.660$ W || |
$((170,65,29), M) = \sqrt{586} = 29.76$
$((175,78,35), M) = \sqrt{1844} = 42.94$
$((185,90,32), M) = \sqrt{3409} = 54.386$
$((170,65,28), W) = \sqrt{899} = 29.98$
$((155,48,31), W) = \sqrt{80} = 8.944$ || |
$((160,55,30), W) = \sqrt{275} = 16.583$ |
$((182,80,30), M) = \sqrt{2354} = 48.518$
$((175,69,28), W) = \sqrt{1290} = 35.916$
$((180,80,27), M) = \sqrt{2289} = 47.843$
$((160,50,31), W) = \sqrt{141} = 11.874$ || |
$((175,72,30), M) = \sqrt{1449} = 38.065$

k = 3 W W W | K = 5 W W W W W

Case 2: $(170, 70, 32)$

$((170,57,32), W) = \sqrt{169} = 13$

$((192,95,28), M) = \sqrt{1125} = 33.541$

$((150,45,30), W) = \sqrt{1029} = 32.078$

$((170,65,29), M) = \sqrt{34} = 5.83$ 	 || |

$((175,78,35), M) = \sqrt{48} = 9.899$

$((185,90,32), M) = \sqrt{625} = 25$

$((170,65,28), W) = \sqrt{41} = 6.403$ 	 || |

$((155,48,31), W) = \sqrt{710} = 26.645$

$((160,55,30), W) = \sqrt{329} = 18.138$

$((182,80,30), M) = \sqrt{248} = 15.748$

$((175,69,28), W) = \sqrt{42} = 6.4807$ 	 |

$((180,80,27), M) = \sqrt{225} = 15$

$((160,50,31), W) = \sqrt{501} = 22.383$

$((175,72,30), M) = \sqrt{33} = 5.7445 \rightarrow M$ 	 || |

$\therefore (170,70,32) = M$

$K = 3$

M, W, M

$\downarrow$

M

$K = 5$

M, M, W, W, M

$\downarrow$

M

Case (3)   (175, 70, 35)

((170, 57, 32), W) = $\sqrt{203}$ = 14.247

((192, 95, 28), M) = $\sqrt{963}$ = 31.032

((150, 95, 30), W) = $\sqrt{1275}$ = 35.707

((170, 65, 29), M) = $\sqrt{86}$ = 9.273   1

11   1

((175, 78, 35), M) = $\sqrt{64}$ = 8

((185, 90, 32), M) = $\sqrt{509}$ = 22.5610

((170, 65, 28), W) = $\sqrt{99}$ = 9.949   1

((155, 48, 31), W) = $\sqrt{900}$ = 30

((160, 55, 30), W) = $\sqrt{475}$ = 21.794

((182, 80, 30), M) = $\sqrt{174}$ = 13.190

11   1

((175, 69, 28), W) = $\sqrt{50}$ = 7.07

((180, 80, 27), M) = $\sqrt{189}$ = 13.747

((160, 50, 31), W) = $\sqrt{641}$ = 25.317

((175, 72, 30), M) = $\sqrt{29}$ = 5.385 → M   11   1

K = 3   M W M → M

K = 5   M M W W M → M

. (175, 70, 35) = M

Case 4 $(180, 90, 20)$

$((170, 57, 32), W) = \sqrt{1333} = 36.510$

$((192, 95, 28), M) = \sqrt{233} = 15.264$

$((150, 45, 30), W) = \sqrt{3025} = 55$

$((170, 65, 29), M) = \sqrt{806} = 28.390$

$((175, 78, 35), M) = \sqrt{394} = 19.849$

$((185, 90, 32), M) = \sqrt{169} = 13$

$((170, 65, 28), W) = \sqrt{789} = 28.089$

$((155, 48, 31), W) = \sqrt{87653} = 296.062$

$((160, 55, 30), W) = \sqrt{1725} = 41.5331$

$((182, 80, 30), M) = \sqrt{204} = 14.28$

$((175, 69, 28), W) = \sqrt{530} = 23.021$

$((180, 80, 27), M) = \sqrt{149} = 12.206$

$((160, 50, 31), W) = \sqrt{2121} = 46.0543$

$((175, 72, 30), M) = \sqrt{449} = 21.189$

K=3	K=5
M, M, M $\downarrow$ M	M, M, M M M $\downarrow$ M

$\therefore (180, 90, 20) = M.$

2 c)

Dataset	(155, 40)	(170, 70)	(175, 70)	(180, 90)
(170, 57) →	$\sqrt{514}$	$\sqrt{169}$	$\sqrt{194}$	$\sqrt{1189}$
(192, 95) →	$\sqrt{4394}$	$\sqrt{1109}$	$\sqrt{914}$	$\sqrt{169}$
(150, 45) →	$\sqrt{50}$	$\sqrt{1025}$	$\sqrt{1250}$	$\sqrt{2925}$
(170, 65) →	$\sqrt{850}$	$\sqrt{25}$	$\sqrt{50}$	$\sqrt{725}$
(175, 78) →	$\sqrt{1844}$	$\sqrt{89}$	$\sqrt{64}$	$\sqrt{169}$
(185, 90) →	$\sqrt{3400}$	$\sqrt{625}$	$\sqrt{500}$	$\sqrt{25}$
(170, 65) →	$\sqrt{850}$	$\sqrt{25}$	$\sqrt{50}$	$\sqrt{725}$
(155, 48) →	$\sqrt{64}$	$\sqrt{709}$	$\sqrt{884}$	$\sqrt{2389}$
(160, 55) →	$\sqrt{250}$	$\sqrt{325}$	$\sqrt{709}$	$\sqrt{1625}$
(182, 80) →	$\sqrt{2329}$	$\sqrt{244}$	$\sqrt{149}$	$\sqrt{104}$
(175, 69) →	$\sqrt{1241}$	$\sqrt{26}$	$\sqrt{1}$	$\sqrt{466}$
(180, 80) →	$\sqrt{2225}$	$\sqrt{200}$	$\sqrt{125}$	$\sqrt{100}$
(160, 50) →	$\sqrt{125}$	$\sqrt{500}$	$\sqrt{625}$	$\sqrt{2000}$
(175, 72) →	$\sqrt{1424}$	$\sqrt{29}$	$\sqrt{4}$	$\sqrt{349}$

(155, 40)

K=1 W

K=3 W, W, W

K=5 W, W, W, W, W

(170, 70)

K=1 → M

K=3 → M, W, W - Women

K=5 → M, W, W, M, M

male

(175, 70)

K=1 → W

K=3 → W, M, W (women)

K=5 → W, M, W, W, M → Male

(80, 90)

K=1. → M male

K=3 → M, M, M

K=5 → M, M, M, M, M

Male

3 ⓐ $N(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-1}{2\sigma^2}(x-\mu)^2\right)$

(pdf)

$$= \ln\left[\frac{1}{2\pi\sigma^2}\exp\left(\frac{-1}{2\sigma^2}(x-\mu)^2\right)\right]$$

$$l(\mu,\sigma) = \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) + \ln\left(\exp\left(\frac{-1}{2\sigma^2}(x-\mu)^2\right)\right]$$

$$= \ln\left[(2\pi\sigma^2)^{-1/2}\right] + \left[\frac{-1}{2\sigma^2}(x-\mu)^2\right]$$

$$= \frac{1}{2\sigma^2}(x-\mu)^2 - \frac{1}{2}\ln(2\pi\sigma^2)$$

$$= \frac{-1}{2\sigma^2}(x-\mu)^2 - \left(\frac{1}{2}\cdot\ln(2\pi) + \frac{1}{2}\ln(\sigma^2)\right)$$

$$= \frac{-1}{2\sigma^2}(x-\mu)^2 - \frac{1}{2}\ln 2\pi - \frac{1}{2}\ln(\sigma^2)$$

Maximum likelihood Estimation $L(\mu,\sigma)$

$$\frac{\partial L}{\partial \mu} = \frac{\partial}{\partial \mu}\left[\frac{-1}{2\sigma^2}\sum_{i=1}^{N}(x_i-\mu)^2 - \frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma^2)\right]$$

$$\mu = \frac{-1}{2\sigma^2} \times 2 \sum_{i=1}^{N}(x_i-\mu)$$

$$\sum_{i=1}^{N} x_i - N\mu = 0$$

$$N\mu = \sum_{i=1}^{N} x_i$$

$$\frac{\partial L}{\partial \sigma}(\mu,\sigma) = \frac{\partial}{\partial \sigma}\left[\frac{-1}{2\sigma^2}\sum_{i=1}^{N}(x_i-\mu)^2 - \frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma^2)\right]$$

$$\sigma = \frac{\partial}{\partial \sigma}\left[\frac{-1}{2}\sigma^{-2}\sum_{i=1}^{N}(x_i-\mu)^2 - \frac{N}{2}\ln(\sigma^2)\right]$$

$$= \frac{-1}{2}\sum_{i=1}^{N}(x_i-\mu)^2 \cdot -2\sigma^{2} - \frac{N}{2}\cdot\frac{1}{\sigma^2}(2\sigma)$$

$$\frac{1}{\sigma^3}\sum_{i=1}^{N}(x_i-\mu)^2 - \frac{N}{\sigma} = 0$$

$$\boxed{\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2}$$