Multifactor Models

Multifactor Models

A multi-factor model is a financial model that employs multiple factors in its calculations to explain asset prices.

- These models introduce uncertainty stemming from multiple sources.
- CAPM, on the other hand, limits risk to one source –covariance with the market portfolio.

Multifactor models can be used to calculate the required rate of return for portfolios as well as individual stocks.

- CAPM uses just one factor to determine the required return –the market factor.

The market factor can be split up even further into different macroeconomic factors.

- These may include inflation, interest rates, business cycle uncertainty, etc.

Multifactor Models

A factor can be defined as a variable which explains the expected return of an asset.

- A factor beta is a measure of the sensitivity of a given asset to a specific factor.
- The bigger the factor, the more sensitive the asset is to that factor. A multifactor appears as follows:

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \cdots + \beta_{ik}F_k + e_i$$

Where

- R_i= Rate of return on stock i
- $E(R_i)$= Expected return on stock I
- β_{ik}= Sensitivity of the stock's return to a one unit change in factor k
- F_k= Macroeconomic factor k
- e_I = the firm specific portion of the stock's return unexplained by macro factors

The expected value of the firm-specific return is always zero.

The Single-factor Model

The single-factor model assumes there's just one macroeconomic factor, and appears as follows:

$$R_i = E(R_i) + \beta_i F + e_i$$

Where E(R_i) is the expected return on stock i.

In case the macroeconomic factor has a value of zero in any period, then the return on the security will equal its initially expected return *E (Ri)* plus the effects of firm-specific events.

Example: Return Using a Single-factor Model

Assume the common stock of BRL is examined with a single-factor model, using unexpected percent changes in GDP as the single factor.

Assume the following data is provided:

- Expected return for BRL = 10%
- GDP factor beta = 1.50
- Unexpected GDP growth = 4%

Compute the required rate of return on BRL stock.

Solution

- $R_i = E(R_i) + \beta_i F + e_i$
- R_i = 10% + 1.5 * 4%= 16%

Example: Return Using a Multi-factor Model

Assume the common stock of BRL is examined using a multifactor model using:

- Unexpected percent change in GDP; and
- Unexpected percent change in interest rates.

Assume the following data is provided:

- Expected return for BRL = 10%
- GDP factor beta = 1.50
- Interest rate factor beta = 2.0
- Unexpected growth in GDP = 4%
- Unexpected growth in interest rates = 1%

Compute the required rate of return on BRL stock.

Solution

- $R_i = E(R_i) + \beta_{i1} F_1 + \beta_{i2} F_2$
- $R_i = 10\% + 1.5 * 4\% + 2.0 * 1\% = 18\%$

The Law of One Price and Arbitrage Opportunities

Identical assets selling in different locations should be priced identically in the different locations.

- In the absence of this law, arbitrage exists.
- Arbitrage refers to the action of buying an asset in the cheaper market and simultaneously selling that asset in the more expensive market to make a risk-free profit.
- A well-diversified portfolio is a portfolio whose firm-specific risk is negligible. Diversification eliminates unsystematic/specific risks.

Check a stock’s price in NSE and BSE

Hedging Exposures to Multiple Factors

Consider an investor who manages a portfolio with the following factor betas:

GDP beta = 0.4

Consumer sentiment beta = 0.2

Example:

Assume the investor wishes to hedge away GDP factor risk yet maintain the 0.2 exposure to consumer sentiment.

How would they achieve this?

The investor should combine the original portfolio with a 40% short position in the GDP factor portfolio.

The GDP factor beta on the 40% short position in the GDP factor portfolio equals -0.4, which perfectly offsets the 0.40 GDP factor beta on the original portfolio.

Hedging Exposures to Multiple Factors

Consider an investor who manages a portfolio with the following factor betas:

- GDP beta = 0.4
- Consumer sentiment beta = 0.2

Example:

- Assume the investor wants to hedge away both factor risks.
- How would they achieve this?
- The investor would have to form a portfolio that's 40% invested in the GDP factor portfolio, 20% in the CS factor portfolio, and 40% in the risk-free asset(note that total = 100%). Let's refer to this portfolio as portfolio H.
- Portfolio H can be used to hedge away all the risk factors of the original portfolio.
- That would involve combining the original portfolio with a short position in portfolio H.
- The original portfolio betas (0.4 and 0.2) would be perfectly offset by the short position in portfolio H, the hedge portfolio.

The Arbitrage Pricing Theory

The APT theory describes expected returns as a linear function of exposures to common macroeconomic risk factors.

$$E(R_i) = R_F + \beta_{i1}RP_1 + \beta_{i2}RP_2 + \beta_{i3}RP_3 + \cdots + \beta_{iK}RP_K$$

Where RP_j represents the risk premium attached to risk factor j.

The Arbitrage Pricing Theory

The risk premiums are derived using the following procedure:

- Step 1: Create well-diversified factor portfolios, where each portfolio has a beta of 1 for a single risk factor and betas of 0 on the remaining factors.
  - Repeat the process for all k factors in the multifactor model.
- Step 2: Derive returns for each factor portfolio.
- Step 3: Calculate risk premiums for each factor portfolio. For example, the risk premium for Factor Portfolio 2 equals *E (R2) − RF*.

Once the risk premiums have been derived, we can rewrite the APT formula as follows:

$$E(R_i) = R_F + \beta_{i1}[E(R_1) - R_F] + \beta_{i2}[E(R_2) - R_F] + \beta_{i3}[E(R_3) - R_F] + \cdots + \beta_{iK}[E(R_K) - R_F]$$

Example: The APT Model

The following data exists for asset A:

- Risk-free rate = 3%
- GDP factor beta = 0.40
- Consumer sentiment factor beta = 0.20
- GDP risk premium = 2%
- Consumer sentiment risk premium = 1%

Calculate the expected return for Asset A using a 2-factor APT model.

Solution

- ERA=0.03+0.40.02+0.20.01=0.04=4%

The Fama-French Three-Factor Model

A major weakness of the APT model is that it's silent on the issue of the appropriate risk factors for use.

The FF three-factor model puts three factors forward :

- Size of firms
  - The firm size factor, also known as SMB(small minus big) is equal to the difference in returns between portfolios of small and big firms ($R_s - R_b$).
- .Book-to-market values
  - The book-to-market value factor, also known as HML(high minus low) is equal to the difference in returns between portfolios of high and low book-to-market firms ($R_H - R_L$).
  - Note: book-to-market value is book value per share divided by the stock price.
- Excess return on the market

The Fama-French Three-Factor Model

Why SMB and HML

- Fama and French put forth the argument that returns are higher on small versus big firms as well as on high versus low book-to-market firms.
- This argument has indeed been validated through historical analysis.
- Fama and French contend that small firms are inherently riskier than big firms.
- High book-to-market firms are undervalued compared to low book-to-market firms.

The equation for the Fama-French three-factor model is:

$$E(R_i)=R_F+\alpha_i+\beta_{i1}MRP_1+\beta_{i2}SMB_2+\beta_{i3}HML_3+\varepsilon_i$$