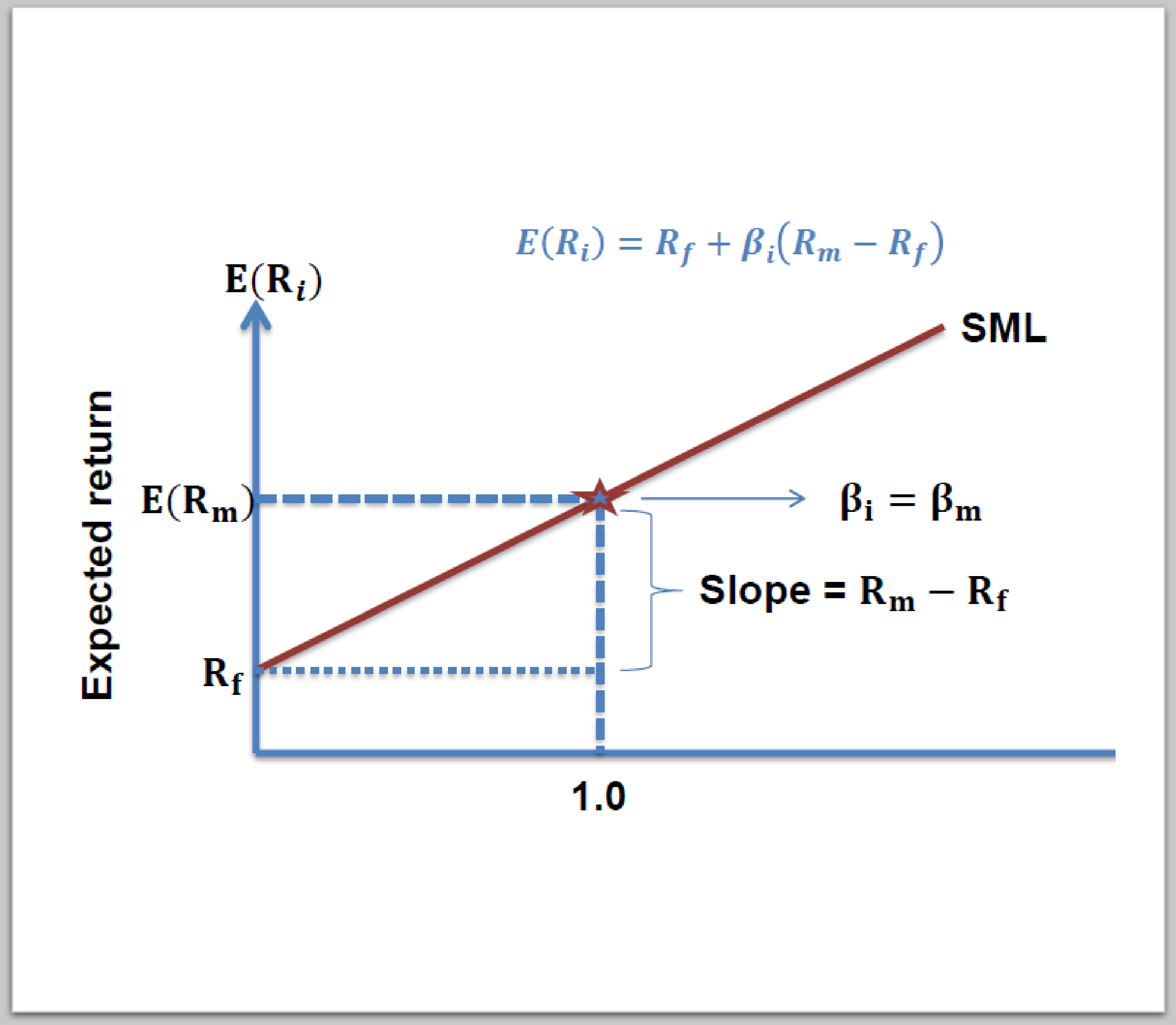
MODERN PORTFOLIO THEORY (MPT)

The Capital Asset Pricing Model

The Capital Asset Pricing Model stipulates that for a single stock:

$$\text{Cost of equity or a stock's expected return} =$$

$$\text{Risk free rate} + \text{Beta for stock} * \text{Equity premium}$$



Assumptions Underlying the CAPM

- Assumptions of the model include:
- There are no transaction costs
- There are no taxes
- Assets are infinitely divisible
- Unlimited short-selling is permissible
- All assets are marketable/liquid
- Investors are price takers whose individual buy and sell transactions have no effect on the price
- Investors’ utility functions are based solely on expected portfolio return and risk
- The only concern among investors are risk and return over a single period, and the single period is the same for all investors

Interpreting Beta

Beta is a measure of the systematic risk associated with a particular stock, asset, or portfolio.

A beta of more than 1 indicates an asset that has amplified the return of the whole market (positive or negative).

A beta close to zero would indicate an asset that provides a more stable return than the market as a whole.

A negative beta would signify an asset whose performance is counter-cyclical, i.e., offsets the overall market experience.

$$\boldsymbol{\beta}_i = \boldsymbol{\rho}_{im} * \boldsymbol{\sigma}_i / \boldsymbol{\sigma}_m$$

Where $\boldsymbol{\rho}_{im}$ is the correlation coefficient between the individual company's stock return and that of the market, and $\boldsymbol{\sigma}_i$ and $\boldsymbol{\sigma}_m$ are the standard deviations of the company's stock return and the market index, respectively.

Example on Beta

How to calculate expected returns for a security using CAPM?

Calculate the expected return for a security, given:

- R_f= 5%
- Std. dev. of security = 40%
- Security correlation with market = 0.80
- Std. dev. of market = 20%
- R_m= 10%

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Solution

First, find Beta;

- Beta = 0.80 x {0.40/0.20} = 1.6

Next, use the CAPM model to find the expected return;

- E(Ri) = 5% + 1.6 x (10% -5%) = 13%

Calculating portfolio beta

Consider the following individual asset weights and betas for a 4-asset portfolio.

Asset	*Portfolio Weight*	*Beta*
1	25%	1.2
2	15%	1.8
3	35%	0.9
4	25%	1.4

Calculate the beta of this 4-asset portfolio.

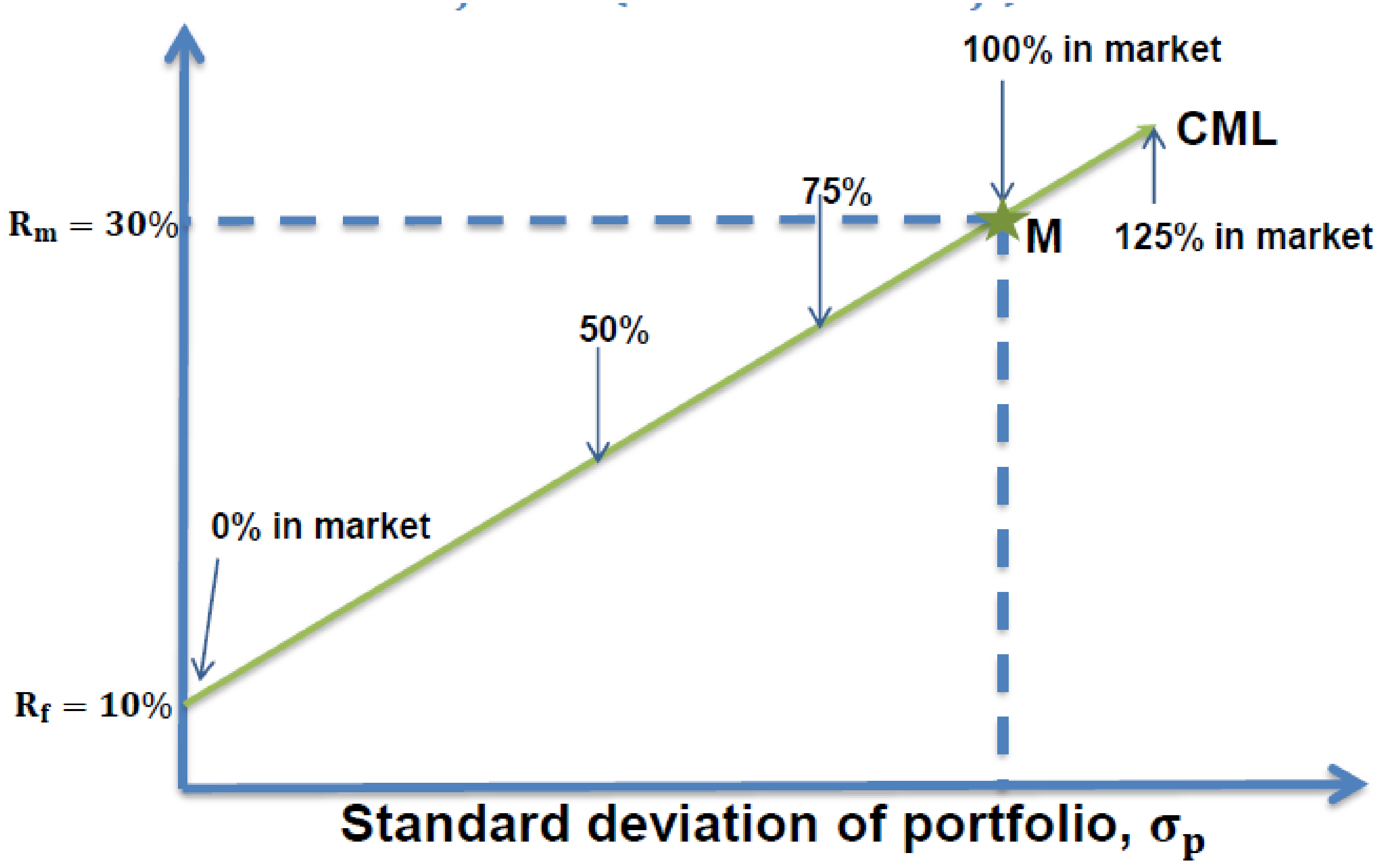
Answer:

- $\beta p = w_1 \beta_1 + w_2 \beta_2 + w_3 \beta_3 + w_4 \beta_4$
- $\beta p = (0.23 \times 1.2) + (0.15 \times 1.8) + (0.35 \times 0.9) + (0.25 \times 1.4)$
- $\beta p = 0.3 + 0.27 + 0.315 + 0.35 = 1.235$

The Capital Market Line

The capital market line expresses the expected return of a portfolio as a linear function of the risk-free rate, the portfolio's standard deviation, and the market portfolio's return and standard deviation.

$E(R_p)=R_f+(E(R_m)-R_f)\times\sigma_p/\sigma_m)$



CML Example

Example

Calculate the expected return from a portfolio which has 130% weight invested in the risky asset, given:

- Risk-free asset: R_f= 3%
- Market portfolio: $E(R_m)$ = 10%
- Standard deviation: σ_m= 26%

Solution

Here, we're borrowing 30% in the risk-free asset and investing the proceeds plus the whole portfolio in the market portfolio.

Return with -30% in the risk-free asset and 130% in the risky asset:

- $E(Ri) = -0.3 \times 3\% + 1.3 \times 10\% = 12.1\%$

The Treynor Measure

Performance metric for determining how much excess return was generated for each unit of risk taken on by a portfolio.

Measures the returns a funds gives with respect to its systematic risk (riskiness).

A higher Treynor ratio means that the fund not only performs well but is less risky than the general market.

$$\textbf{Treynor ratio} = (r_p - r_f) / \beta_p$$

Where:

- r_p= portfolio return
- r_f= risk-free rate
- β_p= beta of the portfolio

The Sharpe Measure

Just like the Treynor ratio, the Sharpe measure is the average return earned in excess of the risk-free rate per unit of volatility.

However, while the Treynor ratio considers only the systematic risk of the portfolio, the Sharpe measure considers the total risk of the portfolio

$$\mathbf{Sharpe ratio} = (\mathbf{r_p} - \mathbf{r_f}) / \boldsymbol{\sigma_p}$$

Where,

- **E**(**R**$_p$) indicates the portfolio's expected return
- **R**$_f$ indicates the risk-free rate
- $\boldsymbol{\sigma}(\boldsymbol{R}_p)$ indicates standard deviation of returns of the portfolio

The Sharpe Measure

Example

- Client ‘A’ currently is holding a $1,000,000 invested in a portfolio with an expected return of 10% and a volatility of 8%.
- The efficient portfolio has an expected return of 17% and a volatility of 10%. The risk-free rate of interest is 5%.
- What is the Sharpe Ratio?

Solution

- Sharpe Ratio of portfolio = (0.10-0.05)/0.08 = 62.5% or **0.63×**
- Sharpe Ratio of efficient portfolio = (0.17-0.05)/0.10 = 120% or **1.2×**
- Therefore, the efficient portfolio has a better Sharpe ratio than the portfolio help by Client ‘A’.

The Jensen Measure (α)

Jensen's alpha is the excess return above or below the security market line. It can be interpreted as a measure of **how much the portfolio "beat the market."**

Jensen's alpha = $\alpha_p = R_p – [R_f + \beta_p(R_m – R_f)]$

- Positive alpha signals superior risk-adjusted returns, and that the manager is good at selecting stocks or predicting market turning points.
- Unlike the Sharpe Ratio, Jensen's method does not consider the ability of the manager to diversify, it only accounts for systematic risk.
- The values of alpha can also be used to rank portfolios or the managers of those portfolios with the alpha representing the maximum an investor should pay for the active management of that portfolio.

Which performance measure should we use?

Sharpe ratio

- Appropriate for the evaluation of an entire portfolio.
- Penalizes a portfolio for being undiversified, because in general, total risk ≈ systematic risk only for relatively well-diversified portfolios.

Treynor ratio and Jensen's alpha

- Appropriate for the evaluation of securities or portfolios for possible **inclusion into an existing portfolio**.
- Both are similar, the only difference?
  - The **Treynor ratio standardizes returns**, including excess returns, relative to beta.
  - Both require a **beta estimate** (and betas from different sources can differ a lot).

The Tracking-Error

Tracking error is the divergence between the price behavior of a position or a portfolio and the price behavior of a benchmark.

It is the standard deviation of the difference between the returns of an investment and its benchmark:

$$\mathbf{TE = \sigma(R_p - R_B)}$$

- Low (tracking) errors indicate that the performance of the portfolio is close to the performance of the benchmark.
- Low errors are common among index funds and ETFs that replicate the composition of stock market indices.
- High (tracking) errors indicate the portfolio performance is significantly different from the benchmark.
- Such a portfolio can either substantially beat the benchmark, or significantly underperform the benchmark.

The information ratio (IR) is a measurement of portfolio returns beyond the returns of a benchmark, usually an index, compared to the volatility of those returns.

The information ratio is the alpha of the managed portfolio relative to its benchmark divided by the tracking error.

$$\textbf{Information Ratio} = (\textbf{Portfolio return} - \textbf{Benchmark return})\ /\textbf{Tracking error}$$

The IR represents a manager’s ability to use information and talent to generate excess returns.

The Information Ratio

The Sortino Ratio

Sortino ratio = $(\mathbf{E(R_P)-R_{min}}) / \sqrt{\mathbf{MSD_{min}}}$

The measure of risk, MSD_{min}, is the square root of the mean squared deviation from R_{min} of those observations in time period t where $R_{Pt} < R_{min}$, else zero.

The Sortino ratio is much like the Sharpe ratio, but there are two differences:

- The Risk-free rate is replaced with a minimum acceptable return, denoted as R_{min}
- The standard deviation is replaced by a semi-standard deviation which measures the variability of only those returns that fall below the minimum acceptable return.

The Sortino ratio is a useful way for investors, analysts and portfolio managers to evaluate an investment's return for a given level of bad risk.

It focuses only on the negative deviation of a portfolio's returns from the mean.

As a result, it is thought to give a better view of a portfolio's risk-adjusted performance since positive volatility is a benefit.