Mid Sem Exam for Intro to Math Finance

10 am - 1pm

duration = 3 hours total = 50 marks

March 9, 2024

1. Consider a one period binomial model, i.e., (a) spot price of $S = 100$ at $t = 0$ and (b) the stock price is either $S \times u$ or $S \times d$ at maturity $t = 1year$ depending on if stock price moved up or moved down. Let $u = 2$ and $d = 1/2$. Also, assume a continuous interest rate of 10%/year.

   (a) **5 marks** Compute the risk neutral probability of the stock going up.

   (b) **10 marks** Take a call option with strike $K = 110$ and maturity $T = 1year$. Compute its price at $t = 0$ using (a) replication portfolio method and (b) risk neutral probabilities.

2. Consider a two period binomial model, with spot price of $S = 100$ at $t = 0$, $u = 2$ and $d = 1/2$. Also, take each period to be one year, i.e, we are only concerned with $t = 0, 1$ and 2 years. Also, assume a continuous interest rate of 10%/year.

   (a) **5 marks** Compute the risk neutral probability of the stock price at 400 at $t = 2y$.

   (b) **10 marks** Take a call option with strike $K = 110$ and maturity $T = 2years$. Compute its price at $t = 0$.

3. I have 4 umbrellas, some at home, some in the office. I keep moving between home and office. I take an umbrella with me only if it rains. If it does not rain I leave the umbrella behind (at home or in the office). It may happen that all umbrellas are in one place, I am at the other, it starts raining and must leave, so I get wet.

   (a) **10 marks** If the probability of rain is p, what is the probability that I get wet, over a long time?

   (b) **10 marks** Current estimates show that $p = 0.6$ in Edinburgh. How many umbrellas should I have so that, if I follow the strategy above, the probability I get wet is less than 0.1?
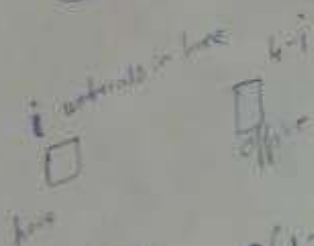
Mid Sem Exam for Intro to Math Finance

1 pm - 4pm

duration = 3 hours; total = 50 marks

April 3, 2024

1. Draw the graphs of the payoffs of the following combinations of options, as a function of the asset price:

   (a) **1 marks** buying one call plus one put option with the same strike price K (this is known as a straddle)

   (b) **3 marks** buying one call option and selling one put option with the same strike price K

   (c) **3 marks** buying one call option with strike price $K1$, and selling another call option with strike price $K2$ (this is known as a bull spread; you will want $K2 > K1$)

2. Consider a two period binomial model, with spot price of $S = 100$ at $t = 0$ and the stock price is either $S \times u$ or $S \times d$ after each 1year period. Take $u = 2$ and $d = 1/2$. We are thus only concerned with $t = 0, 1$ and 2 years. Also, assume an interest rate of 10%/year compounded every 6 month. $r = 0.05$

   (a) **3 marks** Compute the risk neutral probability of the stock price at 400 at $t = 2y$.

   (b) **10 marks** Take a call option with strike $K = 110$ and maturity $T = 2years$. Compute its price at $t = 0$ using (a) replication portfolio method and (b) risk neutral probabilities.

3. **10 marks** Consider a two-period binomial model for the stock price with both periods of length one year. Let the initial stock price be $S(0) = 100$ and assume that the stock pays no dividends. Let the up and down factors be $u = 1.25$ and $d = 0.75$ respectively. Let the continuously compounded interest rate be $r = 5\%$ per annum.

   Roger is interested in purchasing a chooser option with the provision that he can choose if the option is a put or a call after one year. The strike for this option is $100 and the expiry date is two years. Using the binomial tree mentioned in the first para, find the price of the chooser option.

4. Consider the knight's tour on a chess board: A knight selects one of the next positions at random independently of the past.

   (a) **5 marks** Set up the process as a Markov chain. Is it irreducible? Is it aperiodic?

   (b) **10 marks** Find the stationary distribution. Give an interpretation of it: what does it mean, physically?

   (c) **5 marks** Which are the most likely states in steady-state? Which are the least likely ones?