Financial Modelling With Python
Derivative Market

Mousum Dutta

Derivative Market

What is derivative?

- Derivate is financial instrument which is derived from a primitive or fundamental instrument.

- Derivate is financial instrument whose payoffs depend on the underline financial instrument or securities.

Use of Derivatives

- Hedging
- To speculate
- To lock an arbitrage profit
- To change the nature of liabilities

Derivate Instruments

- **Forward** - A forward contract is an agreement between two parties to buy or sell an asset at a specified price at a fixed date in the future.
- **Future** - A futures contract is an agreement to buy or sell something at a future date, for an agreed-upon price. Futures contracts are standardized for quality and quantity to facilitate trading on a futures exchange.
- **Option** - Options contracts are agreements between a buyer and seller which give the buyer the right to buy or sell a particular asset at a later date (expiration date) and an agreed-upon price (strike price). They're often used for securities, commodities, and real estate transactions.
- **Swap** - A swap is a derivative contract through which two parties exchange the cash flows or liabilities from two different financial instruments.

Forward Contracts Vs Future Contracts

Forward Contracts	Future Contracts
Forward contract is a private contract between two parties	Futures contracts are traded on an exchange
Customizable, not standardized	Standardized
One specific delivery date	Multiple delivery dates
Settled at maturity date	Can be closed prior to maturity
Credit Risk	No Credit Risk

Options Vs Forward/Future

- Options and futures are similar trading products that provide investors with the chance to make money and hedge current investments.

- An option gives the buyer the right, but not the obligation, to buy (or sell) an asset at a specific price at any time during the life of the contract.
  duty/bound

- A futures contract gives the buyer the obligation to purchase a specific asset, and the seller to sell and deliver that asset at a specific future date unless the holder's position is closed prior to expiration.

Concepts Used in Derivatives

- **Long Position** – In finance, a long position in a financial instrument means the holder of the position owns a positive amount of the instrument. The holder of the position has the expectation that the financial instrument will increase in value.
- **Short Position** - In finance, being short in an asset means investing in such a way that the investor will profit if the value of the asset falls.
- **Call Option** – Call options are financial contracts that give the option buyer the right, but not the obligation to buy underline assets.
- **Put Option** – A put option is a contract giving the owner the right, but not the obligation, to sell–or sell short–a specified amount of an underlying security

Concepts Used in Derivatives

- **Settlement Price** – Settlement price refers to the price at which an asset closes or at which a derivatives contract will reference on trading days. Closing price is the settlement price on last trading day.
- **Strike Price** – A strike price is the set price at which a derivative contract can be bought or sold when it is exercised.
- **Expiration Date** – An expiration date in derivatives is the last day that derivative contracts, such as options or futures, are valid.
- **Exercise Date** - Exercise date refers to the date on which a trader decides to exercise an option (Call/Put) on an exchange or with a brokerage whether bought or written/sold where 'exercise' means making use of the actual right specified in the contract.

Relationship Between Spot Price and Future Price

- Basis = Current Spot Price – Future Price
- Normal Market
  - Future Price > Spot Price
  - Distance Future Price > Nearby Future Price distant future price (say 2 months) nearby(1 month)
- Inverted Market
  - Future Price < Spot Price
  - Distance Future Price < Nearby Future Price
- At Expiration, Basis is zero

Spread

- Spread is difference between two future prices
- Intra Commodity Spread
- Inter Commodity Spread

Hedge is an investment or contract used to offset potential losses from another asset.

Optimal Hedge Ratio

Stock hedge
Commodity hedge

write

- The hedge ratio compares the amount of a position that is hedged to the entire position.
- The minimum variance hedge ratio helps determine the optimal number of options contracts needed to hedge a position.
- The minimum variance hedge ratio is important in cross-hedging, which aims to minimize the variance of a position's value.

OPTIMAL HEDGE RATIO

How?

If an asset has been hedged, then the movements in its spot price and in the accompanying short hedge should constitute compensating variations. The optimum size of the hedge will be a function of the variances of the spot price and the futures price and of the covariance of the two.

Let $\nabla S = S_{t_2} - S_{t_1}$ denote the change in the spot price between times t_1 and t_2, and let $\nabla F = F_{\tau|t_2} - F_{\tau|t_1}$ be the change in the futures price. Also, let h denote the hedge ratio, which is the ratio of the value of the futures contract at time t to the value of the asset in question. The change in the value of the hedger's position between time t_1 and t_2 is

$$\nabla S - h\nabla F.$$

We may denote the variance of ∇S by σ_S^2 and the variance of ∇F by σ_F^2. Then, the variance of the hedger's position is

$$\nu = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F,$$

where ρ is the correlation between ∇S and ∇F and where, consequently, $\rho\sigma_S^2\sigma_F^2$ is the covariance of ∇S and ∇F.

The value of h which minimises ν is

$$h = \rho\frac{\sigma_S}{\sigma_F}.$$

Options

- **Call Option** – Call options are financial contracts that give the option buyer the right, but not the obligation to buy underline assets.

- **Put Option** – A put option is a contract giving the owner the right, but not the obligation, to sell–or sell short–a specified amount of an underlying security

Options

- American Options

- European Options

Options at Expiration

Buyer won't buy if asset price is lower in call option and if higher in put option.

- On the expiration, if the asset price is lower than the strike price, Call Option would be unexercised.
- On the expiration, if the asset price is higher than the strike price, Put Option would be unexercised.

Example

Option	Exercise price	Stock price	premium Call Option price	Classification	Intrinsic value
1	80	83.5	6.75	In-the-money profit	3.5
2	85	83.5	2.5	Out-of the money loss/nothing gained	0

Future Price: Securities Providing No Income

- S = Spot Price
- F = Future Price of a contract deliver after T years
- r = Risk free Interest rate

- $F = S * (1 + r)^T$

S = 300, T = 2, r = 7% then $F = 300 * (1.07)^2$ = 343.47

Future Price: Securities Providing No Income

- S = Spot Price
- F = Future Price of a contract deliver after T years
- r = Risk free Interest rate

Future Price shhould be as great as of F which is depending on r

- $\boxed{F = S * e^{rT}}$

S = 300, T = 2, r = 7% then $F = 300 * e^{0.07*2}$ = 345.08

Future Price: Securities Providing No Income

You invest according to the values of these calculated formulaes

- $F > S * e^{rT}$

Take a short position in the contract and make profit $F - S * e^{rT}$

- $F < S * e^{rT}$

Take a long position in the contract and make profit $S * e^{rT} - F$

Future Price: Securities Providing Know Income

- S = Spot Price
- F = Future Price of a contract deliver after T years
- r = Risk free Interest rate (Continuous)
- I = Present value of all incoming cashflows during the period

what is this ?

- $F = (S - I) * e^{rT}$

We subtract :

I (the present value of income) because the holder of the futures contract does not receive the income generated by the asset during the contract period

Future Price: Securities Providing Know Income

- S = Spot Price
- F = Future Price of a contract deliver after T years
- r = Risk free Interest rate (Continuous)
- I = Present value of all incoming cashflows during the period

- $F = (S - I) * e^{rT}$
- If y = Average yield provided by the asset

when interest rate is known

- $\boxed{F = S * e^{(r-y)T}}$

Future Price: Storage Cost

- S = Spot Price
- F = Future Price of a contract deliver after T years
- r = Risk free Interest rate (Continuous)
- C = Present value of all negative cashflows due to storage during the period

- $F = (S + C) * e^{rT}$

Carry Pricing Model

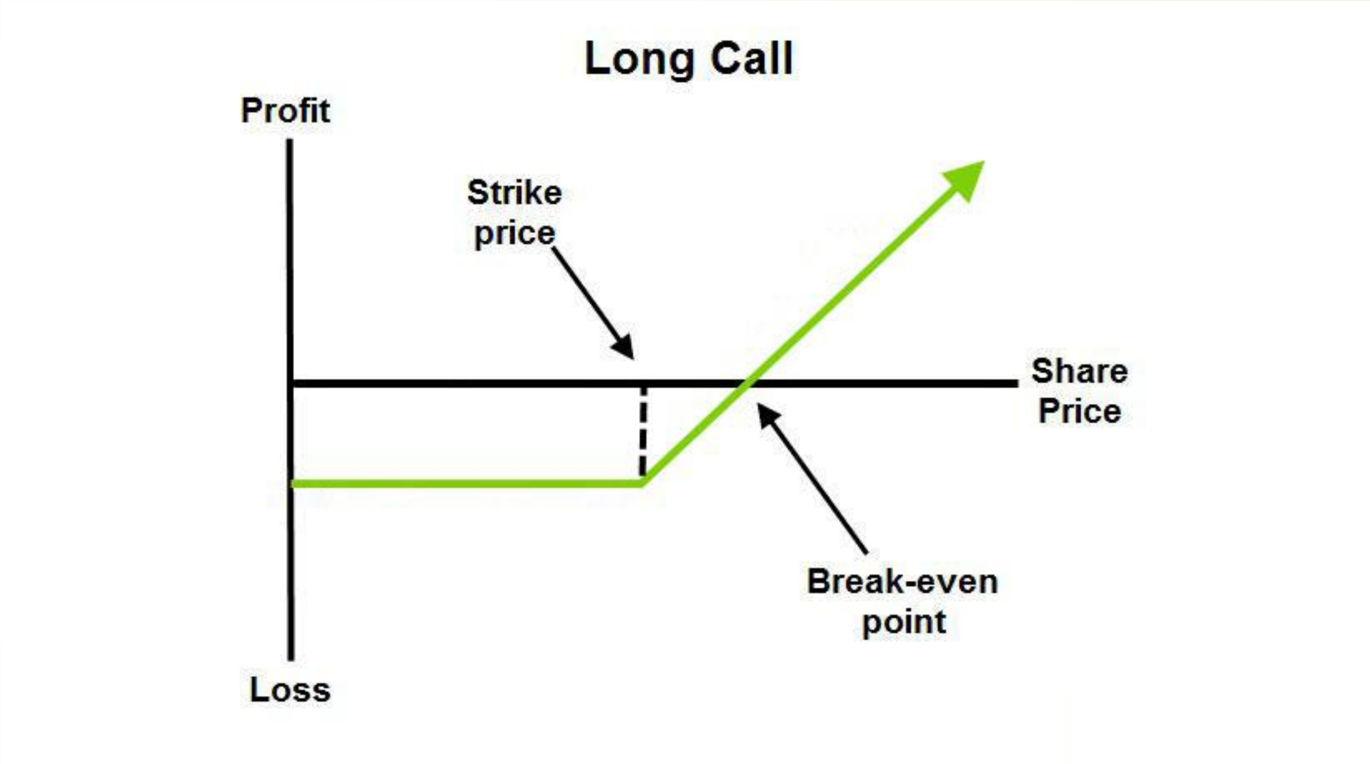
- Forward or Future Price = Spot Price + Carry Cost - Carry Return

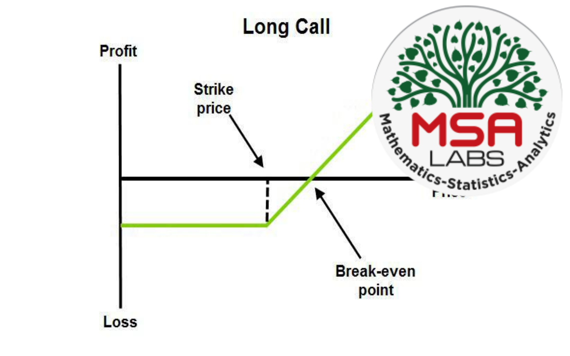
- Carry Cost = Holding costs including interest charges, Insurance costs, storage costs etc.

- Carry Return – Income such as dividend form stocks etc.

Long and Short Positions on Call and Put Options

Long Call

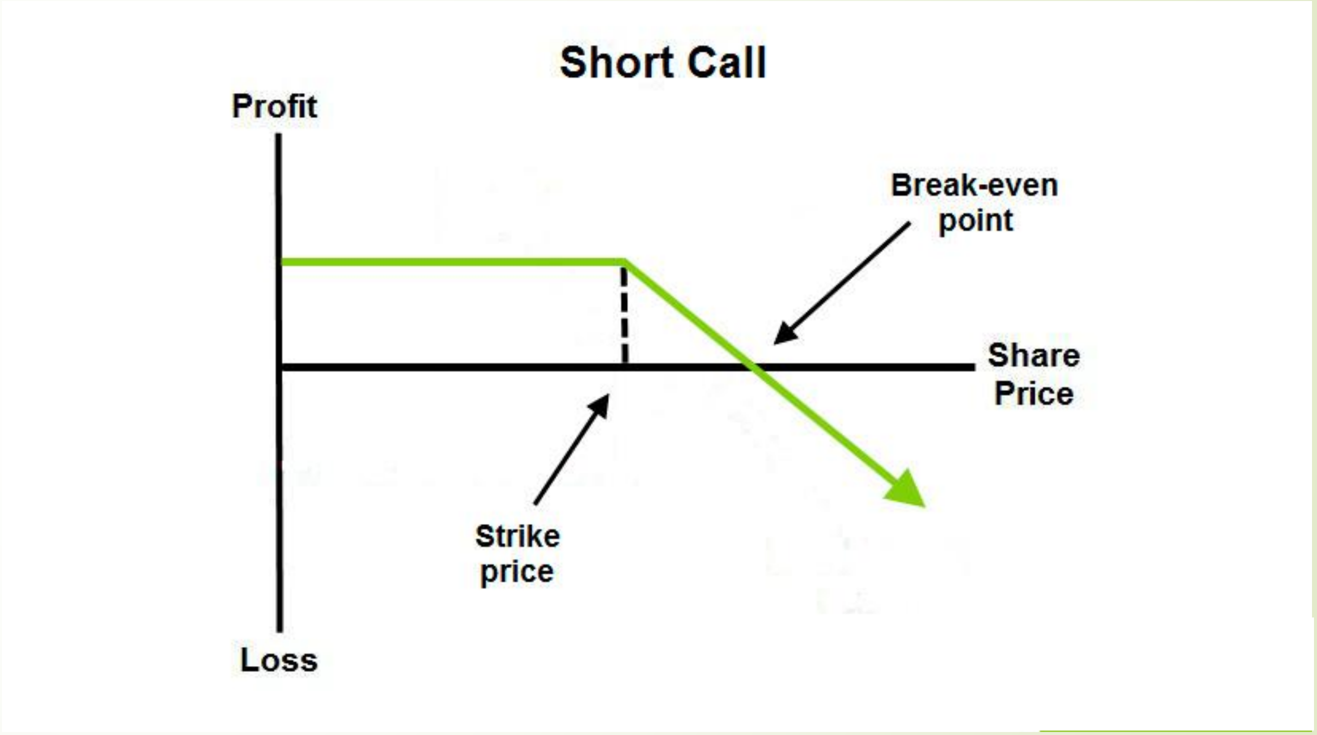

Long Call
Profit
Strike
price
Share
Price
Break-even
point
Loss



Long Call

- In this strategy, a trader expects the market to rise in near future
- This strategy has limited risk and unlimited profit potential
- Buyer pays the option premium in exchange for the right to buy share or index at a fixed price by a certain expiry date
- Breakeven Point = Strike Price + Premium   eg. 20 premium, strike price 80, stock price 100 at expiration
- Max Loss = Limited, Premium Paid
- Profit = Unlimited, Price of Underlying - (Strike Price + Premium Paid)   Imp
- Buying a Call Option instead of the underlying allows you to gain more profits by investing less and limiting your losses to minimum   underlying asset = stock
- Call options have a limited lifespan. So, in case the price of your underlying stock is not higher than the strike price before the expiry date, the call option will expire worthlessly and you will lose the premium paid.

Short Call

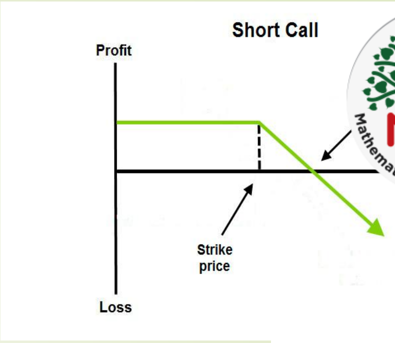


Short Call
Profit
Break-even
point
Share
Price
Strike
price
Loss

more risky

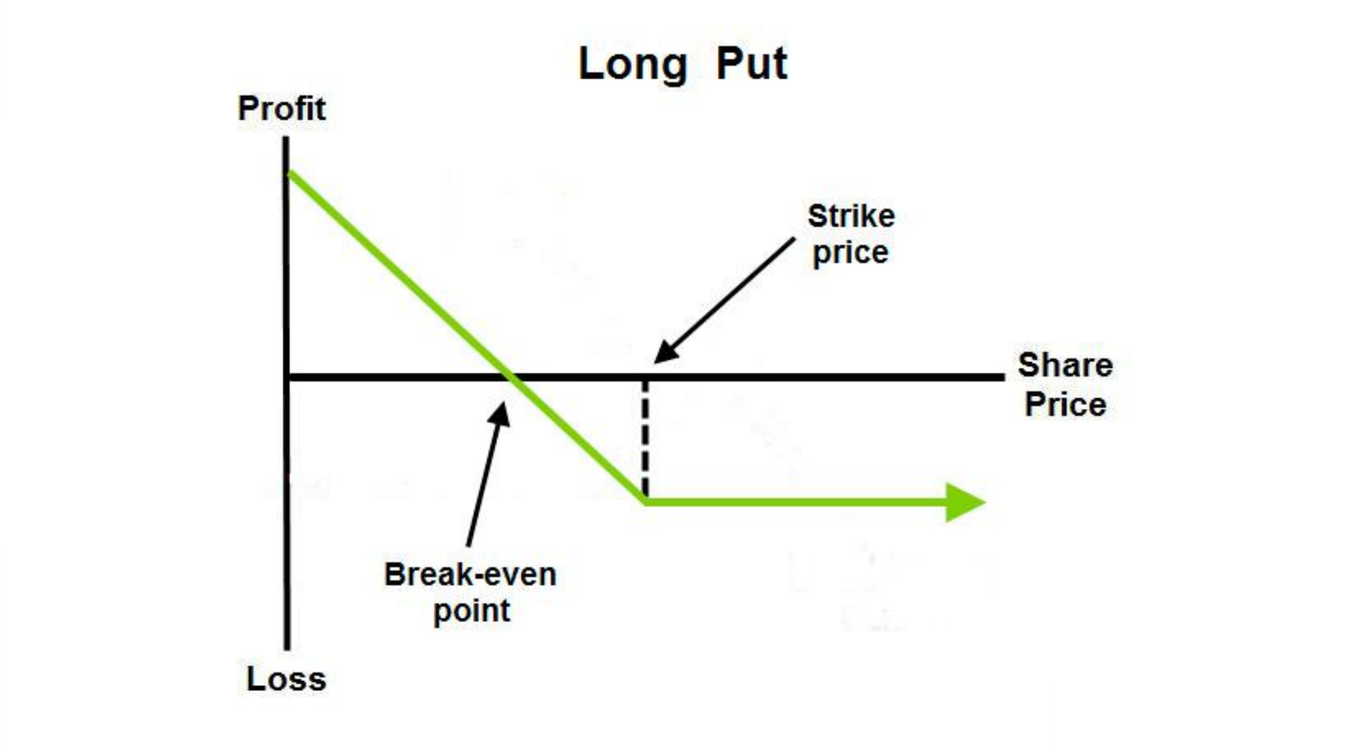
Short Call

long call = purchasing for rise
short call = selling for down



- Short Call (or Naked Call) strategy involves the selling of the Call Options (or writing call option). The pay-off of Short Call is mirror image of Long Call
- In this strategy, a trader expects the price of the underlying asset to go down in near future.
- This strategy is highly risky with potential for unlimited losses
- A trader sells a call option and earn profits if the price of the underlying asset goes down
- Breakeven Point = Strike Price of Short Call + Premium Received
- Max Profit = Premium Received Stock price goes down the strike price. the person will not purchase the stock.
- Maximum Loss = Unlimited, Loss = Price of Underlying – (Strike Price of Short Call - Premium Received)
- This strategy allows you to profit from falling prices in the underlying asset.
- There's unlimited risk on the upside as you are selling Option without holding the underlying.
- Rewards are limited to premium received only

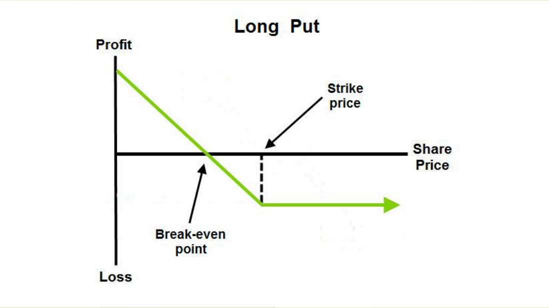
Long Put



Long Put
Profit
Strike
price
Share
Price
Break-even
point
Loss

Premium paid to get the right to sell the product i.ee if price drops other has to purchase

Long Put

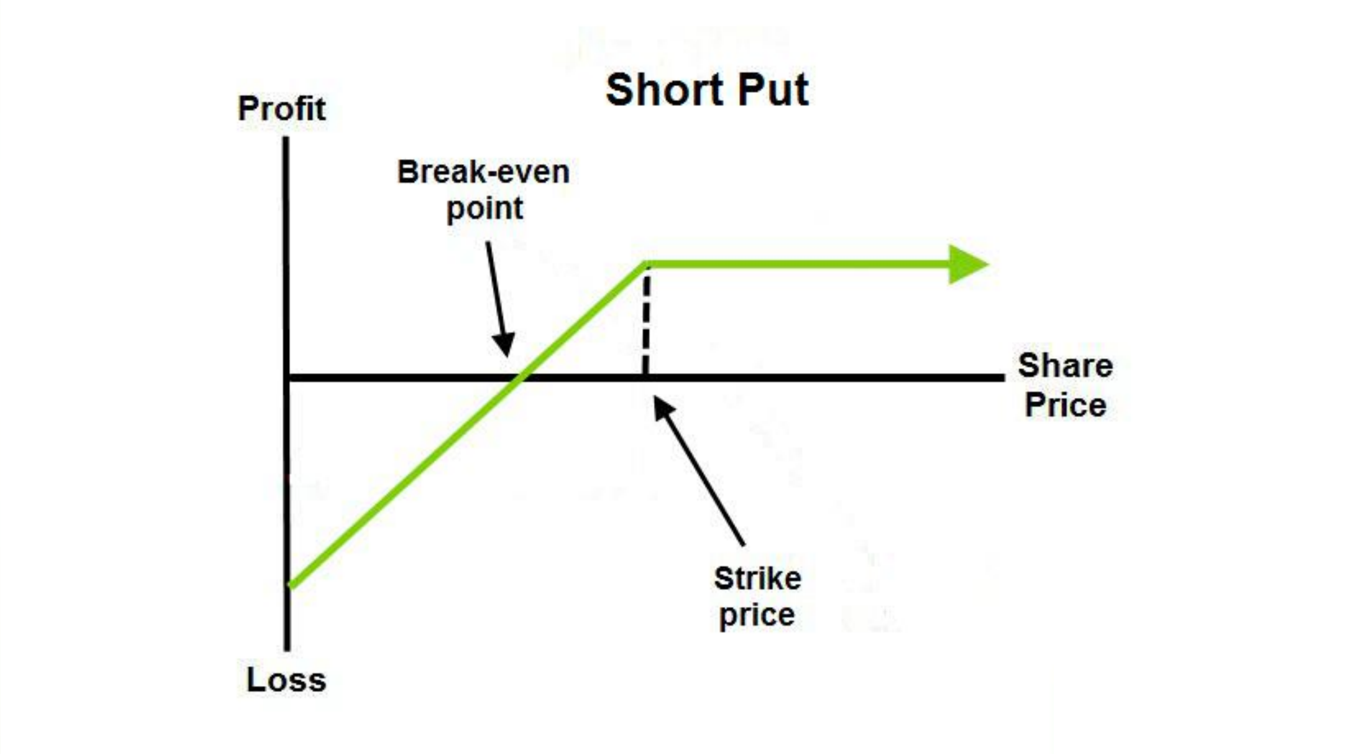


- Long Put is the opposite of Long Call. Traders take a position to benefit from the fall in the price of the underlying asset
- The risk is limited to premium while rewards are unlimited
- Trader pays the option premium in exchange for the right to sell share or index at a fixed price by a certain expiry date
- Breakeven Point = Strike Price of Long Put - Premium Paid

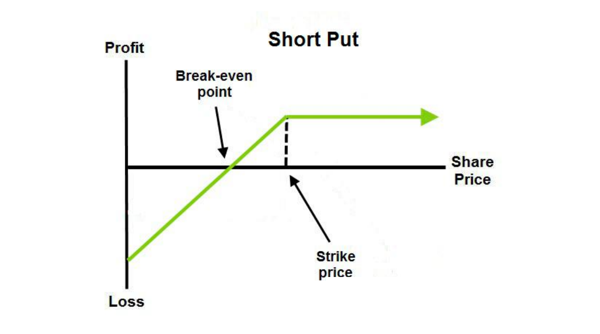
80 Strike , 20 prem, 60 breakevenn i.e. no loss no profit

- Maximum Profit = Achieved When Price of Underlying is 0. Profit = Strike Price of Long Put - Premium Paid
- The risk for this strategy is limited to the premium paid for the Put Option. Maximum loss will happen when price of underlying is greater than strike price of the Put option.
- Unlimited profit potential with risk only limited to loss of premium.
- You may incur 100% loss in premium if the underlying price rises.

Short Put

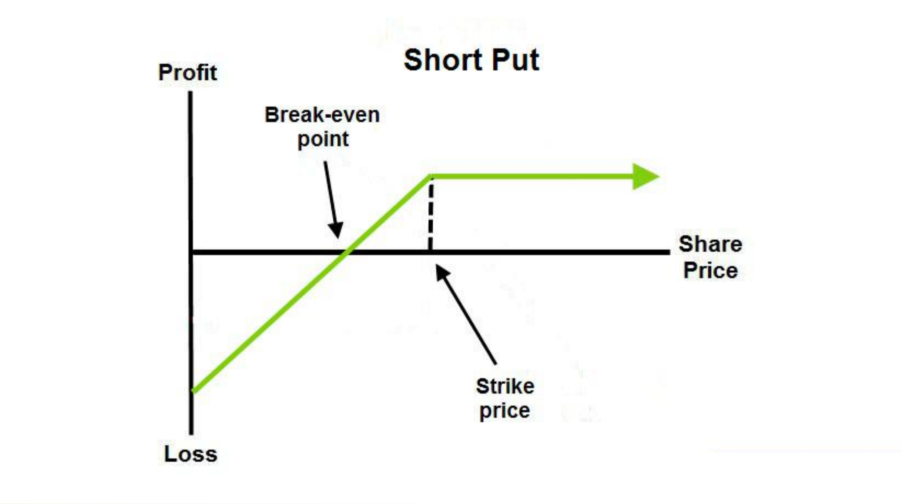

Short Put
Profit
Break-even point
Share Price
Strike price
Loss

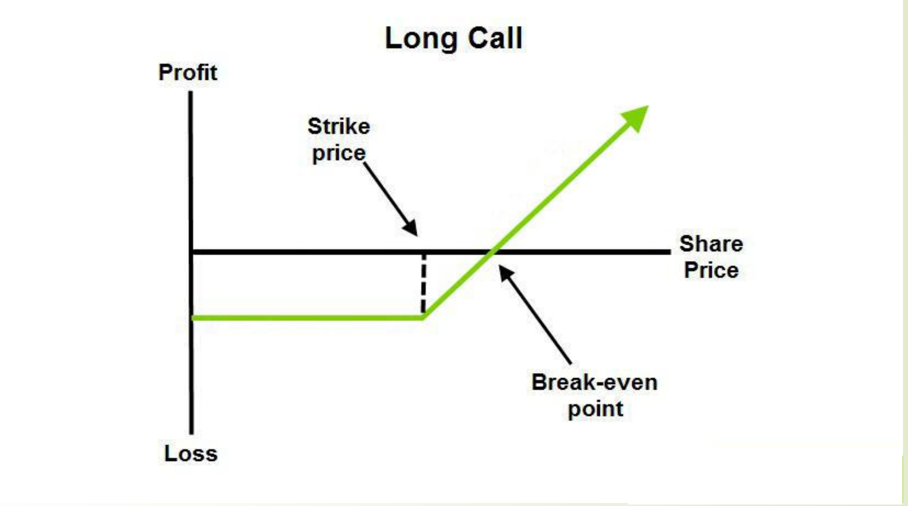
Short Put



- In this trading strategy, your view is that the price of an underlying will not move below a certain level
- The strategy involves entering into a single position of selling or writing a Put Option
- It has low profit potential and is exposed to unlimited risk
- Breakeven Point = Strike Price - Premium
- The risk is when the price of the underlying falls, and the Put is exercised. You are then obliged to buy the underlying at the strike price
- The profit is limited to premium received in your account when you sell the Put Option
- It allows you benefit from time decay. And earn income in a rising or range bound market scenario
- It is a high risk strategy and may cause huge losses if the price of the underlying falls steeply

Put Call Parity





Comparative Profit and Payoffs

Put Call Parity

Type text here

	t = 0	t = T	
Portfolio A		$S_T < K$	$S_T > K$
Buy Call	-c	0	S_T - K
Sell Put	+p	S_T - K	0
Total	**p-c**	**S_T - K**	**S_T - K**

Put Call Parity

	t = 0	**t = T**	
Portfolio B		$S_T < K$	$S_T > K$
Buy Stock	$-S_0$	S_T	S_T
Borrow	$Ke^{-(rT)}$	$-K$	$-K$
Total	$\mathbf{-S_0+Ke^{-(rT)}}$	$\mathbf{S_T - K}$	$\mathbf{S_T - K}$

Put Call Parity

	t = 0	t = T	
Portfolio A	p-c	S_T - K	S_T - K
Portfolio B	$-S_0+Ke^{-(rT)}$	S_T - K	S_T - K

$$p - c = -S_0 + Ke^{-(rT)} \quad => c + Ke^{-(rT)} = p + S_0$$

Arbitrage: $c + Ke^{-(rT)} > p + S_0$

	t = 0	t = T	
Portfolio B		$S_T < K$	$S_T > K$
Short Call	$+c$	0	$-(S_T - k)$
Borrow	$Ke^{-(rT)}$	$-K$	$-K$
Buy Stock	$-S_0$	S_T	S_T
Buy Put	$-p$	$K - S_T$	0
Total	$c+Ke^{-(rT)} - p - S_0 > 0$	**0**	**0**

(European) Call Put Parity

S = Rs 31

K = Rs 30

T = 3 Months

r = 10%

No Dividend

Call option price c = Rs 3

Put option price p = ?

If p = Rs 2.25?

$$c + Ke^{-rT} = p + S$$

$$p = c + Ke^{-rT} - S$$

$$p = 3 + 30e^{-.1*.25} - 31 = 1.26$$

If p = 2.25 then ?

Buy the call and short both put and stock,

-3 + 2.25+ 31 = 30.25, Invest the money for 3 month

30.25 $e^{-.1*.25}$ = 31.02

At expiration, if the stock price of option is greater than 30, the call will be exercised;

Profit = 31.02 – 30 = 1.02

Put Call Parity: Asset Generates Dividend

$$c + Ke^{-(rT)} = p + S_0 - D_0$$

Upper and Lower Bound of Options

If price of an option goes beyond the upper limit or lower limit, there is profitable situation for arbitrageurs

- Upper Bound of Call Option: c <= S
- Upper Bound of Put Option: p <= K
- Lower Bound of Call Option: $c > S - Ke^{-rT}$
- Lower Bound of Put Option: $p > Ke^{-rT} - (S - D)$

Lower Bound of Call Options

$$c > S - Ke^{-rT}$$

c = Rs. 3	S = Rs. 20
T = 1	r = 10%
K = Rs. 18	D = 0

Is there an arbitrage opportunity?

S_0- Ke^{-rt} = Rs. 3.71

Arbitrageur can buy the call and short the stock , Cash Inflow = 20-3 = 17, Invest for one year at 10% per annum, Total money = $17e^{0.1}$=Rs. 18.79

If stock price is greater than 18 he can exercise the option to buy the stock and close the short position and profit will be : Rs. 18.79 – Rs. 18 = 0.79

Lower Bound of Put Options

$$p > Ke^{-rT} - S$$

Suppose that

p	= Rs. 1	S	= Rs. 37
T	= 0.5	r	=5%
K	= Rs. 40	D	= 0

Is there an arbitrage opportunity?

$Ke^{-rt} - S$ = Rs. 2.01, It is more than the put price. The arbitrageurs can borrow Rs. 38 for six months to buy both put and the stock. He is required to pay $38e^{0.05*0.5}$= Rs. 38.96

If the stock price is below 40, the arbitrageur exercises the option to sell the stock for Rs. 40 repays the loan and makes a profit of Rs 40 – Rs. 38.96 = Rs. 1.04

Factors Affecting Option Prices

- Spot Price
- Strike Price
- Time Period
- Risk-free Rate
- Volatility of Asset
- Dividend or other Income

	Call Option	Put Option
Spot Price	+	-
Strike Price	-	+
Time Period	+	+
Risk-free Rate	+	-
Volatility of the Asset	+	+
Dividend or other Income	-	+