VaR Mapping

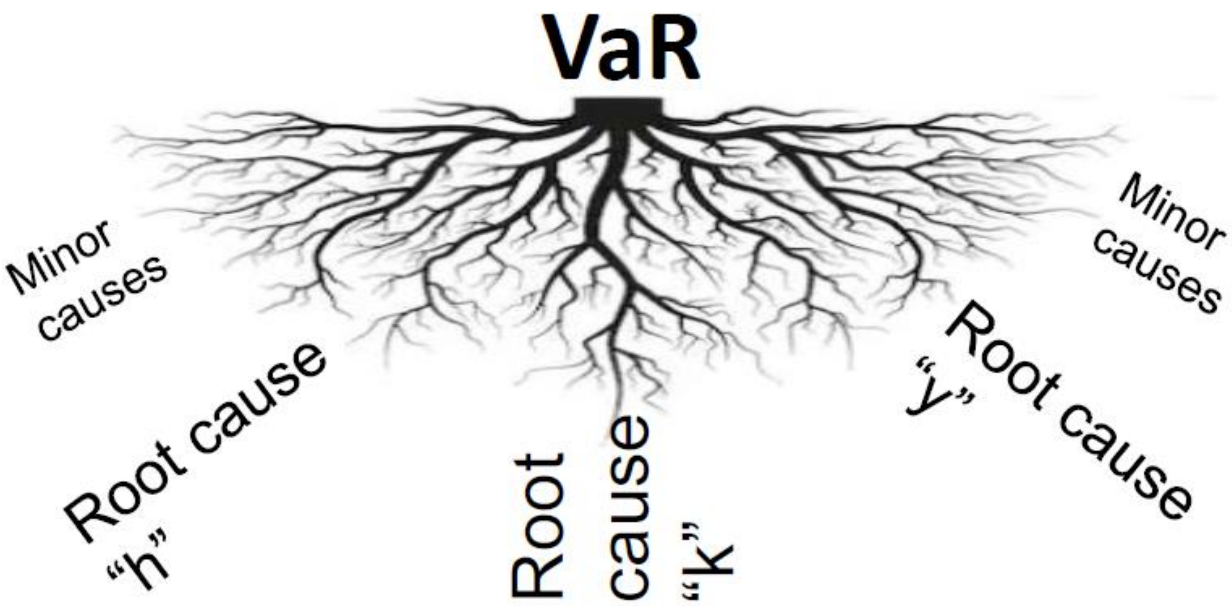
Market Risk

What is VaR Mapping?

Mapping refers to the process of replacing the current values of a portfolio with risk factor exposures.

- More generally, it is the process of replacing each instrument by its exposures on selected risk factors.

Through mapping, a complex portfolio or instrument can be broken down into its constituent elements that determine its value.



Why is Mapping So Important?

1. Mapping is necessary when we do not have sufficient data on positions.
   - Think of an emerging market instrument that has a very short trading record, meaning that there’s barely enough data on it.
   - In such circumstances, we might map our position to some comparable position for which we already have plenty of data and a good understanding of its risk exposure.
2. Mapping helps us to cut down on the dimensionality of covariance matrices and correlations.
   - Given a portfolio comprising of *n* instruments, we would need to gather data on n volatilities and *n(n-1)/2* correlations, resulting in a labyrinth of pieces of information.
   - As *n* increases, so does the amount of information we have to collect and process. It is important to keep the dimensionality of our covariance matrix at a manageable level to avoid computational problems.

Why is Mapping So Important?

3. Mapping helps avoid rank correlation problems
   - By handling many risk factors that are closely correlated (or even perfectly correlated in extreme cases), we might run into rank problems with the covariance matrix and end up producing pathological estimates that might lead to erroneous conclusions.
   - To avoid such problems, it is important that we select an appropriate set of risk factors that are not closely related.
4. Mapping greatly reduces the time needed to carry out risk assessment and related calculations
   - By reducing a portfolio comprised of many different positions to a consolidated set of risk-equivalent positions in basic risk factors, it is possible to conduct calculations at a faster speed.
   - The only downside to such a move is that precision is lost.

Principles Underlying Mapping

- VaR mapping help us to aggregate risk factors in situations where considering each position separately is computationally intensive
- VaR Mapping is useful for measuring changes over time. When managing the risk attached to bonds for instance, risk exposure can be mapped to spot yields that reflects the current position
- VaR mapping quit useful when historical data is not available.

The Mapping Process

Step 1

Mark all positions to market value (in current dollars or reference currency)

This involves establishing the current market value of all positions.

Step 2

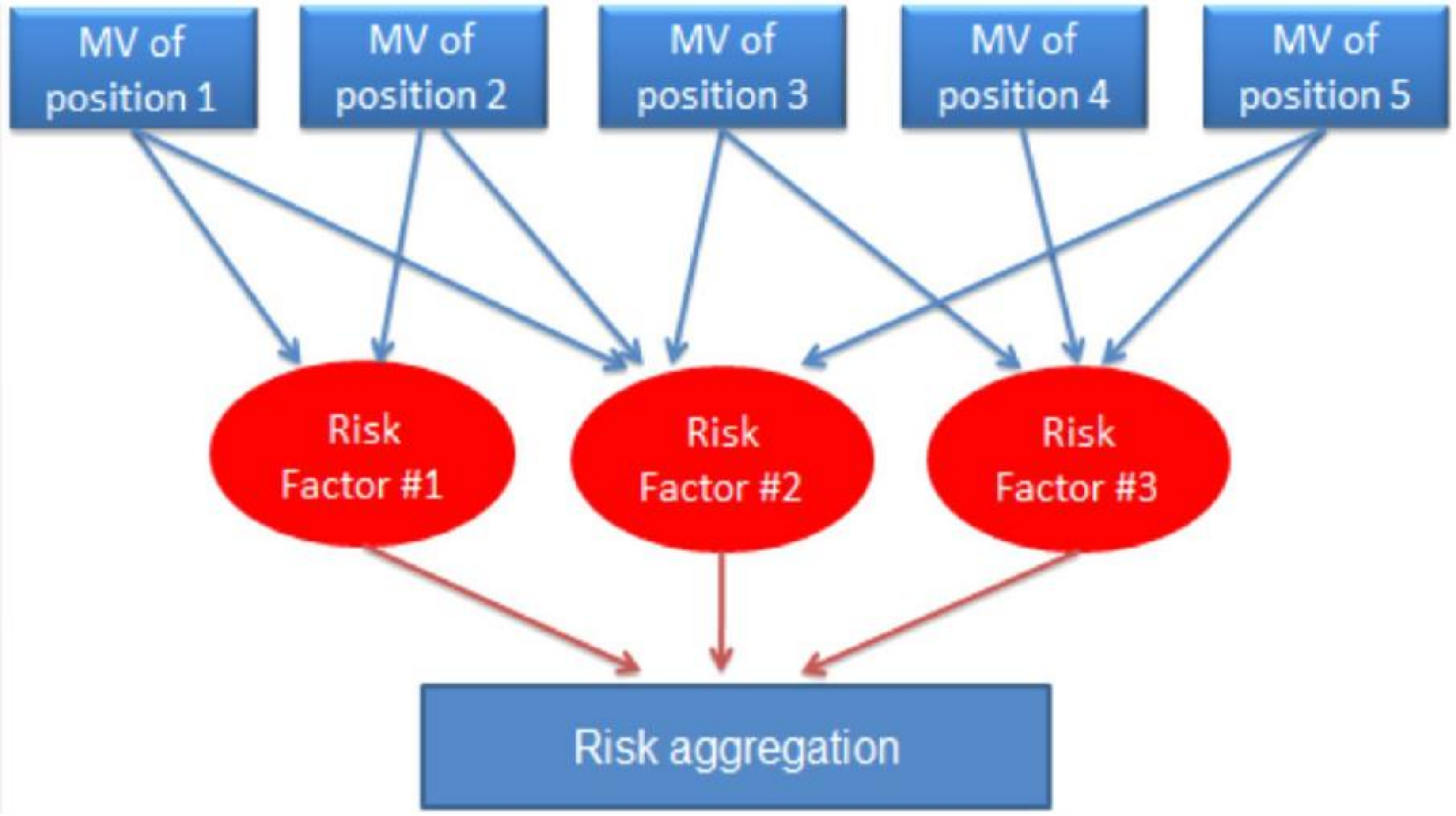
Identify common risk factors for different investment positions

A good example of a common risk factor could be a specific exchange rate, say the Euro versus the dollar.

Step 3

Allocate the market value for each position/instrument to the risk factors

The Mapping Process



The Mapping Process

Step 4

- Construct a risk factor distribution and inputs all data into the risk model
- The market value of position number 1, for example, is allocated to the risk exposures in the first row, $x_{1,1}$, $x_{1,2}$, and $x_{1,3}$.

Step 5

- Sum the risk factors in each column then create a vector consisting of three risk exposures

The Mapping Process

Investment/ Position	Market value	Risk Factor 1	Risk Factor 2	Risk Factor 3
1	MV_1	x_{11}	x_{12}	x_{13}
2	MV_2	x_{21}	x_{22}	x_{23}
3	MV_3	x_{31}	x_{32}	x_{33}
4	MV_4	x_{41}	x_{42}	x_{43}
5	MV_5	x_{51}	x_{52}	x_{53}
Total portfolio	MV	$x_1 = \sum_{i=1}^{5} x_{i1}$	$x_2 = \sum_{i=1}^{5} x_{i2}$	$x_3 = \sum_{i=1}^{5} x_{i3}$

The five positions above could be **any six instruments**, say, forward contracts on the same currency but with different maturities.

Through mapping, these positions can be **replaced by exposures** on three risk factors only – factors 1, 2, and 3.

William Sharpe’s Diagonal Model

- The idea of risk mapping can be seen in William Sharpe’s diagonal model where he attempts to simplify risk measurement in a portfolio made up of many stock positions.
- The model decomposes individual stock return movements into two components: a common index component and an idiosyncratic component.
- The latter disappears as more and more positions are added to the portfolio, leaving the common index component as the main driver of risk.

What’s the Ideal No. of Risk Factors to Work with?

The choice and number of general risk factors will directly affect the size of specific risks. Specific risks are the risks that affect specific assets in the portfolio. They are issuer-specific as opposed to market-related or general risk.

To demonstrate just how the choice and number of general risk factors affect the size of specific risks, consider a portfolio of bonds with different maturities, ratings, terms, and different denominations.

- Let’s say we start with duration as the only risk factor. In this case, there will be a significant proportion of specific risk.
- If we add another risk factor, say, for credit risk, we would expect the amount of specific risk to reduce.
- If we add yet another factor for currency risk, we would chip away at specific risk even further.

Methods of Mapping Portfolios of Fixed Income Securities

Principal Mapping

Bond risk is associated with the maturity of the principal payment only (risk of prepayment of the principal amount, ignoring all other intervening payments). One factor is chosen that corresponds to the average maturity of the portfolio.

Duration Mapping

Bond risk is mapped to a zero-coupon bond with maturity equal to the bond duration.

We calculate VaR by using the risk level of the zero-coupon bond that equals the duration of the portfolio

Cash Flow Mapping

The risk of fixed-income instrument is decomposed into the risk of each of the bond cash flows.

The present values of all cash flows are mapped onto the risk factors for zeros of the same maturities

Example:
How to Map a Fixed Income Portfolio into Positions of Standard Instruments

Suppose a portfolio consists of two par value bonds.

- Bond 1: market value = $100 million; coupon rate = 4%; maturity = 1 year
- Bond 2: market value = $100 million; coupon rate = 6%; maturity = 5 years

The yields, yield VaRs, durations, and return VaRs for zero-coupon bonds with maturities ranging from one to five years (at the 95% confidence level) are as follows:

Maturity (yrs)	Yield	Yield VaR	Mac Dur	Modified dur. (yrs)	Returns VaR (%)
1	5.83%	0.497%	1.0	0.945	0.4697
2	5.71%	0.522%	2.0	1.892	0.9876
3	5.81%	0.523%	3.0	2.835	1.4827
4	5.89%	0.522%	4.0	3.778	1.9721
5	5.96%	0.514%	5.0	4.719	2.4256

Returns VaR = Mod. Duration × Yield VaR
For example, at 4 years, the returns VaR = 3.778 × 0.522% = 1.9721

How VaR can be used as a Performance Benchmark

To benchmark a portfolio, we measure the **VaR of the portfolio** relative to the **VaR of a benchmark**.

- The VaR of the **deviation between the two portfolios** is referred to as a **tracking error VaR**.

If x is the vector position of the portfolio and x_0 the vector position of the index, then the tracking error VaR is given by:

$$\text{Tracking error} = \alpha\sqrt{(x - x_0)'\sum(x - x_0)}$$

If the **tracking error is $y**, the **maximum deviation** between the index and the portfolio is **$y**.

Mapping Derivatives Instruments

To mapping complex or exotic instruments, it is important to decompose the instrument into two or more constituent instruments.

Forwards:

- A long position in a forward contract has, for example, three building blocks:
- A short position in a U.S. Treasury bill.
- A long position in a one-year euro bill.
- A long position in the euro spot market.

Forward rate agreements:

- An FRA is equivalent to a portfolio:
- Long in a zero-coupon bond of one maturity; and
- Short in a zero-coupon bond of a different maturity.

Thus it is possible to map an FRA and estimate its VaR by treating it as a long–short combination of two zeros of different maturities.

Mapping Derivatives Instruments

Options

A change in option price/value can be approximated by taking **partial derivatives**.

A long position in an option can be split into two building blocks:

A long position in the stock equal to delta (Δ); and

A short position in the underlying asset financed by a loan equal to [(Δ shares) – value of the call].

FX forwards

A foreign-exchange forward is the equivalent of:

A long position in a foreign currency zero-coupon bond; and

A short position in a domestic currency zero-coupon bond.