# Risk Neutral Probability

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**Abstract**

Here, we introduce the concept of risk neutral probability (or measure) and contrast it with the real world measure.

## 1 The parable of a book keeper [Baxter and Rennie]

• Imagine a bookkeeper tasked with determining the fair price of bets on an election outcome. There are two possible outcomes: Candidate A wins or Candidate B wins. Also assume that the bookkeeper has an access to a very accurate survey which predicts that

- Candidate A wins with 70% probability
- Candidate B wins with 30% probability

• Suppose P wants to bet an amount $c$ + transaction cost on candidate A winning. The transaction cost is supposed to be the bookkeeper's fee for providing the opportunity for people to bet on their favorite candidates. The bookkeeper has to give P a payoff. He cannot give too low a payoff, as then P will go to a bookkeeper providing higher payoff. At the same time, he cannot offer too high a payoff as well, otherwise he will incur huge loss. One way to proceed is for the bookkeeper to follow the real world probability.

- P has bet an amount $c$ on A.
- The odds of A winning is 70%.
- So, $c = .7 \times$ payoff, implying payoff $= c/.7$.

• Now suppose 400 rupees (+ transaction costs) has been bet on A winning and 200 rupees (+ transaction costs) bet on B winning. There are two possibilities, lets see how much the bookkeeper makes in each case.

- A wins (70% probability): transaction costs + 600 - 400/.7 ≈ transaction costs + 29
- B wins (30% probability): transaction costs + 600 - 200/.3 ≈ transaction costs - 66

• If the transaction costs are low, the bookkeeper will end up loosing money when B wins, which is not desirable. He wants to make a steady amount on transactions irrespective of who wins. A way to achieve this is to forget about real world probabilities and just concentrate on the amount of money bet on each candidate. If A wins, the entire pot (600 rupees) goes to people betting on A. Similarly, if B wins, the entire pot goes to B. Following this, the bookkeeper need not worry about who wins, he makes money either way.

• In this approach, the odds that people get to bet on their favorite candidates happens to be 600/400 for A and 600/200 for B. This can be interpreted as a 'risk-neutral' probability that the market assigns to the win of each candidate:

- Market (or the people betting their money) think that A will win with probability 4/6.
- Similarly, market thinks B will win with probability 2/6.

• In other words, 'risk-neutral' probability is a way to describe what the market thinks about an event rather than an actual real world probability. By assigning the risk-neutral probabilities, we can still use the language and tools from probability. For example, the price of betting on a candidate winning can now be expressed as the expected payoff under this risk-neutral probability.

## 2 Risk Neutral probability in finance

• Here, a financial intermediary or Bank is tasked with determining the fair price of a financial asset or derivative. Roughly speaking, similar to previous case, the bank does not care about the actual probabilities of outcomes. Instead, they focus on creating a pricing framework so that they will make money on transaction costs without worrying about the actual outcomes of events.

• For example, suppose these are the two bets in the market:

- 200 rupees on the stock price going up the next day
- 400 rupees on the stock price going down the next day

The bank will assign 'risk-neutral' probabilities of 2/6 and 4/6 to these two events, so that the prices can now be expressed as expected payoffs under these probabilities.

• **Problem 1.** Given the following bets, how would you compute the risk-neutral probabilities of events?

- 200 rupees on the stock price becoming $> 100$
- 400 rupees on the stock price becoming $< 100$

• **Problem 2.** Given the following bets, how would you compute the risk-neutral probabilities of events?

- 200 rupees on the stock price becoming $> 100$
- 400 rupees on the stock price becoming $< 90$

• **Problem 3.** Given the following bets, how would you compute the risk-neutral probabilities of events?

- 200 rupees on the stock price becoming $> 100$
- 400 rupees on the stock price becoming $< 90$
- 600 rupees on the stock price staying between 80 and 120.