Financial Modelling With Python - Basic Financial Mathematics

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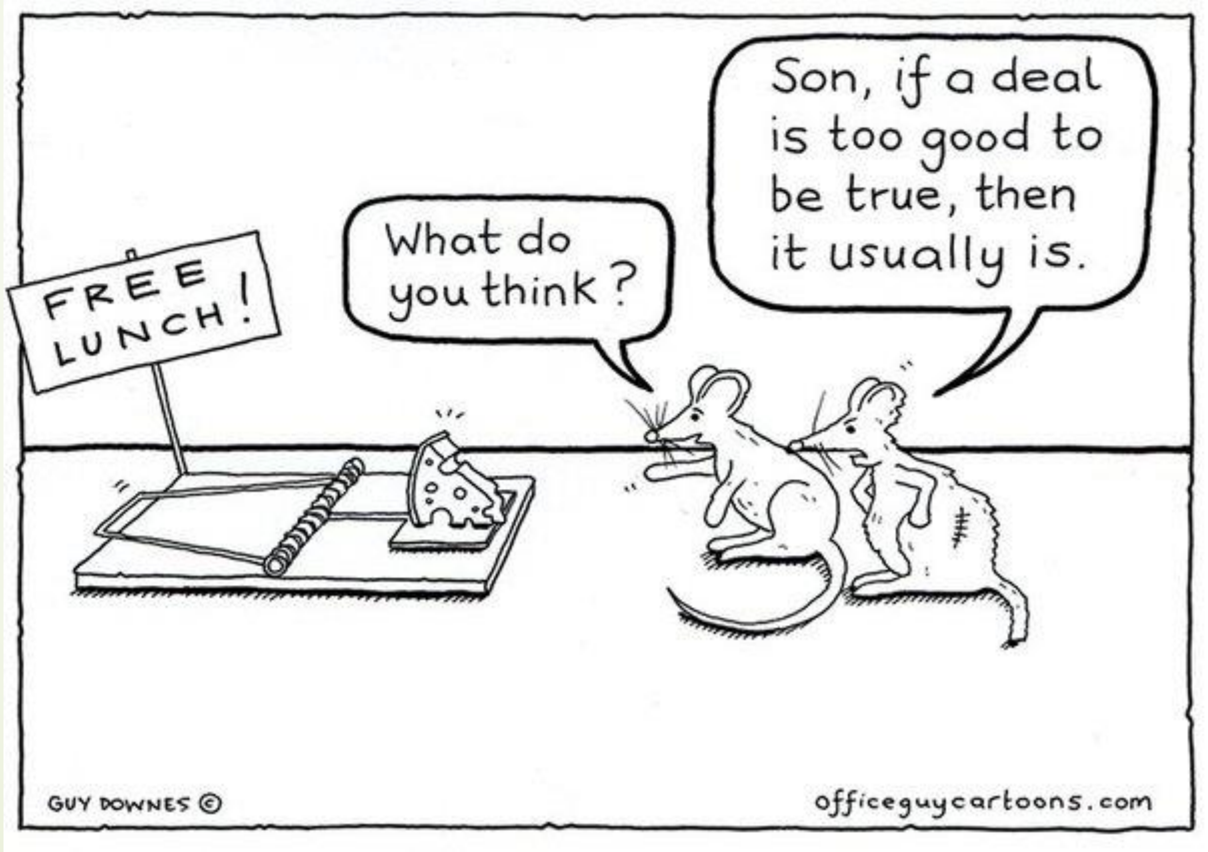
There is No Such Thing as a Free Lunch



Time Value of Money

- $PV = \frac{1}{(1+r)^n} FV$

FV is the future value

PV is the present value

'r' is the annual rate of interest

'n' is the number of years

- Would you prefer to get Rs. 1000 today or Rs. 1050 one year from now? – Interest rate is 6%.
- Present value of Rs. 1050 ≈ Rs. 990.60

$1000 \times (1+.06)^n$

$1000 \times (1+.06) = 1060$

$1060 \times (1+.06) = 1$

$\times (1+.06)$

$\times (1+.06)$

Time Value of Money

- $FV = PV * (1 + r)^n$

FV is the future value

PV is the present value

‘r’ is the annual rate of interest

‘n’ is the number of years

- You have borrowed Rs. 1000 from your friend. How much you should pay after 1 year? Interest rate is 6%.
- Rs. 1060.00

$FV = PV\left(1 + \frac{r}{m}\right)^{mn}$

$r \to m$

$\uparrow 6$

$r \to$ semi

$100\left(1 + \frac{.06}{2}\right)\left(1 + \frac{r}{2}\right)$

$100(1 + 0.03)(1 + 1.03)$

Time Value of Money

If interest is compounded ‘m’ times per annum

- $FV = PV * (1 + \frac{r}{m})^{nm}$

FV is the future value

PV is the present value

‘r’ is the annual rate of interest

‘n’ is the number of years

- You have borrowed Rs. 1000 from your friend. How much you should pay after 1 year? Interest rate is 6%.

- Rs. 1060.00

  Annual compounding, m = 1
- Rs. 1060.90

  Semi-annual compounding, m = 2
- Rs. 1061.68

  Monthly compounding, m = 12
- Rs. 1061.83

  Daily compounding, m = 365

Time Value of Money

If interest is compounded 'm' times per annum

- $FV = PV * (1 + \frac{r}{m})^{nm}$

FV is the future value

PV is the present value

'r' is the annual rate of interest

'n' is the number of years

- If 'm' approaches infinity
- $(1 + \frac{r}{m})^{m}$ tends to e^{r}
- $FV = PV * e^{rn}$
- Continuous Compounding

$\left(1+\frac{1}{x}\right)^{x} \to e$

$x \to \infty$

Time Value of Money

- If you receive Rs. 1000 per year for the next six years, what is the net present value if the interest rate is 6%.

Year	Cash flow	PV
1	1000.00	943.40
2	1000.00	890.00
3	1000.00	839.62
4	1000.00	792.09
5	1000.00	747.26
6	1000.00	704.96
Total	6000.00	4917.32

1000/(~~1~~.06)*1.06

Annuity

- An ordinary annuity pays out the same money at the end of each year for n years.

$$PV = \sum_{i=1}^{n} C \frac{1}{(1+r)^i}$$

$$PV = C\frac{1-(1+r)^{-n}}{r}$$

- The PV of an annuity of Rs. 1000 per annum for 6 years at an annual interest rate of 6% is:

$$1000\frac{1-(1.06)^{-6}}{0.06} = 4917.32$$

Year	Cash flow	PV
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4	1000.00	792.09
5	1000.00	747.26
6	1000.00	704.96
Total	6000.00	4917.32

Annuity

- If m payments of capital C (or money) each are received per year
- A general Annuity

$$PV = C\,\frac{1-\left(1+\frac{r}{m}\right)^{-nm}}{\frac{r}{m}}$$

Amortization

- Amortization is a method of repaying a loan through regular payments of interest and principal

- The size of the loan – the original balance – is reduced by the principal part of the payment

- $C = PV * \frac{\frac{r}{m}}{1-(1+\frac{r}{m})^{-mn}}$

- A Student takes out a 15-year Rs. 10,00,000 loan at an 8.0% interest rate

C is the money which is fixed for all the installments. With this payment everytime the principal and interest is reduced to some extent. Interest = Remaining balance*interest rate/m

Amortization

prinicpal = installment - interest

- A Student takes out a 15-year Rs. 10,00,000 loan at an 8.0% interest rate
- C = 9556.52

After 1st month:

- Interest = 10,00,000 * 0.08/12 = 6666.67
- Principal = 9556.52 - 6666.67 = 2889.85
- Principal Remaining = 10,00,000 - 2889.85 = 997110.15

Month	Interest	Principal	Remaining
1	6666.67	2889.85	997110.15
2	6647.40	2909.12	994201.03
3	6628.01	2928.51	991272.51
4	6608.48	2948.04	988324.47
5	6588.83	2967.69	985356.78
6	6569.05	2987.48	982369.31

9556.52
6666.67
997110.15
t2

Amortization

- A Student takes out a 15-year Rs. 10,00,000 loan at an 8.0% interest rate
- C = 9556.52

After 2nd month:

- Interest = 997110.15 * 0.08/12 = 6647.40
- Principal = 9556.52 - 6647.40 = 2909.12
- Principal Remaining = 997110.15 - 2909.12 = 994201.03

Month	Interest	Principal	Remaining
1	6666.67	2889.85	997110.15
2	6647.40	2909.12	994201.03
3	6628.01	2928.51	991272.51
4	6608.48	2948.04	988324.47
5	6588.83	2967.69	985356.78
6	6569.05	2987.48	982369.31
…	…	…	…
179	126.16	9430.36	9493.23
180	63.29	9493.23	0.00

Internal Rate of Return (IRR)

- The IRR is the interest rate that equates an investment's PV with its price P

- $$Inv = \frac{C_1}{1+y} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \cdots + \frac{C_n}{(1+y)^n}$$

Exercise: Suppose you have given Rs. 1000 to your friend. Your friend will be returning Rs 50

Internal Rate of Return (IRR)

- Exercise: Suppose you have given Rs. 1000 to your friend on 1st January 2021 . Your friend will be returning your money as shown in the table.
- What is the IRR

Date	Amount (Rs)
01-Feb-21	50
01-Mar-21	50
01-Apr-21	50
01-May-21	50
01-Jun-21	100
01-Jul-21	100
01-Aug-21	100
01-Sep-21	100
01-Oct-21	100
01-Nov-21	200
01-Dec-21	200

Practice

- With an annual interest rate of 10% compounded twice per annum, what will be the equivalent interest rate compounded once per annum?
- You are liable to pay Rs 20 million 4 years from now. So, you are thinking of investing some money today to exercise the liabilities. The SBI is offering 7% rate of interest semiannually compounded. How much money you must invest today?
- Verify that, given an annual rate, the effective annual rate is higher the higher the frequency of compounding.
- Suppose that r_1 is the annual rate with continuous compounding and r_2 is the equivalent rate compounded m times per annum. Express r_1 in terms of r_2 and r_2 in terms of r_1.
- An annuity that lasts forever is called a perpetual annuity. Show that

$$PV = \frac{mC}{r}$$

- Write a program to calculate amortization schedules with loan amount L and monthly interested rate r. Validate your program with the example given in slide no 12.