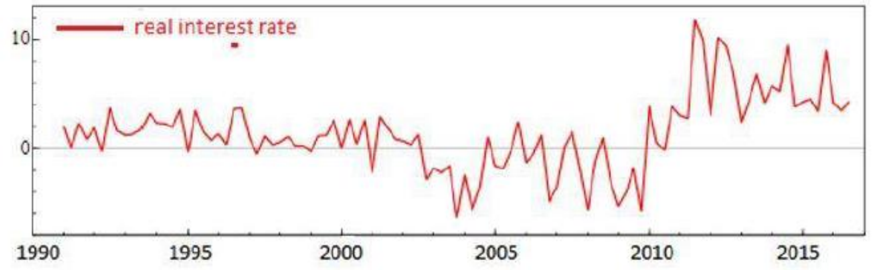
Estimating Volatility

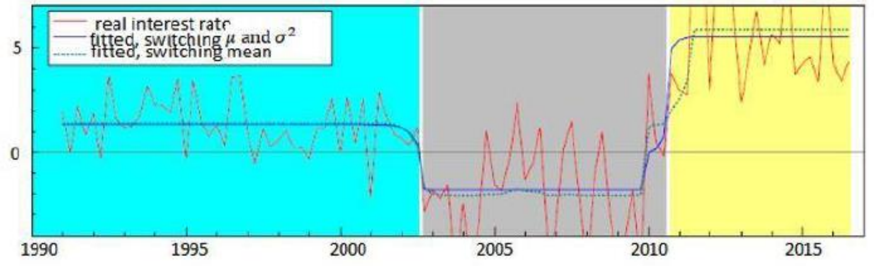
The Implications of Regime Switching on Quantifying Volatility

To better model asset returns, analysts may subdivide a specified time period into regimes.

For illustration, let's use real interest rates of country between 1990 and 2015:



From this econometric model, we can identify **three distinct states of the economy**(or regimes).



Implied Volatility

Volatility of the stock price is the only unobservable parameter in the BSM pricing formula.

The implied volatility of an option is the volatility for which the BSM option price equals the market price.

Implied volatility represents the expected volatility of a stock over the life of the option.

It is influenced by market expectations of the share price as well as by supply and demand of the underlying options.

If we use the observable parameters in the BSM formula and set the BSM formula equal to the market price, then it's possible to solve for volatility that satisfies the equation.

However, there is no closed-form formula for the volatility, and the only way to find it is through iteration.

These calm and stormy periods tend to cluster together—this is called volatility clustering.

Asset Price Volatility:
The Arch And Garch Models

Volatility Clustering: Periods of turbulence in which prices show wide swings and periods of tranquility in which there is relative calm.

calm

Financial time series often exhibit the phenomenon of volatility clustering.

This results in correlation in error variance over time.

Over time, the variance (spread) of these errors is not constant—they're correlated. Big errors tend to follow other big errors.

Use autoregressive conditional heteroscedasticity (ARCH) models to consider such correlation or *time-varying volatility*.

These models account for changing volatility over time (aka "time-varying volatility").

They use past data to estimate how volatile today or tomorrow might be.

The ARCH Model

- This model shows that conditional on the information available up to time *(t-1)*, the value of the random variable *Y* is a function of the variable *X*:

$$Y_t \left| I_{t-1} = \alpha + \beta X_t + u_t \right.$$

- We assume that given the information available up to time *(t – 1)*, the error term is independently and identically normally distributed with mean value of 0 and variance of σ_t^2 (heteroscedastic variance):

$$u_t \left| I_{t-1} = iid\ N(0, \sigma_t^2) \right.$$

- Assume that the error variance at time t is equal to some constant plus a constant multiplied by the squared error term in the previous time period:

$$\sigma_t^2 = \lambda_0 + \lambda_1 u_{t-1}^2, \qquad \text{where } 0 \le \lambda_1 < 1$$

The ARCH Model

- The ARCH(1) model includes only one lagged squared value of the error term

- An ARCH(p) model has p lagged squared error terms, as follows:

$$\sigma_t^2 = \lambda_0 + \lambda_1 u_{t-1}^2 + \lambda_2 u_{t-2}^2 + \ldots + \lambda_p u_{t-p}^2$$

- If there is an ARCH effect, it can be tested by the statistical significance of the estimated coefficients.

- If they are significantly different from zero, we can conclude that there is an ARCH effect.

r sq t-1 : squared return(error term) from previous time step
sigma t-1 sqia

The GARCH Model

- In its simplest form, the variance equation in the GARCH model is modified as follows:

$$\sigma_t^2 = \lambda_0 + \lambda_1 r_{t-1}^2 + \lambda_2 \sigma_{t-1}^2$$

- This is known as the GARCH (1,1) model.

recursive form builds in the effect of many past lags of squared returns indirectly through the variance term

- The ARCH (p) model is equivalent to GARCH (1,1) as p increases. p >> infinity
- Note that in the ARCH (p) we must estimate (p+1) coefficients, whereas in the GARCH (1,1) model given in we must estimate only three coefficients.
- The GARCH (1,1) model can be generalized to the GARCH (p, q) model with p lagged squared error terms and q lagged conditional variance terms, but in practice GARCH (1,1) has proved useful to model returns on financial assets.

$$\sigma_t^2 = a + b_1 r_{t-1,t}^2 + b_2 r_{t-2,t-1}^2 + \cdots + b_p r_{t-p,t-p+1}^2 + c_1 \sigma_{t-1}^2 + c_2 \sigma_{t-2}^2 + \cdots + c_q \sigma_{t-q}^2$$

GARCH and EWMA Model

Suppose,

Weight $=(\mathbf{1}-\boldsymbol{\lambda})\boldsymbol{\lambda}^{t}$

The forecast is a weighted average of the previous forecast, with weight λ, and the latest squared innovation, with weight*(1−λ):*

imp $\boldsymbol{\sigma_t}^{2}=\boldsymbol{\lambda}\boldsymbol{\sigma_{t-1}}^{2}+(\mathbf{1}-\boldsymbol{\lambda})\boldsymbol{r_{t-1,t}}^{2}$ r sq t-1 is the most recent squared log return (from today)

EWMA Model

$$\boldsymbol{\sigma_t}^{2}=(\mathbf{1}-\boldsymbol{\lambda})(\boldsymbol{\lambda}^{0}\boldsymbol{r_{t-1,t}}^{2}+\boldsymbol{\lambda}^{1}\boldsymbol{r_{t-2,t-1}}^{2}+\boldsymbol{\lambda}^{2}\boldsymbol{r_{t-3,t-2}}^{2}+\cdots+\boldsymbol{\lambda}^{N}\boldsymbol{r_{t-N-1,t-N}}^{2})$$

GARCH Model

Example

- The current estimate of daily volatility is 2.3%.
- The closing price of an asset yesterday was INR 46.
- Today, the asset ended the day at INR 47.20.
- Using log returns and the exponentially weighted moving average model with λ=0.94, determine the updated estimate of volatility.

Solution

- Current log-return= ln(47.2) –ln(46) = 3.85439 –3.82864 = 0.02575 = 2.575%
- The updated variance is given by:

r sq t-1 is the most recent squared log return (from today)

- σ_t=λ(current volatility)^2+(1−λ)(current return)^2
- sq σ_t =0.94(0.023)^2+0.06(0.02575)^2=0.000537
- Updated estimate of volatility = 0.000537=2.32%

take the square root

Nonparametric Approaches for VaR

Historical Simulation Method returns are weighted equally based on *k*, the number of observations used*(Weight=1/k).*

The hybrid approach uses historical simulation to produce estimates of return. The steps required for implementation of the hybrid approach are:

*Step 1:*Assign weights for historical realized returns to the most recent k returns using an exponential smoothing process:

$$\left[\frac{1-\lambda}{1-\lambda^k}\right]\lambda^0, \left[\frac{1-\lambda}{1-\lambda^k}\right]\lambda^1, \left[\frac{1-\lambda}{1-\lambda^k}\right]\lambda^2, \ldots, \left[\frac{1-\lambda}{1-\lambda^k}\right]\lambda^{k-1}$$

*Step 2:*Order the returns

*Step 3:*Determine the VaR for the portfolio by starting with the lowest return and accumulating the weights until percentage is reached.

Nonparametric vs. Parametric VaR methods

Advantages of nonparametric models:

- They do not require any assumptions regarding the distribution of returns to estimate VaR
- Multivariate density estimation allows for weights to vary, regardless of the timing of the data
- Problems posed by fat tails, skewness, and other deviations from the assumed distribution are avoided
- Multivariate density estimation exhibits a lot of flexibility in introducing dependence on economic variables

Disadvantages of nonparametric approaches:

- Subdividing the full sample data into different market regimes reduces the amount of data available for historical simulations
- Their use of data is less efficient compared to parametric models
- Multivariate density estimation requires a large amount of data that has a direct relationship with the number of conditioning variables incorporated in the model
- Multivariate density estimation may lead to data snooping or overfitting when working to produce weighting scheme assumptions