

Exercises:

1. Prove that expectation of sum of random variables = sum of expectations [ linearity of expectation, we do not need independence ]
2. Prove that variance of sum of independent variables = sum of variances [ here, we do need independence ]
3. We spoke about two ways to estimate the value of an integral via Monte Carlo: (a) Enclosing by rectangle (b) Computing expectation. Do error analysis numerically, and find out which one is better. Also compute  $\text{sig}/\sqrt{n}$  analytically and confirm the numerical findings.

A nice source to read on Monte Carlo methods:

<https://artowen.su.domains/mc/>

Questions on Prob:

1 - [https://www.probabilitycourse.com/chapter11/11\\_2\\_7\\_solved\\_probs.php](https://www.probabilitycourse.com/chapter11/11_2_7_solved_probs.php)

Markov Chains :

2 - <https://bookdown.org/jkang37/stochastic-process-lecture-notes/hw02.html>

Beyond mid :

Some notebooks on numerical integration and Ito Lemma.

Task:

1. Play around with Integration wrt Brownian Motion. Is Left sum - Right sum always equal to  $T$ ?
2. We motivated why  $\sum (\Delta B^2)$  goes to  $\sum (\Delta t)$ . Check this numerically as well.
3. Geometric Brownian Motion solution -- Get the histogram numerically via Monte Carlo and compare with the derived closed form solution. Do they match ?

I would also suggest you to go through this youtube video:

<https://youtu.be/A5w-dEgIU1M?si=15pWE5RJdJYoWgWa>

Some solved problems on Brownian Motion and Ito Lemma:

<https://www.math.drexel.edu/~song/Gene%20Golub%20Summer%20School/Song/Exercise%202.pdf>

<https://math.nyu.edu/~goodman/teaching/StochCalc2018/notes/Lesson4.pdf>

<https://uregina.ca/~kozdrn/Teaching/Regina/441Fall14/Notes/L28-Nov10.pdf>