Simulations in Multivariate Setup

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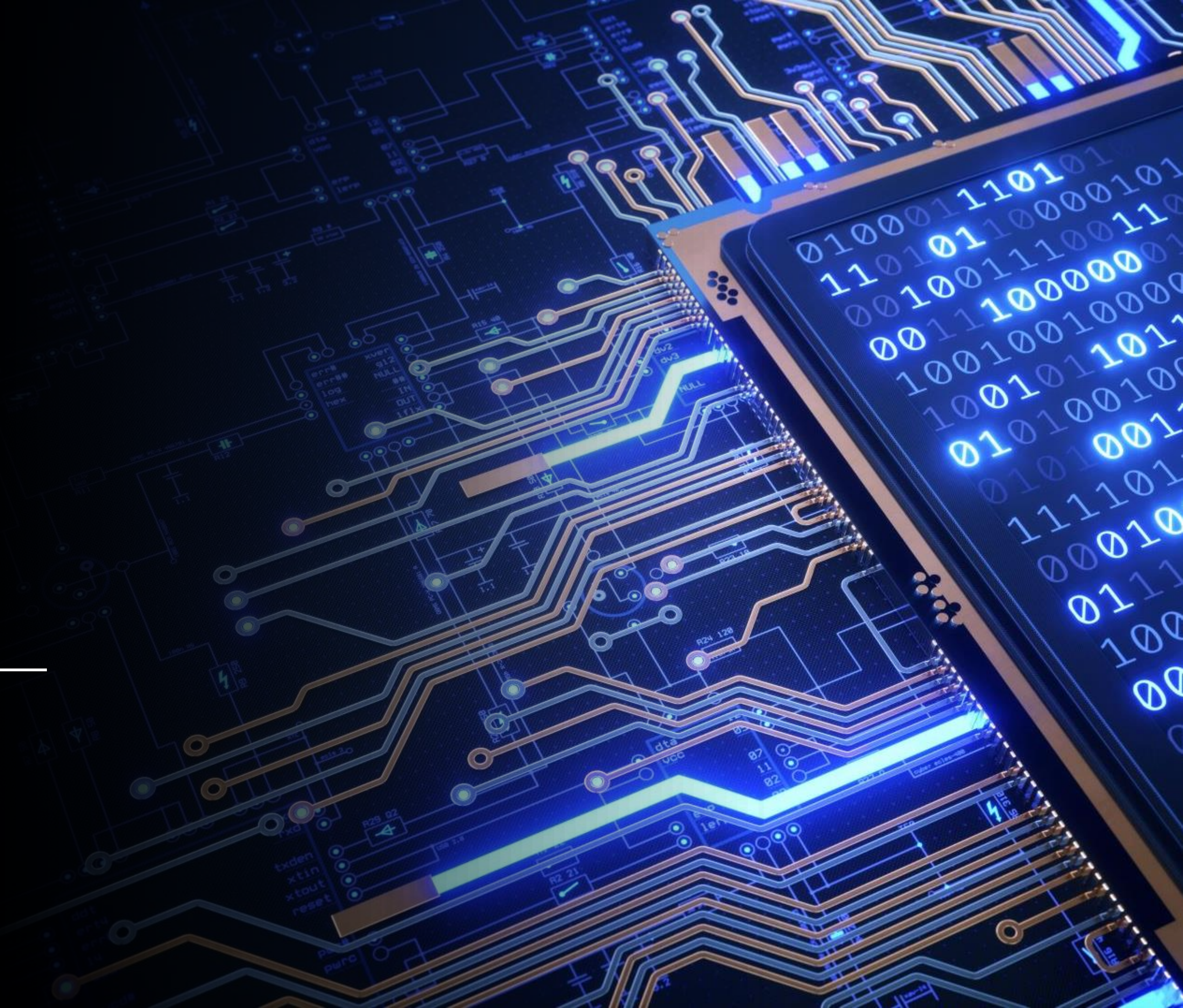

Toy Example

Fixed Cost	Cost Per Unit	Average Sales Per Month	Average Price Per Unit	Profit
₹ 10,000	₹ 200	100	₹ 320	₹ 2,000

What is the probability that you have some profit?

What is the probability that your profit will exceed ₹2000?

Toy Example

Fixed Cost	Cost Per Unit	Average Sales Per Month	Average Price Per Unit	Profit
₹ 10,000	₹ 200	100	₹ 320	₹ 2,000
		Normal(100,10)		Normal(2000,10)

What is the probability that you have some profit?

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Toy Example

Fixed Cost	Cost Per Unit	Average Sales Per Month	Average Price Per Unit	Profit
₹ 10,000	₹ 200	100	₹ 320	₹ 2,000
		Normal(100,10)	Normal(320,5)	Simulation

What is the probability that you have some profit?

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Toy Example

Fixed Cost	Cost Per Unit	Average Sales Per Month	Average Price Per Unit	Profit
₹ 10,000	₹ 200	100	₹ 320	₹ 2,000
		Normal(100,10)	Normal(320,5)	??

Price	Sales
317.92	103.78
326.91	105.54
326.78	102.76
320.13	105.57

What is the probability that you have some profit?

What is the probability that your profit will exceed ₹2000?

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		Normal(100,10)	Normal(320,5)	??
		Correlation = -0.7		

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Toy Example

Fixed Cost	Cost Per Unit	Sales Per Month	Average Price Per Unit	Profit
₹ 10,000	₹ 200	100	₹ 320	₹ 2,000
		Hypergeometric	Normal(320,5)	??
		Correlation = -0.7		

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Copula

Copula

The word copula is derived from the Latin noun for a link or a tie (as is the English word “couple”)

Its purpose is to describe the dependence structure between two variables.

Sklar’s theorem states that “Any multivariate joint distribution can be written in terms of univariate marginal distribution functions and a copula which describes the dependence structure between the two variables”.

How Copula Works?

$$Let:$$

$$X_1 \ and \ X_2 \ be \ RVs$$

$$U_1 = F(X_1); \ \ U_2 = F(X_2)$$

Then:

$$U_1 \sim uniform$$

$$U_2 \sim uniform$$

How Copula Works?

$$Let:$$
$$U_1 = F(X_1);\;\; U_2 = F(X_2)$$

$$F(X_1, X_2) = P(X_1 \leq x_1 \;\cap X_2 \leq x_2)$$
$$= P(F^{-1}(U_1) \leq x_1 \cap F^{-1}(U_2) \leq x_2)$$
$$= P(U_1 \leq F(x_1) \cap U_2 \leq F(x_2))$$
$$= \boldsymbol{P(U_1 \leq u_1 \cap U_2 \leq u_2)} = \boxed{\mathbf{C(U_1, U_2)}}$$

Gaussian Copula - Simulation

For the normal copula, the input of the simulation is the correlation matrix Σ. The normal copula can be simulated by the following steps, in which $\boldsymbol{U} = (U_1, \ldots, U_m)$ denotes one random draw from the copula:

1. Generate a multivariate normal vector $\boldsymbol{Z} \sim N(0, \Sigma)$, where Σ is an m-dimensional correlation matrix.
2. Transform the vector $\boldsymbol{Z}$ into $\boldsymbol{U} = (\Phi(Z_1), \ldots, \Phi(Z_m))^T$, where Φ is the distribution function of a univariate standard normal.

The Student t Copula

Let $\Theta = \{(\nu, \Sigma) : \nu \in (1, \infty), \Sigma \in \mathbb{R}^{m \times m}\}$, and let t_ν be a univariate t distribution with ν degrees of freedom.

The Student's t copula can be written as

$$C_\Theta(u_1, u_2, \ldots, u_m) = \boldsymbol{t}_{\nu,\Sigma}\left(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2), \ldots, t_\nu^{-1}(u_m)\right)$$

where $\boldsymbol{t}_{\nu,\Sigma}$ is the multivariate Student's t distribution that has a correlation matrix Σ with ν degrees of freedom.

The input parameters for the simulation are (ν, Σ). The t copula can be simulated by the following steps:

1. Generate a multivariate vector $\boldsymbol{X} \sim t_m(\nu, 0, \Sigma)$ that follows the centered t distribution with ν degrees of freedom and correlation matrix Σ.
2. Transform the vector $\boldsymbol{X}$ into $\boldsymbol{U} = (t_\nu(X_1), \ldots, t_\nu(X_m))^T$, where t_ν is the distribution function of a univariate t distribution with ν degrees of freedom.

Gaussian Copula Vs t - Copula