Financial Modelling Using Python
Chennai Mathematical Institute
Practice Set 2

1. The price of a six-month, INR 24.00 strike, European put option on a stock is INR 3.00. The stock price is INR 26.00. A dividend of INR 1.00 is expected in three months. The continuously compounded risk-free rate for all maturities is 5% per year. find the value of a European call option on the same underlying stock with a strike price of INR 25.00 and a time to maturity of six months.

2. A 12-month futures contract on an equity index is currently trading at USD 3,700. The underlying index is currently valued at USD 3,600 and has a continuously-compounded dividend yield of 2% per year. The continuously compounded risk-free rate is 4.5% per year. Assuming no transactions costs, what is the potential arbitrage profit per contract and the appropriate strategy?

3. Suppose the BSE Index has an expected annual return of 8% and volatility of 8.%. Suppose the ABC Fund has an expected annual return of 6% and volatility of 9.0% and is benchmarked against the BSE Index. If the risk-free rate is 4% per year, what is the beta of the ABC Fund according to the CAPM?

4. A risk manager is evaluating the price sensitivity of an investment-grade callable bond using the firm's valuation system. The table below presents information on the bond as well as on the embedded option. The current interest rate environment is flat at 5%.

Interest Level	Callable Bond Price	Call Option
4.98%	102.07848	2.08719
5.00%	101.61158	2.05010
5.02%	100.92189	2.01319

Estimate the convexity of the bond.

5. An analyst wants to price a 1-year, European-style call option on company ABC's stock using the Black-Scholes-Merton (BSM) model. The relevant information for the BSM model inputs are in the following table. What is the price of the 1-year call option on the stock?

Current stock price	USD 40
Stock price volatility	10% per year
Risk-free rate	5% per year
Call option exercise price	INR 40
N(d1)	0.5750
N(d2)	0.5116

1. We know that,

$$c + Ke^{-rT} + De^{-r\tau} = p + S_0$$

given, $S_0 = 26$ $\quad p = 3$

strike price, $K = 25$ $\quad T = 0.5$

$r = 0.05$ $\quad \tau = 0.25$

$D = 1$

$$\therefore c = 3 + 26 - 25 \times e^{-0.05 \times 0.5} - 1 \times e^{-0.05 \times 0.25}$$

$$= 3.6245$$

2. Using the cost-carry model, let's compute the price of future contract first,

$$F = Se^{(r-q)T};$$

$S = 3600$

$r = 0.095$

$q = 0.02$

$T = 1$

$\therefore F = 3691...$

which is lesser than the price of the contract

So, (i) we should sell the future contract.

(ii) Borrow 3600 and invest.

$$\text{Profit} = 3700 - 3600 \times e^{(0.095 - 0.02)}$$

$$= 8.866$$

3. From the CAPM Model,

expected return = Risk free-rate + $\beta \times$ Equity Premium

$= \beta \times$ (Market Return − Risk-Free Rate)

$\therefore$ using this,

$$6 = 4 + \beta(8 - 4)$$

$$\Rightarrow \beta = 0.5$$

4.

$$C = \frac{1}{P}\left[\frac{P_+ + P_- - 2P}{(\Delta r)^2}\right]$$

$$= \frac{1}{101.61158}\left[\frac{102.67898 + 100.92189 - 2\times 101.61158}{0.0002^2}\right]$$

$$= -54819.1265$$

5. So, we've.

$S_0 = 40$ USD $\quad c = 40$ INR

$\sigma = 0.1 \quad N(d_1) = 0.575$

$r = 0.05 \quad N(d_2) = 0.5116$

$T = 1$

from BSM,

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$= 40 \times 0.575 - 40 \times e^{-0.05\times 1} \times 0.5116$$

$$\simeq 3.539$$

6. Go through the examples of
   a. CAPM,
   b. Jensen's Alpha,
   c. The Treynor Measure,
   d. Sharp Ratio,
   e. Tracking error etc.

7. Concentrate on theory of put call parity.

8. Go through the pricing theory of callable bond and example, find the call price.

9. Go though the relationships between spot curve and forward curve.

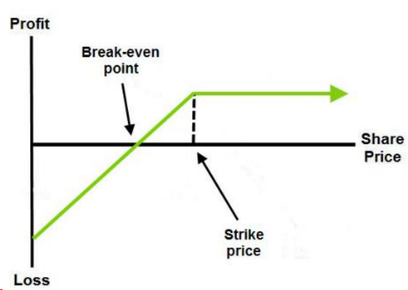
10. Assuming there are 252 trading days in a year, for an analysis of 95.0% daily VaR what is the expected exceptions?

$$\text{Expected exceptions} = \#\text{ trading days} \times \alpha$$
$$= 252 \times 0.05 \simeq 12.6 < \underline{13}$$

11. Explain the following 5 Greeks related to options
Delta, Gamma, Theta, Vega and Rho

12. Suppose that the time to expiration is 6 months, the strike price is $105, the call premium is $9, the put premium is $7, the current stock price is $94, and the continuously compounded annual interest rate is 10%. Is there any opportunity for riskless profit? If yes, then how to earn a riskless profit?

13. A stock price follows the Geometric Brownian motion with an expected return of 21% and a volatility of 46.35%. The current price is 38. What is the probability that a European call option on the stock with an exercise price of 40 and a maturity date in 6 months will be exercised?



14. Write down the name of the strategy (based on position of options) for the above profit-loss function. As an investor when you would like to follow this strategy. Explain the pros and cons of this strategy. short put

15. Consider a 2-year European call with a strike price of 52 on a stock whose current price is 50. In each time step (of one year) the stock price either moves up by 20% or moves down by 20%. Let the risk-free interest rate be 5%. Calculate the premium of the call option.

12. We know, from the put call parity equation,

$$p + S_0 = c + Ke^{-rT}$$

given,

$T = 0.5$ $\quad S_0 = 99$

$K = 105$ $\quad r = 0.1$

$c = 9$

$p = 7$

$\therefore p + S_0 = 103$

$c + Ke^{-rT} = 9 + 105 \times e^{-0.05} \simeq 108.88$

So we will,

(i) sell the call

(ii) borrow the put and stock and invest

$p + S_0 = 101$

He's required to pay $101 e^{0.1 \times 0.5} = 106.18$

since it's more than strike, put option can't be exercised.

profit $= 9 + (106.18 - 105)$

$= 10.18$

13. Let, the stock price in six-months be S_T.

$$\ln(S_T) \sim N\left(\ln S_0 + \left(r - \frac{1}{2}\sigma^2\right)T,\ \sigma^2 T\right)$$

$$\therefore \ln(S_T) \sim N\left(\ln 38 + \left(0.21 - \frac{0.4635^2}{2}\right) \times 0.5,\ 0.4635^2 \times 0.5\right)$$

$$\sim N(3.689,\ 0.107)$$

now, $\ln(40) = 3.689$

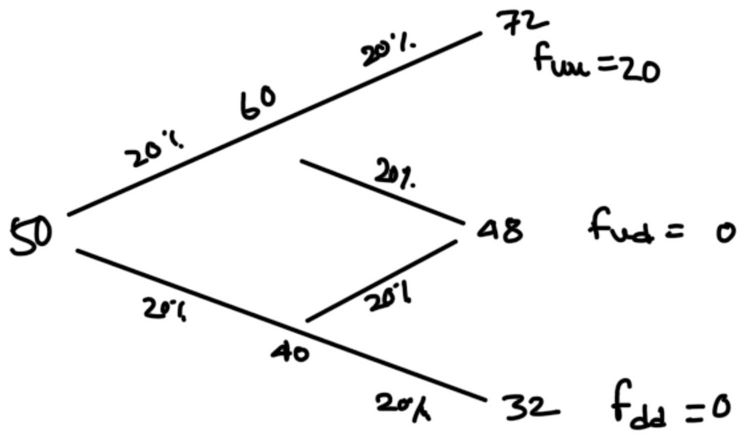
$\therefore$ the required prob $= \Phi(X > 3.689)$

$= 1 - \Phi(X \leq 3.689)$

$= 1 - \Phi\left(\frac{X - 3.689}{\sqrt{0.107}} \leq 0\right)$

$= 0.5$

15.



given,

strike price = 52

$r = 0.05$

$u = 1 + 0.2 = 1.2$

$d = 1 - 0.2 = 0.8$

(i) $f_0 = e^{-rT}\left[f_u q_u + f_d q_d\right]$

(ii) $q_u = \dfrac{e^{rT} - d}{u - d} = \dfrac{e^{0.05} - 0.8}{1.2 - 0.8} = 0.6282$

$q_d = 1 - q_u = 0.3718$

$f_u = e^{-rT}\left[f_{uu} q_u + f_{ud} q_d\right]$

$= e^{-0.05} \times 20 \times 0.6282$

$= 11.95$

$f_d = e^{-rT}\left[f_{ud} q_u + f_{dd} q_d\right]$

$= 0$

$f_0 = e^{-rT}\left[f_u q_u + f_d q_d\right] = e^{-0.05}\left[11.95 \times 0.6282\right]$

$= 7.142$

16. A bank's position in options on a stock has a delta of 200 and a gamma of 200. An option X that is being traded has a delta of 0.50 and a gamma of 0.80. How to make the portfolio delta and gamma neutral?

same as 11

17. Explain the following 5 Greeks related to options
Delta, Gamma, Theta, Vega and Rho

18. The price of a commodity moves according to a BM, X(t) = σB(t) + μt, with variance term σ2 = 4 and drift μ = −3. Given that the price is 4 at time t = 8, what is the probability that the price is below 1 at time t = 9?

16. $\delta = 200$ $\qquad \delta_x = 0.5$

$\gamma = 200$ $\qquad \gamma_x = 0.8$

i) First we'll make the portfolio gamma neutral.

we'll short $\frac{\gamma}{\gamma_x}$ units $= 250$ units of option X

ii) Delta of our portfolio is now,

$$200 - 250 \times 0.5 = 75$$

Now short 75 units of stock, which will result in delta neutrality.

18. $X(t) = \sigma B(t) + \mu t$

$\sigma^2 = 4, \ \mu = -3$

$\therefore \ X(1) \sim N(-3, 4) \Rightarrow X(1) = 2Z - 3$ unit normal

to find, $P(X(9) < 1 \mid X(8) = 4)$

$= P(X(9) - X(8) < -3 \mid X(8) = 4)$

$= P(X(9) - X(8) < -3)$ [$\because$ increments are independent]

$= P(X(1) < -3)$ [stationary increment]

$= P(2Z - 3 < -3)$

$= P(2Z < 0) = P(Z < 0) = \Phi(0) = 0.5$