Market Risk

Estimating Market Risk Measures

Estimating VaR using a Historical Simulation Approach

Important steps:

- Order (sort) the daily profit/loss observations.
- Find the $[(1 - CI\%)\, n + 1]^{th}$ lowest observation.
- Locate the loss corresponding to the specified confidence level; e.g., 95.0%, 99.0%.

Estimating VaR using a Historical Simulation Approach

Example

Over the last 300 trading days, the five worst daily losses (in millions) were:

-30; -27; -23; -21; and -19

What is the 99% daily HS VaR?

Solution

1 perc constitute -23,-21,-19 i.e. highest losses

The 99% VaR would be given by the (1 –0.99) 300 + 1 = 4th highest value. This would be -21.

Note that the 4th highest observation would separate the 1% of largest losses from the remaining 99% of returns.

Estimating Parametric VaR

When estimating VaR using the historical simulation approach, we do not make any assumption regarding the distribution of returns.

We shall be looking at:

1. VaR for returns that follow the normal distribution; and
2. VaR for returns that follow the lognormal distribution.

Estimating Parametric VaR

Normal VaR

Data: Profit/Loss

Example

Let P/L for ABC Ltd. over a specified period be normally distributed with a mean of $12 million and a standard deviation of $24 million.

Calculate the 95%VaR.

Solution

95% VaR: $\alpha=95$

$VaR95\%=-\mu(PL)+\sigma(PL)\times z95$

$=-12+24\times 1.645=27.48$

ABC expects to lose at most $27.48 million over the next year with 95% confidence. Equivalently, ABC expects to lose more than $27.48 million with a 5% probability.

Estimating Parametric VaR

Normal VaR

- Data: Arithmetic

When using **arithmetic data** rather than P/L data, VaR calculation follows a similar format.

Assuming the arithmetic returns follow a normal distribution,

$$r_t = \frac{p_t + D_t - p_{t-1}}{p_{t-1}}$$

- Where p_t = asset price at the end of periods;
- D_t: interim payments

The VaR is:

$$VaR(\alpha\%) = [-\mu_r + \sigma_r \times z_\alpha] p_{t-1}$$

Estimating Parametric VaR

Logormal VaR

look diag

Unlike the normal distribution, the lognormal distribution is **bounded by zero** and is also **skewed to the right**.

- It is the favored distribution when **modeling the prices of assets** such as stocks (which can never be negative).

If we assume that geometric returns follow a normal distribution (μ_R, σ_R), then the **natural logarithm of asset prices follows a normal distribution** and P_t follows a lognormal distribution

It can be shown that:

what is log normal ?

$$VaR(\alpha\%) = (1 - e^{\mu_R - \sigma_R \times z_\alpha})$$

Estimating the Expected Shortfall Given P/L or Return Data

Despite the significant role VaR plays in risk management, it stops short of telling us the amount or magnitude of the actual loss.

- We do not know what amount the actual loss would be beyond the VaR.
- To have an idea of the magnitude of expected loss, we need to compute the expected shortfall.

Expected shortfall(ES) is the expected loss given that the portfolio return already lies below the pre-specified worst case quantile return, e.g., below the 5^{th} percentile return.

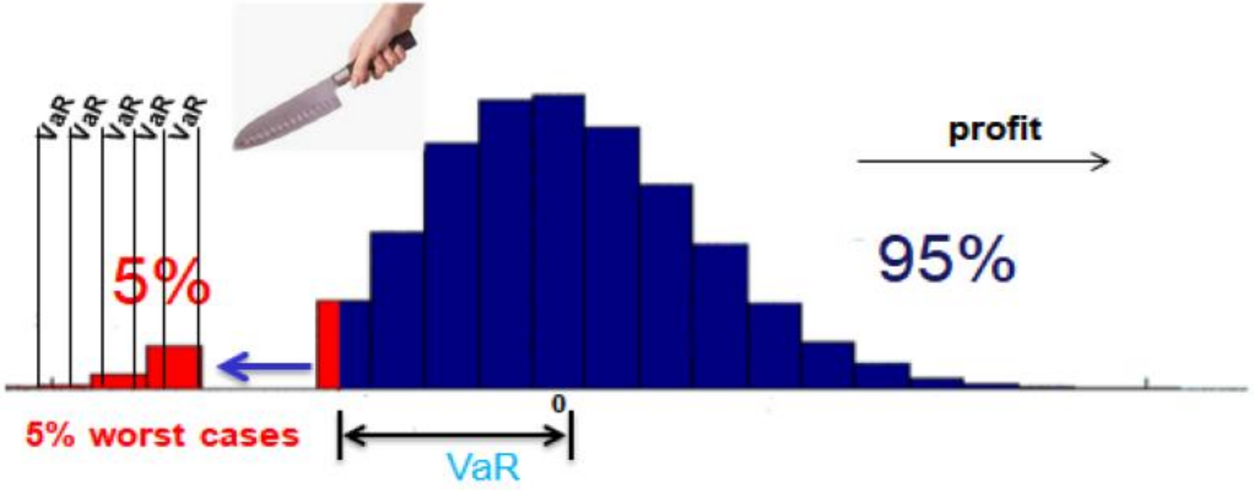
It is the probability-weighted average of tail losses; the expected loss conditional on the loss exceeding VaR.

Estimating the Expected Shortfall Given P/L or Return Data

As a probability-weighted average of tail losses, we can estimate ES as an average of ‘tail VaRs.’

To determine the ES:

- Divide the tail mass into *n* equal slices each of which has the same probability mass
- Estimate the VaR for each slide (a total of *n*–1 VaRs)
- Estimate the ES as an average of the computed VaRs



Evaluating Estimators of Risk Measures by Estimating their Standard Errors

Crucially, bear in mind that any risk measure estimates are only as useful as their precision.

Hence, it is important to supplement risk measure estimates with some indicator that gauges their precision.

The standard error is a useful indicator of precision.

More generally, confidence intervals (built with the help of standard errors) can be used.

Key Question: How do we go about determining standard errors and establishing confidence intervals?

Let's start with a **sample size of *n*** and arbitrary **bin width of *h*** around quantile, q.

- **Bin width** refers to the width of the intervals, or what we usually call "bins," in a (statistical) histogram.

The **square root of the variance of the quantile** is equal to the standard error of the quantile.

- Once the **standard error has been specified**, a confidence interval for a risk measure can be constructed:

$$[q + se(q) \times z_\alpha] > \mathrm{VaR} > [q - se(q) \times z_\alpha]$$

- Where $se(q) = \frac{\sqrt{\frac{p(1-p)}{n}}}{f(q)}$

Evaluating Estimators of Risk Measures by Estimating their Standard Errors

Example(1/3)

- Construct a **90% confidence interval** for **5% VaR** (the 95% quantile) drawn from a standard normal distribution.
  - Assume bin width = 0.1; and
  - The sample size is equal to 1,000.

Step 1*: determine the value of *q

- The quantile value, q, corresponds to the 5% VaR. For the normal distribution, the 5% VaR occurs at 1.645 (implying that q = **1.645**). So in crude form, the confidence interval will take the following shape:

$$[q + se(q) \times z_\alpha] > \mathrm{VaR} > [q - se(q) \times z_\alpha]$$

Step 2*: determine the range of *q

- For the bin width of 0.1, we know that q falls in the bin spanning:

$$1.645 \pm \frac{0.1}{2} = [1.595, 1.695]$$

Evaluating Estimators of Risk Measures by Estimating their Standard Errors

Step 3*: determine the probability mass *f(q)

- We wish to calculate the **probability mass between 1.595 and 1.695**, represented as *f(q).*
  - From the normal distribution table, the probability of a **loss exceeding 1.695 is 4.5%** (which is also equal to p); and
  - The probability of **profit or a loss less than 1.595 is 94.46%.**
- Hence, *f(q)* = 1 – 0.045 – 0.9446 = 1.032%

***Step 4*: calculate the standard error of the quantile from the variance approximation of *q*.**

$$se(q) = \frac{\sqrt{\frac{p(1-p)}{n}}}{f(q)} = \frac{\sqrt{\frac{0.045 \times 0.955}{1000}}}{0.01032} = 0.63523$$

- Thus, the following gives us the required CI:

$$[1.645 + 0.63523 \times 1.645] > \text{VaR} > [1.645 - 0.63523 \times 1.645]$$

$$2.69 > \text{VaR} > 0.6$$

Evaluating Estimators of Risk Measures by Estimating their Standard Errors

Using Resampling

Step 1

- Generate a random subsample (say of 80%) from the original sample or
- Generate a random sample from the distribution

Step 2

- Compute VaR

Step 3

- Repeat the above for several times

Step 4

- Compute mean and SD of VaRs