Part A:

when	amount of runs	At end i is	2^x	2^{2^x}
n = 2	0	2	2^1	2^{2^0}
n = 3	1	4	2^2	2^{2^1}
n = 5	2	16	2^4	2^{2^2}
n = 17	3	256	2^8	2^{2^3}
n = 257	4	65,536	2^{16}	2^{2^4}

2^{2^x} is how much n needs to change in order for the # of runs to increment.

$$2^{2^k} < n$$

$$2^k < \log(n)$$

$$k < \log(\log(n))$$

$O(1) + \underbrace{O(\log(\log(n)))}_{\text{bigger}}$

$$\boxed{\text{Runtime: } \Theta(\log(\log(n)))}$$

Part B: Look at worst, so consider if statement evaluates to true.

when	amount outer • inner	# if trues	equation
n = 1	$1 \cdot 1 = 1$	1	$1 \cdot (\sqrt{1} \times 1)^3$
n = 4	$4 \cdot (8 + 64) = 288$	2	$4 \cdot ((\sqrt{4} \times 1)^3 + (\sqrt{4} \times 2)^3)$
n = 9	$9 \cdot (27 + 216 + 729) = 8748$	3	$9 \cdot ((\sqrt{9} \times 1)^3 + (\sqrt{9} \times 2)^3 + (\sqrt{9} \times 3)^3)$
n = 16	$16 \cdot (64 + 512 + 1728 + 4096) = 102{,}400$	4	$16 \cdot ((\sqrt{16} \times 1)^3 + (\sqrt{16} \times 2)^3 + (\sqrt{16} \times 3)^3 + (\sqrt{16} \times 4)^3)$

Equation: $n \sum_{i=1}^{\sqrt{n}} (\sqrt{n} \times i)^3$

$$= n \cdot \sqrt{n}^3 \cdot \sum_{i=1}^{\sqrt{n}} i^3$$

1
9
36
100
225

$$= n \cdot \sqrt{n}^3 \cdot \left(\frac{(\sqrt{n})^2 (\sqrt{n}+1)^2}{4}\right)$$

$$= n^1 \cdot n^{3/2} \cdot n^2 = n^{9/2}$$

$$\text{Runtime} = \Theta(n^{9/2})$$

Part C:

amount
outer · mid · inner loops

Evaluate for when if statement is true.

$m = m + m$
$\downarrow$
$m = 2m$

$n \cdot n \cdot$ (# of times $\overset{m}{\overline{m}} \cdot 2$ until $m > n$)

n	times
$n=2$	1
$n=4$	2
$n=8$	3
$n=16$	4

$\downarrow$
$\log n$

$\sum_{i=1}^{n} \sum_{k=1}^{n} (O(1) + \log(n))$ → max amount of times this for loop can execute is n so xn.

$= \sum_{i=1}^{n} \sum_{k=1}^{n} O(1) + n\log(n))$

$= \Theta(n^2) + O(n\log(n))$

$\downarrow$
higher power takes over, so

Runtime: $\Theta(n^2)$

Part D:

$n = 30$

size → 10, 15, 22, 33
newsize → 15, 22, 33

	i → same as times excecuted for j
0	0
1	10
2	15
3	22

b first 10 = a first 10

a is 15 long but has 10 filled

a[10] = 100

$10 + 15 + 22 = 47$

$10\left(\frac{3}{2}\right)^0 + 10\left(\frac{3}{2}\right)^1 + 10\left(\frac{3}{2}\right)^2$

$= \sum_{K=10}^{10(\frac{3}{2})^K < n} \left(10 \times \left(\frac{3}{2}\right)^K\right) = \sum_{K=10}^{\log_{3/2}(\frac{n}{10})} 10 \times \left(\frac{3}{2}\right)^K$

newsize

find: $10\left(\frac{3}{2}\right)^K < n$

$\left(\frac{3}{2}\right)^K < \frac{n}{10}$

$K \log_{\frac{3}{2}}\left(\frac{3}{2}\right) < \log_{\frac{3}{2}}\left(\frac{n}{10}\right)$

$K < \log_{\frac{3}{2}}\left(\frac{n}{10}\right)$

$= \Theta\left(\frac{3}{2}\right)^{\log_{\frac{3}{2}} \frac{n}{10}}$

$= \Theta\left(\frac{n}{10}\right)$

$= \Theta(n)$

Runtime: $\Theta(n)$