

**Set 11 - 2D Diffusion, ADI, Thomas algorithm**

Issued: December 07, 2018

Hand in (optional): December 14, 2018 23:59

**Question 1: Diffusion in 2D using ADI scheme**

Heat flow in a medium can be described by the diffusion equation of the form

$$\frac{\partial \rho(x, y, t)}{\partial t} = D \nabla^2 \rho(x, y, t) \quad (1)$$

where  $\rho(x, y, t)$  is a measure for the amount of heat at position  $\mathbf{r}$  and time  $t$  and the diffusion coefficient  $D$  is constant. Let's define the domain  $\Omega$  in two dimensions as  $x, y \in [-1, 1]$ . We will use the boundary condition:

$$\rho(x, y, t) = 0 \quad \forall t \geq 0 \text{ and } (x, y) \notin \Omega \quad (2)$$

and an initial distribution:

$$\rho(x, y, 0) = \begin{cases} 1 & |x, y| < 1/2 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

- Discretize equation (1) using the Alternating Direction Implicit (ADI) scheme. Write down the resulting system in matrix form. What do you observe? Comment on your choice of method/algorithm for the solution of the resulting implicit scheme and explain why this choice is justified.
- Use the provided skeleton code (`diffusion2d_adi_openmp.cpp`) to solve the 2D diffusion problem on a uniform grid. Implement the missing code parts in all sections marked by `TODD`. Use Thomas algorithm for the solution of the implicit systems resulting from the ADI scheme.
- Parallelize your code using OpenMP. Comment on any complexity that would arise if you chose to parallelize the ADI scheme with MPI.
- Compute an approximation to the integral of  $\rho$  over the entire domain in `compute_diagnostics` and plot the result as a function of time using the parameters  $D = 1$ ,  $L = 1$  and  $N = 256$ .