

LENTES GRAVITACIONALES EN ASTROFÍSICA Y COSMOLOGÍA

SEMANA - 3

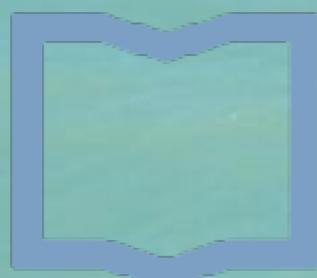
MARTÍN MAKLER

ICAS/IFICI/CONICET & UNSAM Y CBPF

ICAS

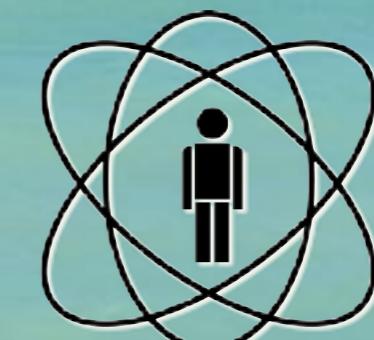


CONICET



Instituto de
Ciencias Físicas

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Aplicaciones de microlentes

Ejemplos: descubierta de planetas y agujeros negros, búsqueda de materia oscura, caracterización de la distribución de objetos compactos...

- Eventos individuales

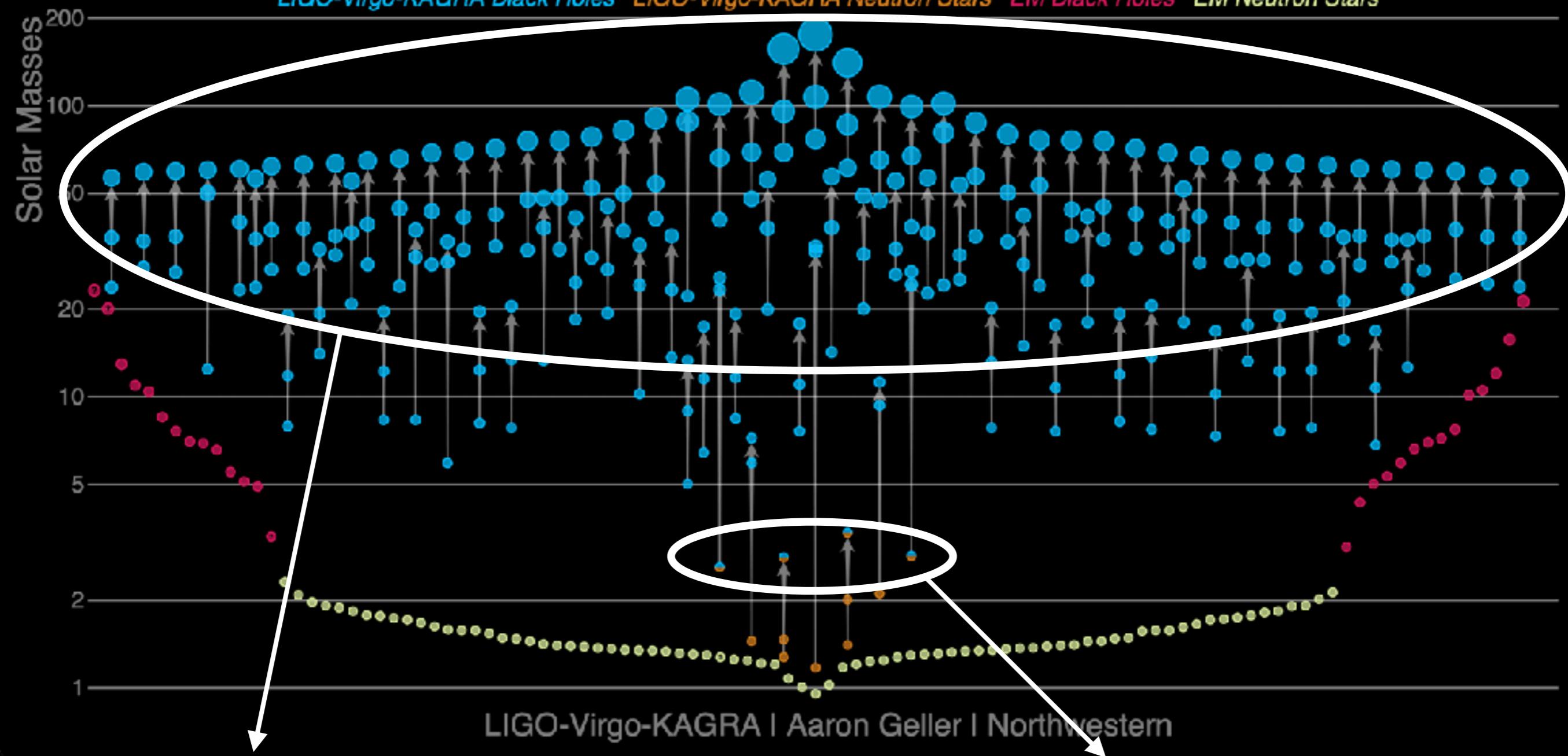
- Informaciones adicionales: romper degeneraciones
- Eventos de lentes binarias y múltiples (incluso exoplanetas y QSO *microlensing*)

- Estadística

- Materia oscura, poblaciones

Masses in the Stellar Graveyard

LIGO-Virgo-KAGRA Black Holes LIGO-Virgo-KAGRA Neutron Stars EM Black Holes EM Neutron Stars



¿Podemos detectar esos agujeros negros de alta masa por otros medios?

¿Cuál es la población de agujeros negros libres?

¿Son realmente objetos en la brecha de masas? ¿O están amplificados por lentes gravitacionales?

¿Podemos encontrar tales objetos por otros medios?

El *microlensing* puede ayudarnos a construir un censo más completo de objetos compactos

Mas allá de la curva de Paczynski

- 90% de los eventos de *microlensing* se pueden describir con una curva de lente y fuente puntuales y movimientos uniformes

$$t_E = \frac{\theta_E D_{OL}}{v_{\perp}}$$

$$\theta_E = \sqrt{\frac{D_{LS}}{D_{OS} D_{OL}} \frac{4GM}{c^2}}$$

Mas allá de la curva de Paczynski

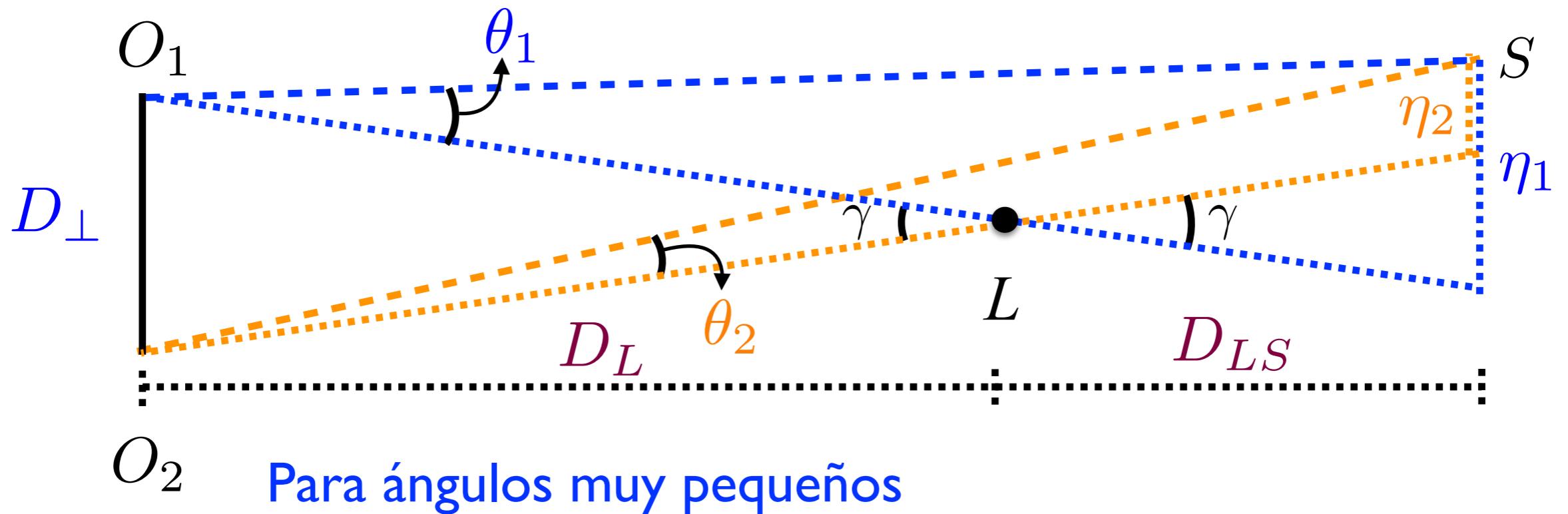
- 90% de los eventos de *microlensing* se pueden describir con una curva de lente y fuente puntuales y movimientos uniformes

$$t_E = \frac{\theta_E D_{OL}}{v_{\perp}} \quad \mu_{\text{rel}} = \frac{v_{\perp}}{D_L} \quad \text{relative proper motion} \quad t_E = \frac{\theta_E}{\mu_{\text{rel}}}$$

- Los otros 10% requieren la inclusión de efectos adicionales, lo que puede permitir quebrar algunas de las degeneraciones de *microlensing*:
 - Fuentes binarias: superposición de dos curvas de luz (si se puede despreciar la rotación)
 - Fuente finita: cuando el tamaño de la fuente es relevante (importante si $u_0 \lesssim r = \theta_R/\theta_E$)
 - Movimiento del observador (*parallax*) o de la fuente (*xallarap*)
 - Lentes compuestas (incluso planetas!)

Paralaje (geometría)

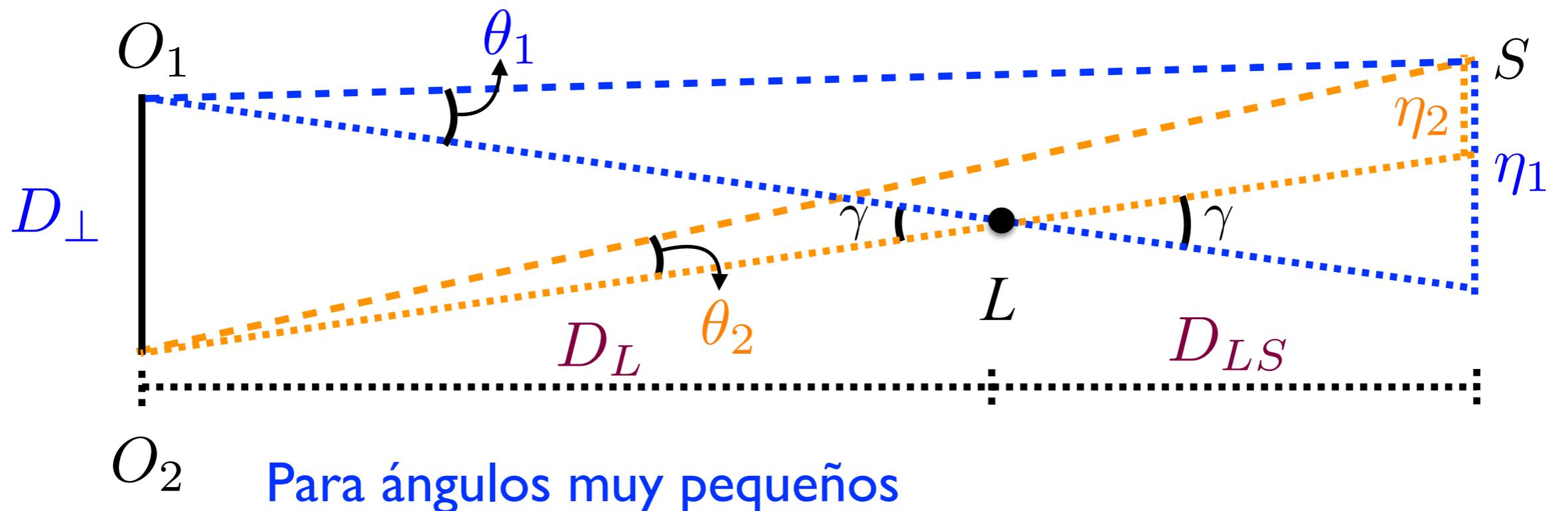
- Variación de la posición angular de una fuente S relativa a un objeto L por el desplazamiento de un observador O :



$$\begin{aligned}\pi_{\text{rel}} &:= |\theta_1 - \theta_2| = \frac{|\eta_1 - \eta_2|}{D_S} = \frac{\gamma D_{LS}}{D_S} = \frac{(D_{\perp}/D_L) D_{LS}}{D_S} \\ &= D_{\perp} \frac{D_{LS}}{D_L D_S}\end{aligned}$$

Paralaje (geometría)

- Variación de la posición angular de una fuente S relativa a un objeto L por el desplazamiento de un observador O :



$$\pi_{\text{rel}} = D_{\perp} \frac{D_{LS}}{D_L D_S} = D_{\perp} \left(\frac{1}{D_L} - \frac{1}{D_S} \right)$$

Definiendo:

$$\pi_L = \frac{D_{\perp}}{D_L}, \pi_S = \frac{D_{\perp}}{D_S}, \boxed{\Delta\theta = \pi_{\text{rel}} = \pi_L - \pi_S}$$

Paralaje en *microlensing*

- En *microlensing*, consideramos el efecto de la paralaje en el movimiento relativo lente-fuente, que se refleja en la curva de luz
- Ese efecto temporal depende de la orientación del movimiento propio relativo $\hat{\mu}_{\text{rel}}$ en relación a la separación entre los puntos de observación (ya sea por el movimiento del observador o por puntos de observación distintos en un mismo momento)
- Por eso es conveniente definir el vector paralaje como

$$\vec{\pi}_{\text{rel}} = \pi_{\text{rel}} \hat{\mu}_{\text{rel}}$$

- Además, la dependencia observable es siempre en unidades del radio de Einstein, por lo que definimos

$$\vec{\pi}_E = \frac{\pi_{\text{rel}}}{\theta_E} \hat{\mu}_{\text{rel}}$$

Nota: a veces se usa la notación $\hat{\mu}_{\text{rel}} = \hat{e}_{LS}$ para la dirección de la velocidad relativa lente-fuente

Efectos de paralaje en microlentes

- Efecto temporal (secular) en la curva de luz

$$\begin{aligned} |\vec{u}_\oplus(t)|^2 &= |\vec{u}_\odot(t) + \vec{u}(t)|^2 = \\ &= u_0^2 + \omega_E^2(t-t_0)^2 + 2\pi_E \{ \sin \zeta(t) [u_0 \cos \psi + \omega_E(t-t_0) \sin \psi] + \\ &+ \sin \beta \cos \zeta(t) [u_0 \sin \psi - \omega_E(t-t_0) \cos \psi] \} + \\ &+ \pi_E^2 [\sin^2 \zeta(t) + \sin^2 \beta \cos^2 \zeta(t)]. \end{aligned}$$

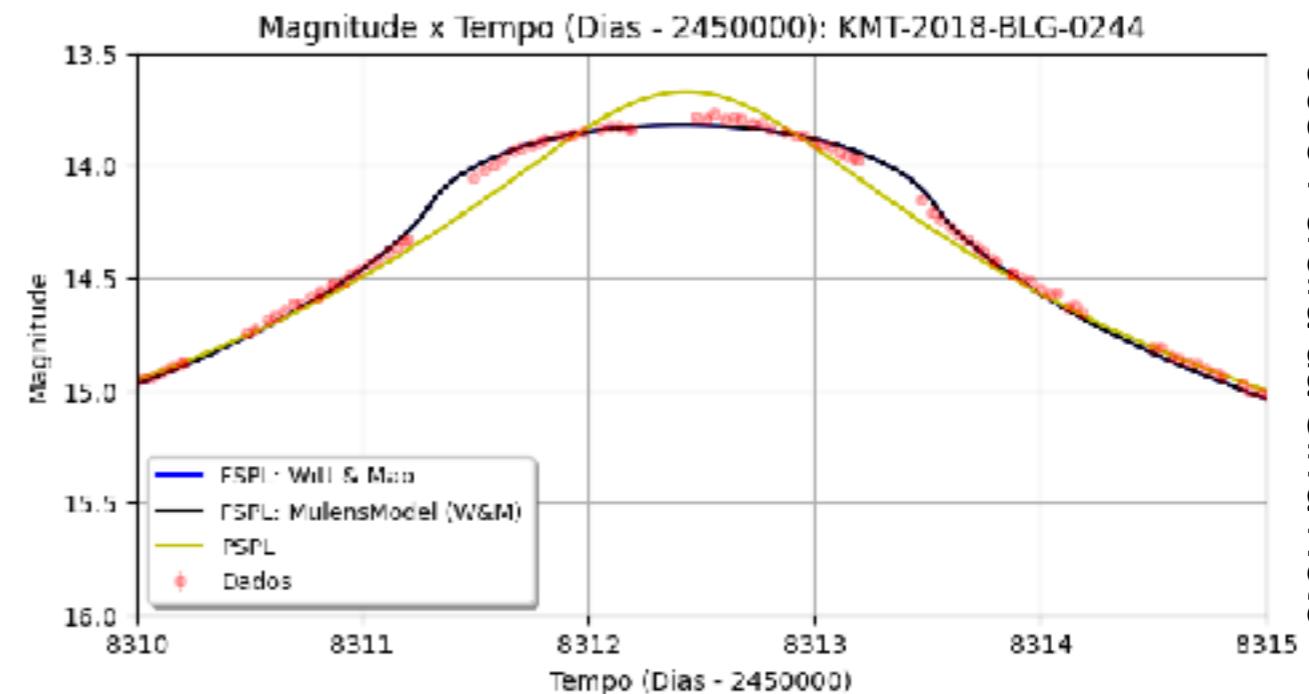
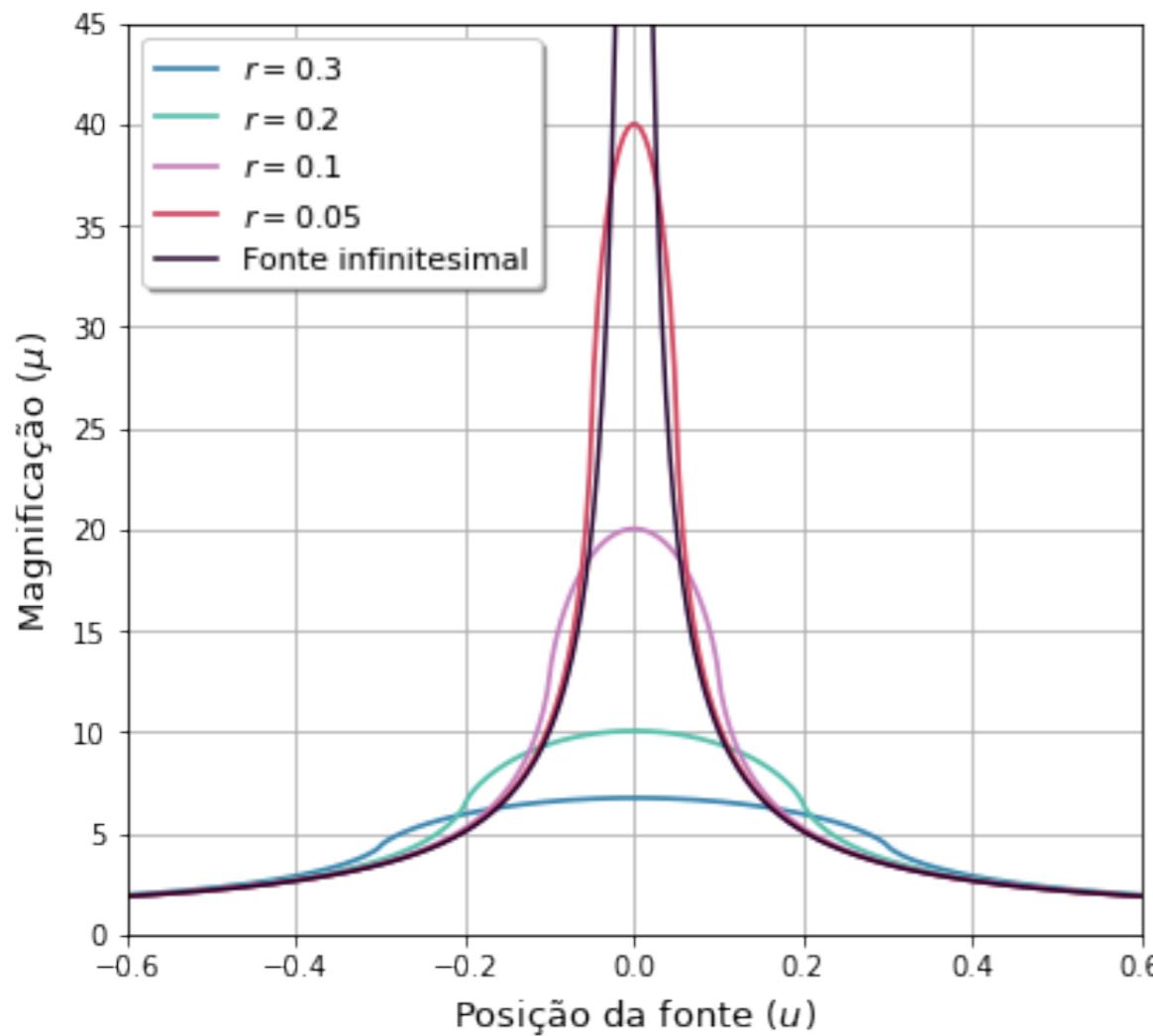
- Paralaje satelital

$$\pi_E = \frac{1}{D_\perp} \left(\frac{\Delta t_0}{t_E}, \Delta u_0 \right)$$

Sea com en efecto de la paralaje en una a curva de luz o desde dos observatórios, se puede medir π_E !

Efecto de fuente finita

- Efecto en la curva de luz



$$r = \frac{R}{R_E} = \frac{R}{D_{OS} \theta_E} = \frac{\theta_R}{\theta_E}$$

Si sabemos la dimensión de la fuente y su distancia, obtenemos θ_E !

Quebrar degeneraciones de *microlensing* con efectos de orden más alta

De la definición del paralaje:

$$\pi_L = \frac{D_\perp}{D_L}, \quad \pi_S = \frac{D_\perp}{D_S}, \quad \Delta\theta = \pi_{\text{rel}} = \pi_L - \pi_S$$

$$\pi_E = \frac{\pi_{\text{rel}}}{\theta_E}$$

Y del radio de Einstein: $\theta_E = \sqrt{\frac{D_{LS}}{D_{OS}D_{OL}} \frac{4GM}{c^2}}$

Tenemos

$$M = \frac{c^2}{4G} \frac{\theta_E}{\pi_E}$$

Quebrar degeneraciones

Otras formas de quebrar degeneraciones, mas allá de los datos de curvas de luz:

- medir *microlensing astrométrico* (i.e., desplazamiento del centroide)
- medidas/estimaciones de distancia (lente y/o fuente)
 - directas (paralaje! e.g. Gaia)
 - posición de la fuente (e.g. nube de Magallanes)
 - relaciones empíricas con color (o espectro) y magnitud
- medidas directas de la velocidad propia (e.g. Gaia, HST, AO)
- dimensión de la fuente: relación empírica color-brillo superficial (o con espectro)
- estimación de la masa de la lente con su luminosidad y distancia

Microlensing events from all over the sky from Gaia

Łukasz Wyrzykowski ✦ Astronomical Observatory, University of Warsaw

LUKASZ WYRZYKOWSKI - GAIA AND MICROLENSING BLACK HOLES

HOW TO FIND A GALACTIC BLACK HOLE WITH MICROLENSING?

Black hole = large mass + dark object

mass

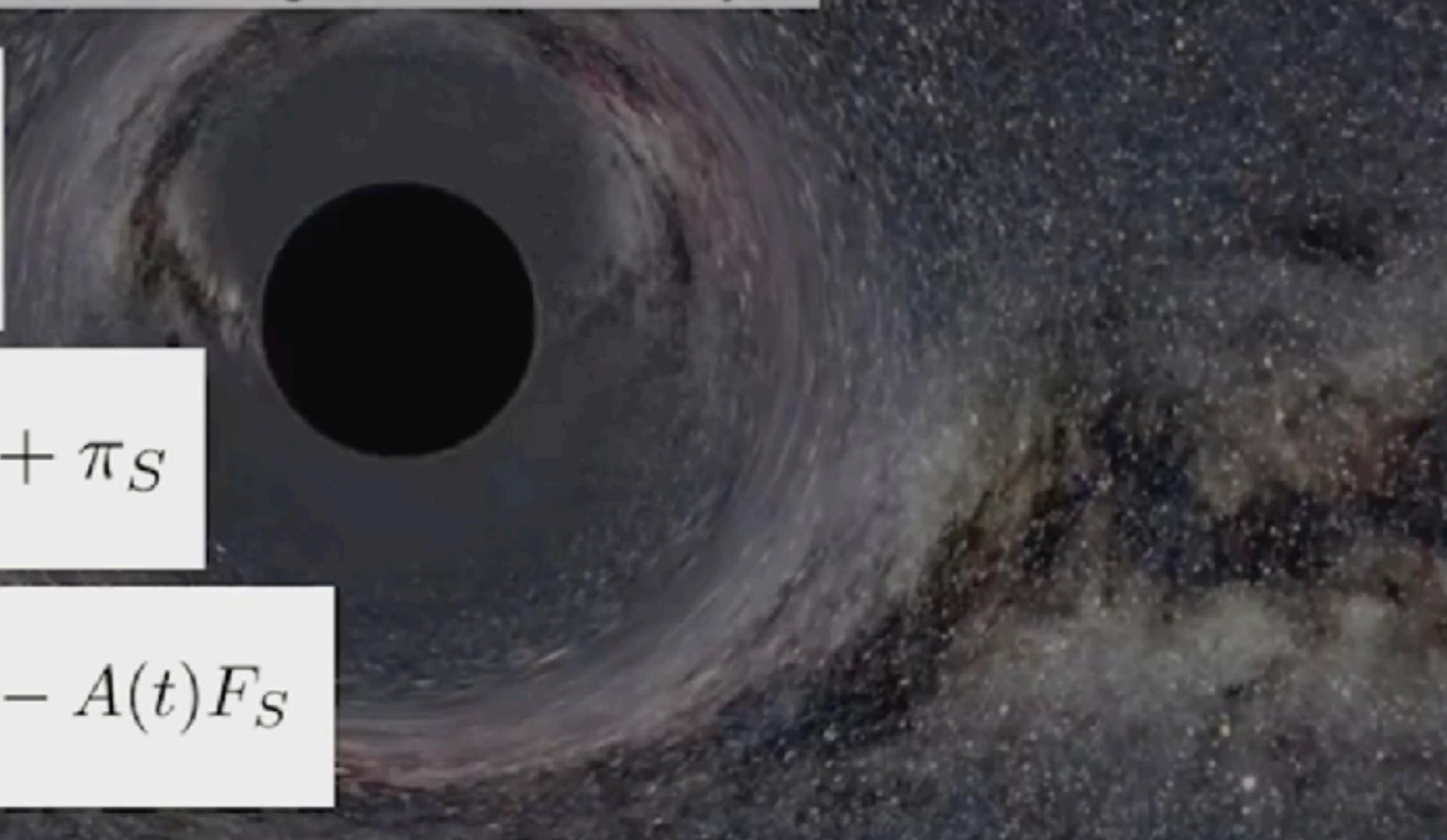
$$M = \frac{\theta_E}{\kappa \pi_E}$$

distance

$$\pi_L = \theta_E \pi_E + \pi_S$$

flux
(luminosity)

$$F_L = F_{obs}(t) - A(t)F_S$$



HOW TO FIND A GALACTIC BLACK HOLE WITH MICROLENSING?

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Photometry

Astrometry

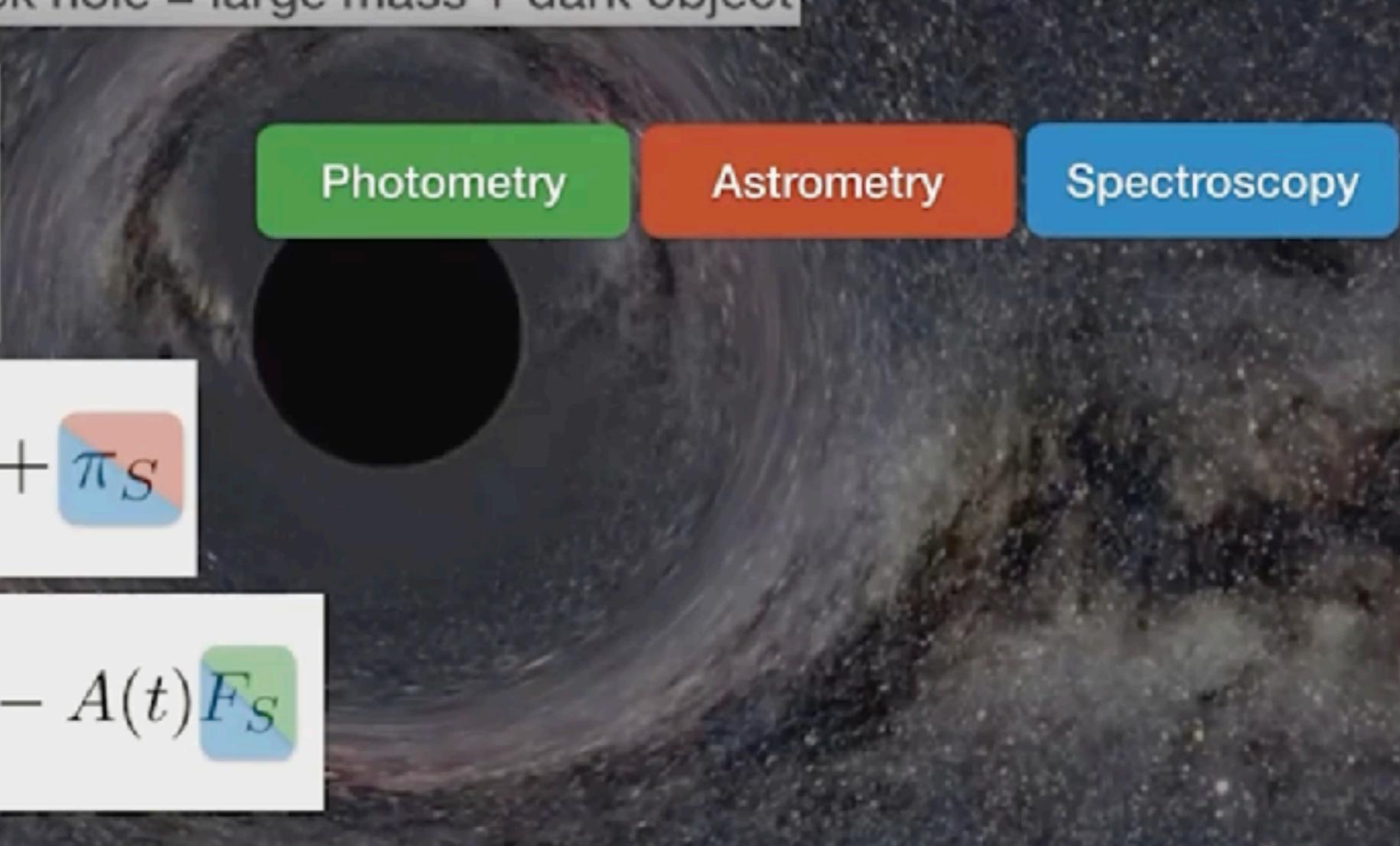
Spectroscopy

distance

$$\pi_L = \theta_E \pi_E + \pi_S$$

flux
(luminosity)

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HOW TO FIND A GALACTIC BLACK HOLE WITH MICROLENSING?

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Photometry



Astrometry



Spectroscopy

distance

$$\pi_L = \theta_E \pi_E + \pi_S$$

Gaia, OGLE,
Spitzer, LCO,
*VRO/LSST

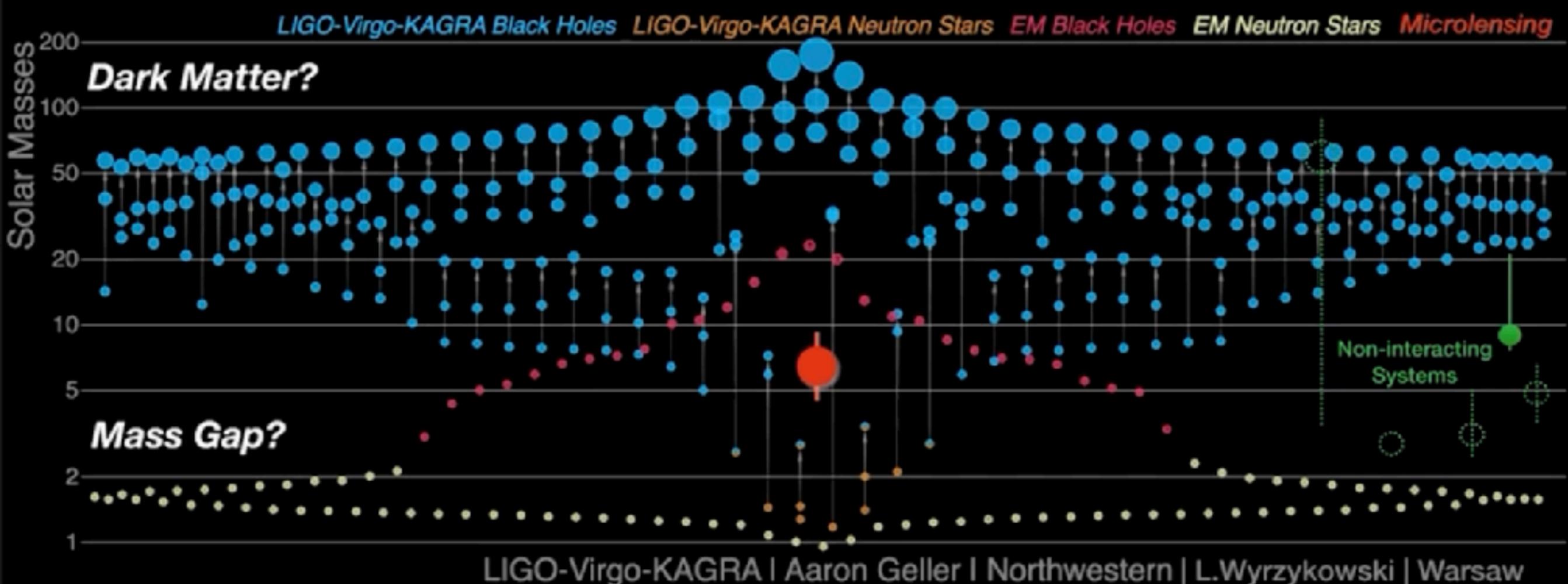
Gaia,
VLTI/GRAVITY,
VLTI/PIONIER,
AO, JWST,
*ELT,
*RST/WFIRST

Gaia, VLT,
Gemini,
SOAR, SALT,
*ELT

flux
(luminosity)

$$F_L = F_{obs}(t) - A(t) F_S$$

MASSES OF BLACK HOLES AND NEUTRON STARS



Abbott+2021

Thompson et al. 2019, Liu+2020, Shenar+2022

Sahu+2022

Corral Santana+2016

Rivinius+2020, Jayasinghe+2021

Lam+2022, Mróz+2022

Ejemplo:

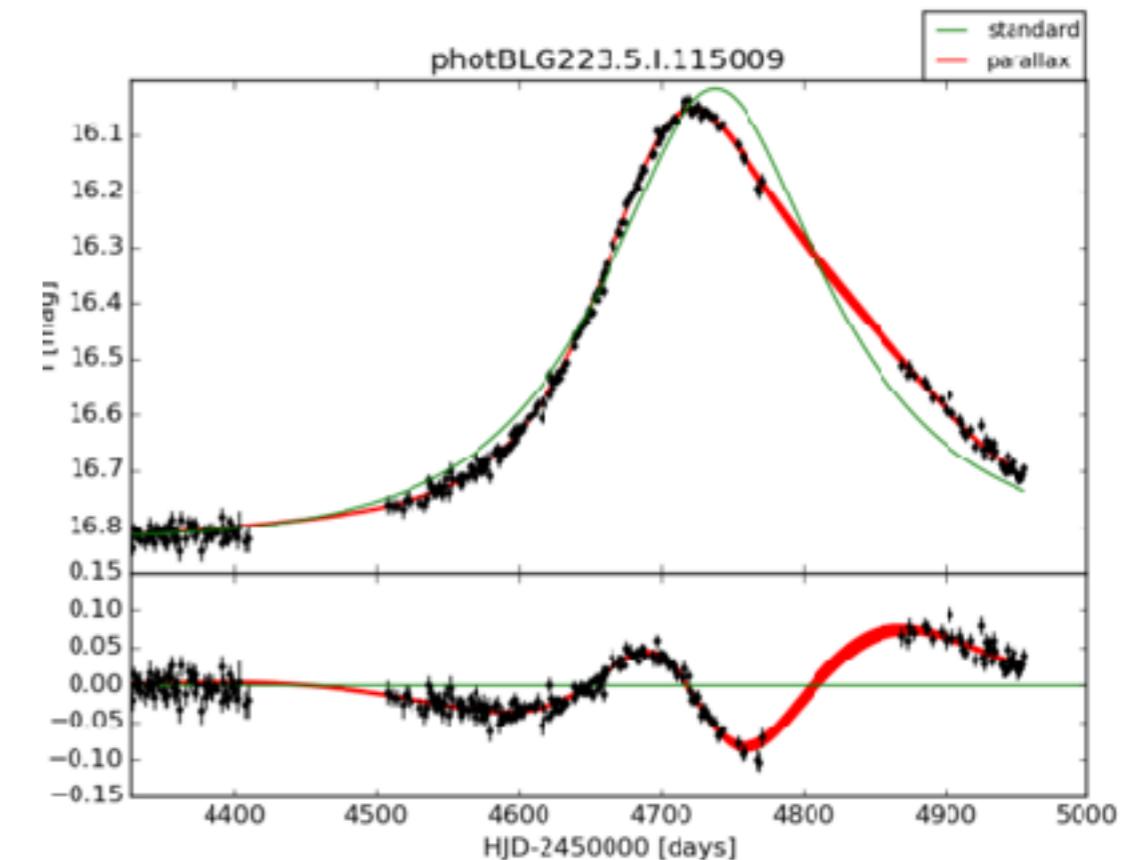
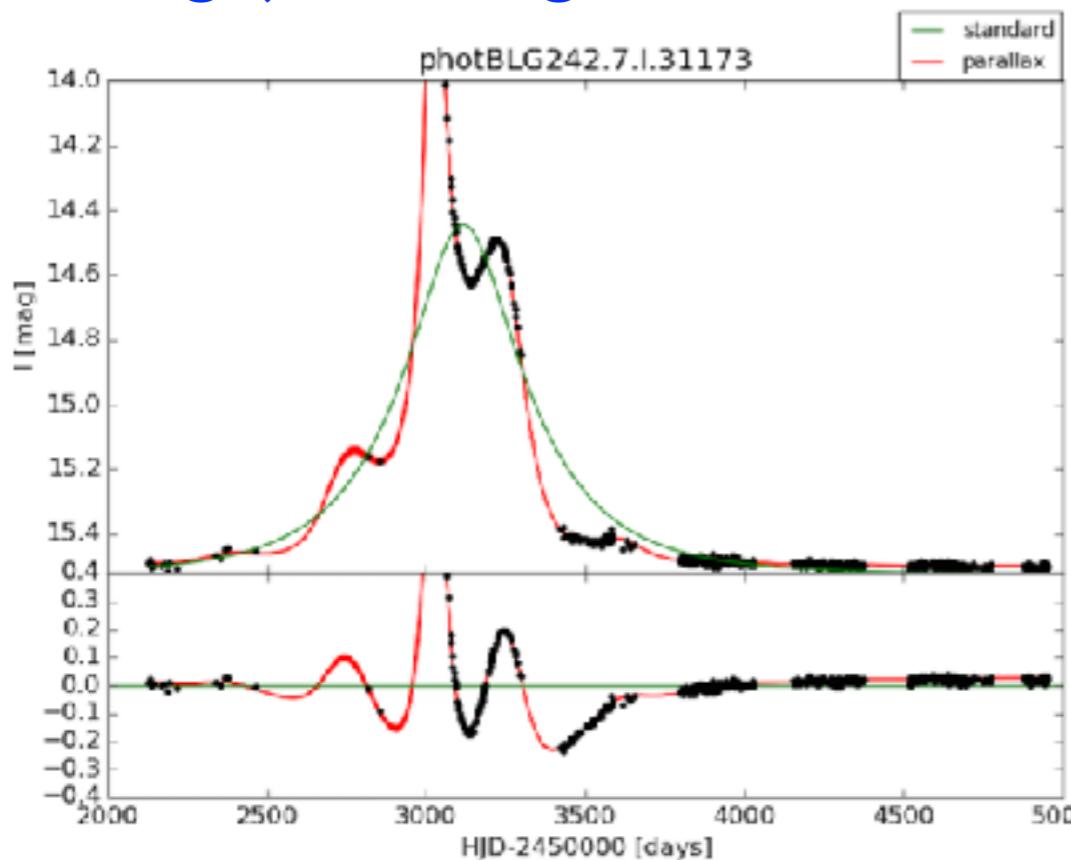
Constraining the masses of microlensing black holes and the mass gap with Gaia DR2

arXiv:1904.07789

Łukasz Wyrzykowski^{1, *} and Ilya Mandel^{2, 3, 4}

Masas para eventos en que no se detecta la luz de la lente:

- dirección del bulbo con paralaje bien medido con OGLE-III
- medidas directas de la distancia de la fuente y del movimiento propio con Gaia
- 18 eventos con esa combinación, con bajo *blending*
- 8 candidatos en el *mass gap* entre estrellas de neutrones y agujeros negros



Lentes binarias

- Ecuación de la lente binaria

$$y = x - \frac{\mu_A}{\bar{x} - \bar{x}_A} - \frac{\mu_B}{\bar{x} - \bar{x}_B}$$

$$q := \mu_A / \mu_B$$

$$d := |x_A - x_B| = 2|x_A|$$

- Para un planeta como Júpiter: $q = 0.001$
- Para un planeta como la Tierra: $q = 0.000003$

Lentes binarias

- Ecuación de la lente binaria

$$y = x - \frac{\mu_A}{\bar{x} - \bar{x}_A} - \frac{\mu_B}{\bar{x} - \bar{x}_B}$$

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- Cáusticas y curvas críticas:

$$J = \det \begin{pmatrix} \frac{\partial y}{\partial x} & \frac{\partial y}{\partial \bar{x}} \\ \frac{\partial y}{\partial \bar{y}} & \frac{\partial y}{\partial \bar{x}} \end{pmatrix} = \left| \frac{\partial y}{\partial x} \right|^2 - \left| \frac{\partial y}{\partial \bar{x}} \right|^2 = 0$$

BINARY LENSES

- Lens equation:

$$z_s = z - \frac{m_1}{z^* - z_1^*} - \frac{m_2}{z^* - z_2^*}$$

- determinant of the Jacobian:

$$\det A = 1 - \left| \frac{\partial z_s}{\partial z^*} \right|^2$$

$$\frac{\partial z_s}{\partial z^*} = \frac{m_1}{(z^* - z_1^*)^2} + \frac{m_2}{(z^* - z_2^*)^2}$$

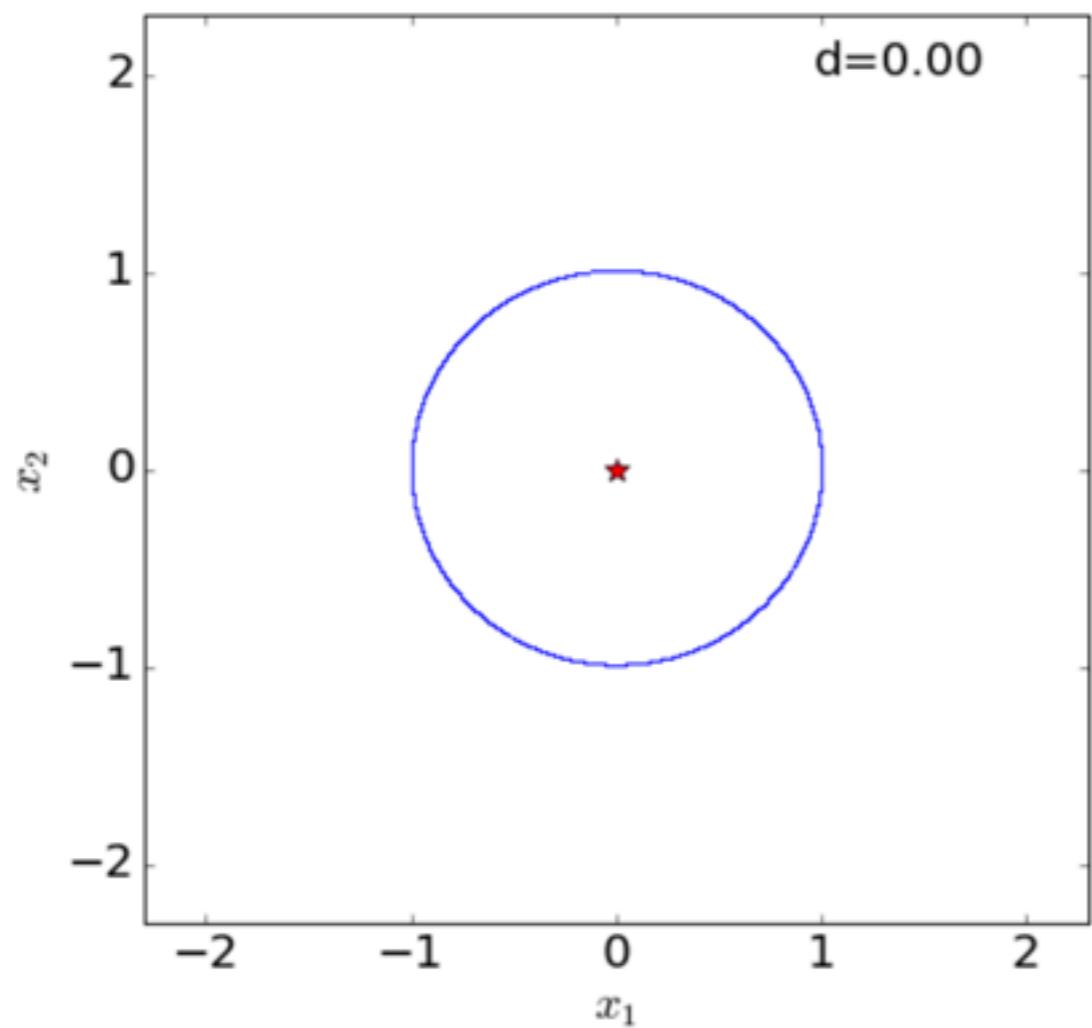
- condition for critical points:

$$\frac{\partial z_s}{\partial z^*} = e^{i\phi}$$

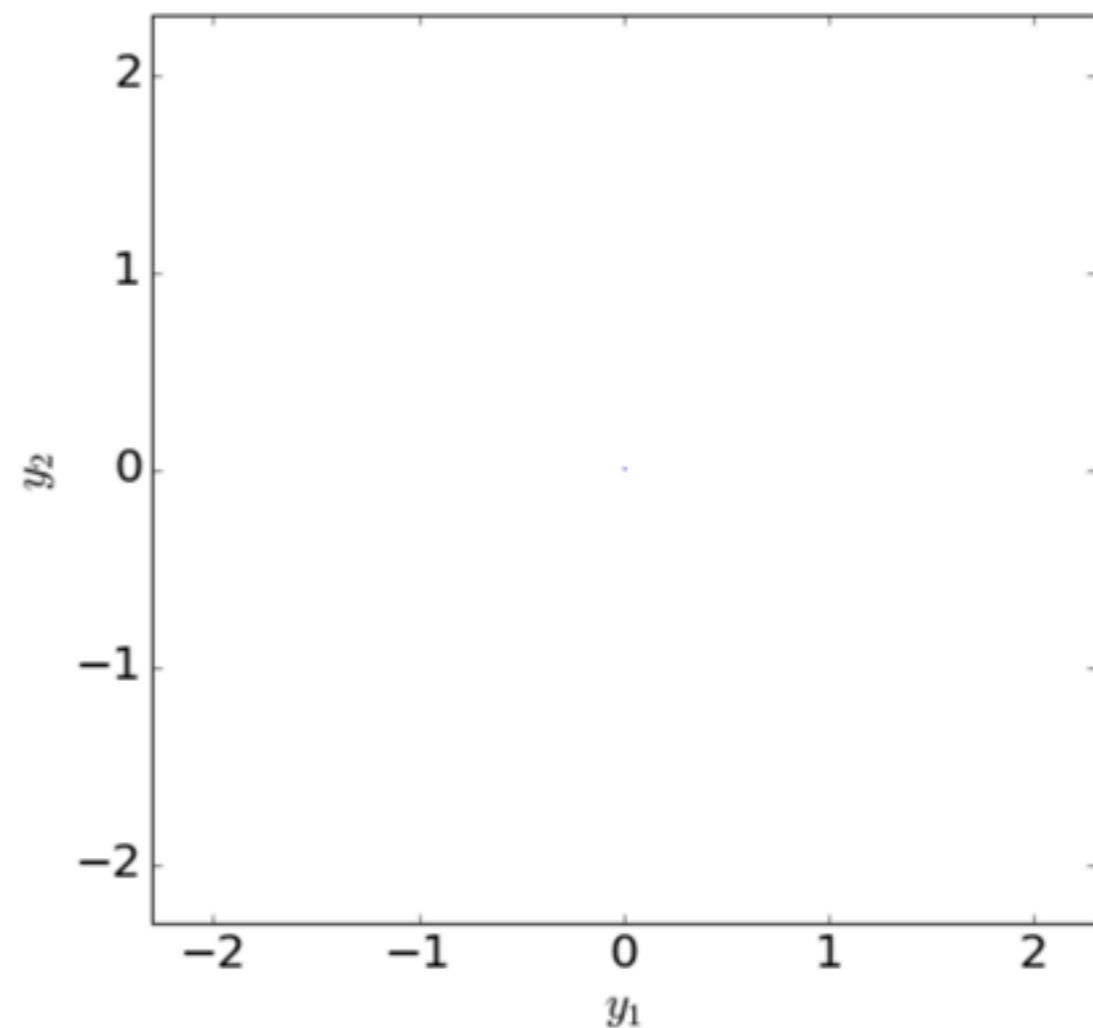
- resulting fourth grade polynomial:

$$z^4 - z^2(2z_1^{*2} + e^{i\phi}) - zz_1^*2(m_1 - m_2)e^{i\phi} + z_1^{*2}(z_1^{*2} - e^{i\phi}) = 0$$

BINARY LENSES: TWO LENSES WITH THE SAME MASS ($Q=1$) AND VARYING DISTANCE

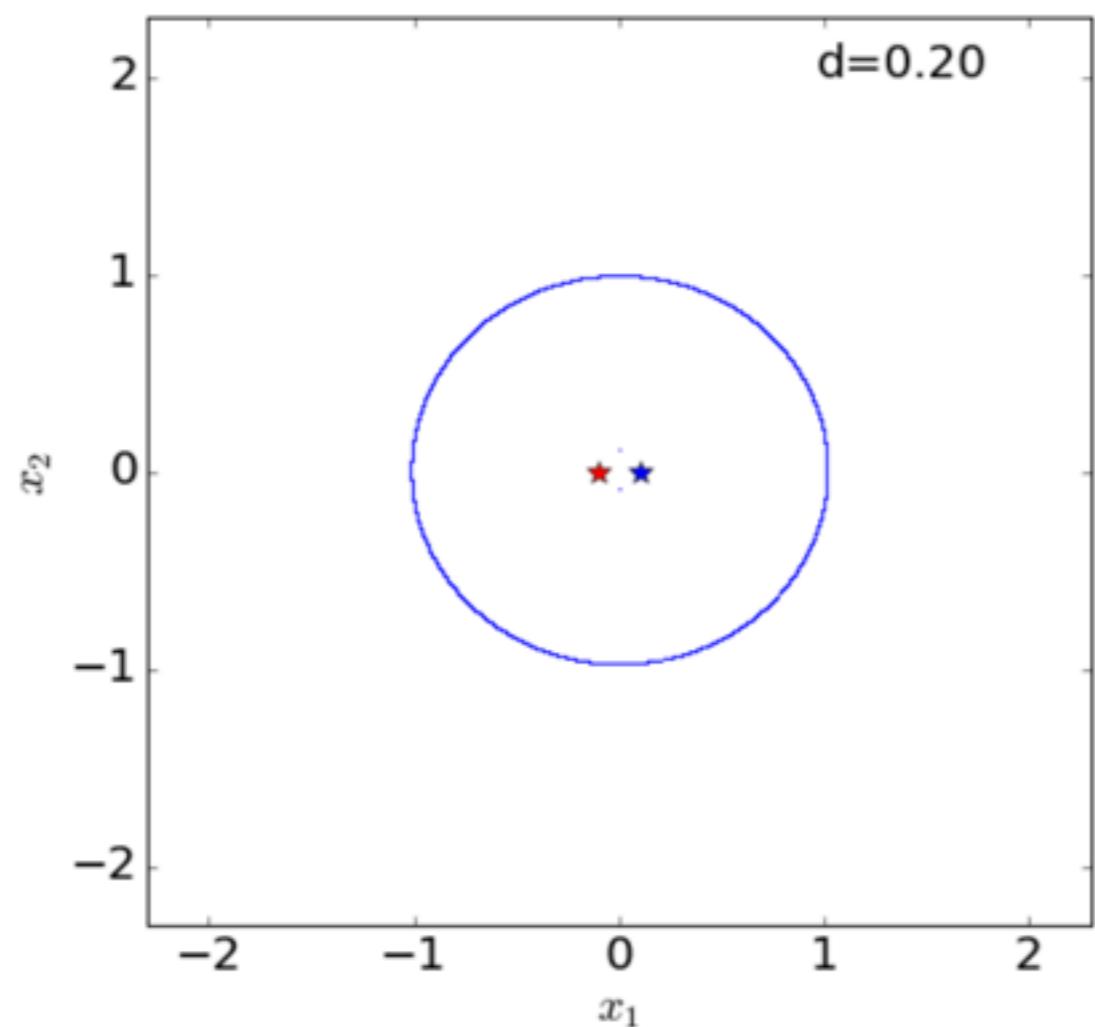


critical lines

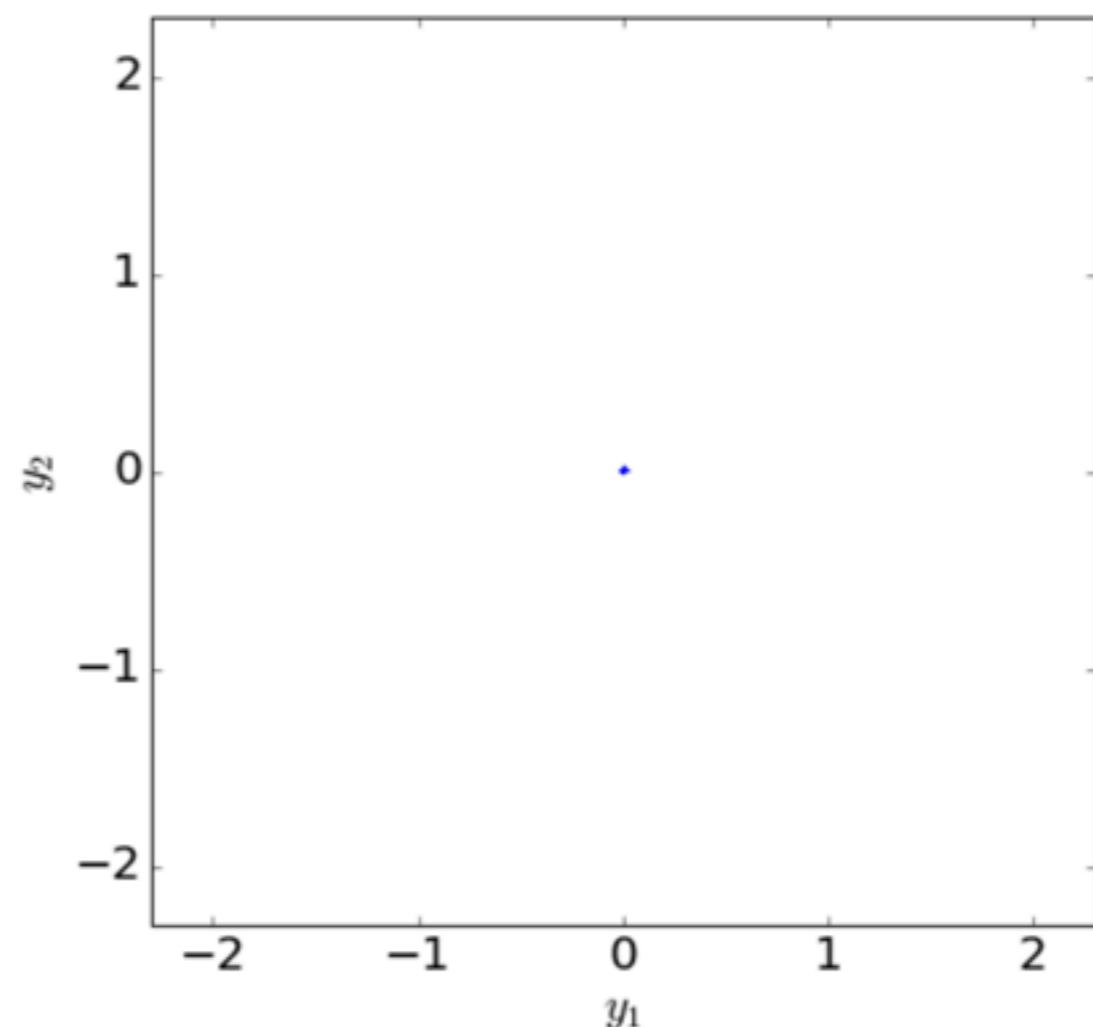


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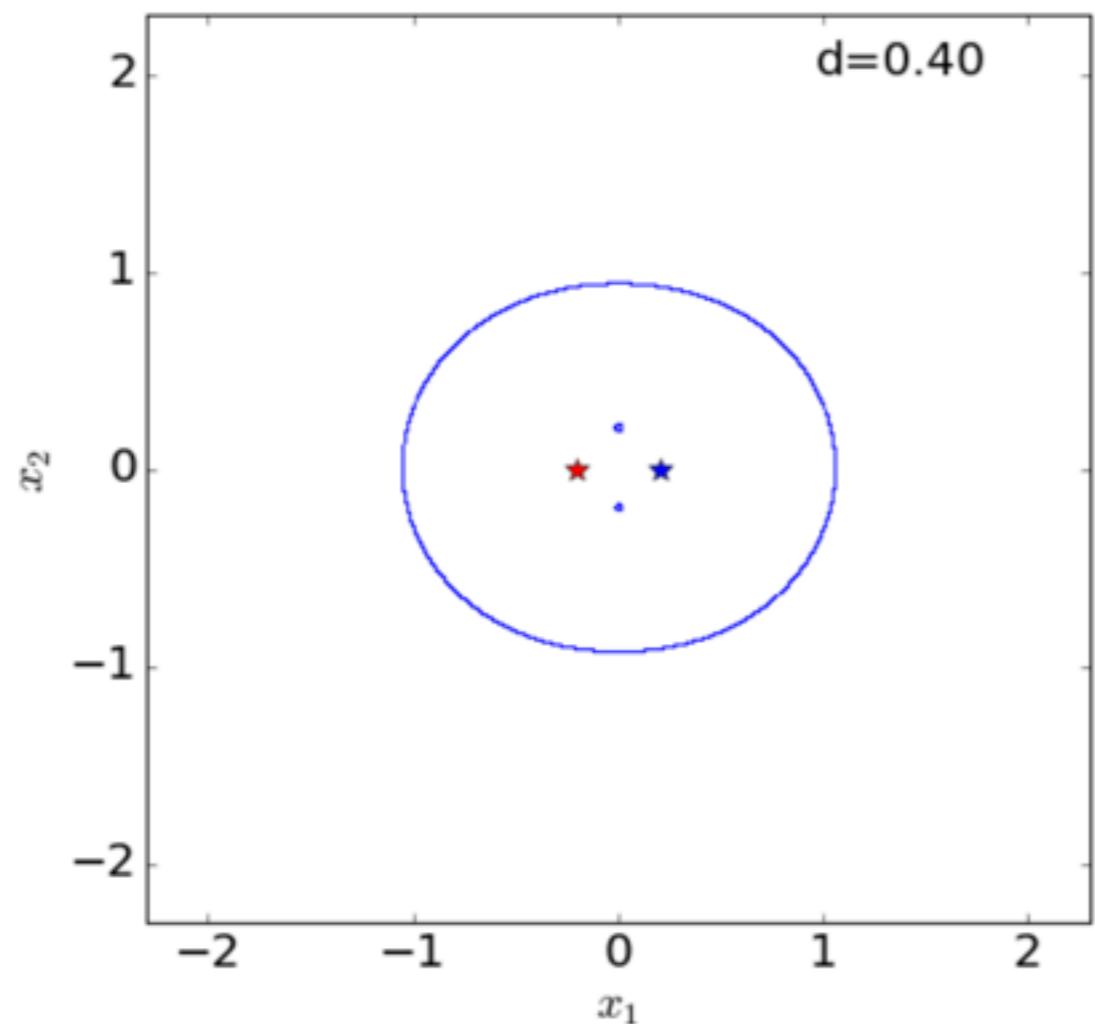


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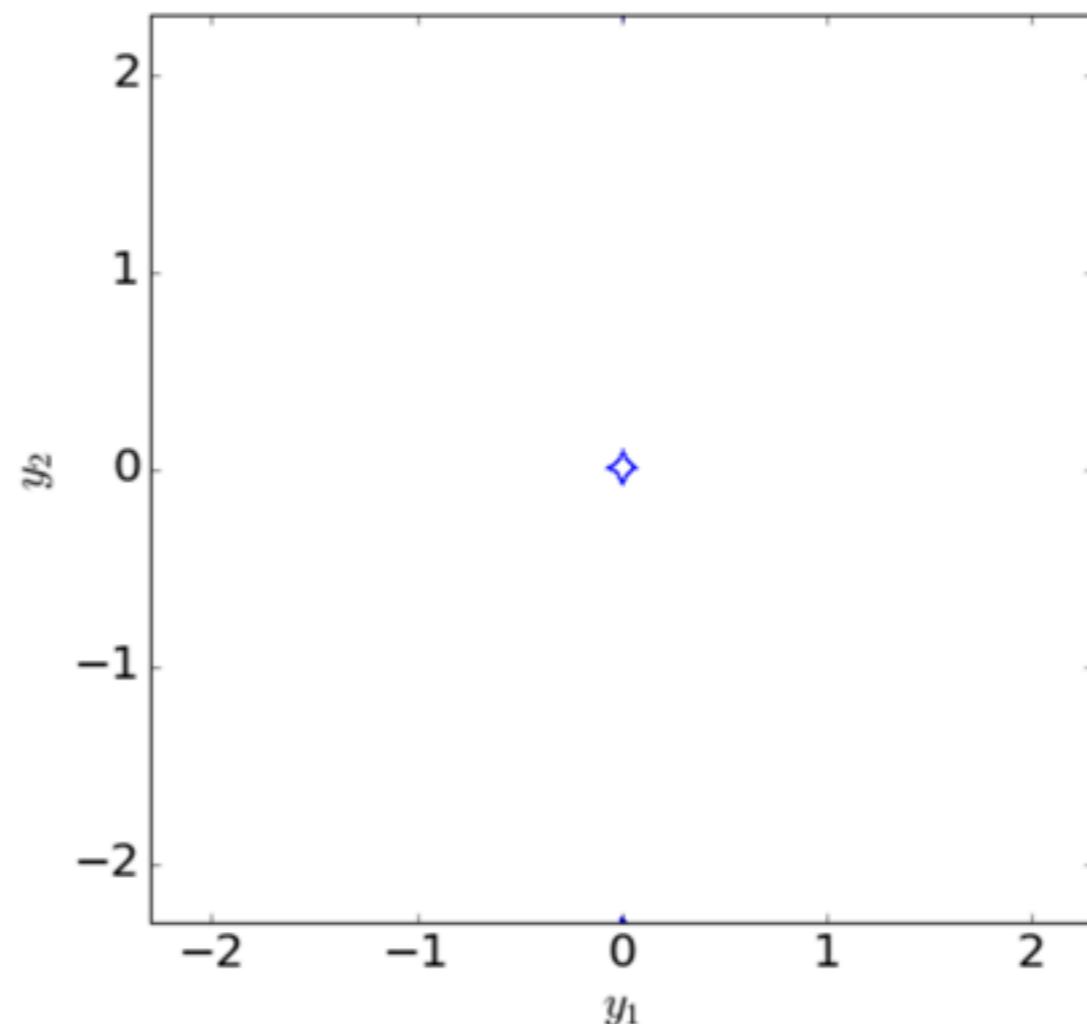


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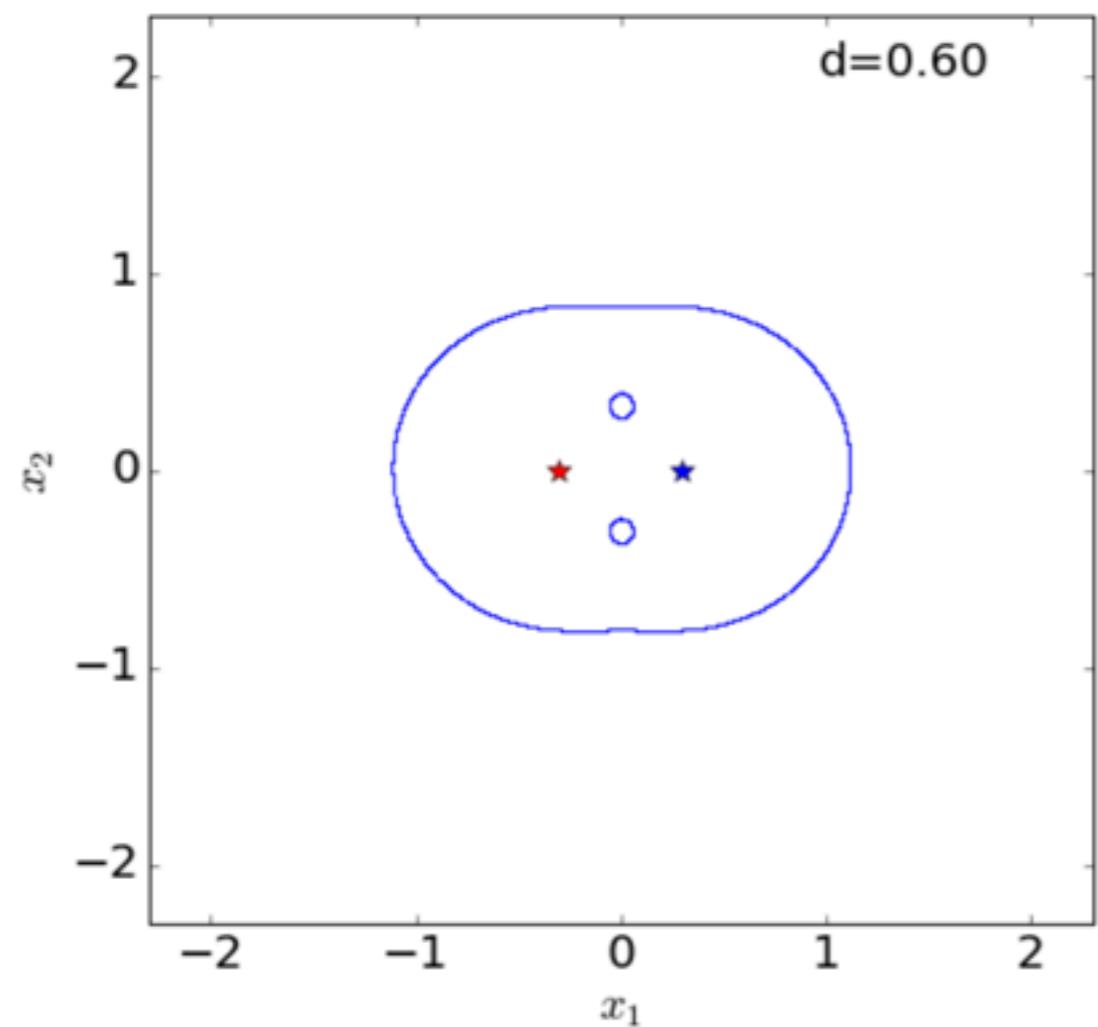


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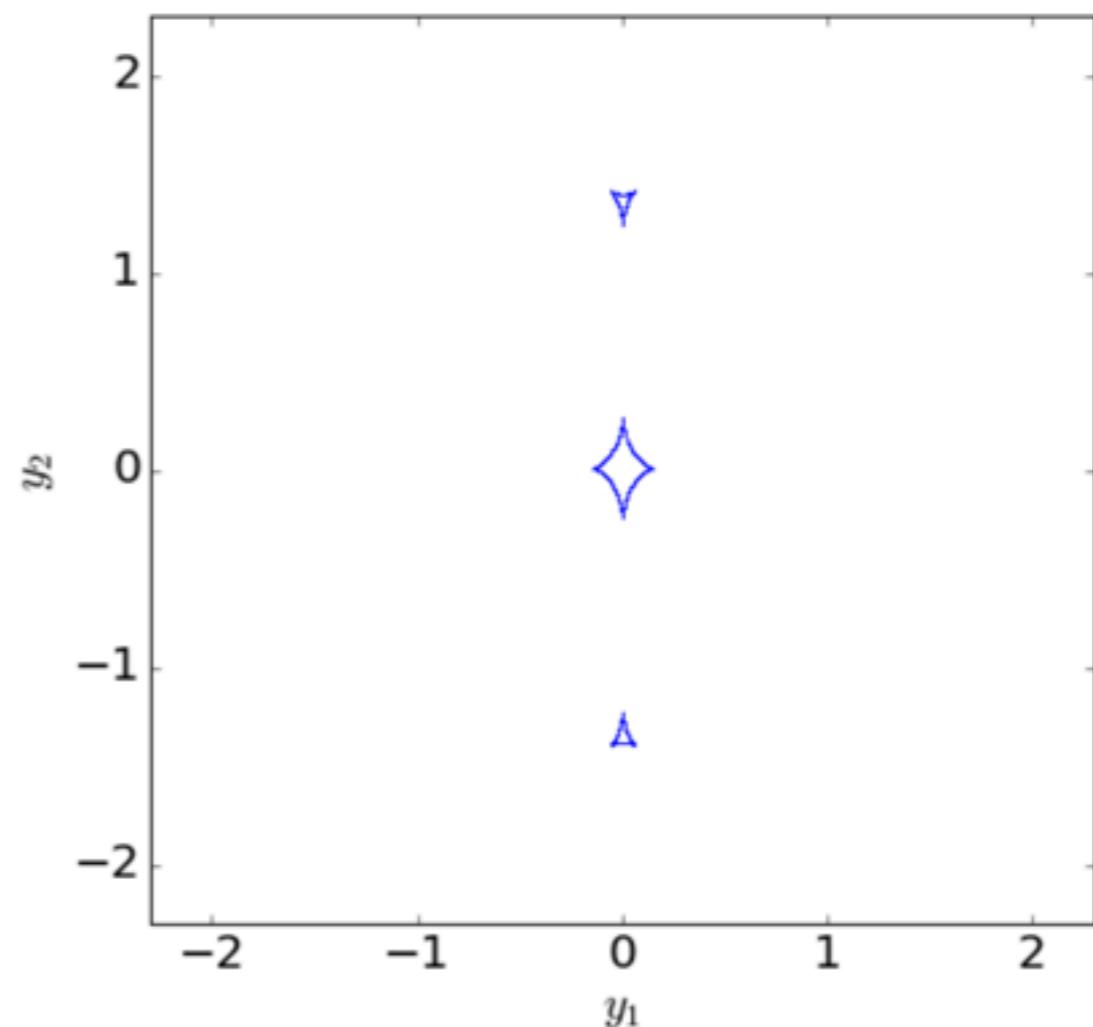


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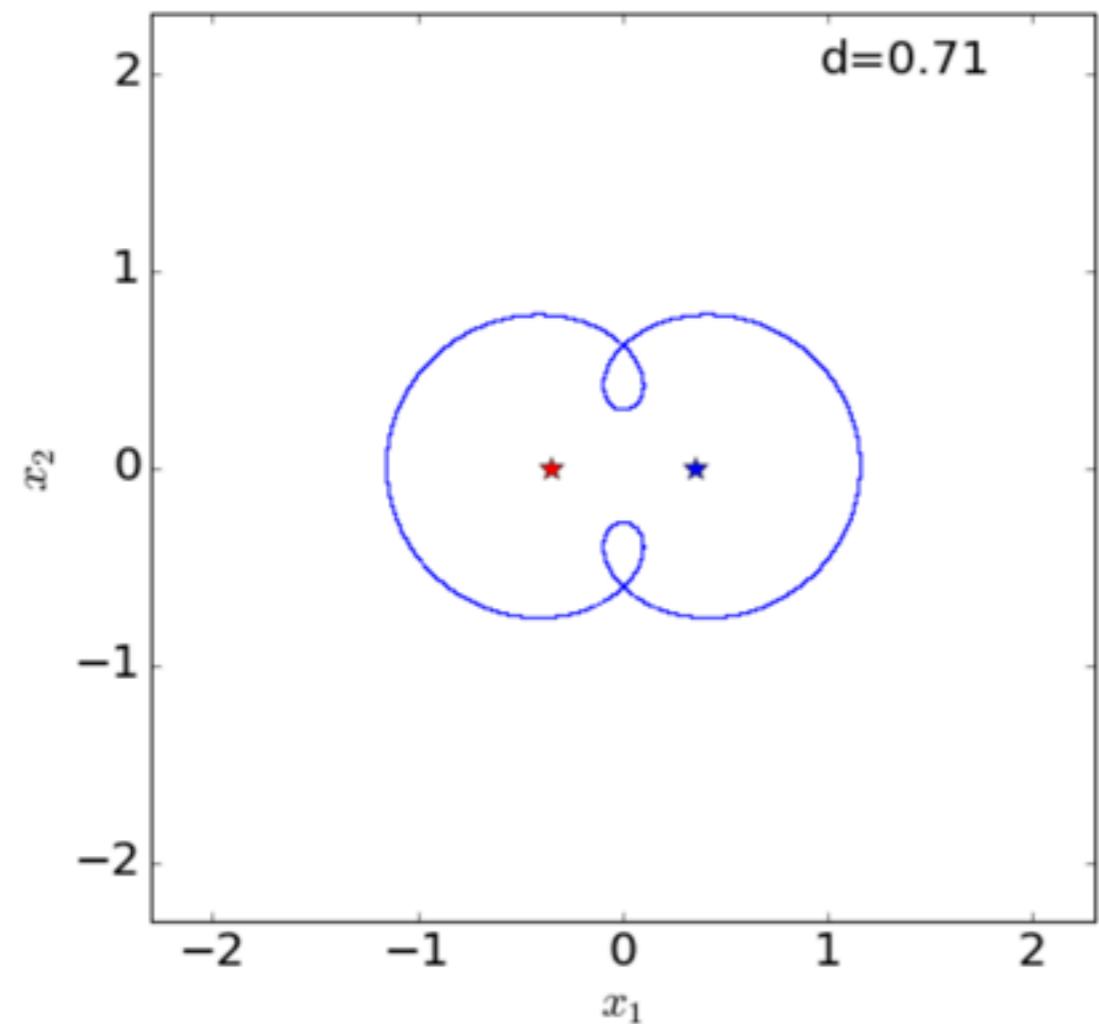


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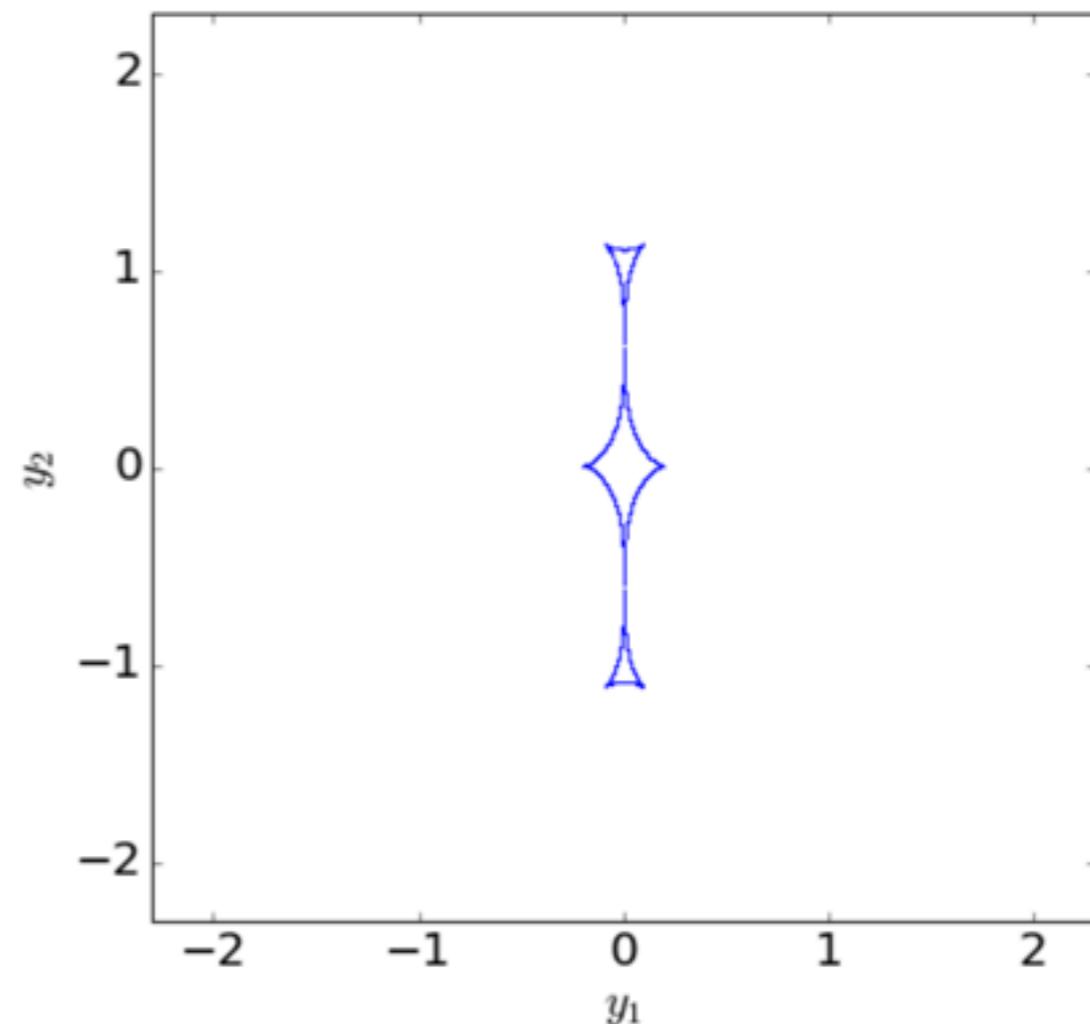


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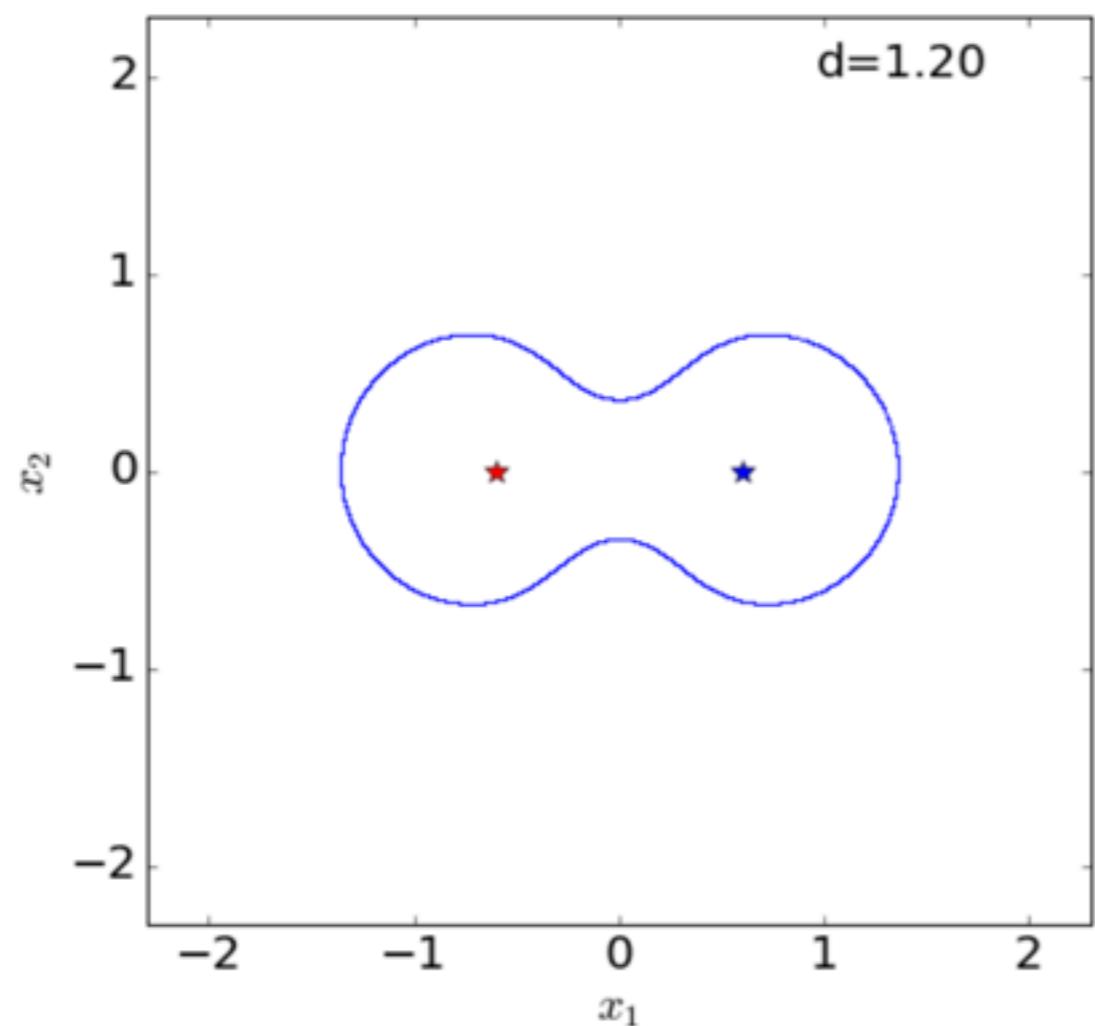


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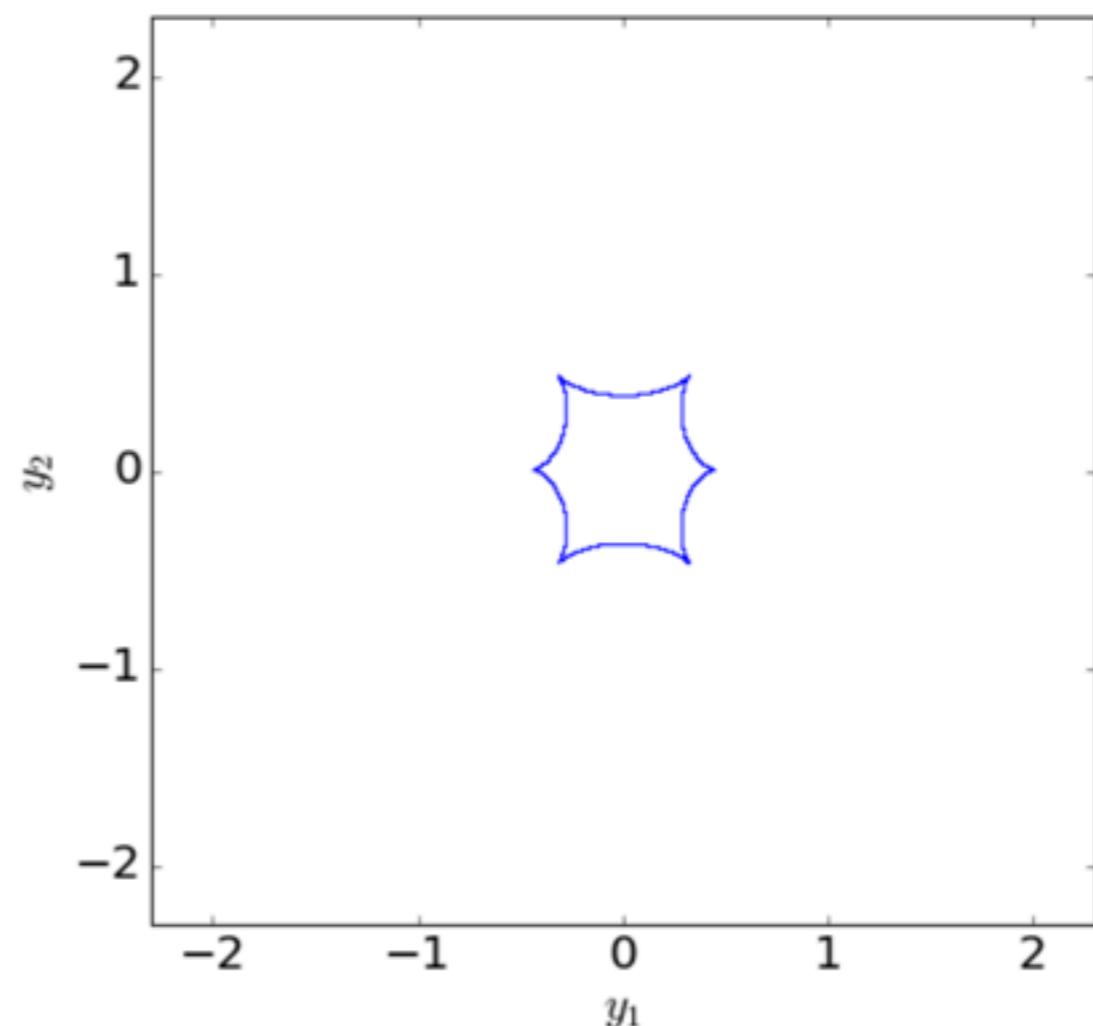


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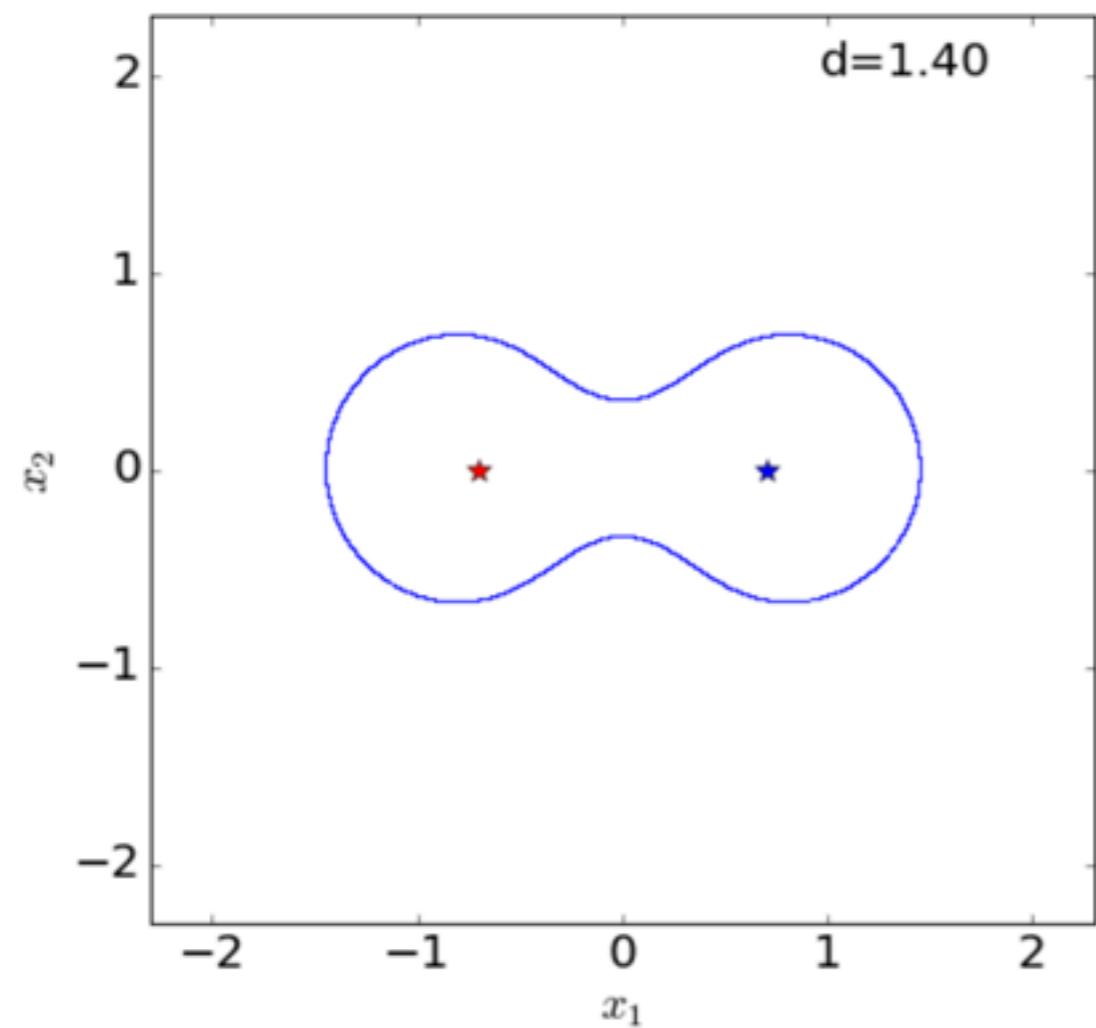


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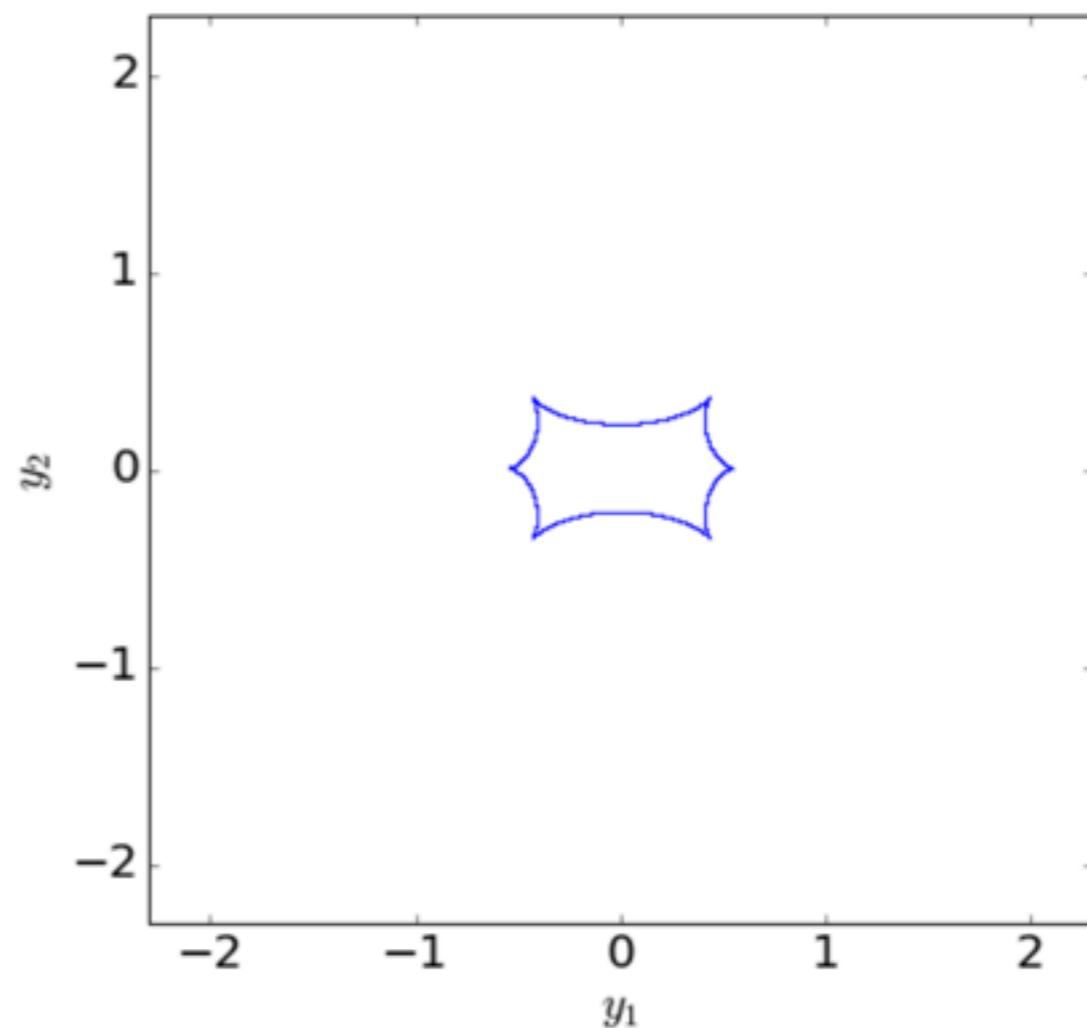


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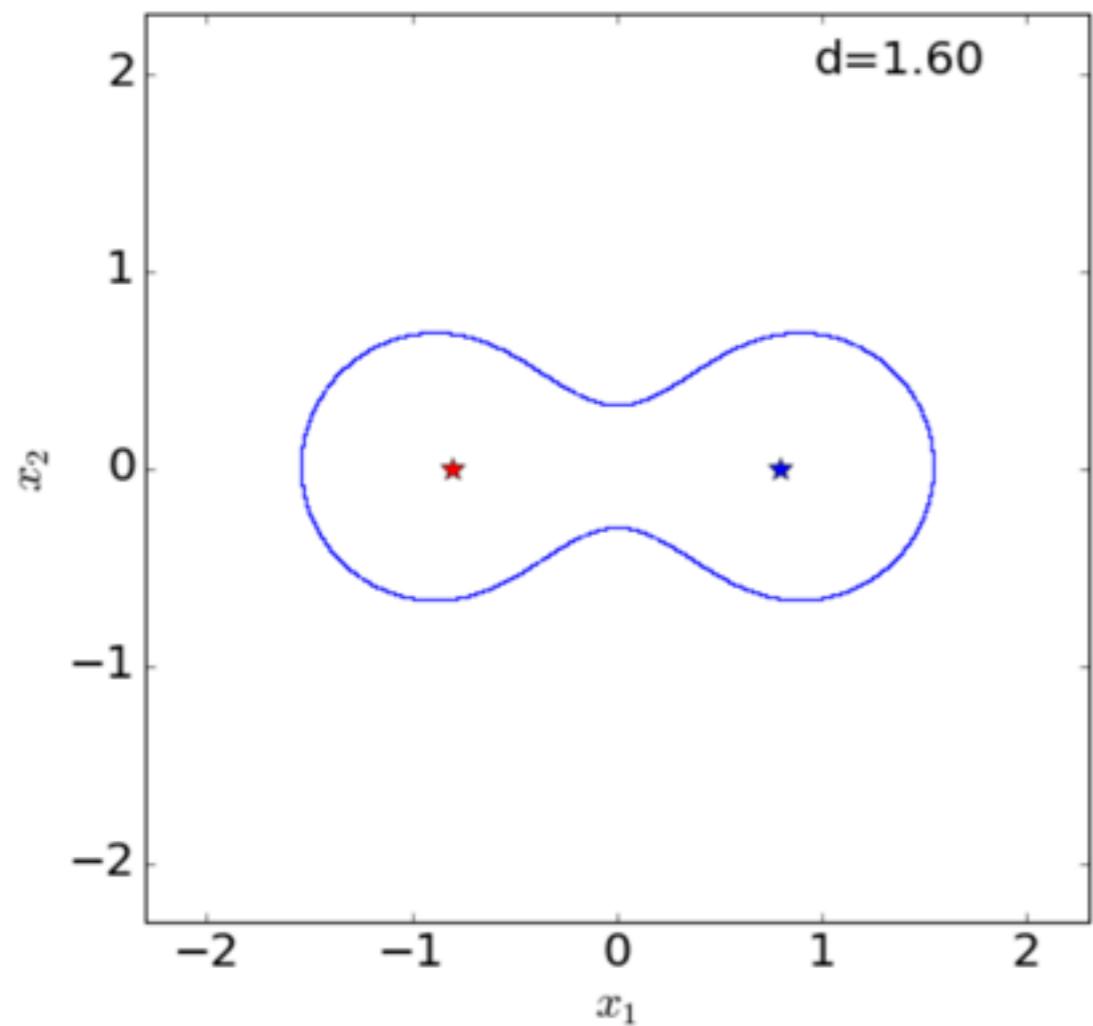


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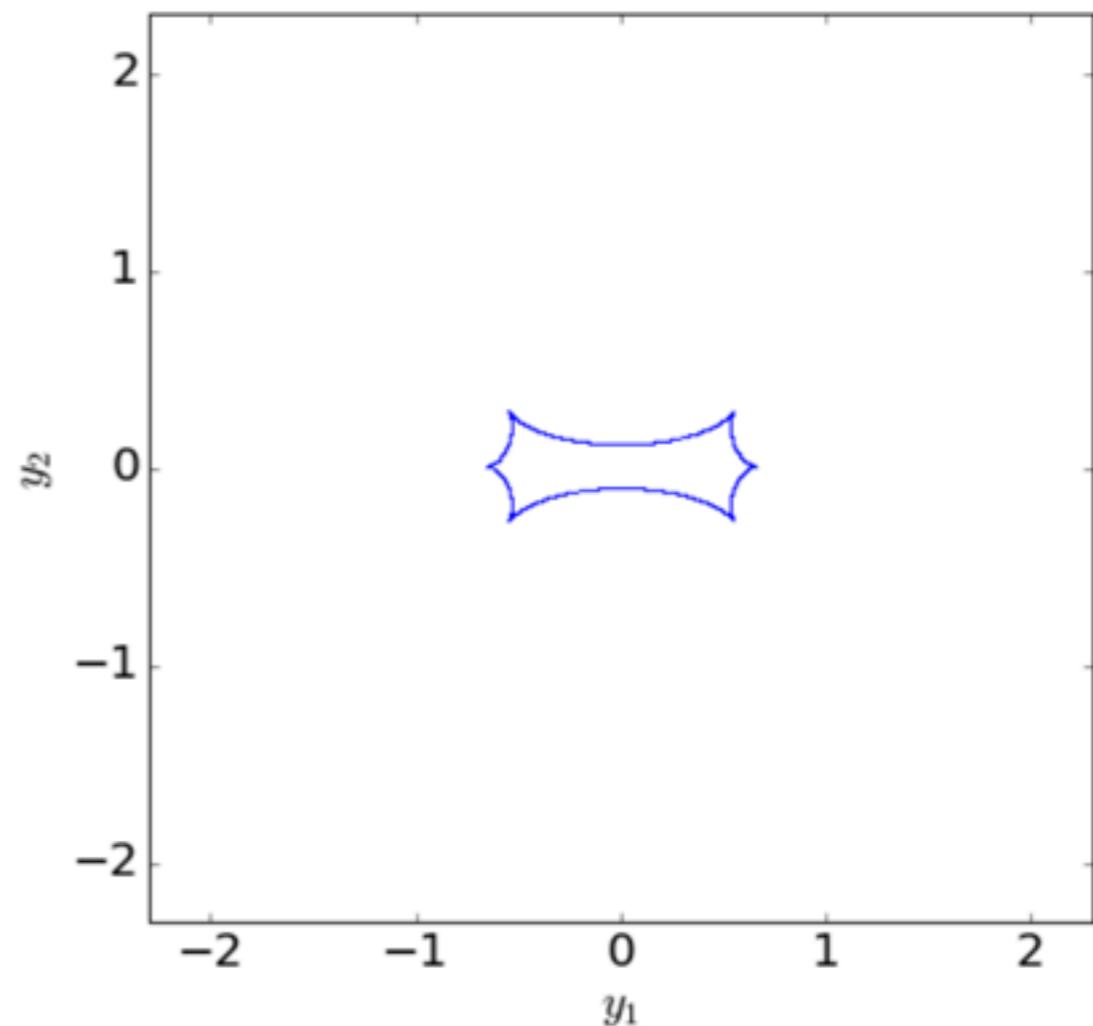


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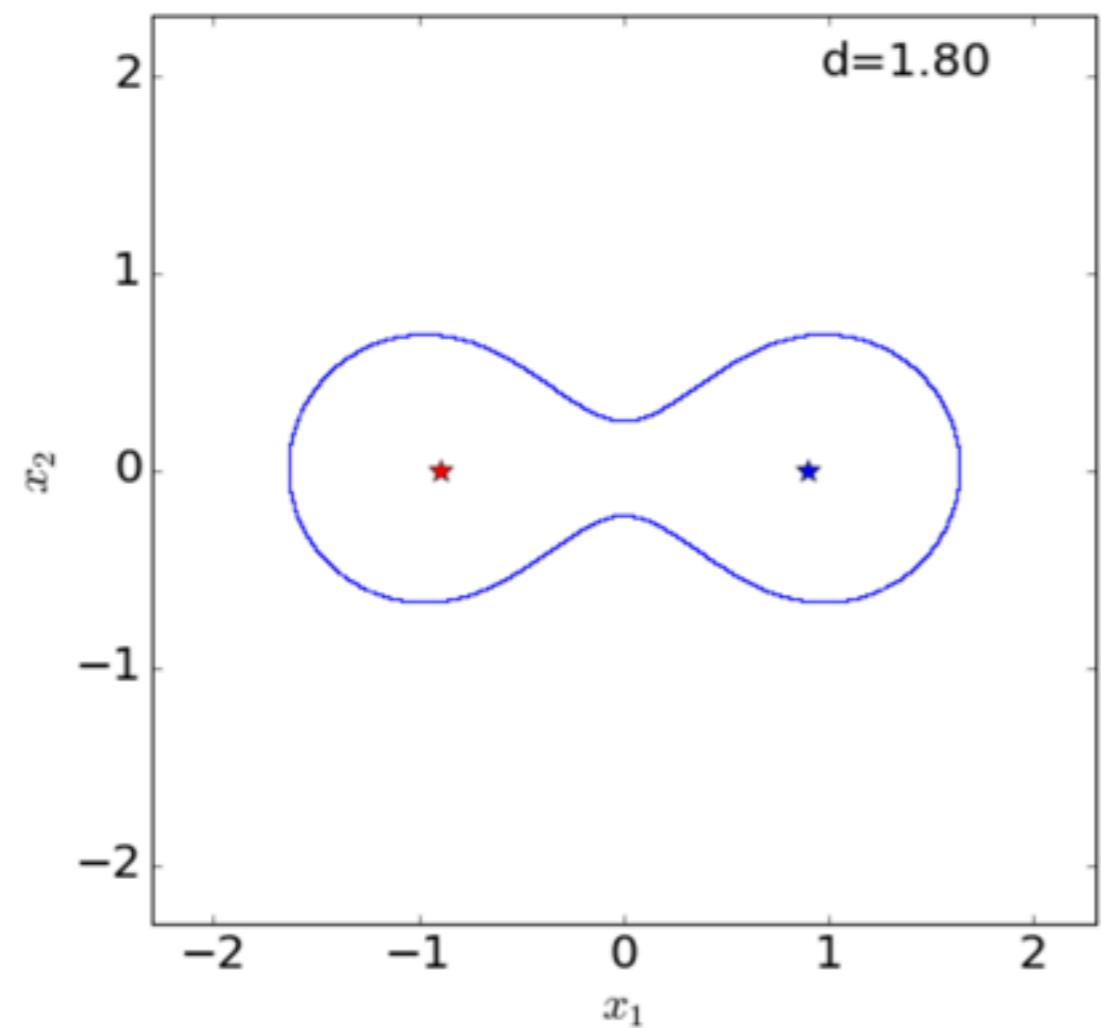


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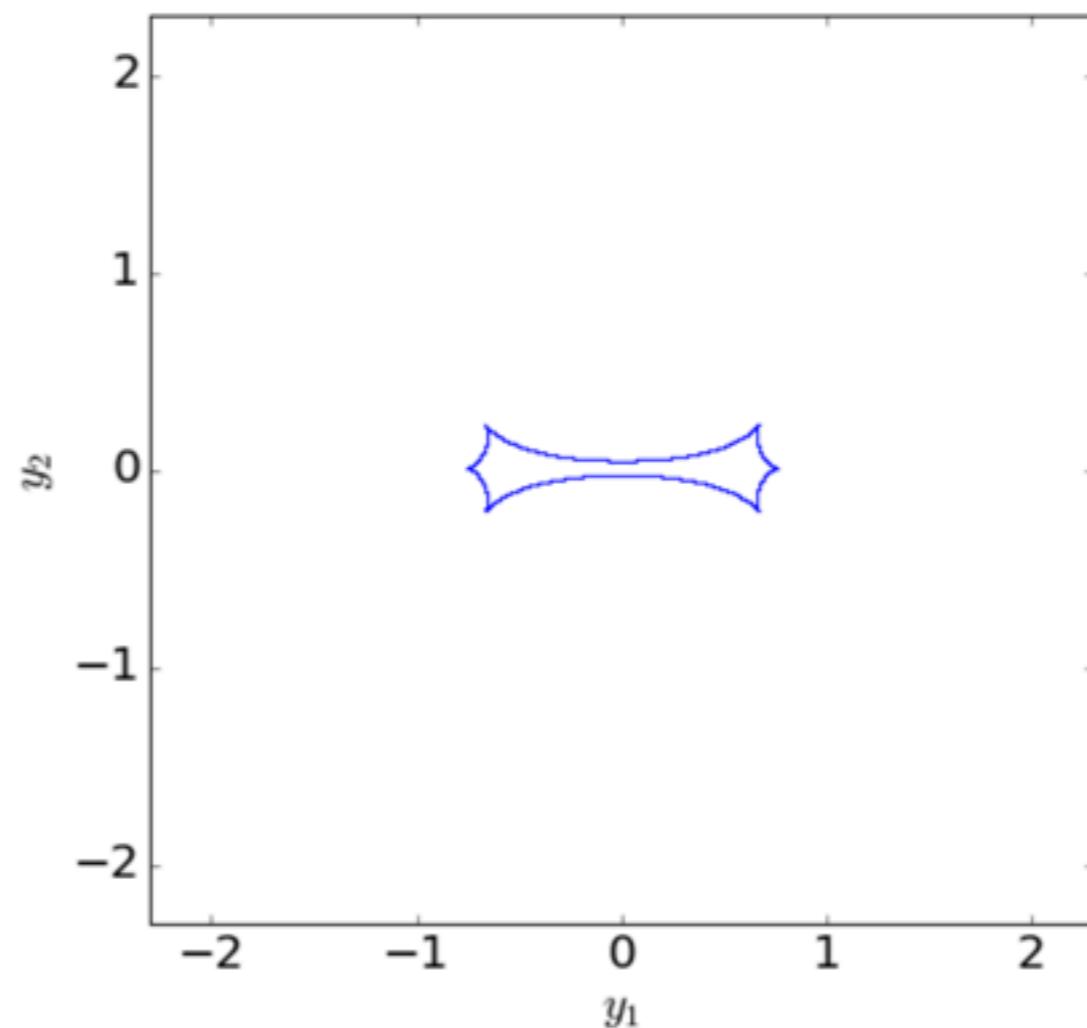


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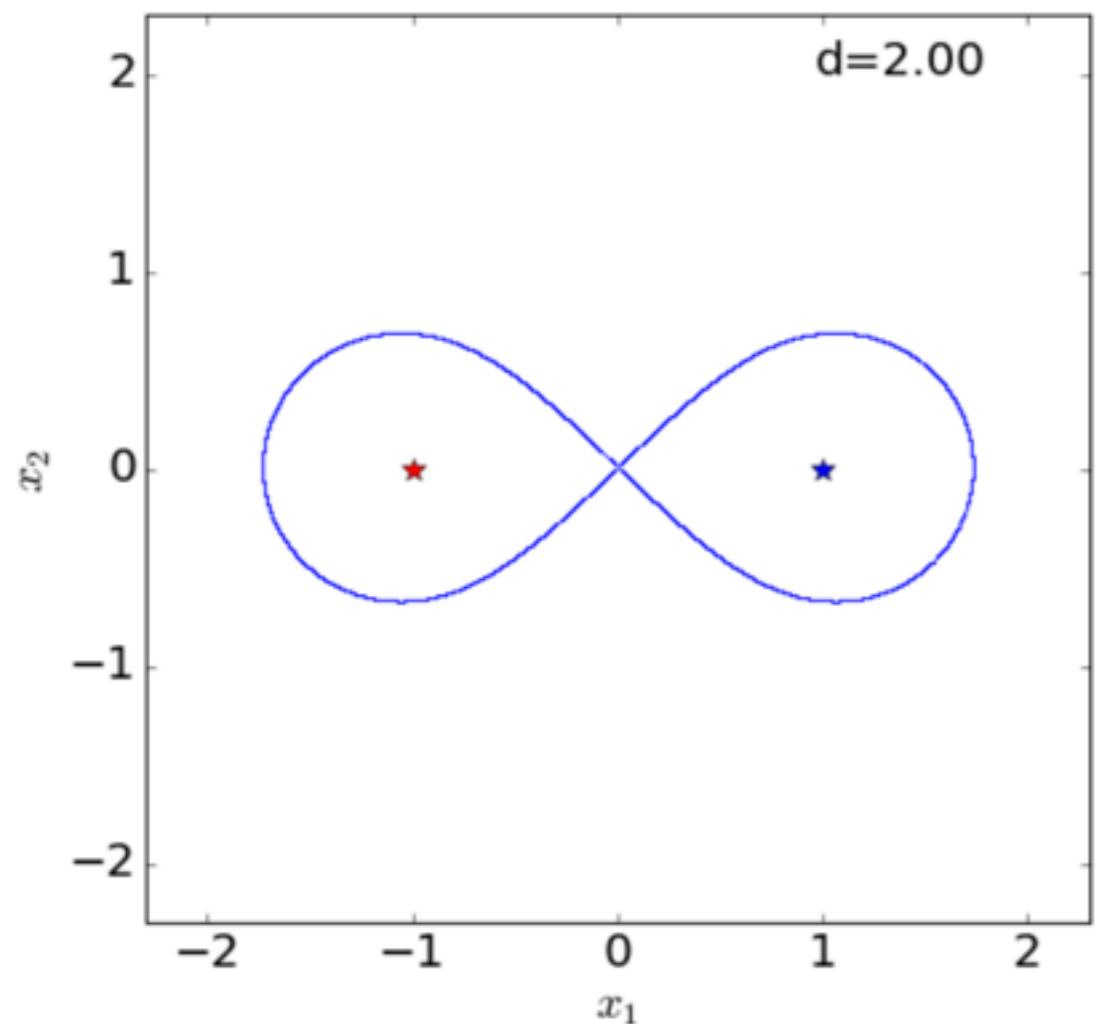


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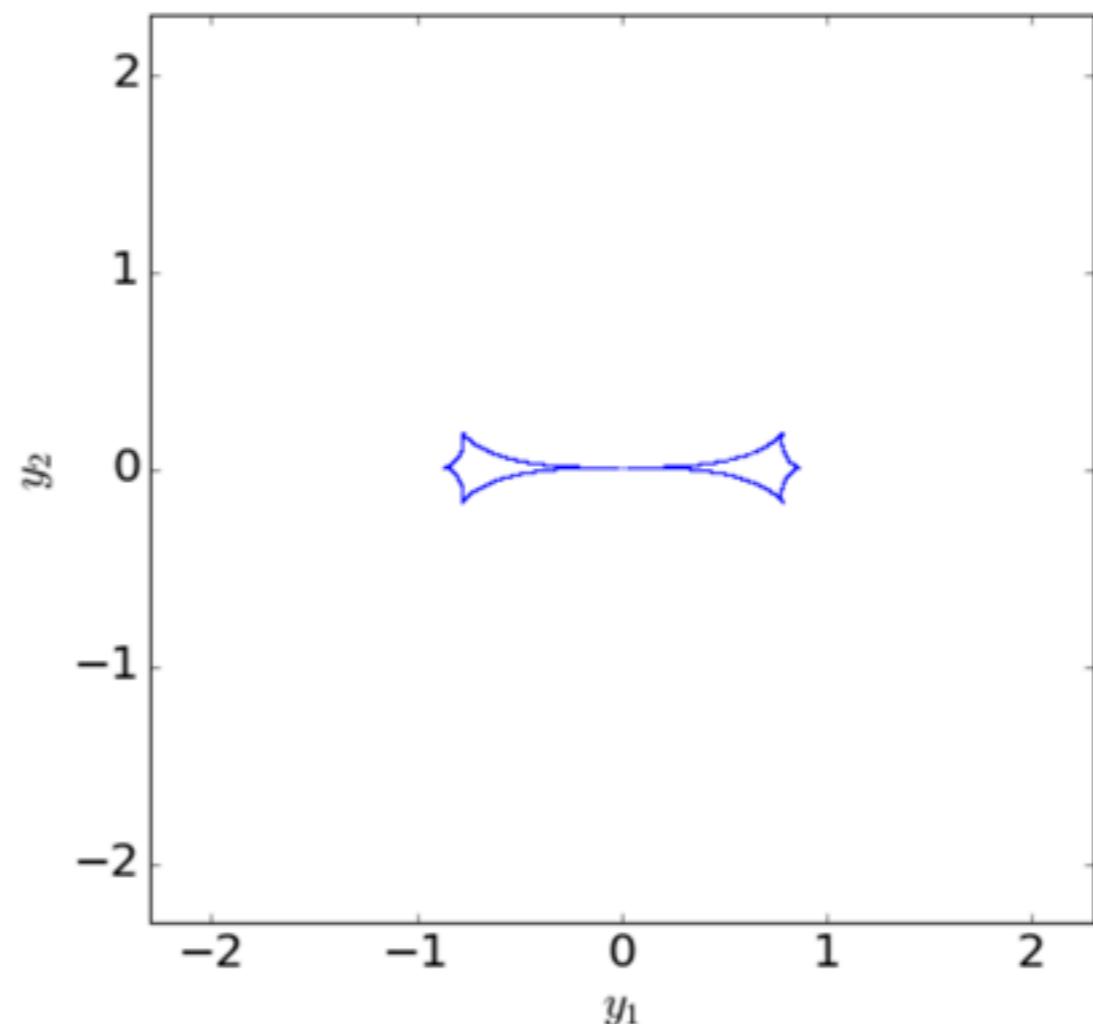


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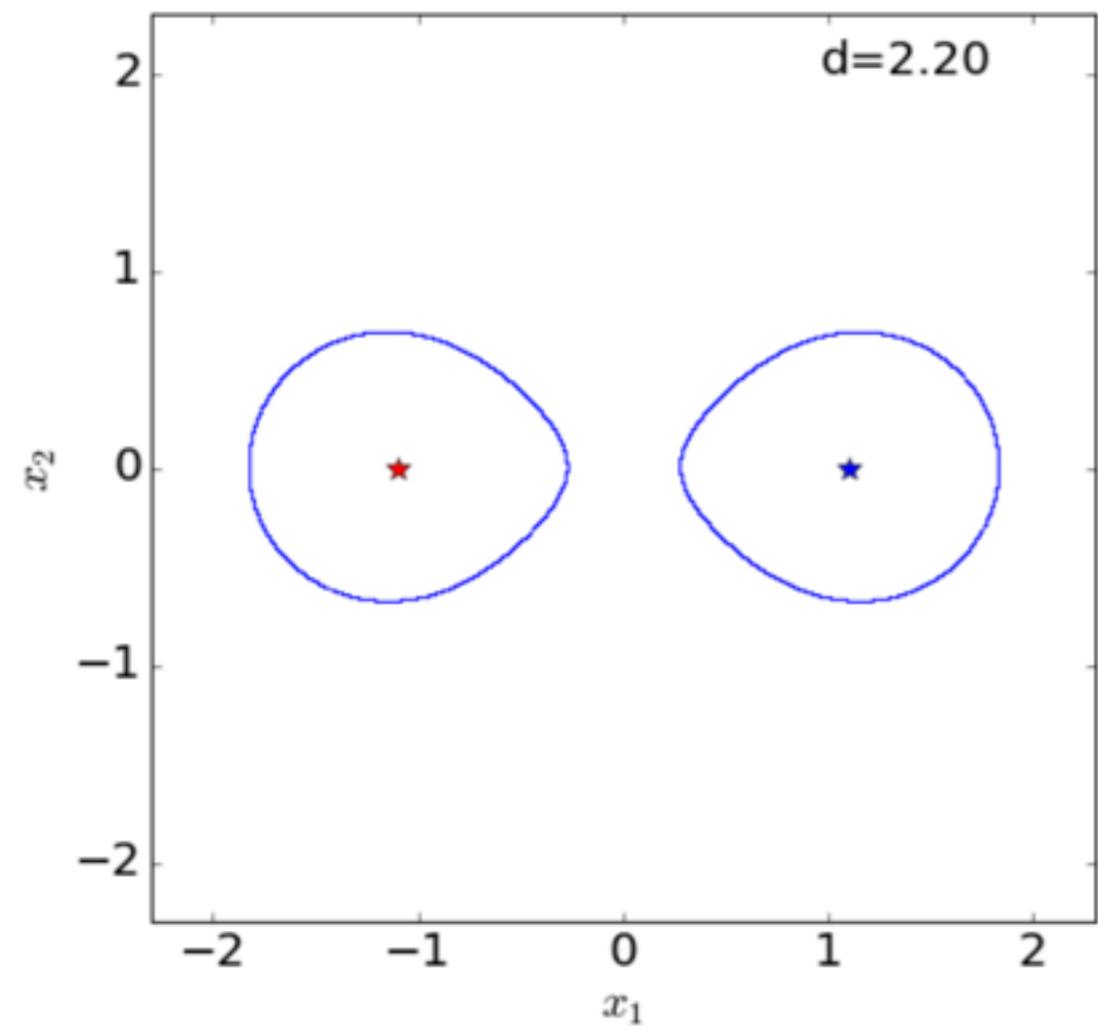


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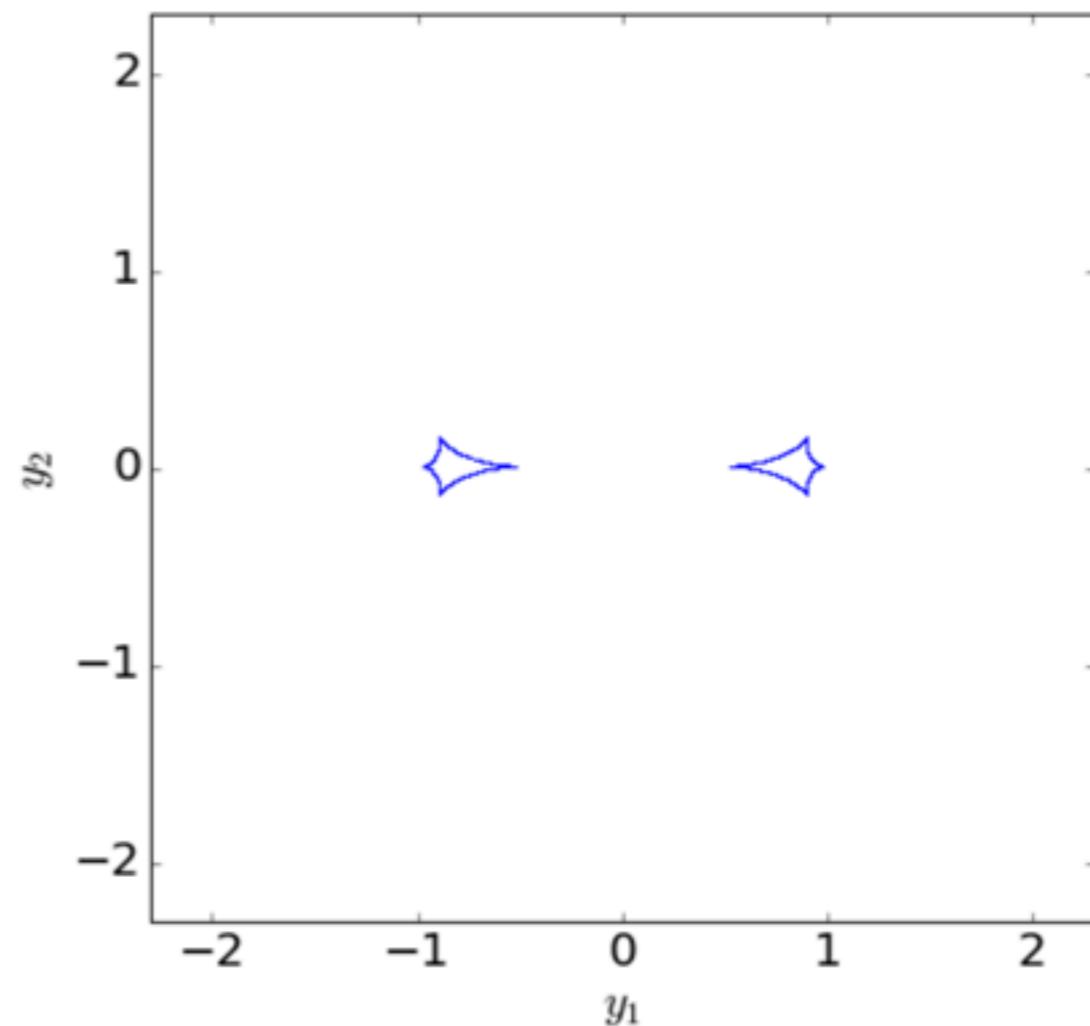


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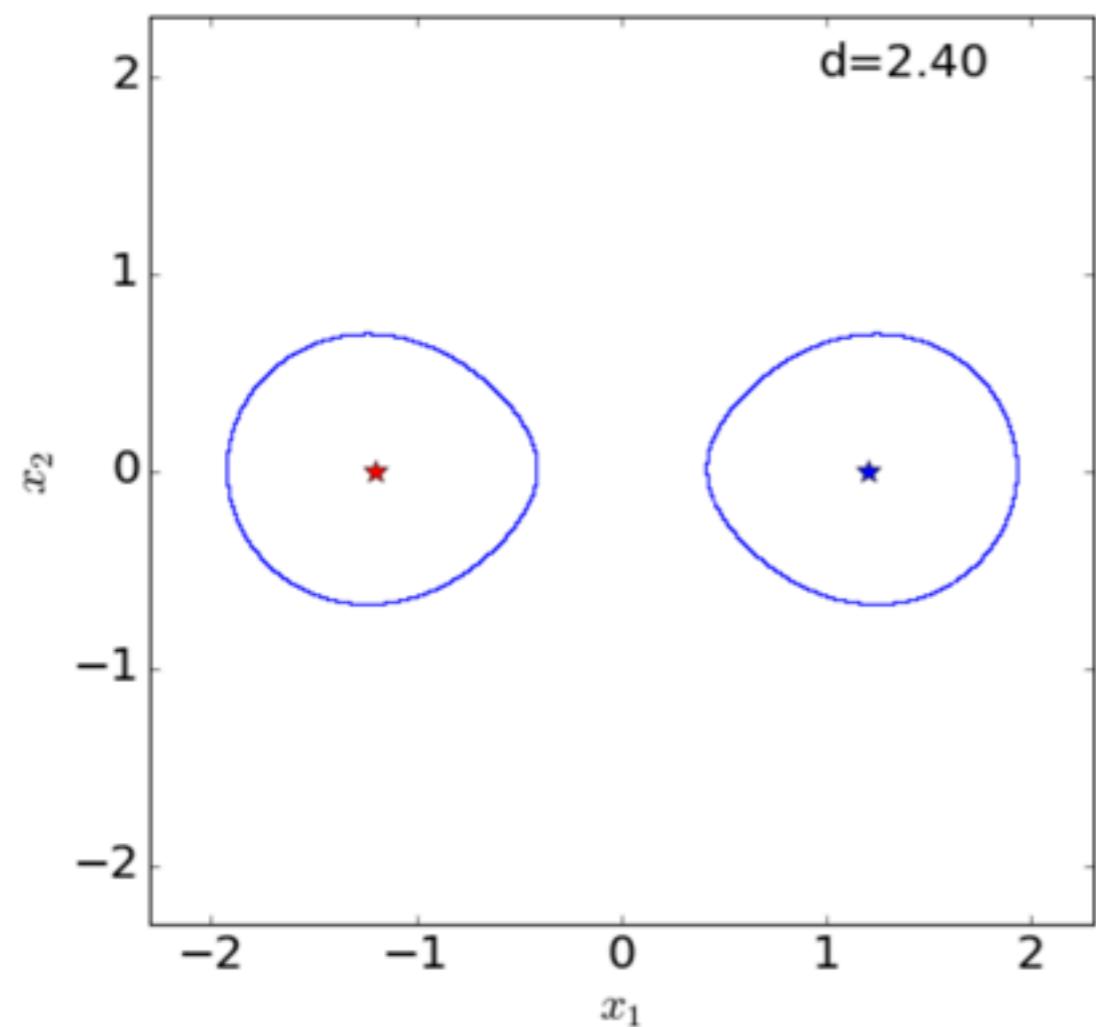


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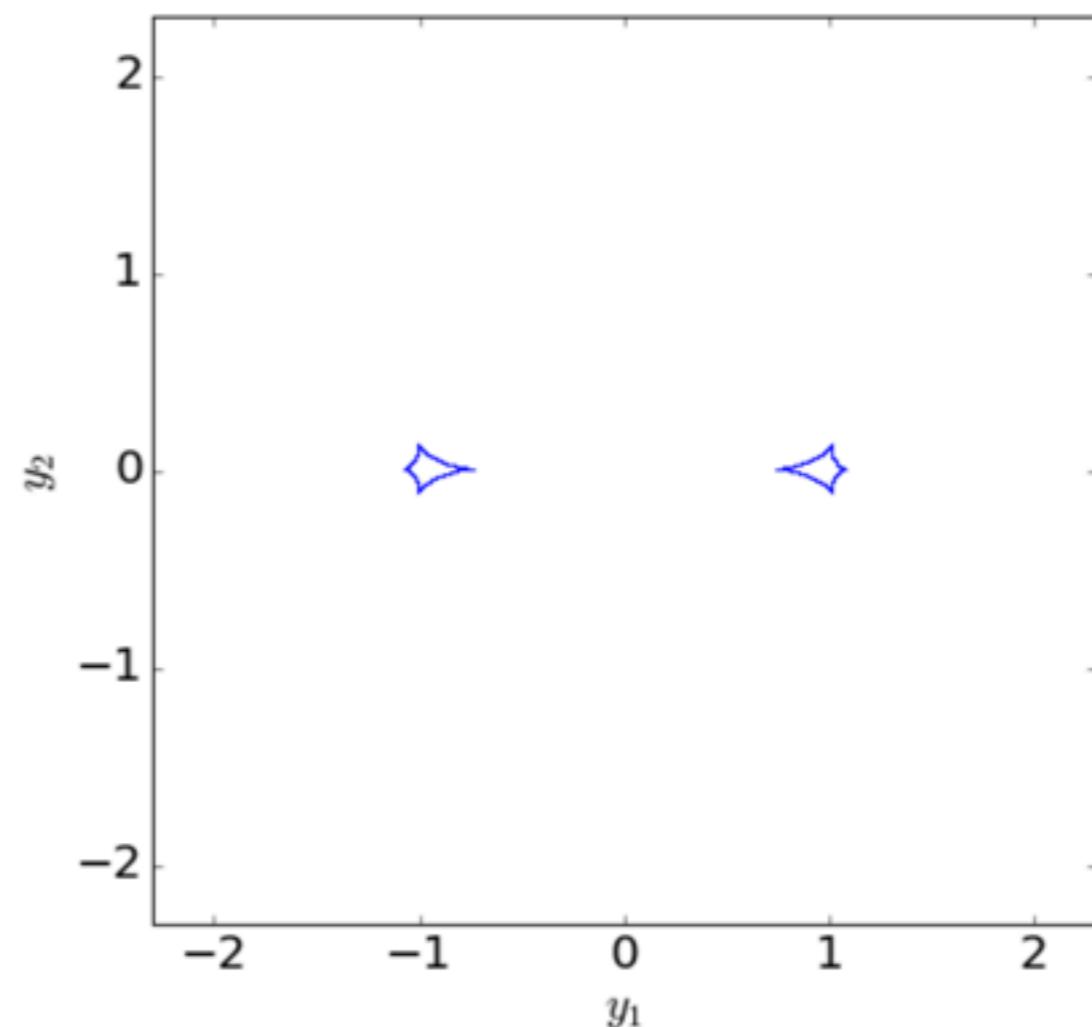


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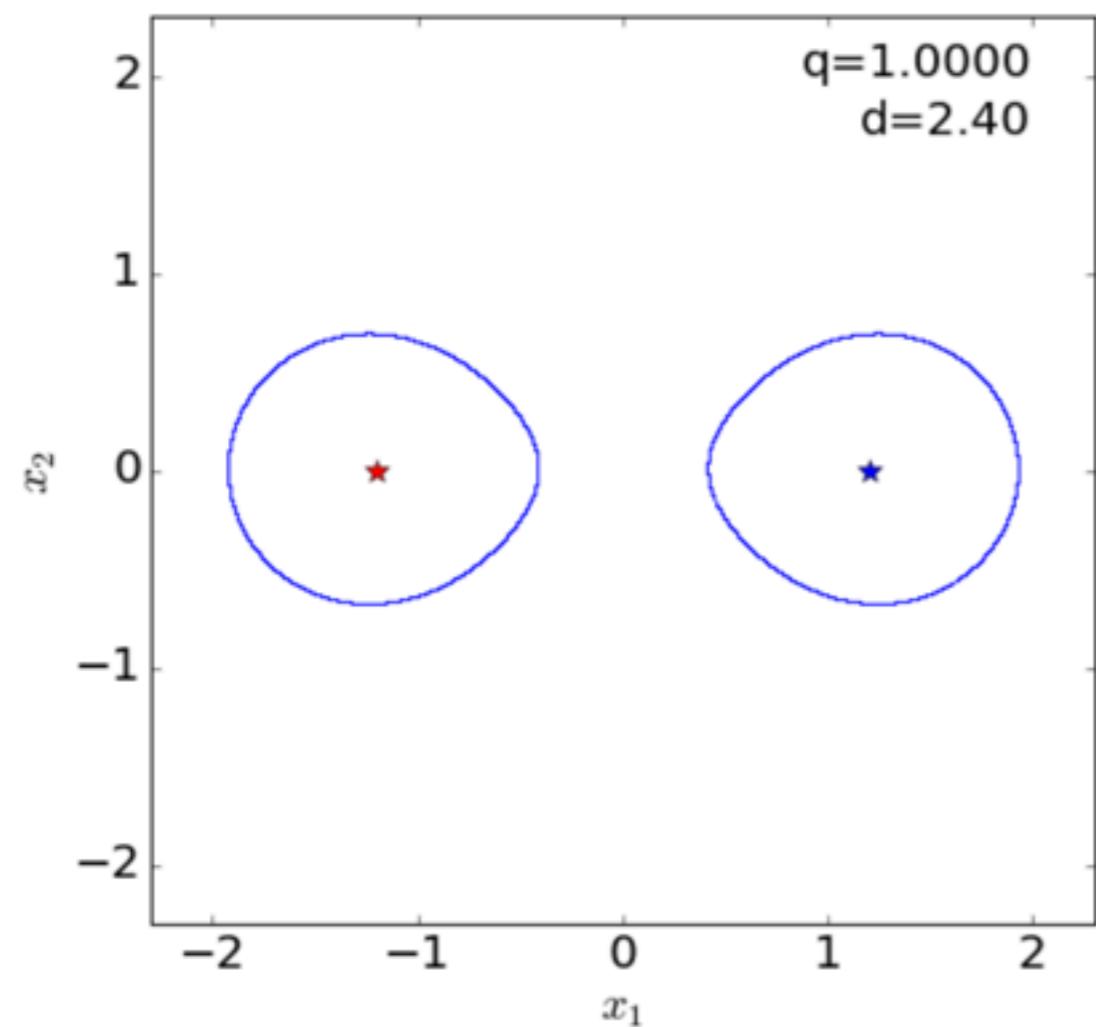


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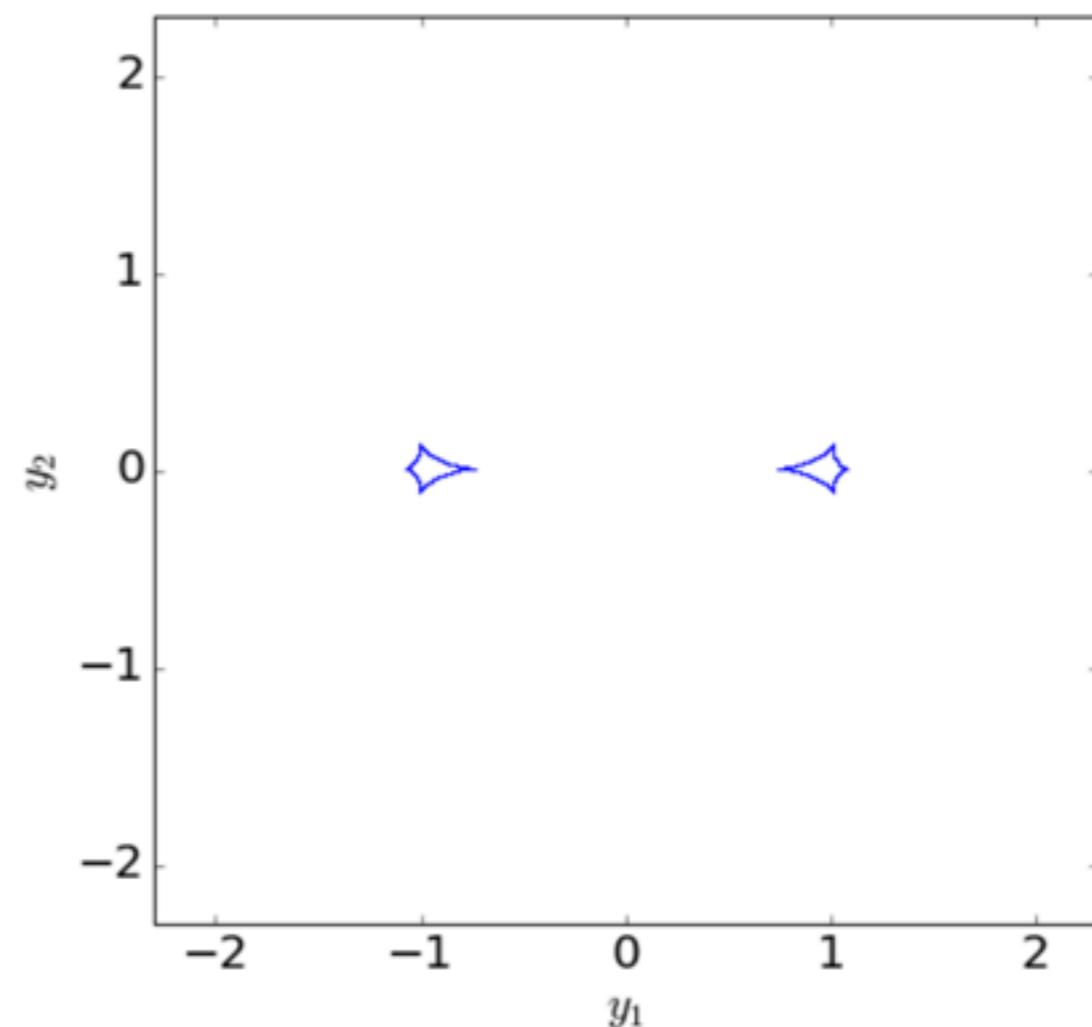


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BINARY LENSES: TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

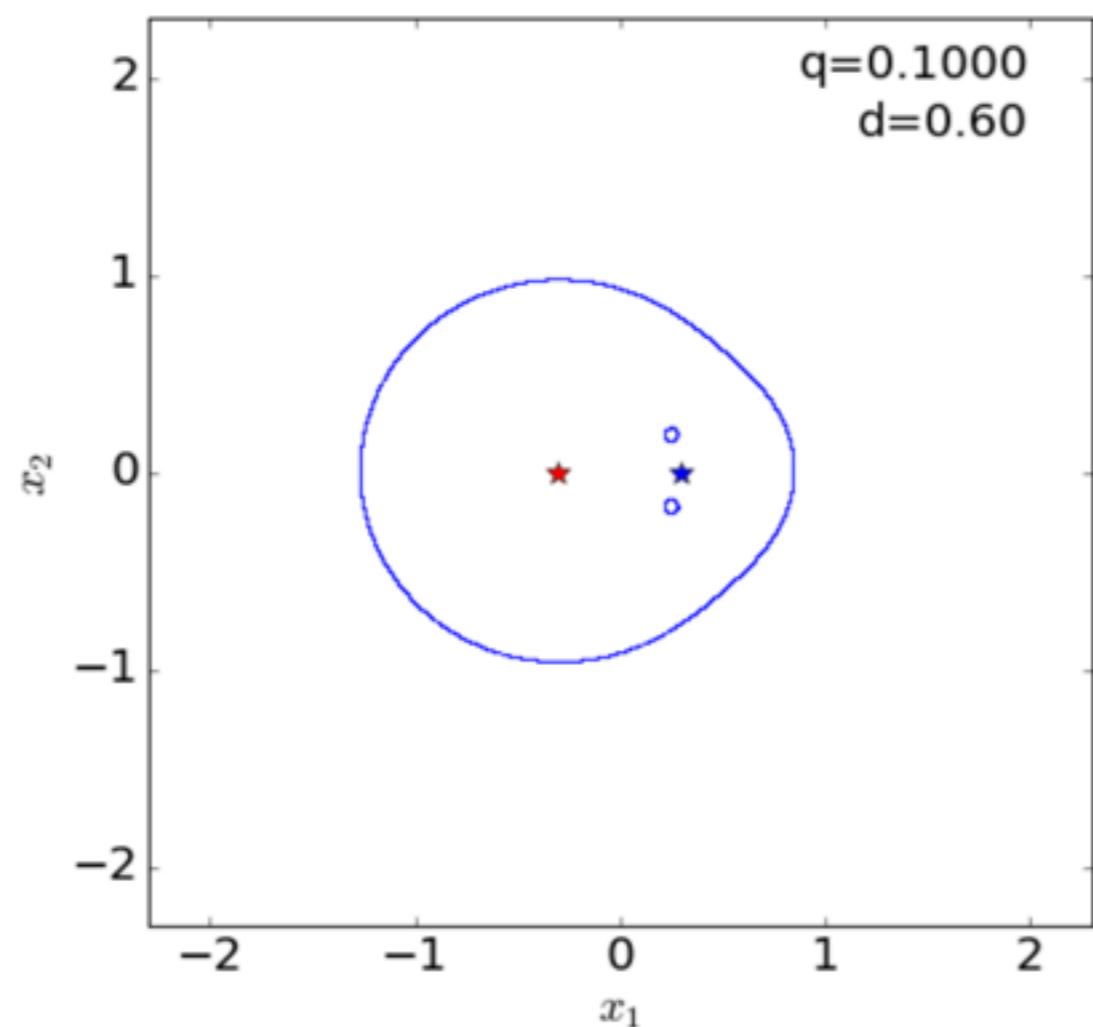


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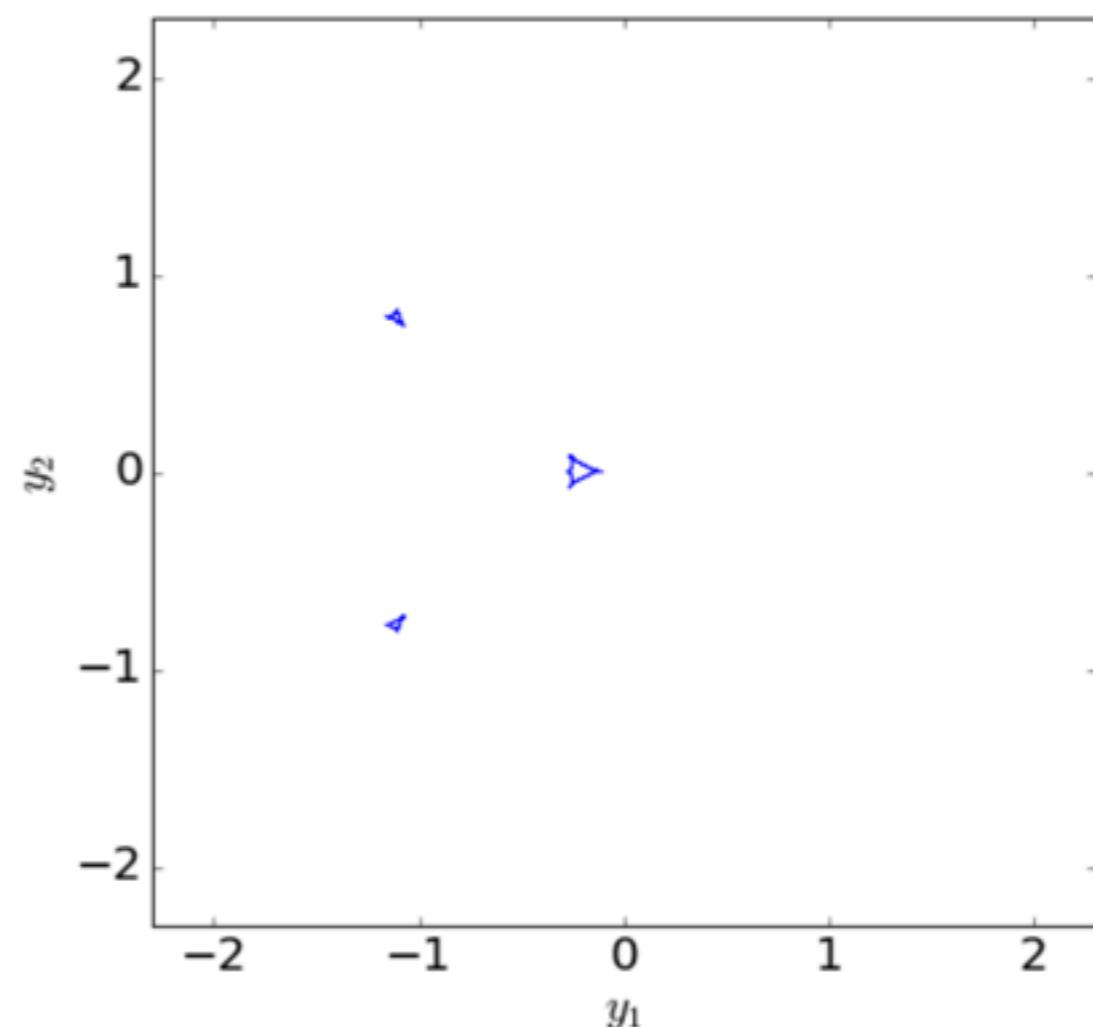


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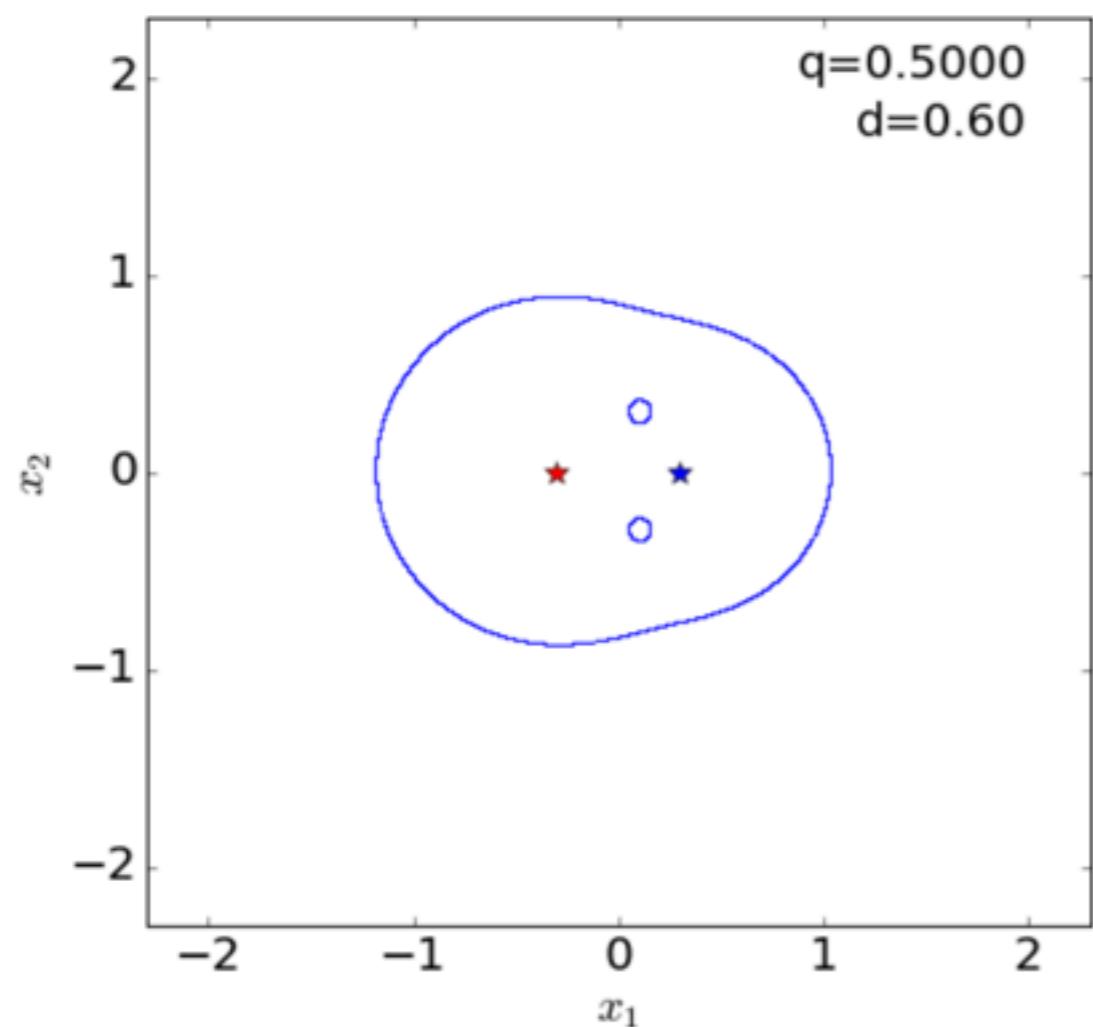


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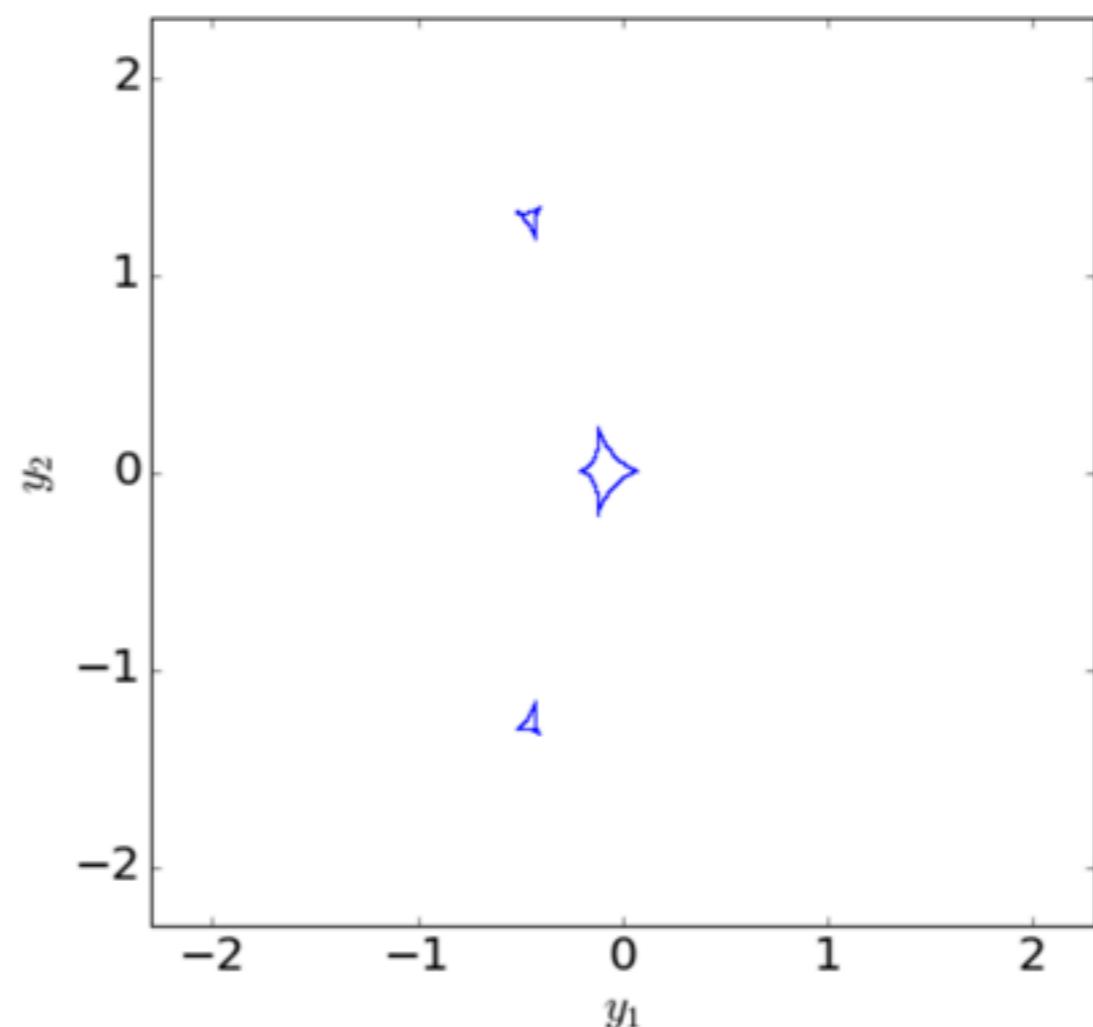


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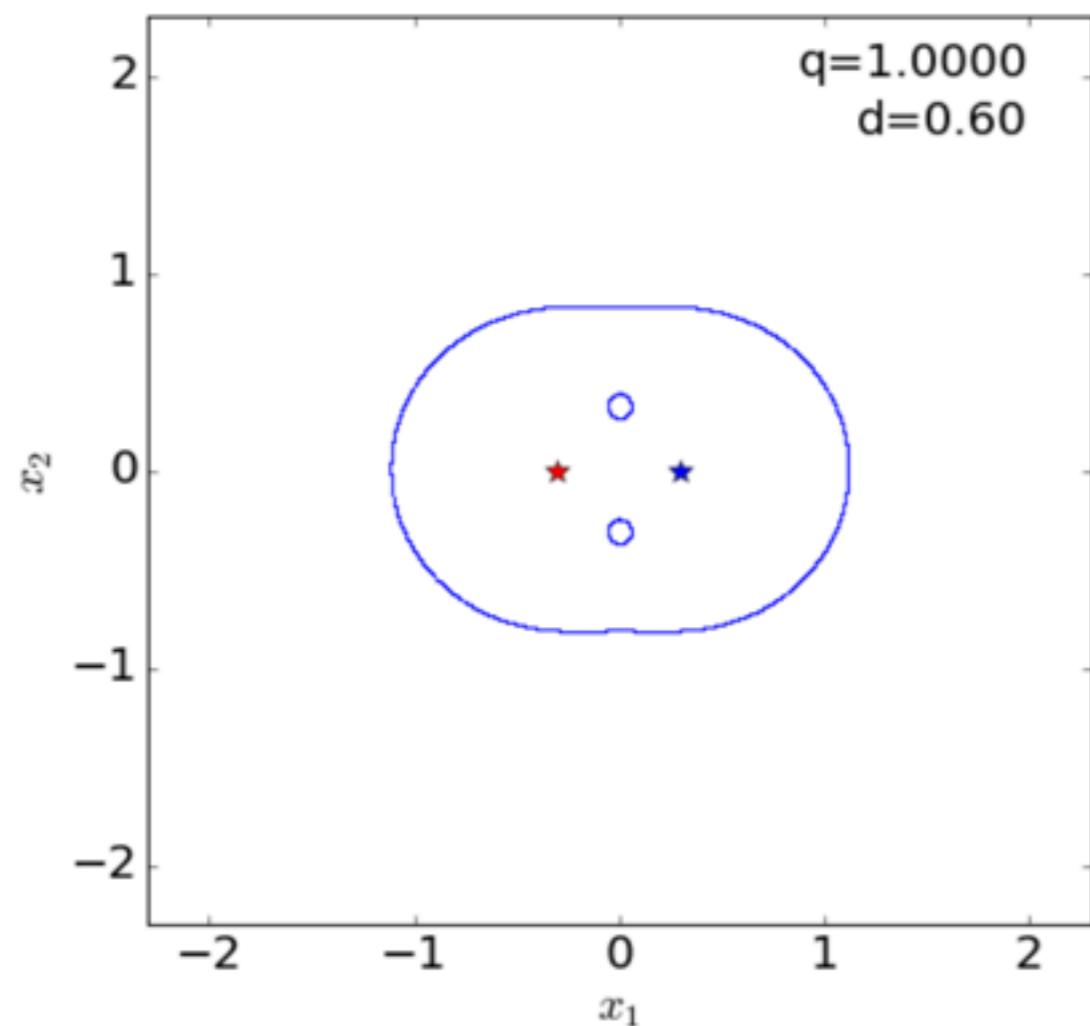


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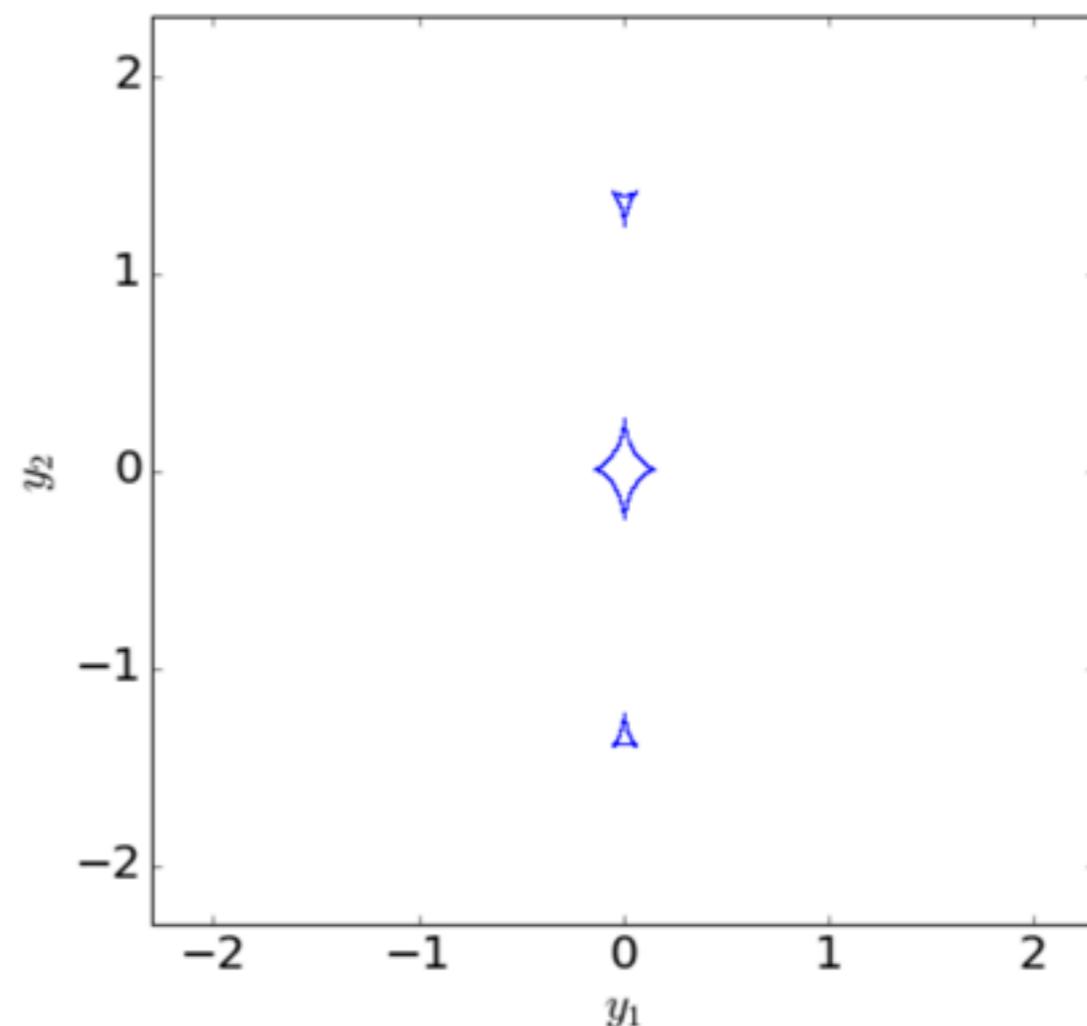


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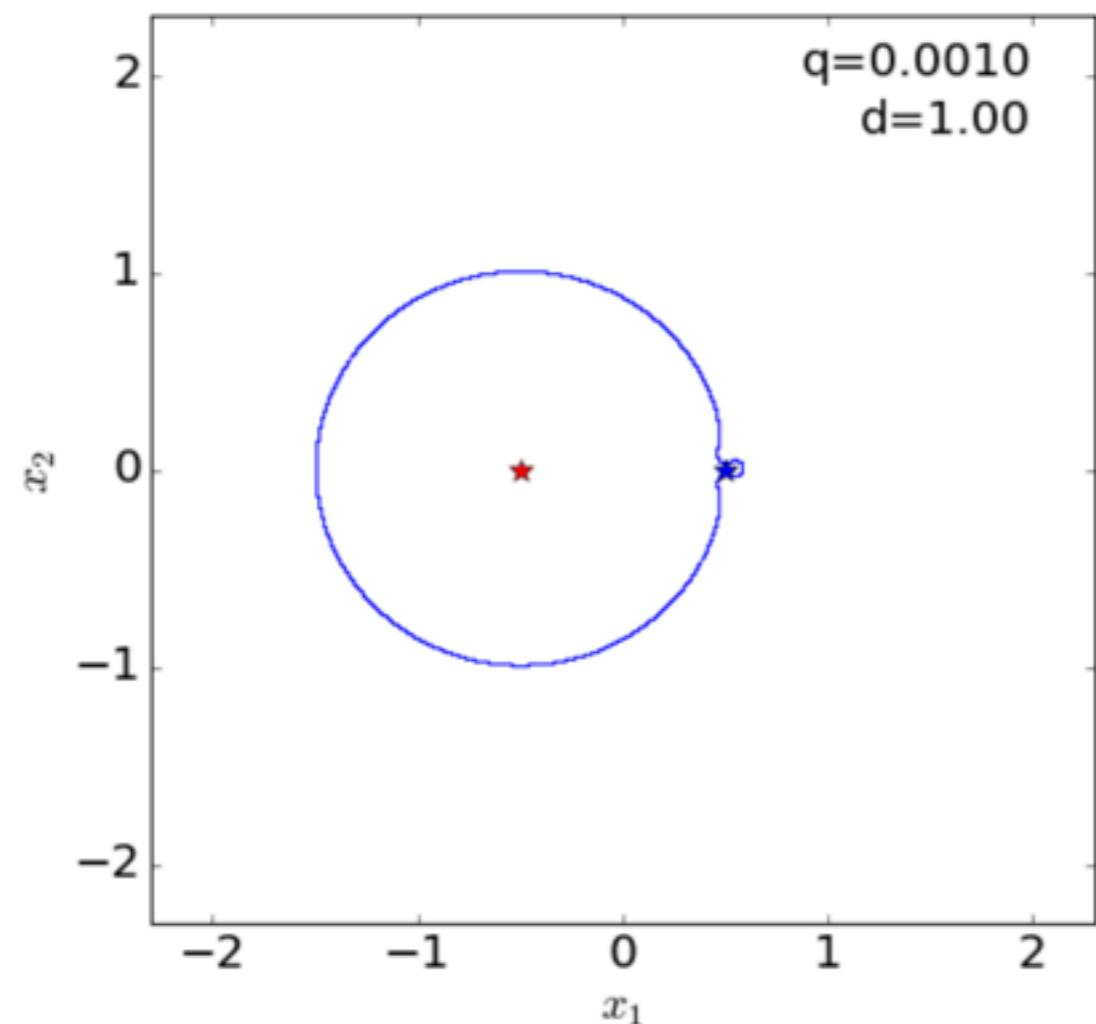


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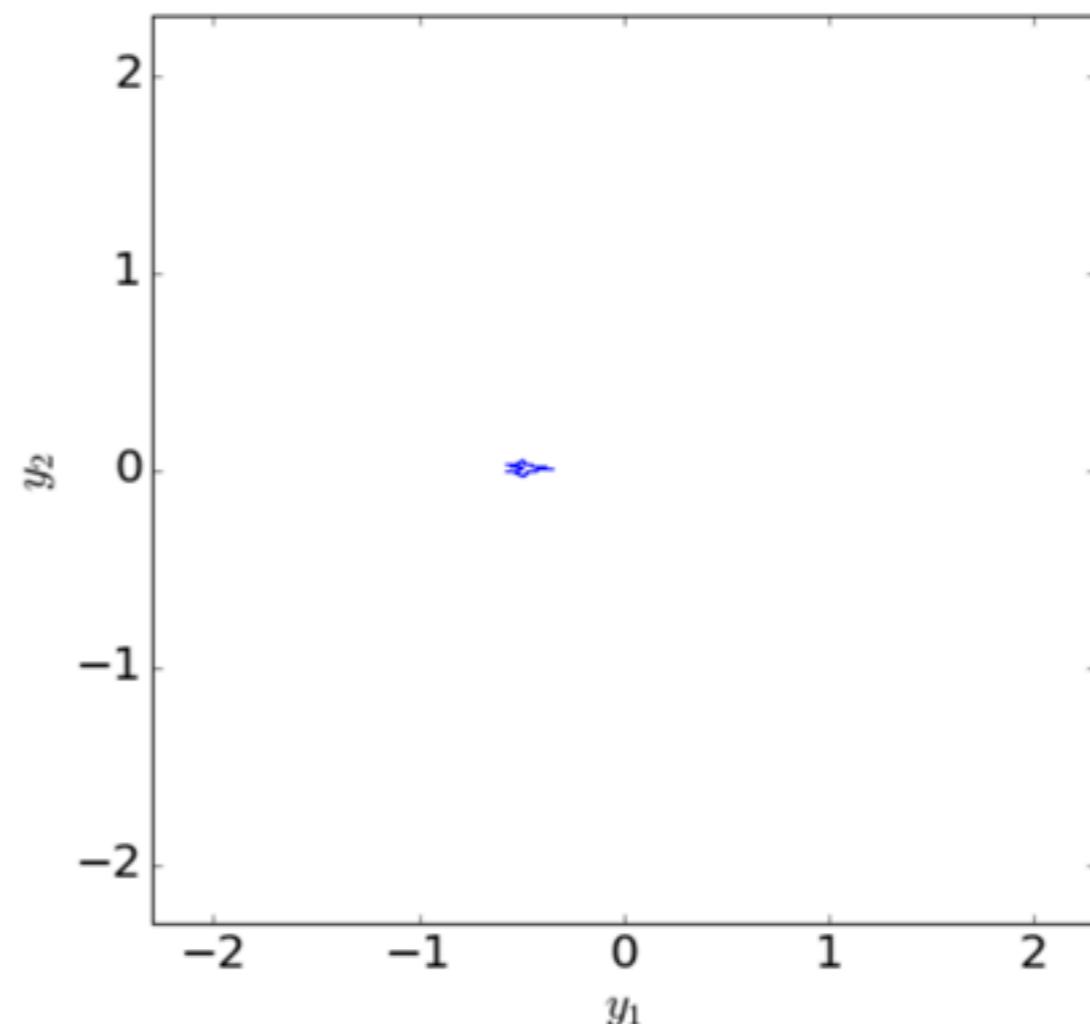


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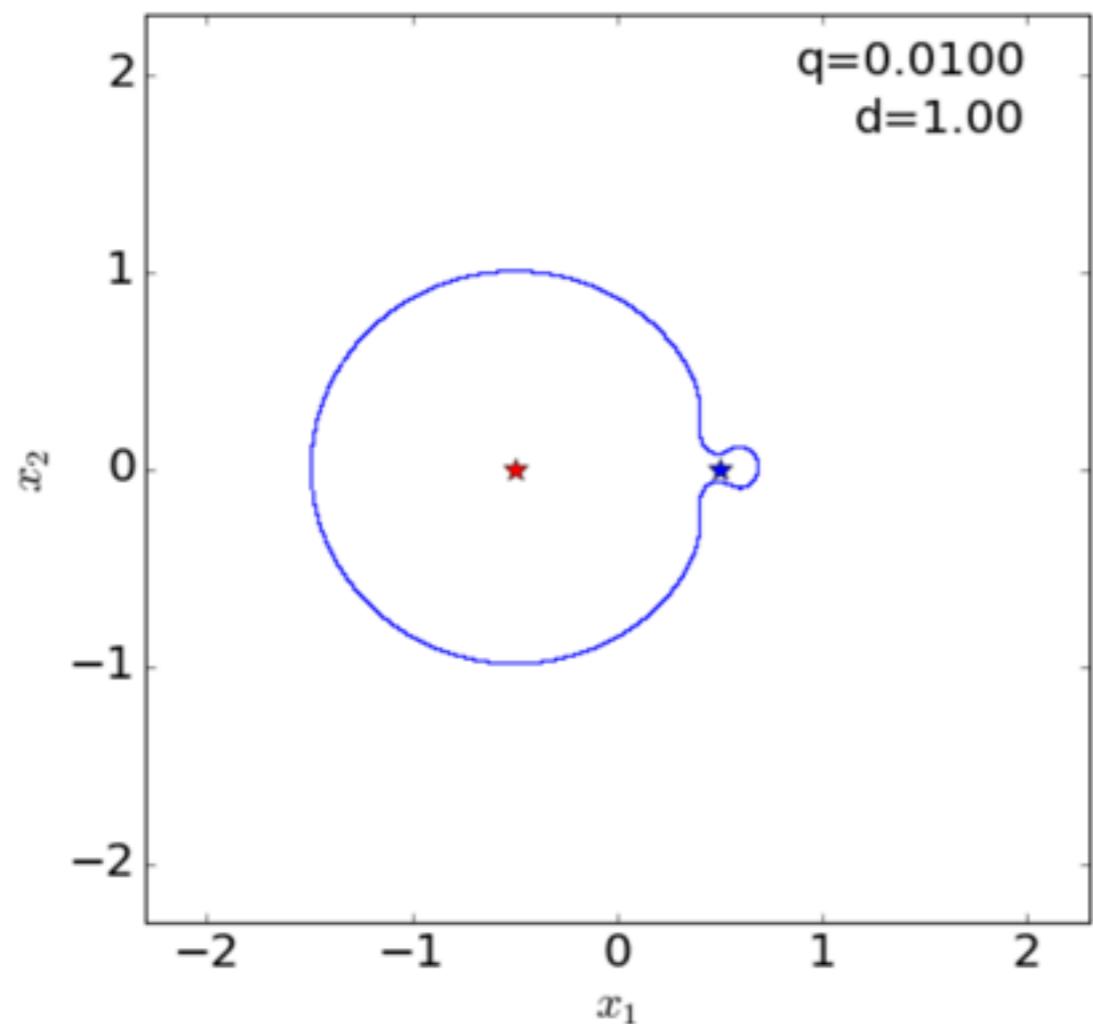


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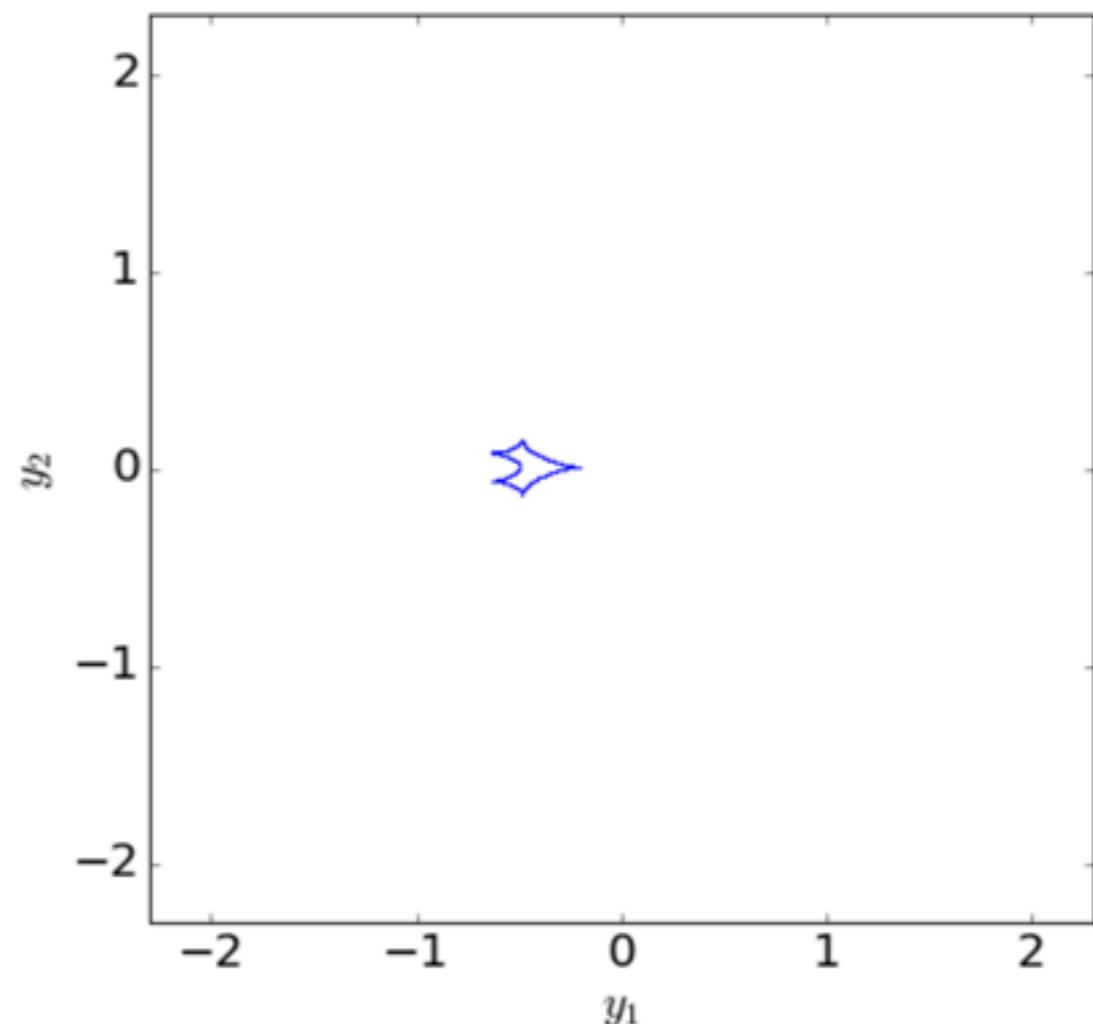


caustics

BINARY LENSES: TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

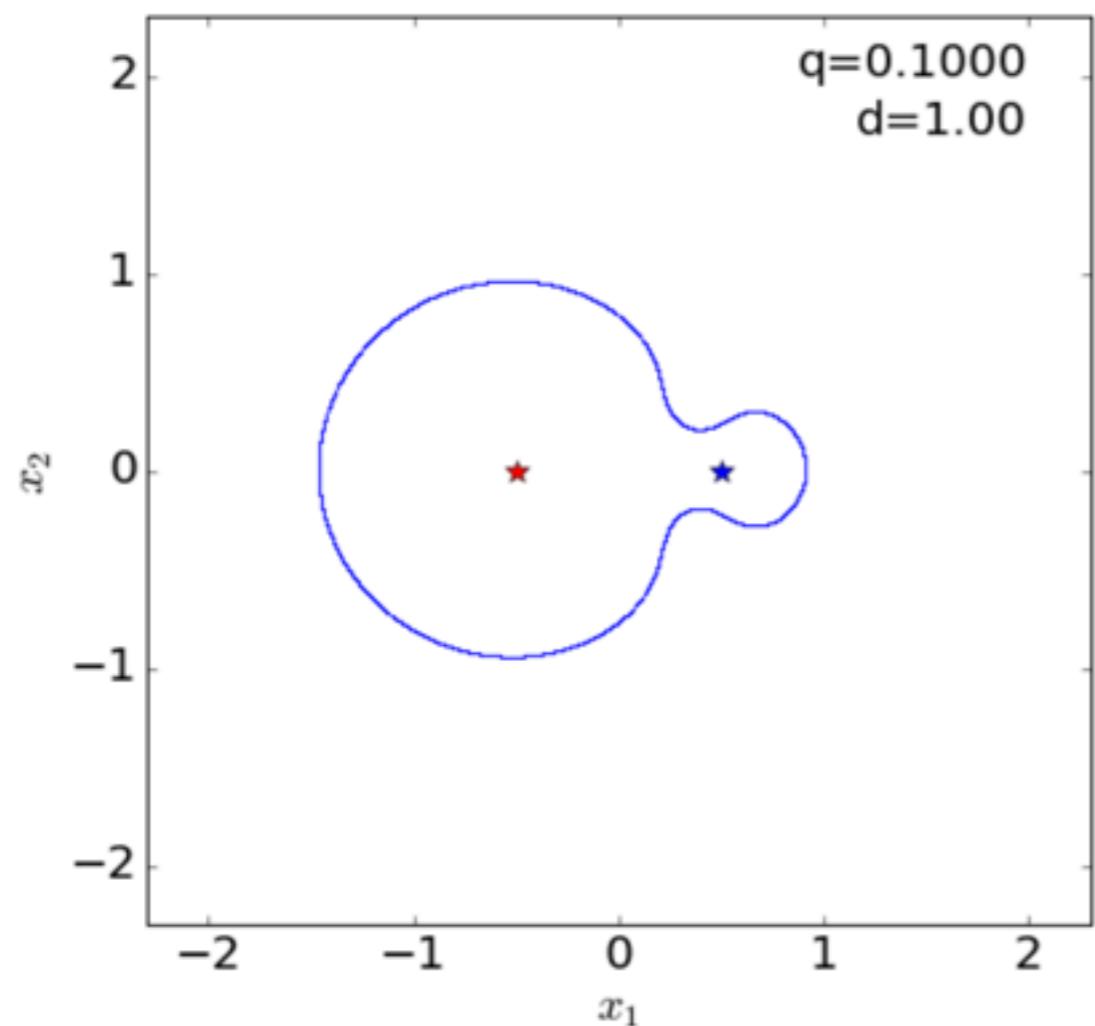


critical lines

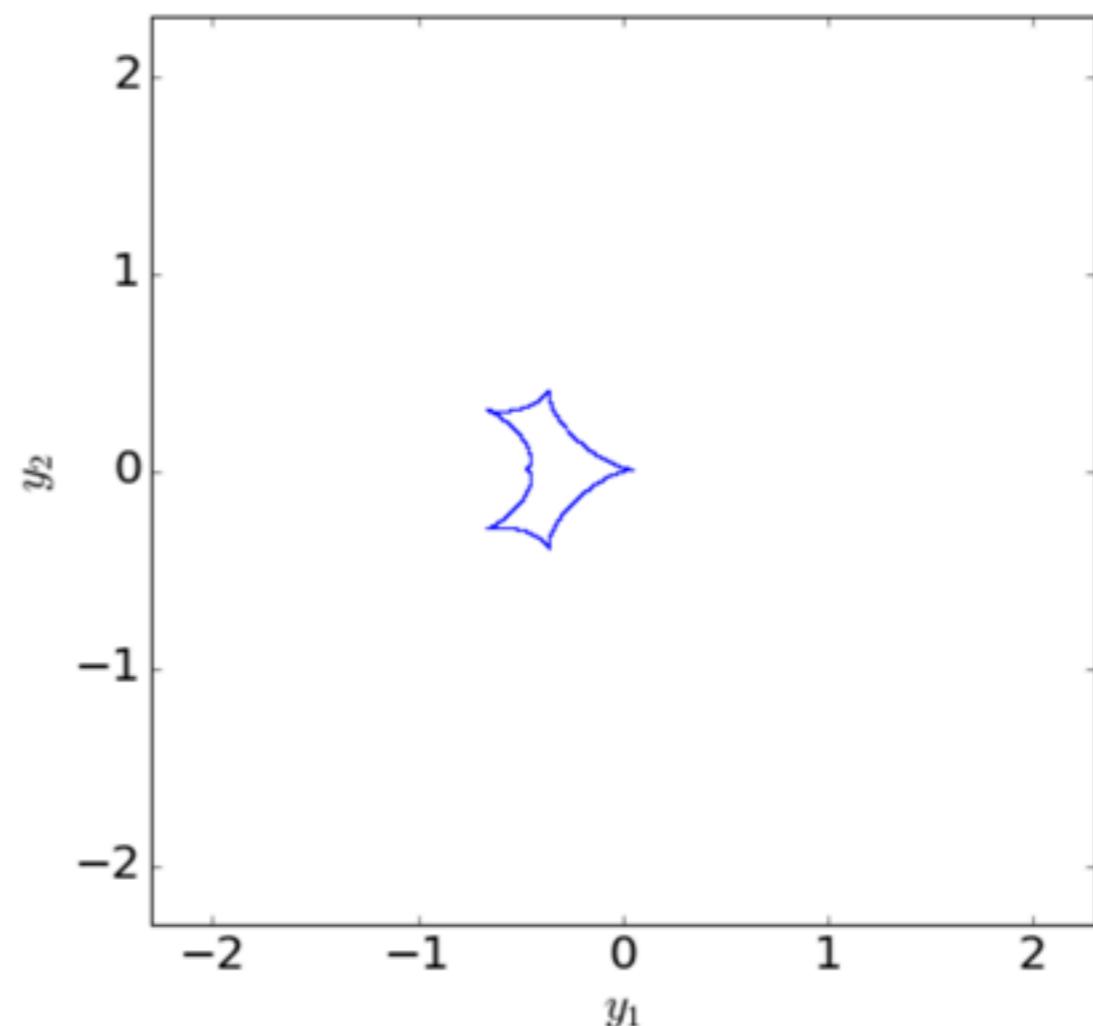


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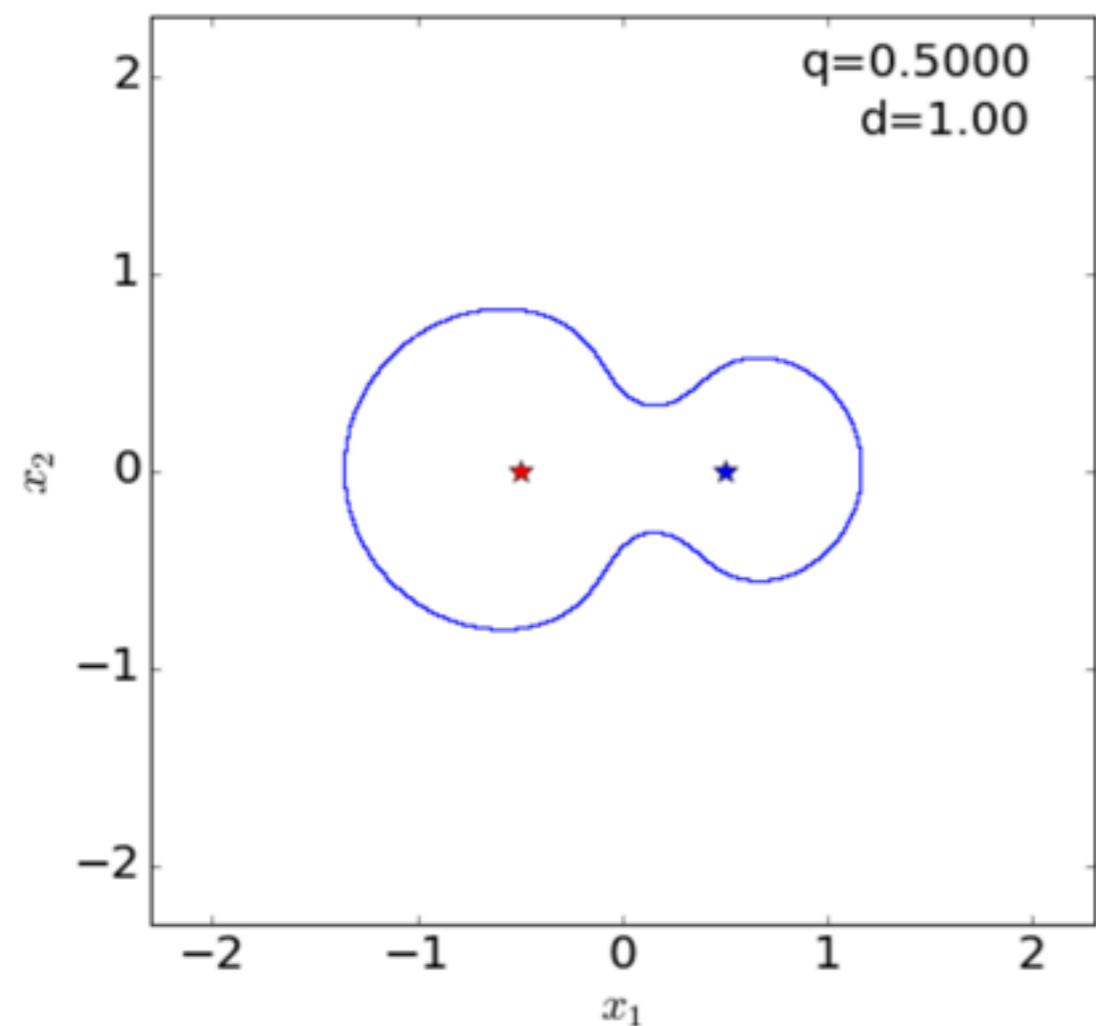


critical lines

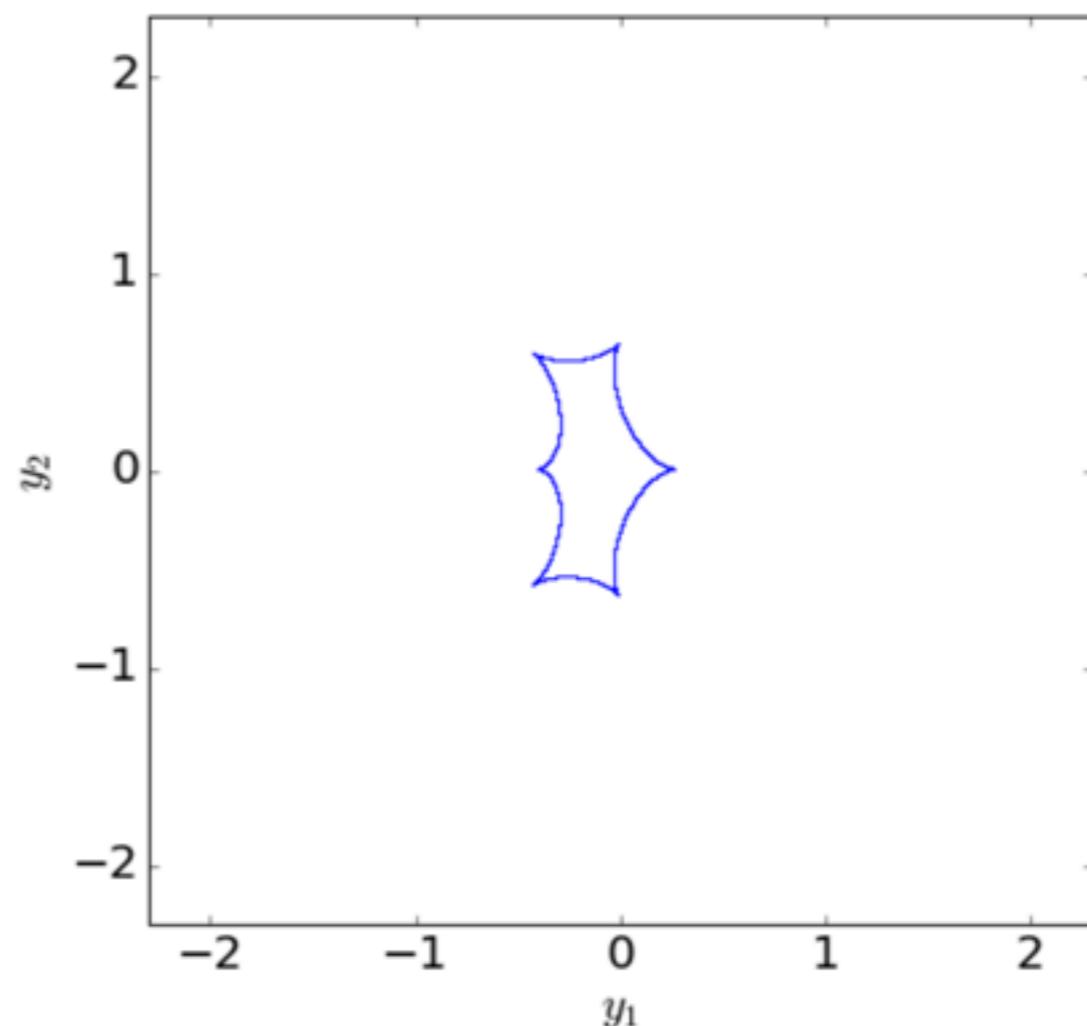


caustics

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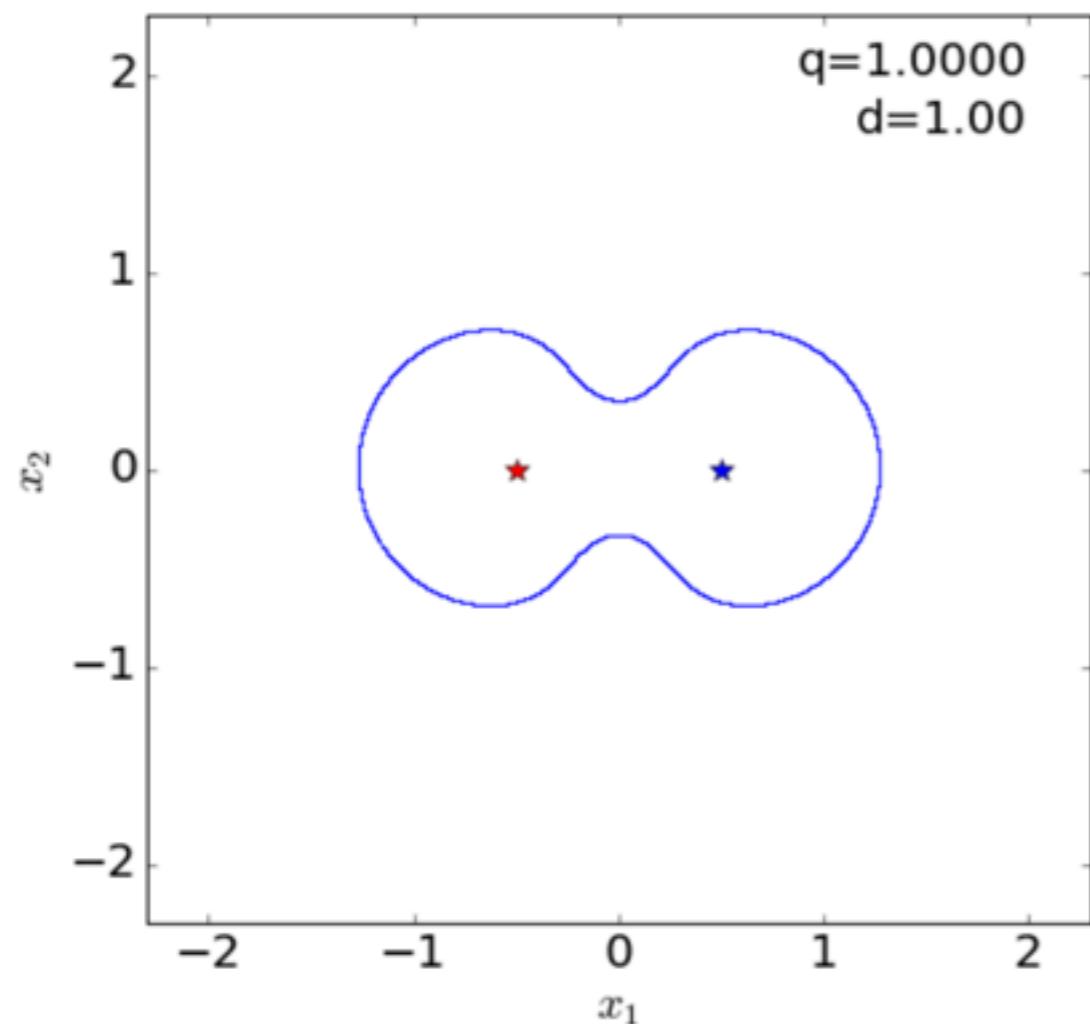


critical lines

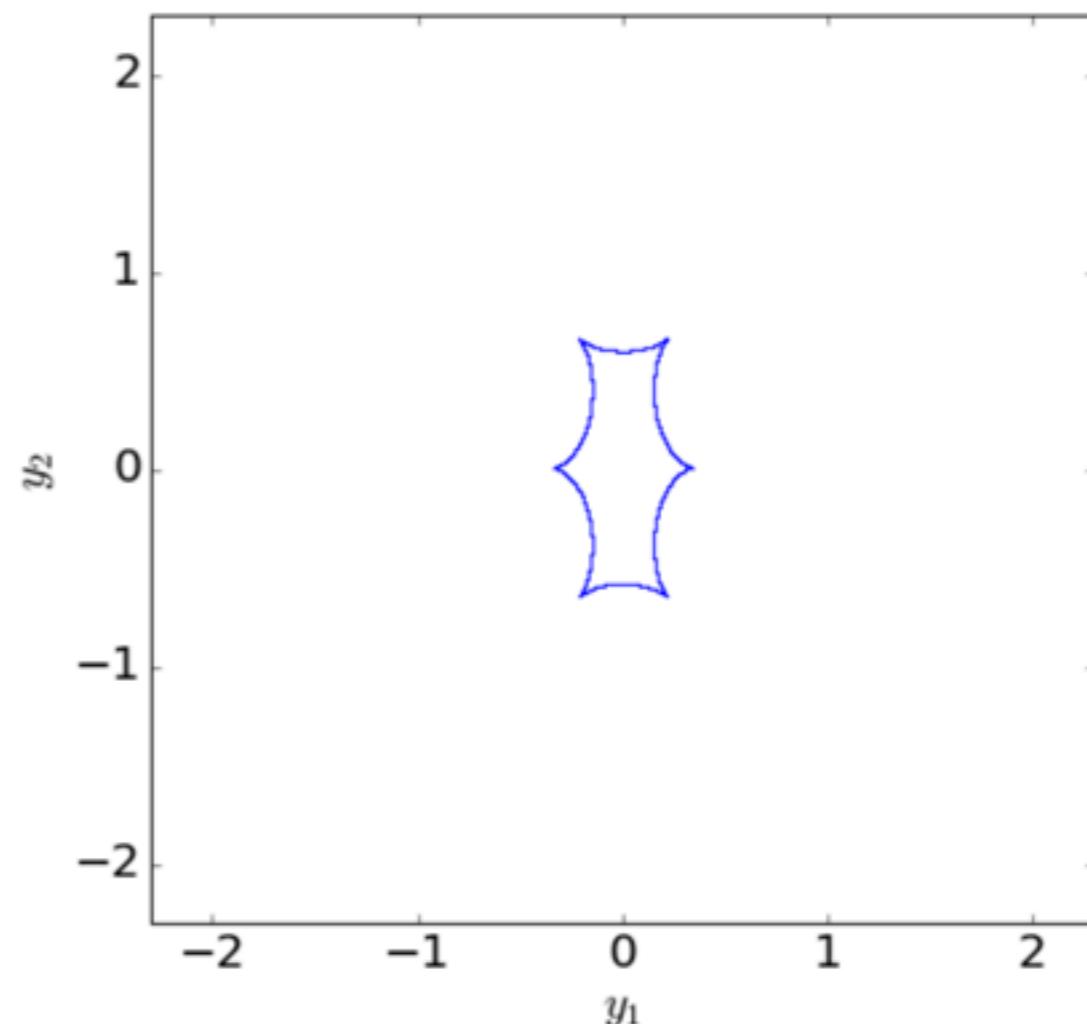


caustics

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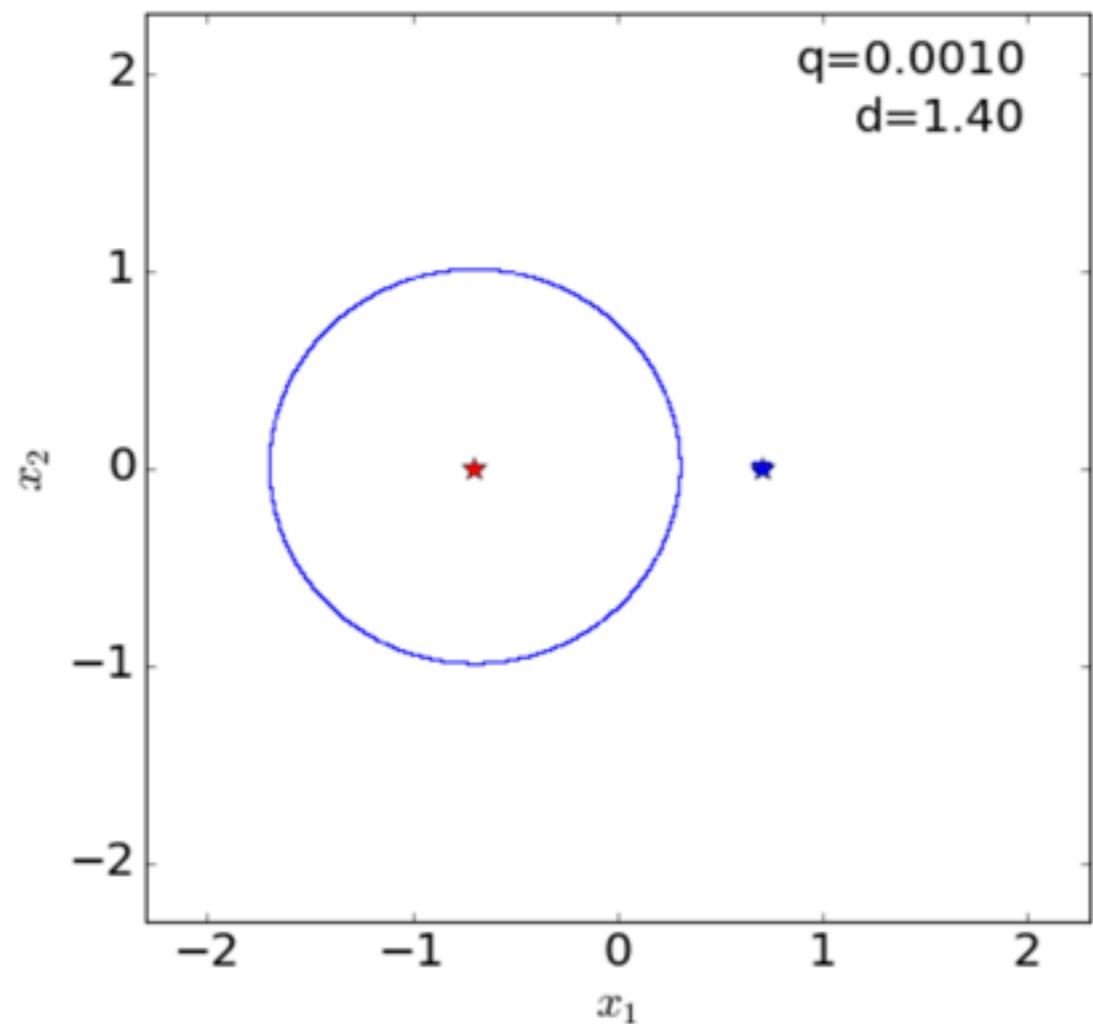


critical lines

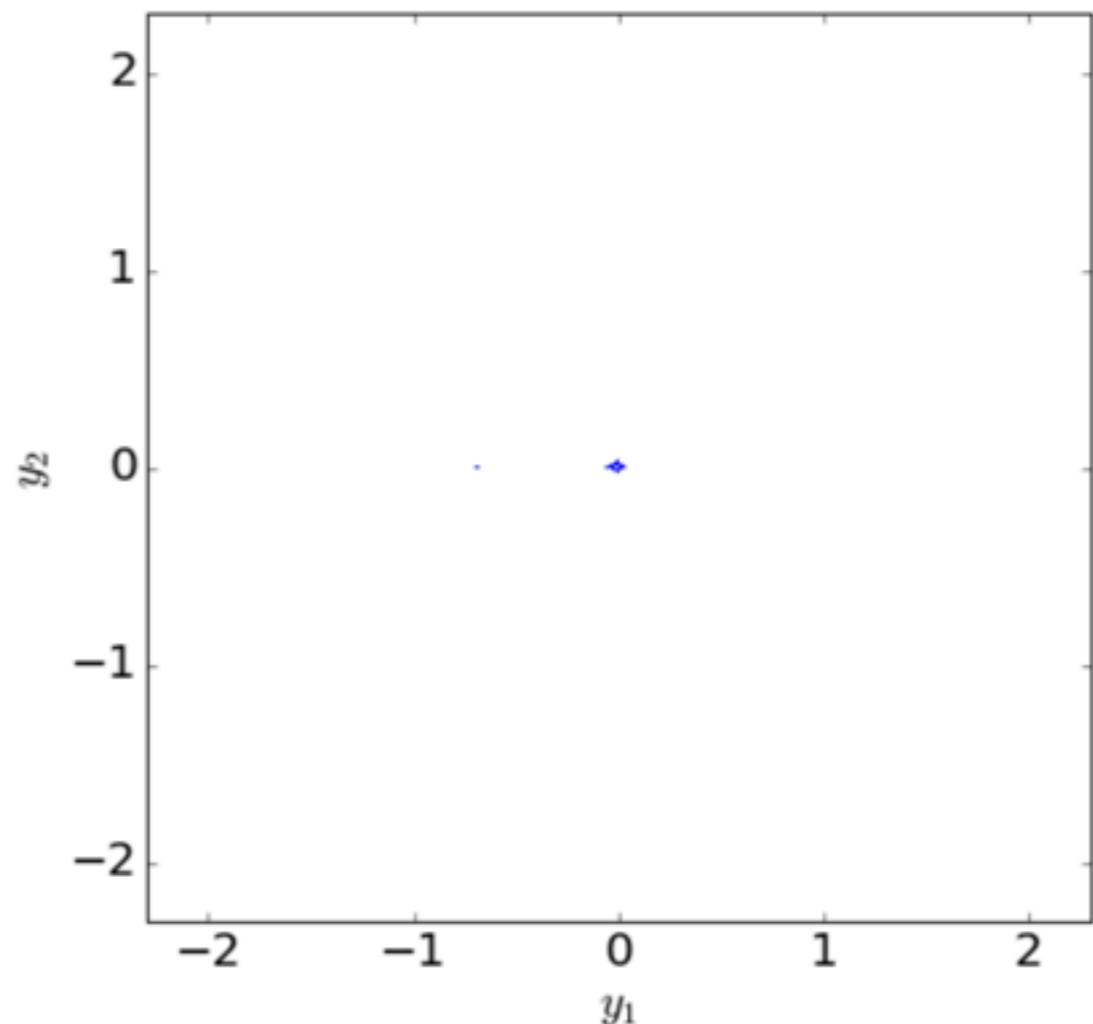


caustics

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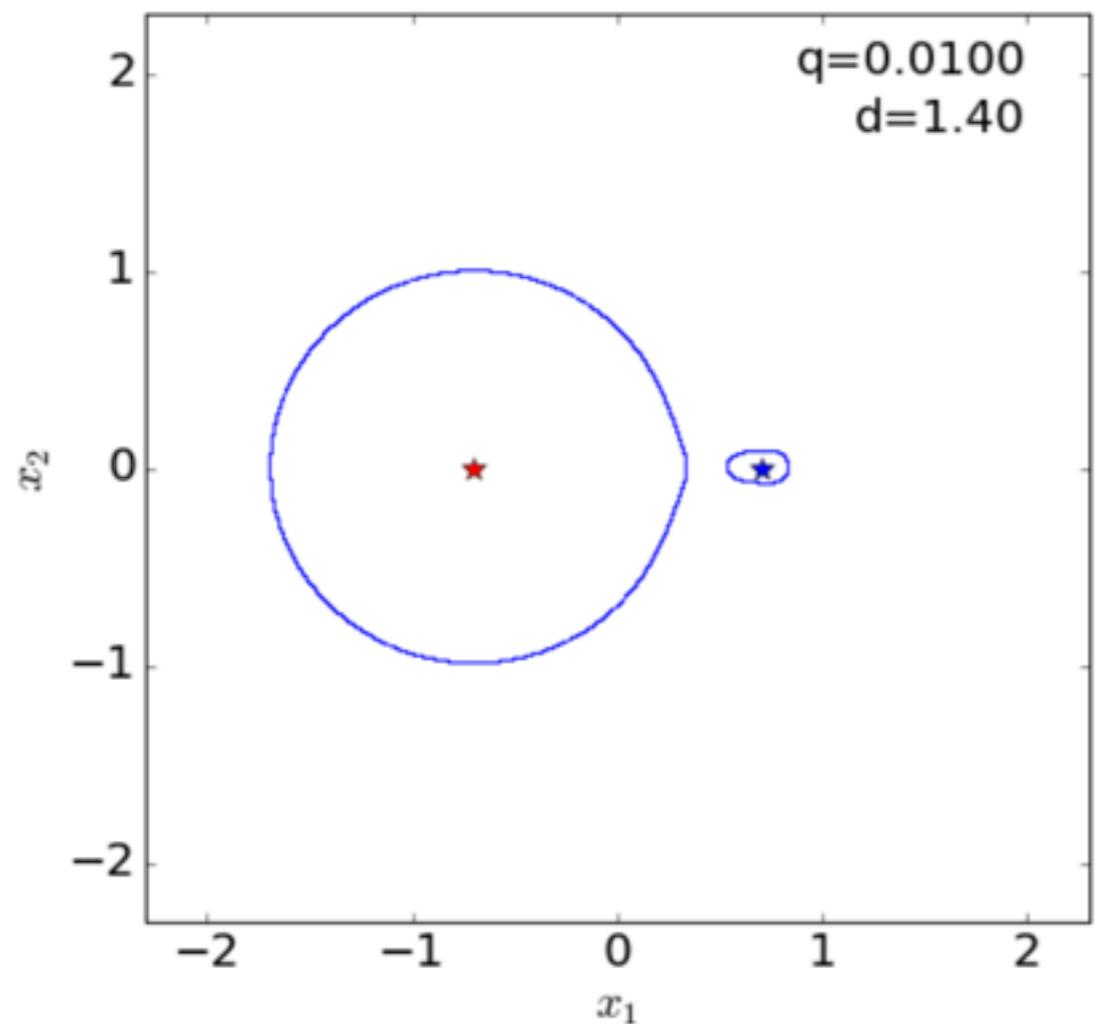


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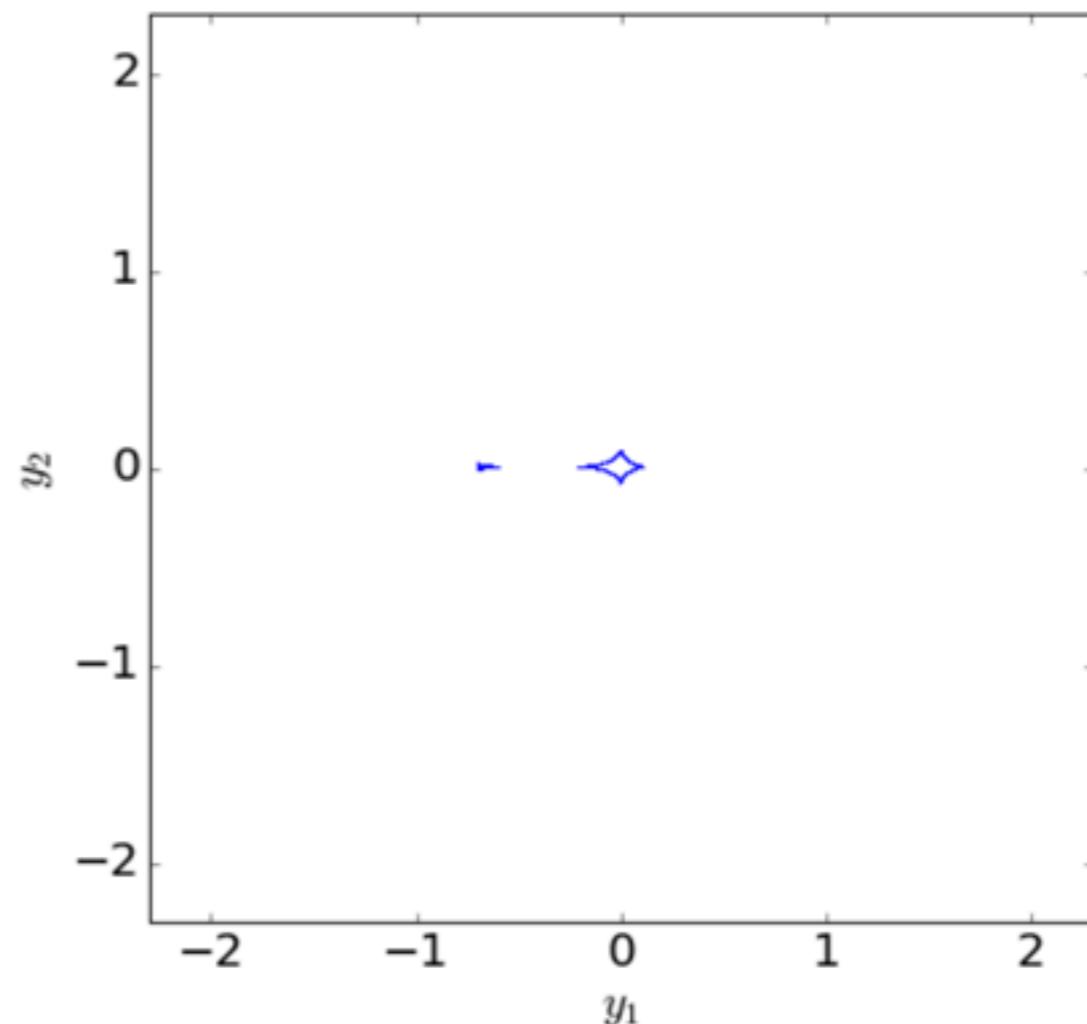


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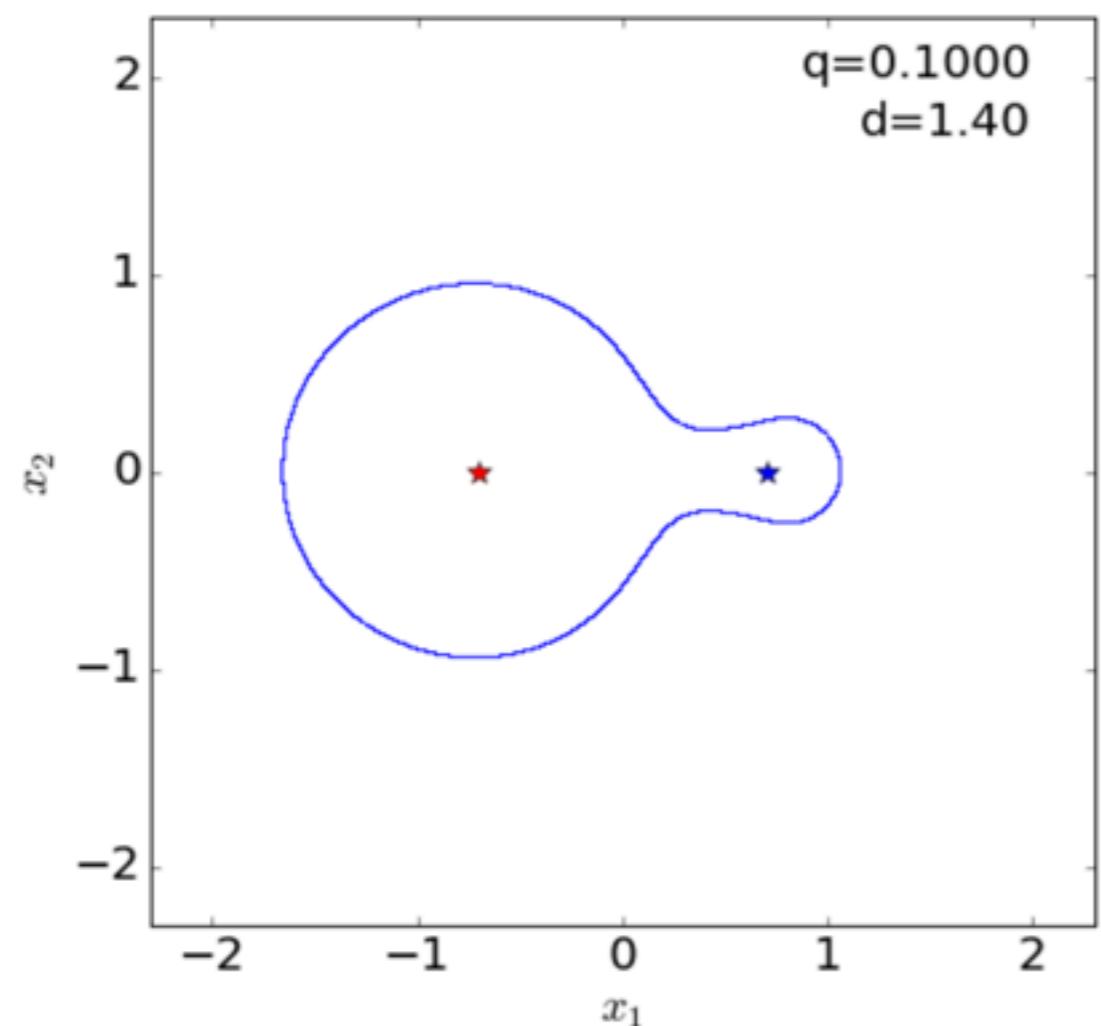


critical lines

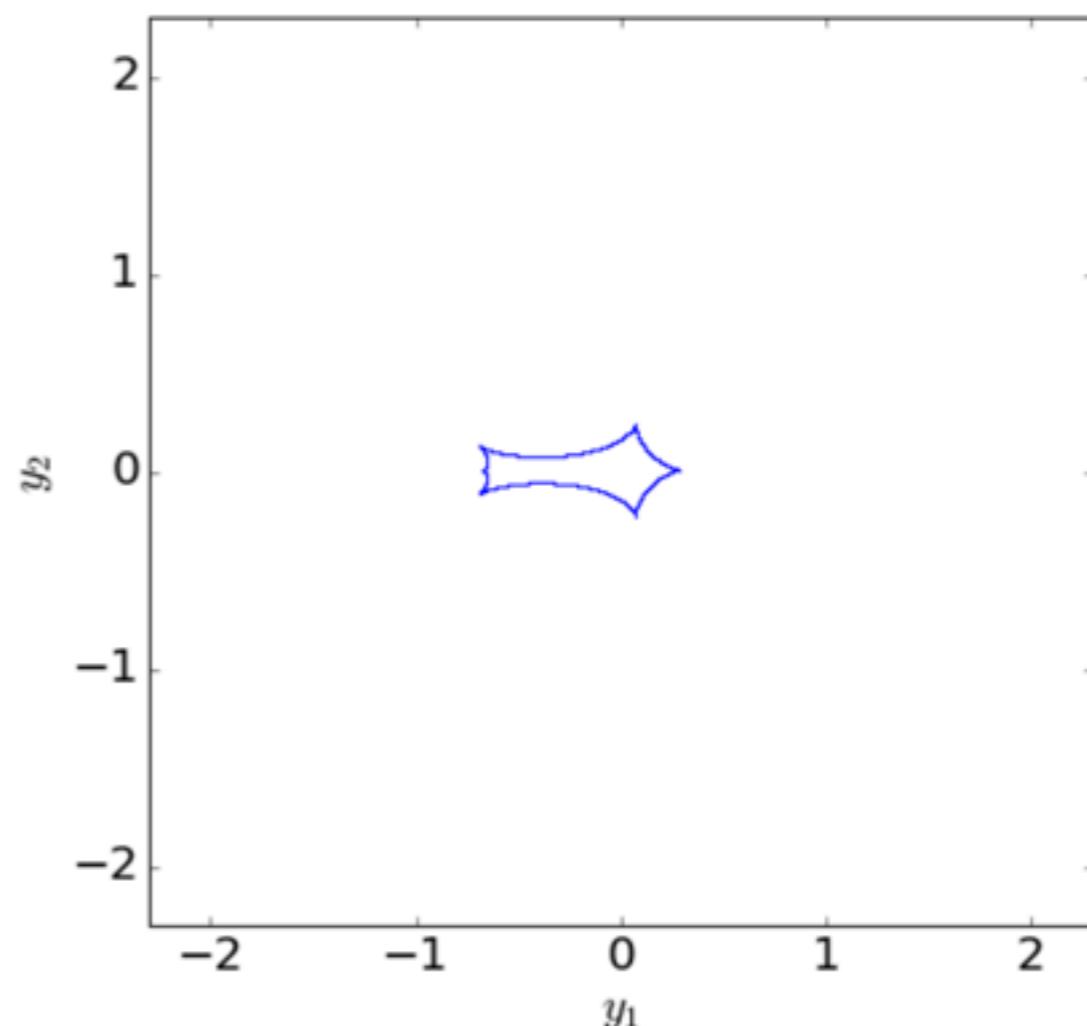


caustics

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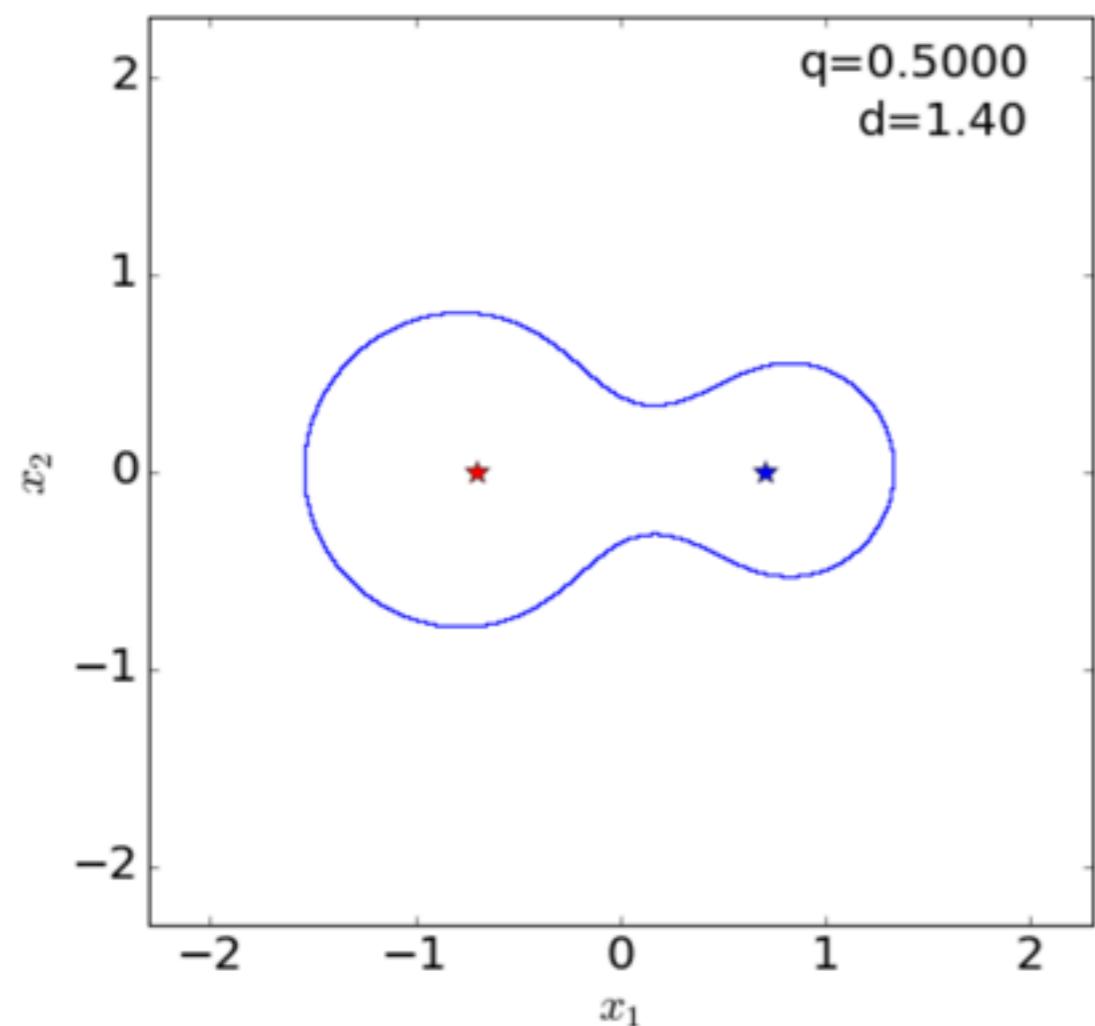


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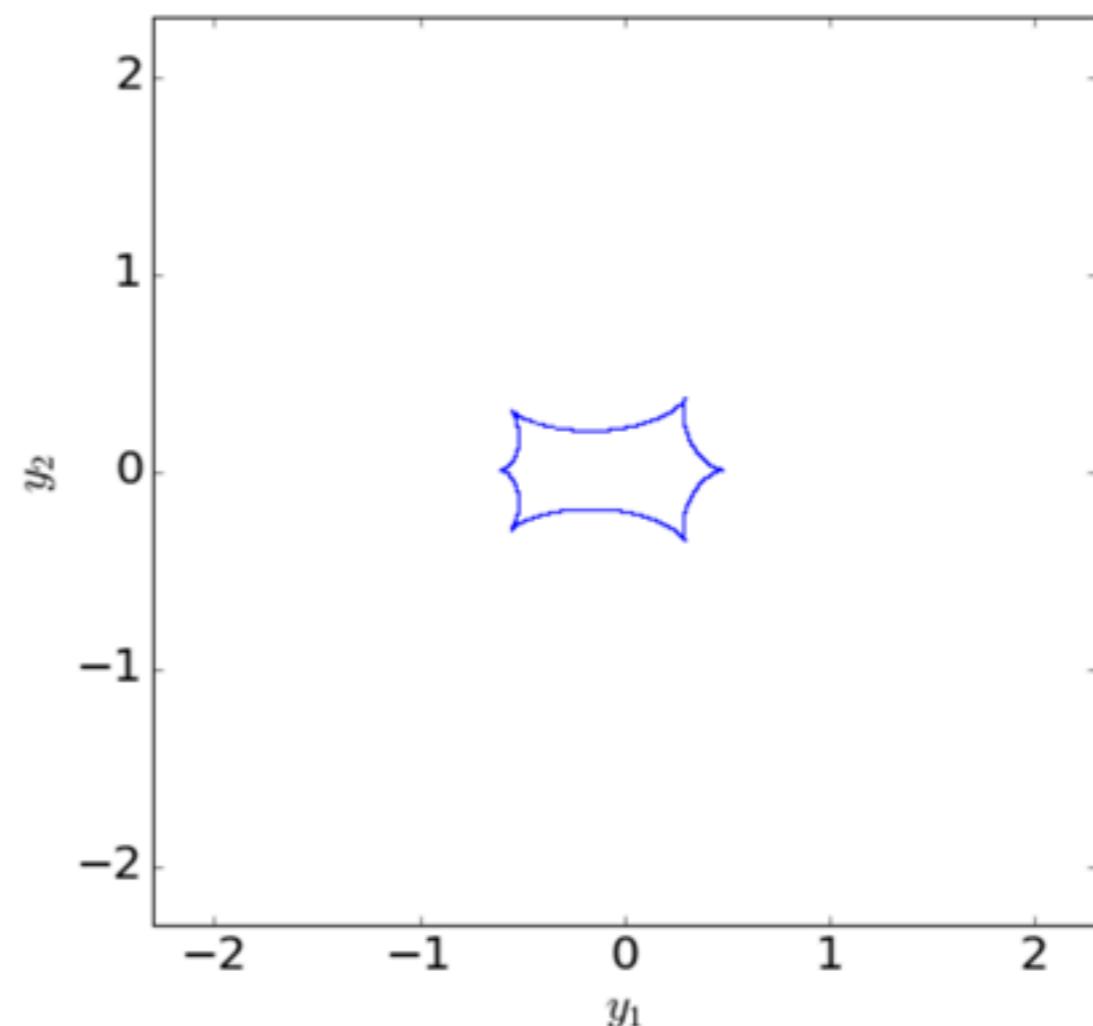


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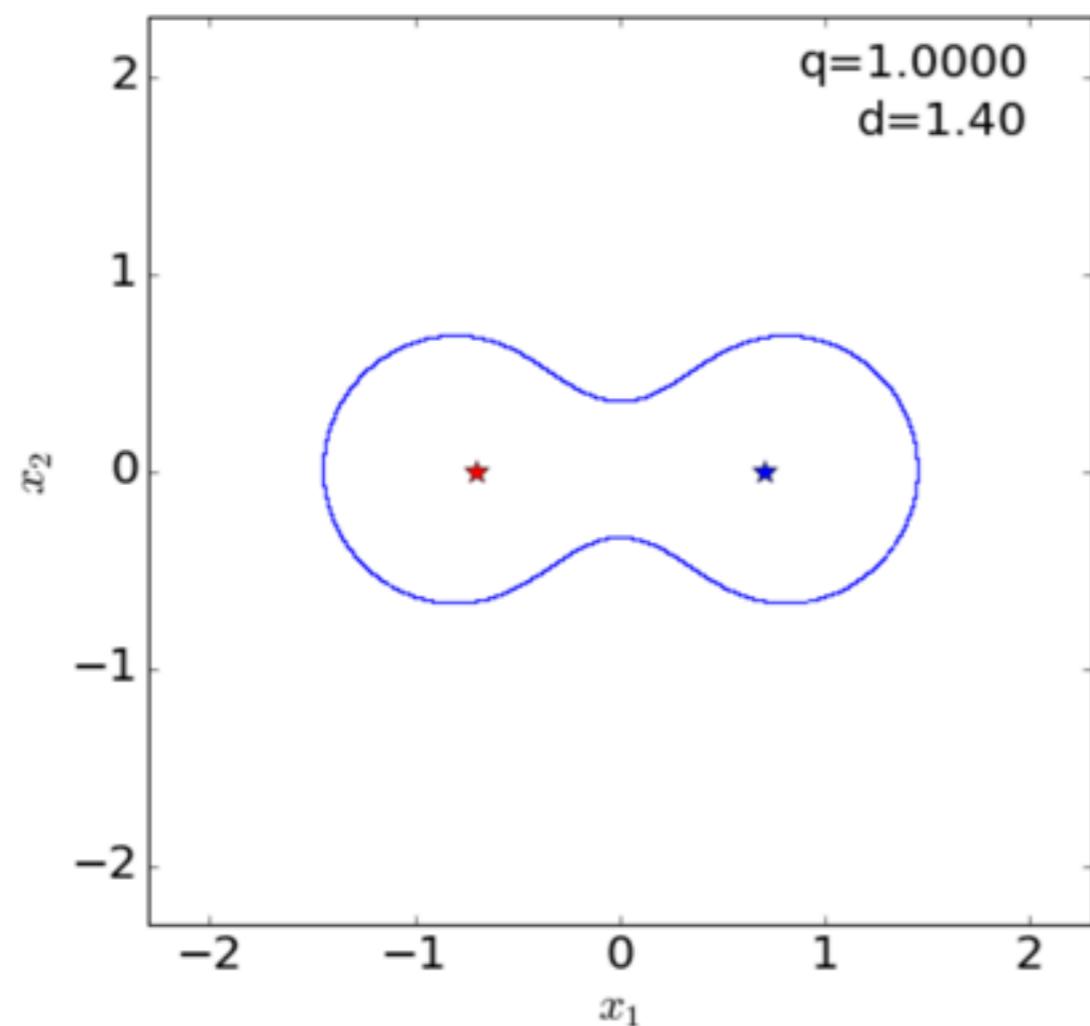


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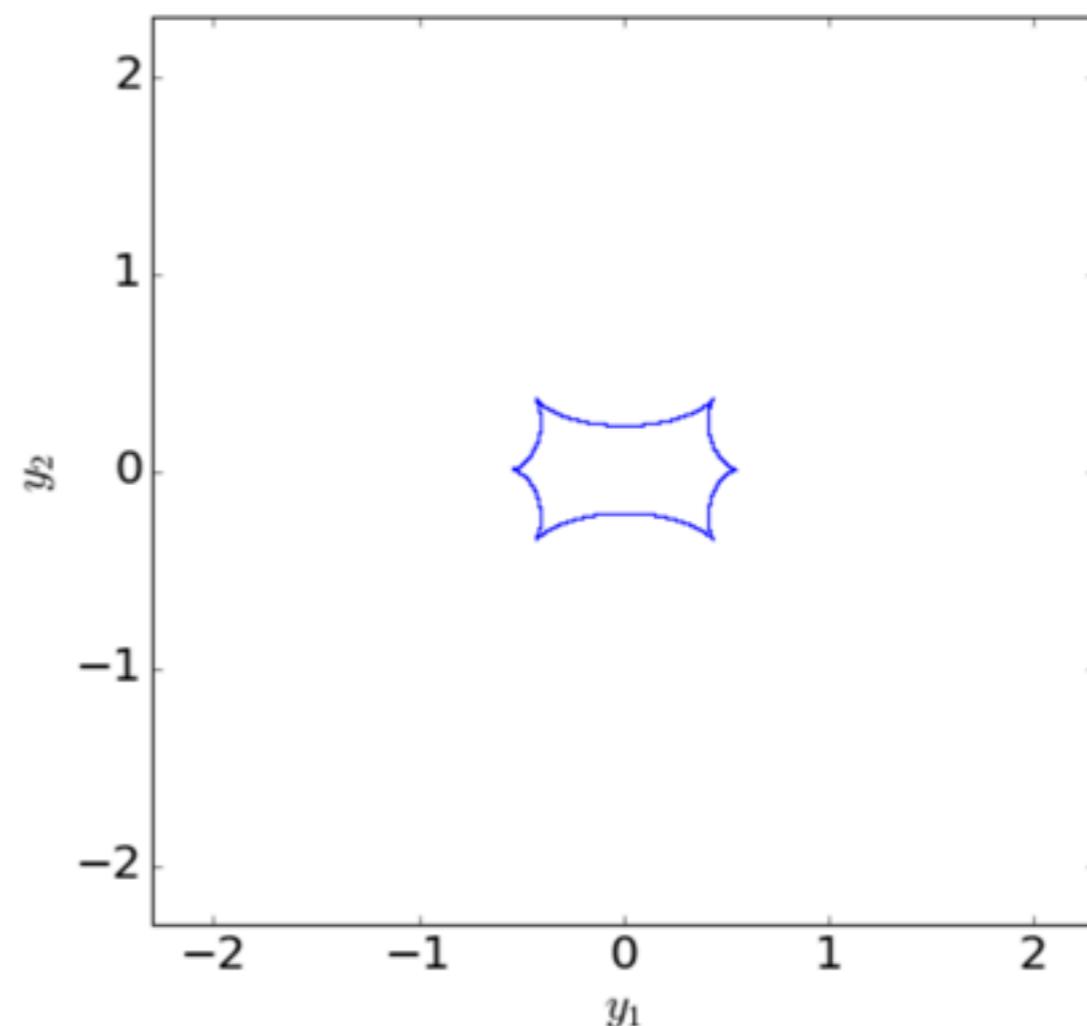


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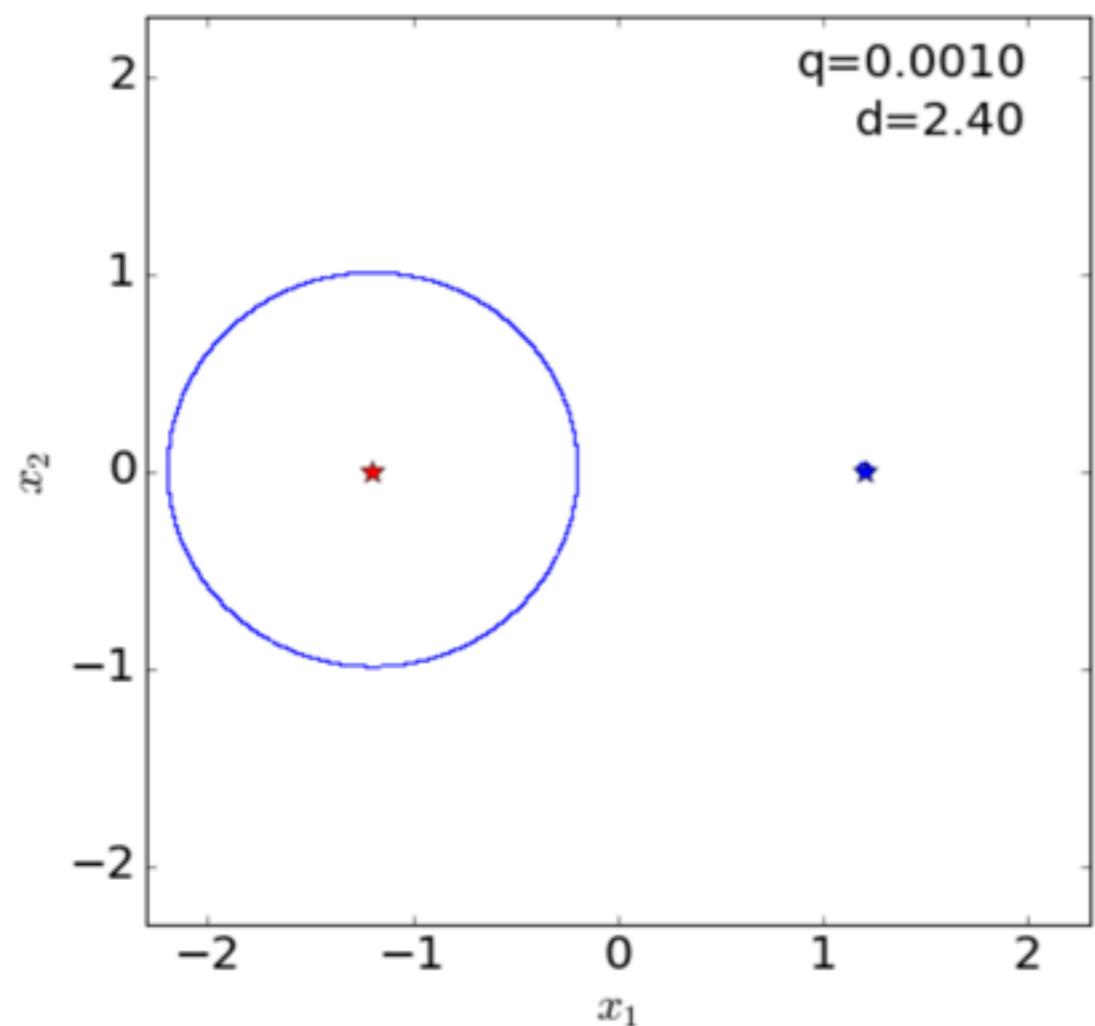


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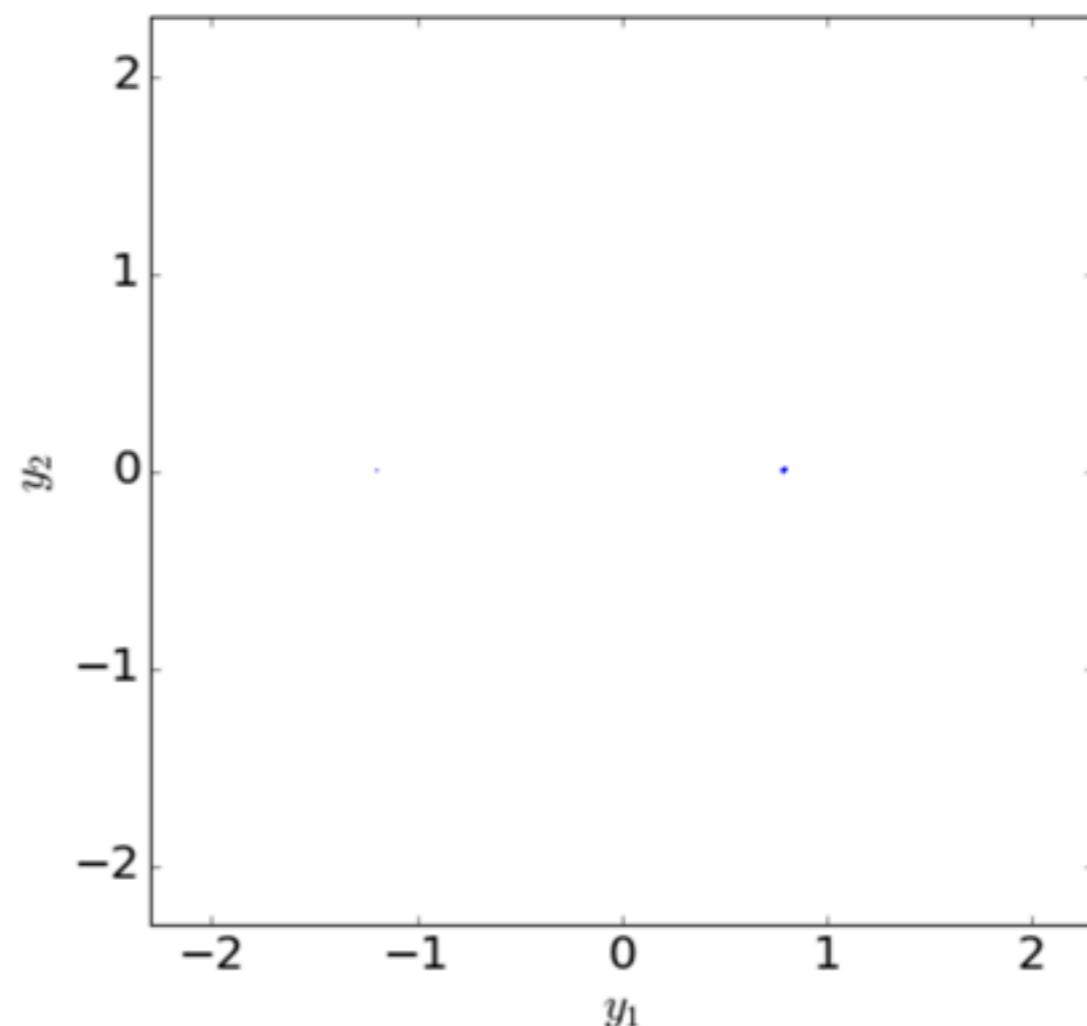


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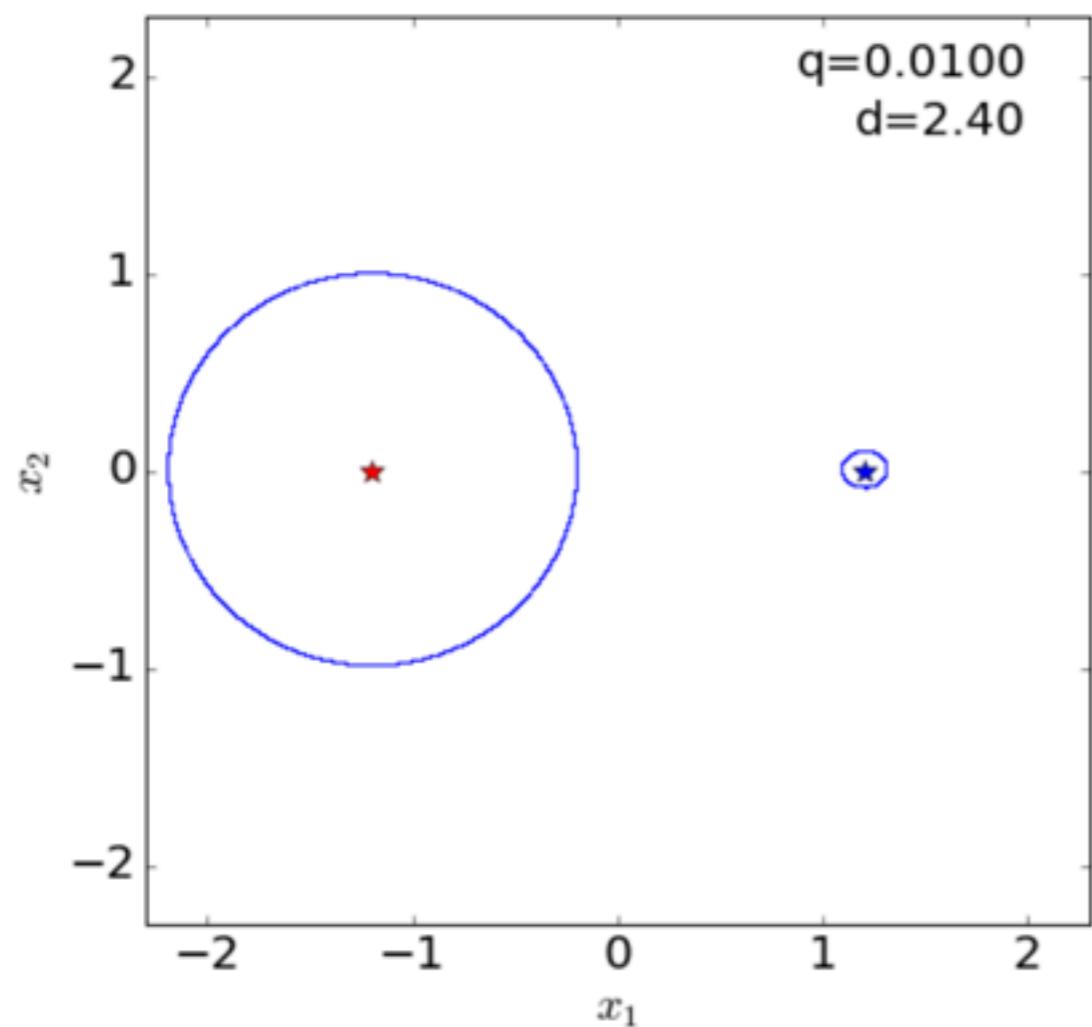


critical lines

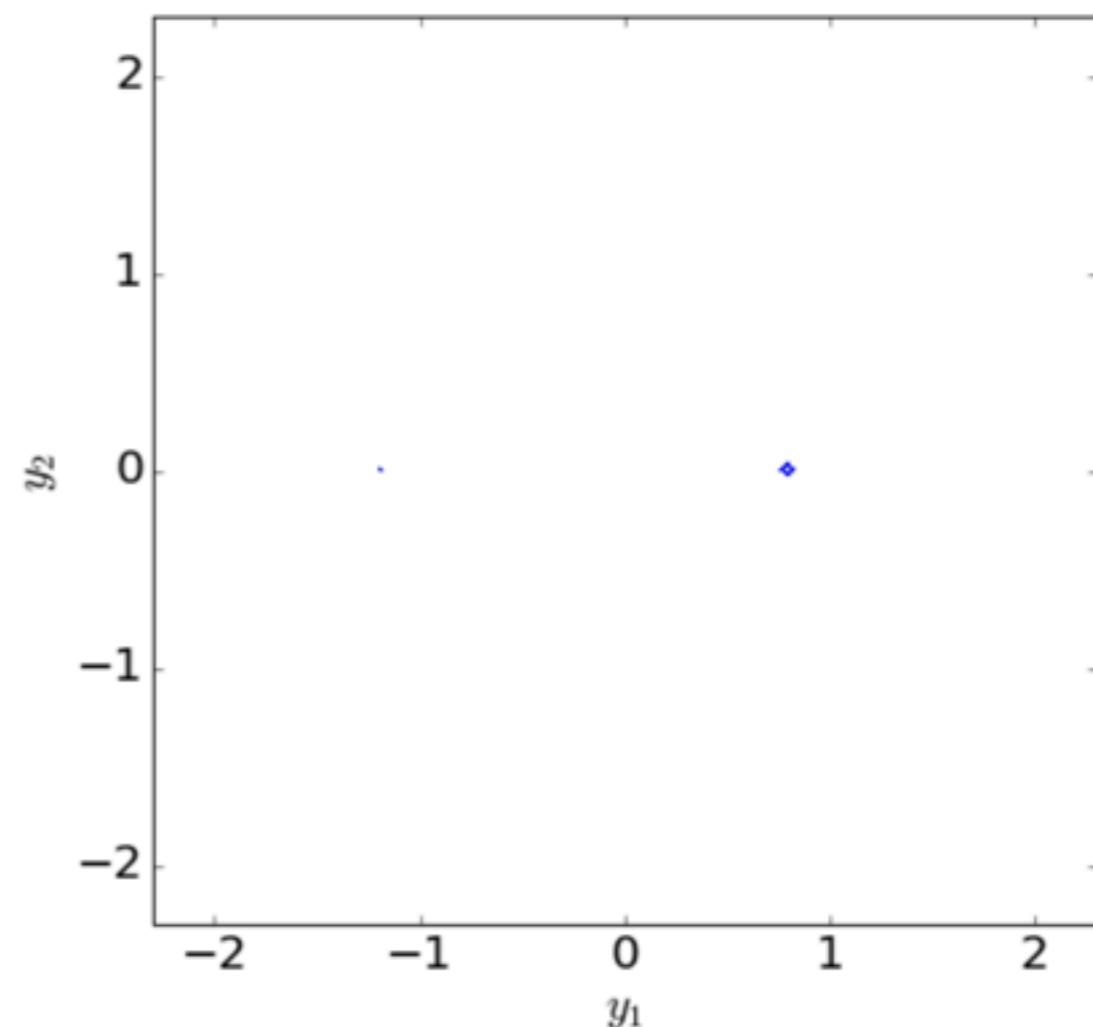


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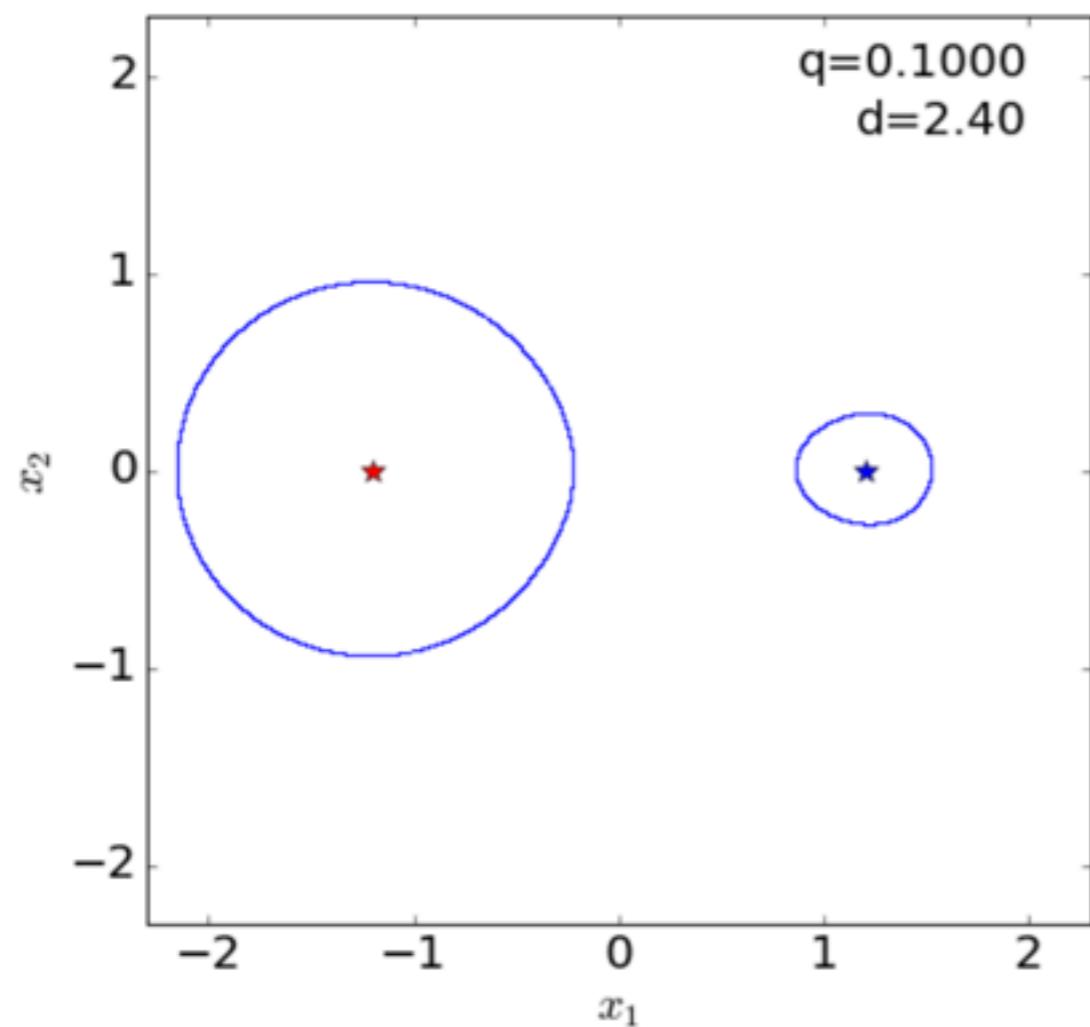


critical lines

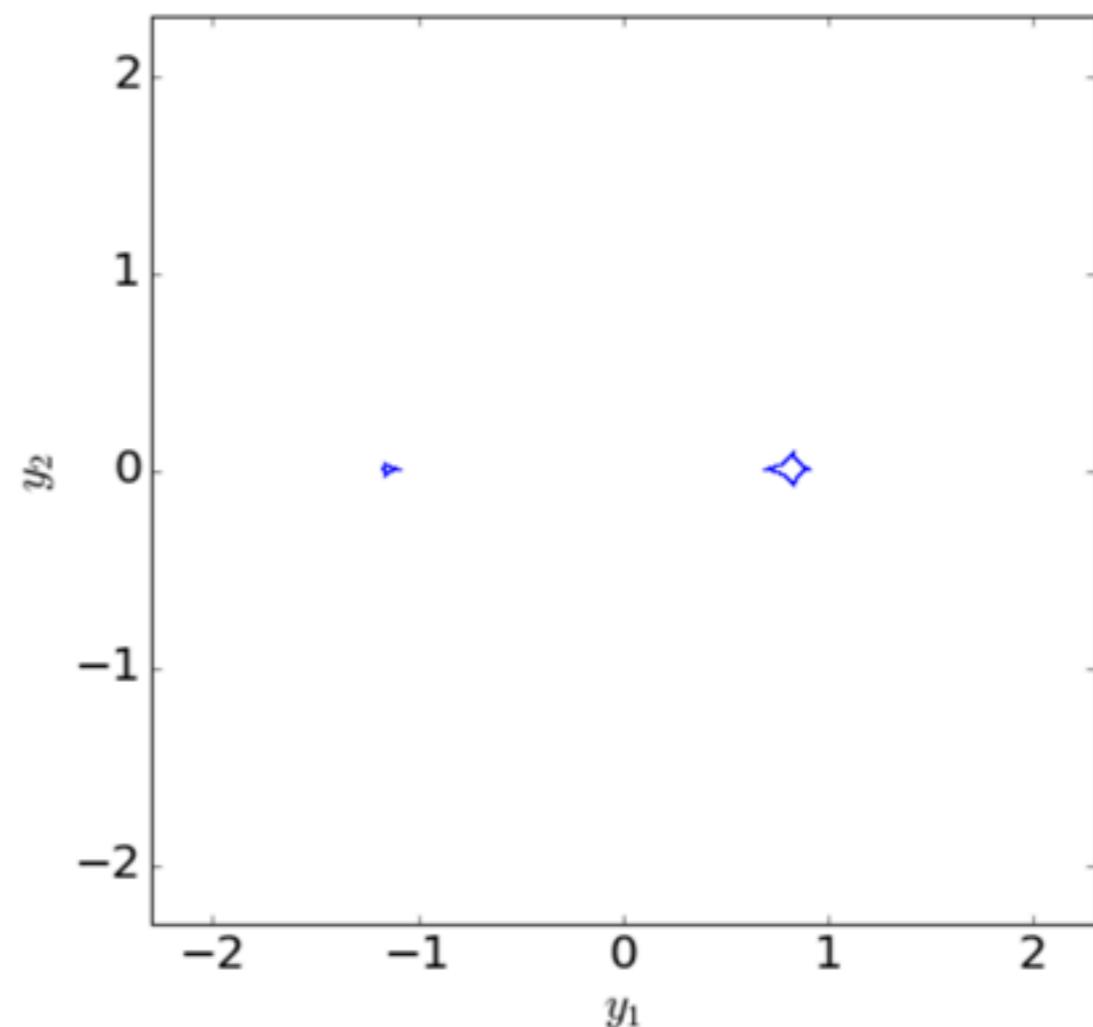


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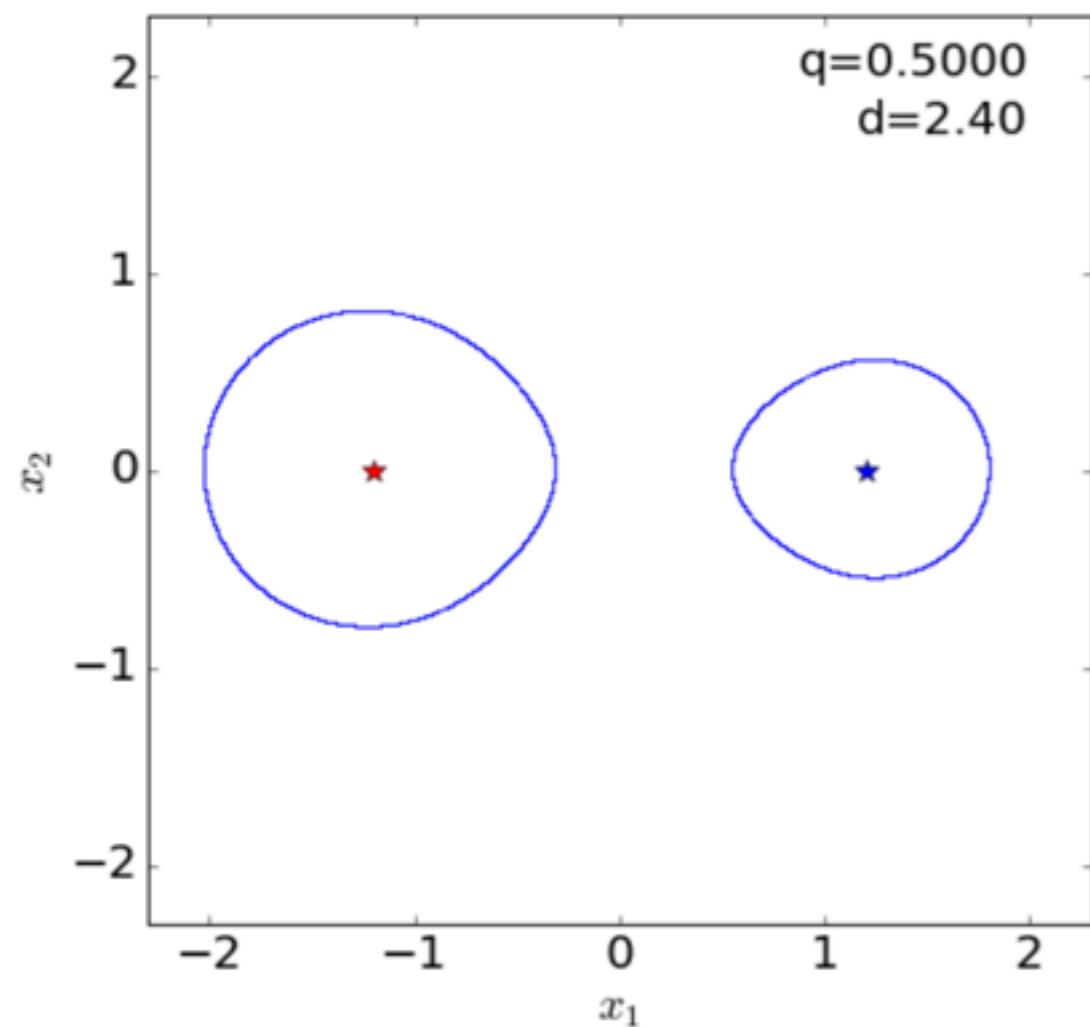


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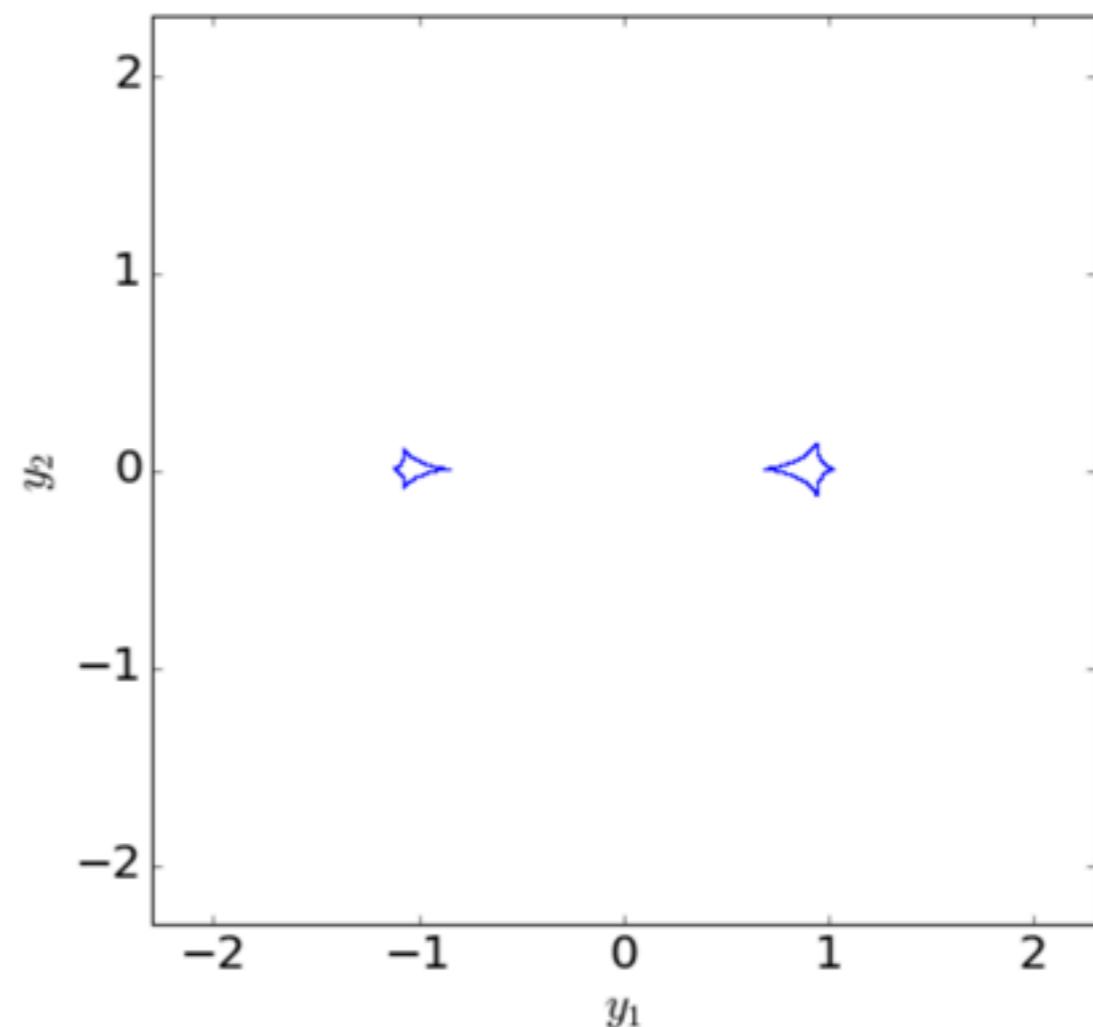


caustics

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critical lines

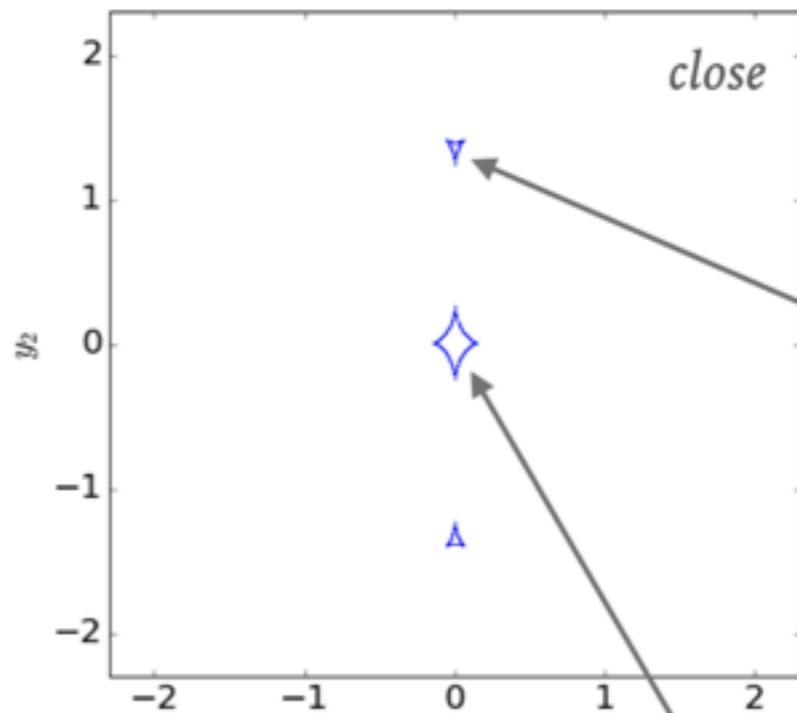
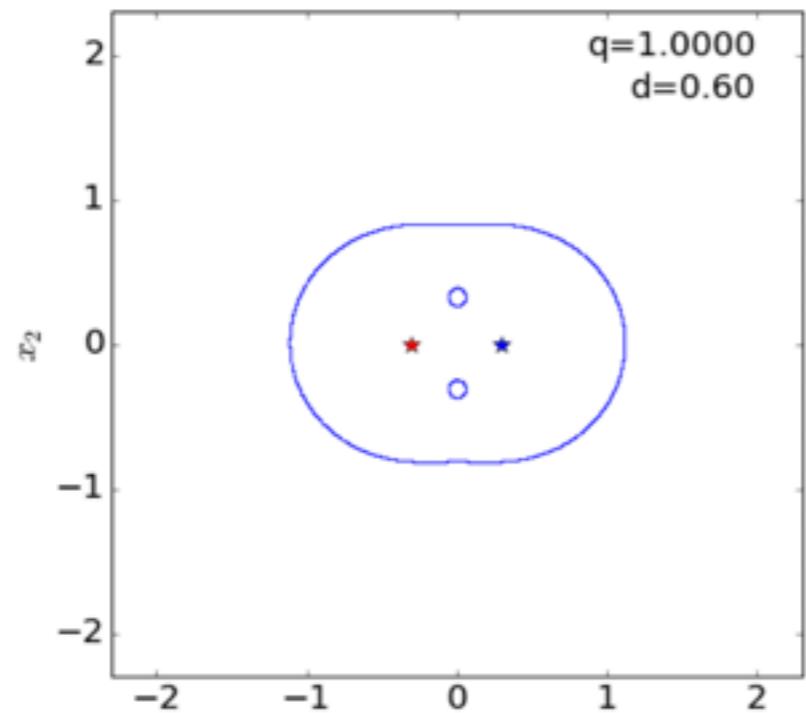


caustics

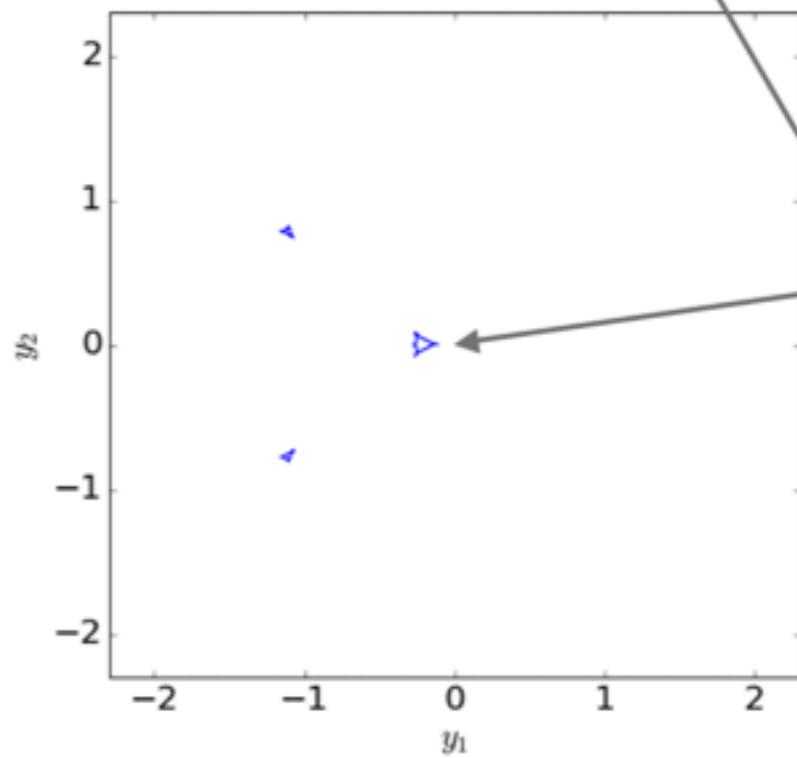
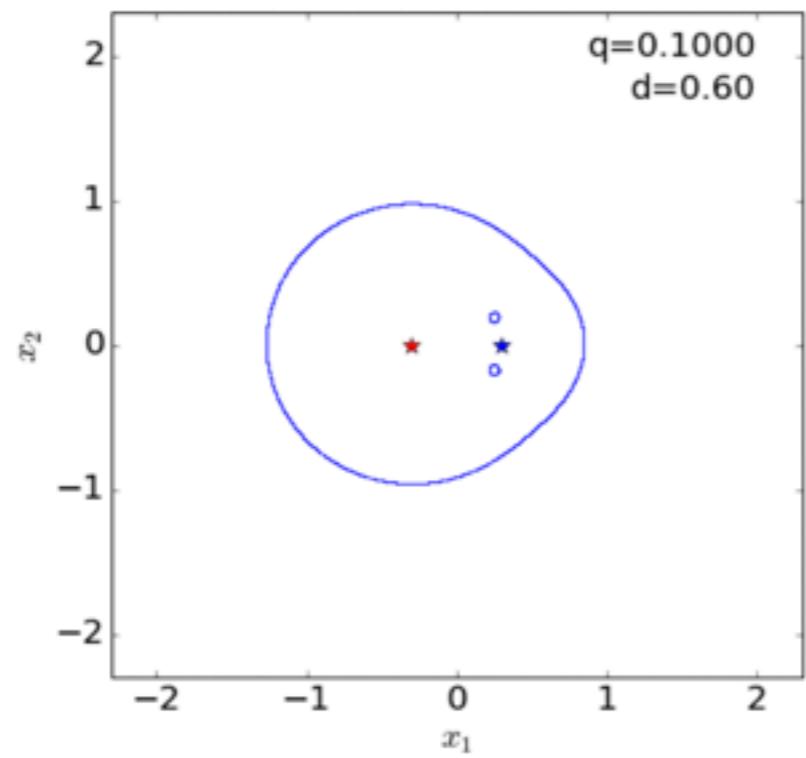
Ejercicio

- Ver como cambia la forma/estructura/ topología de las cáusticas y curvas críticas valiendo el espacio 2D de d y q
- <http://pico.oabo.inaf.it/~massimo/teaching/notebooks/Lecture%2011.ipynb>

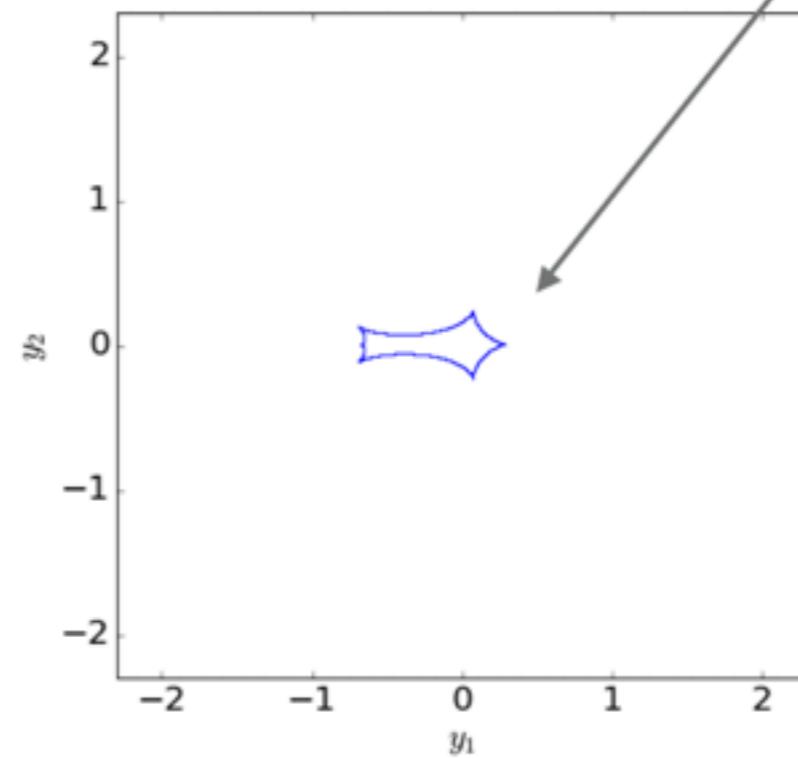
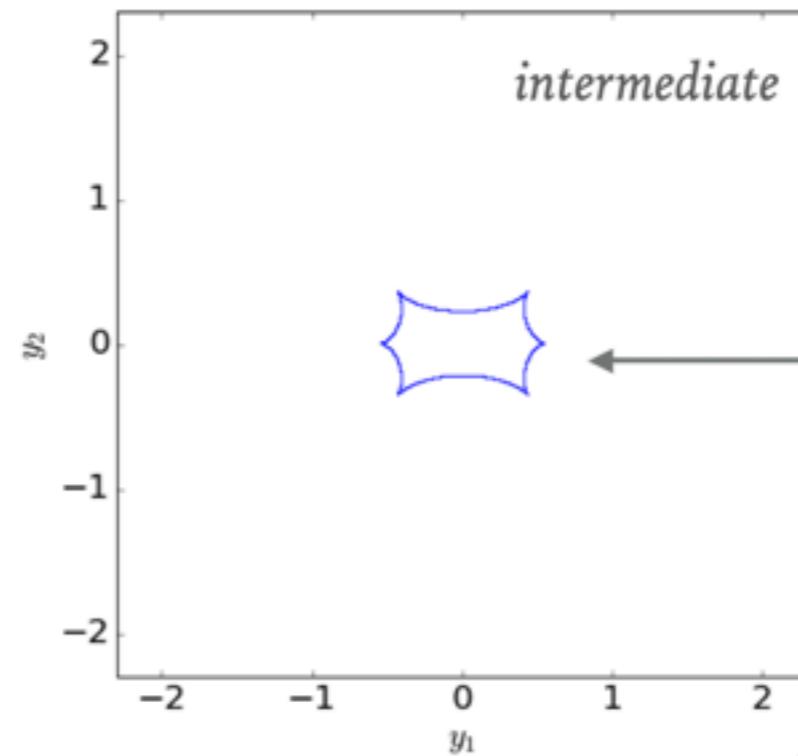
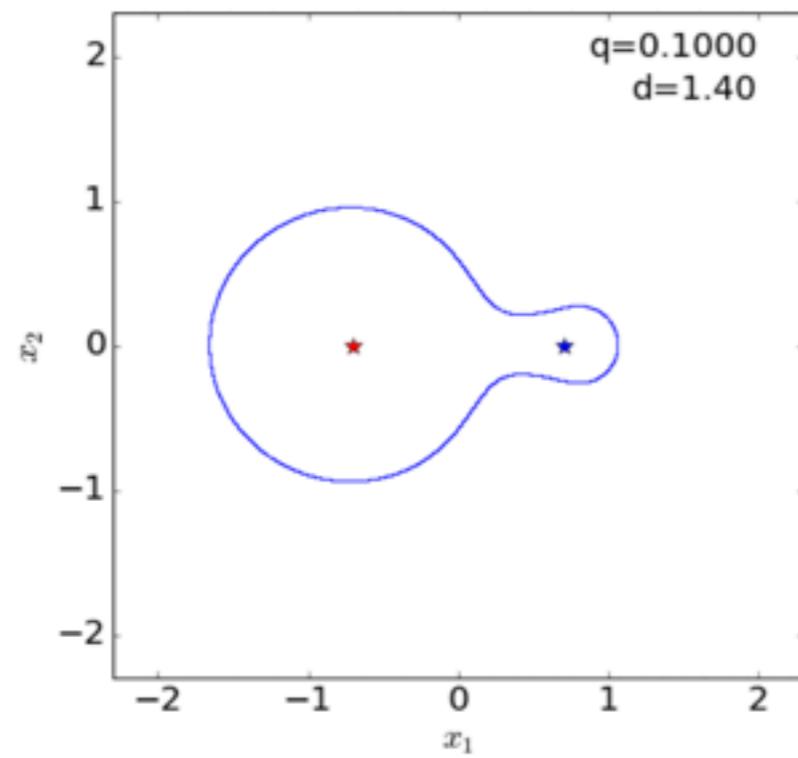
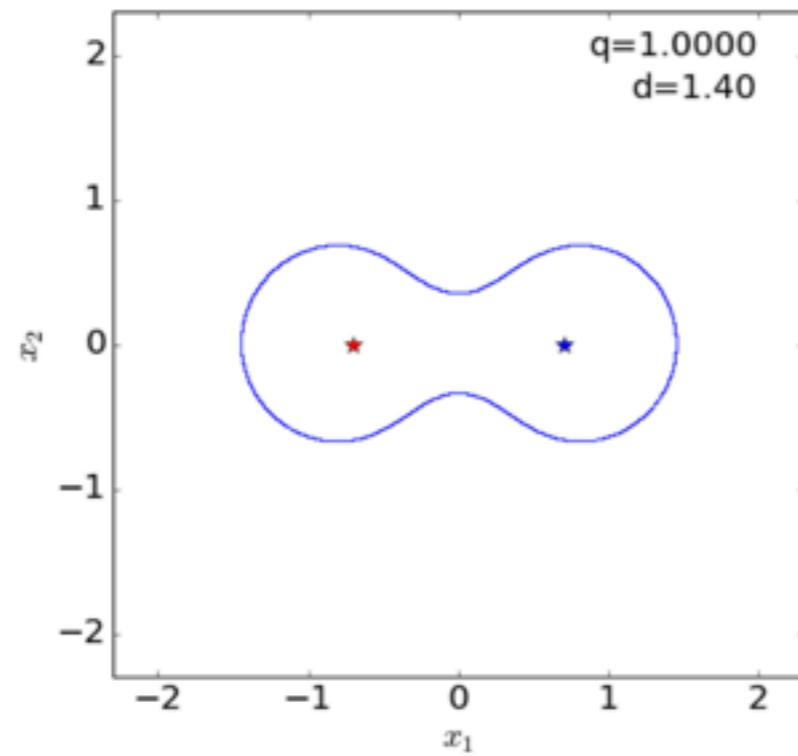
BINARY LENSES: TOPOLOGY CLASSIFICATION



*two
triangular
caustics*



BINARY LENSES: TOPOLOGY CLASSIFICATION

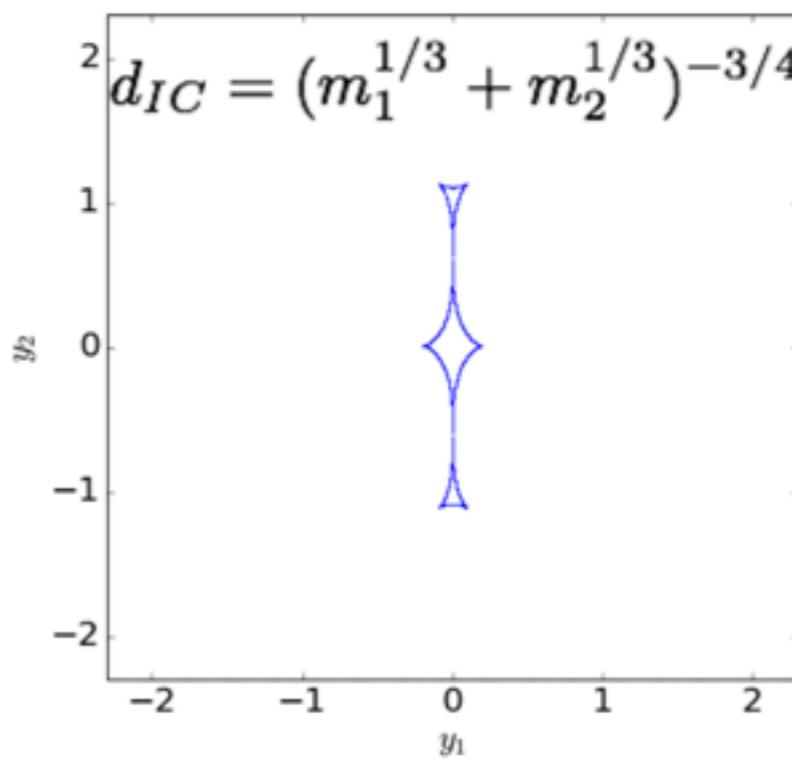
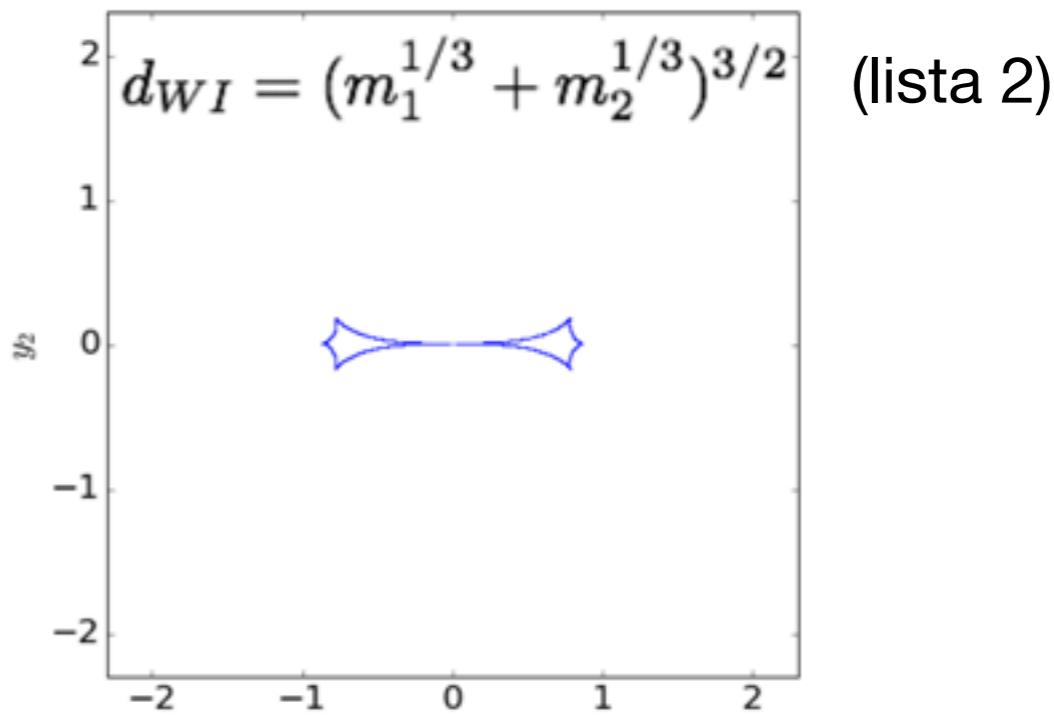
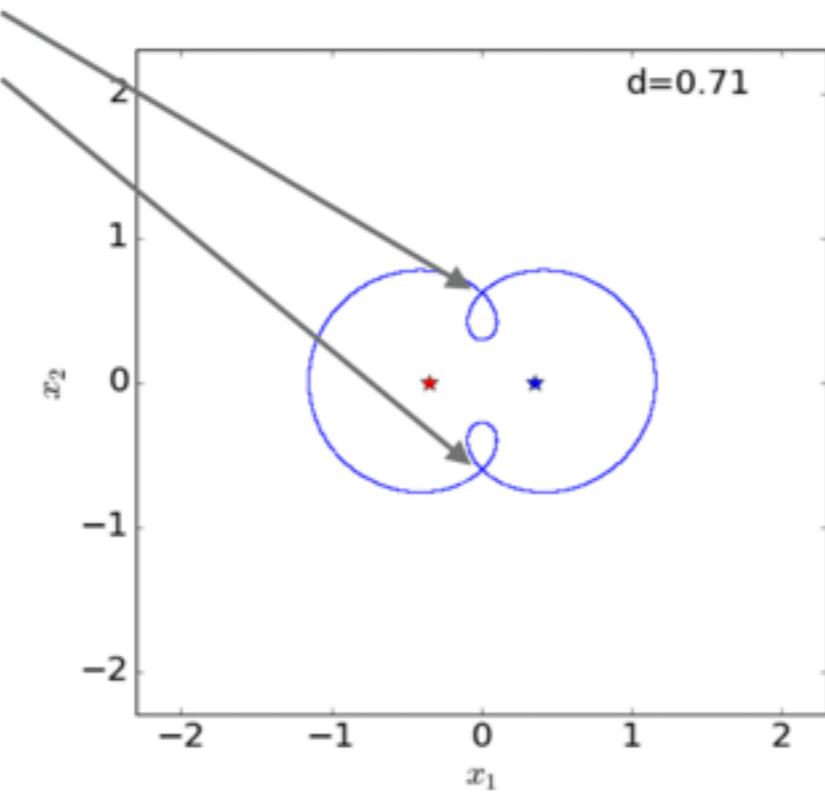
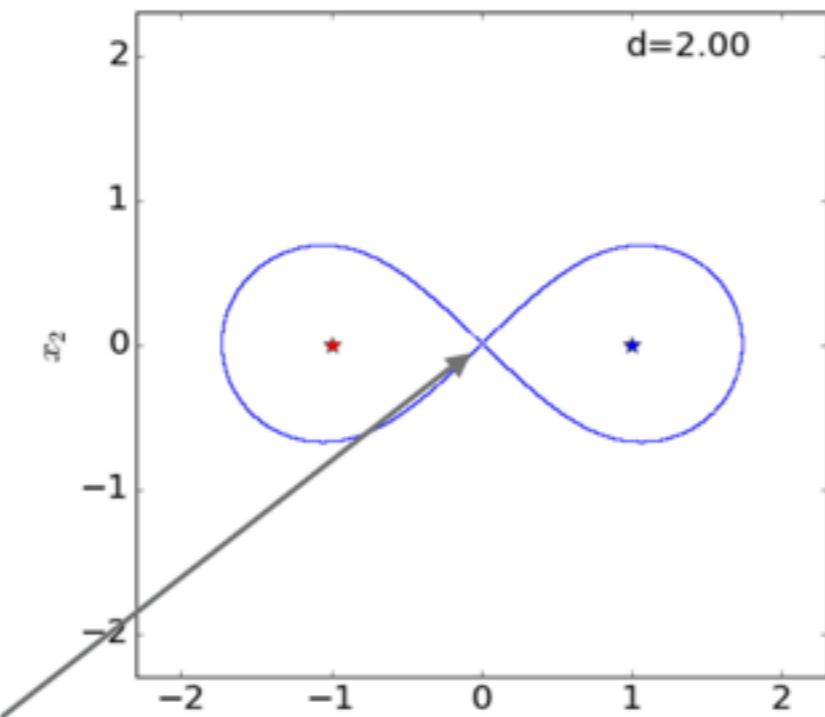


*single 6-cusp
caustic*

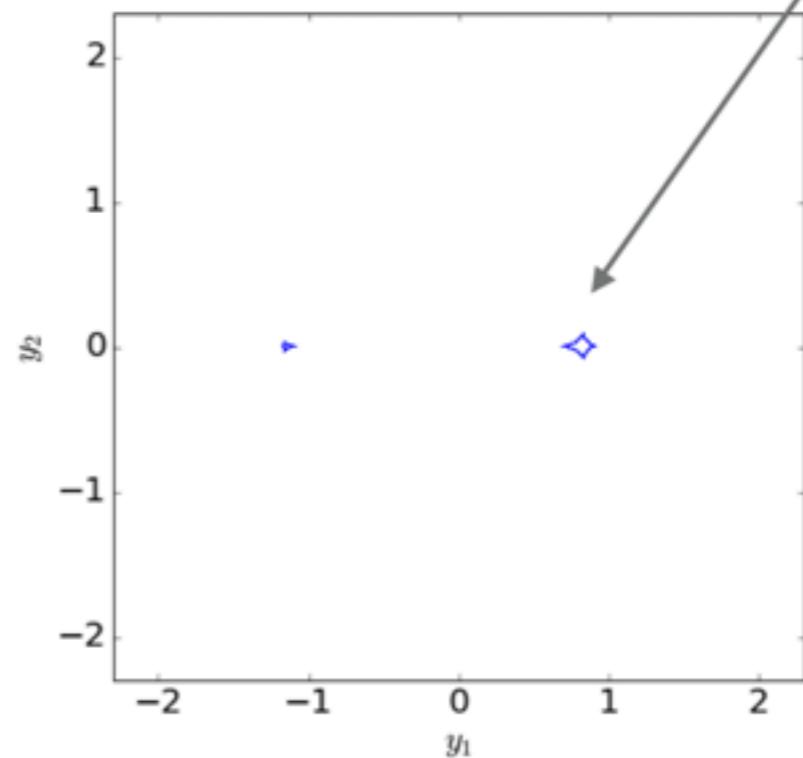
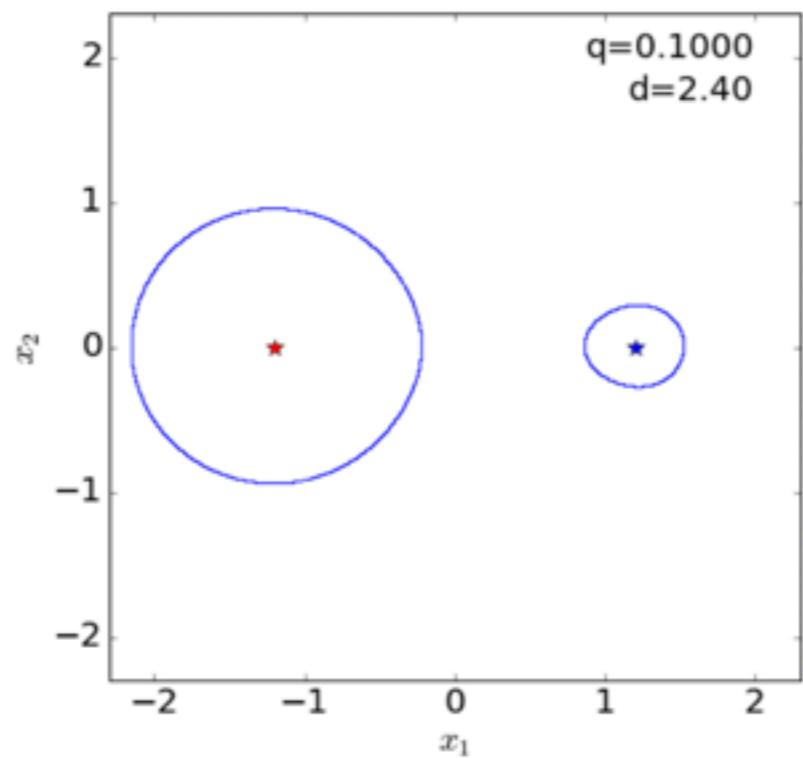
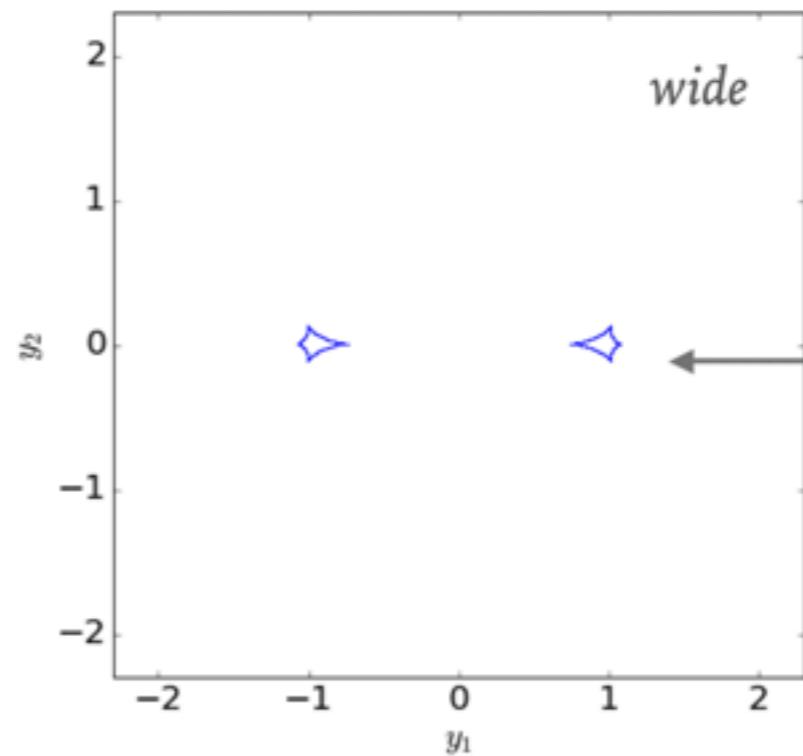
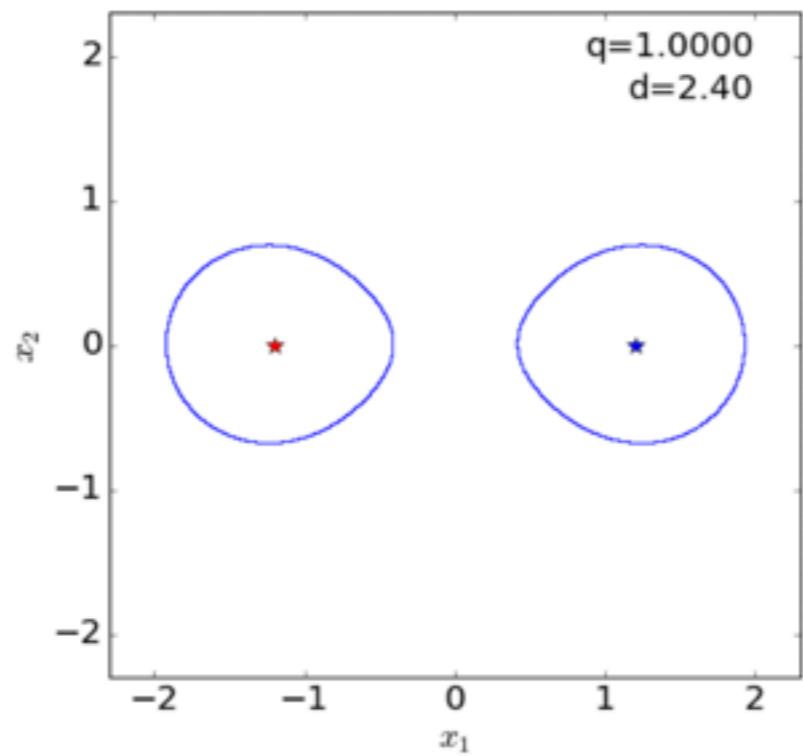
TRANSITIONS

*Touching
critical lines*

$$\det A = 0$$
$$\frac{\partial \det A}{\partial z^*} = 0$$



BINARY LENSES: TOPOLOGY CLASSIFICATION



separate 4-cusp caustics

MULTIPLE IMAGES

- Lens equation:

$$z_s = z - \frac{m_1}{z^* - z_1^*} - \frac{m_2}{z^* - z_2^*}$$

- complex polynomial:

$$p_5(z) = \sum_{i=0}^5 c_i z^i$$

$$\Delta m = \frac{m_1 - m_2}{2} \quad m = \frac{m_1 + m_2}{2} \quad z_2 = -z_1 \quad z_1 = z_1^*$$

$$c_0 = z_1^2 [4(\Delta m)^2 z_s + 4m\Delta m z_1 + 4\Delta m z_s z_s^* z_1 + 2mz_s^* z_1^2 + z_s z_s^* z_1^2 - 2\Delta m z_1^3 - z_s z_1^4]$$

$$c_1 = -8m\Delta m z_s z_1 - 4(\Delta m)^2 z_1^2 - 4m^2 z_1^2 - 4mz_s z_s^* z_1^2 - 4\Delta m z_s^* z_1^3 - z_s^* z_1^4 + z_1^6$$

$$c_2 = 4m^2 z_s + 4m\Delta m z_1 - 4\Delta m z_s z_s^* z_1 - 2z_s z_s^* z_1^2 + 4\Delta m z_1^3 + 2z_s z_1^4$$

$$c_3 = 4mz_s z_s^* + 4\Delta m z_s^* z_1 + 2z_s^* z_1^2 - 2z_1^4$$

$$c_4 = -2mz_s^* + z_s z_s^* z_1^2 - 2\Delta m z_1 - z_s z_1^2$$

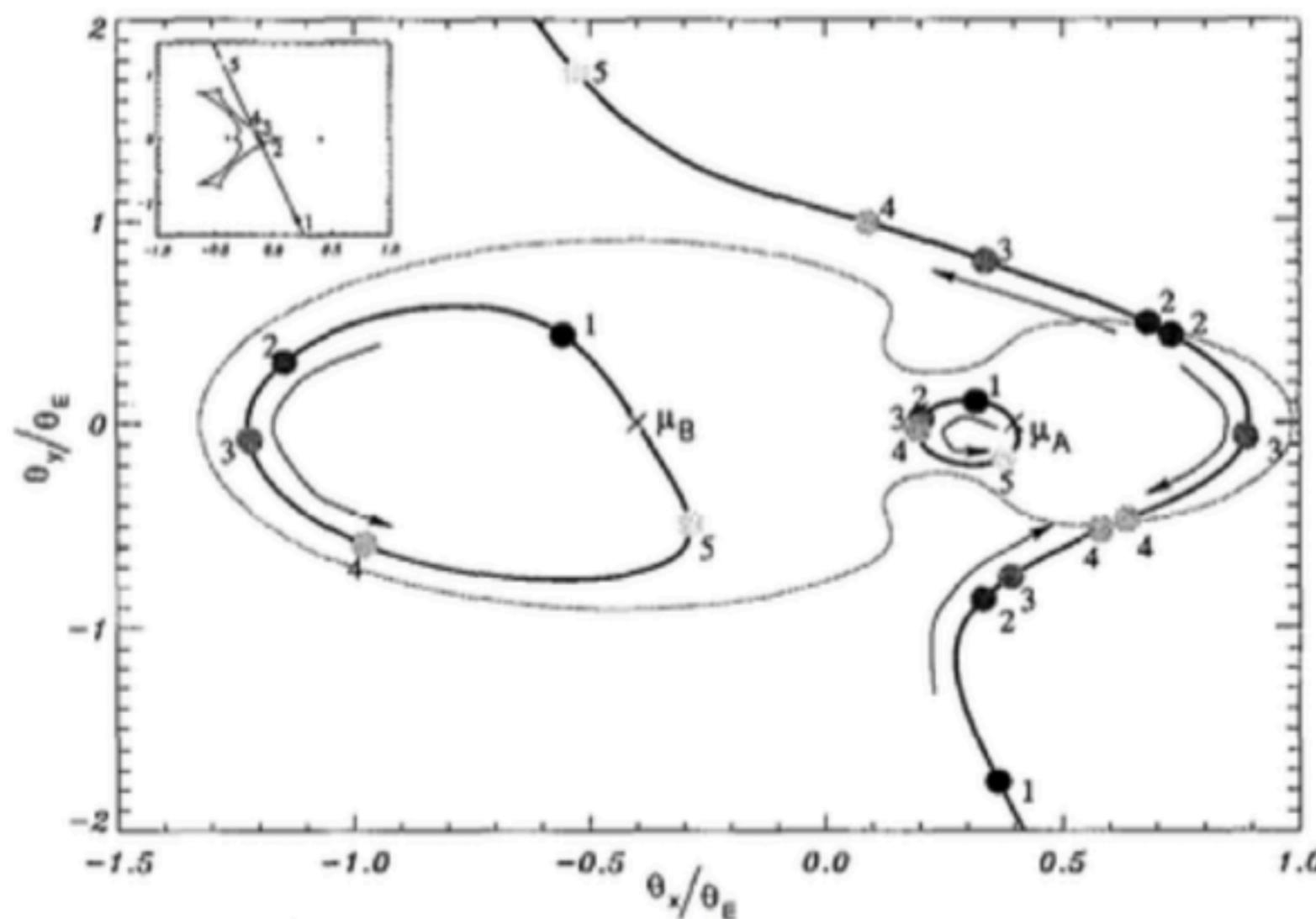
$$c_5 = z_1^2 - z_s^* z_1^2$$

Witt & Mao, 1995,

ApJ, 447, L105

- 3 or 5 images

MULTIPLE IMAGES



Mollerach & Roulet, "Gravitational Lensing and Microlensing"

IMAGE MAGNIFICATION

- magnification at the image position:

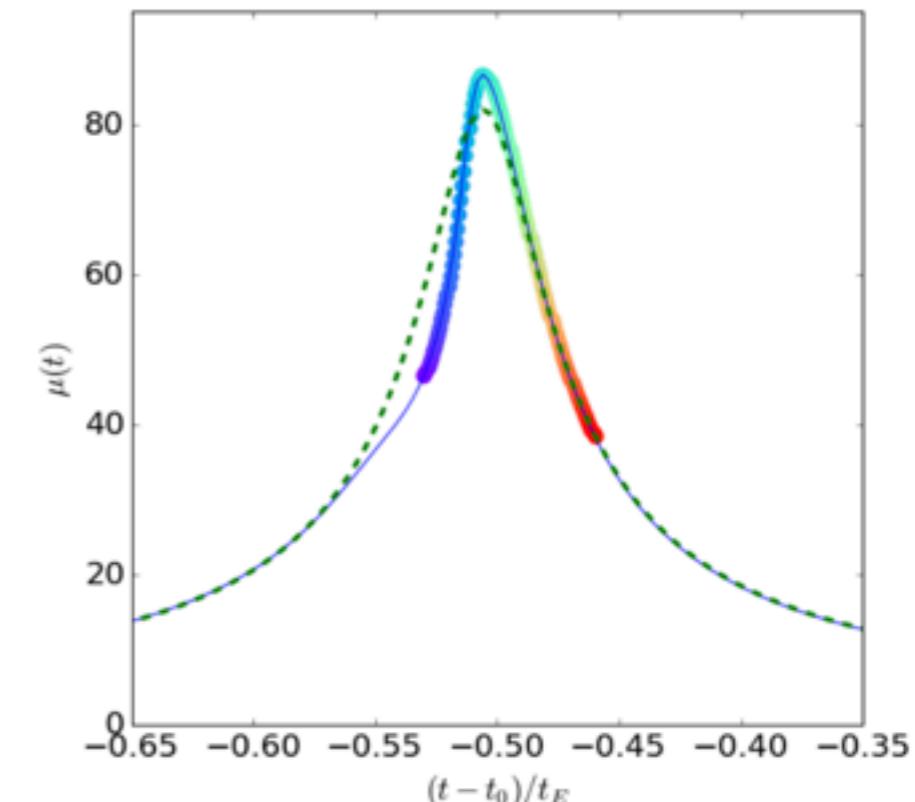
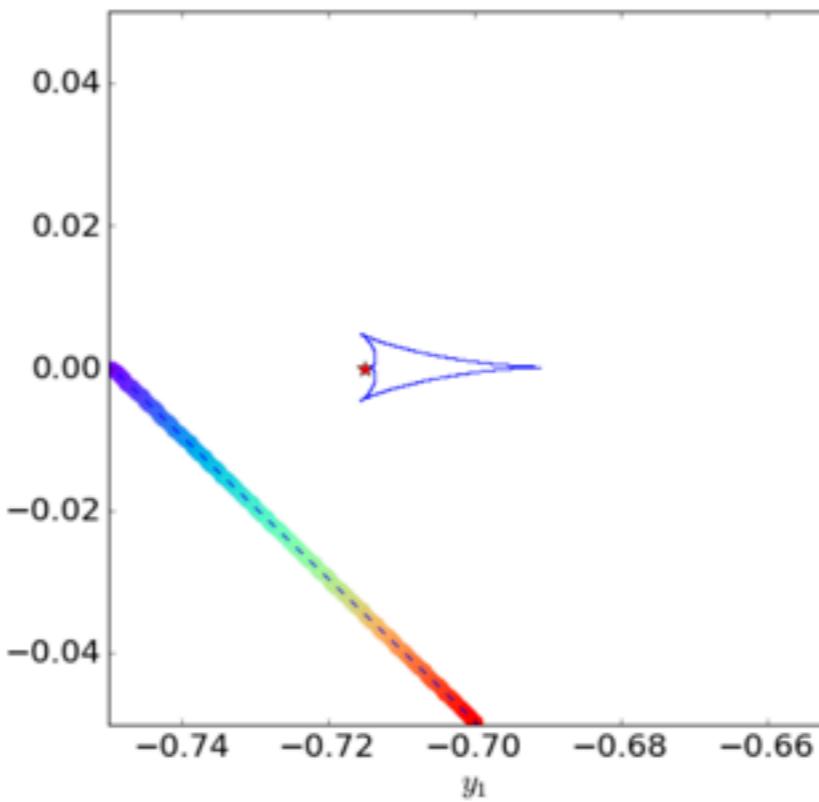
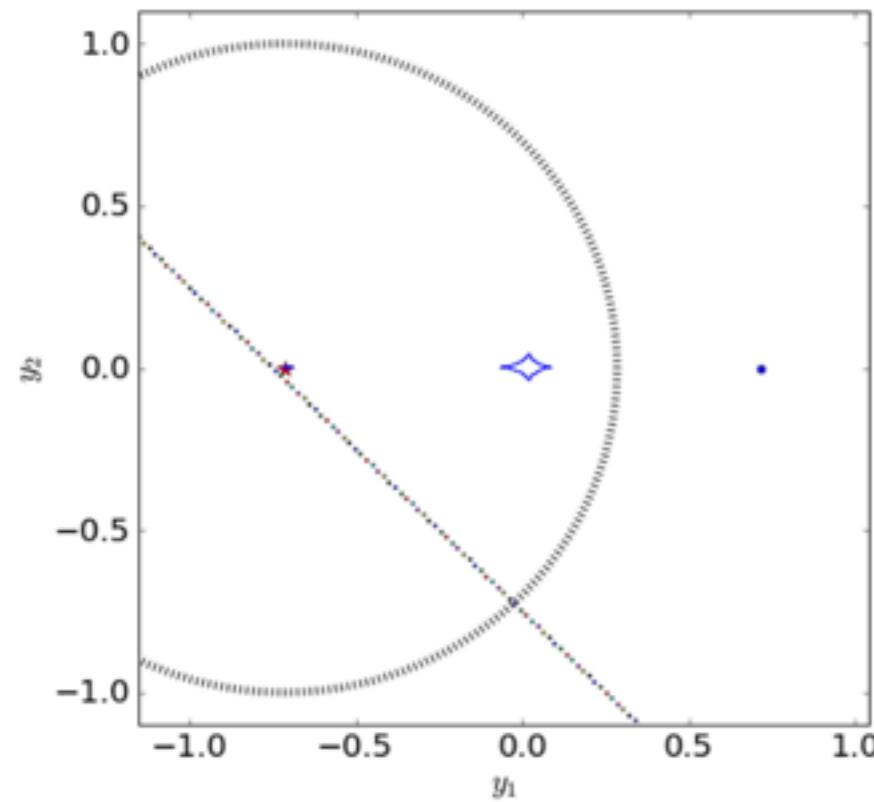
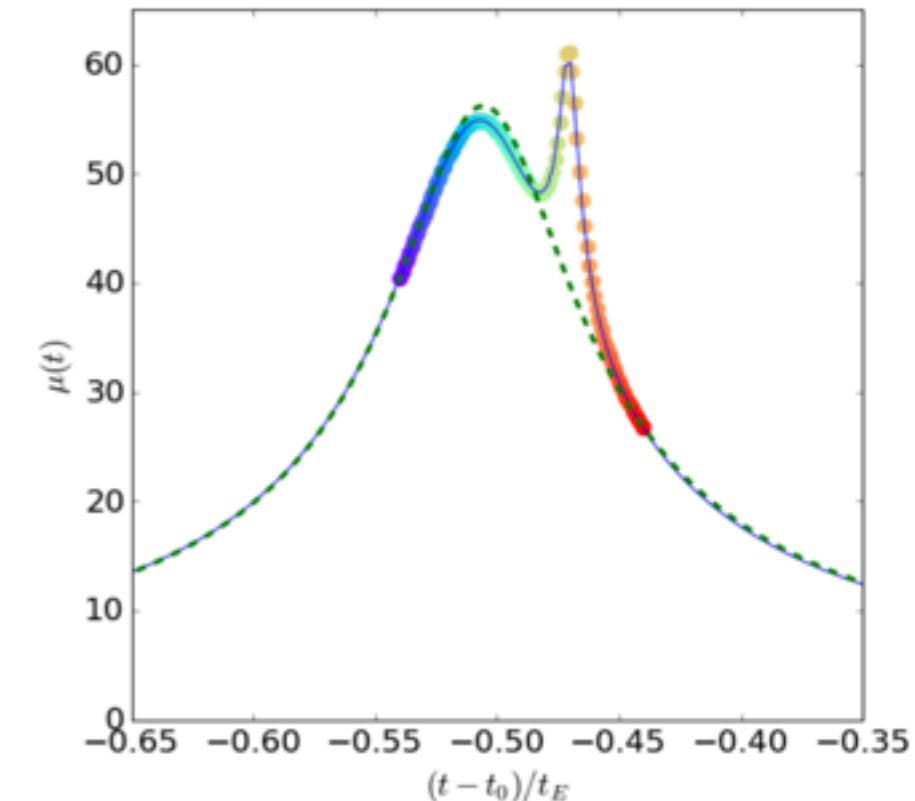
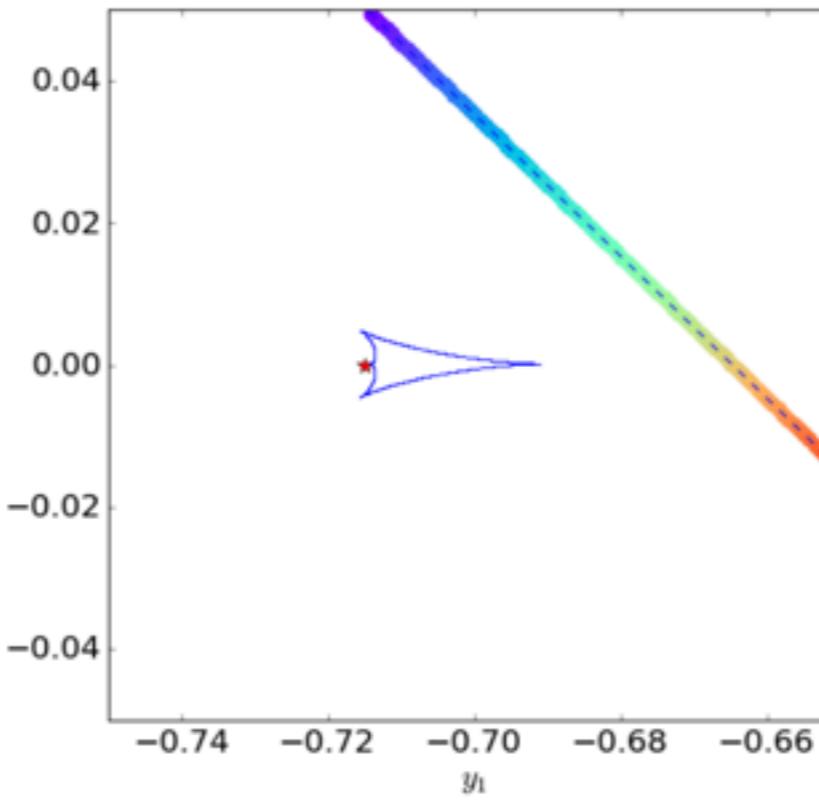
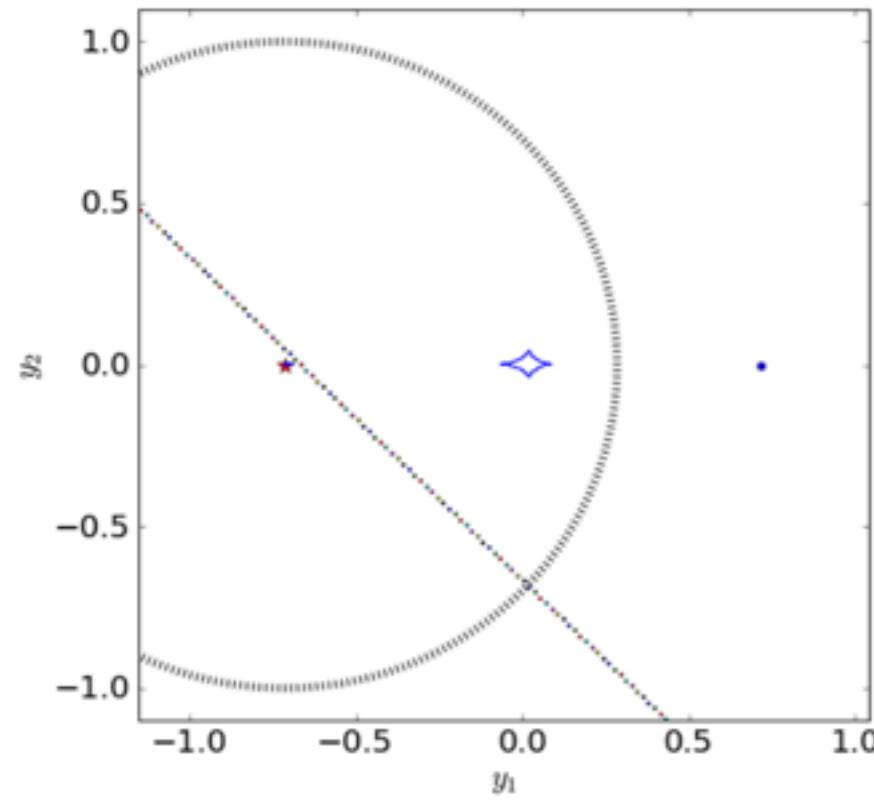
$$\mu = \det A^{-1} = \left[1 - \left| \frac{m_1}{(z^* - z_1^*)^2} + \frac{m_2}{(z^* - z_2^*)^2} \right| \right]^{-1}$$

- total magnification:

$$\mu_{tot} = \sum_{i=1}^{n_i} |\mu_i|$$

- of course, the magnification varies as a function of z ...

CENTRAL CUSP PERTURBATIONS



— Close

