Lentes gravitacionales en astrofísica y cosmología
Semana - 7
Parte II: lentes por galáxias y cúmulos de galáxias

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ICAS

CONICET

Instituto de Ciencias Físicas
ICIFI-ECYT_UNSAM-CONICET

CBPF

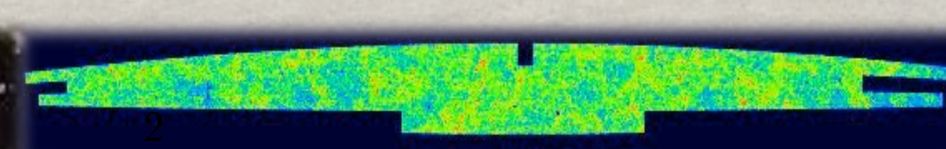
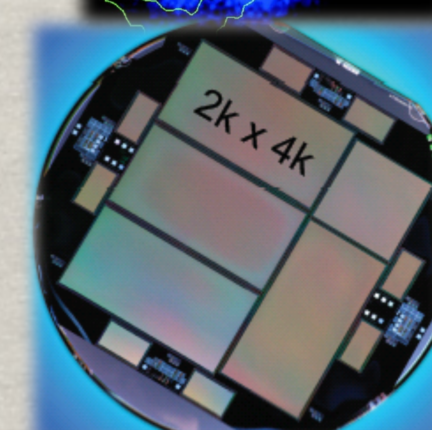
Instituto de Tecnología SABATO 30 años

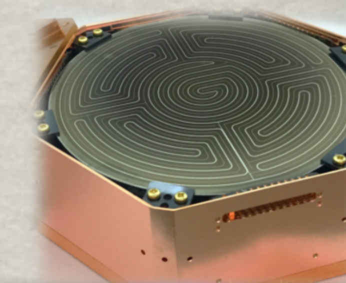
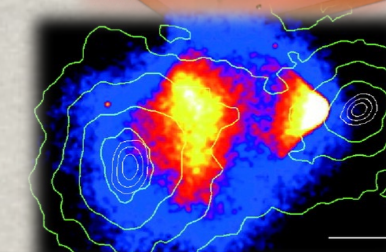
Plan

Hoy: Cosmología en ~~un~~ dos días
(casi) sin relatividad general

- ☑ Redshift y expansión del Universo
- ☑ Dinámica y parámetros cosmológicos
- ☐ Métrica y distancias
- ☐ Energía oscura
- ☐ Propagación de la luz y ecuación de la lente




La Geometría del Cosmos

"The visions of space and time that I will present here originate from experimental physics and this is why they are so powerful"

H. Minkowski

The metric

- Invariante en la relatividad restringida

$$\Delta S^2 = (\Delta x_0)^2 - (\Delta x_1)^2 - (\Delta x_2)^2 - (\Delta x_3)^2 = \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

- En coordinadas esféricas

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}.$$

La métrica

- Invariante en la relatividad restringida

$$\Delta S^2 = (\Delta x_0)^2 - (\Delta x_1)^2 - (\Delta x_2)^2 - (\Delta x_3)^2 = \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix} .$$

- Elemento de línea en el espacio-tiempo curvo

invariante $\longrightarrow$ $dS^2 = g_{\mu\nu} dx^\mu dx^\nu$ interpretación física

- Las propiedades geométricas (curvatura) son dadas por el tensor de Riemann

The (Spatial) Metric of the Homogeneous Universe

- Metric: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
- Spatial section, isotropy:

$$(d\vec{r})^2 = f(r)dr^2 + g^2(r)\left(r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)$$

- Area element perpendicular to r

$$dA = (g(r) r d\theta)\,(g(r) r \sin\theta d\phi) = (r' d\theta)\,(r' \sin\theta d\theta)$$

- Area of the sphere centered at $r = 0$

$$A = 4\pi g(r)^2 r^2 = 4\pi r'^2$$

Construcción de "cantidades físicas" con la métrica

The (Spatial) Metric of the Homogeneous Universe

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- Spatial section, isotropy:

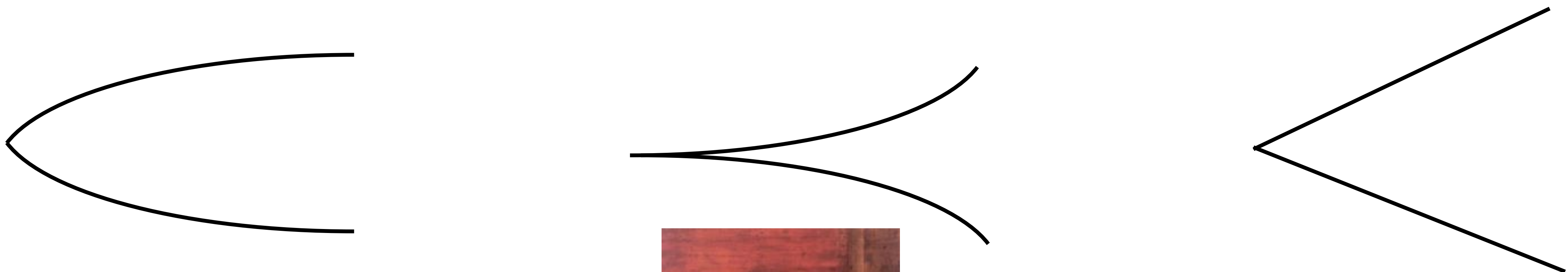
$$(d\vec{r})^2 = f(r)dr^2 + g^2(r)\left(r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)$$

- r' defined by the area (will call simply r)
- Constant curvature: $f = \dfrac{1}{1 - Kr^2}$

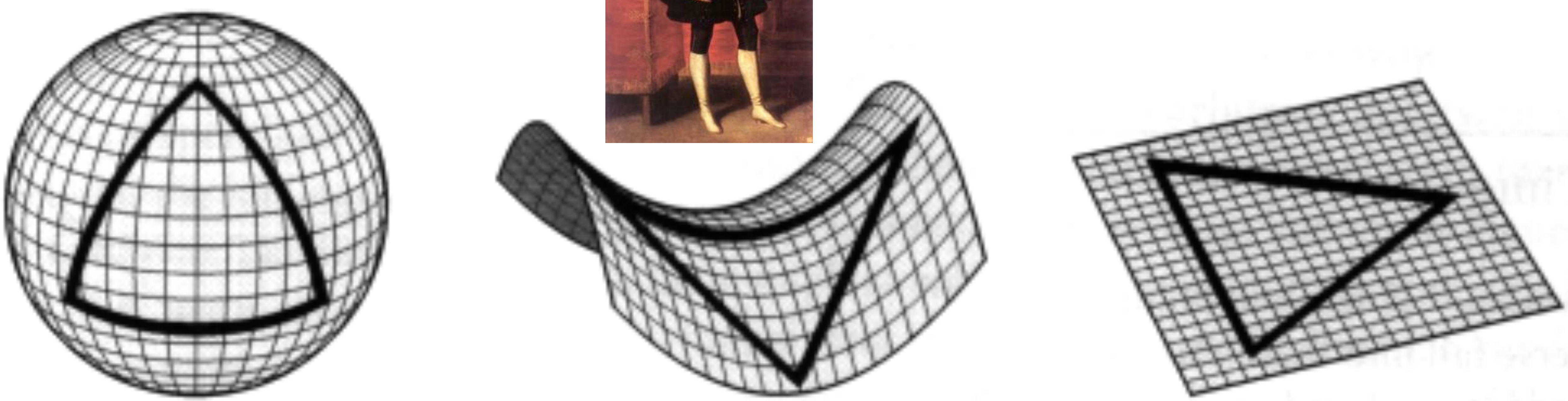
$$\boxed{(d\vec{r})^2 = \frac{1}{1 - Kr^2} dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2}$$

Geometry

Angles:



2D analogy:



Spherical **Hyperbolic** **Flat**

Space-time Geometry of the Homogeneous Universe

- Invariant: $ds^2 = dt^2 - \left(d\vec{x}\right)^2$
- All scales expand with $a(t)$:

$$\left(d\vec{x}\right)^2 = a^2(t)\left(d\vec{r}\right)^2 = a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)$$

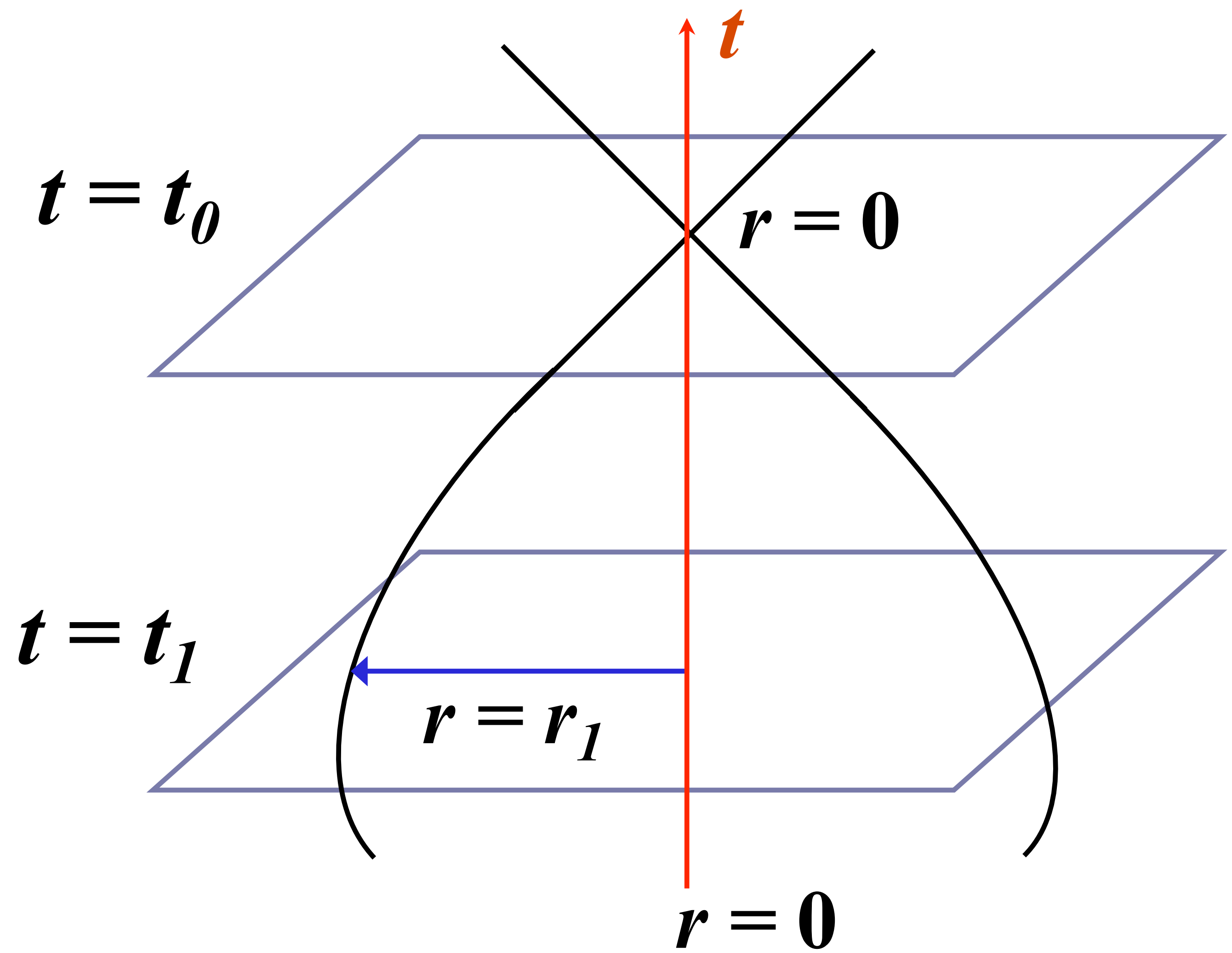
$$\boxed{ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)}$$

Friedmann metric (Robertson-Walker)

Light Propagation

- Integrating a null geodesic in the radial direction:

$$dt^2 = a^2(t)\left(\frac{1}{1-Kr^2}dr^2\right)$$



Thus

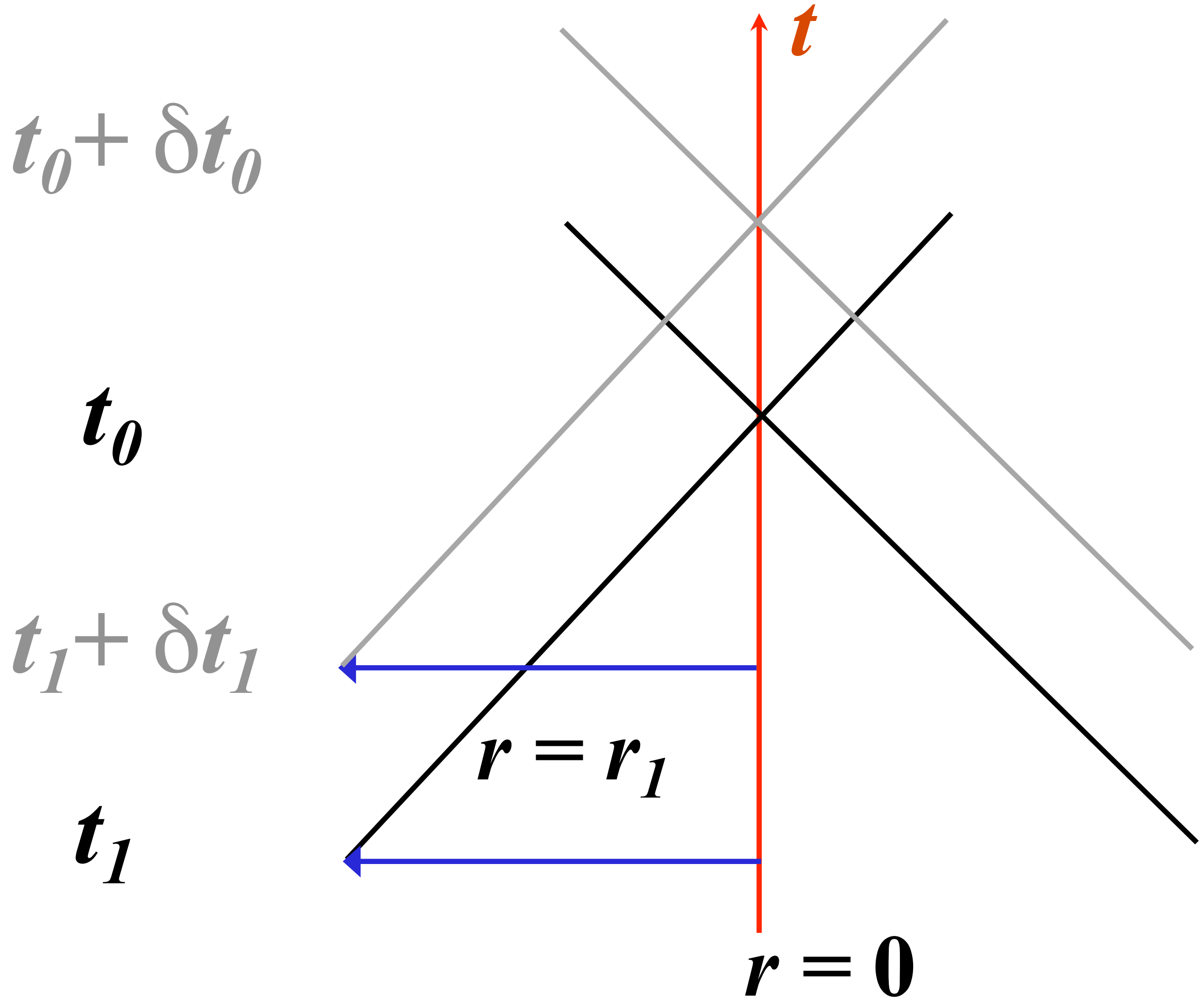
$$\boxed{\int_{t_1}^{t_0}\frac{dt}{a(t)} = \int_0^{r_1}\frac{dr}{\sqrt{1-Kr^2}} \equiv f(r_1)}$$

Light propagation and redshift

- Two “light pulses” emitted by a galaxy:

$$\int_{t_1}^{t_0} \frac{\mathrm{d}t}{a(t)} = f(r_1) = \int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{\mathrm{d}t}{a(t)}$$

$$\Rightarrow \quad \frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)}$$



Redshift $\Rightarrow$

$$\boxed{z := \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1}$$

$$\boxed{\frac{a(t)}{a_0} = \frac{1}{1+z}}$$

Recapitulando....

$$\boxed{ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)}$$

Parámetro de Hubble:

$$H^2(a) = H_0^{\,2}\left[\Omega_r a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda\right] \qquad \Omega_k = -\frac{K}{H_0^{\,2}}$$

Trayectória de la luz (geodésica nula)

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1-Kr^2}} \equiv f(r_1) \quad \Rightarrow \quad \frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)}$$

Corrimiento al rojo $\Rightarrow$ $\boxed{z := \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1}$ $\boxed{a(t) = (1+z)^{-1}}$

Luminosity Distance

$$d_L^2 := \frac{L}{4\pi F}$$

F: energy per unit area per unit time (at the detector)

Area of S^2 centered on the source at the time of detection t_0: $4\pi a^2(t_0) r_1^2$

Variation of the photon energy: $h\nu_1 / h\nu_0 = (1+z)^{-1}$

Time difference: $\delta t_0 / \delta t_1 = a(t_0)/a(t_1) = 1+z$

Thus: $$F = \frac{L'}{4\pi a^2(t_0) r_1^2} = \frac{L}{4\pi a^2(t_0) r_1^2 (1+z)^2}$$

L': energía total por unidad de tiempo a una distancia r_1 de la fuente **hoy**

$$\Rightarrow \quad \boxed{d_L^2 = a^2(t_0) r_1^2 (1+z)^2}$$

Luminosity Distance

$$\boxed{d_L = a(t_0) r_1 (1+z)}$$

As

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1-Kr^2}} = \begin{cases} \sin^{-1}\left(\sqrt{|K|}r_1\right) \Big/ \sqrt{|K|}, \text{ para } K > 0 \\ r_1, \text{ for } K = 0 \\ \sinh^{-1}\left(\sqrt{|K|}r_1\right) \Big/ \sqrt{|K|}, \text{ para } K < 0 \end{cases}$$

and

$$\int_{t_1}^{t_0} \frac{dt}{a} = \int_{t_1}^{t_0} \frac{da}{\dot{a}a} = -\int_z^0 \frac{dz'}{H} \quad \text{and} \quad |K| = \sqrt{|1-\Omega_0|}H_0$$

we have

$$d_L = (1+z)\,\mathrm{sen}_K\left(H_0\sqrt{|1-\Omega_0|}\int_0^z \frac{dz'}{H(z')}\right) \Big/ H_0\sqrt{|1-\Omega_0|}$$

Distancia de Luminosidad

Recordando que

$$\boxed{H^2(a) = H_0^2 \left[\Omega_r \left(\frac{a}{a_0} \right)^{-4} + \Omega_M \left(\frac{a}{a_0} \right)^{-3} + \Omega_K \left(\frac{a}{a_0} \right)^{-2} + \Omega_\Lambda \right]}$$

$$\Omega_i := \frac{\rho_{i0}}{\rho_{crit}} \quad \text{con} \quad \rho_{crit} = \frac{3H_0^{\ 2}}{8\pi G} \quad \text{y} \quad \Omega_K = -\frac{K}{a_0^2 H_0^2}$$

$$\text{y} \quad \frac{a}{a_0} = (1+z)^{-1}$$

$$\boxed{d_L = (1+z)\,\mathrm{sen}_K \left(H_0 \sqrt{|1-\Omega_0|} \int_0^z \frac{dz'}{H(z')} \right) \Big/ H_0 \sqrt{|1-\Omega_0|}}$$

Luminosity Distance

Flat case:

$$d_L = (1+z)\int_0^z \frac{dz'}{H(z')}$$

Recall that (in the ΛCDM model)

$$H^2(a) = H_0^2\left[\Omega_r a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda\right]$$

then

$$d_L = \frac{(1+z)}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_r(1+z')^4 + \Omega_M(1+z')^3 + \Omega_\Lambda}}$$

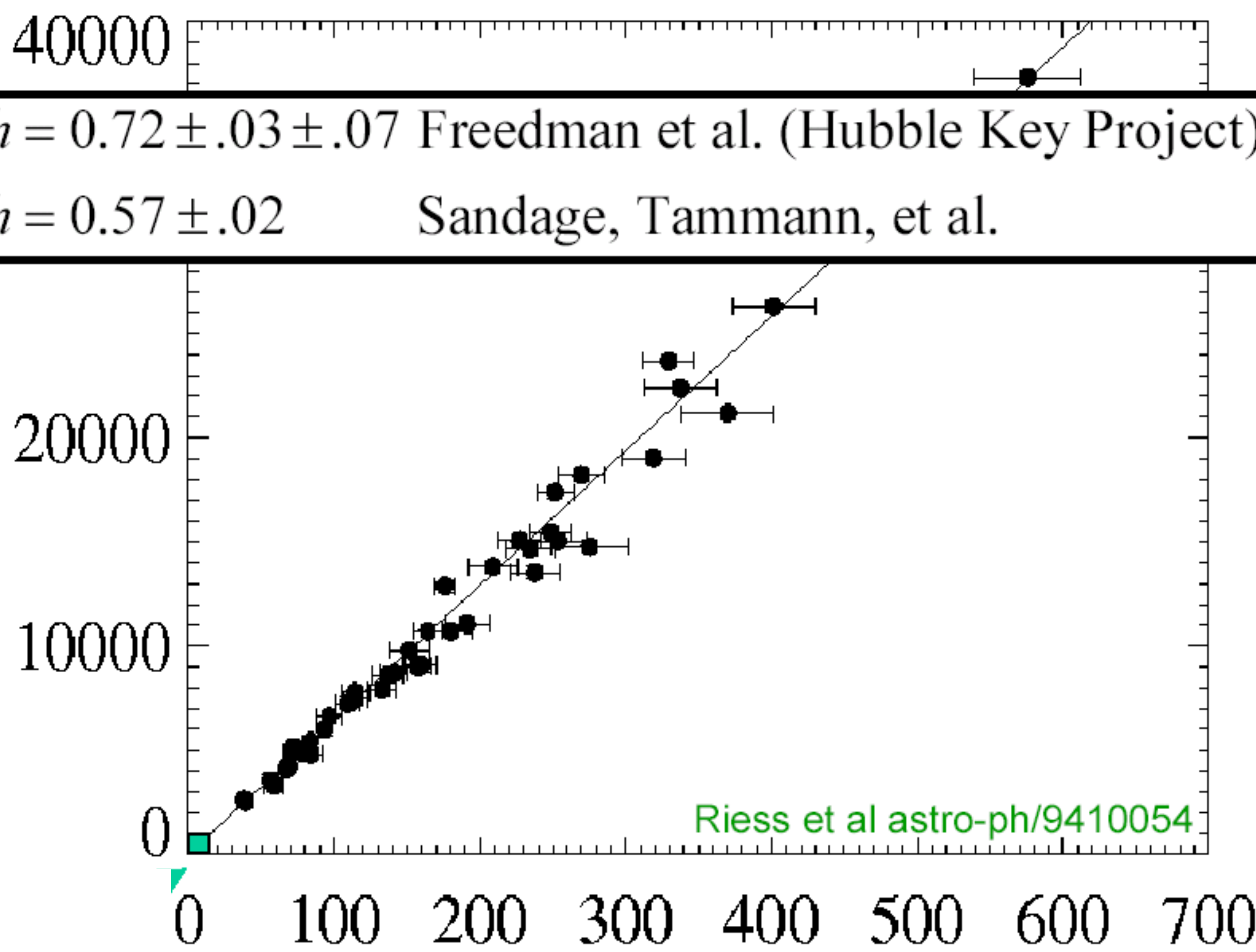
Luminosity Distance

Series Expansion (model independent):

To 1st order

$$H_0 d_L(z) = cz$$

⇦ **Hubble's "law"**



$h = 0.72 \pm .03 \pm .07$ Freedman et al. (Hubble Key Project)

$h = 0.57 \pm .02$ Sandage, Tammann, et al.

Luminosity Distance

Series Expansion (**model independent**):

To 1st order

$$H_0 d_L(z) = cz$$ **Hubble's "law"**

Exercise: obtain, at 2nd order

$$H_0 d_L(z) = cz\left[1 + \frac{1}{2}(1 - q_0)z\right]$$

Hint: expand $\dfrac{1}{a(t)}$ in power series

The Deceleration Parameter

At 2nd order

$$H_0 d_L(z) = cz\left[1 + \frac{1}{2}(1 - q_0)z\right]$$

where

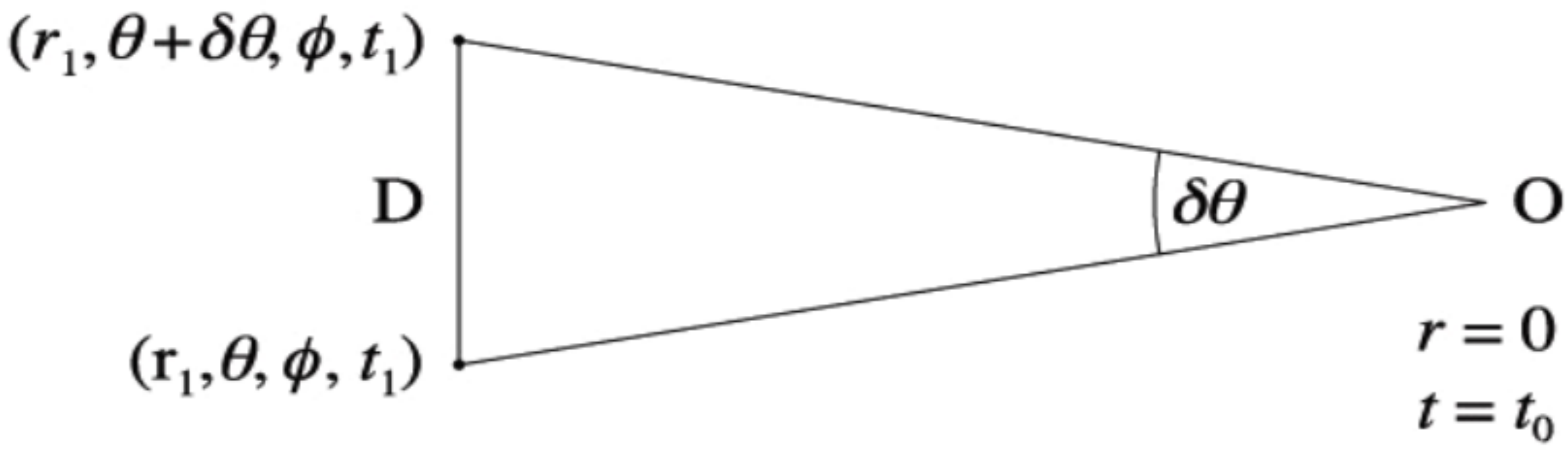
$$q_0 = -\frac{\ddot{a}_0}{a_0 H_0^2}$$

Exercise:

1) obtain the equation for $\ddot{a}$ and connect to the cosmological parameters

2) derive the series expansion to 4th order in z

Distancia de diámetro angular



$$d_A \equiv \frac{D}{\delta\theta}$$

$$(\text{para}\, \delta\theta \ll 1)$$

Regla standard

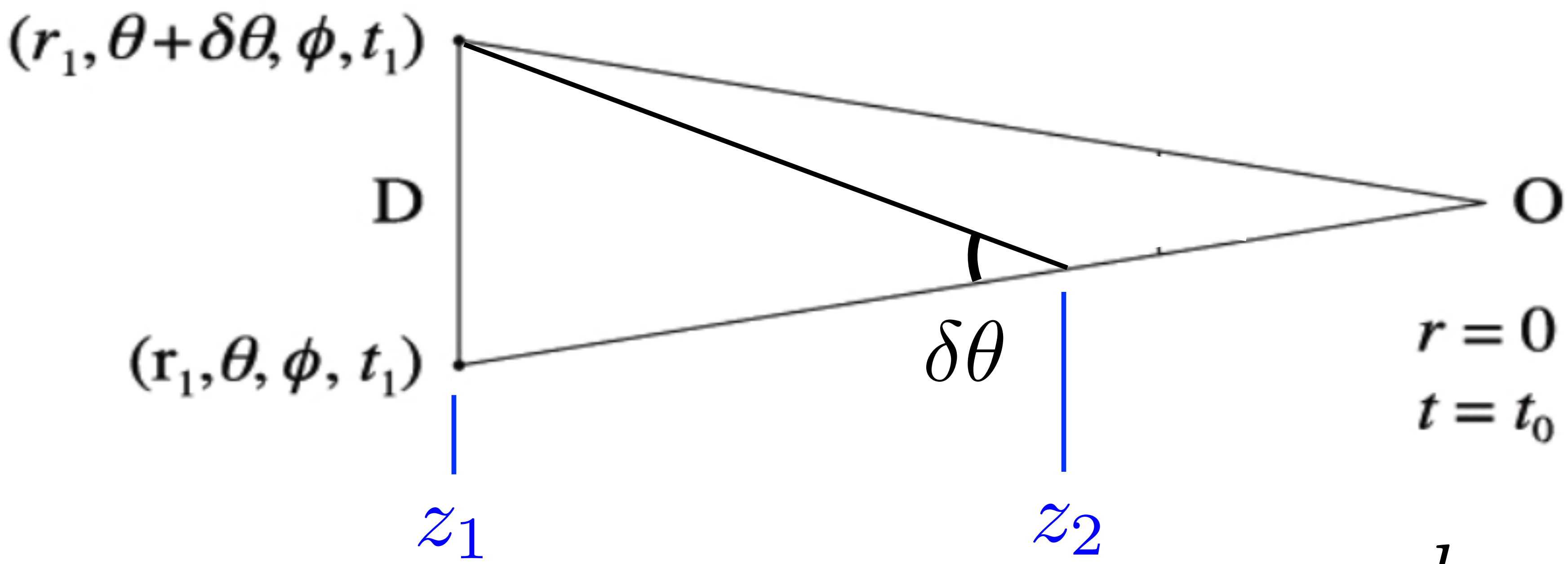
$$\boxed{ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)}$$

luego $D = a(t_1)\, r_1 \delta\theta$ entonces $\boxed{d_A = a(t_1)\, r_1}$

$$\boxed{d_A = (1+z)^{-1} \mathrm{sen}_K \left(H_0 \sqrt{|1-\Omega_0|} \int_0^z \frac{dz'}{H(z')} \right) \Big/ H_0 \sqrt{|1-\Omega_0|}}$$

d_A puede disminuir con z!

Distância de diámetro angular entre dos puntos



$$d_A \equiv \frac{D}{\delta\theta}$$

(para $\delta\theta \ll 1$)

$$d_A = a(t_2) r_{12} = a(z_2) r_{12}$$

$$\boxed{d_A = (1+z_2)^{-1} \operatorname{sen}_K \left(H_0 \sqrt{|1-\Omega_0|} \int_{z_1}^{z_2} \frac{dz'}{H(z')} \right) \bigg/ H_0 \sqrt{|1-\Omega_0|}}$$

Caso plano: $D_A(z_1, z_2) = \dfrac{1}{1+z_2} \displaystyle\int_{z_1}^{z_2} \frac{dz}{H(z)}$

Distância de diámetro Angular entre dos puntos

Caso plano:

$$D_A(z_1, z_2) = \frac{1}{1+z_2} \int_{z_1}^{z_2} \frac{dz}{H(z)}$$

En el modelo wCDM $\quad p = w\rho$

$$H^2(a) = H_0^2 \left[\Omega_r a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_{DE} a^{-3(1+w)}\right]$$

de forma que

$$D_A(z_1, z_2) = \frac{(1+z_2)^{-1}}{H_0} \int_{z_1}^{z_2} \frac{dz'}{\sqrt{\Omega_M (1+z')^3 + (1-\Omega_M)(1+z')^{3(1+w)}}}$$

$D_{LS} = D_A(z_L, z_S)$, etc.

Otras definiciones de distancia

- Distancia de diámetro angular

$$D_L(z) = (1+z)^2 D_A(z)$$

Relación de dualidad de distancias de Etherington

- Distancia del viaje de la luz

$$d_T(z) = c\,t(z) = \int_0^z \frac{dz'}{(1+z')H(z')}$$

Otros sistemas de coordenadas

- Distancia transversa comóvil

$$ds^2 = -dt^2 + a^2(t)\left(d\chi^2 + F(\chi)(d\theta^2 + sen^2\theta\, d\phi^2)\right).$$

- Tiempo conforme

$$dt = a(\tau)d\tau$$

- Gautreau

$$ds^2(R,\tau) = -K^{-2}\left[\alpha\left(K^2 - A\right)^{1/2} d\tau - dR\right]^2 + d\tau^2 - R^2 d\Omega^2$$

Summary of part II: the homogeneous Universe

Two paths: Newtonian | General Relativity

Energy conservation (of test particle) in spherical gravitational potential

Homogeneous and isotropic line element

$$ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)$$

$+ \quad G_{\mu\nu} = 8\pi G T_{\mu\nu}$ Einstein's equation

First law of thermodynamics

(+ special relativity)

Friedmann's Equation

$$\dot{a}^2 = \frac{8\pi G}{3} a^2 \rho - K$$

Energy Conservation

$$\frac{d\rho}{da} + \frac{3}{a}(\rho + p) = 0$$

$$\Downarrow$$

$$H^2(a) = H_0^2\left[\Omega_r a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda\right]$$

Summary of part II: the homogeneous Universe

Two paths: Newtonian | General Relativity

Light propagation in co-moving coordinates ($c = 1$)

$$a(t)dr = dt$$

Homogeneous and isotropic line element

$$\boxed{ds^2 = dt^2 - a^2(t)\left(\frac{1}{1-Kr^2}dr^2 + r^2 d\theta^2 + r^2 sen^2\theta\, d\phi^2\right)}$$

+ Null geodesic: $dt^2 = a^2(t)\left(\frac{1}{1-Kr^2}dr^2\right)$

$$\frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)} \Rightarrow \boxed{z := \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1}$$

Flux: $F = \dfrac{L}{4\pi a^2(t_0) r_1^2 (1+z)^2}$

$$\boxed{d_L = a(t_0) r_1 (1+z)}$$

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1-Kr^2}} \equiv f(r_1)$$

$$d_L = (1+z)\,\mathrm{sen}_K\left(H_0\sqrt{|1-\Omega_0|}\int_0^z \frac{dz'}{H(z')}\right) \Big/ H_0\sqrt{|1-\Omega_0|}$$

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Homogeneous and isotropic line element

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+ Null geodesic: $dt^2 = a^2(t)\left(\frac{1}{1-Kr^2}dr^2\right)$

$$\frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)} \Rightarrow \boxed{z := \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1}$$

Flux: $$F = \frac{L}{4\pi a^2(t_0) r_1^2 (1+z)^2}$$

For $K = 0$

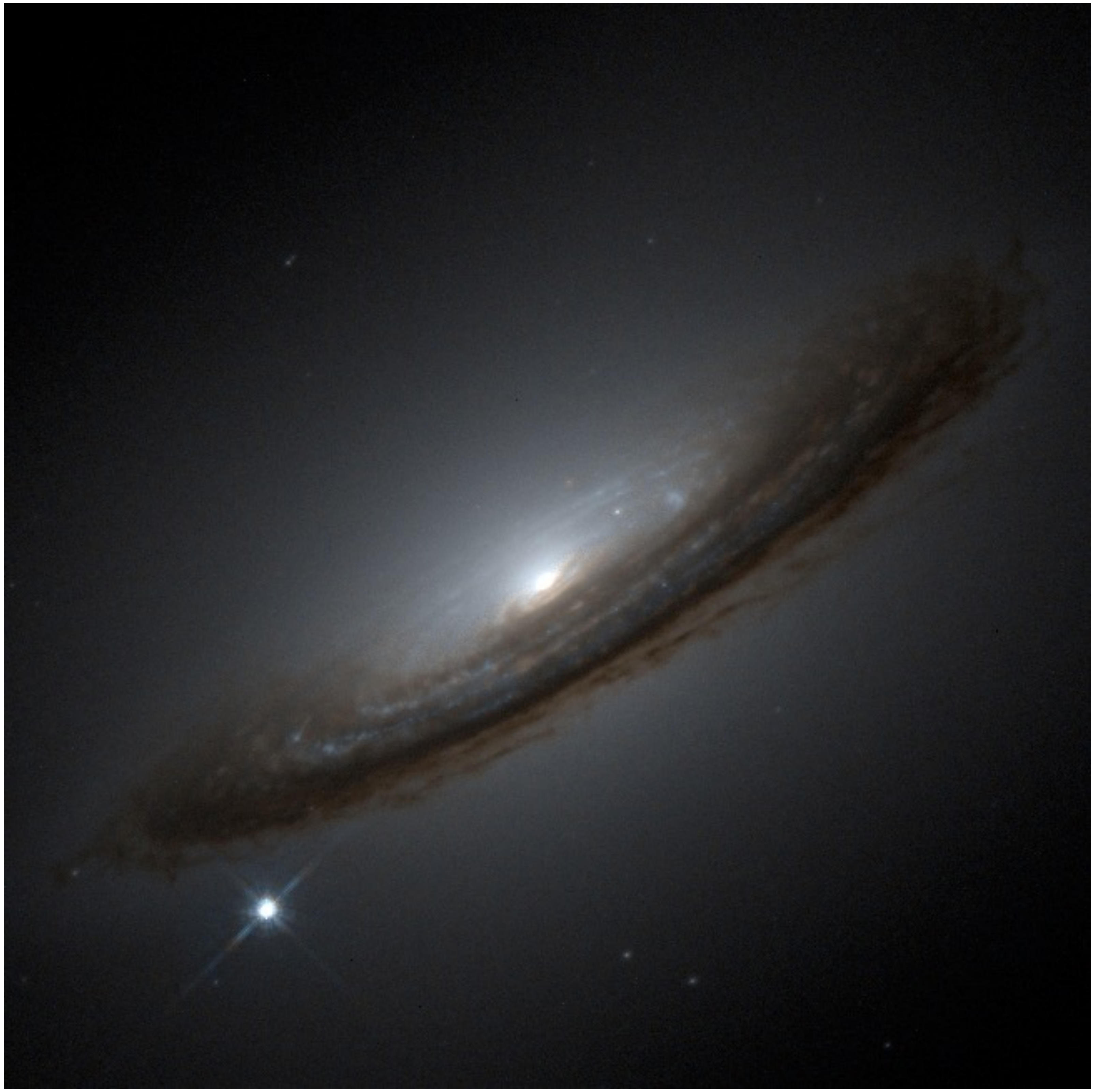
$$d_L = \frac{(1+z)}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_r (1+z')^4 + \Omega_M (1+z')^3 + \Omega_\Lambda}}$$

The Dark Side of the Universe

Episode II

The Accelerating Universe

Type Ia Supernovae and Cosmology



Advantages:

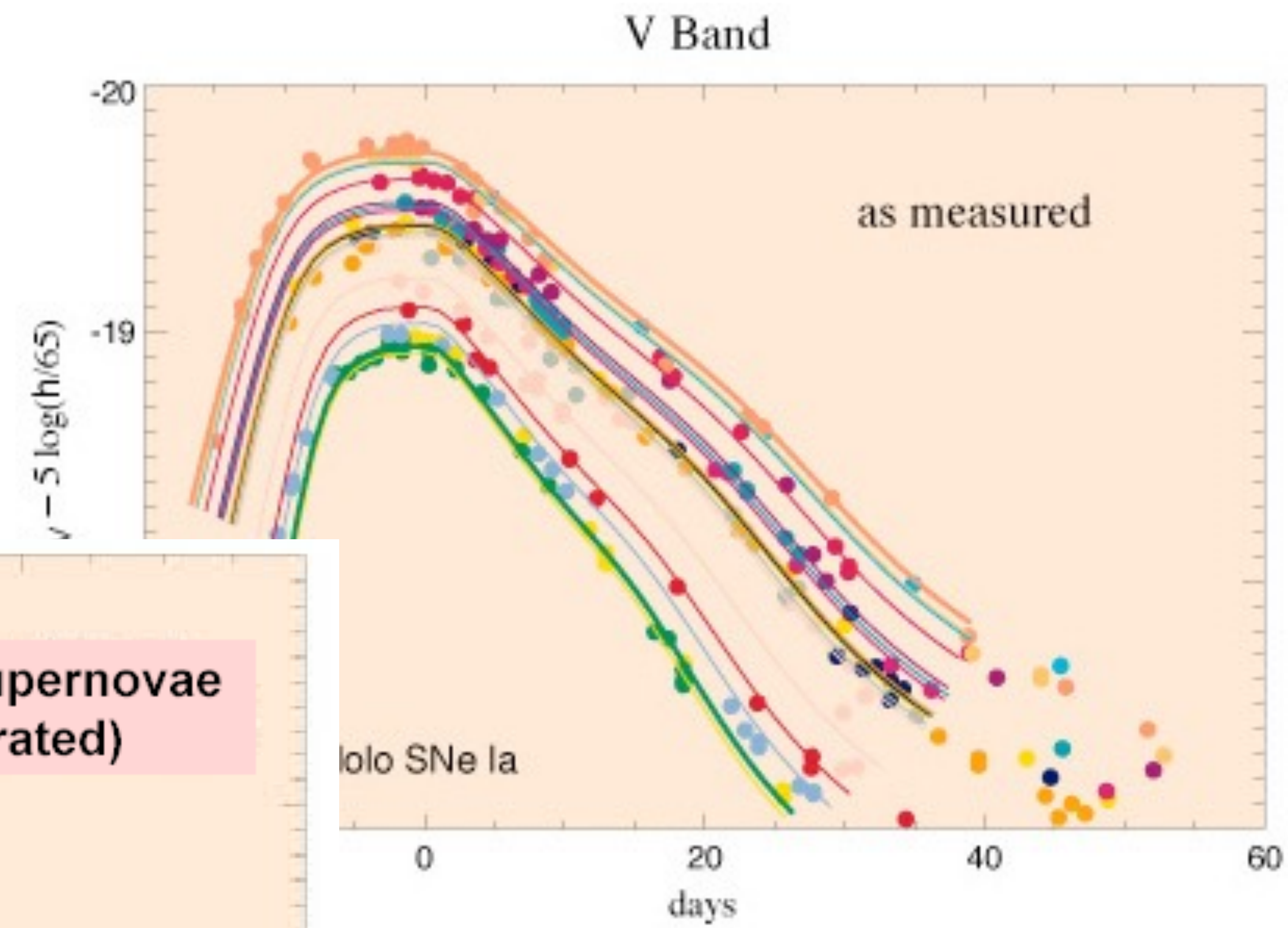
- Extreme Luminosities (10^9 - $10^{10}\, L_\odot$)

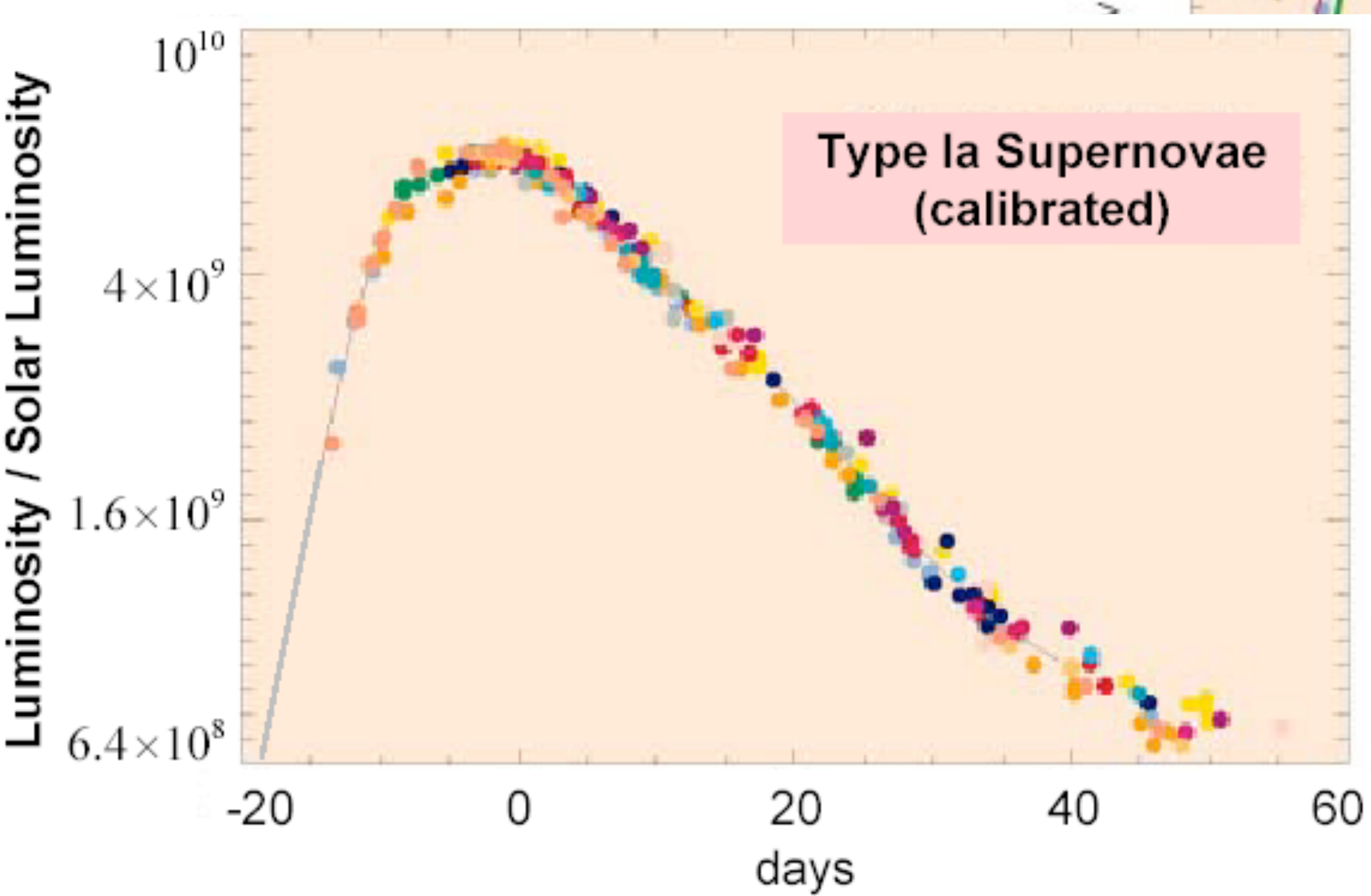
➔ May be detected at large distances

Type Ia Supernovae Light Curves

Very homogeneous

➔ Standardizable candles





Type Ia Supernovae and Cosmology

Advantages:

- Extreme Luminosities (10^9 - 10^{10} $L_\odot$)
- Very homogeneous

➔ Standardizable candles

Disadvantages:

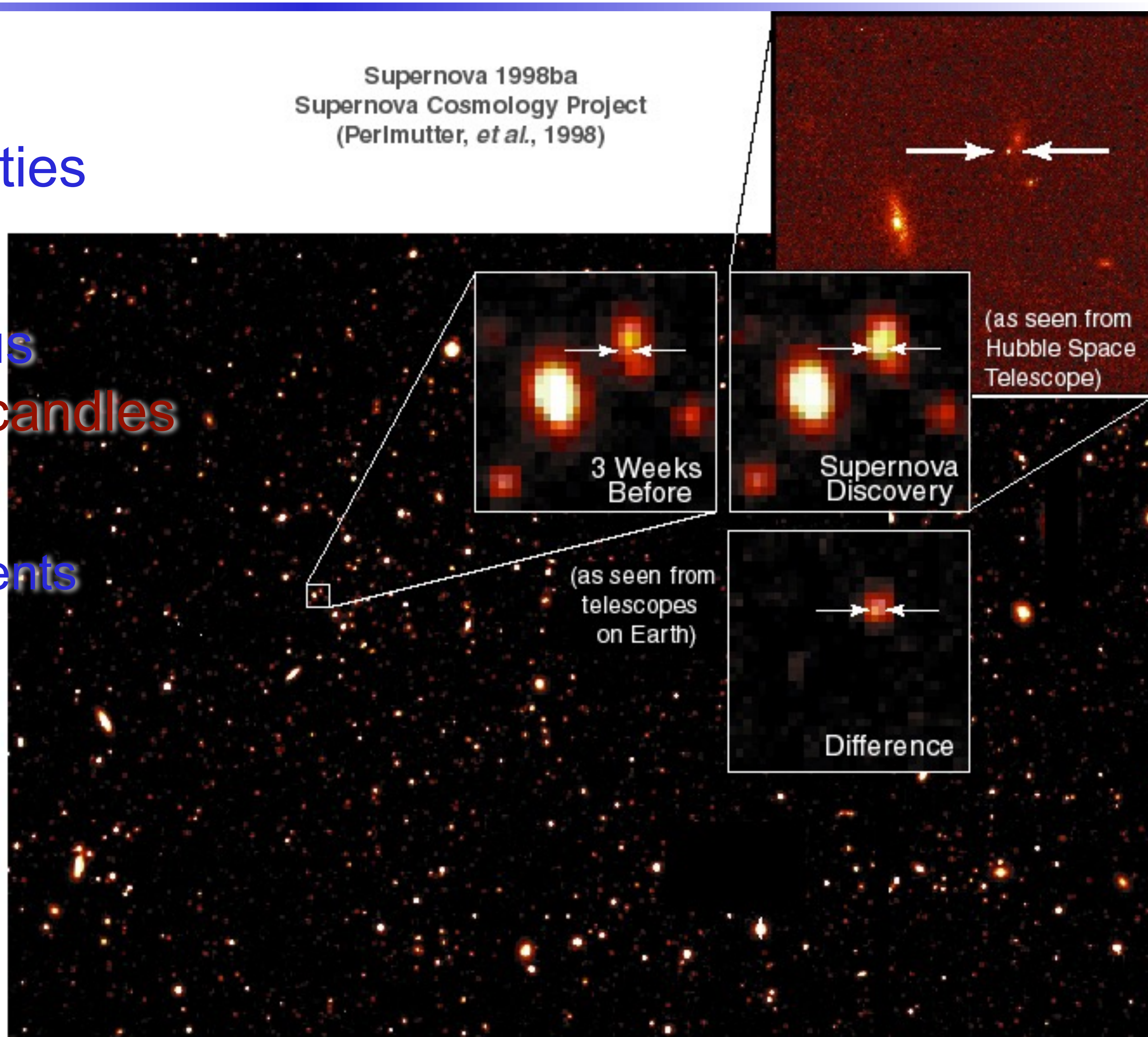
- Rare and random events

~ 1/500 yr/galaxy

- Short duration

Solution:

- Automated search
- SCP, High-z team

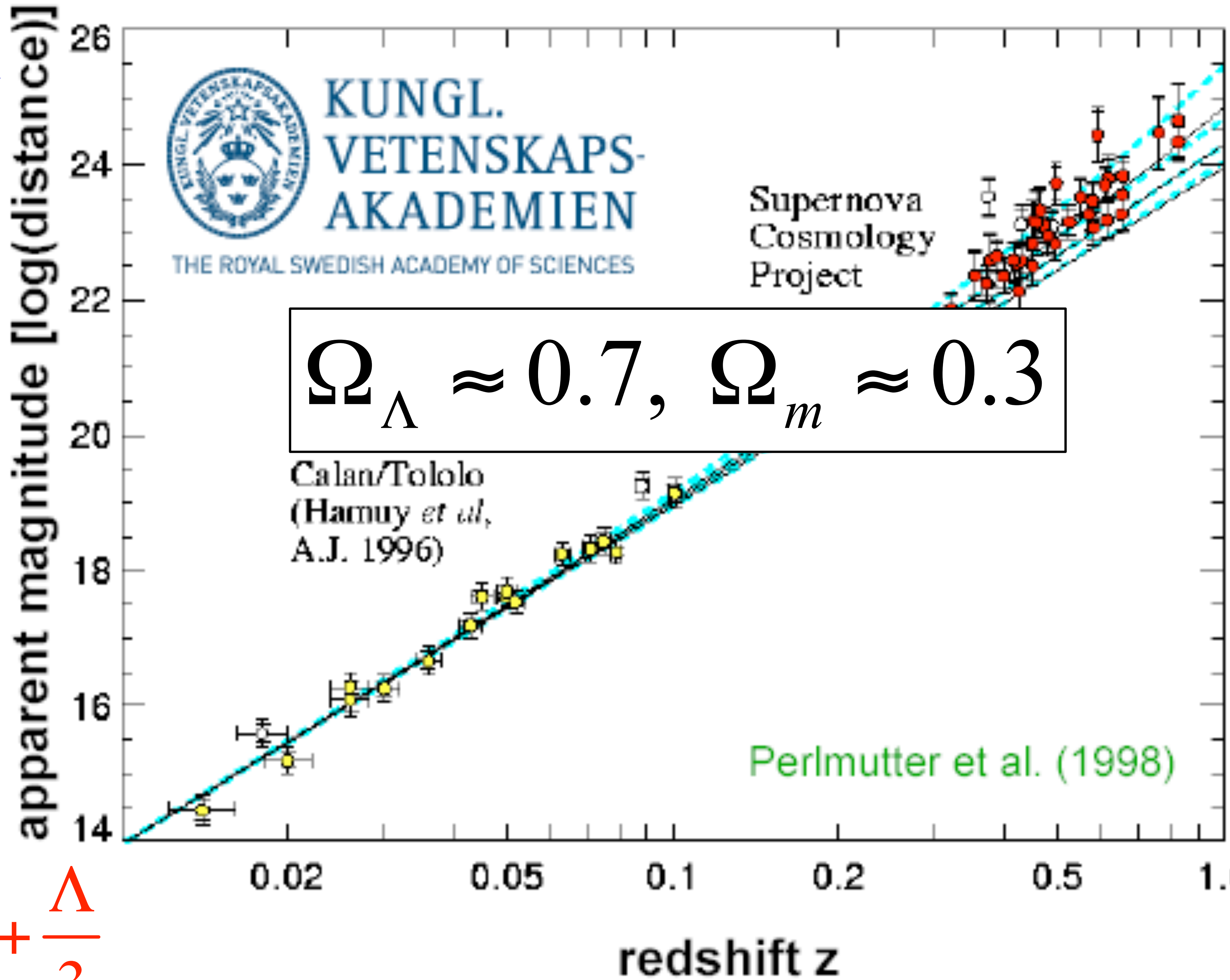


The Accelerating Universe

- Hubble diagram for large distances

➔ The expansion is accelerating

$$q_0 = -\frac{\ddot{a}_0}{a_0 H_0^2}$$

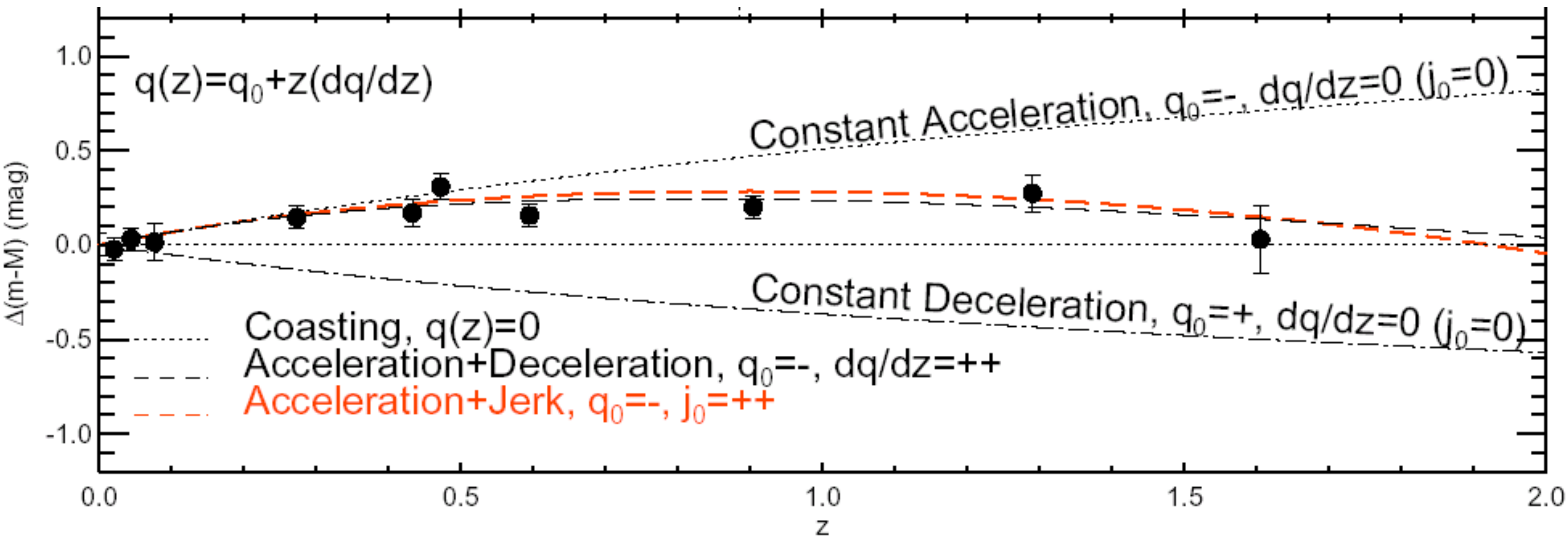


But

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

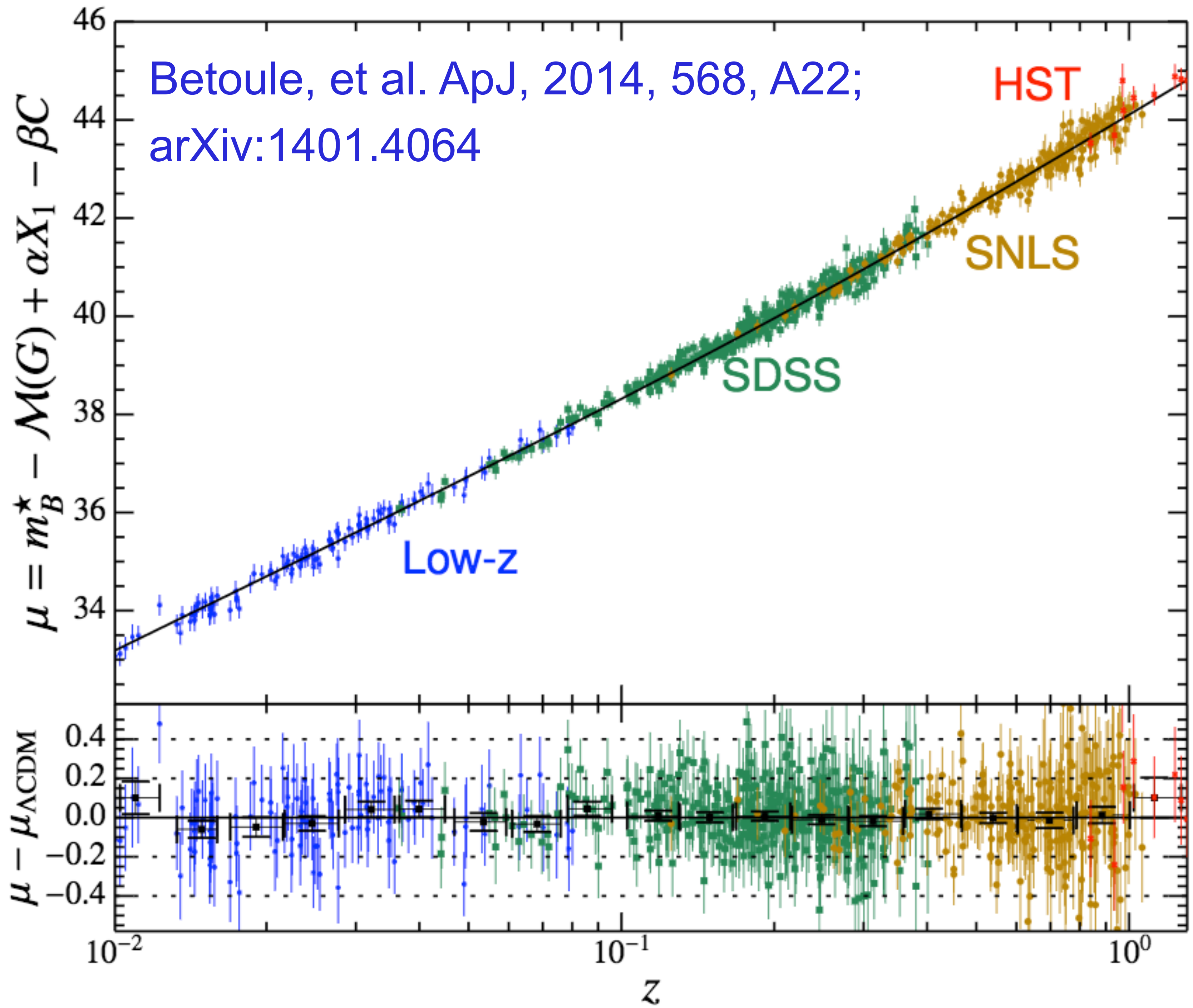
⇒ **Dark Energy or Cosmological Constant**

The Decelerated Universe!



Current Results

Joint light-curve analysis (JLA)



$\Omega_m = 0.295 \pm 0.034$

$$p = w\rho$$

Combined with CMB (+flat):

$w = -1.027 \pm 0.055$

Combined with BAO (+flat):

$w = -1.018 \pm 0.057$

Age of the Universe

Age x H_0^{-1}

Evolution of the Universe ($\Lambda = 0$)

$$\dot{a}_0 = a_0 / t_{0\mathrm{est}}$$

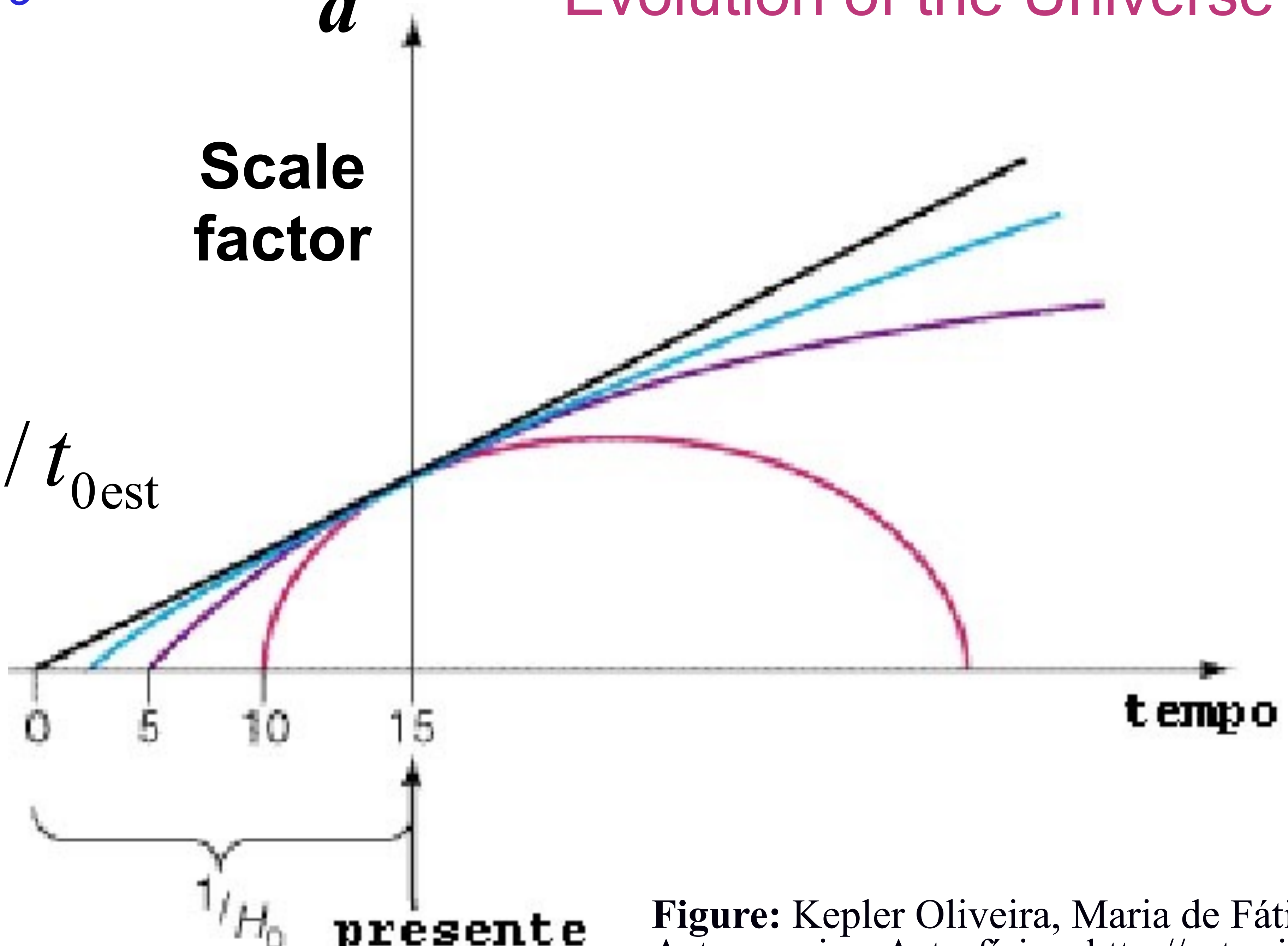


Figure: Kepler Oliveira, Maria de Fátima Saraiva Astronomia e Astrofísica, http://astro.if.ufrgs.br/

Age of the Universe

- Acceleration solves the age problem

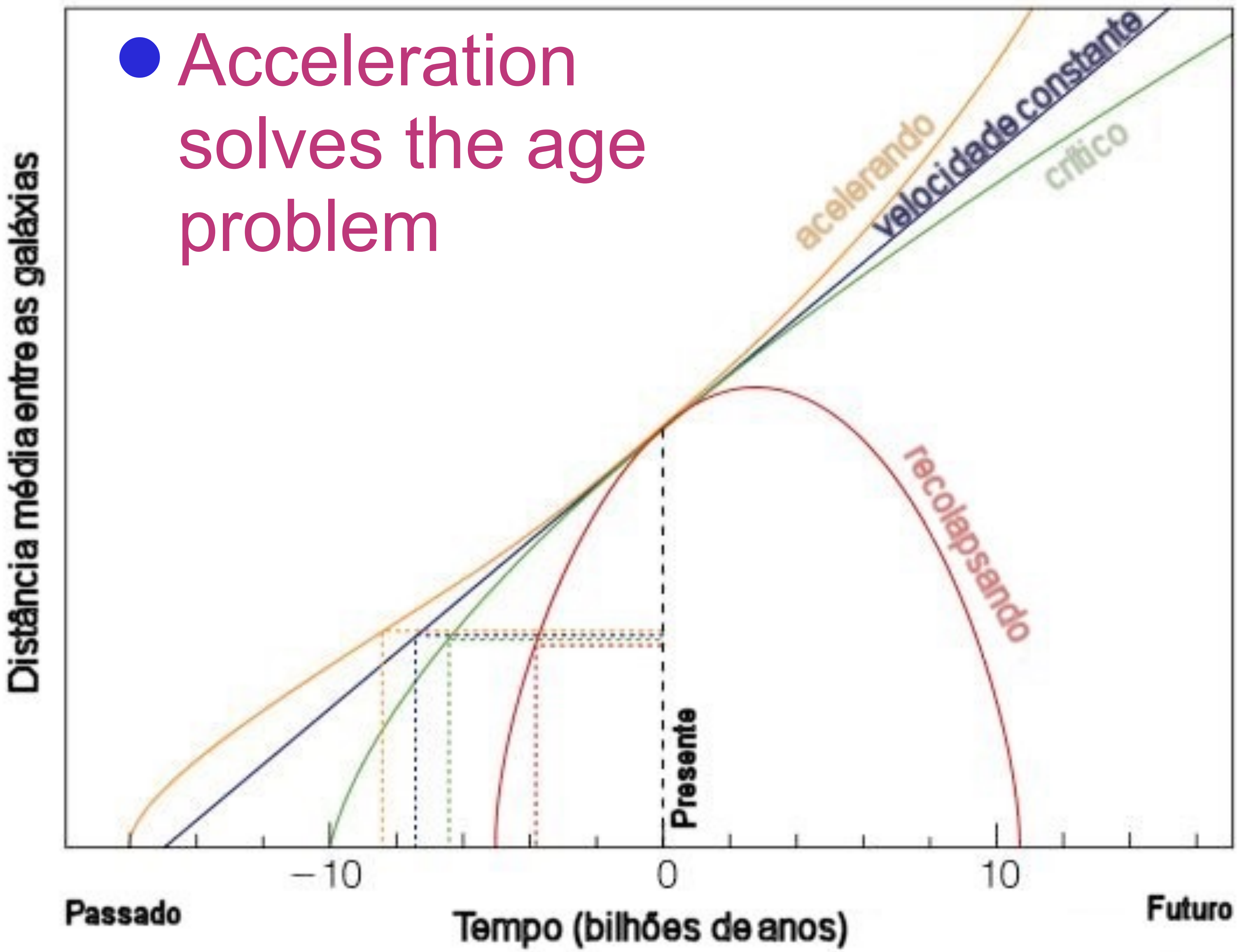


Figure: Kepler Oliveira, Maria de Fátima Saraiva, Astronomia e Astrofísica, http://astro.if.ufrgs.br/

Dark Energy

2/3 of the energy density of the Universe are in the form of Dark Energy! (or $\Lambda \neq 0$)

Evidences:

- Accelerated expansion of distant galaxies
- Age of the Universe
- Small curvature
- Integrated Sachs-Wolfe effect
- Combined analyses of cosmological observables (cosmic concordance)

Candidates (Taxonomy of Dark Energy):

- Cosmological constant
- Scalar field:
  - Quintessence
  - Quartessence, k-essence, spintessencia, snot...

Modified gravity

Lado oscuro del Universo I: materia oscura

Evidencias y propiedades:

- Nuestra própia galaxia
- Curvas de rotación de galaxias
- Movimientos de galaxias y cúmulos (virial y gran escala)
- Flujos de raios-X em cúmulos
- Lentes gravitacionales (veremos…)
- Efecto Sunyaev-Zel'dovich, Radciación cósmica, etc. etc.

Hay ~5x más *materia oscura* que materia bariónica!

La materia oscura es mas suavemente distribuída que la visible (relacionado a sua **física**)

Física: generación, pequeñas escalas, detección