

LENTES GRAVITACIONALES EN ASTROFÍSICA Y COSMOLOGÍA

SEMANA - 9

PARTE II: LENTES POR GALÁXIAS Y CÚMULOS DE GALÁXIAS

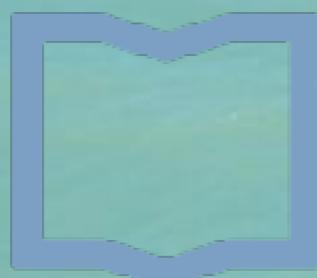
MARTÍN MAKLER

ICAS/IFICI/CONICET & UNSAM Y CBPF

ICAS

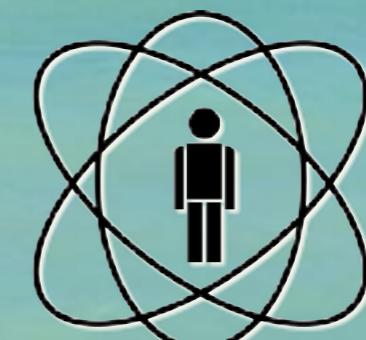


CONICET



Instituto de
Ciencias Físicas

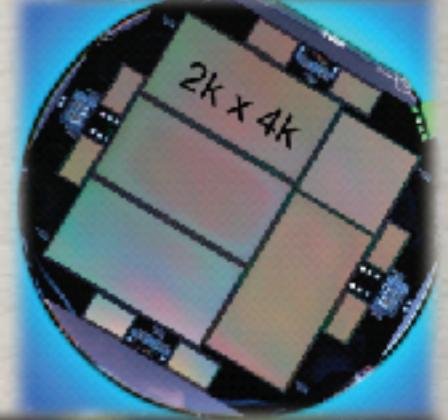
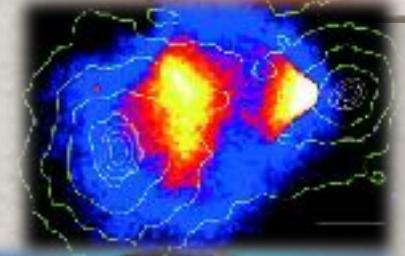
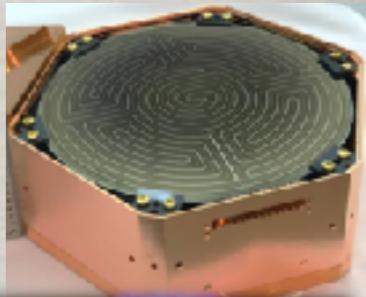
ICIFI-ECYT_UNSAM-CONICET

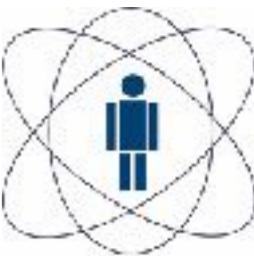


CBPF

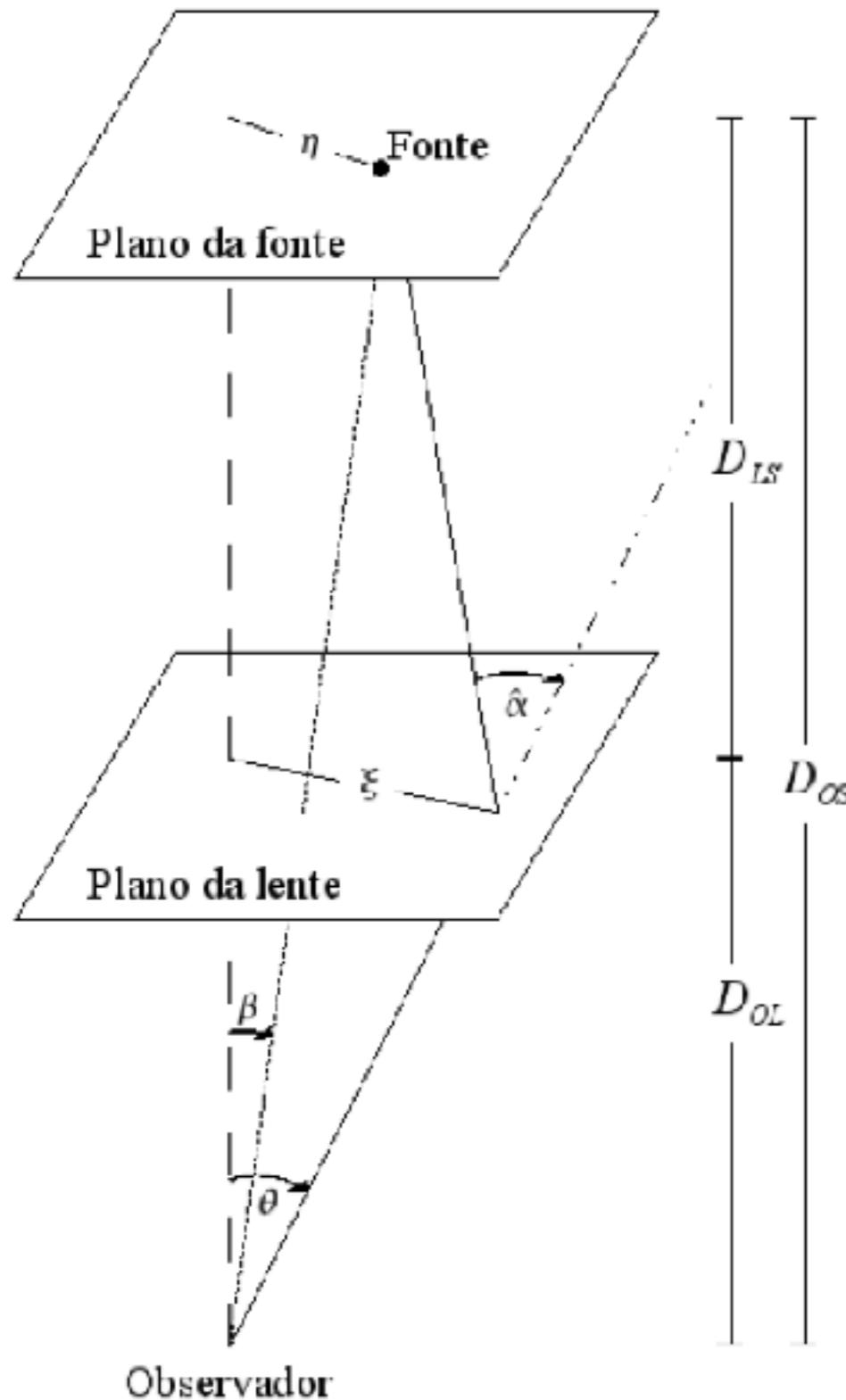
PLAN DE LA PARTE II

- Redshift y expansión del Universo
- Dinámica y parámetros cosmológicos
- Métrica y distancias
- Energía oscura
- Propagación de la luz y ecuación de la lente
- Lentes extendidas
- Jacobiana de la transformación:
cáusticas y curvas críticas
- Modelos de lentes extendidas y aplicaciones
- Retraso temporal y aplicaciones
- Efecto débil de lentes





Summary



Lens equation $\vec{\beta} = \vec{\theta} - \hat{\vec{\alpha}}(\vec{\theta})$

$$= \vec{\theta} - \vec{\nabla}_{\theta} \Psi(\vec{\theta})$$

Lens potential $\Psi(\vec{x}) \equiv \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \psi$

Deflection angle

$$\hat{\vec{\alpha}} = \frac{4G}{c^2} \int d^2 \xi' \Sigma(\vec{\xi}') \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$$

**Multiple images, magnification,
Einstein rings, arcs
Examples: point mass, SIS**

Jacobiana de la transformación

$$\vec{\beta} = \vec{\theta} - \vec{\nabla}_{\theta} \Psi(\vec{\theta})$$

$$J_{ij} = \left(\frac{\partial \vec{\beta}}{\partial \vec{\theta}} \right)_{ij} = \delta_{ij} - \frac{\partial^2 \Psi}{\partial \theta_i \partial \theta_j}, \quad \text{definiendo } \Psi_{ij} = \frac{\partial^2 \Psi}{\partial \theta_i \partial \theta_j}$$

tenemos $\mathbf{J} = \begin{pmatrix} 1 - \Psi_{11} & -\Psi_{12} \\ -\Psi_{21} & 1 - \Psi_{22} \end{pmatrix}$

como $\nabla_{\theta}^2 \Psi = \Psi_{11} + \Psi_{22} = 2\kappa$ Recordar que $\kappa(\vec{\theta}) = \frac{\Sigma(\vec{\theta})}{\Sigma_{\text{crit}}}$

$$\mathbf{J} = (1 - \kappa)\mathbf{I} - \begin{pmatrix} \frac{1}{2}(\Psi_{11} - \Psi_{22}) & \Psi_{12} \\ \Psi_{21} & -\frac{1}{2}(\Psi_{11} - \Psi_{22}) \end{pmatrix}$$

Cizalladura

$$\mathbf{J} = (1 - \kappa)\mathbf{I} - \begin{pmatrix} \frac{1}{2}(\Psi_{11} - \Psi_{22}) & \Psi_{12} \\ \Psi_{21} & -\frac{1}{2}(\Psi_{11} - \Psi_{22}) \end{pmatrix}$$

Cizalladura:

$$\gamma_1(\vec{\theta}) = \frac{1}{2}(\Psi_{11} - \Psi_{22}) \quad \gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$$
$$\gamma_2(\vec{\theta}) = \Psi_{12} = \Psi_{21}$$

En términos de la cizalladura y la convergencia tenemos

$$\mathbf{J} = (1 - \kappa)\mathbf{I} + \begin{pmatrix} -\gamma_1 & -\gamma_2 \\ -\gamma_2 & \gamma_1 \end{pmatrix}$$

Autovalores de \mathbf{J}^{-1}

$$\mu_1 = \frac{1}{1 - \kappa - \gamma}, \quad \mu_2 = \frac{1}{1 - \kappa + \gamma}$$

A wide-angle photograph of a tropical beach. The foreground is a light tan sandy beach with some small debris. The middle ground is filled with the vibrant turquoise water of the ocean, with gentle waves breaking near the shore. The background is a clear, pale blue sky meeting the horizon.

MODELOS DE LENTES

Dos caminos

Ecuación de la lente $\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}) = \vec{\theta} - \vec{\nabla}_{\theta}\Psi(\vec{\theta})$

→ Obtener $\vec{\alpha}(\vec{\theta})$ o $\Psi(\vec{\theta})$

→ Solución de la ecuación de la lente y jacobiana

$$\hat{\vec{\alpha}} = \frac{4G}{c^2} \int d^2\xi' \Sigma(\vec{\xi}') \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}.$$

$$\psi(\vec{\xi}) = 2G \int d^2\xi' \Sigma(\vec{\xi}') \ln |\xi - \xi'|$$

Camino I: Elegir $\Sigma(\vec{\xi}) \circ \rho(\vec{r})$

Camino II: Modelo para $\Psi(\vec{\theta})$

Modelos de potencial

Esfera isotérmica singular $\Psi = \theta_E |\vec{\theta}|$

Esfera isotérmica com “caroço” $\Psi = \theta_0 \sqrt{\theta^2 + \theta_c^2}$

Convergência $\kappa = \theta_0 \frac{\theta^2 + 2\theta_c^2}{2(\theta^2 + \theta_c^2)^{3/2}}$

Perfil de densidade

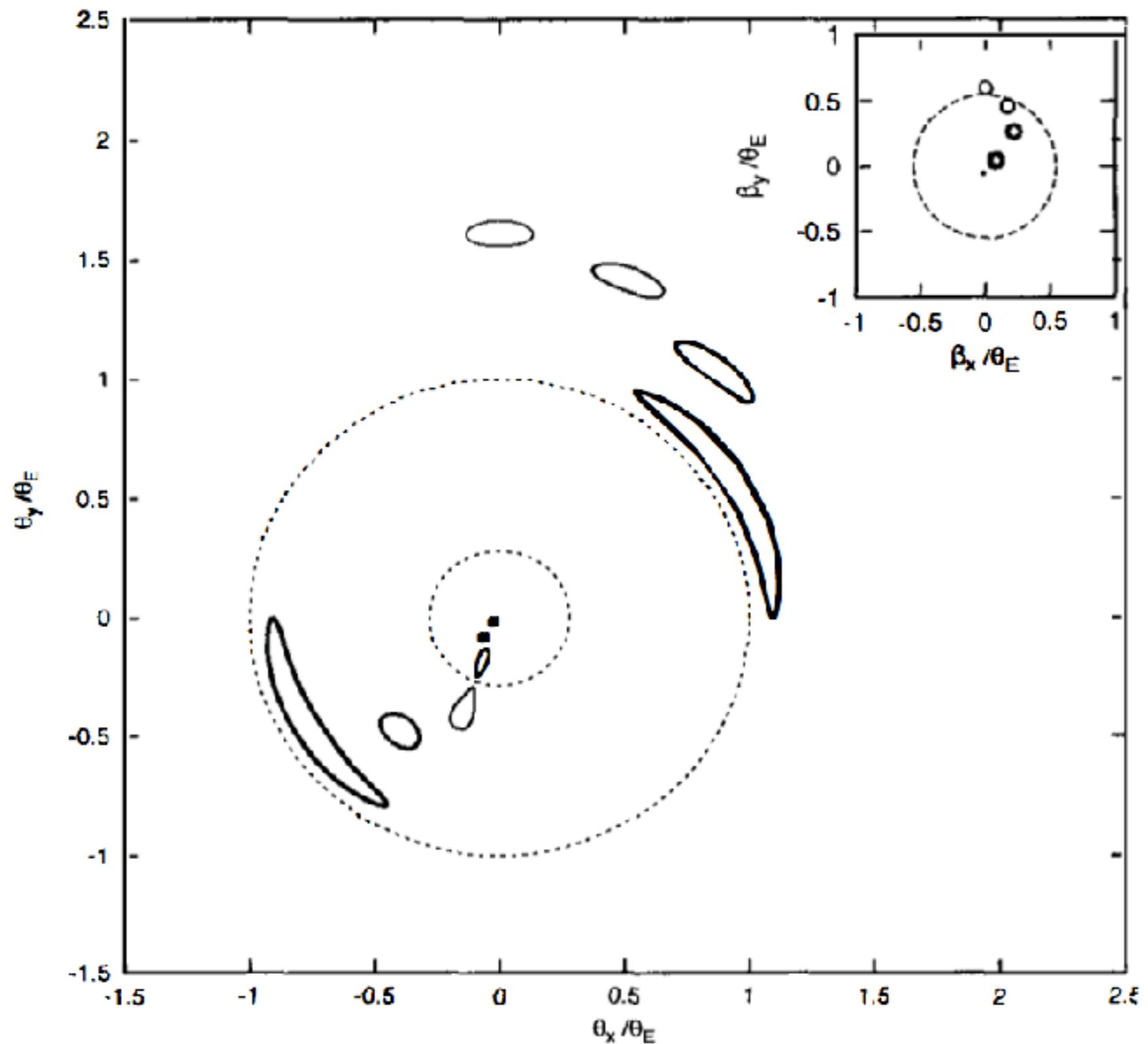
Esfera isotérmica suavizada $\rho(r) = \frac{\sigma^2}{2\pi G(r^2 + r_c^2)}$

Densidade projetada $\Sigma(\theta) = \frac{\sigma^2}{2GD_{OL}} \frac{1}{\sqrt{\theta^2 + \theta_c^2}}$

Potencial de lente

$$\Psi(\vec{\theta}) = \theta_0 \left[\sqrt{\theta^2 + \theta_c^2} - \theta_c \ln \left(\sqrt{\theta^2 + \theta_c^2} + \theta_c \right) \right]$$

Modelo axial não singular



Modelo Navarro-Frenk-White

Simulaciones computacionales de *N*-cuerpos en un contexto cosmológico

Densidad media radial: $\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$

Universalidad de los perfiles de materia oscura!

El perfil de Navarro-Frenk-White



https://wwwmpa.mpa-garching.mpg.de/~swhite/pictures/NFW_profile1.jpg

LOS SIMULADORES



Modelo Navarro-Frenk-White

Densidade média radial: $\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$

Convergência:

$$\kappa(x) = 2\kappa_s \frac{1 - F(x)}{x^2 - 1}, \quad x = \frac{\xi}{r_s} \quad \text{e} \quad k_s = \frac{\rho_s r_s}{\Sigma_{cr}}$$

$$F(x) = \begin{cases} \frac{\text{ArcTan}(\sqrt{x^2-1})}{\sqrt{x^2-1}} & , \quad x > 1 \\ 1 & , \quad x = 1 \\ \frac{\text{ArcTanh}(\sqrt{1-x^2})}{\sqrt{1-x^2}} & , \quad x < 1 \end{cases}$$

Potencial de lente:

$$\Psi(x) = 2\kappa_s r_s^2 \left[\ln^2 \frac{x}{2} - \text{ArcTanh}^2 \sqrt{1-x^2} \right]$$

Potencial mais suave que o da Esfera Isotérmica:

curvas críticas menores, maior amplificação, menor separação angular

Other lens models

Implemented in *gravlens* and several public codes (+ Sérsic, Einasto, etc.)

Model	N_r	Density $\rho(r)$	Surface Density $\kappa(r)$
Point mass	0	$\delta(\mathbf{x})$	$\delta(\mathbf{x})$
Power law or α -models	2	$(s^2 + r^2)^{(\alpha-3)/2}$	$(s^2 + r^2)^{(\alpha-2)/2}$
Isothermal ($\alpha = 1$)	1	$(s^2 + r^2)^{-1}$	$(s^2 + r^2)^{-1/2}$
$\alpha = -1$	1	$(s^2 + r^2)^{-2}$	$(s^2 + r^2)^{-3/2}$
Pseudo-Jaffe	2	$(s^2 + r^2)^{-1} (a^2 + r^2)^{-1}$	$(s^2 + r^2)^{-1/2} - (a^2 + r^2)^{-1/2}$
King (approximate)	1	...	$2.12 (0.75r_s^2 + r^2)^{-1/2}$ $-1.75 (2.99r_s^2 + r^2)^{-1/2}$
de Vaucouleurs	1	...	$\exp [-7.67(r/R_e)^{1/4}]$
Hernquist	1	$r^{-1} (r_s + r)^{-3}$	see eq. (47)
NFW	1	$r^{-1} (r_s + r)^{-2}$	see eq. (53)
Cuspy NFW	2	$r^{-\gamma} (r_s + r)^{\gamma-3}$	see eq. (57)
Cusp	3	$r^{-\gamma} (r_s^2 + r^2)^{(\gamma-n)/2}$	see eq. (64)
Nuker	4	...	see eq. (71)
Exponential disk	1	...	$\exp[-r/R_d]$
Kuzmin disk	1	...	$(r_s^2 + r^2)^{-3/2}$

The background of the slide is a photograph of a tropical beach. The water is a vibrant turquoise color, transitioning to a darker shade of blue towards the horizon. The sky above is a clear, pale blue with no visible clouds. The sandy beach in the foreground is light tan and appears relatively clean.

MODELOS ELÍPTICOS Y PSEUDO-ELÍPTICOS

Modelos elípticos

Sustituir $\theta \rightarrow \sqrt{q_1 \theta_1^2 + q_2 \theta_2^2}$

- en el potencial (modelos pseudo-elípticos)
- en la densidad proyectada (modelos elípticos)

Ejemplo: Esfera isotérmica com core
pseudo-elíptico

$$\Psi(\theta_1, \theta_2) = \frac{\Psi_0}{\theta_c} \sqrt{(1 - \epsilon)\theta_1^2 + (1 + \epsilon)\theta_2^2 + \theta_c^2}$$

Elipticidad en la densidad proyectada

Ejemplo: Esfera isotermica eliptica con core

$$\Sigma(\theta_1, \theta_2) = \frac{\Sigma_0 \theta_c}{\sqrt{(1 - \epsilon)\theta_1^2 + (1 + \epsilon)\theta_2^2 + \theta_c^2}}$$

Solución para modelos elípticos

$$\phi(x, y) = \frac{q}{2} I(x, y)$$

$$\phi_{,x}(x, y) = q x J_0(x, y)$$

$$\phi_{,y}(x, y) = q y J_1(x, y)$$

$$\phi_{,xx}(x, y) = 2 q x^2 K_0(x, y) + q J_0(x, y)$$

$$\phi_{,yy}(x, y) = 2 q y^2 K_2(x, y) + q J_1(x, y)$$

$$\phi_{,xy}(x, y) = 2 q x y K_1(x, y)$$

$$I(x, y) = \int_0^1 \frac{\xi}{u} \frac{\phi_{,r}(\xi(u))}{[1 - (1 - q^2)u]^{1/2}} du$$

$$J_n(x, y) = \int_0^1 \frac{\kappa(\xi(u)^2)}{[1 - (1 - q^2)u]^{n+1/2}} du$$

$$K_n(x, y) = \int_0^1 \frac{u \kappa'(\xi(u)^2)}{[1 - (1 - q^2)u]^{n+1/2}} du$$

$$\xi(u)^2 = u \left(x^2 + \frac{y^2}{1 - (1 - q^2)u} \right)$$

Campos externos

Expansión hasta segundo orden del potencial

$$\Psi(\theta_1, \theta_2) = \frac{\kappa}{2} (\theta_1^2 + \theta_2^2) + \frac{\gamma_1}{2} (\theta_1^2 - \theta_2^2) + \gamma_2 \theta_1 \theta_2$$

Model for the source

Surface brightness distribution of the source

Example: Sérsic profile

$$I(R) = I_0 \exp \left\{ -b_n \left(\frac{R}{R_e} \right)^{1/n} \right\}$$

Elliptical brightness distribution
source centered at (S_1, S_2)

$$\begin{aligned} R^2 &= (1 - \varepsilon_S)[(\beta_1 - S_1) \cos \phi_e + (\beta_2 - S_2) \sin \phi_e]^2 \\ &\quad + (1 + \varepsilon_S)[(\beta_2 - S_2) \cos \phi_e - (\beta_1 - S_1) \sin \phi_e]^2 \end{aligned}$$

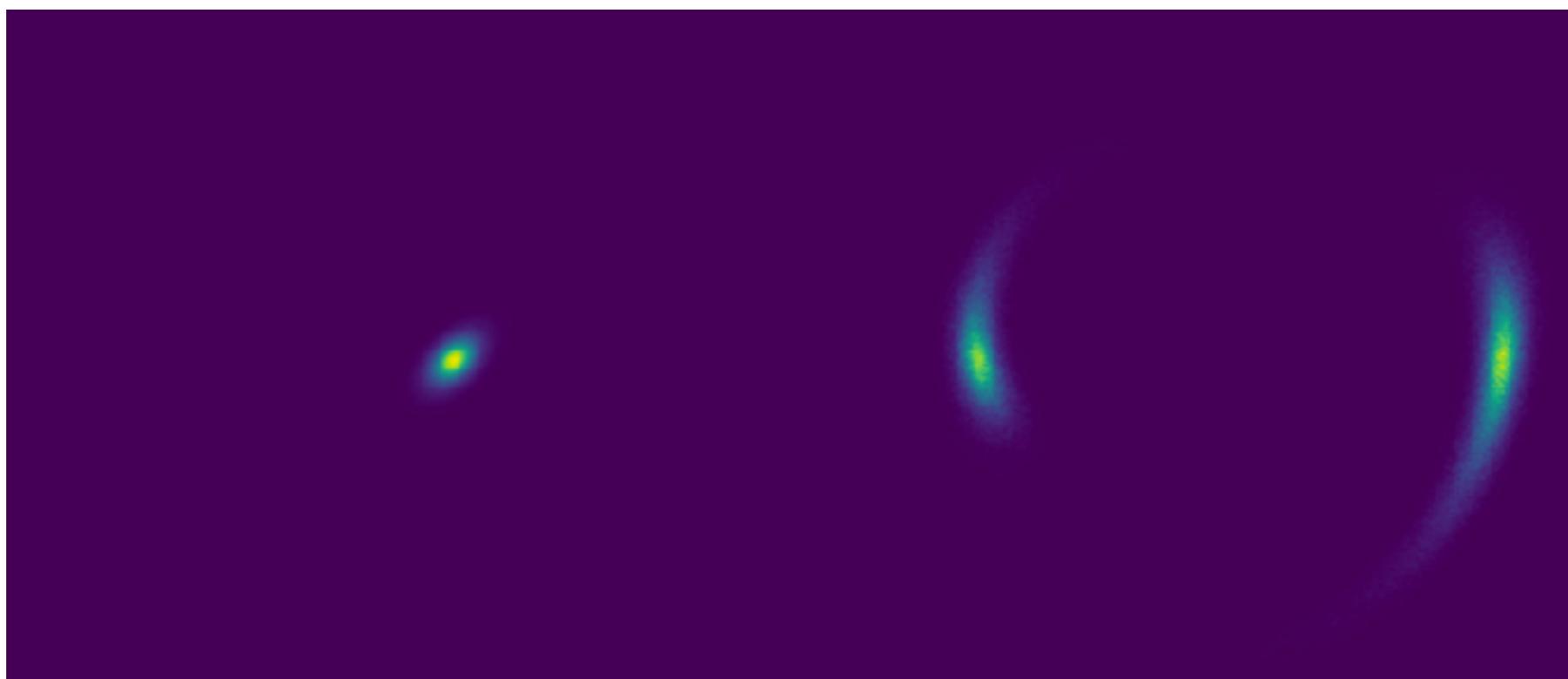
Simulating Strong Lensing Images

Map the brightness distribution of the source to the lens plane

$$I(R) = I(\vec{\beta}) = I(\vec{\beta}(\vec{\theta}))$$

Use the lens equation (no need to solve it!)

Add PSF and noise



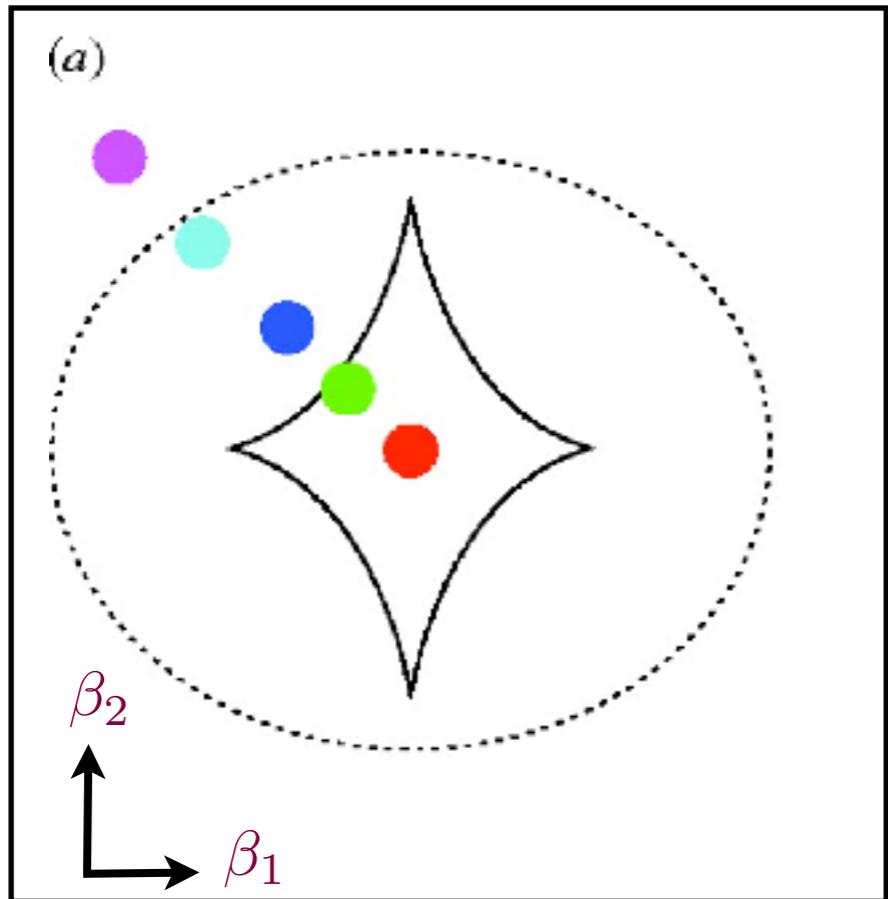
AddArcs v2.0

Actividad Práctica 7

Lensing Mapping

▶ image → source mapping:

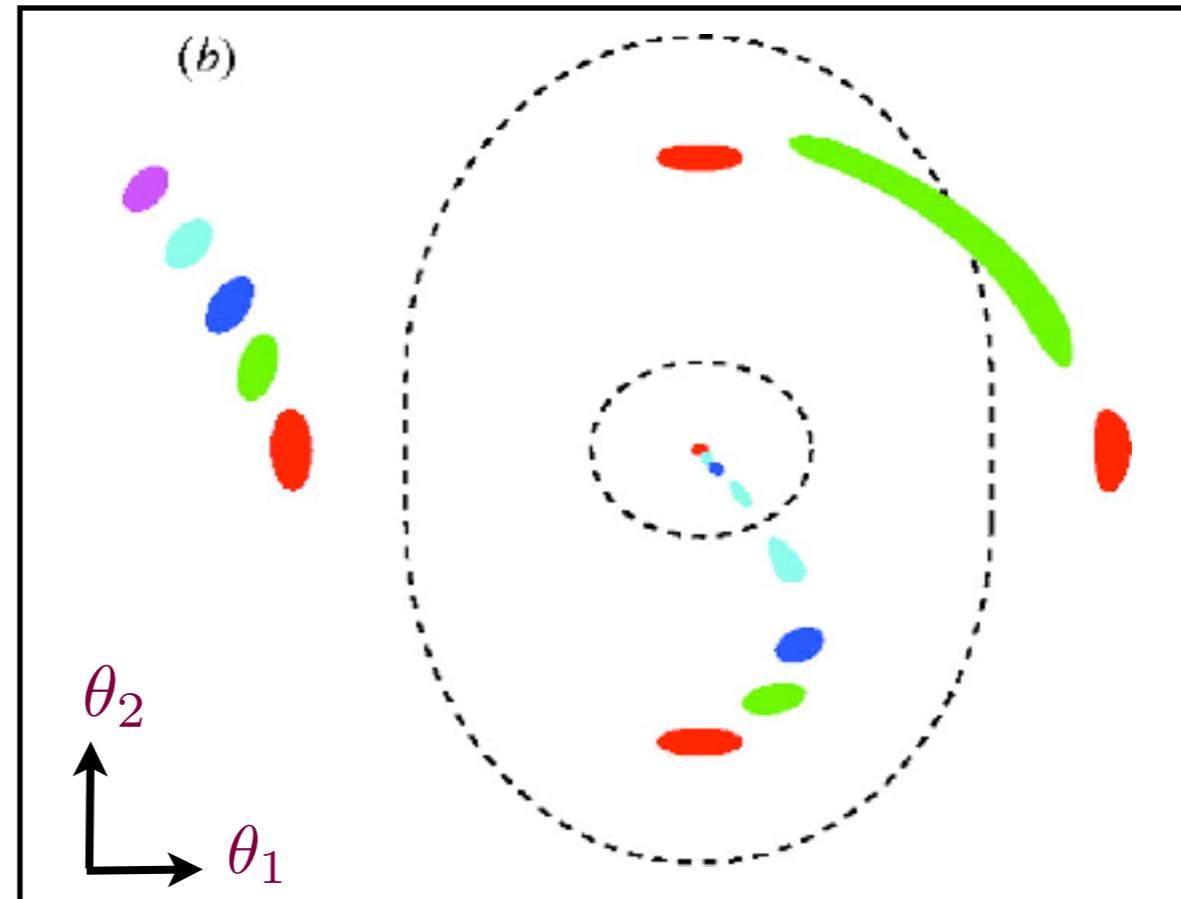
$$\frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}$$



Source plane

▶ eigenvalues:

$$\mu_1 = \frac{1}{1-\kappa+\gamma}, \mu_2 = \frac{1}{1-\kappa-\gamma}$$



Lens/image

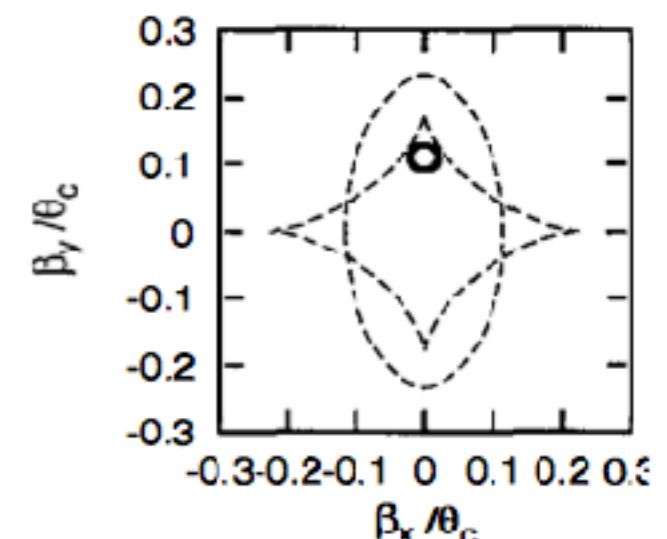
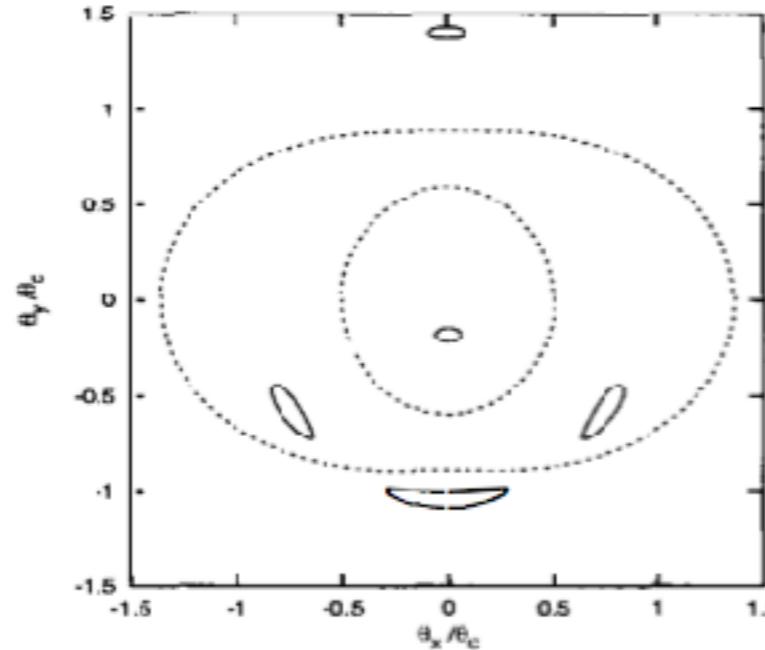
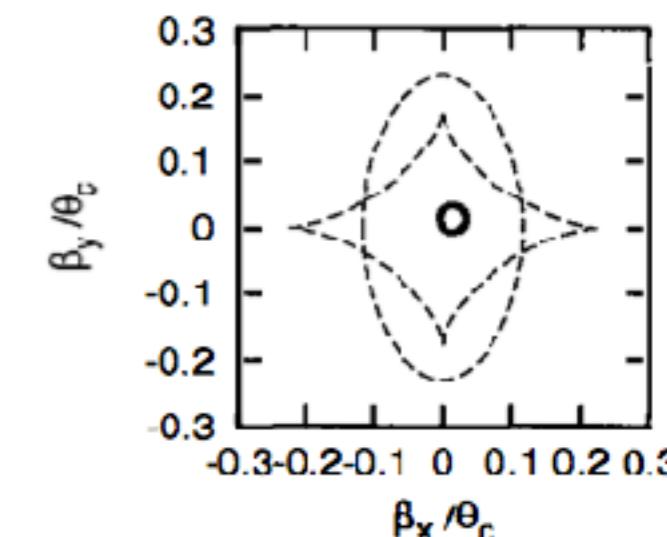
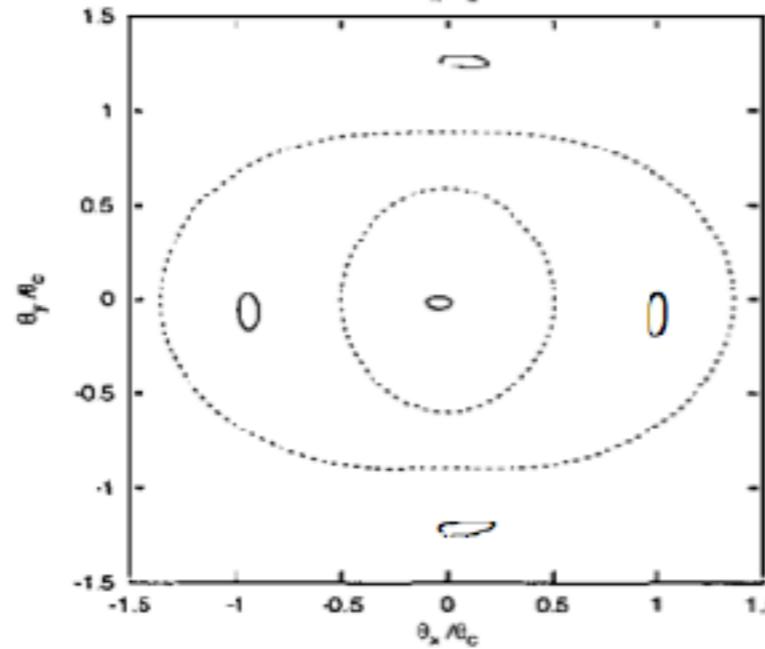
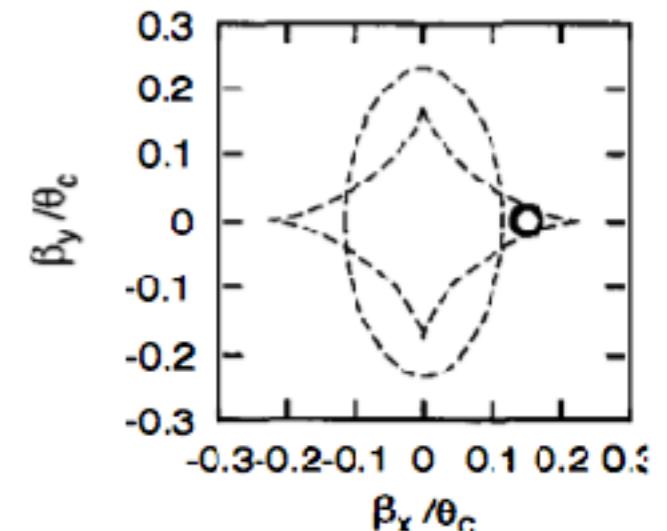
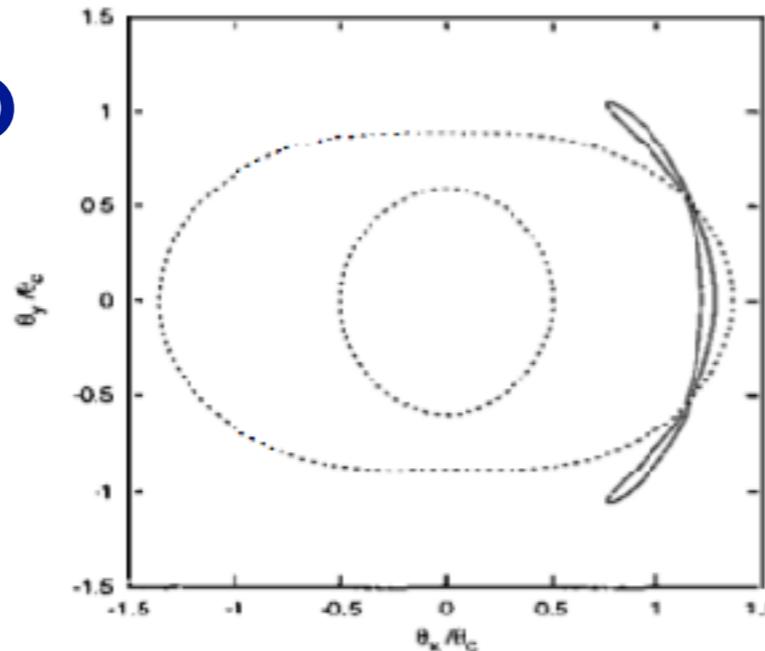
Comportamento geral

Cáusticas:

- **tangencial:** astróides com cúspides
- **radial:** “oval”

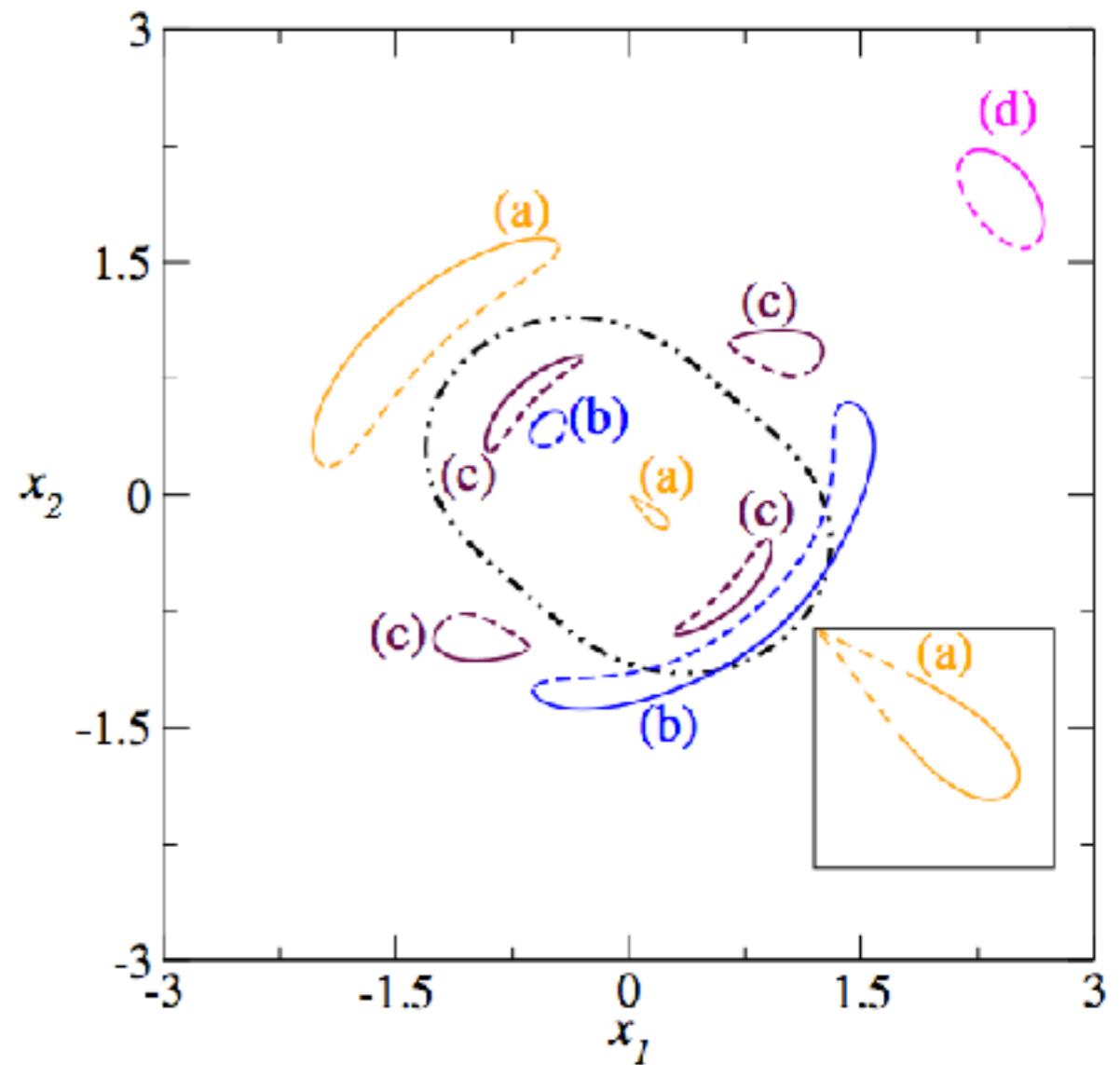
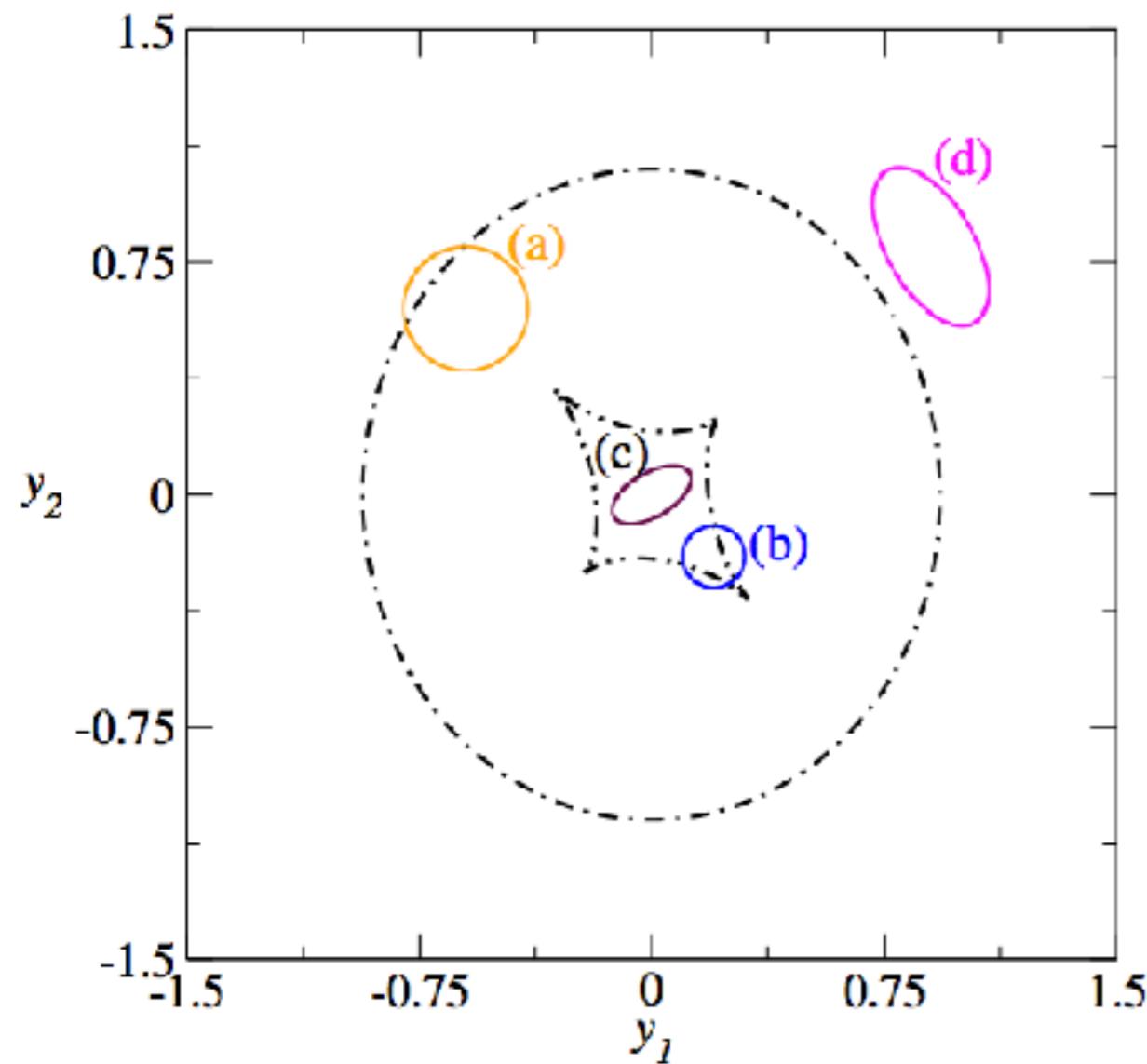
Curvas críticas:

- **tangencial:** “oval”
- **radial:** “circular”



Arcos em modelos elípticos

Soluções analíticas para esfera isotérmica elíptica e pseudo-elíptica com campos externos e fontes elípticas



The background of the slide is a photograph of a tropical beach. The water is a vibrant turquoise color, transitioning to a darker shade of blue towards the horizon. The sky above is a clear, pale blue with no visible clouds. The sandy beach in the foreground is light tan and appears relatively clean.

RECONSTRUCCIÓN DE LA DISTRIBUCIÓN DE MATERIA EN LA LENTE

Inverse Modeling

Use systems of multiple images to determine the lensing potential

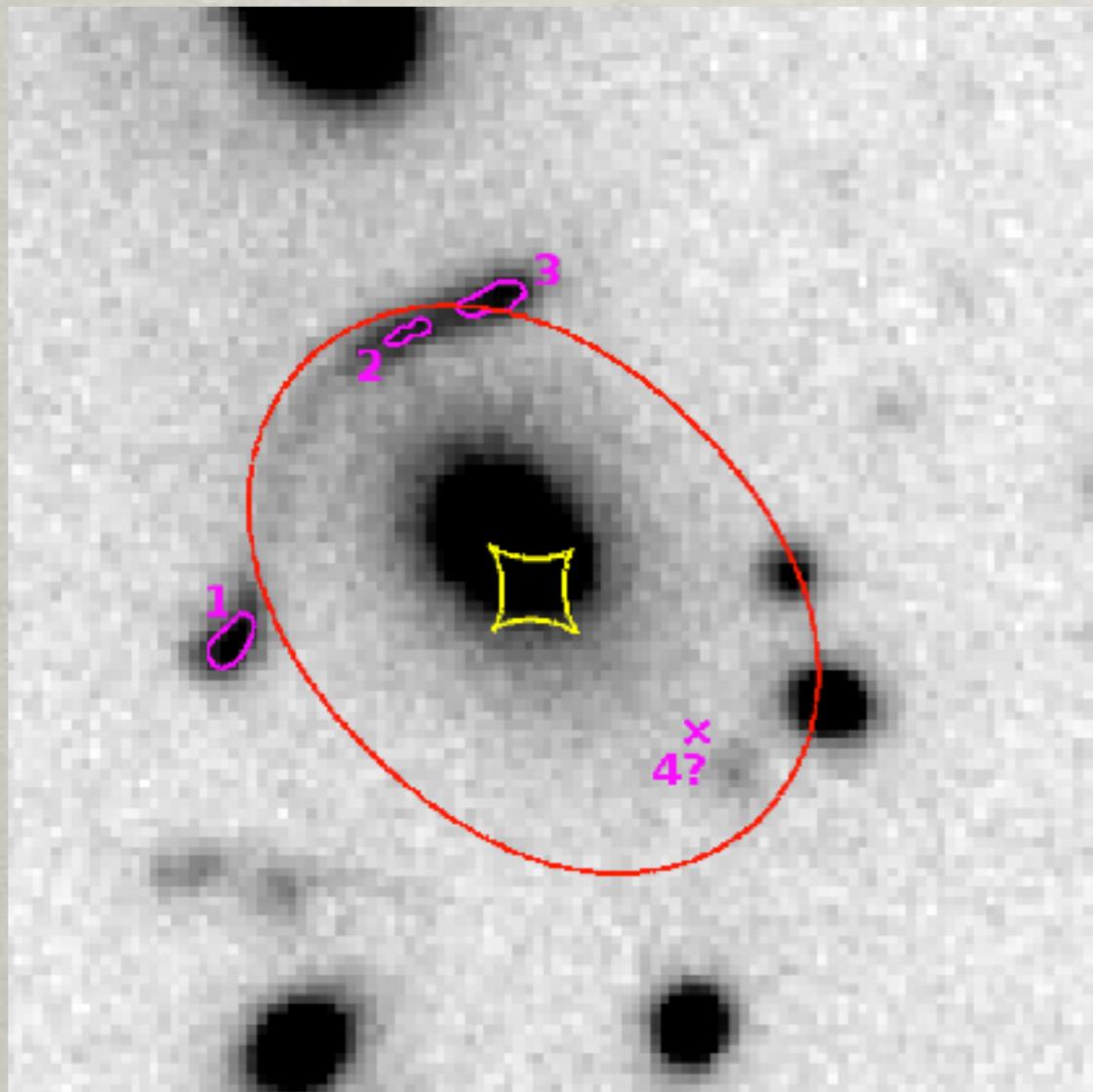
$$\chi_{\text{lente}}^2 := \sum_i \left(\frac{\vec{\theta}_i^{\text{obs}} - \vec{\theta}^{\text{mod}}(\vec{\beta}, \vec{\Pi})}{\sigma_i^{\text{obs}}} \right)^2$$

Position of the multiple images Uncertainty on the image positions

Available codes: *lenstool*, *gravlens*, *glafic*, etc. + “home-made”

- Parameters that minimize this function (or maximize the likelihood) are the best fitting lens model
- Combination with independent mass constraints (e.g., x-ray, Sunyaev Zel'dovich, velocity dispersions) yields limits on cosmology or gravity

INVERSE MODELING: MAPPING THE MASS



Use systems of multiple images to determine the lensing potential

$$\chi^2_{\text{lente}} := \sum_i \left(\frac{\vec{\theta}_i^{\text{obs}} - \vec{\theta}^{\text{mod}}(\vec{\beta}, \vec{\Pi})}{\sigma_i^{\text{obs}}} \right)^2$$

Multiple image positions

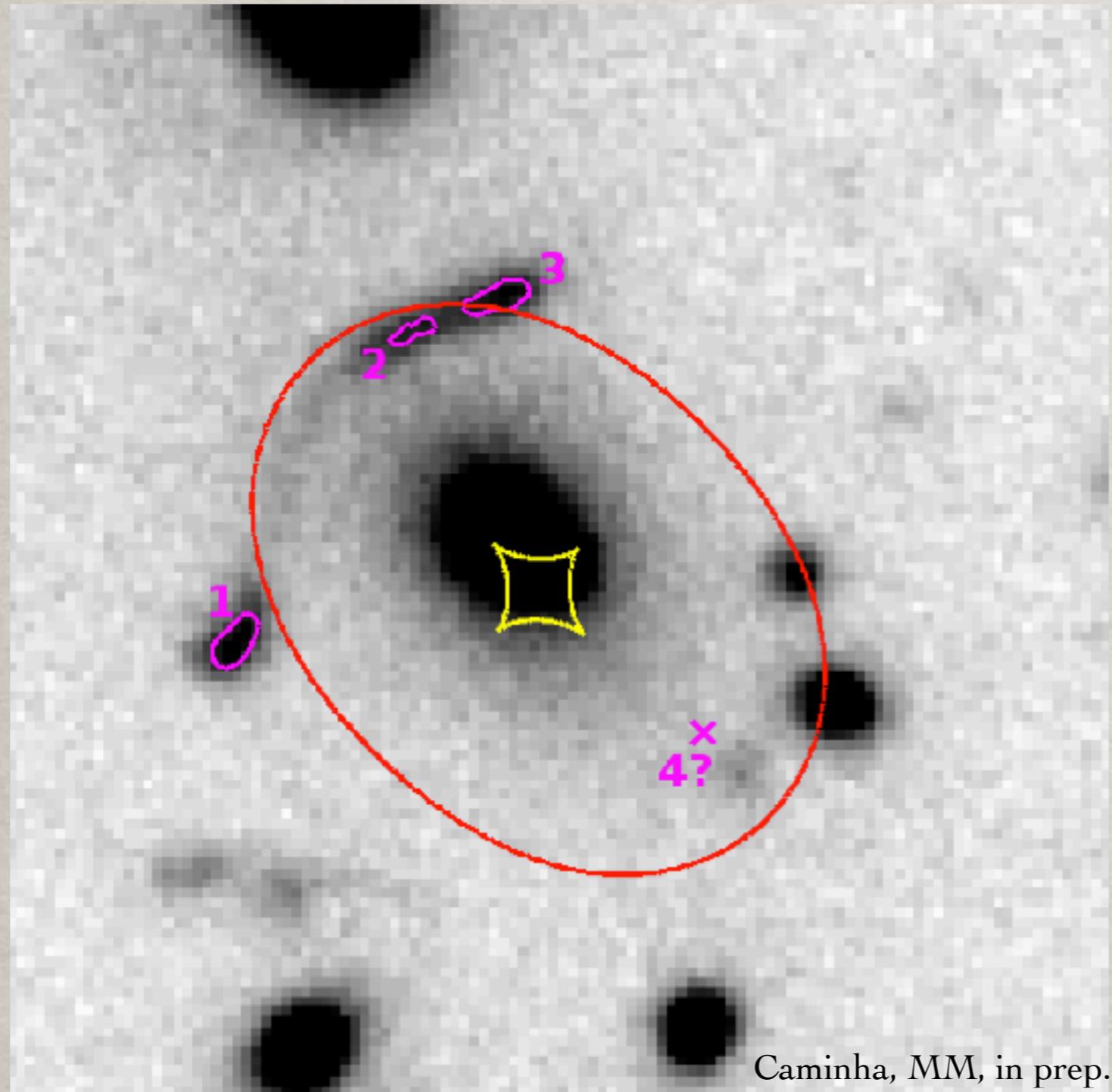
Error on image positions

Methods: parametric (often “mass traces light”), free form

The more multiple images, the more constraints
Cluster x Galaxy scales

- Combination with independent mass constraints (e.g., x-ray, Sunyaev Zel'dovich, velocity dispersions) yields limits on cosmology or gravity

INVERSE MODELING FOR SYSTEM SOGRASO04 1-OO43



$$2.35_{-0.14}^{+0.03} \times 10^{14} M_{\odot}$$

Modelling with lenstool
(Jullo, Kneib)

Fit 1: 3 images

Fit 2: 4 images

	Fit 1	Fit 2
$\sigma_v [km/s]$	622_{-13}^{+11}	642_{-3}^{+3}
$\theta_{\text{or}} [\circ]$	$135.2_{-0.8}^{+0.7}$	$135.2_{-1.3}^{+1.5}$
$x_{\text{lente}} ["]$	$0.50_{-0.2}^{+0.2}$	$0.50_{-0.06}^{+0.05}$
$y_{\text{lente}} ["]$	$-0.76_{-0.15}^{+0.17}$	$-0.86_{-0.03}^{+0.06}$
ε	—	$0.13_{-0.03}^{+0.02}$

~ 0.9" displacement between central galaxy and center of mass distribution
(Zitrin et al. 2012)

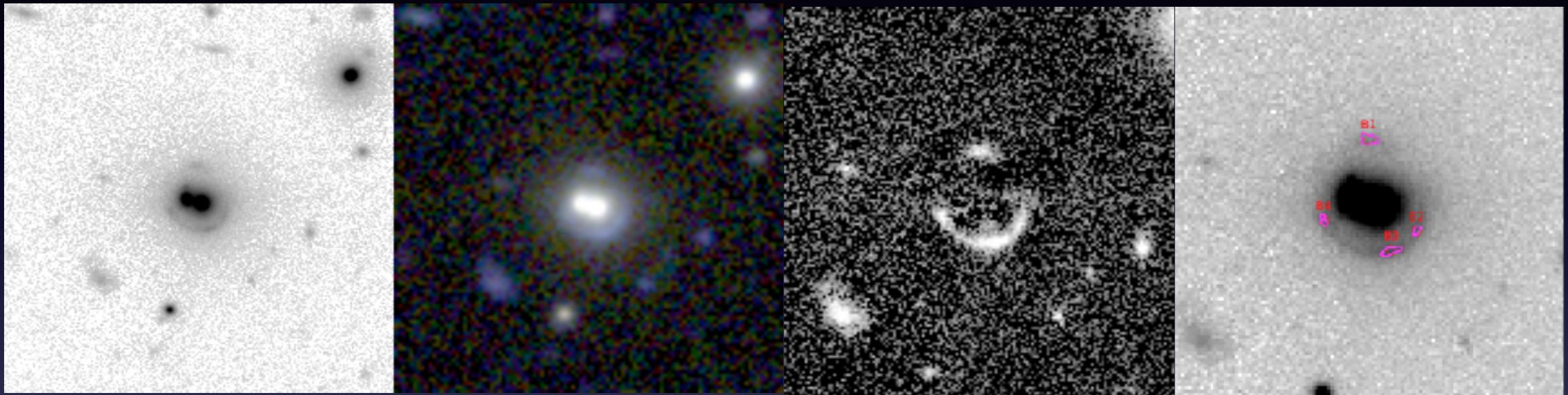
Error estimate from simulations
(Caminha et al. in prep.):

~ 8% bias in mass

~ 5% statistical errors

Modeling the full light distribution of the images

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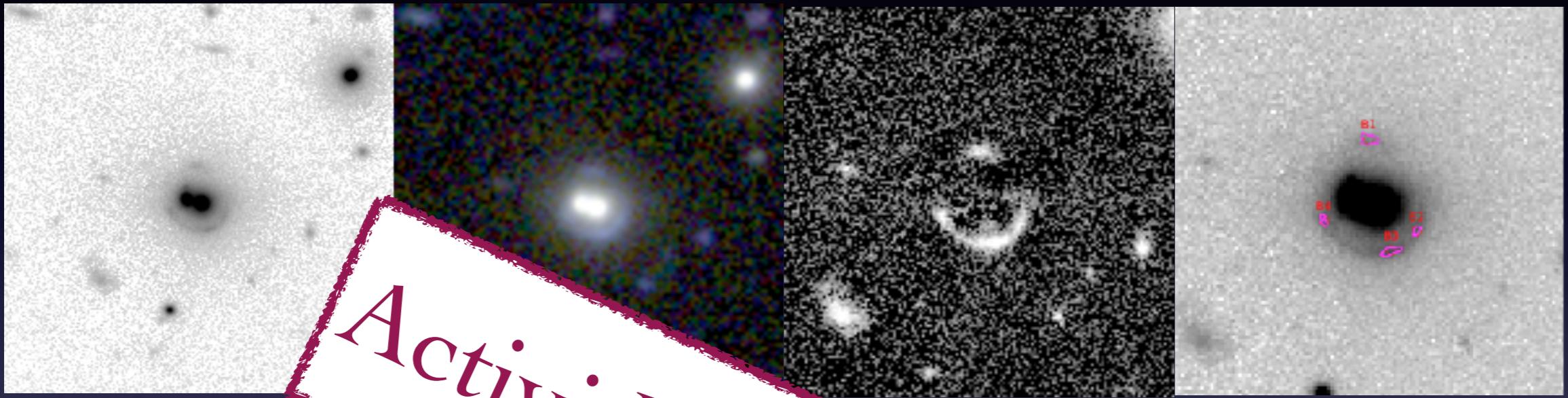


Anna Niemiec

- Instead of peak/multiple images, use the full information of the images
- Allows one to reconstruct source properties (often parametrically)
- Remove contamination from lens galaxy (with `galfit`) and mask other objects

Modeling the full light distribution of the images

CS82SL01:36:39+00:08:18



Anna Niemiec

- Instead of peaking at the central point, use the full information of the image to model the light distribution
- Allows one to reconstruct source properties (often parametrically)
- Remove contamination from lens galaxy (with `galfit`) and mask other objects

Actividad Práctica 7

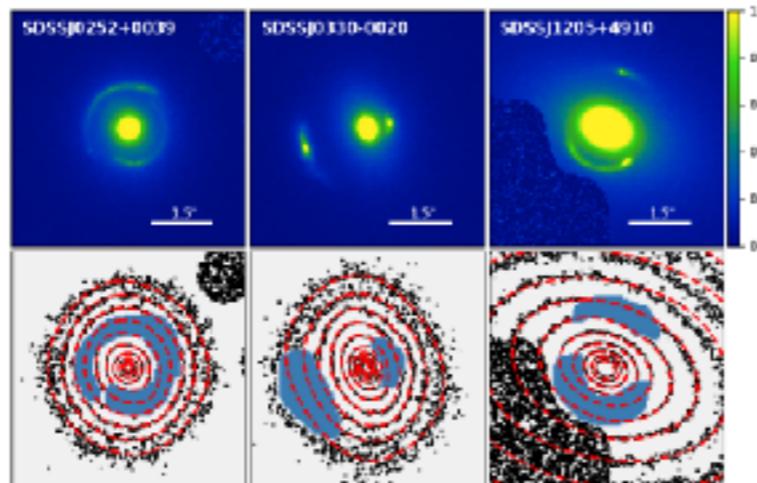
Inverse modeling with extended sources

Example code: PyAutoLens

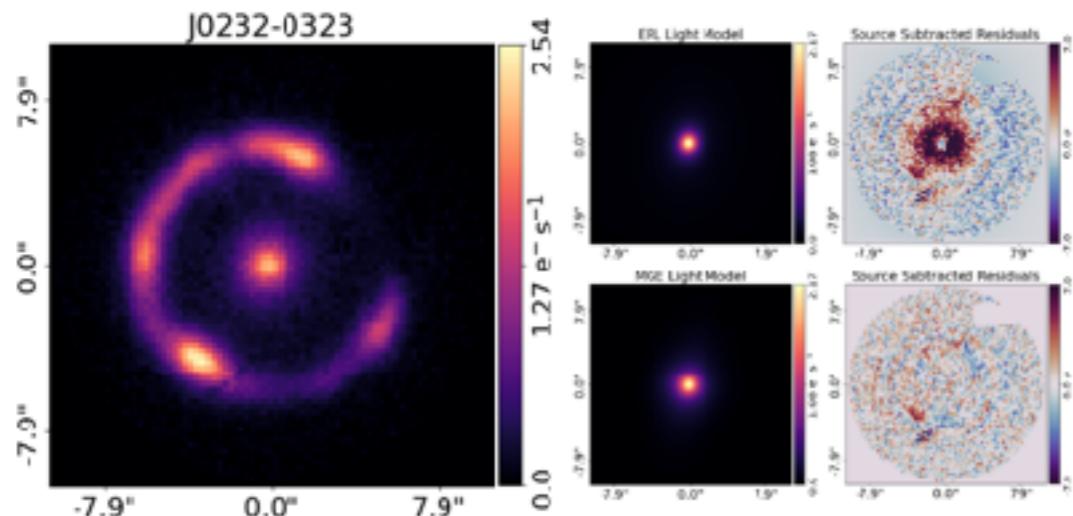
Parametric light distribution for lenses

$$I_T(R, \theta) = \sum_{j=1}^N I_j \cdot \text{Exp}\left(-\frac{R_i^2(x, y)}{2\sigma_i^2}\right)$$

$$R_i(x, y) = \sqrt{x'^2 + \left(\frac{y'}{q_i}\right)^2}$$



MGE vs EPL on Lens light fitting on ground-based image modeling



Elliptical Power-Law (EPL) projected mass density

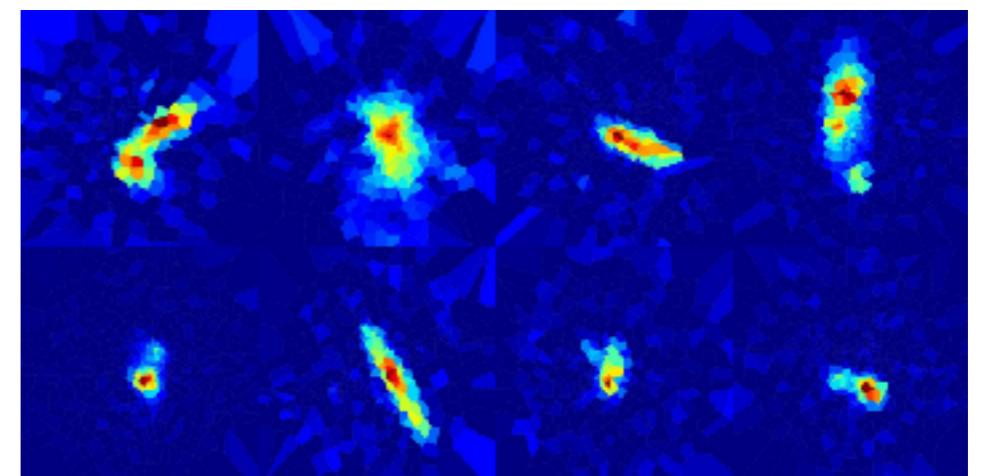
$$k_\alpha(x, y) = \frac{\Sigma(\xi)}{\Sigma_{\text{crit}}} = \frac{3 - \alpha}{1 + q} \left(\frac{b}{\xi}\right)^{\alpha - 1}$$

External shear

$$\gamma_1 = \frac{1}{2} \left(\frac{\partial^2 \psi}{\partial \theta_1^2} - \frac{\partial^2 \psi}{\partial \theta_2^2} \right)$$

$$\gamma_2 = \frac{\partial^2 \psi}{\partial \theta_1 \partial \theta_2}$$

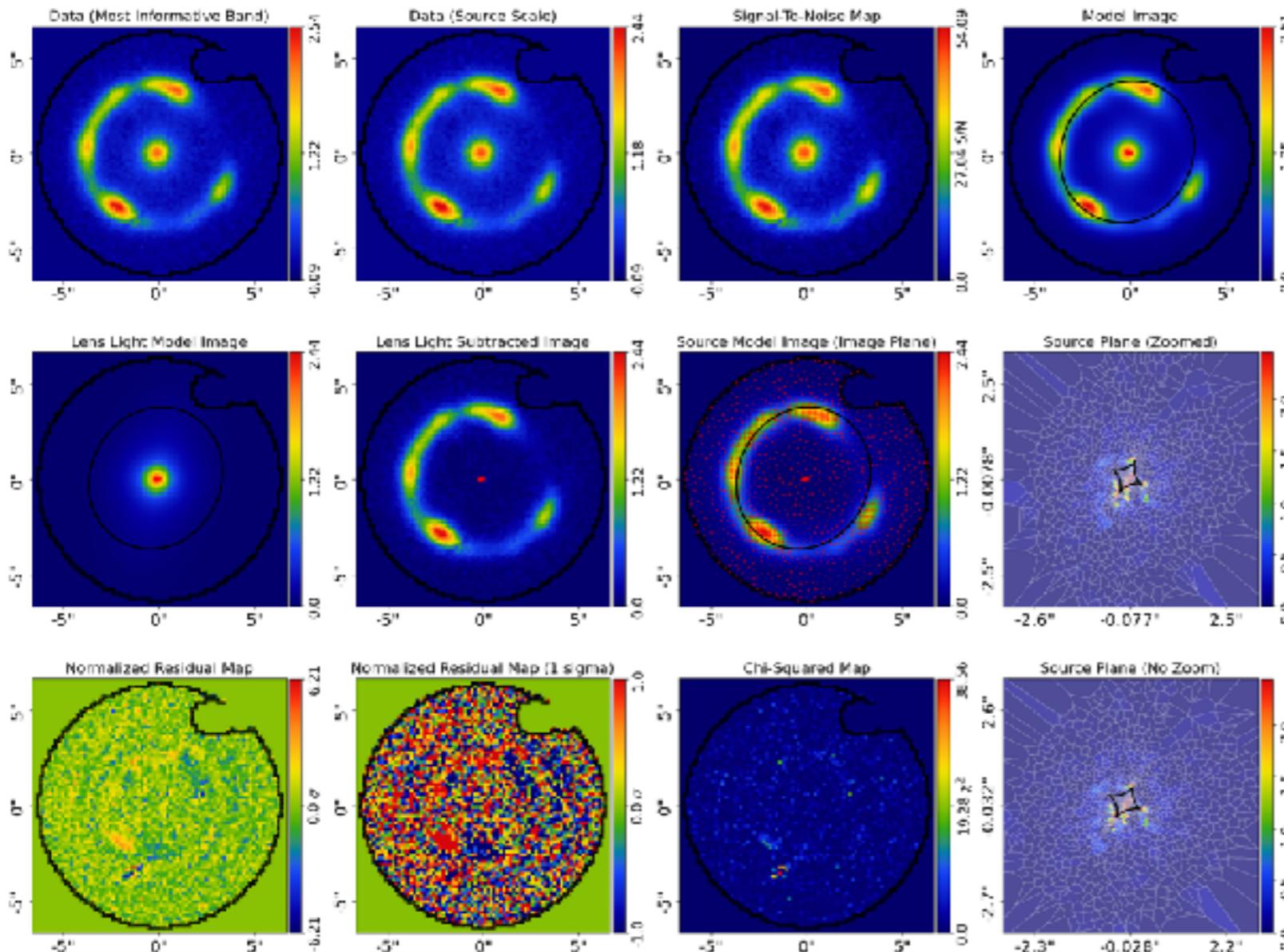
Total mass model is the sum of the two
Voronoi adaptive pixelization for sources



Galan+; arxiv:2012.02802

Source Lens and Mass (SLaM) pipelines

with PyAutoLens



Session	Phase	Component	Model	Prior Info
Source parametric	SP	Lens mass	SIE+Shear	-
		Lens light	MGE	-
		Source light	MGE	-
Source Inversion	SI1	Lens mass	SIE+Shear	SP
		Lens light	MGE	SP
		Source light	MPR	-
Light Parametric	SI2	Lens mass	SIE+Shear	SI1
		Lens light	MGE	SP
		Source light	BPR	-
Mass Total	LP	Lens mass	SIE+Shear	SI1
		Lens light	MGE	SP
		Source light	BPR	SI2
	MT	Lens mass	EPL+Shear	SI1
		Lens light	MGE	SP
		Source light	BPR	SI2

REGIMES AND CHALLENGES

● Galaxy cluster scale

- Complex mass models, but many families of multiple images
- HST Data: Frontier Fields, CLASH, RELICS, etc.
- + Massive spectroscopic follow-up (IFU/MUSE)
- Cosmology from single systems + high- z sources
- Limited to very massive lenses and few systems
- Promising for particle DM properties!

● Galaxy scale systems

- Simple mass modes, but few constraints
- Also HST imaging, but few IFU data (e.g. Gemini/NIFS)
- Promising for Modified Gravity!

● Statistics

- Spin-off from cosmological surveys: $\mathcal{O}(10^{1-2}) \rightarrow \mathcal{O}(10^{3-5})$
- Need to find these systems!
 - Automated arc finders
- Understand selection function.
- Use simulations
- Need to automate analyses!

● Challenge: Systematics

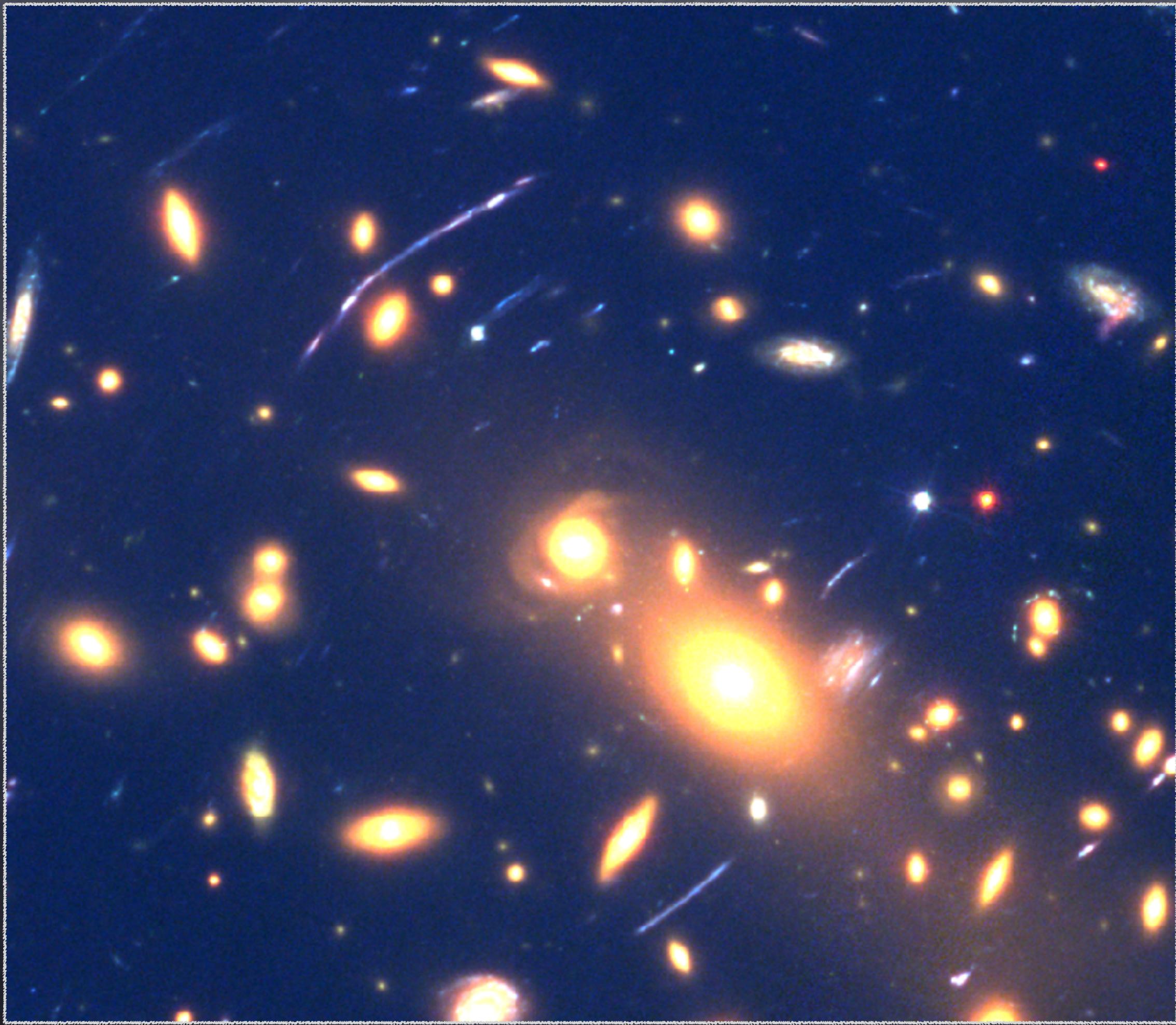
- Use simulations → end-to-end approach
- Test modeling at the data level.
Independent tests: predictions × SN!
- Improve models from data: LSS/los, IFU, etc.

Cúmulos de galaxias



Arcos Gravitacionales





THE HST FRONTIER FIELDS

THE DEEPEST DATA EVER OBTAINED FOR LENSING GALAXY CLUSTERS !!!

Abell 2744 - z = 0.308

Fully observed

Atek et al. 2014a, *ApJ*, 786, 60 ; Laporte et al. 2014, *A&A*, 562, 8; Zitrin et al. 2014, *ApJ*, 793, 12; Ishigaki et al. 2015, *ApJ*, 799, 12; Atek et al. 2015, *ApJ*, 800, 18; Jauzac et al. 2015, *MNRAS*, 452, 437 ; Wang et al., *ApJ*, 811, 29

MACS J0416 - z = 0.396

Fully observed

Jauzac et al. 2014, *MNRAS*, 443, 1549; Lam et al. 2014, *ApJ*, 797, 98; Jauzac et al. 2015, *MNRAS*, 446 4132; Grillo et al. 2015, *ApJ*, 800, 38 ; Harvey et al. 2016, *MNRAS*, 458, 660 ; Caminha et al., 2016, arXiv1607.0346

MACS J1149 - z = 0.543

Fully observed

Kelly et al. 2015, *Science*, 347, 1123 ; Sharon & Johnson 2015, *ApJ*, 800, 26 ; Oguri 2015, *MNRAS*, 449, 86 ; Diego et al. 2016, *MNRAS*, 459, 344 ; Jauzac et al. 2016, *MNRAS*, 457, 2029 ; Treu et al. 2016, *ApJ*, 817, 60 ; Grillo et al. 2016, *ApJ*, 822, 78

Abell S1063 - z = 0.348

Fully observed

Diego et al. 2016, *MNRAS* 459, 3447

MACS J0717 - z = 0.545

Fully observed

Diego et al. 2015, *MNRAS*, 451, 3920 ; Limousin et al. 2016, *A&A*, 588, 99; Kawamata et al. 2016, *ApJ*, 819, 14

Abell 370 - z = 0.375

ACS to go

¿Cómo se arma un modelo de cúmulos de galaxias?

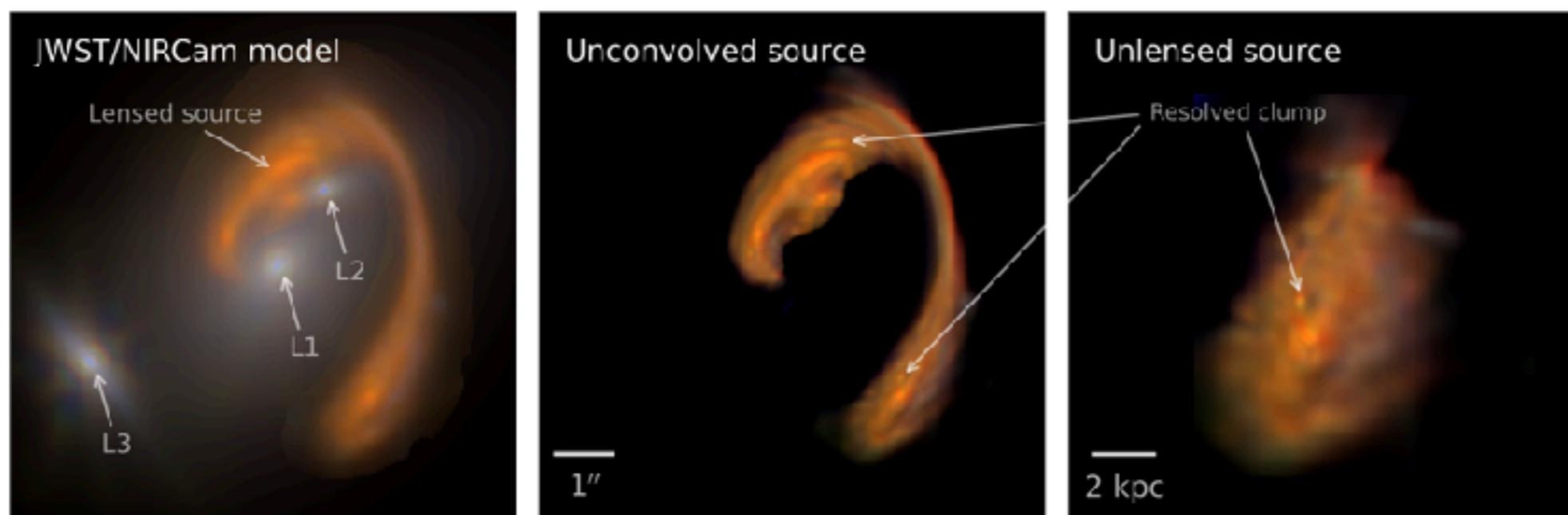
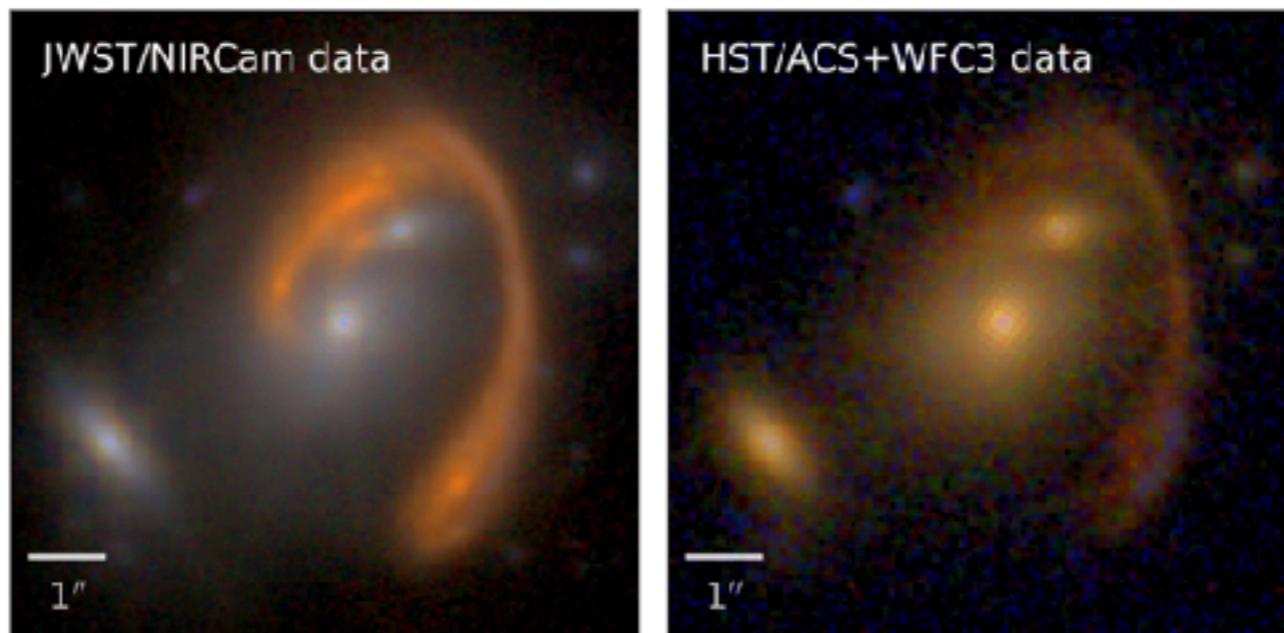
- Modelos paramétricos: distribución de masa compuesta de
 - Uno o más halos de materia oscura para el cúmulo
 - Un halo para cada galaxia con relaciones empíricas de escaleo (exponente, radio de corte, etc.) $M = M_c(L/L_\star)^\gamma$
- Datos: posiciones de las imágenes múltiples (más parámetros: posiciones de las fuentes), o distribución de brillo superficial
- Hoy: datos de HST para los cúmulos más masivos, $\mathcal{O}(10)$ sistemas + JWST
- Futuro/actualidad: Euclid, Roman + ULTRASET, CSST, CASTOR
- Software: LensPerfect, LensTool, GLAFIC, Gravlens, Glee, HERCULENS...

Cúmulo de galaxias El Gordo (ACT-CL J0102-4915, SPT-CL J0102-4915) observado con el JWST



El Gordo needs El Anzuelo: Probing the structure of cluster members with multi-band extended arcs in JWST data

A. Galan^{1,2,*}, G. B. Caminha^{1,2}, J. Knollmüller^{1,3,4}, J. Roth^{5,2,6}, and S. H. Suyu^{1,2}



Reconstrucción
de la fuente
usando el modelo
de lente



DARK ENERGY FROM FAMILIES OF MULTIPLE IMAGES

GALAXY CLUSTER SCALE COSMOLOGICAL CONSTRAINTS AND MORE

$$\vec{\theta} = \vec{\beta} + \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \nabla_{\theta} \psi(\vec{\theta})$$

Families of images with sources at different *redshifts*

constraints on cosmology, in addition to the matter distribution

The ratio of angular diameter distances for 2 (or more) images with sources at different redshifts defines a ratio of families

$$\Xi(z_1, z_{s1}, z_{s2}; \Omega_M, \Omega_X, w_X) = \frac{D(z_1, z_{s1})}{D(0, z_{s1})} \frac{D(0, z_{s2})}{D(z_1, z_{s2})}$$

- Jullo et al. 2010, Science: example of competitive limits in cosmological parameters from the Abell 1689 system
- 8 families of sources with $z = 1.15$ to 4.86
- Caminha et al. 2016: RXC J2248.7-4431 (Abell S1063), 17 sources, 47 images
- Magaña, Motta, Cárdenas, Verdugo, Jullo, 2015: Dark Energy models

REMINDER: ANGULAR DIAMETER DISTANCE

In the wCDM model $p = w\rho$

$$H^2(a) = H_0^2 \left[\Omega_r a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_{DE} a^{-3(1+w)} \right]$$

In the flat case

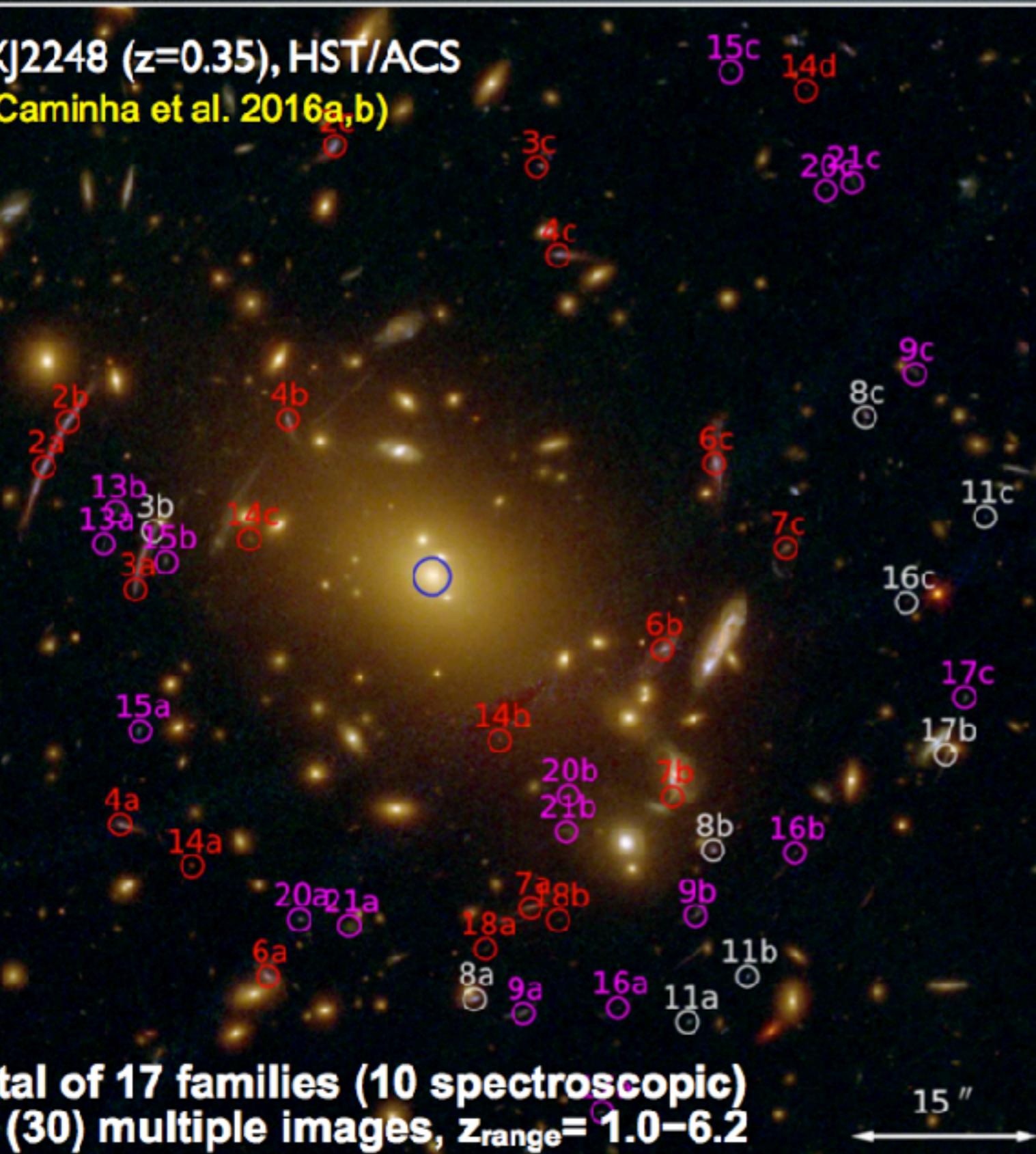
$$D_A(z_1, z_2) = \frac{(1+z_2)^{-1}}{H_0} \int_{z_1}^{z_2} \frac{dz'}{\sqrt{\Omega_M(1+z')^3 + (1-\Omega_M)(1+z')^{3(1+w)}}}$$

$$D_{LS} = D_A(z_L, z_S)$$

COSMOLOGICAL CONSTRAINTS

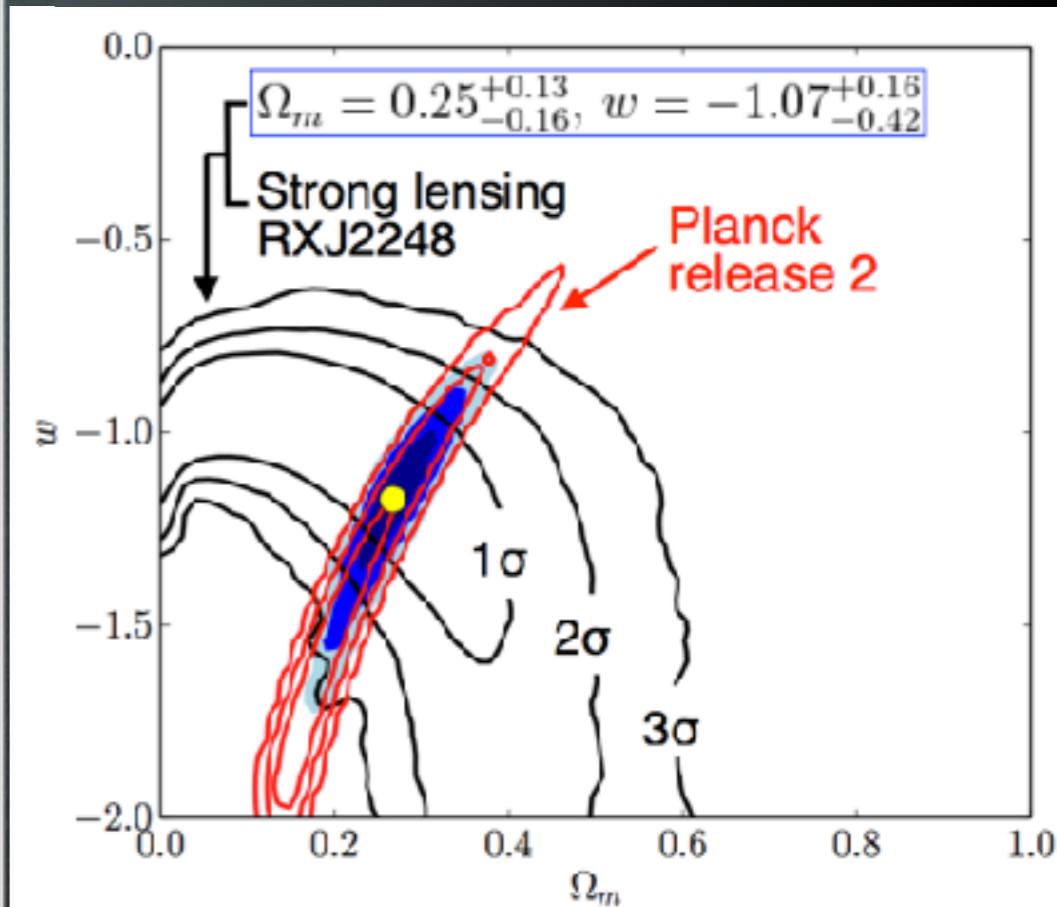
Frontier Field Cluster AS1063 (aka RXJ2248)

RXJ2248 ($z=0.35$), HST/ACS
(B.Caminha et al. 2016a,b)



Caminha et al., 2016

MUSE SV programme + GO (PI: K.Caputi)
(Karman et al. 2015)
(W.Karman et al. 2016, arXiv/160601471)

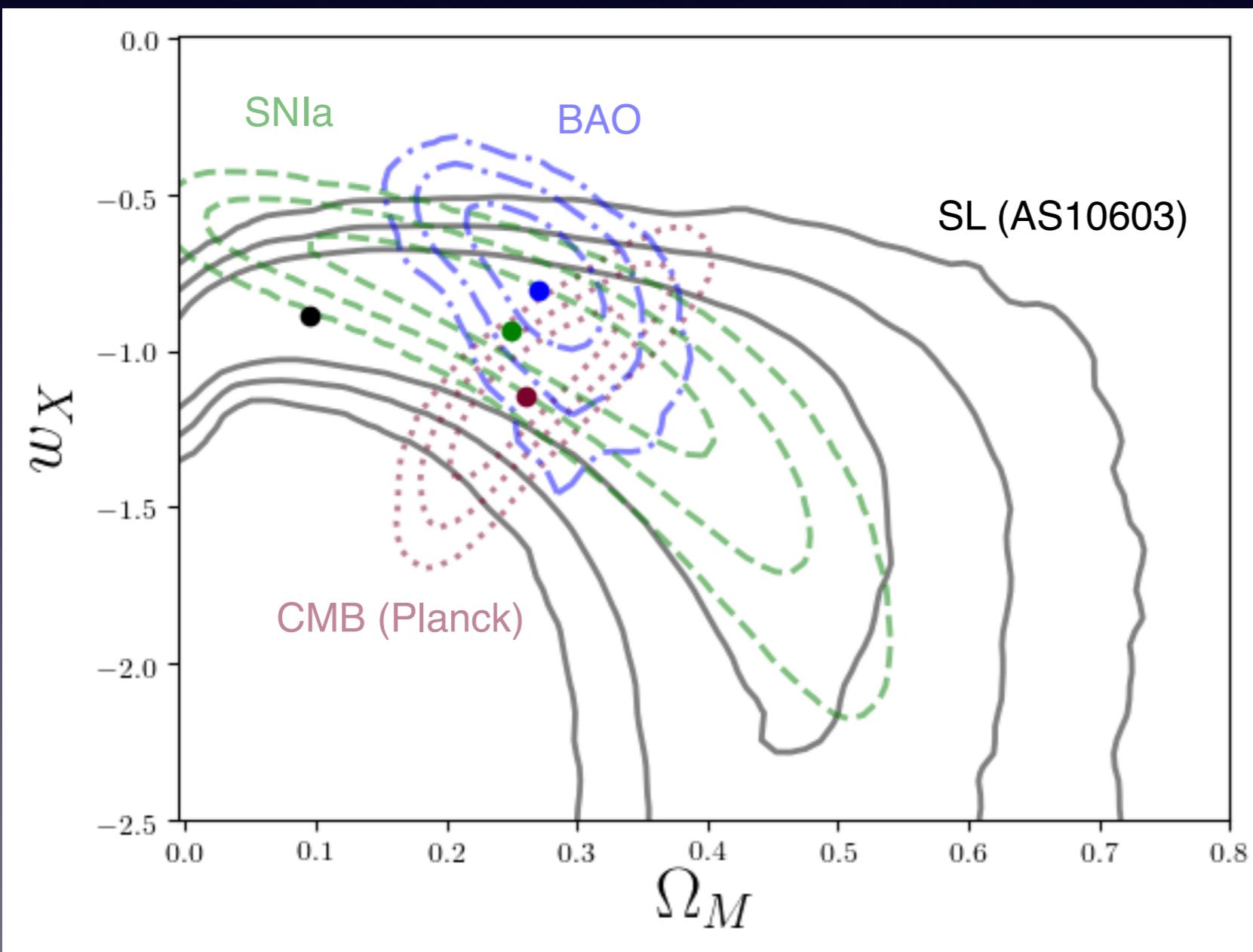


1 arcmin² FoV
2.6 Å resolution (4800-9300 Å)
0.2 arcsec/pxl
Exp. = 5 hrs

New constraints from Abell S1063

- Doubling the number of families of multiple images
- Comparison and combination with other observables

Bom, MM, Caminha, Vitenti, Penna-Lima (2022, in prep.)



Limits from Strong
Lensing only:

$$\Omega_M = 0.422^{+0.062}_{-0.274}$$

$$w = -0.911^{+0.030}_{-1.356}$$

Single system!

Improvement by combining
with SL over each probe alone:

Probe	$\Delta\sigma_w$	$\Delta\sigma_{\Omega_M}$	ΔA_σ
SNIa	27%	23 %	31 %
BAO	29%	17%	28 %
CMB	44%	37%	36 %