Lentes gravitacionales en astrofísica y cosmología
Semana - 8
Parte II: lentes por galáxias y cúmulos de galáxias

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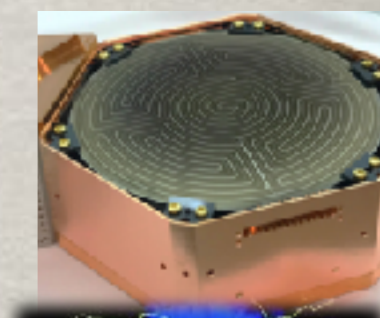
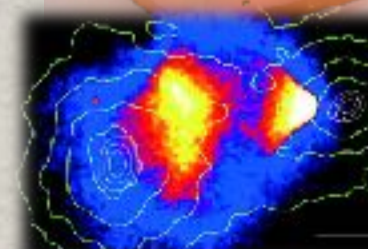
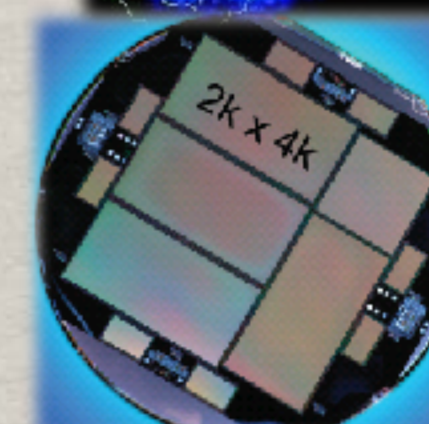
Instituto de Tecnología SABATO 30 años

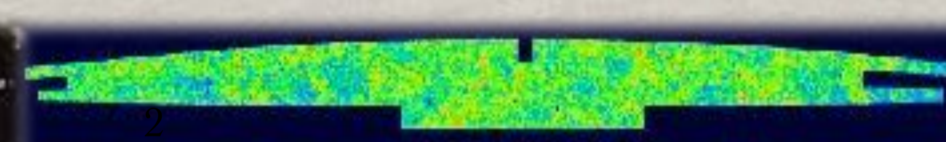
Plan de la Parte II

- [x] Redshift y expansión del Universo
- [x] Dinámica y parámetros cosmológicos
- [x] Métrica y distancias
- [x] Energía oscura
- [ ] Propagación de la luz y ecuación de la lente
- [ ] Lentes extendidas
- [ ] Jacobiana de la transformación: cáusticas y curvas críticas
- [ ] Modelos de lentes extendidas y aplicaciones
- [ ] Retraso temporal y aplicaciones
- [ ] Efecto débil de lentes









Límite newtoniano y teoria de perturbación relativista

- Límite newtoniano: $\dfrac{\phi}{c^2} \ll 1$ e $\left(\dfrac{v}{c}\right)^2 \ll 1$
- Correcciones relativistas: formalismo pos-newtoniano (e.g. sistema solar)

Límite newtoniano y teoria de perturbación relativista

- Límite newtoniano: $\frac{\phi}{c^2} \ll 1$ e $\left(\frac{v}{c}\right)^2 \ll 1$
- Correcciones relativistas: formalismo pos-newtoniano (e.g. sistema solar)
- Teoria de perturbación cosmológica: métrica de Robertson-Walker perturbada

$$\begin{aligned} ds^2 &= \left[g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu}\right] dx^\mu dx^\nu \\ &= a^2(\tau)\left[-d\tau^2 + \gamma_{ij}(\vec{x})dx^i dx^j + h_{\mu\nu}(\vec{x},\tau)dx^\mu dx^\nu\right] \quad , \end{aligned}$$

- Desacople entre los modos en el regimen lineal
- Perturbaciones escalares:

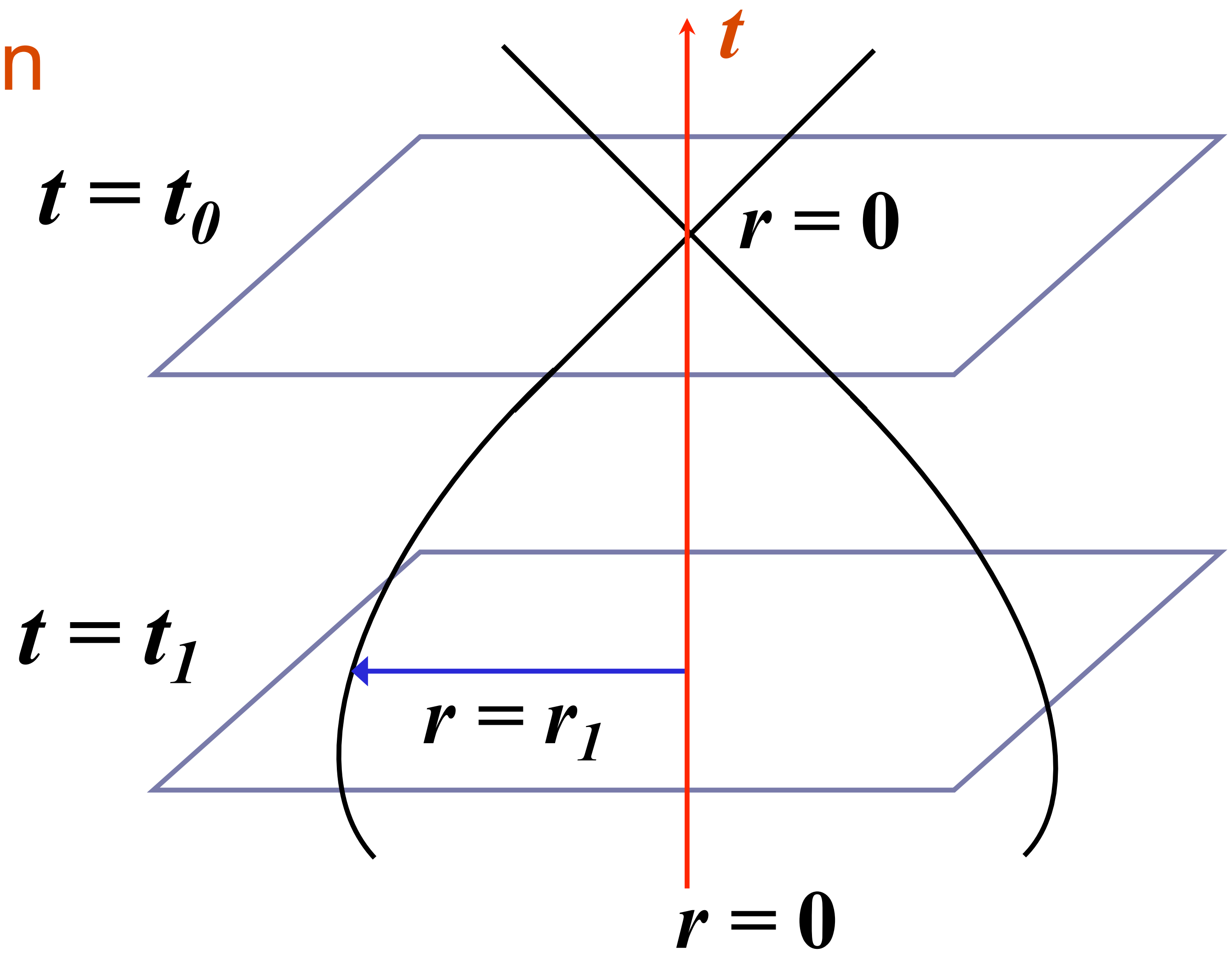
$$\boxed{ds^2 = a^2(\tau)\left[-(1+2\Psi)d\tau^2 + (1-2\phi)\gamma_{ij}\, dx^i dx^j\right]}$$

(para um fluido perfecto $\Phi = \Psi$)

- Nuevamente, límite newtoniano para $\frac{\phi}{c^2} \ll 1$ $\left(\frac{v}{c}\right)^2 \ll 1$

Comentario/recordatorio

- Observaciones se realizan en el llamado cono de luz
- Relación entre distancia y tiempo
- Relación entre distancia y corrimiento al rojo z
- Utilizados de forma intercambiable
- z es directamente observable,
- t y r dependen del modelo



$$\boxed{z = \frac{\lambda_r - \lambda_e}{\lambda_e}}$$

$$d_i = f(z; \text{cosmologia})$$

Ecuación de Friedmann

- Todas las escalas cosmológicas se expanden, en promedio, por el factor de escala a
- Relación entre a y z: $\boxed{a(t) = (1+z)^{-1}}$

Equação de Friedmann: $\boxed{\dot{a}^2 = \frac{8\pi G}{3} a^2 \rho - K}$

- Conociendo $\rho(a)$, podemos obtener $a(t)$
  - Ejemplo I: materia (partículas): $\rho \propto a^{-3}$
    - $K = 0 \longrightarrow a \propto t^{2/3}$ Einstein - de Sitter
    - $K \neq 0$
- Tasa de expansión del Universo: $H = \frac{\dot{a}}{a}$

$$H(a), H(t), H(z)$$

Resumen del Universo Homogéneo

- El Universo está en expansión (alejamiento de objetos distantes), de forma homogénea e isótropa **en promedio**, descripta por el factor de escala a
- Corrimiento al rojo (*redshift*)

$$\boxed{z := \frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1+v/c}{1-v/c}} - 1 \approx v/c \quad \text{para } v << c}$$

- Diagrama de Hubble para z << 1 (expansión **média** del Universo)

$$\boxed{v \simeq H_0 d_L} \qquad \boxed{H_0 = 100 h Km/s/Mpc}$$

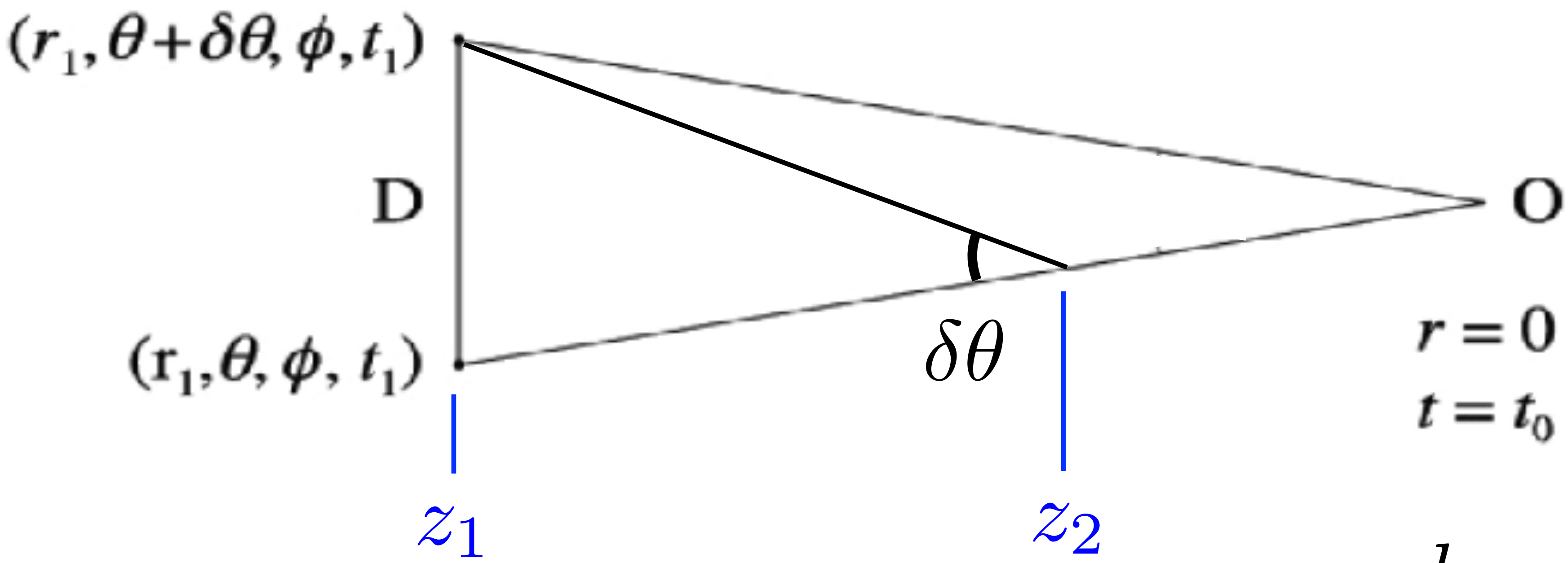
- Observaciones son realizadas en el llamado cono de luz
- Relación entre r, t y z
- Relación entre a y z:

$$\boxed{a(t) = (1+z)^{-1}}$$

- Tasa de expansión del Universo:

$$\boxed{H = \frac{\dot{a}}{a}}$$

Distância de diámetro angular entre dos puntos



$$d_A \equiv \frac{D}{\delta\theta}$$

(para $\delta\theta \ll 1$)

$$d_A = a(t_2) r_{12} = a(z_2) r_{12}$$

$$\boxed{d_A = (1+z_2)^{-1} \mathrm{sen}_K \left(H_0 \sqrt{|1-\Omega_0|} \int_{z_1}^{z_2} \frac{dz'}{H(z')} \right) \Big/ H_0 \sqrt{|1-\Omega_0|}}$$

Caso plano: $D_A(z_1, z_2) = \dfrac{1}{1+z_2} \displaystyle\int_{z_1}^{z_2} \frac{dz}{H(z)}$

Distância de diámetro angular entre dos puntos

Caso plano:

$$D_A(z_1, z_2) = \frac{1}{1+z_2} \int_{z_1}^{z_2} \frac{dz}{H(z)}$$

En el modelo wCDM $\qquad p = w\rho$

$$H^2(a) = H_0^2 \left[\Omega_r a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_{DE} a^{-3(1+w)} \right]$$

de forma que

$$D_A(z_1, z_2) = \frac{(1+z_2)^{-1}}{H_0} \int_{z_1}^{z_2} \frac{dz'}{\sqrt{\Omega_M (1+z')^3 + (1-\Omega_M)(1+z')^{3(1+w)}}}$$

$D_{LS} = D_A(z_L, z_S)$, etc.

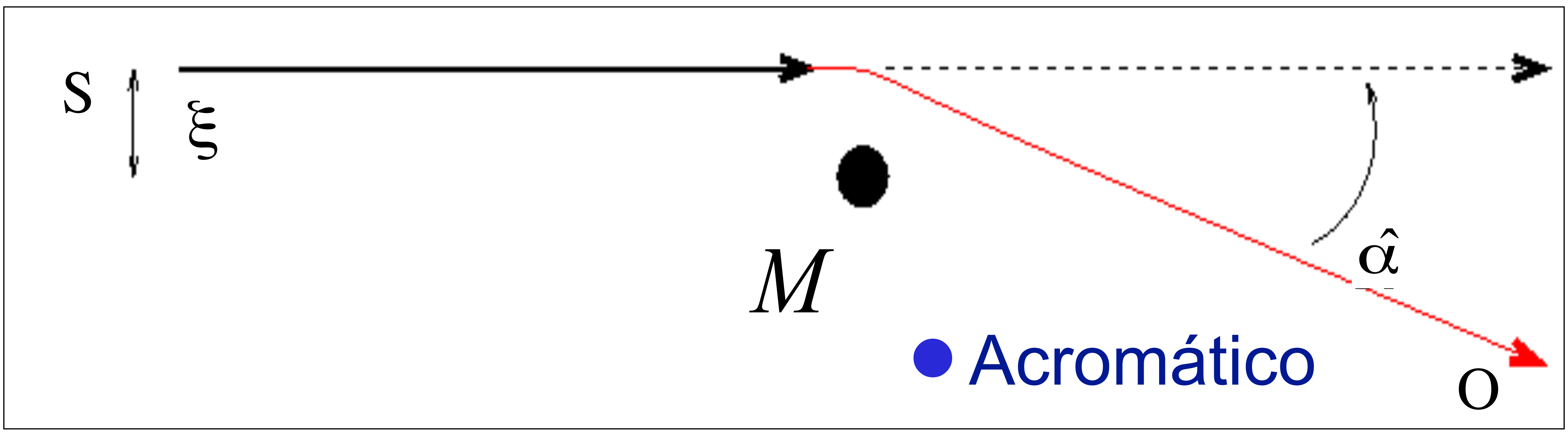
DESVIACIÓN DE LA LUZ POR LA GRAVEDAD

Geodésica nula,
Princípio de Fermat

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\phi}{c^2}\right) d\sigma^2$$

Parte espacial de la métrica de FLRW

$$\frac{d\sigma}{dt} := c' = \sqrt{\frac{1 + 2\phi/c^2}{1 - 2\phi/c^2}} \simeq c\left(1 + \frac{2\phi}{c^2}\right)$$



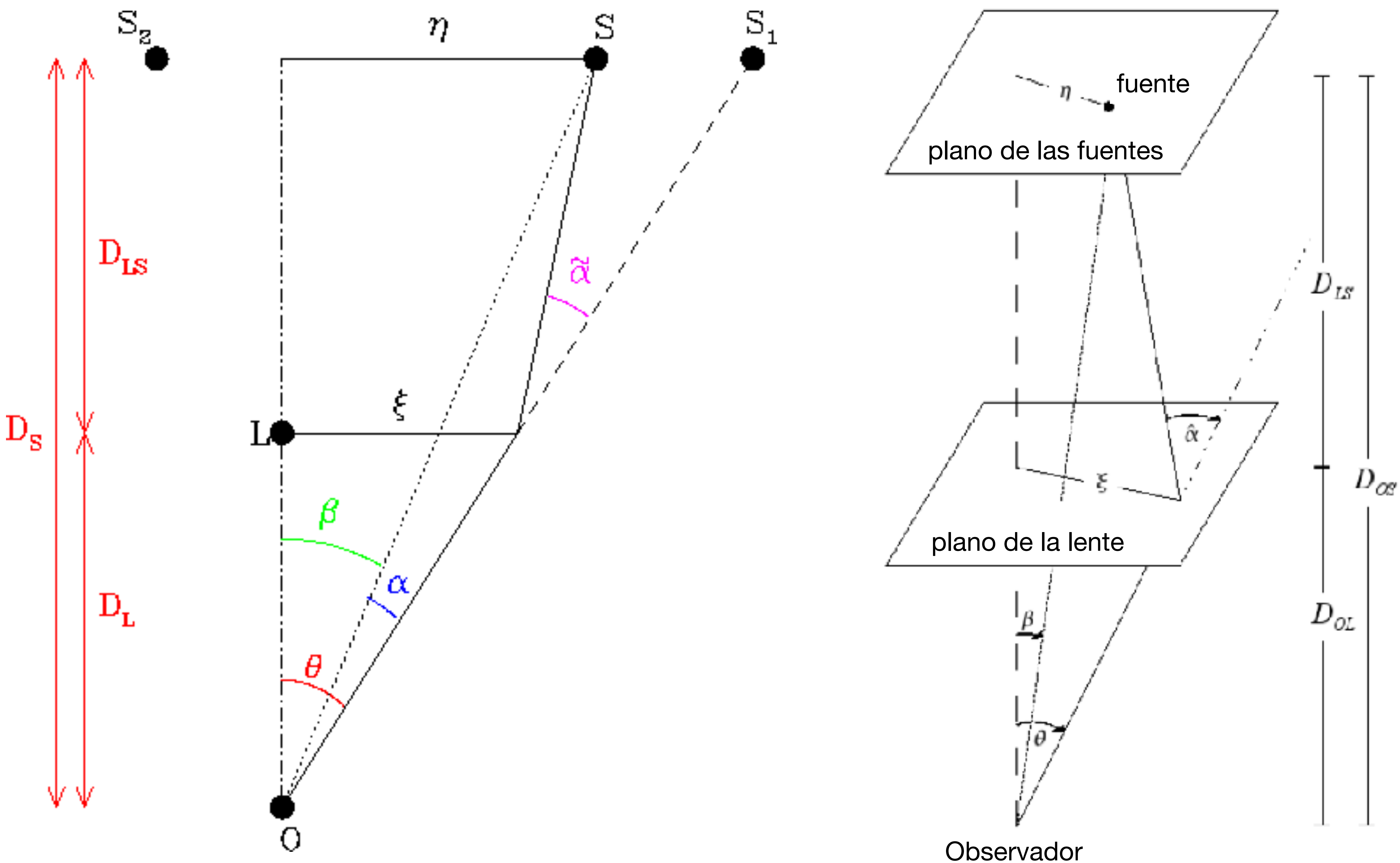
- Acromático

Desvío causado por una lente puntual:

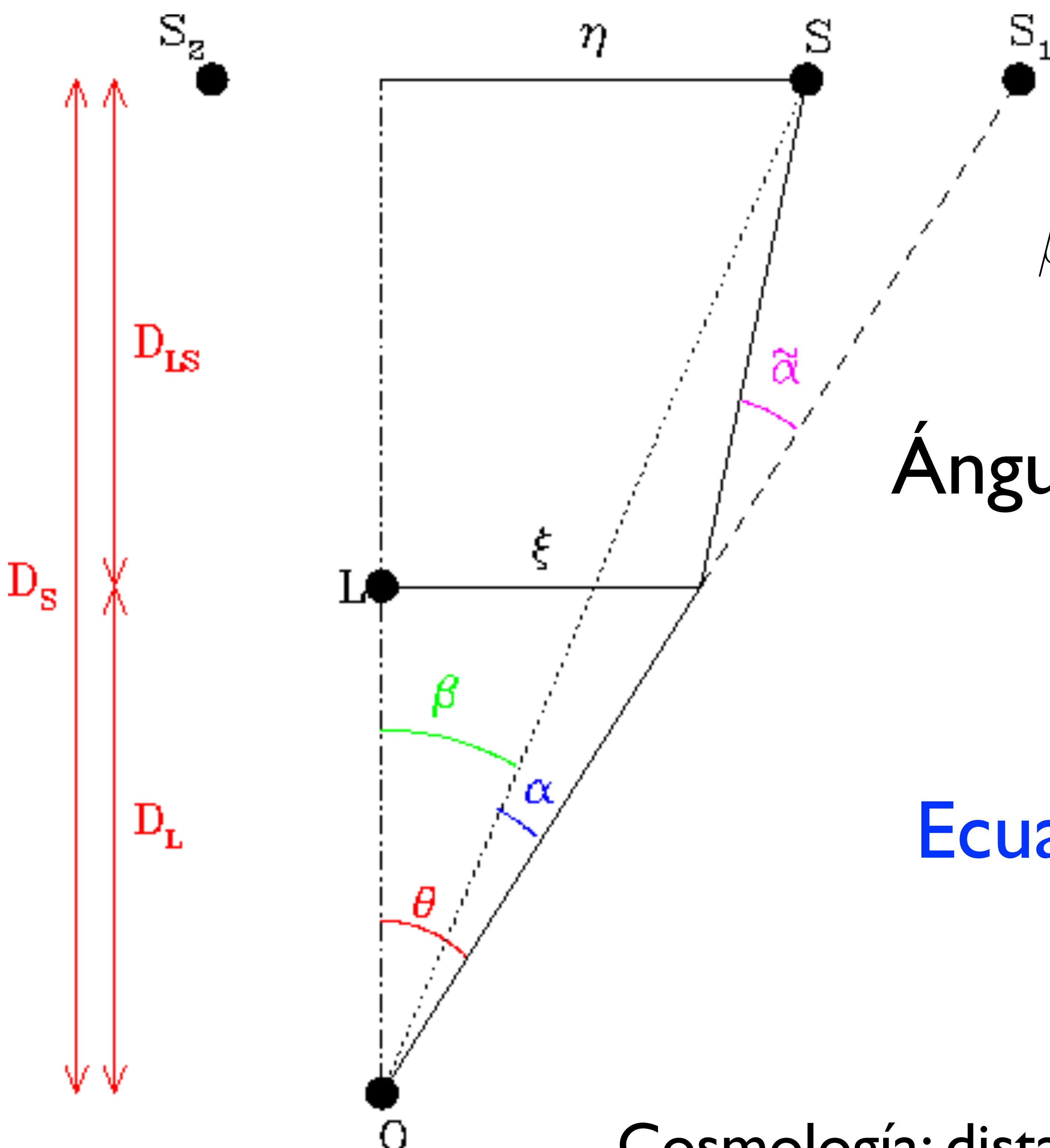
$$\hat{\alpha} = \frac{4GM}{c^2}\frac{1}{\xi}$$

(factor 2 en comparación con "Newton")

Geometría del efecto de lentes por plano único



La ecuación de la lente



¡(*Strong*) *lensing* es resolver la ecuación de la lente!

$$\vec{\beta} D_{OS} = \vec{\theta} D_{OS} - \hat{\vec{\alpha}} D_{LS} \left(\vec{\theta}\right)$$

Ángulo de deflexión reducido

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}}$$

Ecuación de la lente

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}\left(\vec{\theta}\right)$$

Cosmología: distancias de diámetro angular!

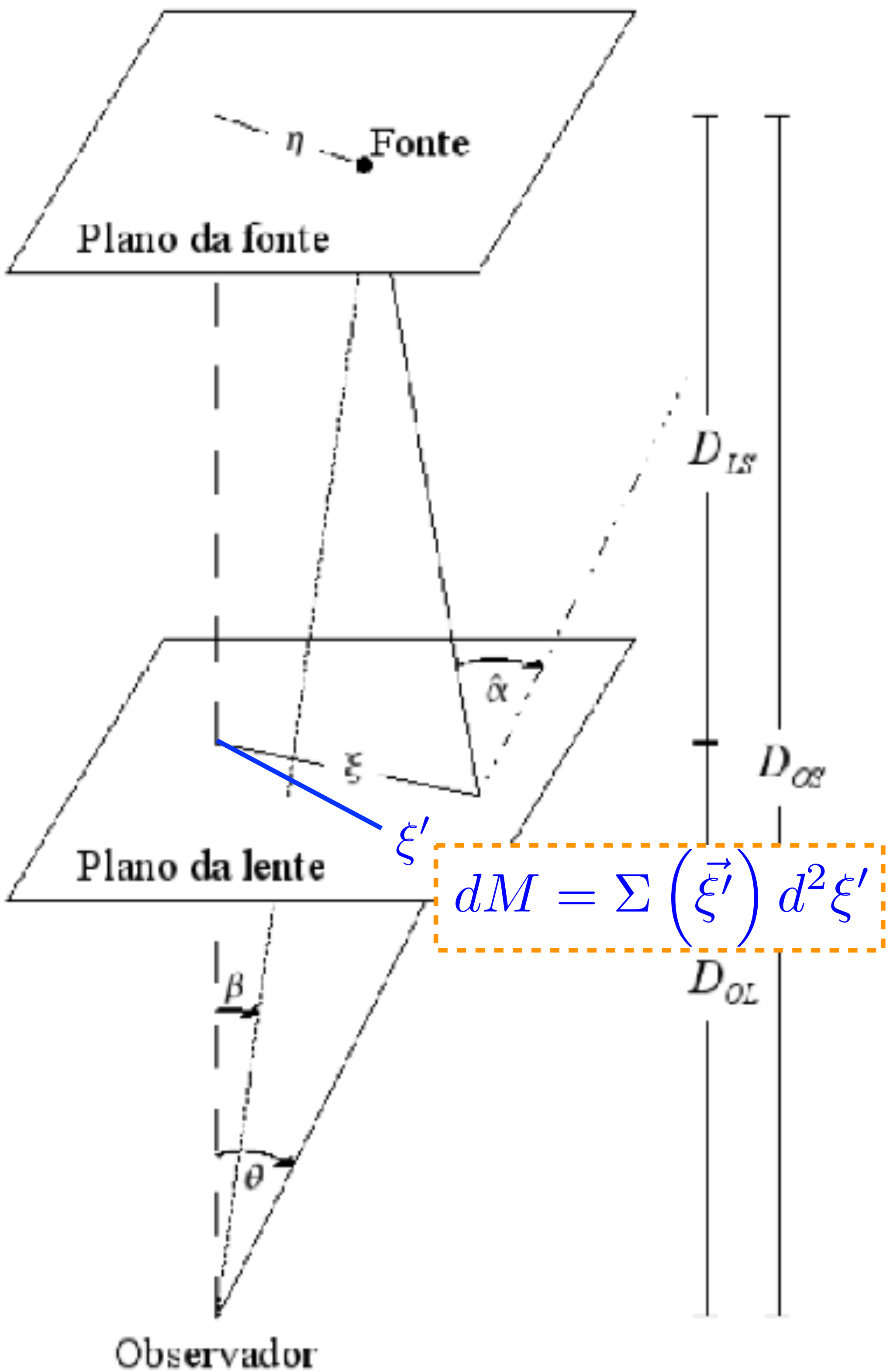
Deducciones formales

- Lentes gravitacionales en relatividad general: Schneirder, Ehlers, Falco: capítulos 3 (*Optics in curved spacetime*) y 4 (contexto cosmológico)

- Ángulo de reflexión en las 3 geometrías de fondo: Petters, Lavine, Wambsganss

Lentes Extensas

Extended Lenses



Point mass $\hat{\vec{\alpha}} = \dfrac{4GM}{c^2\xi}$

Surface mass density

$$\Sigma\left(\vec{\xi}\right) = \int_0^\infty dz\rho\left(\vec{\xi}, z\right)$$

Contribution of the area element

$$d\hat{\vec{\alpha}} = \frac{4G}{c^2}\Sigma\left(\vec{\xi'}\right) d^2\xi' \frac{\vec{\xi} - \vec{\xi'}}{|\vec{\xi} - \vec{\xi'}|^2}$$

Deflection angle

$$\hat{\vec{\alpha}} = \frac{4G}{c^2}\int d^2\xi'\Sigma\left(\vec{\xi'}\right) \frac{\vec{\xi} - \vec{\xi'}}{|\vec{\xi} - \vec{\xi'}|^2}$$

Extended Lenses

Projected potential $\psi\left(\vec{\xi}\right) = \int dz\varphi(\vec{\xi}, z)$

Poisson equation $\nabla_{\xi}^{2}\psi\left(\vec{\xi}\right) = 4\pi G\Sigma\left(\vec{\xi}\right)$

Using the 2D Green function

$$\psi\left(\vec{\xi}\right) = 2G\int d^{2}\xi'\Sigma\left(\vec{\xi'}\right)\ln|\xi - \xi'|$$

Comparing with the deflection angle

$$\hat{\vec{\alpha}} = \frac{4G}{c^{2}}\int d^{2}\xi'\Sigma\left(\vec{\xi'}\right)\frac{\vec{\xi} - \vec{\xi'}}{|\vec{\xi} - \vec{\xi'}|^{2}}$$

We obtain $\hat{\vec{\alpha}} = \frac{2}{c^{2}}\vec{\nabla}_{\xi}\psi\left(\vec{\xi}\right)$

Extended Lenses

Projected potential $$\psi\left(\vec{\xi}\right) = \int dz \varphi(\vec{\xi}, z)$$

Deflection angle $$\hat{\vec{\alpha}} = \frac{2}{c^2} \vec{\nabla}_\xi \psi\left(\vec{\xi}\right)$$

Reduced deflection angle

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \vec{\nabla}_\theta \psi\left(\vec{\theta}\right)$$

Lensing potential $$\Psi(\vec{x}) \equiv \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \psi$$

Lens equation $$\vec{\beta} = \vec{\theta} - \vec{\alpha}\left(\vec{\theta}\right) = \boxed{\vec{\theta} - \vec{\nabla}_\theta \Psi\left(\vec{\theta}\right)}$$

Extended Lenses

Deflection angle $$\hat{\vec{\alpha}} = \frac{4G}{c^2} \int d^2\xi' \Sigma\left(\vec{\xi'}\right) \frac{\vec{\xi} - \vec{\xi'}}{|\vec{\xi} - \vec{\xi'}|^2}$$

Projected potential $$\psi = \int_{-\infty}^{\infty} \phi(\vec{\xi}, z) dz$$

Poisson equation $$\nabla^2_{\xi} \psi\left(\vec{\xi}\right) = 4\pi G \Sigma\left(\vec{\xi}\right)$$

Lens equation $$\vec{\beta} = \vec{\theta} - \vec{\alpha}\left(\vec{\theta}\right) = \boxed{\vec{\theta} - \vec{\nabla}_{\theta} \Psi\left(\vec{\theta}\right)}$$

Lensing potential $$\Psi \equiv \frac{2}{c^2} \frac{D_{\mathrm{LS}}}{D_{\mathrm{OS}} D_{\mathrm{OL}}} \psi$$

Lentes extensas

Ángulo de deflexión $$\hat{\vec{\alpha}} = \frac{4G}{c^2} \int d^2\xi' \Sigma \left(\vec{\xi'} \right) \frac{\vec{\xi} - \vec{\xi'}}{|\vec{\xi} - \vec{\xi'}|^2}$$

Ecuación de Poisson $\nabla^2_\xi \psi \left(\vec{\xi} \right) = 4\pi G \Sigma \left(\vec{\xi} \right)$

Ecuación de la lente $$\vec{\beta} = \vec{\theta} - \vec{\alpha} \left(\vec{\theta} \right) = \boxed{\vec{\theta} - \vec{\nabla}_\theta \Psi \left(\vec{\theta} \right)}$$

Potencial de Lente(amiento) $\Psi \equiv \frac{2}{c^2} \frac{D_{\mathrm{LS}}}{D_{\mathrm{OS}} D_{\mathrm{OL}}} \psi$

convertiendo variables $d\xi_i = D_{OL} d\theta_i$

$$\nabla^2_\theta \Psi = \frac{2}{c^2} \frac{D_{\mathrm{LS}}}{D_{\mathrm{OS}} D_{\mathrm{OL}}} D^2_{\mathrm{OL}} 4\pi G \, \Sigma$$

Convergencia

Ecuación de Poisson

$$\nabla_\theta^2 \Psi = \frac{2}{c^2} \frac{D_{\mathrm{LS}}}{D_{\mathrm{OS}} D_{\mathrm{OL}}} D_{\mathrm{OL}}^2 4\pi G \, \Sigma\left(\vec{\theta}\right) = 2 \frac{\Sigma\left(\vec{\theta}\right)}{\Sigma_{\mathrm{crit}}}$$

Densidad superficial crítica

$$\boxed{\Sigma_{\mathrm{crit}} = \frac{c^2}{4\pi G} \frac{D_{\mathrm{OS}}}{D_{\mathrm{OL}} D_{\mathrm{LS}}}}$$

Convergencia

$$\kappa\left(\vec{\theta}\right) = \frac{\Sigma\left(\vec{\theta}\right)}{\Sigma_{\mathrm{crit}}}$$

Ecuación de Poisson $\boxed{\nabla_\theta^2 \Psi = 2\kappa\left(\vec{\theta}\right)}$

Jacobiana de la transformación

$$\vec{\beta} = \vec{\theta} - \vec{\nabla}_\theta \Psi \left(\vec{\theta}\right)$$

$$J_{ij} = \left(\frac{\partial \vec{\beta}}{\partial \vec{\theta}}\right)_{ij} = \delta_{ij} - \frac{\partial^2 \Psi}{\partial \theta_i \partial \theta_j},$$ definiendo $\Psi_{ij} = \dfrac{\partial^2 \Psi}{\partial \theta_i \partial \theta_j}$

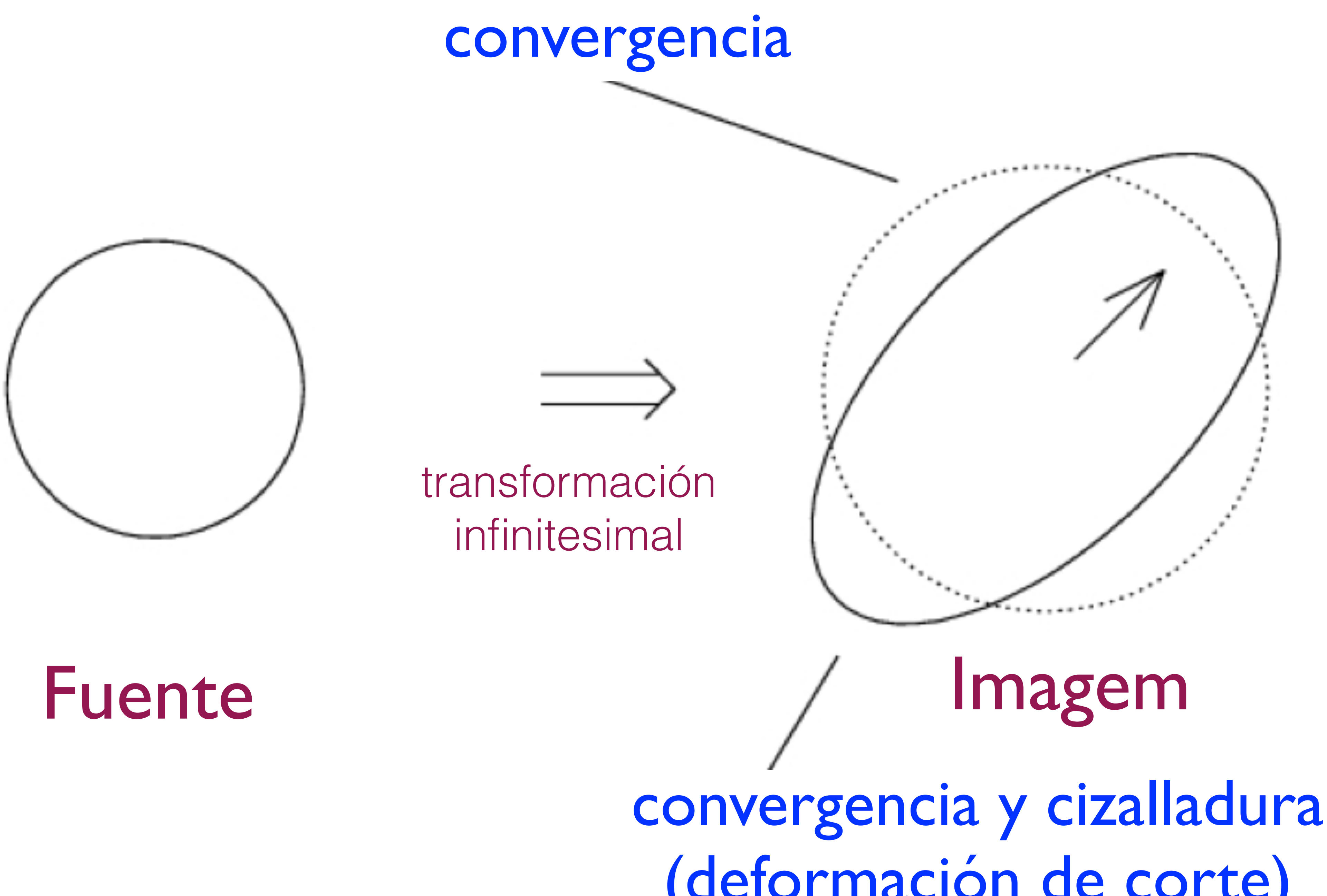
tenemos $\mathbf{J} = \begin{pmatrix} 1 - \Psi_{11} & -\Psi_{12} \\ -\Psi_{21} & 1 - \Psi_{22} \end{pmatrix}$

como $\nabla^2_\theta \Psi = \Psi_{11} + \Psi_{22} = 2\kappa$

$$\mathbf{J} = (1-\kappa)\mathbf{I} - \begin{pmatrix} \frac{1}{2}\left(\Psi_{11} - \Psi_{22}\right) & \Psi_{12} \\ \Psi_{21} & -\frac{1}{2}\left(\Psi_{11} - \Psi_{22}\right) \end{pmatrix}$$

Jacobiana de la transformación

$$\mathbf{J} = (1-\kappa)\mathbf{I} - \begin{pmatrix} \frac{1}{2}\left(\Psi_{11} - \Psi_{22}\right) & \Psi_{12} \\ \Psi_{21} & -\frac{1}{2}\left(\Psi_{11} - \Psi_{22}\right) \end{pmatrix}$$



Cizalladura

$$\mathbf{J} = (1-\kappa)\mathbf{I} - \begin{pmatrix} \frac{1}{2}\left(\Psi_{11} - \Psi_{22}\right) & \Psi_{12} \\ \Psi_{21} & -\frac{1}{2}\left(\Psi_{11} - \Psi_{22}\right) \end{pmatrix}$$

Cizalladura:

$$\gamma_1(\vec{\theta}) = \frac{1}{2}(\Psi_{11} - \Psi_{22})$$

$$\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$$

$$\gamma_2(\vec{\theta}) = \Psi_{12} = \Psi_{21}$$

Em términos de la cizalladura y la convergencia tenemos

$$\mathbf{J} = (1-\kappa)\mathbf{I} + \begin{pmatrix} -\gamma_1 & -\gamma_2 \\ -\gamma_2 & \gamma_1 \end{pmatrix}$$

Autovalores de $\mathbf{J}^{-1}$

$$\mu_1 = \frac{1}{1-\kappa-\gamma}, \quad \mu_2 = \frac{1}{1-\kappa+\gamma}$$

Mapeo de Lentes

- mapeo imagem → fuente

$$\frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}$$

- plano único

$$\Psi = \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \int \phi(\xi, z) dz$$

potencial gravitacional

distâncias cosmológicas

$$D_{LS} = D_A(z_L, z_S)\ldots$$

- autovalores:

$$\mu_1 = \frac{1}{1-\kappa+\gamma}, \mu_2 = \frac{1}{1-\kappa-\gamma}$$

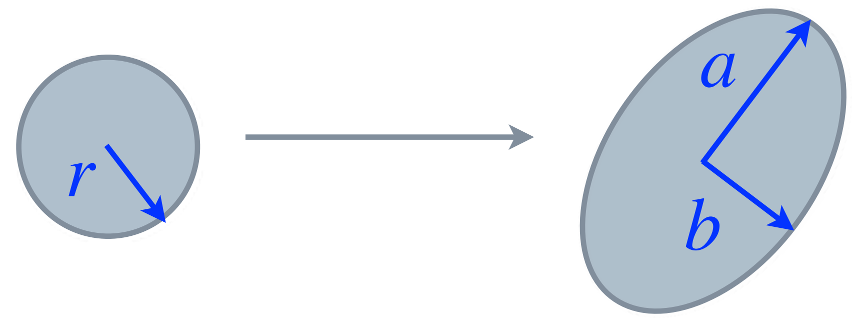
- magnificación local y razón axial:

$$\mu = \mu_1 \, \mu_2$$

$$r = \left| \frac{\mu_1}{\mu_2} \right|$$

Mapeo Lineal

Fuente circular



$$a = \left(\frac{1}{1-\kappa-\gamma}\right) r$$

$$b = \left(\frac{1}{1-\kappa+\gamma}\right) r$$

- Magnificación

$$\mu = \frac{A_{\text{imagem}}}{A_{\text{fonte}}} = \left[(1-\kappa)^2 - \gamma^2\right]^{-1}$$

- Elipticidad

$$\boxed{\epsilon := \frac{a-b}{a+b} = \frac{\gamma}{1-\kappa} =: g}$$

Mapeo de Lentes

- mapeo imagem → fuente

$$\frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}$$

- plano único

$$\Psi = \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \int \phi(\xi, z) dz$$

potencial gravitatorio

distancias cosmológicas

$$D_{LS} = D_A(z_L, z_S) \ldots$$

- autovalores:

$$\mu_1 = \frac{1}{1-\kappa+\gamma}, \mu_2 = \frac{1}{1-\kappa-\gamma}$$

- magnificación local y razón axial:

$$\mu = \mu_1 \mu_2 \qquad r = \left| \frac{\mu_1}{\mu_2} \right|$$

- densidad superficial crítica

$$\Sigma_{\rm crit} = \frac{c^2}{4\pi G} \frac{D_{OS}}{D_{OL} D_{LS}}$$

- convergencia

$$\kappa = \frac{\Sigma}{\Sigma_{\rm crit}}$$

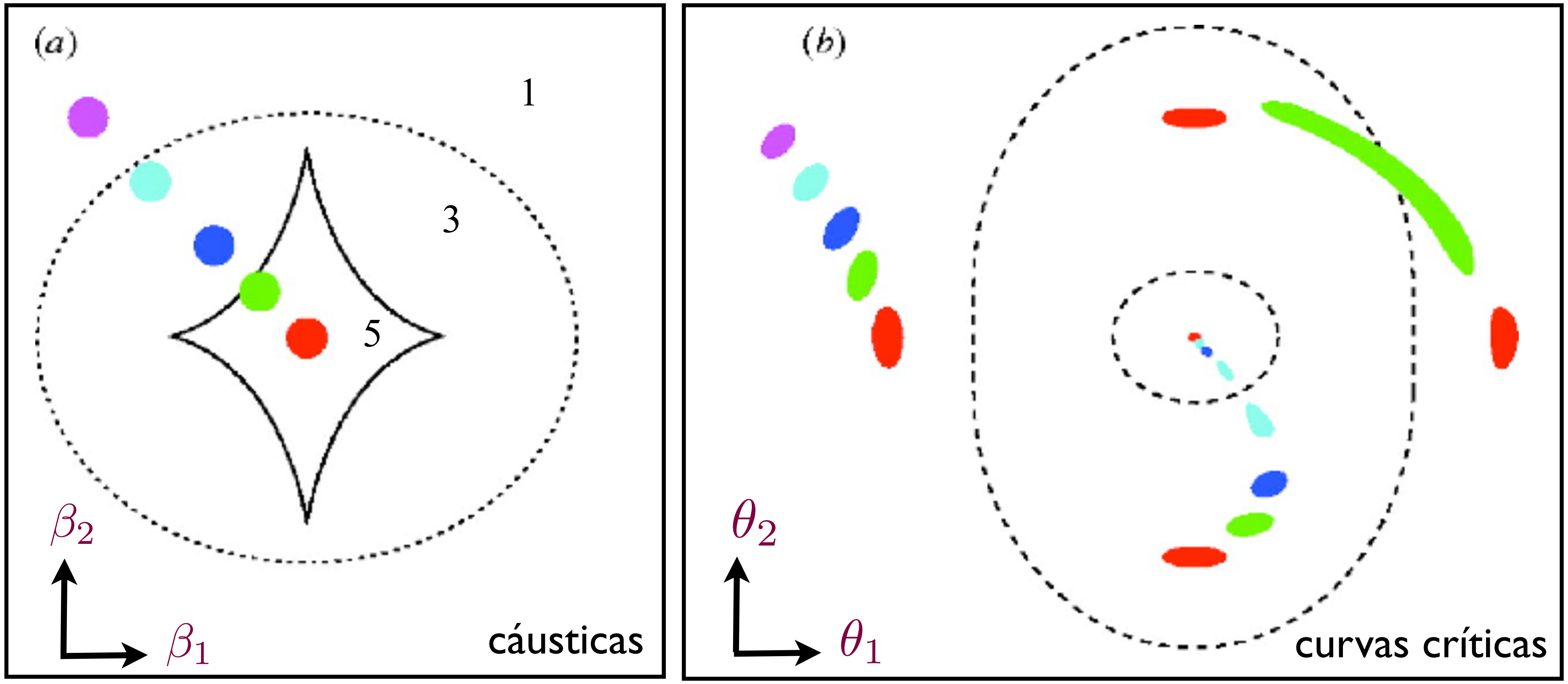
Mapeo de Lentes

- mapeo imagem → fuente

$$\frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \Psi(\vec{\theta})}{\partial \theta_i \partial \theta_j}$$

- autovalores:

$$\mu_1 = \frac{1}{1-\kappa+\gamma}, \mu_2 = \frac{1}{1-\kappa-\gamma}$$



Plano de las fuentes | Lente/imágenes

Los números en el plano de las fuentes indicam la multiplicidad de las imaágenes

Esfera isoterma singular

Esfera isoterma singular

- Modelo más simple de lente extendida
- Caso típico para soluciones más generales
- Mas allá de su valor didáctico, es un modelo realista para la distribución de massa en la escala de galáxias (*bulge-halo conspiracy*)
- Muy usado en el caso de galáxias de tipo temprano (elípticas)
- Quizás aplicabilidad más amplia (e.g. arXiv: 2406.0965)

Esfera Isotérmica Singular

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

Densidad proyectada (superficial)

$$\Sigma(\xi) = \frac{\sigma_v^2}{2\pi G} 2 \int_0^\infty \frac{dz}{\xi^2 + z^2} = \frac{\sigma_v^2}{2G\xi}$$

Masa contenida en radio ξ

$$M(\xi) = \int_0^\xi \Sigma(\xi')\, 2\pi\xi' \,\mathrm{d}\xi' = \sigma_v^2 \frac{\pi}{G} \xi$$

Ángulo de deflexión

$$\Rightarrow \quad \hat{\vec{\alpha}} = \frac{4GM(\xi)}{c^2\xi} \hat{\xi} = 4\pi \left(\frac{\sigma_v}{c}\right)^2 \hat{\xi}$$

constante!

Lens Equation

Reduced deflexion angle

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} \qquad \hat{\vec{\alpha}} = 4\pi \left(\frac{\sigma_v}{c} \right)^2 \hat{\xi}$$

$$\vec{\alpha} = \hat{\vec{\alpha}} \frac{D_{LS}}{D_{OS}} = \boxed{4\pi \left(\frac{\sigma_v}{c} \right)^2 \frac{D_{LS}}{D_{OS}}} \hat{\xi} = \theta_E \, \hat{\xi}$$

$$\vec{\beta} = \vec{\theta} - \theta_E \, \hat{\theta} = \left(1 - \frac{\theta_E}{\theta} \right) \vec{\theta}$$

$$\Rightarrow \quad \beta = \left| 1 - \frac{\theta_E}{\theta} \right| \theta$$

Solutions: 1) if $\quad 1 - \dfrac{\theta_E}{\theta} > 0$

then $\quad \beta = \left(1 - \dfrac{\theta_E}{\theta} \right) \theta = \theta - \theta_E \Rightarrow \boxed{\theta = \beta + \theta_E}$

Lens Equation

$$\beta = \left|1 - \frac{\theta_E}{\theta}\right| \theta$$

Solutions: I) if $\quad 1 - \dfrac{\theta_E}{\theta} > 0$

then $\quad \beta = \left(1 - \dfrac{\theta_E}{\theta}\right)\theta = \theta - \theta_E \Rightarrow \boxed{\theta = \beta + \theta_E}$

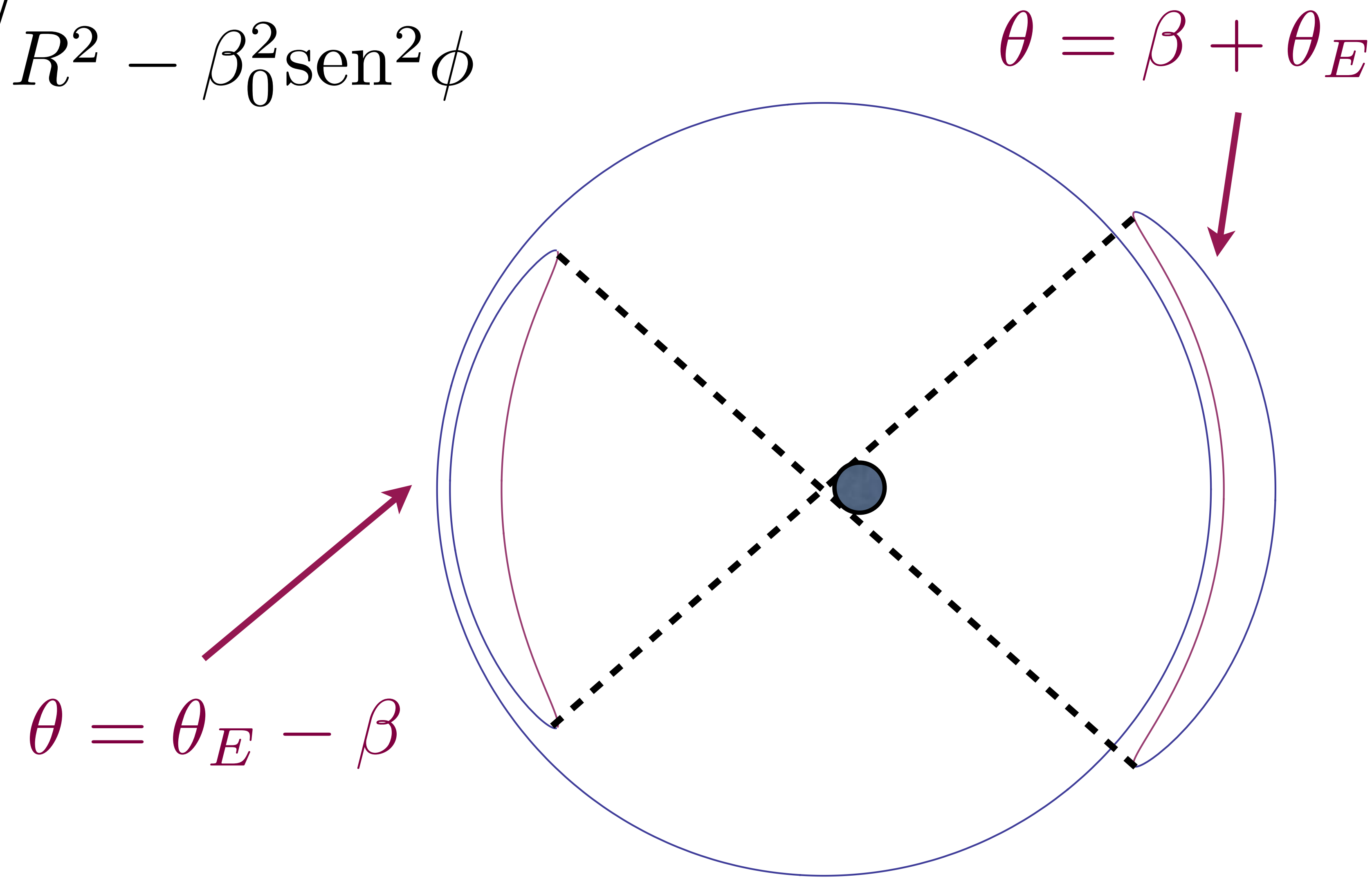
II) if $\quad 1 - \dfrac{\theta_E}{\theta} < 0$

then $\quad \beta = -\left(1 - \dfrac{\theta_E}{\theta}\right)\theta = \theta_E - \theta \Rightarrow \boxed{\theta = \theta_E - \beta}$

Fuente circular

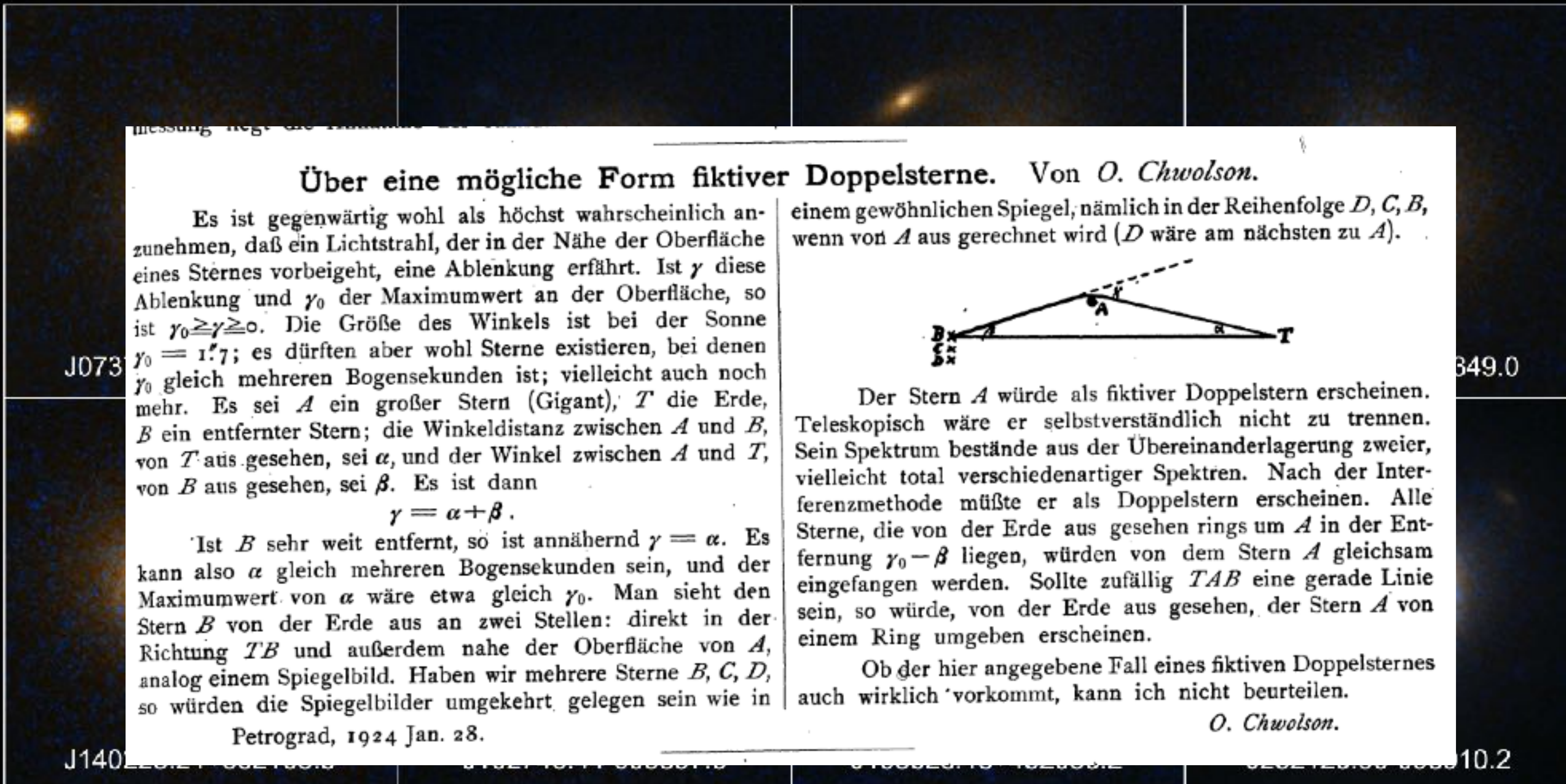
Círculo en el plano de las fuentes: $\left|\vec{\beta} - \vec{\beta}_0\right| = R$

$$\beta = \beta_0 \cos\phi \pm \sqrt{R^2 - \beta_0^2 \mathrm{sen}^2\phi}$$



Caso general $\beta = \beta_0 \cos(\phi - \phi_0) \pm \sqrt{R^2 - \beta_0^2 \mathrm{sen}^2(\phi - \phi_0)}$

“Anillos de Chwolson-Einstein”



Über eine mögliche Form fiktiver Doppelsterne. Von *O. Chwolson.*

Es ist gegenwärtig wohl als höchst wahrscheinlich anzunehmen, daß ein Lichtstrahl, der in der Nähe der Oberfläche eines Sternes vorbeigeht, eine Ablenkung erfährt. Ist γ diese Ablenkung und γ_0 der Maximumwert an der Oberfläche, so ist $\gamma_0 \geq \gamma \geq 0$. Die Größe des Winkels ist bei der Sonne $\gamma_0 = 1''.7$; es dürften aber wohl Sterne existieren, bei denen γ_0 gleich mehreren Bogensekunden ist; vielleicht auch noch mehr. Es sei A ein großer Stern (Gigant), T die Erde, B ein entfernter Stern; die Winkeldistanz zwischen A und B, von T aus gesehen, sei α, und der Winkel zwischen A und T, von B aus gesehen, sei β. Es ist dann

$$\gamma = \alpha + \beta\,.$$

Ist B sehr weit entfernt, so ist annähernd $\gamma = \alpha$. Es kann also α gleich mehreren Bogensekunden sein, und der Maximumwert von α wäre etwa gleich γ_0. Man sieht den Stern B von der Erde aus an zwei Stellen: direkt in der Richtung TB und außerdem nahe der Oberfläche von A, analog einem Spiegelbild. Haben wir mehrere Sterne B, C, D, so würden die Spiegelbilder umgekehrt gelegen sein wie in einem gewöhnlichen Spiegel, nämlich in der Reihenfolge D, C, B, wenn von A aus gerechnet wird (D wäre am nächsten zu A).

Der Stern A würde als fiktiver Doppelstern erscheinen. Teleskopisch wäre er selbstverständlich nicht zu trennen. Sein Spektrum bestände aus der Übereinanderlagerung zweier, vielleicht total verschiedenartiger Spektren. Nach der Interferenzmethode müßte er als Doppelstern erscheinen. Alle Sterne, die von der Erde aus gesehen rings um A in der Entfernung $\gamma_0 - \beta$ liegen, würden von dem Stern A gleichsam eingefangen werden. Sollte zufällig TAB eine gerade Linie sein, so würde, von der Erde aus gesehen, der Stern A von einem Ring umgeben erscheinen.

Ob der hier angegebene Fall eines fiktiven Doppelsternes auch wirklich vorkommt, kann ich nicht beurteilen.

Petrograd, 1924 Jan. 28.

O. Chwolson.

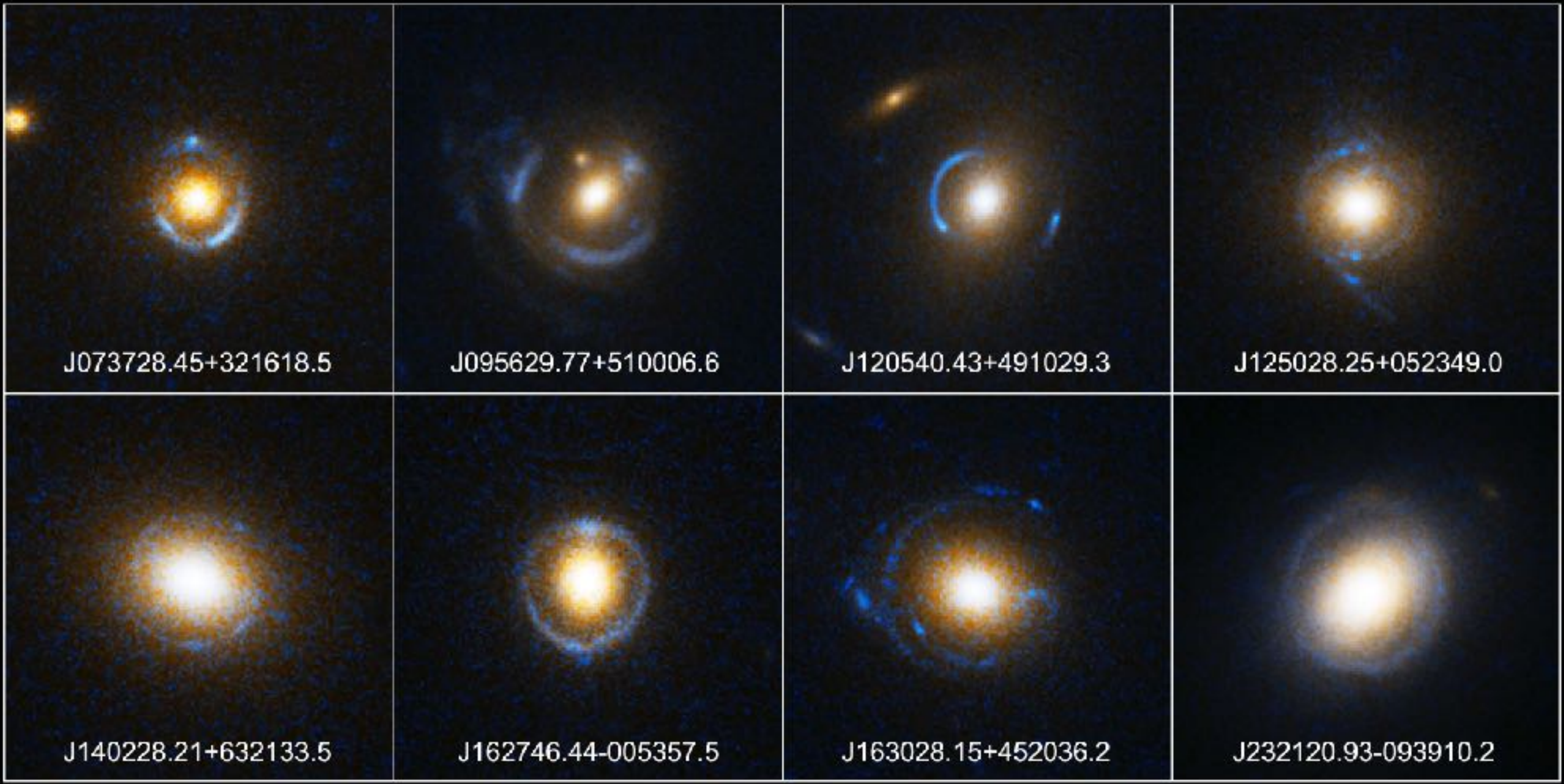
Einstein Ring Gravitational Lenses

Hubble Space Telescope • Advanced Camera for Surveys

NASA, ESA, A. Bolton (Harvard-Smithsonian CfA), and the SLACS Team

STScI-PRC05-32

"Anillos de Chwolson-Einstein"



Einstein Ring Gravitational Lenses

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STScI-PRC05-32

A Relativistic Eclipse

What Might be Seen from a Planet Conveniently Placed Near the Companion of Sirius . . . Perfect Tests of General Relativity that are Unavailable

By HENRY NORRIS RUSSELL, Ph. D.

Chairman of the Department of Astronomy and Director of the Observatory at Princeton University. Research Associate of the Mount Wilson Observatory of the Carnegie Institution of Washington. President of the American Astronomical Society.

It is a familiar aphorism that the theory of relativity, despite its enormous importance, both in physics and philosophy, may be forgotten in ordinary practical life. There is a good reason. In almost every known case its results agree so closely with those of the older "classical" theories that very accurate observations are required to distinguish between them. Thus, for example, the famous Michelson-Morley experiment requires long series of the

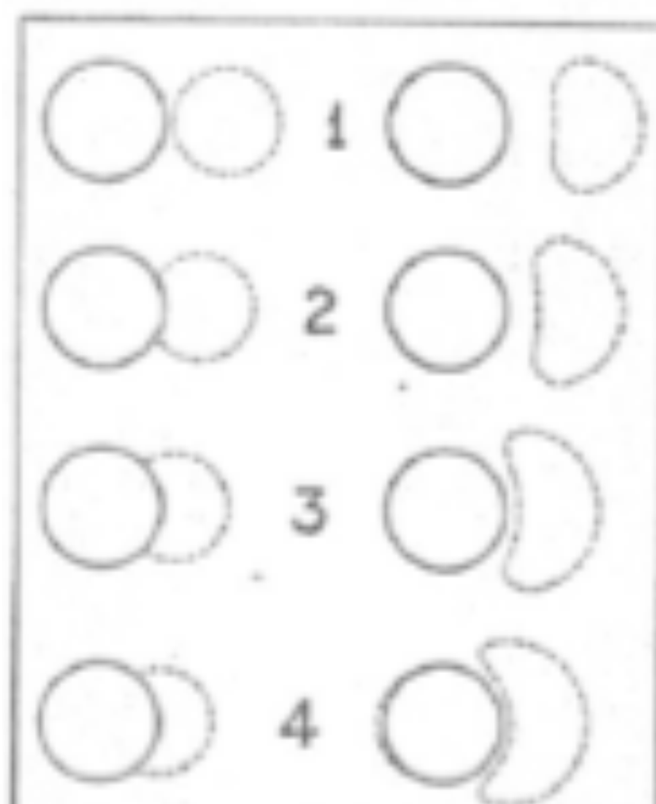
cludes gravitation in its scope) are of this sort. The advance of the perihelion of Mercury ... as an increase ... but definitely calculable, amount due to

stein himself.[1] From a point exactly in line ... be resolved by even the greatest tele-

for it. The focusing effect, however, would ... the light of the dis- ... For a star ... mathematical point, with no angular diameter at all, the increase in brightness would be infinite. But any real star must have a finite angular size, however small, and for such a star the increase in brightness, though it might be large, would have a limit.

If a bright star passed in front of a fainter one, its own light would drown out the effect; but if one of the faint red stars, which are really more abundant than any other sort, should get directly

No one predicted gravitational arcs?



My hearty thanks are due to Professor Einstein, who permitted me to see the manuscript of his note before its publication.—*Princeton University Observatory, December 2, 1936.*

is, a million miles or so—from the line through the center of the two stars. The

Lentes com Simetria Axial

Teorema de Gauss

Equación de Poisson $\nabla_{\xi}^{2}\psi\left(\vec{\xi}\right)=4\pi G\Sigma\left(\vec{\xi}\right)$

Ángulo de deflexión $\hat{\vec{\alpha}}=\frac{2}{c^{2}}\vec{\nabla}_{\xi}\psi\left(\vec{\xi}\right)$

Así: $\vec{\nabla}\cdot\hat{\vec{\alpha}}=\frac{8\pi G}{c^{2}}\Sigma\left(\vec{\xi}\right)$

Teorema de Gauss 2D

$$\int_{S_{\xi}}\vec{\nabla}\cdot\hat{\vec{\alpha}}\,dS=\oint_{C_{\xi}}\left(\hat{\vec{\alpha}}\cdot\hat{n}\right)dl$$

Como en la lente puntual!
Radio de Einstein, etc.

$$\frac{8\pi G}{c^{2}}\underbrace{\int_{S_{\xi}}\Sigma\left(\xi\right)dS}_{M(\xi)}=\int_{0}^{2\pi}\hat{\alpha}\,\xi\,d\phi\;\Rightarrow\;\hat{\vec{\alpha}}=\frac{4GM(\xi)}{c^{2}\xi}\hat{\xi}$$

Lens equation with axial symmetry

Deflection angle $$\hat{\vec{\alpha}} = \frac{4GM(\xi)}{c^2\xi}\hat{\xi}$$

Lens equation $$\vec{\beta} = \vec{\theta} - \hat{\vec{\alpha}}\frac{D_{LS}}{D_{OS}} = \vec{\theta} - \frac{4GM(\theta)}{c^2\theta^2 D_{OL}}\vec{\theta}\frac{D_{LS}}{D_{OS}}$$

$$\vec{\beta} = \left(1 - \frac{4GM(\theta)D_{LS}}{c^2 D_{OL} D_{OS}}\frac{1}{\theta^2}\right)\vec{\theta}$$

Einstein angle $\left(\vec{\beta} = 0\right)$

$$\theta_E = \sqrt{\frac{D_{LS}}{D_{OS}D_{OL}}\frac{4GM\left(\theta_E\right)}{c^2}} \quad \Longrightarrow$$

Mass estimate at $\theta < \theta_E$

Ecuación de la Lente

Ecuación de la Lente $\vec{\beta} = \left(1 - \frac{4GM(\theta)D_{LS}}{c^2 D_{OL} D_{OS}} \frac{1}{\theta^2}\right)\vec{\theta}$

Densidad superficial crítica $\Sigma_{\rm crit} = \frac{c^2}{4\pi G}\frac{D_{\rm OS}}{D_{\rm OL}D_{\rm LS}}$

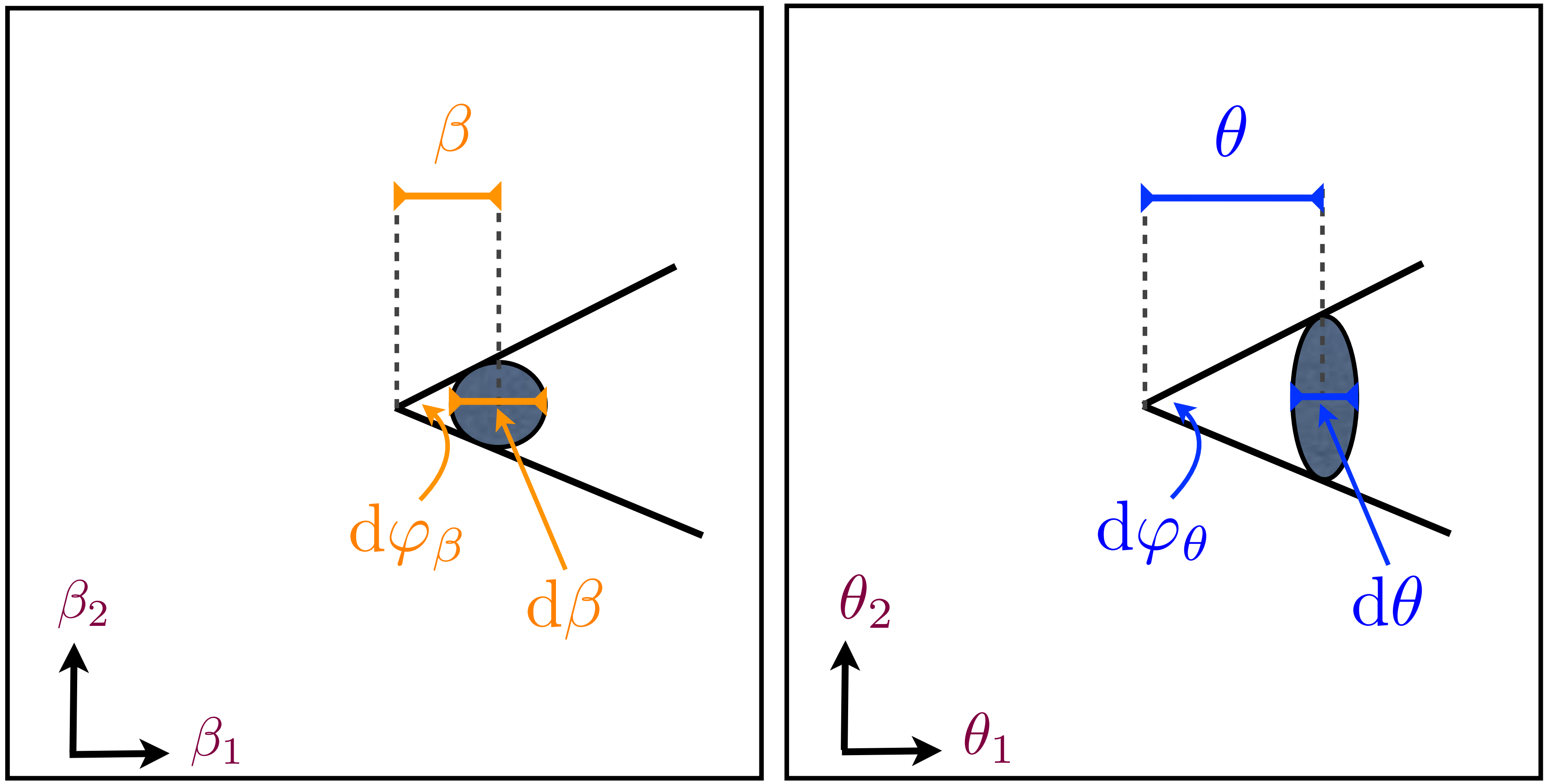
Ecuación de la Lente $\vec{\beta} = \left(1 - \frac{M(\theta)}{\pi\Sigma_{\rm crit}\left(D_{OL}\theta\right)^2}\right)\vec{\theta}$

$$\frac{\text{Massa contida em}\,\theta}{\text{Massa se}\,\Sigma = \Sigma_{\rm crit}}$$

Anillo de Einstein e imágenes múltiplas si

$\Sigma(\theta = 0) > \Sigma_{\rm crit}$ (se densidade monotónicamente descresciente)

Magnificación



Plano das fontes

Lente/imagens

Longitud radial	$\mathrm{d}\beta$	$\mathrm{d}\theta$	Magnificación radial	$\dfrac{\mathrm{d}\theta}{\mathrm{d}\beta}$
Longitud tangencial	$\beta\,\mathrm{d}\varphi_\beta$	$\theta\,\mathrm{d}\varphi_\theta$	Magnificación tangencial	$\underbrace{\dfrac{\mathrm{d}\varphi_\theta}{\mathrm{d}\varphi_\beta}}_{=1}\dfrac{\theta}{\beta}$

Magnificación

Ecuación de la Lente $\vec{\beta} = \left(1 - \frac{4GM(\theta)D_{LS}}{c^2 D_{OL} D_{OS}} \frac{1}{\theta^2}\right) \vec{\theta}$

$$\beta = \theta - \frac{M(\theta)}{\pi \Sigma_{\rm crit} D_{OL}^2 \theta}$$

Magnificación tangencial

$$\frac{\theta}{\beta} = \left(\frac{\beta}{\theta}\right)^{-1} = \left(1 - \frac{M(\theta)}{\pi \Sigma_{\rm crit} D_{OL}^2 \theta^2}\right)^{-1}$$

Magnificación radial

$$\frac{\mathrm{d}\theta}{\mathrm{d}\beta} = \left(\frac{\mathrm{d}\beta}{\mathrm{d}\theta}\right)^{-1} = \left[1 - \frac{1}{\pi \Sigma_{\rm crit} D_{OL}^2} \frac{\mathrm{d}}{\mathrm{d}\theta} \left(\frac{M(\theta)}{\theta}\right)\right]^{-1}$$

Ejercicio: $\gamma_{1,2}$ e κ, variables adimensionales

Esfera isoterma singular

$$M = \sigma_v^2 \frac{\pi}{G} \xi = \sigma_v^2 \frac{\pi}{G} D_{OL} \theta$$

Magnificación tangencial

$$\frac{\theta}{\beta} = \left(1 - \frac{M(\theta)}{\pi \Sigma_{\mathrm{crit}} D_{OL}^2 \theta^2}\right)^{-1} = \left(1 - \overset{\theta_E}{\boxed{4\pi \left(\frac{\sigma_v}{c}\right)^2 \frac{D_{LS}}{D_{OS}}}} \frac{1}{\theta}\right)^{-1}$$

Magnificación radial

$$\frac{\mathrm{d}\theta}{\mathrm{d}\beta} = \left[1 - \frac{1}{\pi \Sigma_{\mathrm{crit}} D_{OL}^2} \frac{\mathrm{d}}{\mathrm{d}\theta} \left(\frac{M(\theta)}{\theta}\right)\right]^{-1} = 1$$

No hay cáustica/
curva crítica
radial