# Course Introduction

## Course Plan

### WEEK 1 INTRODUCTION

Overview / Maths & Cryptography introduction
ZKP Theory 1
Use Cases of ZKPs / L2
ZCash / Zokrates

### WEEK 2 - STARKNET AND CAIRO

Starknet / Cairo 1
Games Development
Cairo 2
Cairo 3

### WEEK 3 MINA AND DEFI

MINA / ZkApps
Snarky.js / Practical
DeFi / Aztec
zkRollups

### WEEK 4 - THEORY AND CRYPTOGRAPHY

STARK Theory
Cryptographic alternatives
Data Privacy
Research and review

## Practical Details

All lessons will be conducted via zoom.
The format will usually be 45 mins of theory followed by 45 mins practical
You can work in teams if you wish
We use Sli.do to provide Q&A and polls : link

## About us


Free Developer Workshops

- Basic
- Enterprise
- Advanced EVM
- Zero Knowledge Proofs

Business Workshops

Website : https://extropy.io

Email : info@extropy.io

Twitter : @extropy

## General Resources

Zero Knowledge Podcast
Alex Pinto's Blog
Intro to ZKPs

# Introduction

## Context

"Human dignity demands that personal information, like medical and forensic data, be hidden from the public. But veils of secrecy designed to preserve privacy may also be abused to cover up lies and deceit by institutions entrusted with Data, unjustly harming citizens and eroding trust in central institutions." - Starkware

"ZK gives out similar vibe as ML. More and more people just mention ZK as a magic solution that fixes everything with no context of its current limitation." - 0xMisaka

# Introductory Maths

## Numbers

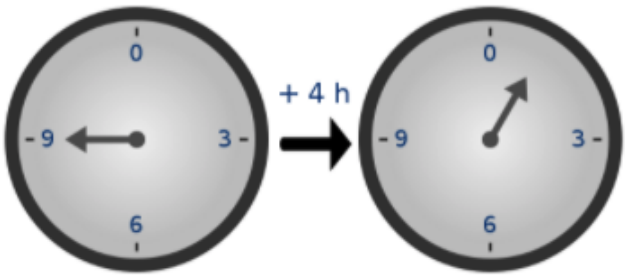
The set of Integers is denoted by $\mathbb{Z}$ e.g. $\{\cdots,-4,-3,-2,-1,0,1,2,3,4,\cdots\}$
The set of Rational Numbers is denoted by $\mathbb{Q}$ e.g. $\{\ldots 1, \frac{3}{2}, 2, \frac{22}{7}\ldots\}$
The set of Real Numbers is denoted by $\mathbb{R}$ e.g. $\{2, -4, 613, \pi, \sqrt{2}, \ldots\}$

Fields are denoted by $\mathbb{F}$, if they are a finite field or $\mathbb{K}$ for a field of real or complex numbers we also use $\mathbb{Z}_p^*$ to represent a finite field of integers mod prime p with multiplicative inverses.

We use finite fields for cryptography, because elements have “short”, exact representations and useful properties.

## Modular Arithmetic

See this introduction




Because of how the numbers "wrap around", modular arithmetic is sometimes called "clock math"

When we write n mod k we mean simply the remainder when n is divided by k. Thus
25 mod 3 = 1
15 mod 4 = 3
The remainder should be positive.

## Group Theory

Simply put a group is a set of elements {a,b,c,...} plus a binary operation, here we represent this as •
To be considered a group this combination needs to have certain properties

1. Closure
   For all a, b in G, the result of the operation, a • b, is also in G
2. Associativity
   For all a, b and c in G, (a • b) • c = a • (b • c)
3. Identity element
   There exists an element e in G such that, for every element a in G, the equation e • a = a • e = a holds. Such an element is unique and thus one speaks of the identity element.
4. Inverse element
   For each a in G, there exists an element b in G, commonly denoted $a^{-1}$ (or −a, if the operation is denoted "+"), such that a • b = b • a = e, where e is the identity element.

### SUB GROUPS

If a subset of the elements in a group also satisfies the group properties, then that is a subgroup of the original group.

### CYCLIC GROUPS AND GENERATORS

A finite group can be cyclic. That means it has a generator element. If you start at any point and then apply the group operation with the generator as argument a certain number of times, you go around the whole group and end in the same place,

### FINDING AN INVERSE

From Fermat's little theorem

$a^{-1} \equiv a^{p-2} (mod p)$
Let p = 7 and a = 2. We can compute the inverse of a as:
$a^{p-2} = 2^5 = 32 \equiv 4$ mod 7.

This is easy to verify: 2 x 4 ≡ 1 mod 7.

### EQUIVALENCE CLASSES

Since
6 mod 7 = 6
13 mod 7 = 6
20 mod 7 = 6
...
we can say that 6, 13, 20 ... form an equivalence class
more formally

modular arithmetic partitions the integers into N equivalence classes, each of the form $i + kN \mid k \in \mathbb{Z}$ for some $i$ between $0$ and $N - 1$.

Thus if we are trying to solve the equation
x mod 7 = 6
x could be 6, 13, 20 ...
This gives us the basis for a one way function.

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## Fields

A field is a set of say Integers together with two operations called addition and multiplication.
One example of a field is the Real Numbers under addition and multiplication, another is a set of Integers mod a prime number with addition and multiplication.
The field operations are required to satisfy the following field axioms. In these axioms, a, b and c are arbitrary elements of the field $\mathbb{F}$.

1. Associativity of addition and multiplication: a + (b + c) = (a + b) + c and a · (b · c) = (a · b) · c.
2. Commutativity of addition and multiplication: a + b = b + a and a · b = b · a.
3. Additive and multiplicative identity: there exist two different elements 0 and 1 in $\mathbb{F}$ such that a + 0 = a and a · 1 = a.
4. Additive inverses: for every a in F, there exists an element in F, denoted −a, called the additive inverse of a, such that a + (−a) = 0.
5. Multiplicative inverses: for every a ≠ 0 in F, there exists an element in F, denoted by $a^{-1}$ , called the multiplicative inverse of a, such that a ·$a^{-1}$ = 1.
6. Distributivity of multiplication over addition: a · (b + c) = (a · b) + (a · c).

### FINITE FIELDS AND GENERATORS

A finite field is a field with a finite set of elements, such as the set of integers mod p where p is a prime.

To try out operations on finite fields, see https://asecuritysite.com/encryption/finite

The **order** of the field is the number of elements in the field's set.

For a finite field the order must be either

- prime ( a prime field)
  or
- the power of a prime (an extension field)

An element can be represented as an integer greater or equal than 0 and less than the field's order: {0, 1, ..., p-1} in a simple field.

Every finite field has a generator. A generator is capable of generating all of the elements in the set by exponentiating the generator .
So for generator g we can take $g^0$, $g^1$,$g^2$ and eventually this will give us all elements in the group

For example. taking the set of integers and prime p = 5, we get the group $\mathbb{Z}_5^*$ = {0,1,2,3,4}. In the group $\mathbb{Z}_5^*$, operations are carried out modulo 5; hence, we don't have 3 × 4 = 12 but instead have 3 × 4 = 2, because 12 mod 5 = 2.

$\mathbb{Z}_5^*$ is cyclic and has two generators, 2 and 3, because $2^1 = 2, 2^2 = 4, 2^3 = 3, 2^4 = 1$, and $3^1 = 3, 3^2 = 4, 3^3 = 2, 3^4 = 1$

In a finite field of order $q$, the polynomial $X^q - X$ has all $q$ elements of the finite field as roots.

### GROUP HOMOMORPHISMS

A homomorphism is a map between two algebraic structures of the same type, that preserves the operations of the structures.

This means a map $f : A \to B$ between two groups A, B equipped with the same structure such that,

if $\cdot$ is an operation of the structure (here a binary operation), then
$f(x \cdot y) = f(x) \cdot f(y)$

To try out operations on finite fields, see https://asecuritysite.com/encryption/finite

For a great introduction see http://coders-errand.com/zk-snarks-and-their-algebraic-structure/

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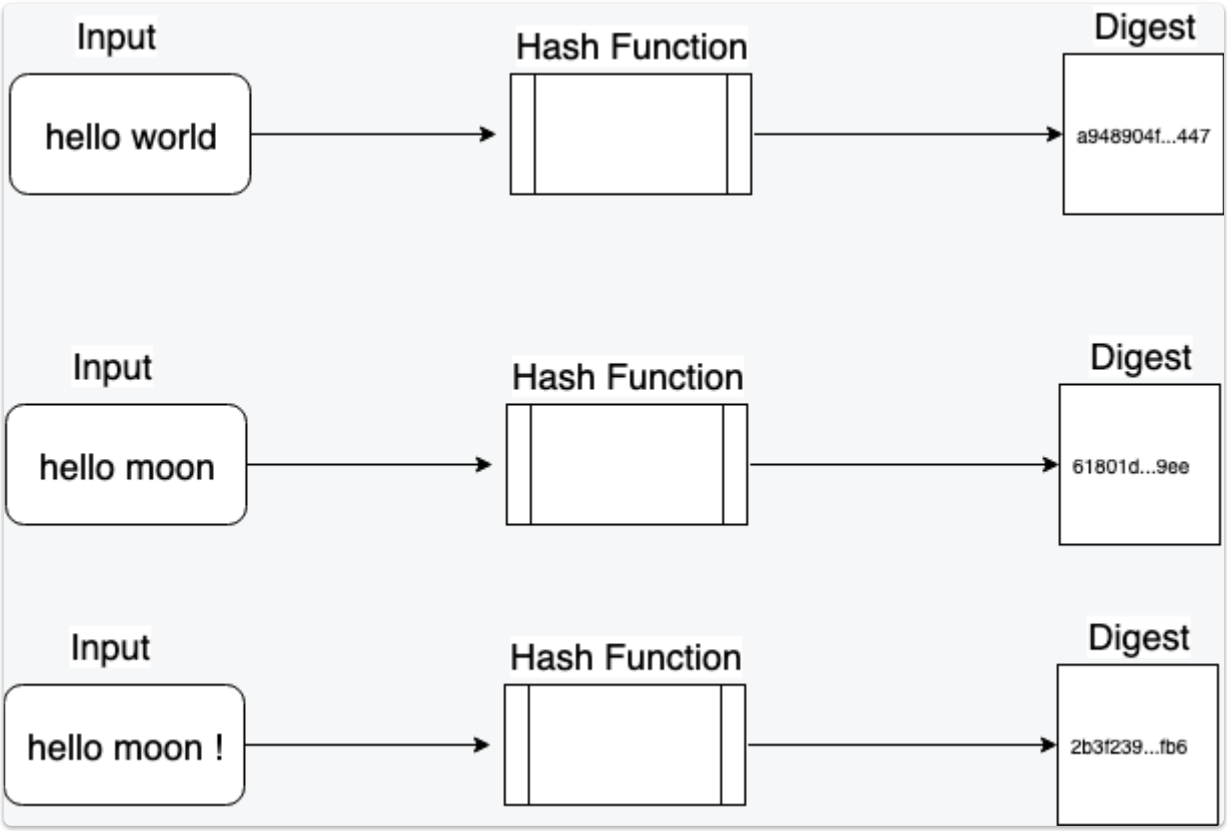
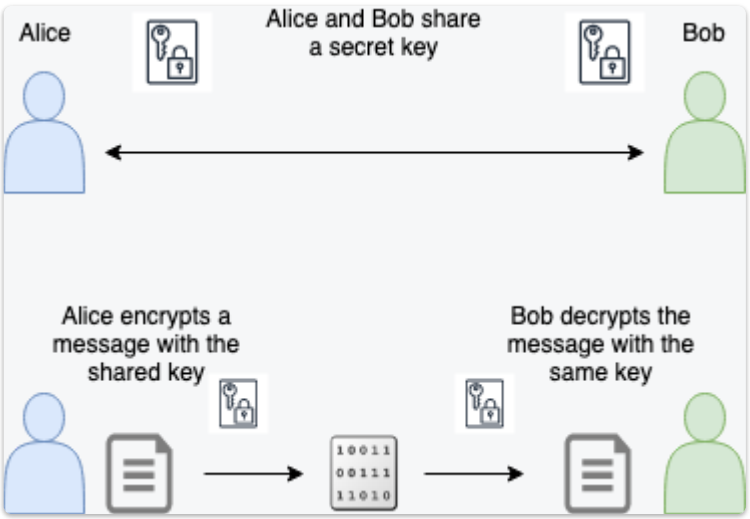
# Cryptography Background

## Hash Functions



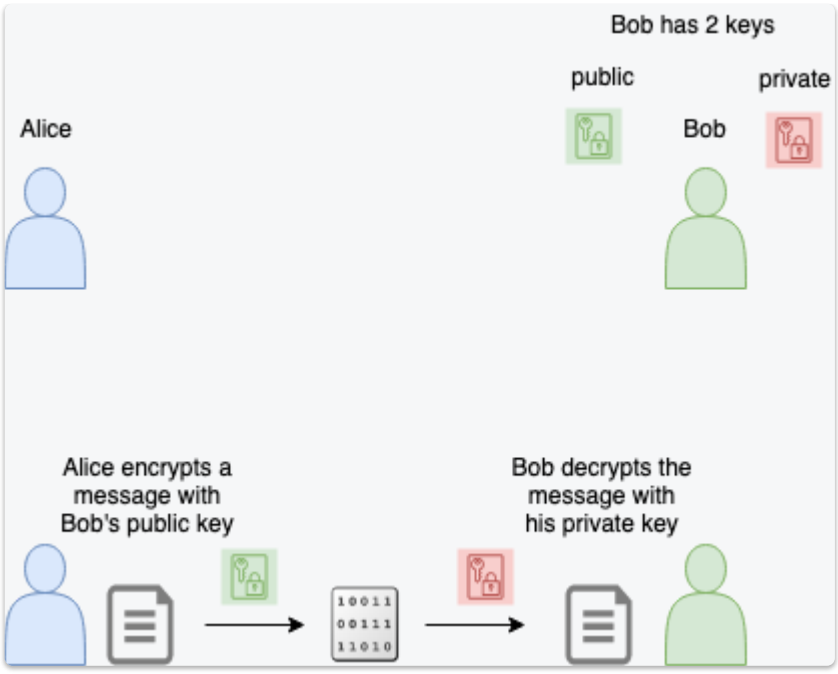

## Encryption

### SYMMETRIC ENCRYPTION

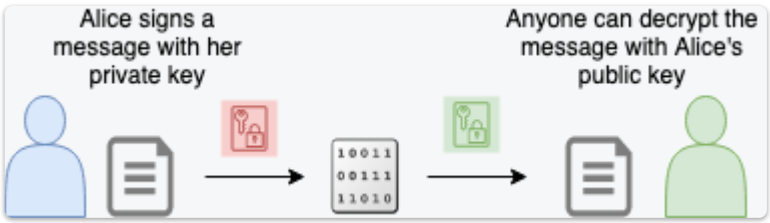



### ASYMMETRIC ENCRYPTION

Sending a secret message




Proving ownership (knowledge of) of a private key

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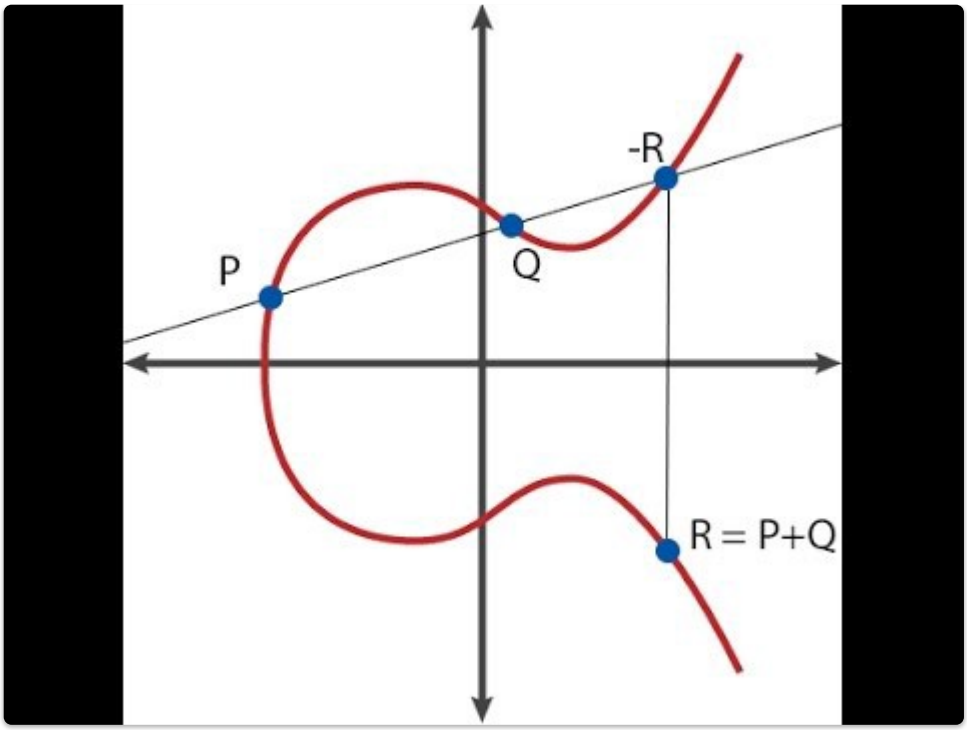
## Elliptic Curves

The defining equation for an elliptic curve is for example $y^2 = x^3 + ax + b$
For certain equations they will satisfy the group axioms

- every two points can be added to give a third point (closure);
- it does not matter in what order the two points are added (commutatitivity);
- if you have more than two points to add, it does not matter which ones you add first either (associativity);
- there is an identity element.

We often use 2 families of curves :

MONTGOMERY CURVES




For example curve 22519 with equation $y^2 = x^3 + 486662x^2 + x$

Curve 25519 gives 128 bits of security and is used in the Diffie–Hellman (ECDH) key agreement scheme
BN254 / BN_128 is the curve used in Ethereum for ZKSNARKS
BLS12-381 is the curve used by ZCash

## EDWARDS CURVES

The general equation is $ax^2 + y^2 = 1 + dx^2y^2$ with a = 1
for some scalar d which can be 0 or 1.

If a <> 1 they are called Twisted Edwards Curves
Every twisted Edwards curve is birationally equivalent to a Montgomery curve

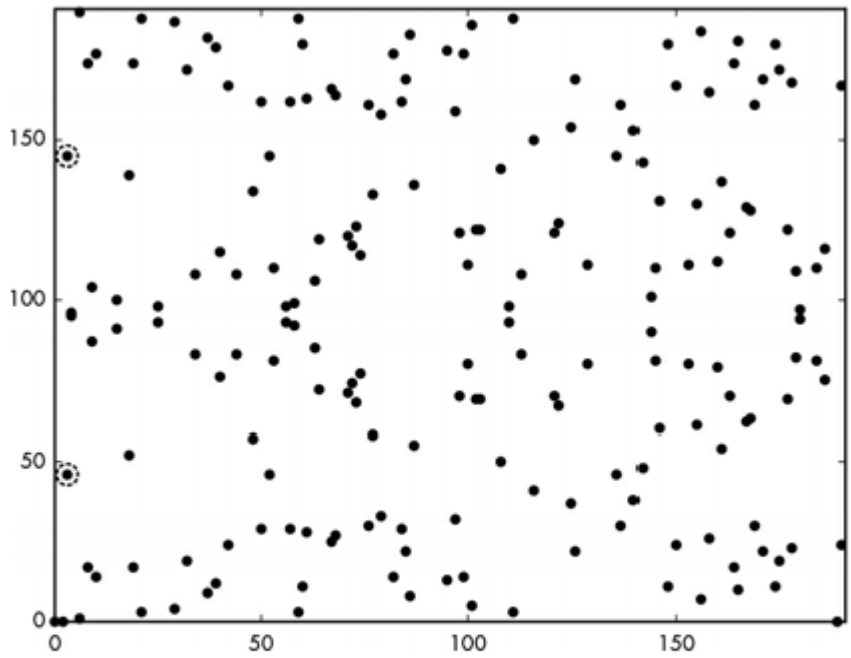



*Figure 12-2: The elliptic curve with the equation* $y^2 = x^3 - 4x$ *over* $\mathbf{Z}_{191}$*, the set of integers modulo 191*

From
Serious Cryptography Jean-Philippe Aumasson

## Verifiable Random Functions

From Algorand VRFs :

A Verifiable Random Function (VRF) is a cryptographic primitive that maps inputs to verifiable pseudorandom outputs. VRFs were Introduced by Micali, Rabin, and Vadhan in '99.

Given an input value x, the knowledge of the secret key SK allows one to compute $y = F_{SK}(x)$ together with the proof of correctness $\pi_x$. This proof convinces every verifier that the value $y = F_{SK}(x)$ is indeed correct with respect to the public key of the VRF. We can view VRFs as a commitment to a number of random-looking bits

The owner of a secret key can compute the function value as well as an associated proof for any input value. Everyone else, using the proof and the associated public key or verification key can check that this value was indeed calculated correctly, yet this information cannot be used to find the secret key.

Algorand have released it as an extension to the libsodium library

Such functions are ideal to find block producers in a blockchain in a trustless verifiable way.

# Intuitive grasp of Zero Knowledge Proofs

## Introduction

It is difficult to find zero knowledge resources that avoid the extremes of either over simplyfying the subject, or presenting so much mathematical detail that the reader gets bogged down and loses interest.
In this course I hope to find an accessible but informative middle ground.

We start with some examples to show how zero knowledge proofs can proceed, and the situations where they could be used.

## What is a zero knowledge proof

A loose definition
It is a proof that there exists or that we know something, plus a zero knowledge aspect, that is the person verifying the proof only gains one piece of information - that the proof is valid or invalid.

## Actors in a Zero Knowledge Proof System

- Creator - optional, maybe combined with the prover
- Prover - I will call her Peggy
- Verifier - I will call him Victor

# Examples to give an Intuitive grasp of zero-knowledge proofs

1. Colour blind verifier
   This is an interactive proof showing that the prover can distinguish between a red and a green billiard ball, whereas the verifier cannot distinguish them.

   ```
   • The prover wants to show the verifier that they have different colours but does not want him to learn which is red and which is green.

   • Step 1: The verifier takes the balls, each one in each hand, holds them in front of the prover and then hides them behind his back. Then, with probability 1/2 either swaps them (at most once) or keeps them as they are. Finally, he brings them out in front.

   • Step 2: The prover has to say the verifier switched them or not.

   • Step 3: Since they have different colours, the prover can always say whether they were switched or not.
   But, if they were identical (the verifier is inclined to believe that), the prover would be wrong with probability 1/2.
   ```

```
• Finally, to convince the verifier with very high probability, the prover could repeat Step 1 to Step 3 k times to reduce the probability of the prover being successful by chance to a extremely small amount.
```

2. Wheres Wally
   Based on the pictures of crowds where Wally is distinctivly dressed, the aim being to find him within a sea of similar people.
   The proof procedes as follows :
   Imagine the Peggy has found Wally in the picture and wants to prove this fact to Victor, however if she just shows him, Victor is liable to cheat and claim he also found Wally.
   In order to prove to Victor that she has indeed found Wally, without giving away his location in the picture

```
1. Peggy cuts a hole in a (very) large sheet of paper, the hole should be the exact shape of Wally in the underlying picture.
2. Peggy places the paper sheet over the original picture, so that the location of the picture beneath the paper is obscured.
3. Victor can then see throught he hole that Wally has indeed been found, but since the alignment with the underlying picture cannot be seen, he doesn't gain any information about the location of Wally.
```

3. Sudoku
   An interactive proof can be created to prove the knowledge of a solution to a sudoku puzzle by placing cards in the sudoku grid. The process is descrived here Sudoku Proof

Quote from Vitalik Buterin
"You can make a proof for the statement "I know a secret number such that if you take the word ‘cow', add the number to the end, and SHA256 hash it 100 million times, the output starts with `0x57d00485aa`". The verifier can verify the proof far more quickly than it would take for them to run 100 million hashes themselves, and the proof would also not reveal what the secret number is."

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## Zero Knowledge Proof Timeline

Changes have occurred because of

- Improvements to the cryptographic primitives (improved curves or hash functions for example)
- A fundamental change to the approach to zero knowledge
  See the excellent blog post from Starkware :
  The Cambrian Explosion

1984 : Goldwasser, Micali and Rackoff - Probabilistic Encryption.
1989 : Goldwasser, Micali and Rackoff - The Knowledge Complexity of Interactive Proof Systems
1991 O Goldreich, S. Micali and A. Wigderson. Proofs that Yield Nothing but their Validity or All Languages in NP Have Zero-Knowledge Proof Systems. Preliminary version in 1986. (Graph colouring problem)

....

2006 Groth, Ostrovsky and Sahai introduced pairing-based NIZK proofs, yielding the first linear size proofs based on standard assumptions.

2010 Groth combined these techniques with ideas from interactive zero-knowledge arguments to give the first constant size NIZK arguments.

2016 : Jens Groth - On the Size of Pairing-based Non-interactive Arguments

From Matthew Green

Prior to Goldwasser et al., most work in this area focused the soundness of the proof system. That is, it considered the case where a malicious Prover attempts to ‘trick’ a Verifier into believing a false statement. What Goldwasser, Micali and Rackoff did was to turn this problem on its head. Instead of worrying only about the Prover, they asked: what happens if you don’t trust the Verifier?

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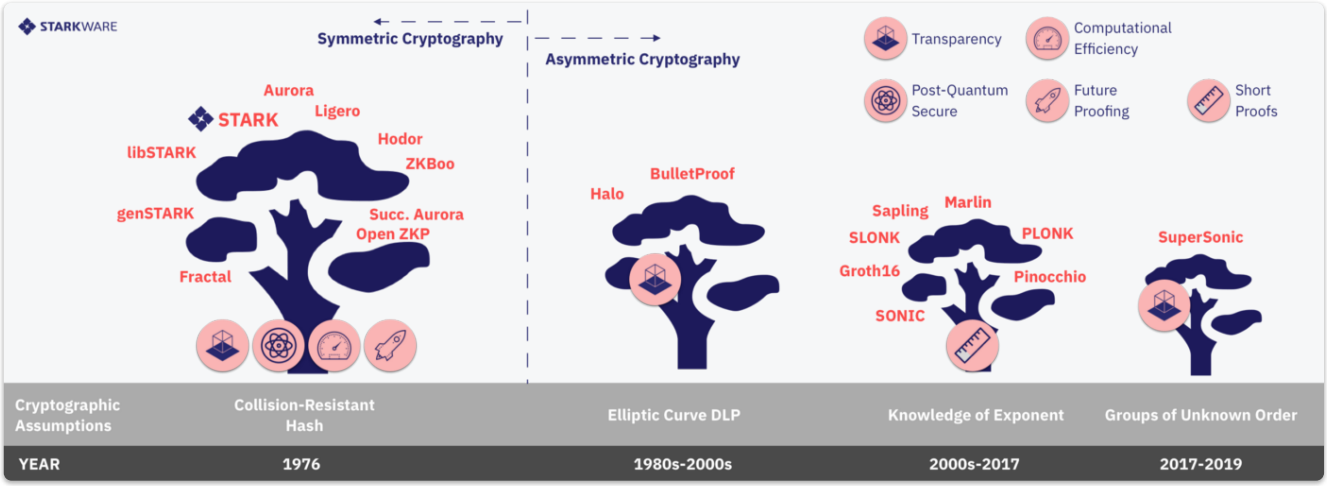
## ZKP ECOSYSTEM




from

The Cambrian Explosion

## Proving Systems

A statement is a proposition we want to prove. It depends on:

- Instance variables, which are public.
- Witness variables, which are private.

Given the instance variables, we can find a short proof that we know witness variables that make the statement true (possibly without revealing any other information).

What do we require of a proof ?

- Completeness: there exists an honest prover P that can convince the honest verifier V of any correct statement with very high probability.
- Soundness: even a dishonest prover P running in super-polynomial time cannot convince an honest verifier V of an incorrect statement. Note: P does not necessarily have to run in polynomial time, but V does.

To make our proof zero knowledge we also need 'zero knowledginess'

To oversimplify: represented on a computer, a ZKP is nothing more than a sequence of numbers, carefully computed by Peggy, together with a bunch of boolean checks that Victor can run in order to verify the proof of correctness for the computation.

A zero-knowledge protocol is thus the mechanism used for deriving these numbers and defining the verification checks.

### INTERACTIVE V NON INTERACTIVE PROOFS

Non-interactivity is only useful if we want to allow multiple independent verifiers to verify a given proof without each one having to individually query the prover.

In contrast, in non-interactive zero knowledge protocols there is no repeated communication between the prover and the verifier. Instead, there is only a single "round", which can be carried out asynchronously.
Using publicly available data, Peggy generates a proof, which she publishes in a place accessible to Victor (e.g. on a distributed ledger).
Following this, Victor can verify the proof at any point in time to complete the “round”. Note that even though Peggy produces only a single proof, as opposed to multiple ones in the interactive version, the verifier can still be certain that except for negligible probability, she does indeed know the secret she is claiming.

### SUCCINT V NON SUCCINT

Succinctness is necessary only if the medium used for storing the proofs is very expensive and/or if we need very short verification times.

### PROOF V PROOF OF KNOWLEDGE

A proof of knowledge is stronger and more useful than just proving the statement is true. For instance, it allows me to prove that I know a secret key, rather than just that it exists.

### ARGUMENT V PROOF

In a proof, the soundness holds against a computationally unbounded prover and in an argument, the soundness only holds against a polynomially bounded prover.
Arguments are thus often called "computationally sound proofs".

The Prover and the Verifier have to agree on what they’re proving. This means that both know the statement that is to be proven and what the inputs to this statement represent.

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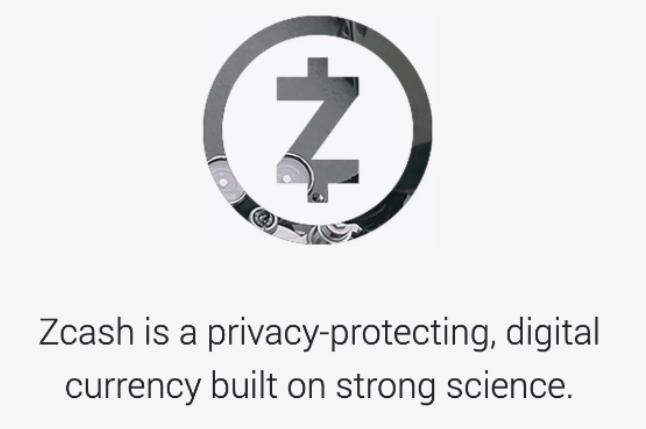
## ZKP Use Cases

### Privacy preserving cryptocurrencies





Also Nightfall , ZKDai

### Blockchain Scalability

For example
Rollups on Ethereum

"The scalability of ZK rollup will increase by up to 4x, pushing theoretical max TPS of such systems well over 1000." - Vitalik

ZkSync
ZK Sync is designed to bring a VISA-scale throughput of thousands of transactions per second to Ethereum.

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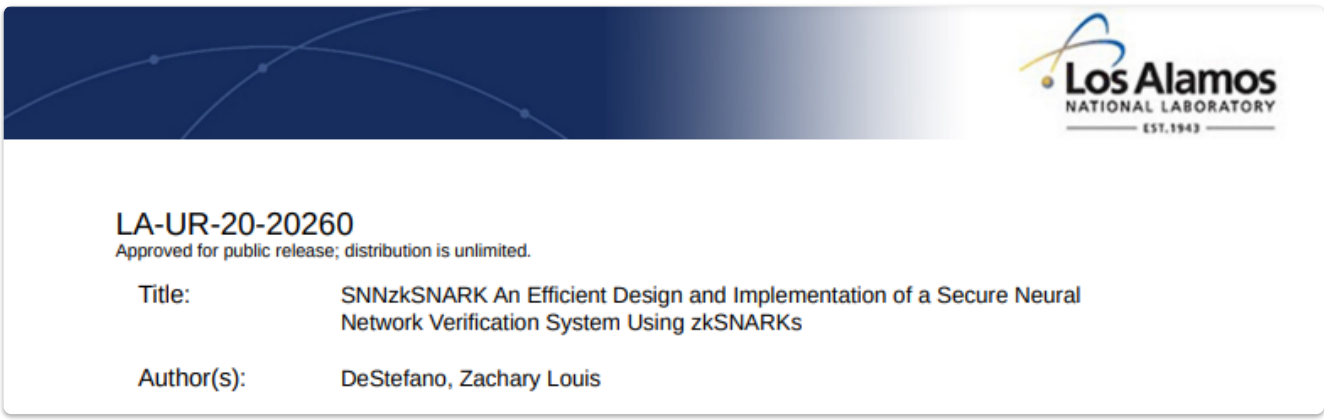
### Nuclear Treaty Verification

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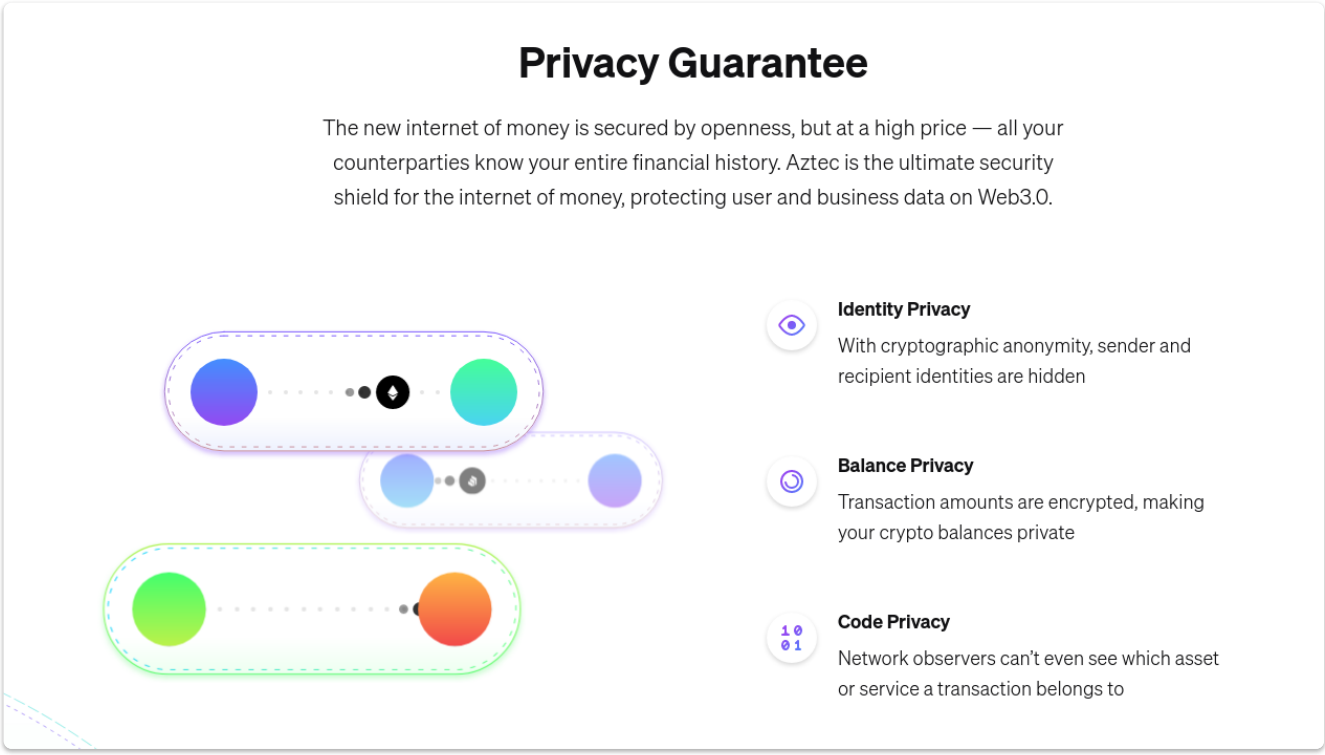
## Privacy Preserving Financial Systems

### Aztec




### Identity and Privacy