



DESIGN OF MECHANICAL SYSTEM

Mechanical Engineering , Semester –VIII, University of Mumbai



PROF. SANJAY W. RUKHANDE
FR. C. RODRIGUES INSTITUTE OF TECHNOLOGY, VASHI
NAVI MUMBAI

Preface

It gives me immense pleasure to present this compilation on Design of Mechanical system. This content has been compiled specially for Final Year Semester VIII students of Mechanical Engineering in University of Mumbai. Numerous solved design have been added for the benefit of student community and teaching faculty. Design Data Book by PSG and IC Engine Design Data Book by Kale and Khandare is referred for design.

Despite my best efforts, should some mistakes have crept in, these may kindly be brought to my notice. I welcome constructive criticism for further improvement of this compilation.

I would like to express my thanks to all those people who directly or indirectly contributed. My special thanks to all my students whose continuous feedback is the source of inspiration. A feedback in the form of suggestion and comments from the readers will be highly appreciated.

- **Sanjay W. Rukhande**

SR. No.	CONTENTS	Page No.
1	Syllabus	3
2	Module 1. Introduction	4
3	Design Methodology	4
4	Morphology of design	6
5	Optimum design and system concept in design	8
6	Module 2. Design Of Hoisting Mechanism	10
7	Numerical 1 on Design of Hoisting Mechanism	38
8	Numerical 2 on Design of Hoisting Mechanism	60
9	Module 3: Design of Belt Conveyor System	82
10	Numerical 1 on Design of Belt Conveyor System	106
11	Numerical 2 on Design of Belt Conveyor System	116
12	Numerical 3 on Design of Belt Conveyor System	128
13	Module 4: Design of Internal Combustion Engines	138
14	Numerical 1 on Design of Diesel Engine	163
15	Numerical 2 on Design of Petrol Engine	174
16	Numerical 3 on Design of Petrol Engine	184
17	Module 5: (I) Design of Centrifugal Pump	197
18	Numerical 1 on Design a centrifugal pump	217
19	Numerical 2 on Design a centrifugal pump	228
20	Module 5: (I) Design of Gear Pump	239
21	Numerical 1 on Design of a Gear pump	250
22	Numerical 2 on Design of a Gear pump	262
23	Module 6: Design Of Machine Tool Gear Box	275
24	Numerical 1 on Design of a Machine Tool Gear Box	295
27	Numerical 2 on Design of a Machine Tool Gear Box	296
28	Numerical 3 on Design of a Machine Tool Gear Box	305

SYLLABUS

Course Code: MEC801/Subject: Design of Mechanical Systems/Credits: 4+1

Objectives

1. To study system concepts and methodology of system design.
2. To study system design of various systems such as snatch block, belt conveyors, engine system, pumps and machine tool gearbox.

Outcomes: Learner will be able to...

1. Design material handling systems such as hoisting mechanism of EOT Crane, belt conveyors.
2. Design engine components such as cylinder, piston, connecting rod and crankshaft from system design point of view.
3. Design pumps for the given applications.
4. Prepare layout of machine tool gear box and select number of teeth on each gear.

Modules	Detailed Content	Hrs.
01	Methodology & Morphology of design. Optimum design, System concepts in design.	04
02	Design of Hoisting mechanism: Design of Snatch Block assembly including Rope selection, Sheave, Hook, Bearing for hook, cross piece, Axle for sheave and shackle plate, Design of rope drum, selection of motor with transmission system.	10
03	Design of belt conveyors -- Power requirement, selection of belt, design of tension take up unit, idler pulley.	06
04	Engine Design (Petrol & Diesel): Design of Cylinder, Piston with pin and rings, Connecting Rod & Crank Shaft with bearings.	10
05	Design of pump : Design of main components of gear pump: 1. Motor selection 2. Gear design 3. Shaft design and bearing selection 4. Casing and bolt design 5. Suction and delivery pipe. Design of main components of centrifugal pump: 1. Motor selection 2. Suction and delivery pipe 3. Design of Impeller, Impeller shaft, 4. Design of Volute casing.	10
06	Design of gear boxes for machine tool applications: (Maximum three stages and twelve speeds): Requirements of gear box, determination of variable speed range, graphical representation of speeds, structure diagram, ray diagram, selection of optimum ray diagram, estimation of numbers of teeth on gears, deviation diagram, layout of gear box.	08

Module 1

Methodology and Morphology of Design

1.1 Introduction

A collection of components with interrelated performance is called a system. Also system is defined as a collection of various entities put together to get desired output. A large system are made of several subsystem. There are many complex system, examples are power plants, bridges, highways, automobiles, plane, satellite, supercomputer, missiles, space shuttle, pump, human body, a group of interacting mechanical and electrical components and many more. For design and development of a system lot of issues need to take into accounts. Design and development has been developed for ages. It took so many years to evolve from civilization. System design made systematic approach to reduce error or avoid accident. In a complex system design life cycle approach is very important where design, development, deployment, maintenance and disposal of the system are taken care.

A mechanical system consists of

1. a power source and actuators that generate forces and movement
2. a system of mechanisms that shape the actuator input to achieve a specific application of output forces and movement, and
3. a controller with sensors that compares the output to a performance goal and then directs the actuator input.

This can be seen in EOT crane, where the power is provided by motor to hoist drum through gear box and the rotation of the drum converted to linear movement of the rope and hence hoisting load. Similarly Belt Conveyor system, Centrifugal pump, Gear pump, IC Engine and Gear Box system are designed for desire output.

Power flow through a mechanical system provides a way to understand the performance of devices ranging from levers and gear trains to automobiles and robotic systems.

1.2 Design methodology

The design is the use of imagination, scientific principles and technical information of any mechanical structure or system to perform pre specified function with maximum efficiency and economy. The design methodology flow chart for system design is shown in figure 1.1. It begins with the preparation of specification list and ends with blue prints of design. The designer have various option in between as per availability of resources.

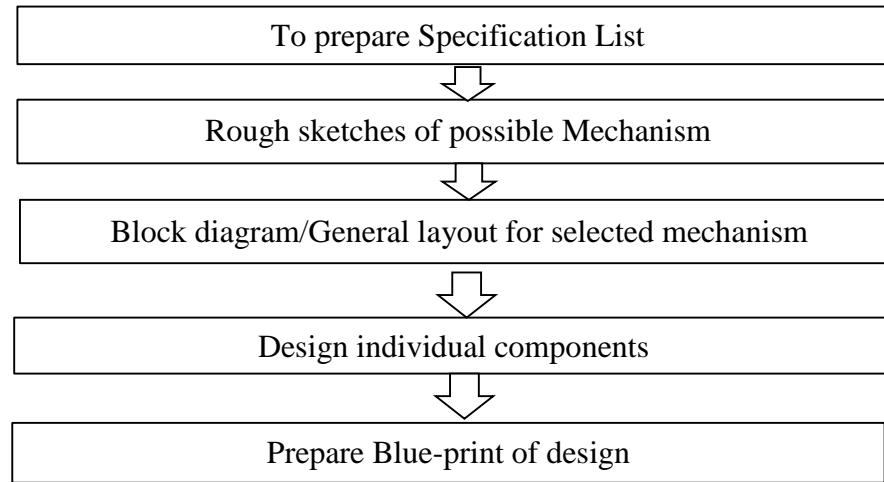


Fig. 1.1 Flow chart for Design Methodology

Step1

To prepare list of specification including the requirement of the products. These requirements include output capacity of product, its service life, Cost, Reliability for some cases. It may also include the dimensions and weights of the components. For example: In the design of Motor Bike, the list of the specification will be,

- a) Fuel Consumption
- b) Maximum Speed
- c) Carrying Capacity
- d) Overall Dimensions (Length, Width and Height)
- e) Weight
- f) Cost

Step 2

To prepare rough sketches of different possible mechanism. After carefully study of requirements of the product, the designer prepare rough sketches of different possible mechanism. For Example: In the design of punching and blanking, the following mechanism are possible.

- a) Mechanism involving crank and connecting rod mechanism, converting rotary motion of the shaft into the translatory motion of the punch.
- b) Mechanism involving nut and screw. It is simple and cheap configuration but having poor efficiency.
- c) Mechanism involving Hydraulic Cylinder, Piston and Valves which is costly but highly efficient.

All possible mechanism are compared with each other depending upon their cost competitiveness, availability of raw materials and manufacturing facility and Best mechanism is selected.

Step 3

To prepare a Block diagram of general layout of selected configuration. For example the layout of EOT crane will include following components.

- a) Motor for the supply of power
- b) Clutch to connect and disconnect the motor at the will of operator
- c) Coupling to connecting main shaft and clutch shaft.
- d) Gear Box to reduce speed from motor speed to required speed.
- e) Rope Drum to convert rotary motion to translatory motion of the wire rope.
- f) Rope, pulley system and Hook to attach the load.

Step 4

To design individual components which involve following steps.

- a) Determine the forces acting on the components.
- b) Select the material of the components, depending upon the functional requirement such as strength, rigidity, hardness, wear resistance, fatigue resistance.
- c) Determine the most likely mode of failure and depending upon it select the criteria of the failure such as yield strength, ultimate tensile strength, fatigue strength and permissible deflection.
- d) Determine the dimensions of the components by assuming suitable factor of safety and the dimensions for the manufacturing facility.

Step 5

To prepare blue print of the design of assembly and components. The dimensions, tolerances surface finish and manufacturing methods are specified on the drawing.

1.3 Morphology of Design

Morphology of design implies step by step procedure involved in the phase of design process. The flow chart for the morphology is shown in figure 1.2.

Design process begin with the realization of unfulfilled need of the society and ends with satisfying them. Engineers and designers are concerned with the application of technology to satisfy human need. Need statements may not be available straight forward always. There may be various options available. In design process the resources are transformed into needed system. Sometimes possible solution are precluded by stating need improperly.

Feasibility study, Preliminary design and detailed design are the primary design phases and remaining are phases in production consumption cycle.

The phases in the morphology of design begins with the primitive need which involves the main objective of design and market survey.

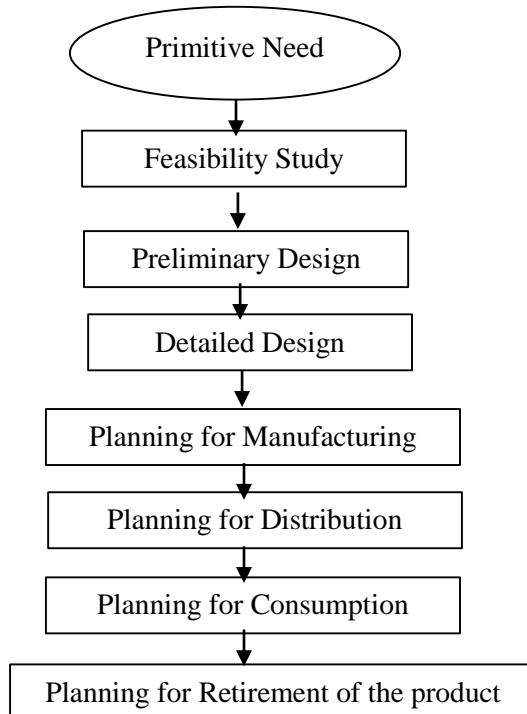


Fig. 1.2 Flowchart for the phases in the morphology of design

1.3.1 Feasibility Study involves

- a) Verifying current existence of need.
- b) Exploring design problem with constraints.
- c) Effort to seek number of feasible solution.
- d) Sorting of potential useful solution from feasible set.

1.3.2 Preliminary design involve

- a) Formation of mathematical model.
- b) Sensitivity analysis, Compatibility analysis, Stability analysis.
- c) Formal optimization.
- d) Projection in future.
- e) Testing design concept and simplification.

(Methods like FEA, CFD are used for design analysis)

1.3.3 Detailed Design involves

- a) Exploration in the large scale come to an end.
- b) Simulation analysis may be tried.
- c) Components and partial prototype can be tested.

1.3.4 Planning for Manufacturing involves

- a) The decision to produce.
- b) Economic commitment and financial capabilities
- c) Business condition before arriving at a final decision.

1.3.5 Planning for Distribution involves

- a) Designing the packaging of the product.
- b) Planning warehouse system.
- c) Planning Promotional activities.

1.3.6 Planning for Consumption involves

- a) Design for maintenance.
- b) Design for reliability.
- c) Design for safety.
- d) Aesthetic feature and operational economy.
- e) Adequate duration of service.

1.3.7 Planning for Retirement

- a) Often goods are retired more frequently because of technical obsolescence than for physical deterioration.
- b) Sometime it may be deliberate attempt from the industry so as to bring in new improved product range.

1.4 Optimum design and system concept in design

Optimum design is the process of selecting the best possible design satisfying certain criteria among many feasible design or adequate design. Some are always better than other. Design of complex system requires large calculation often repetitively for various combinations of design variable. The computer can perform exceedingly complex calculations and process large data efficiently. Many optimization tools are available today. Prediction of future behaviour is not deterministic. The development of probabilistic design and optimization techniques like Artificial Neural Network (ANN), Genetic Algorithm (GA), Particle Swarm Optimization (PSO) etc. have enable designer to predict the future behaviour. In optimum design desirable effects are maximized and undesirable effects are minimised. For Optimisation minimising parameters are cost, weight, size, stress, deflection etc. and maximizing parameters are Power transmitting capacity, load carrying capacity, energy storing capacity etc.

There are three design parameters:

1. Functional required parameter
2. Material parameter and
3. Geometrical parameters.

For Optimum design, Johnson method is popular method. In this method Johnson suggested three different equation.

1. Primary Design Equation (PDE): These are most important equations which takes care of cost, weight and significant undesirable effect to be minimised.
2. Subsidiary Design Equation (SDE): These are other than primary design equations like stress equation.
3. Limit Equation (LE): Ranges of certain parameters are expressed in Limit Equation.

There are three categories of optimum design,

1. Normal Specification
2. Redundant Specification and
3. Independent Specification

1.5 Optimum design procedure for Normal Specification

1. Selection of geometrical parameters.
2. Decide basis for optimum design.
3. Write Primary Design Equations (PDE).
4. Write all Subsidiary Design Equation (SDE) and Limit Equation (LE).
5. Classified all parameters from PDE, SDE and LE.
6. Combine SDE with PDE and eliminate common unspecified parameter.
7. Combined PDE with LE and eliminate common parameters.
8. Using PDE, find the material selection parameters.
9. Determine optimum values of eliminated parameters using original SDE.

Module 2

Design of Hoisting Mechanism

2.1 Introduction

For lifting and conveying material within the unit of an industry, an overhead crane or a bridge crane is used. An overhead crane consists of snatch block, trolley, trolley travelling mechanism and cross travel mechanism to cover an entire area of unit. A hoist of a crane, travels along the bridge. An overhead cranes are used for either manufacturing or maintenance applications. These cranes are either Human operated or remote operated cranes.

2.2 Application

In manufacturing plants at every process the material is handled by crane till finished product leaves a factory. For pouring Raw materials into a furnace, for rolling hot metal to specific thickness, for tempering, annealing and for storing purpose an overhead crane is used. For lifting finished product and loading in truck or train an overhead crane is used. Many industries including automobile uses an overhead crane to handle the steel, raw material and finished product in the factory. Small cranes, such as jib cranes handle lighter loads in a work area, such as CNC mill. An overhead crane is used in the refinement plants of metals like steel, copper, aluminium etc.

For regular maintenance in paper mills like removal of heavy press rolls and other equipment bridge cranes are required. For installing drying drums and other massive equipment, the bridge cranes are used.

2.3 Configuration

Based on applications, Overhead cranes are manufactured in a number of configurations. Some are mentioned below.

2.3.1 EOT Crane (Electric Overhead Traveling Crane)

These cranes are electrically operated by a control pendant, radio/IR remote pendant or from an operator cabin attached with the crane itself. In most factories EOT crane is used which is most common type of overhead crane.

2.3.2 Rotary overhead crane

Rotary overhead crane has one end of the bridge mounted on a fixed pivot while the other end carried on an annular track; the bridge traverses the circular area beneath. This crane is used for longer reach and to eliminate lateral strains on the walls.

2.4 History

In 1876 Sampson Moore in England designed and supplied the first electric overhead crane to hoist guns at the Royal Arsenal in Woolwich, London. This crane was in service till 1980, and is now in a museum in Birmingham, Alabama. Over the years, important innovations, such as the Weston load brake and the wire rope hoist were implemented. The original hoist contained components mated together in the built-up style hoist. They also provide for easier maintenance. Now many hoists are package hoists, built as one unit in a single housing, generally designed for ten-year life, but the life calculation is based on an industry standard. The true life calculation is based on load and hours used. Fig. 2.1 show an example of steam powered overhead crane from 1875.

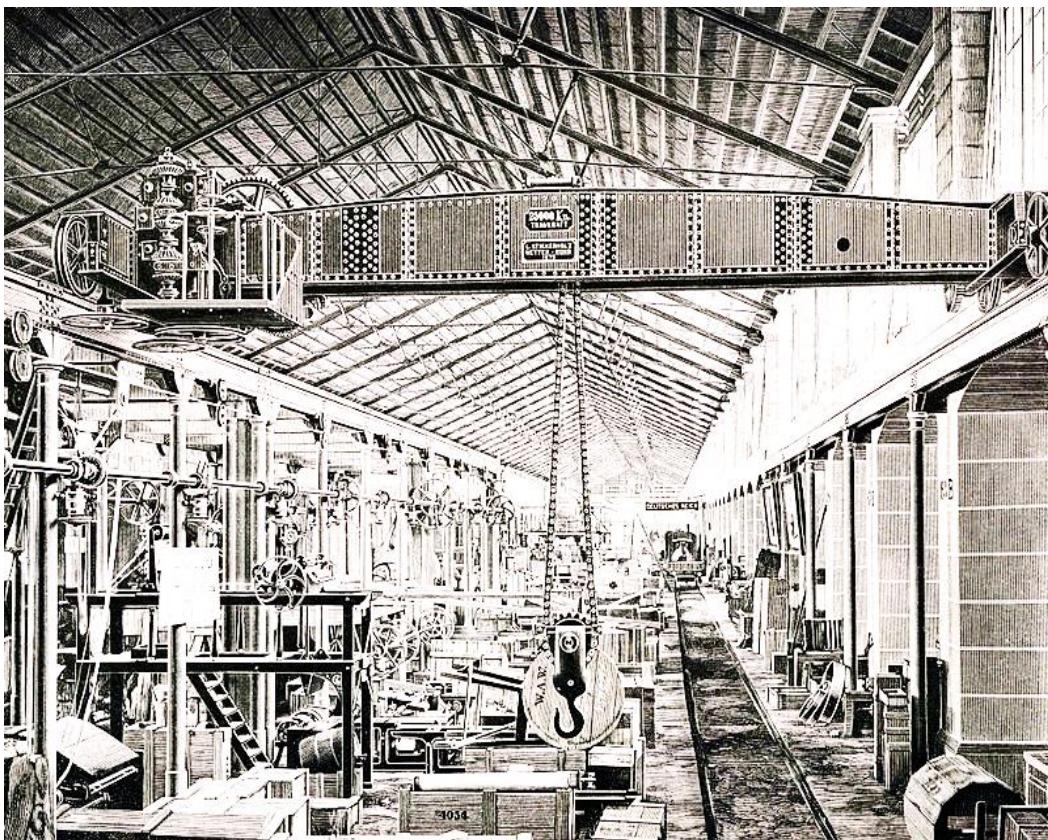


Fig. 2.1 Example of steam powered overhead crane from 1875, produced by Stuckenholz AG, Wetter an der Ruhr, Germany. Design developed by Rudolf Bredt from an original installation at Crew [1]

2.5 Introduction to Hoisting

Hoisting is the process of lifting and lowering some material or load or person from lower position to higher position with the help of some device or mechanism. Fig. 2.2 shows an electrically operated overhead travelling crane mechanism where hook block is attached to the trolley which can move on the bridge girder and bridge girder can move in cross way on runway rail.

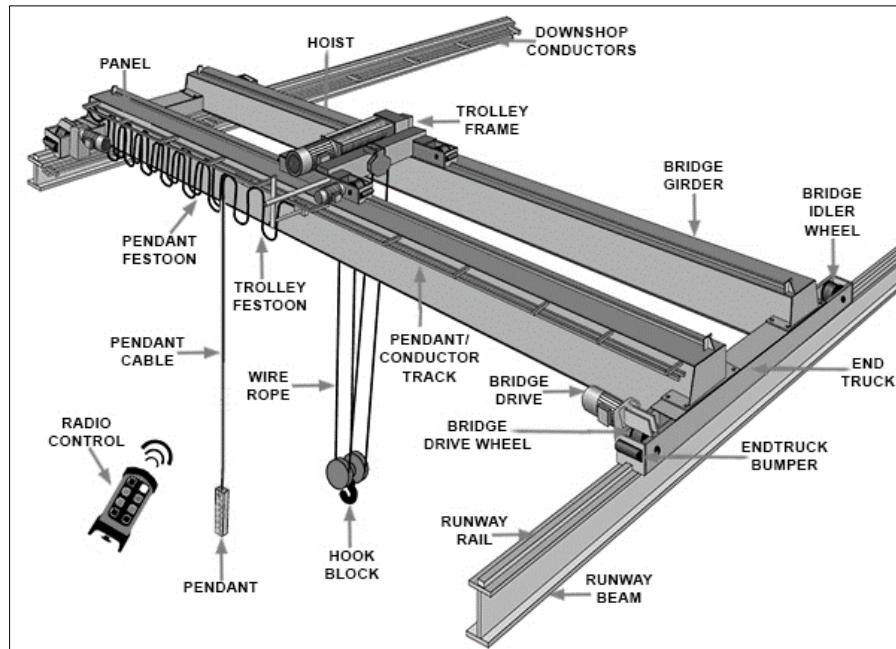


Fig. 2.2 Electrically operated overhead travelling crane

2.5.1 Hoisting Devices

A hoisting device is used for lifting or lowering a load by means of a drum or lift-wheel with the help of rope or chain. It may be manually operated, electrically or pneumatically driven and may use chain, fibre or wire rope as its lifting medium. Examples: Elevators, crane etc.

The hoisting part of the EOT crane consists of the following parts,

1. Hoist motor, 2. Gear box, 3. Drum, 4. Pulleys, 5. Wire rope, 6. Hook

A hoist motor is used as a driving system for the mechanism. The motor is coupled to a gearbox and the gear box is coupled to the rope drum. The rope is wounded on the rope drum. The pulleys are arranged with some rope falls. At the bottom of the pulley the hook is attached with the help of a thrust bearing and cross piece. Figure 2.3 show the schematic block diagram for the hoisting mechanism with four fall system.

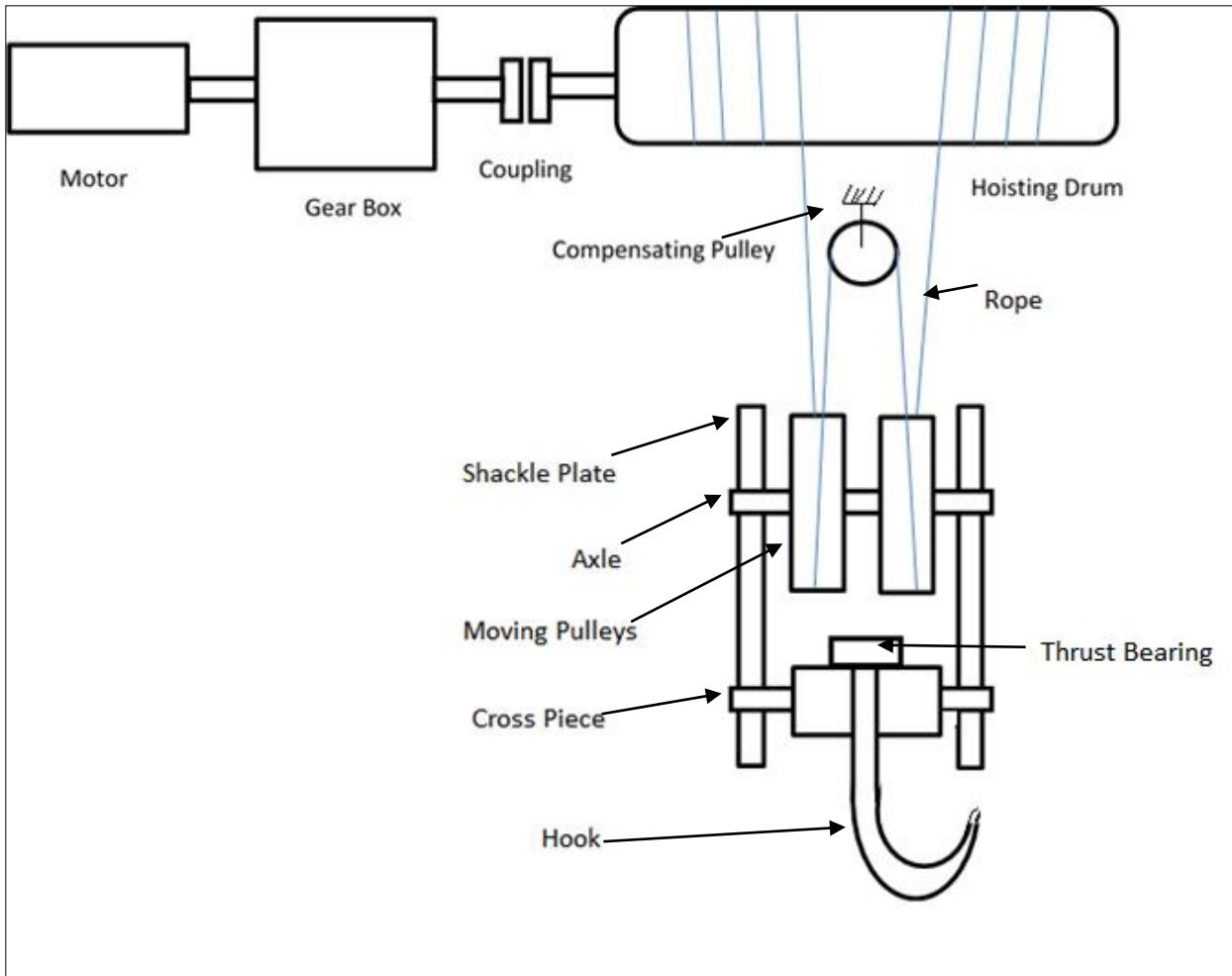


Fig. 2.3 Schematic view of the hoisting Mechanism

2.5.2 Requirement of Hoisting Mechanism

- 1. Kind of properties of loads to be handled:** For unit loads – their form, weight, convenient bearing surface or parts by which they can be suspended like brittleness, temperature, etc. for bulk loads – lump size, volume weight, friability and the amount of crumbling liable to occur during shipments like temperature, chemical properties, etc.
- 2. Required hourly capacity of unit:** A practically unlimited hourly load moving capacity can be easily obtained with certain types of devices as with some continuous-action conveyors. On the other hand, there are devices such as power driven trucks or overhead travelling cranes following a definite cycle of movements with a return idle run.
- 3. Direction and length of travel:** Various types of devices can carry loads in a horizontal or vertical direction or at an angle to the horizontal. Some devices can easily negotiate track curves while others move only rectilinearly, in one direction.
- 4. Methods of stacking loads at the initial, final and intermediate points:** Loading in vehicles and unloading at their destination differ considerably because some handling

machines can be loaded mechanically while others require special auxiliary fixture or manual power.

5. **Characteristics of production processes involved in moving loads:** This is the most important factor essentially influences the choice of the type of transporting facility. As a rule, the movements of the materials handling equipment are closely linked with and depend on manufacturing process; sometimes these movements are directly involved in the performance of certain processing operation.
6. **Specific local conditions:** It includes the size and shape of the area, type and design of the building, ground relief, possible arrangement of the processing units, dust or humidity conditions in the premises, temperatures etc.

2.5.2 Type of hoisting mechanism

a. Hoisting Machines

Hand Trolley Hoists, Portable power-operated hoists, travelling power-operated hoists, Winches, Crane Trolleys

b. Cranes

Stationary Rotary Cranes, Cranes travelling on guide rails, Trackless Cranes, Bridge type Cranes, Cable Cranes

c. Elevators

Portable air-operated hoists, Manually Propelled Stackers, Vertical skip elevators, Mast-type elevators, Funiculars.

2.6 Snatch Block Assembly

A snatch block is essentially a pulling block assembly which is used specifically to increase the load pulling capacity of a winch. A Snatch Block is a pulley system to aid with winching. The unit is essentially a pulley block assembly which opens to allow the easy connection of a looped rope. The block or 'sheave' is a wheel with a grooved edge which carries the rope or cable. The side plates or check plates house the wheel assembly. A snatch block can effectively allow double the line which in turn doubles the lifting capacity of the snatch block and winch arrangement. A single snatch block or multiple snatch blocks can be deployed together to maximize lifting load capacity. Figure 2.4 shows the snatch block for four fall system. It consists of two movable pulleys, pulley bearings , pulley axle, side plate, shackle plate, crane Hook, cross piece, thrust bearing, washer, nut, spacer etc.

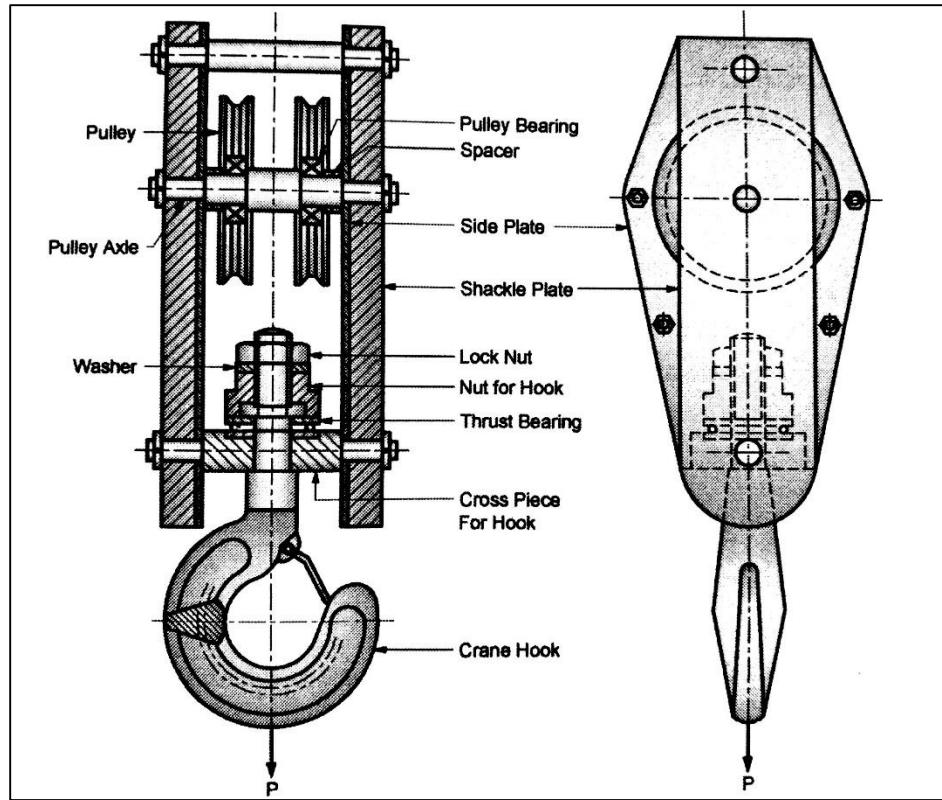


Fig. 2.4 Snatch Block Assembly

2.6.1 Pulleys

Pulleys are manufactured in fixed and movable designs. Pulleys with fixed axles are guiding because they change the direction of flexible hoisting appliance. A rope and pulley system is called a pulley block which is characterized by the use of a single continuous rope to transmit a tension force around one or more pulleys to lift or move a load. This system is also called as a simple machine.

If the rope and pulley system does not dissipate or store energy, then its mechanical advantage is given by the number of parts of the rope that act on the load. This is explained as follows.

Consider the set of pulleys that form the moving block and the parts of the rope that support this block. If there are p of these parts of the rope supporting the load W , then a force balance on the moving block shows that the tension in each of the parts of the rope must be W/p . This means the input force on the rope is $T=W/p$. Thus, the block and tackle reduces the input force by the factor p .

Different types of pulleys are as follows:

1. **Fixed:** A fixed pulley has an axle mounted in bearings attached to a supporting structure. A fixed pulley changes the direction of the force on a rope or belt that moves along its circumference. Mechanical advantage is gained by combining a fixed pulley with a movable pulley or another fixed pulley of a different diameter.
2. **Movable:** A movable pulley has an axle in a movable block. A single movable pulley is supported by two parts of the same rope and has a mechanical advantage of two.
3. **Compound:** A combination of fixed and some movable pulleys forms a block and tackle. A block and tackle can have several pulleys mounted on the fixed and moving axles, further increasing the mechanical advantage.



Fig. 2.5 (a) The gun tackle “rove to advantage” has the rope attached to the moving pulley. The tension in the rope is $W/3$ yielding an advantage of three. (b) The Luff tackle add a fixed pulley “rove to disadvantage.” The tension in the rope remains $W/3$ yielding an advantage of three.

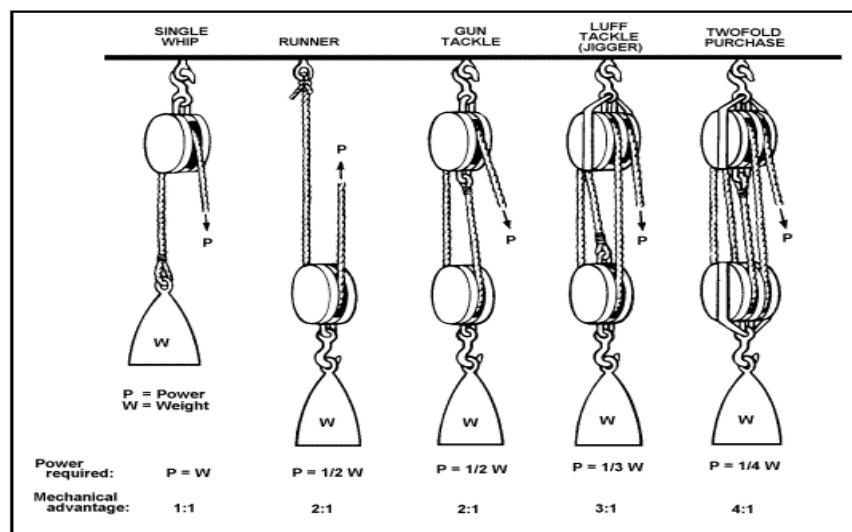


Fig. 2.6 Different pulley system with power weight relation and mechanical advantage

2.6.1.1 Pulley Systems

A pulley system is a combination of several movable and fixed pulleys. There are system for gain in speed and force. Hoisting devices employ pulley for a gain in force predominantly and only rarely for gain in speed. For example, in hydraulic or pneumatic lifts, pulleys are used for gain in speed. As independent lifting appliances pulley system are of secondary importance, they are mainly used for power transmission in winches and cranes.

a) Fixed Pulleys

In this, one end of rope passing around the pulley is loaded with weight and the other with pulling force. The path of pulling force is equal to height to which load is raised. Neglecting the resistance in the pulley, the pulling force becomes equal to weight.

b) Movable pulley

These pulleys have movable axles to which either a load or force (effort) is applied. Accordingly, there are pulleys which are used for gaining in speed and force. The efficiency of a movable pulley is higher than of a fixed pulley.

i) Pulley for gain in force

The distance travelled by a point of the rope where the effort is applied is equal to twice the height to which the load raised.

$S=2h$, $C=2V$, where C - Speed of the efforts, V - Speed of the load

$$Z+S_o = Q,$$

$$Z = \varepsilon S_o = \varepsilon(Q - Z),$$

$$Z = \frac{\varepsilon}{1+\varepsilon} Q,$$

$$\text{Efficiency } \eta = \frac{Z_o}{Z} = \frac{\left(\frac{Q}{2}\right)}{\frac{\varepsilon}{1+\varepsilon} Q} = \frac{1+\varepsilon}{2\varepsilon}$$

When $\varepsilon = 1.05$, efficiency = 0.975

That is Efficiency of the movable pulley is somewhat higher than that of fixed pulley.

ii) Pulley for gain in Speed

The distance travelled by a point of the rope where the effort is applied is equal to half the height to which the load raised.

$S= h/2$, $C=V/2$, With regards to resistance of the pulley,

$$Z = Q + S_o = Q + Q \varepsilon = Q(1 + \varepsilon)$$

$$\text{Efficiency } \eta = \frac{Z_o}{Z} = \frac{2Q}{Q(1 + \varepsilon)} = \frac{2}{1 + \varepsilon}$$

When $\varepsilon = 1.05$, efficiency = 0.9756

That is in this case too, Efficiency of the movable pulley is somewhat higher than that of fixed pulley.

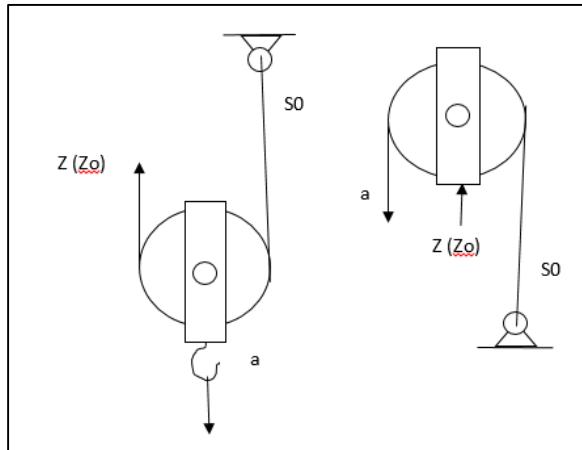


Fig. 2.7 Movable Pulley

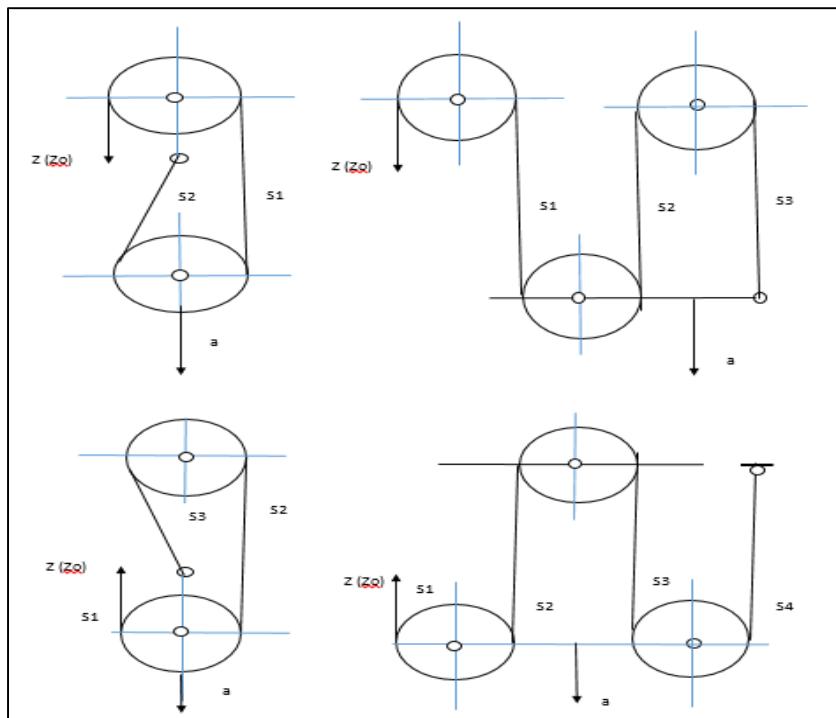


Fig. 2.8 Pulley system with gain in force

2.6.1.2 Multiple Pulley Systems

Due to direct suspension of loads from the rope end or from employing simple pulleys for a gain in force in hoisting appliances the following shortcomings can be pointed out.

1. The rope parts in one plane and this may cause the load to sway/ move.
2. Large diameter of ropes and pulleys are required.
3. The load being lifted moves in a horizontal direction because a rope coiling on a drum moves along its length.

These shortcomings can be avoided, especially in the hoisting mechanisms of winches and cranes with an electric drive, by using multiple pulley systems which raise the load in strictly vertical direction and keep it more stable. These systems carry the load with twice capacity as compared to a similar pulley system.

In crane design, always multiple pulley system are used. Multiple pulley system is also known as multi-fall system. It is done to get mechanical advantage and to reduce the load per strand of rope. It enables us to use small cross-section of rope. This in turn reduces the size and cost of pulley. So that for assembly we can go for light weight construction.

Figure 2.9 shows the types of multifall system.

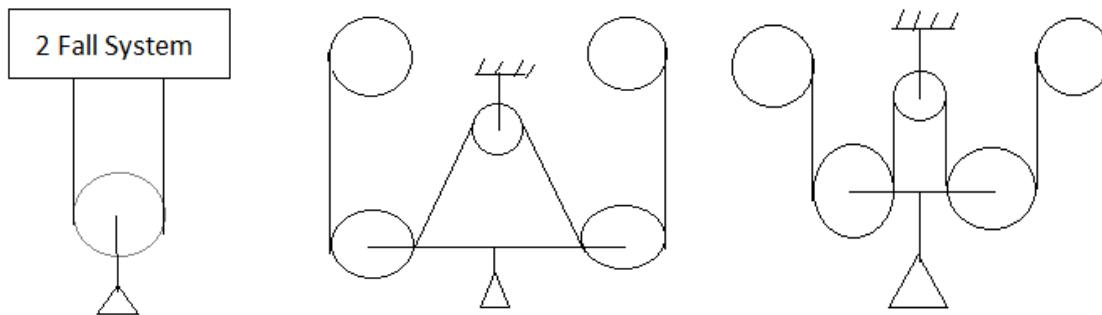


Fig. 2.9 a) 2 fall system, b) 4 fall system with 3 bends, c) 4 fall system with 4 bends

2.6.1.3 Efficiency of pulley systems

Efficiency of the pulley is defined as a ratio of ideal efforts to actual efforts applied. The efficiency of the pulley system can be expressed in terms of transmission ratio ' ϵ ', and number of pulleys in the system 'n'.

Consider a simple pulley system as shown below.

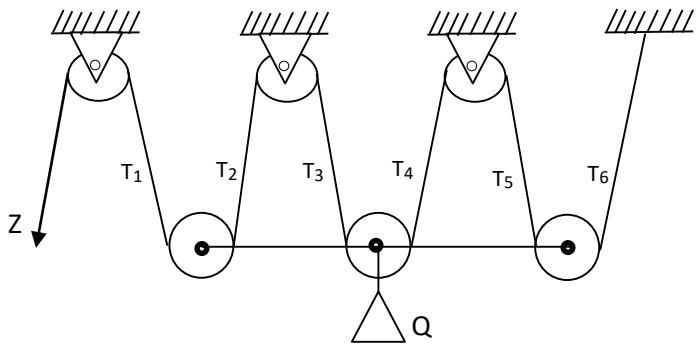


Fig. 2.10 Pulley system

Let, Q is a load to be lifted with efforts Z and $T_1, T_2, T_3, T_4, T_5, T_6$ are the tensions in the rope shown.

$$\text{Now, } T_1 + T_2 + T_3 + T_4 + T_5 + T_6 = Q$$

If ε be the transmission ratio, then, $T_5 = T_6 \times \varepsilon$, similarly,

$$T_4 = T_5 \times \varepsilon = T_6 \times \varepsilon^2$$

$$T_3 = T_6 \times \varepsilon^3$$

$$T_2 = T_6 \times \varepsilon^4$$

$$T_1 = T_6 \times \varepsilon^5, \text{ Hence,}$$

$$T_6 [\varepsilon^5 + \varepsilon^4 + \varepsilon^3 + \varepsilon^2 + \varepsilon + 1] = Q,$$

$$T_6 \left[\frac{\varepsilon^6 - 1}{\varepsilon - 1} \right] = Q$$

$$T_6 = Q \left[\frac{\varepsilon - 1}{\varepsilon^6 - 1} \right],$$

$$Z = T_1 \times \varepsilon = T_6 \times \varepsilon^6$$

$$Z = Q \cdot \varepsilon^6 \left[\frac{\varepsilon - 1}{\varepsilon^6 - 1} \right]$$

Now Ideal effort is given by,

$$Z_0 = \frac{Q}{6} = \frac{\text{Load}}{\text{Number of pulleys}};$$

Efficiency of the pulley,

$$\eta = \frac{Z_0}{Z} = \frac{Q}{6} \cdot \frac{\varepsilon^6 - 1}{Q \cdot \varepsilon^6 [\varepsilon - 1]} = \frac{\varepsilon^6 - 1}{6 \cdot \varepsilon^6 [\varepsilon - 1]}$$

Hence, general equation for the efficiency of n number of pulleys,

$$\eta = \frac{\varepsilon^n - 1}{n \cdot \varepsilon^n \cdot [\varepsilon - 1]}$$

There are different fall system used for different loads which is shown in table 2.1.

Table 2.1: fall system corresponding to different loads and efficiency of pulley system

Load to be Lifted kN	No. of Fall System	Efficiency of Pulley
Up to 40	2	0.97
40-200	4	0.95
200-250	6	0.92
250-750	8	0.90
750-1000	10	0.85
1000-1200	12	0.8



Fig. 2.11 Pulley/Sheave

Figure 2.11 shows a model of a pulley.

2.6.2 Sheave Maintenance

Examination of the sheave grooves for wear and proper diameter is essential. To check the size, contour and amount of wear, a sheave gauge is used. The gauge should contact the groove for about 150° of arc. Inspection of the fleet angle for poor sheave alignment is required. The fleet angle is the side, or included angle between a line drawn through the middle of a sheave and a

drum, perpendicular to the axis of each, and a line drawn from the intersection of the drum and its flange to the base of the groove in the sheave. The intersection of the drum and its flange represents the farthest position to which the rope can travel across the drum. There are left and right angles, measured to the left or right of the centre line of the sheave, respectively. It is important to maintain a proper fleet angle on installations where wire rope passes over a lead sheave and onto a drum. A fleet angle larger than recommended limits can result in excessive rubbing of the rope against the flanges of the sheave groove, or crushing and abrasion of the rope on the drum.

This fleet angle, for maximum efficiency and service, should not be more than $1\frac{1}{2}^\circ$ for a smooth drum, not more than 2° if the drum is grooved. The minimum angle which ensures that the rope will cross back and start a second layer in a normal manner, without mechanical assistance, should be $0^\circ 30'$. For smooth faced drums, this works out to a distance of 38 feet for each foot of side travel from the centre line of the sheaves to the flange of the drum. For a grooved drum, the distance is 29 feet. Figure 2.12 shows a fleet angle

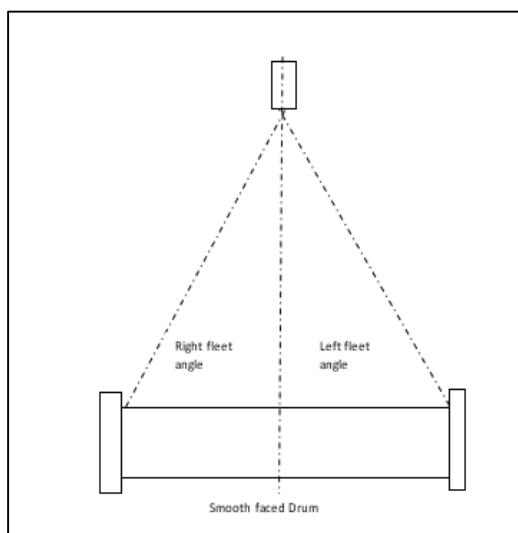


Fig 2.12 Fleet angle

2.6.2.1 Synthetic Sheaves

When synthetic sheaves or synthetic-lined steel sheaves are used, the inspector must carefully examine the rope for diameter reduction or lengthening of lay, even if no visible damage is observed. Synthetic sheaves greatly increase the contact area between the wire rope and sheave, by cushioning the rope. This cushioning effect causes wire rope to wear internally before the damage is noted on the outer wires. This situation places the inspector at a great disadvantage;

therefore, one must be diligent in the detection of diameter reduction and lay lengthening to prevent catastrophic failure from internal core damage in case of synthetic sheaves.

2.6.3 Bend

Bend is considered as a point where there is relative motion between the rope and the pulley or where rope either moves over the pulley or leaves the pulley. A bend is considered as double bend where direction of pulley changes and rope undergoes complete reversal of spaces. Life of rope is always depends on number of bends. Hence the system with minimum number of bends is selected so that life can be increased. In order to reduce the stresses in the rope it would be necessary to increase the diameter of pulley. Hence as number of bends increases, $\frac{D_p}{D_r}$ ratio also increases. Where, D_p and D_r are diameters of pulley and rope respectively.

Individual wire in loaded bend rope experiences a complex stresses consisting of tension, bending, twisting stresses combined with material compression and rubbing of the wires and strands. As a result the total stress can be determined analytically only to a certain degree of approximation. As they run over the pulleys and drums outer wire are subjected to abrasion which in turn reduces the total strength of the rope. It has been found that each rope can withstand during its life only definite number of bends after which rapid disintegration sets in. Depending on the number of bends, the life can be found from the ratio D_{min}/d . Investigation have shown that at the same D_{min}/d ratio, the rope life is approximately inversely proportional to the number of bends. One bend is assumed to mean the transition of the rope from its straight position to bent one or from a bent position into a straight one. Reverse bending reduces the life approximately by one half or it is equal to two single bends towards the same side. In determining the number of bends for multiple pulleys, the compensating pulley is not considered since it remains stationary when load is being raised or lowered. To obtain the same rope life the effect of number of bends should be compensated for by an approximate change in the ratio D_{min}/d . Figure 3.13 show the bend measurement for two cases.

1. Bend toward same side, 2. Reverse bend.

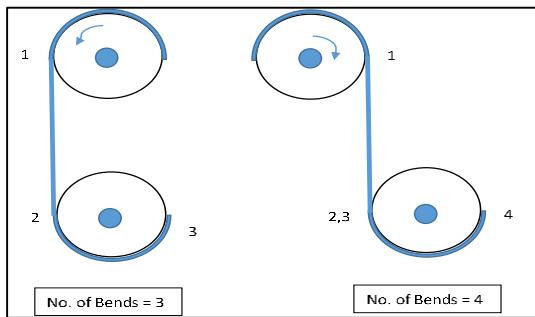


Fig. 2.13 Measurement of bends in rope

2.6.4 Compensating Pulley

Compensating pulley is located at the centre of the system and in normal work, compensating pulleys do not rotate. If for lifting purpose, the hook assembly is pulled on one side then only the

compensating pulley will rotate and adjust the length of rope. Sometimes for higher heights, the weight of the ropes in the suspension will cause an imbalance in the driving mechanisms. The ropes over the compensating pulley help in maintaining appropriate balance of assembly. Compensating pulley diameter can be considered as 60 percent of the movable pulley diameter.

2.6.5 Wire Rope

A hoisting device uses chain, fibre or wire rope as its lifting medium. Wire rope consists of several strands laid (or ‘twisted’) together like a helix. Each strand is likewise made of metal wires laid together like a helix. Abrasion resistance increases for larger outside wires per strand and fatigue resistance increase with more outside smaller wires per strand.

Running rope is bent over sheaves and drums. Therefore, they are stressed mainly by bending and by tension. Stationary ropes or stay ropes have to carry tensile forces and therefore mainly loaded by static and fluctuating tensile stresses. Ropes used for suspension are often called cables. Track ropes have to act as rails for the rollers of cabins or others loads in aerial ropeways and cable cranes. In contrast to running ropes, track ropes do not take on the curvature of the rollers. Under the roller force, a so called free bending radius of the rope occurs. This radius increases with the tensile force and decreases with the roller force. Standard ropes are used to harness various kinds of goods. These slings are stressed by the tensile forces but first of all by bending stresses when bent over the more or less sharp edges of the goods.

2.6.5.1 Advantages of wire rope over chain

Steel wire ropes are extensively used in hoisting machinery as flexible lifting appliances. As compare to chains they have the following advantages:

- Lighter weight.
- Less susceptibility to damage from jerk.
- Silent operation even at high working speed.
- Greater reliability in operation.

2.6.5.2 Construction of wire rope and Designation

Wire rope are manufactured from steel wire with an ultimate strength of $\sigma = 1600 \text{ to } 2000 \text{ N/mm}^2$. In the process of manufacturing the wire is subjected to special heat treatment which, combined with cold drawing, imparts high mechanical properties to the wire.

A rope is a group of yarns, plies, or strands that are twisted or braided together into a larger and stronger form. Ropes have tensile strength and so can be used for dragging and lifting, but are too flexible to provide compressive strength. As a result, they cannot be used for pushing or similar compressive applications. Rope is thicker and stronger than similarly constructed cord, line, string, and twine. Ropes are made from metal strands which are called wire rope as shown in figure 2.14.

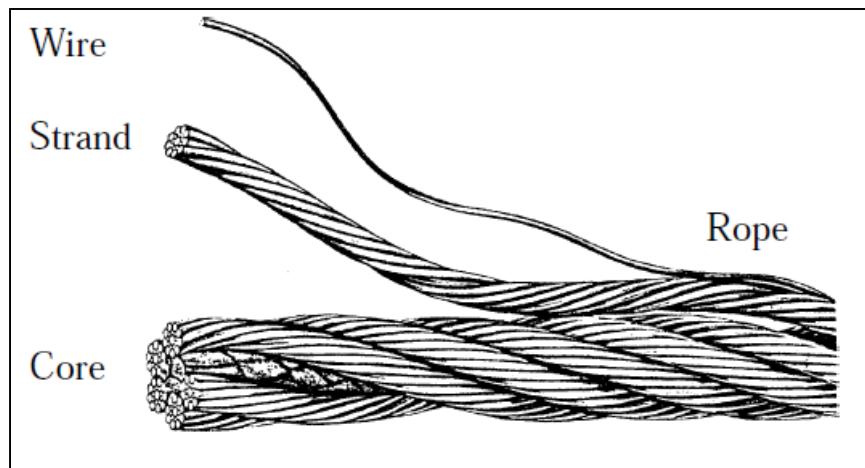


Fig. 2.14 Rope construction

Rope may be constructed of any long, stringy, fibrous material, but generally is constructed of certain natural or synthetic fibres. Material for steel wire ropes are generally high carbon steel. Most of the wire ropes are made up of IPS (Improved Plow Steel) with ultimate tensile strength (σ_{ut}) of 2000 N/mm^2 . Several other grades are also used, plow steel ($\sigma_{ut} = 1600 \text{ N/mm}^2$) and mild plow steel ($\sigma_{ut} = 1200 \text{ N/mm}^2$). Steel wires are manufactured by special machines. Initially separate wires are twisted to form the strands and strands are again twisted to form rope.

Construction of ropes is indicated by two numbers, 6×7 , 6×19 , 6×37 , where 6 for No. of strands and 7, 19, 37 for No. of wires in each strands. More the number of wires in each strand, more the flexible the rope will be. If number of wire is less the rope is stiffer.

6×7 rope is made up of heavy wires and provides maximum resistance to wear and vibration.

6×19 rope is good compromise between flexibility and wear. It is most popular and widely used.

6×37 rope is extra flexible are used where abrasion and wear are not very severe. Relatively sharp bends can be tolerated.

The constructions of wire rope are shown in Figure 2.15 (a) and (b). The wire rope consists of a number of strands, each strand comprising several steel wires. The number of wire in each strand is generally 7, 19 or 37, while the number of strands is usually six. The individual wires are first twisted into the strand and then the strands are twisted around a fibre or steel core.

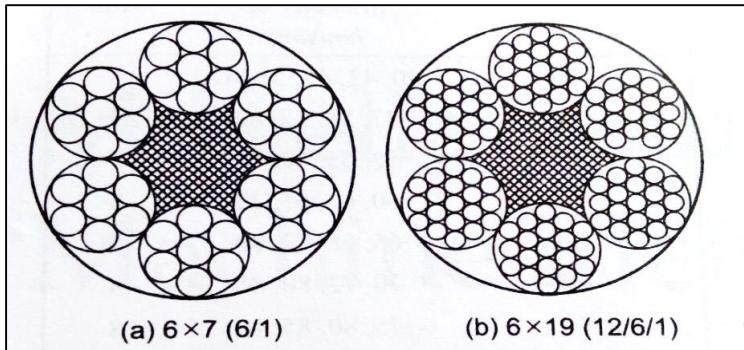


Fig. 2.15 Wire Rope and its strands

The specifications of wire rope include two numbers, such as 6x7 or 6x19. The first number indicates the number of strand in the wire rope, while the second gives the number of steel wires in each strand. The popular constructions of steel wire ropes are as follows:

6×7 (6/1), 6×19 (12/6/1), 6×37 (18/12/6/1)

The central portion of the rope is called core. There are three types of cores-fibre, wire and synthetic materials. The fibre core consists of natural fibre like sisal, hemp, jute or cotton. The fibre core is flexible and suitable for all conditions except when the rope is subjected to severe crushing.

The lay of the rope refers to the manner in which the wires are helically laid into strands and strand into the rope. If the wires in the strand are twisted in the same directions as the strands, then the rope is called a Lang's lay rope. When the wire in the strand is twisted in directions opposite to that of strands, the rope is said to be regular-lay or ordinary-lay. There are mainly three types of ropes parallel, composite and crossed. The lays of wire rope is shown in the figure 2.16 (a), (b) and (c).

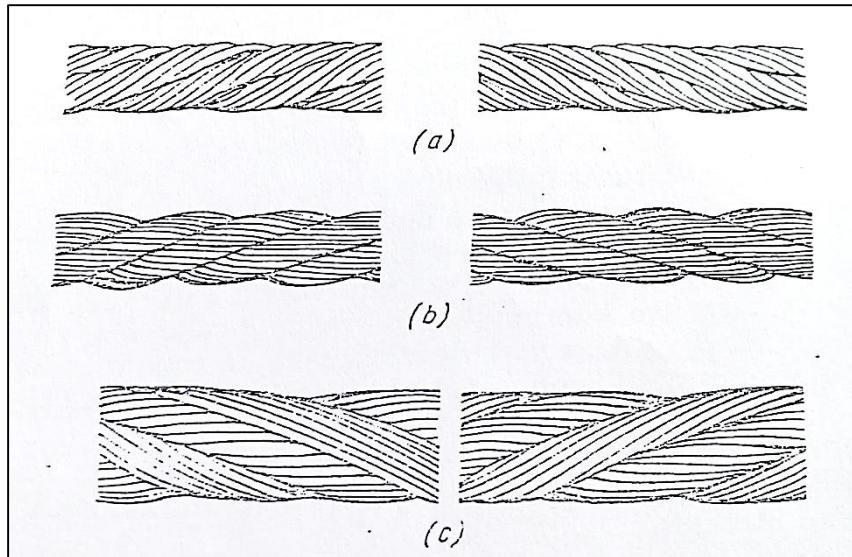


Fig. 2.16 (a) parallel, (b) cross, (c) Composite lays of wire Rope

2.6.5.3 Selection of steel wires

Extremely complex phenomena are involved in the operation of ropes which in some parts are indeterminate. Individual wire in a loaded bent rope experience a complex stress consisting of tension, bending and twisting stresses combined with mutual compression and rubbing of wires and strands. As a result, the total stress can be determined analytically only to a certain degree of approximation. Besides as they run over the pulleys and drums, the outer wire is subjected to abrasion which, in turn, reduces the total strength of ropes. Ropes are selected based on bending and tensile stresses in the rope.

Experimentally it is seen that the life of wire rope is greatly affected by fatigue. Ropes are used in various fashions. One can use different number of rope falls. Increase in number of rope fall helps in decreasing the tension in the wire rope. Size of the rope depends on ultimate tensile strength of the material used, Maximum tension carried by rope fall, Number of Bends, D_{min}/d , types of rope, duty factor and factor of safety.

The relation between the diameter of rope and diameter of wire is given by,

$$d_{rope} = 1.5 \times d_w \times \sqrt{No.\ of\ wires}$$

Where, d_w – Diameter of wire, d_{rope} – Diameter of rope.

2.6.5.4 Calculation of size of rope based on direct and bending stresses

Breaking strength of the wire rope and hence cross sectional area of the rope is depends on parameters like, Maximum tension per fall, $\frac{D_{min}}{d_{rope}}$ ratio, stress factor and types of rope.

Let, T_{max} = Maximum tension per fall, Q = Maximum load to lifted, η_f = No. of pulleys and

η_p = Efficiency of pulley

$$T_{max} = \frac{Q}{\eta_f \times \eta_p}$$

Considering direct and bending stresses,

$$\frac{T_{max}}{A} + \frac{E'd_w}{D_{min}} \leq \frac{\sigma_{ut}}{n}$$

Where,

A = cross sectional area (Metal content)

$$A = \frac{\pi}{4} \times d_{wire}^2 \times \text{no. of wire} = 0.4 \times \frac{\pi}{4} \times d_{rope}^2$$

E' = Modified Young modulus = 0.8×10^6 N/mm² (Considering twisting which causes reduction in bending stress)

d_w = wire diameter

D_{min} = minimum diameter of sheave

σ_{ut} = Ultimate strength of rope material

n = stress factor = n' x duty factor

n' = factor of safety.

$$d_{rope} = 1.5 \times d_w \times \sqrt{\text{No. of wires}}$$

Rearranging the above equation,

$$\frac{T_{max}}{A} + \frac{0.8 \times 10^6 \times d_w \times d_{rope}}{D_{min} \times d_{rope}} = \frac{\sigma_{ut}}{n}$$

$$\frac{T_{max}}{A} + \frac{0.8 \times 10^6 \times d_w \times d_{rope}}{D_{min} \times 1.5 \times d_w \times \sqrt{\text{No. of wires}}} = \frac{\sigma_{ut}}{n}$$

$$\frac{T_{max}}{A} = \frac{\sigma_{ut}}{n} - \frac{0.8 \times 10^6}{1.5 \times \sqrt{\text{No. of wires}}} \times \frac{d_{rope}}{D_{min}}, \text{ Hence,}$$

$$A = \frac{T_{max}}{\frac{\sigma_{ut}}{n} - \frac{0.8 \times 10^6}{1.5 \times \sqrt{\text{No. of wires}}} \times \frac{d_{rope}}{D_{min}}}$$

For 6 x 37 wire designation,

$$A = \frac{T_{max}}{\frac{\sigma_{ut}}{n} - \frac{0.8 \times 10^6}{1.5 \times \sqrt{6 \times 37}}} \times \frac{d_{rope}}{D_{min}} = \frac{T_{max}}{\frac{\sigma_{ut}}{n} - 36000 \times \frac{d_{rope}}{D_{min}}}$$

For 6×19 designation,

$$A = \frac{T_{max}}{\frac{\sigma_{ut}}{n} - 50000 \times \frac{d_{rope}}{D_{min}}}$$

Breaking strength of the wire rope is given by,

$$P = A \times \sigma_{ut} = \frac{T_{max} \times \sigma_{ut}}{\frac{\sigma_{ut}}{n} - \frac{0.8 \times 10^6}{1.5 \times \sqrt{No. \text{ of wires}}} \times \frac{d_{rope}}{D_{min}}}$$

(Breaking strength for different standard diameter of wire rope is given in PSG 9.4)

2.6.5.6 Check for life

Usually failure is by fatigue and it depends on stress and ratio $\frac{d_{rope}}{D_{min}}$

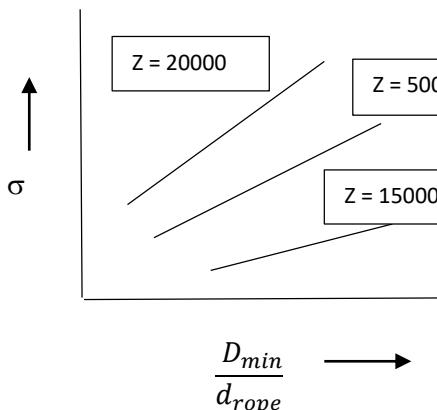


Fig. 2.17 Graph of stress σ vs $\frac{D_{min}}{d_{rope}}$

No. of Cycles, $Z = F_1(\sigma_{ut})$ and, $Z = F_2\left(\frac{D_{min}}{d_{rope}}\right)$, for Z , stress $\sigma = F_3\left(\frac{D_{min}}{d_{rope}}\right)$

$$\frac{D_{min}}{d_{rope}} = m \cdot \sigma_{ut} \cdot C \cdot C_1 \cdot C_2 + 8$$

Constant 8 is added as X – intercept of the graph

C – strength and rope construction factor,

C_1 – Rope size factor

C_2 – Material and surface finish factor, (Refer PSG 9.7 and PSG 9.8)

Value of Z are mentioned corresponding to values of m, m- a constant relating to life of rope

(Multiplication factor for m is 0.01 and for Z is 1000)

$$\text{Life in No. of months, } N = \frac{0.4 Z}{a \beta Z_2},$$

Where,

Z = no. of cycles

a = No. of working cycles per month,

β = Endurance factor

Z_2 = No. of bends

2.6.6 Crosspiece

Cross piece is secured in cross plate and casing with fastener. The main body is rectangular while the ends are modified in cylindrical form called trunion. The trunion provides swinging effect, a provision is made to house the thrust ball bearing which allows the loaded hook to turn easily in the handling load. The recess for the seating rings is made to a depth from 3 to 10mm depending on the size of the bearing. A cross piece is pivoted in the side plate or shackle plate made of steel. This enables the hook to be turned in two mutually perpendicular directions. The cross piece is forged from the steel and turned trunion at the ends are provided. The hole diameter should be little large than the hook shank diameter. Figure 2.18 and figure 2.19 shows the model of cross piece and types of cross piece.

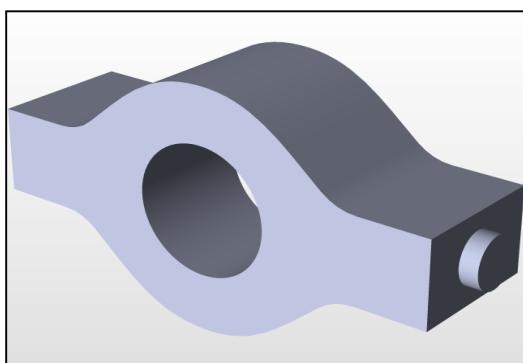


Fig. 2.18 model of cross piece

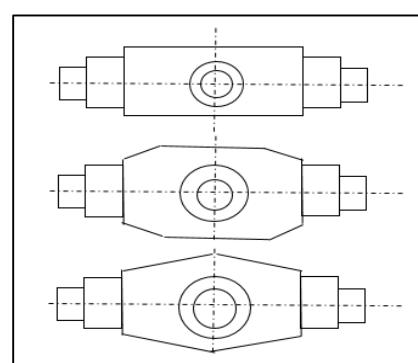


Fig. 2.19 Types of crosspiece

2.6.6.1 Cross piece design structure

1. First Select the material of crosspiece and determine the design stresses.
2. Determination of length of crosspiece based on pulley axle length.
3. Trunion diameter calculation based on shear stress criteria.
4. Checking trunion for bending and bearing failure.

5. The width of crosspiece can be calculated from bearing, nut, and margin consideration. The sectional modulus can be found out.
6. Maximum Bending moment can be calculated considering UDL on cross piece and the crosspiece has to be checked under bending failure.

(Note: For Cross piece design refer numerical)

2.6.7 Shackle plate

The Shackle plates or check plates are form the housing for the snatch block. A shackle plate is piece of metal used with a locking mechanism in padlocks. In hoisting mechanism two shackle plates are used along with side plates. Shackle plates are used for securing crosspiece trunion and axle ends. It is subjected to tension, double shear and crushing failure. Figure 2.20 shows a model of a shackle plate.

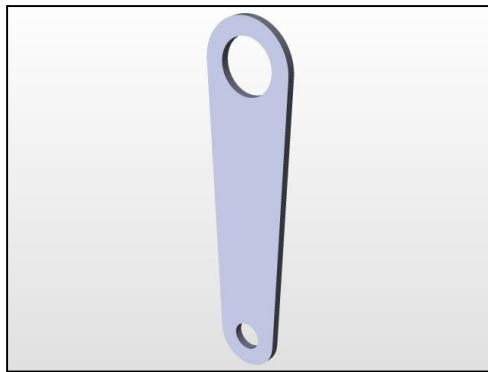


Fig. 2.20 Shackle Plate

2.6.7.1 Shackle Plate Design procedure

1. The crosspiece is secured in the side plates which are strengthened with shackles or straps. As a rule, only shackles are checked for strength neglecting the plates in view of their relatively small thickness.
2. Material for shackle plate is decided.
3. The width of shackle plate and height can be calculated based on empirical relation with the other components dimensions.
4. Shackle plate has to be checked under various failures like tensile, double shear and crushing failure.

(Note: For design refer numerical)

2.6.8 Hook

A hook as shown in figure 2.21 is a component consisting of a length of material that contains a portion that is curved or indented, so that this portion can be used to hold another object. In a number of uses, one end of the hook is pointed, so that this end can pierce another material, which

is then held by the curved or indented portion. Hook is made up of mild steel or high tensile steel. It is subjected to different types of stresses at different cross sections.

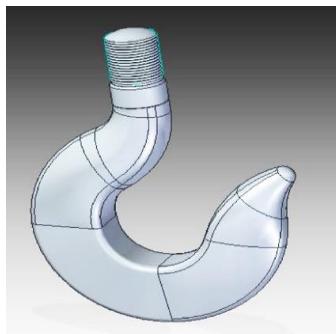


Fig. 2.21 Hook

2.6.8.1 Manufacturing process of crane hook

Forging is an effective method of producing many useful shapes. The process is generally used to produce discrete parts. The forged parts have good strength and toughness. They can be used reliably for highly stressed and critical applications. Crane hooks are manufactured generally by open die forging or press forging. Open die forging does not confine the flow of metal, the operator obtains the desired shape of forging by manipulating the work material between blows. Some specially shaped tools or a simple shaped die between the work piece and the hammer or anvil to assist in shaping the required sections (round, concave, or convex), making holes, or performing cut – off operations may be used. Press forging, which is mostly used for forging of large sections of metal, uses hydraulic press to obtain slow and squeezing action instead of a series of blows as in drop forging. The continuous action of the hydraulic press helps to obtain uniform deformation throughout the entire depth of the work piece.

The design of the hook is based on curved beam theory. The inner fiber of the hook is subjected to maximum stress which is combined effect of tensile stress and bending stress. Hence inner fiber is more critical than outer fiber, to make the hook equally stronger at inner and outer fiber, more material is required at inner side. Hence Crane Hook generally made with trapezoidal cross with bigger side of trapezoid at inner side and smaller side of trapezoid at outer side.

2.6.8.2 Types of crane hook

Figure 2.22 shows different types of crane hook. Figure A is a Sling hook, the curve in the hook is designed to allow the hook to hold a rope loop securely and it also allows the hook to be hooked over its own chain. Figure B is a common design with a large eye to take hemp rope, these are often used in docks. These hooks are used in cranes and also attached to lengths of chain or rope to make slings or 'spotter'. Figure C is a Liverpool hook, only used on cranes not on slings, the projecting upper part is designed to prevent the hook catching on the sides of a ship's hold when a

load is being lifted, it also serves to reduce the risk of the rope or chain sling jumping free if the load is jolted. Figure D shows a Sling hook fitted with a hand loop used on cranes, this allowed a operator to release the load without risk of getting fingers trapped under the sling.

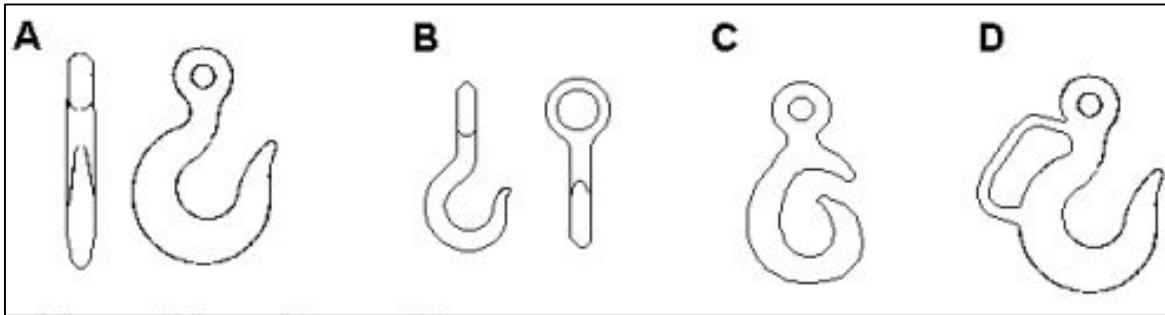


Fig. 2.22 Crane hook types

2.6.8.3 Material of hook

Material used for hook is mild steel and high tensile steel.

For HTS material design stresses,

Threaded part $[\sigma_t] = 100\text{N/mm}^2$, Saddle part $[\sigma_t] = 180\text{N/mm}^2$

For mild steel design stresses can be used as,

Threaded part $[\sigma_t] = 80\text{N/mm}^2$, Saddle part $[\sigma_t] = 150\text{N/mm}^2$

2.6.8.4 Design procedure for hook

The Bed diameter of throat diameter (C) is a very important dimension of the hook which is depends on the design load and a constant based on the material and manufacturing process.

$$C = K \sqrt{W_d}$$

K—(12-13) for forged steel

1. Select the material for hook and determined the stresses for shank and threaded part.
2. Decide the value of throat diameter C of the hook based on the safe load to be lifted and the material of hook.
3. Then using data calculate the other hook dimensions. (Refer PSG 9.11 and PSG 6.3)
4. Find the design load for the hook.
5. Check the hook shank for tensile stresses in the threaded portion.
6. Check the stresses (Tensile, bending and shear stresses) at different sections using different failure theories.
7. The most critical section at inner and outer fibre is require to checked for direct tensile and bending stresses.

8. If stresses are beyond safe limit then either change the material of the hook or redesign it.

2.6.8.5 Important Sections of hook

These are mainly 4 important cross sections of hook where stresses are checked for safe hook design. These sections are shown in figure 2.23.

Section 1-1 is threaded part and it is checked for tensile failure.

Section 2-2 is critical section and checked for combined tensile and bending failure at inner and outer fibre. Inner fibre is most critical as magnitude of total stress induced at inner fibre is maximum there.

Section 3-3 is critical stress region as various stresses are acting in this section. It is subjected to tensile, shear and bending hence, maximum principal stress theory (MPST) and maximum shear stress theory (MSST) are applied at this section to calculate induced stresses.

Section 4-4 is subjected to direct shear stress, hence checked for shear failure.

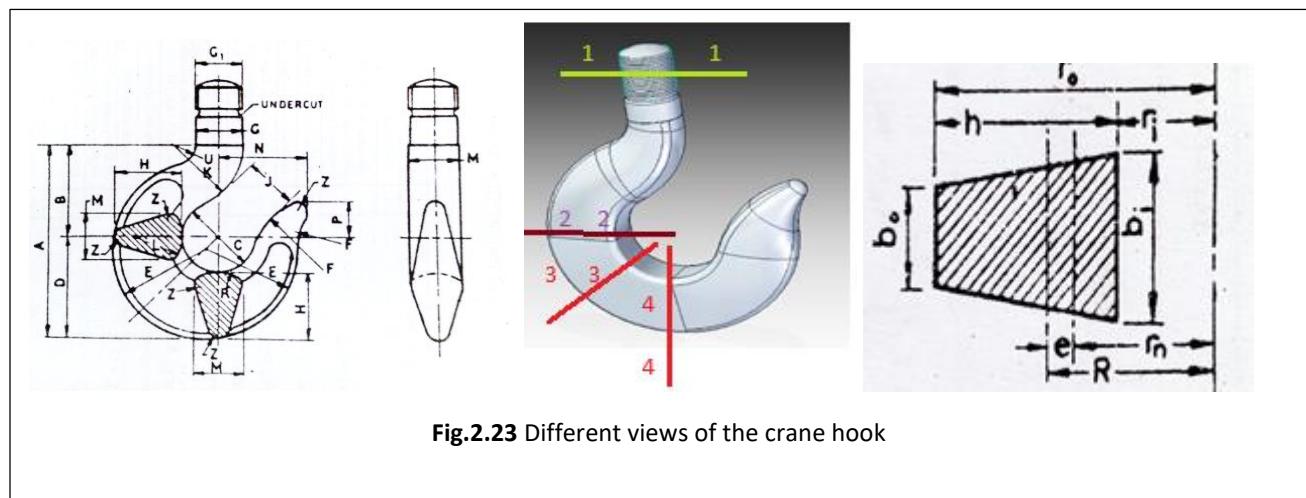


Fig.2.23 Different views of the crane hook

Fig.2.23 Different views of the crane hook

2.6.9 Thrust Bearing

Thrust bearing allows a loaded hook to turn easily in handling loads. It is mounted on crosspiece and it supports the hook nuts. Following steps can be followed to select a thrust bearing.

1. Select a bearing series based on hook thread diameter.
2. The required dynamic capacity is calculated based on radial and axial load acting on hook.
3. A standard bearing is selected from the catalogue which has dynamic capacity more than required dynamic capacity and inner diameter equal to hook thread diameter.

2.6.10 Rope Drum

There are two types of construction of rope drums namely drum with helical grooves and plain cylindrical drums without grooves. In most hoisting installation preference is given to grooved drums instead of plain drums. The drums are usually made up of grey cast iron FG 200. Welded drums are rarely used. The drum is provided with helical grooves so that rope winds up uniformly on the drum.

The rope drum should be made of seamless pipe machined & grooved accurately, to ensure proper seating of wire rope in a proper layer. The drum should be fitted with two heavy duty Ball / Roller bearings of reputed make for smooth operation & longer life.

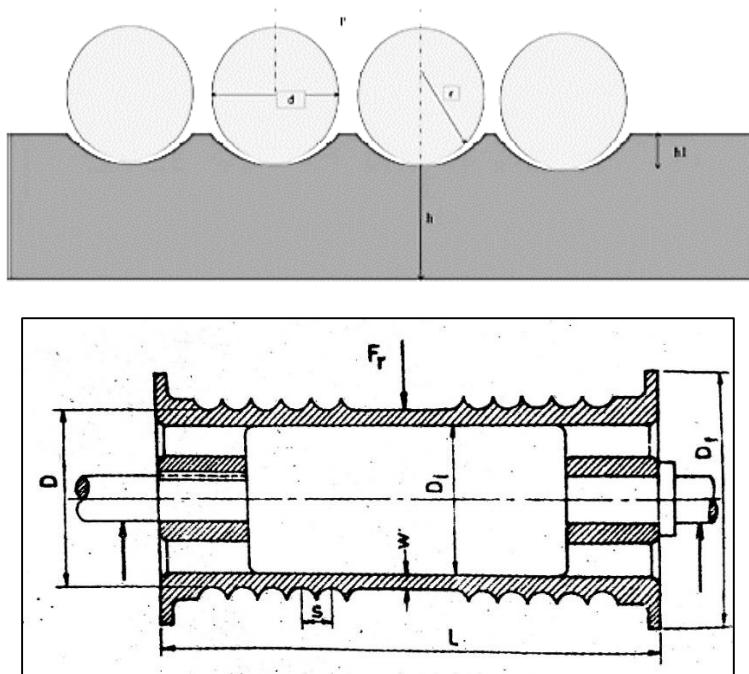


Fig. 2.24 Rope drum cross section

Figure 2.24 shows a cross section of a rope drum where, L is a length of a drum, D is diameter, D_f is flange diameter, D_i – inner diameter, s is a pitch and w is thickness of a drum. The drum is subjected to crushing stress, bending stress and torsion. Compressive stress or crushing stress is more critical compared to shear and bending. Hence the thickness of the drum can be decided on compressive failure criteria. The length of the drum is a function of drum diameter, maximum lift, and ratio of pulley system, pitch of the drum and ungrooved length of the drum.

$$\text{It is given by, } L_D = \left(\frac{2 \times H \times i}{\pi \times D} + 12 \right) \times S + l_1$$

Figure 2.25 shows a rope drum with bearing block.

The minimum permissible diameter of drum or pulley is given by, $D \geq e_1 e_2 d$

Where, e_1 is a factor depends upon the types of hoisting devices and its service condition.

e_2 depends on the rope construction.

Hoisting Mechanism	Drive	Operating Condition	Factor e_1
Locomotive, Crane at construction site, temporary set up	Hand/Power	Light	16
	Power	Medium	18
	Power	Heavy	20
Crane and Hoisting Mechanism	Hand/Power	Light	18-20
	Power	Medium	25
	Power	Heavy	30
Hand operated winches upto 1 Tonne capacity	12
Trolley Hoist	20

For constant e_2 ,

Rope Type	Cross Lay	Parallel Lay
6x19, Ordinary Rope	1	0.9
6x19, Warrington Rope	0.9	0.85
6x19, Seale Rope	0.95	0.85
6x37	1	0.9

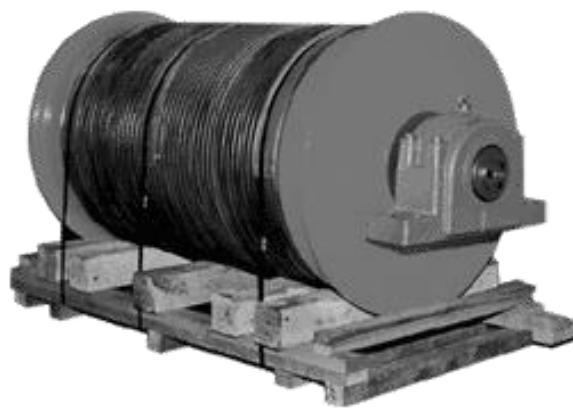


Fig. 2.25 Rope drum

2.6.11 Bearing

A bearing is a machine element that constrains relative motion to only the desired motion, and reduces friction between moving parts. The design of the bearing may, for example, provide for free linear movement of the moving part or for free rotation around a fixed axis; or, it may prevent a motion by controlling the vectors of normal forces that bear on the moving parts. Many bearings also facilitate the desired motion as much as possible, such as by minimizing friction. Bearings are classified broadly according to the type of operation, the motions allowed, or to the directions of the loads (forces) applied to the parts. For EOT crane design roller bearing and thrust bearing is used as per the load lifted. In the cross piece, for hook shank thrust ball bearing is used. For the pulley roller bearing or deep groove ball bearing are suitable. There is large radial load ($2T_{max}$) acting on the pulleys and very small or negligible axial load because of rope run off horizontally. It is better to use two bearing for each pulley so that the load will be shared and proper balancing will also be maintained.

[Note: For details design of Bearing, refer Numerical]

NUMERICALS

2.1 Design a Hoisting Mechanism for lifting a load of 100KN.

Solution:

Design includes Rope, Sheave, Bearing, Axle, Hook, Thrust Bearing, Nut, Cross piece, Shackle Plate, Drum, Hoisting motor, Drum shaft, Bearing for drum shaft.

Note: PSG design data book is referred and suitable data is assumed.

Step 1: Selection of number of falls and pulley system

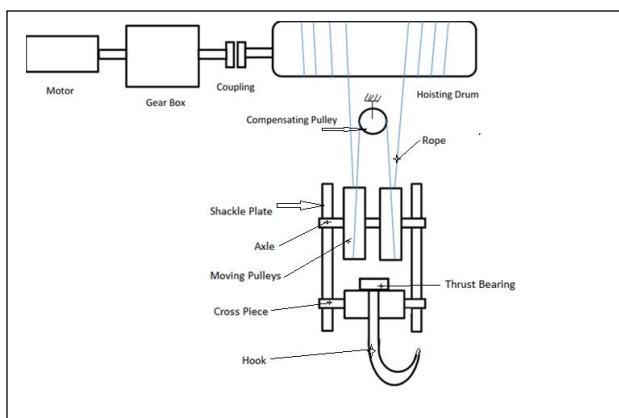


Fig. 2.26 Block diagram of hoisting mechanism

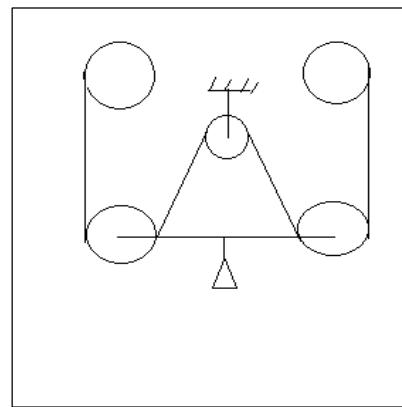


Fig. 2.27 Four Fall System

Selecting 4 fall system as load to be lifted = 100KN

Selecting pulley system as shown in above figures 2.26 and 2.27 with No. of Bends = 3

$$\frac{D_{min}}{d} = 23 \quad (\text{for 3 bends}) \dots \text{PSG 9.1}$$

Step 2: Selection of Rope

Selecting ordinary cross lay type of rope for hoisting, rope is suspended and not guided for economy and avoid spinning, crossed lay type rope is selected.

Selecting 6x37 rope for greater flexibility.

Selecting Material for Rope as IPS (Improved Plow Steel) with $\sigma_{ut} = 1800 \text{ N/mm}^2$

Step 3: Finding Maximum Tension in the Rope

$$T_{\max} = F = \frac{Q}{n_f \times \eta_p}$$

Where,

Q – Load capacity

n_f – No. of fall = 4

η_p – Efficiency of pulley system = 0.95

$$F = \frac{100}{4 \times 0.95}$$

Hence, $T_{\max} = F = 26.315$ KN

Step 4: Rope Selection

Rope diameter can be found from either Cross-Sectional Area of Rope or Strength of the rope

$$\text{Stress factor } n = n' \times \text{Duty factor}$$

Where,

n' (FOS) = 5 for Class II application PSG 9.1

Duty Factor = 1.2 (Based on Strength and Life of crane of 20 years) PSG 9.2

Therefore,

$$n = 5 \times 1.2$$

$$n = 6$$

For 6x37 type of rope, Rope strength P is given by PSG 9.1

$$P = \frac{F \cdot \sigma_u}{\frac{\sigma_u}{n} - \frac{d}{D_{min}} 3600}$$

and area of cross section of the rope is given by ,

$$A = \frac{F}{\frac{\sigma_u}{n} - 3600 \left(\frac{d}{D_{min}} \right)}$$

$$A = \frac{26.315 \times 1000}{\frac{1800}{6} - 3600 \left(\frac{1}{23} \right)}$$

$$A = 183.44 \text{ mm}^2$$

As there are voids, the useful Area = $0.4 \times \frac{\pi}{4} d^2$

$$A = 183.44 = 0.4 \times \frac{\pi}{4} d^2$$

$$d = 24.162 \text{ mm} \cong 25 \text{ mm}$$

\therefore Selecting Standard Diameter = 25 mmPSG 9.4

$\therefore d = 25 \text{ mm}$ and $D_{\min} = 575 \text{ mm}$

Step 5: Rope Life Calculation

Using the equations from PSG 9.7

Life of rope in months is given by,

$$N = \frac{0.4Z}{a\beta Z_2} \quad \text{and} \quad \frac{D}{d} = m \cdot \sigma \cdot c \cdot c_1 \cdot c_2 + 8$$

Where:

N- Life of rope in months of 25 days,

Z- Number of repeated bends corresponding to constant m

a- Working cycle per months (Table No.2, PSG 9.8)

β - Endurance Factor (Table No.2, PSG 9.8)

Z_2 - Number of repeated bends per cycle (Table No.2, PSG 9.8)

m- constant value corresponds to Z (Table No.1, PSG 9.8)

σ - tensile stress in rope in kgf/mm²

c- Strength and rope construction factor (Table No.4, PSG 9.8)

c_1 -Rope size factor (Table No.3, PSG 9.8)

c_2 -Material and surface finish factor (0.63 to 1.15)

and Drum or Pulley Diameter $D \geq e_1 \cdot e_2 \cdot d$

Where,

e_1 = Factor depends on types of hoisting device and its service condition (For power device medium operating condition $e_1 = 25$)

e_2 = Factor depends on Rope Construction = 1 for 6x37 cross laid rope.

$$D = e_1 \times e_2 \times d$$

$$D = 25 \times 1 \times 25 = 625 \text{ mm}$$

Now,

$$\sigma = \frac{F}{\frac{\pi}{4} d^2 * 0.4}$$

$$\sigma = \frac{10F}{\pi d^2} = \frac{10 \times 2632}{\pi \times 25^2} = 13.41 \text{ kgf/mm}^2$$

Assuming,

$$C = 1.02 \text{ strength and rope construction factor for cross } 6 \times 37 \text{ type and } 180 \text{ Kgf/mm}^2 \text{ strength} \quad \dots \dots \text{ PSG 9.8}$$

$$C_1 = 1.09 \text{ Rope size factor for } d= 25\text{mm} \quad \dots \dots \text{ PSG 9.8}$$

$$C_2 = 1 \text{ Material and surface finished factor (0.63 to 1.15)} \quad \dots \dots \text{ PSG 9.7}$$

$$\frac{D}{d} = m \cdot \sigma \cdot c \cdot c_1 \cdot c_2 + 8,$$

$$\frac{625}{25} = (m \times 13.41 \times 1.09 \times 1 \times 1.02) + 8$$

$$m = 1.1402 \text{ in Hundred}$$

$$\therefore m = 114.02 \quad \dots \dots \text{ PSG 9.8}$$

Calculating Z by Interpolation,

Z	150	170
m	107	118

$$\frac{118 - 114.02}{114.02 - 107} = \frac{170 - x}{x - 150}$$

$$x = 162.76379$$

For $m = 114.02$, $Z = 162.763 \times 10^3$

$$\text{Life of rope in months, } N = \frac{0.4Z}{a \cdot \beta \cdot Z_2}$$

Considering, medium duty 16hrs/day

$a = 3400$ working cycles/month

$\beta = 0.4$ Endurance factor

$Z_2 = 3$ no. of repeated bend/cycle

$$N = \frac{0.4 \times 162.76 \times 10^3}{3400 \times 0.4 \times 3}$$

N = 15.95 months

Step 6: Selection of Pulley/ Sheave

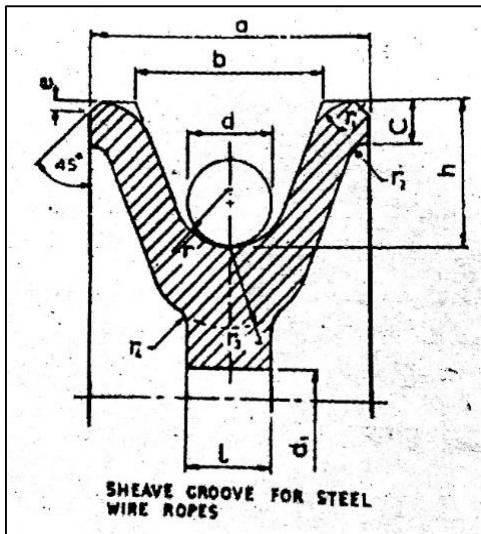


Fig 2.28 Sheave Cross Section

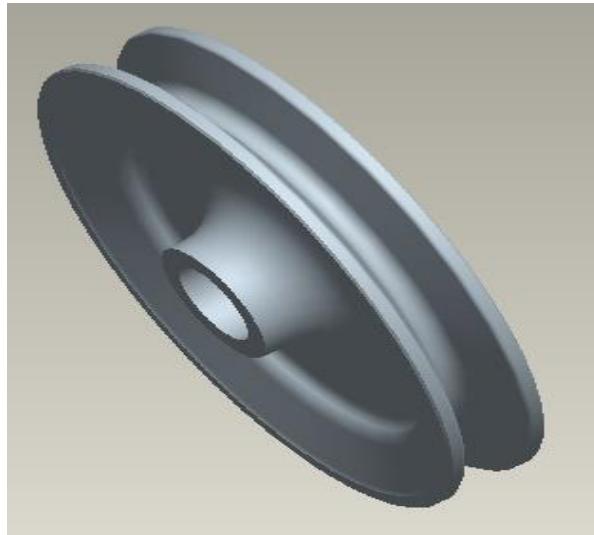


Fig. 2.29 Sheave

For $d=28\text{mm}$, selecting standard pulley from PSG 9.10,

Where,

$a= 80\text{mm}$

$b= 60\text{mm}$

$c= 45\text{mm}$

$h= 12\text{mm}$

$l= 20\text{mm}$

Checking for fleet angle:

$$\tan \alpha = \frac{\left(\frac{b-d}{2}\right)}{\left(\frac{D_o}{2}\right)}, \quad \alpha = \tan^{-1} \frac{\left(\frac{60-25}{2}\right)}{\left(\frac{D_o}{2}\right)}$$

Dimensions of sheave corresponding to next size of rope selection. Fleet angle α permissible will increase due greater width of the groove and it would give more space or width for bearing.

$$D_o = D_{min} + 2h$$

$$D_o = 575 + 2(45)$$

$$D_o = 665 \text{ mm,}$$

$$\alpha = \tan^{-1} \frac{\left(\frac{60 - 25}{2}\right)}{\left(\frac{665}{2}\right)}$$

$$\alpha = 3.01^\circ < 5^\circ \text{ hence accepted.}$$

Step 7: Speed of Sheave (N)

$$\text{Velocity of sheave} = \pi DN$$

$$V = \text{Velocity of hoisting} \times \frac{n}{2}$$

where n- no. of fall

$$N = \frac{V \times \frac{n}{2}}{\pi D}$$

$$N = \frac{6 \times \frac{4}{2}}{\pi \times 625 \times 10^{-3}} = 6.11 \text{ rpm}$$

Step 8: Selection of bearing

Bearings are used between shaft and rope drum to permits relative motion between them. Generally deep groove ball bearings are used because there is a negligible small axial force present. Selecting two bearing per pulley so that the radial load acting on bearing is shared and small bearing can be used also the balancing of load will be automatically taken care.

Now, forces acting on each bearing are,

$$F_r = 26320 \text{ N}$$

$$F_a = 0 \text{ N}$$

$$P_{eq} = (X \cdot V \cdot F_r + Y \cdot F_a) S$$

Assuming, Radial load factor X = 1,

Race rotation factor V = 1.2

Service factor S = 1

Equivalent load, $P_{eq} = 31584 \text{ N}$

Assuming life of the bearing = 10,000 hours

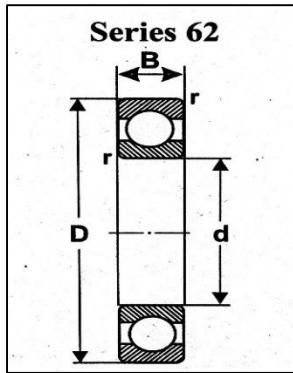


Fig. 2.30 Bearing

Life in millions of revolutions is given by,

$$L_{mr} = \frac{L_h \times N \times 60}{10^6}$$

$$L_{mr} = \frac{10000 \times 6.11 \times 60}{10^6} = 3.666 \text{ mr}$$

Dynamic Capacity,

$$C = P_{eq} (L_{mr})^{\frac{1}{K}}, \text{ Where } K = 1/3 \text{ for ball bearing}$$

Dynamic capacity,

$$C = 31584 \times 3.66^{\frac{1}{3}}$$

$$C = 48673.677 \text{ N} = 4867.4 \text{ kgf}$$

∴ Selecting DGBB SKF 6215PSG 4.13

Having, $d = 75 \text{ mm}$, $D = 130 \text{ mm}$, $B = 25 \text{ mm}$, $C = 5200 \text{ kgf}$

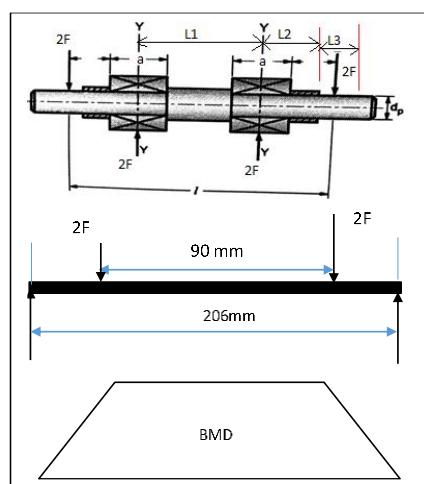


Fig. 2.31 Sheave axle, Force and Bending Moment diagram

Step 9: Design of Axle

Considering, Sheave axle material: C-30 steel with,

$$[\sigma_b] = 100 \frac{N}{mm^2}; \quad [\tau] = 60 \frac{N}{mm^2}$$

From bearing design, $d = 75\text{mm}$

From figure 2.31,

$$L_1 = a + \text{margin} = 80 + 10 = 90 \text{ mm}$$

$$L_2 = a/2 + \text{margin} = 40 + 10 = 50 \text{ mm}$$

$$L_3 = \text{casing thickness} + \text{shackle plate thickness} = 4 + 12 = 16 \text{ mm}$$

$$L = \text{support span} = L_1 + 2L_2 + L_3 = 90 + 100 + 16 = 206 \text{ mm}$$

Checking for bending failure,

$$B.M_{\max} = 2F \times \left(\frac{206-90}{2}\right)$$

$$B.M_{\max} = (26320 \times 2) 58$$

$$B.M_{\max} = 3053.12 \text{ KN.mm}$$

And

$$(\sigma_b) = \frac{M}{Z} = \frac{3053.12 \times 10^3}{\frac{\pi}{32}d^3}$$

$$(\sigma_b) = \frac{3053.12 \times 10^3}{\frac{\pi}{32}(75)^3} = 73.75 \frac{N}{mm^2} < [\sigma_b]$$

\therefore Safe in bending, hence $d = 75 \text{ mm}$

Step 10: Design of Hook

Hooks are made of mild steel or High tensile steel. After forging and machining operations hooks are carefully annealed and cleaned from scale. The inner diameter of hooks should be sufficient to accommodate two strands of chain or rope which carry the load. More often hooks have a trapezoidal section, made wider on the inside. A trapezoidal section makes for better utilization of the material and less complicated design. On the top, the hook ends in a round shank operating only in tension. The upper part of die-forged hooks is threaded for suspension from crosspieces of load carrying devices.

Let material used for hook be mild steel,

For 25 tonnes load the dimensions of trapezoidal section point hooks

10.1 Design Load

Assuming 10% extra load,

$$[W_d] = W_1 + (0.1W) = 100 + 10 = 110 \text{ kN}$$

C = 134

$$A = 2.75 \times C = 368.5 \text{ mm}$$

$$B = 1.31 \times C = 175.54 \text{ mm}$$

$$D = 1.44 \times C = 192.96 \text{ mm}$$

$$H = 0.93 \times C = 124.62 \text{ mm}$$

$$Z = 0.12 \times C = 16.08 \text{ mm}$$

$$M = 0.6 \times C = 80.4 \text{ mm}$$

$$b_0 \equiv 2 \times Z \equiv 32.16 \text{ mm}$$

10.3 Cross Sectional Area

Theoretical Area is given by,

$$(A) = \frac{1}{2}(b_i + b_o)H$$

$$A = \frac{1}{2}(80.4 + 32.16) \times 124.62 ,$$

Actual Area is given by, $a' = 0.90 \times 7013.61 = 6312.25 \text{ mm}^2$

10.4 Locational Parameters

$$r_i \equiv C/2 \equiv 67 \text{ mm}$$

$$r_o \equiv r_i + H = 67 + 124.62 = 191.62 \text{ mm}$$

$$R = r_i + \frac{H}{3} \left(\frac{b_i + 2b_o}{b_i + b_o} \right) \quad \dots \dots \text{PSG 6.3}$$

$$R_N = \frac{\frac{1}{2} \times (b_i + b_o) H}{\left(\frac{b_i r_o - b_o r_i}{H} \right) \ln \left(\frac{r_o}{r_i} \right) - (b_i - b_o)}$$

$\therefore R = 120.408 \text{ mm}$ and $R_N = 110.45 \text{ mm}$

$$h_o = H - h_i = 81.17 \text{ mm}$$

$$e = R - R_N = 120.408 - 110.45 = 9.958 \text{ mm}$$

$$h_i = R_N - r_i = 110.45 - 67 = 43.45 \text{ mm}$$

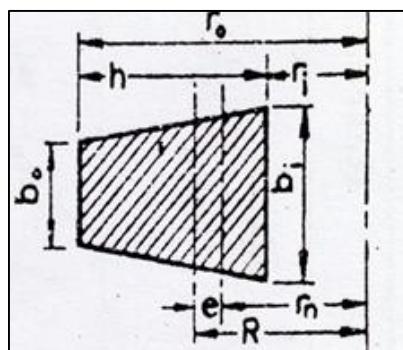


Fig. 2.32 Cross Section of Hook

10.5 Failures at Different cross Section of hook

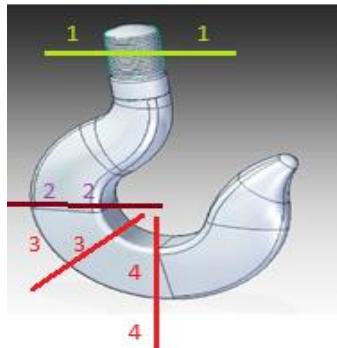


Fig. 2.33 Hook with different cross sections

1) At section 1-1,

Tensile stress is given by,

$$(\sigma_t) = \frac{[W_d]}{\frac{\pi}{4} d^2} = \frac{110}{\frac{\pi}{4} (70)^2} = 28.5829 \frac{\text{N}}{\text{mm}^2} < [\sigma_t]$$

Hence design is safe at section 1-1.

2) At section 2-2

(a) Inner fibre (tensile and bending stress) (2-2 is Most critical section)

$$(\sigma_t)_{\text{total}} = (\sigma_t)_i + (\sigma_b)_i$$

$$(\sigma_t) = \frac{[W_d]}{a'} = \frac{110 \times 1000}{6312.25} = 17.426 \text{ N/mm}^2$$

Since,

$$M_b = [W_d] \times R = 13.24 \times 10^6 \text{ N.mm}$$

$$(\sigma_b) = \frac{M_b \times h_i}{a' \times e \times r_i} = \frac{13.24 \times 10^6 \times 43.45}{6312.25 \times 9.958 \times 67}$$

$$(\sigma_b) = 136.598 \text{ N/mm}^2$$

$$(\sigma_t)_{\text{total}} = 17.426 + 136.598 = 154.024 \text{ N/mm}^2$$

$$(\sigma_t)_{\text{total}} < [\sigma_t] = 180 \text{ N/mm}^2$$

Safe at inner fibre that is at most critical section.

(b) Outer Fibre (Compressive and bending stress)

$$(\sigma_c)_{\text{total}} = (\sigma_{b2}) - (\sigma_{t1})$$

$$(\sigma_c)_{\text{total}} = \frac{M_b \times h_o}{a' \times e \times r_o} - \frac{W_d}{a'}$$

$$(\sigma_c)_{\text{total}} = \frac{13.24 \times 10^6 \times 81.17}{6312.25 \times 9.958 \times 191.62} - 17.426$$

$$(\sigma_c)_{\text{total}} = 71.7989 \text{ N/mm}^2$$

3) At section 3-3 (tensile, bending and shear stress)

Section 3-3 is critical Section with regards to nature of stressed induced,

$$W_1 = W_d \cos 45$$

$$W_2 = W_d \cos 45$$

$$M_b = R \cdot W_d \cos 45$$

a) Tensile stress

$$(\sigma_{t1}) = \frac{W_d \cdot \cos 45}{a'} = 12.3223 \text{ N/mm}^2$$

b) Shear stress

$$(\tau) = \frac{W_d \cdot \cos 45}{a'} = \frac{110 \times 1000 \cos 45}{6312.25} = 12.3223 \text{ N/mm}^2$$

c) Bending stress

$$(\sigma_b) = \frac{M_b \times h_i}{a' \times e \times r_i} = \frac{R \cos 45 \times W_d \times h_i}{a' \times e \times r_i} = \frac{120.408 \times \cos 45 \times 110 \times 1000 \times 43.45}{6312.25 \times 9.958 \times 67},$$

$$\therefore (\sigma_b) = 96.625 \text{ N/mm}^2$$

Net stress at inner fibre at section 3-3

a) By Maximum Shear Stress Theory

$$(\tau) = \sqrt{\left(\frac{\sigma_t + \sigma_b}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{12.32 + 96.62}{2}\right)^2 + 12.32^2} = 55.84 \text{ N/mm}^2$$

$$(\tau) < [\tau] = 90 \text{ N/mm}^2, \text{ Hence, design is safe.}$$

b) By Maximum Principle Stress Theory

$$(\sigma_t) = \left(\frac{\sigma_b + \sigma_t}{2}\right) + \sqrt{\left(\frac{\sigma_t + \sigma_b}{2}\right)^2 + \tau^2} = \left(\frac{12.32 + 96.62}{2}\right) + \sqrt{\left(\frac{12.32 + 96.62}{2}\right)^2 + 12.32^2}$$

$$(\sigma_t) = 110.32 \text{ N/mm}^2 < [\sigma_t] = 180 \text{ N/mm}^2, \text{ Hence, design is safe.}$$

4) At section 4-4 (Shear Stress)

$$(\tau) = \frac{[W_d]}{a'} = \frac{110 \times 1000}{6312.25} = 17.426 \text{ N/mm}^2 < [\tau], \text{ hence safe in shear failure.}$$

Step 10.6: Design of Nut for hook

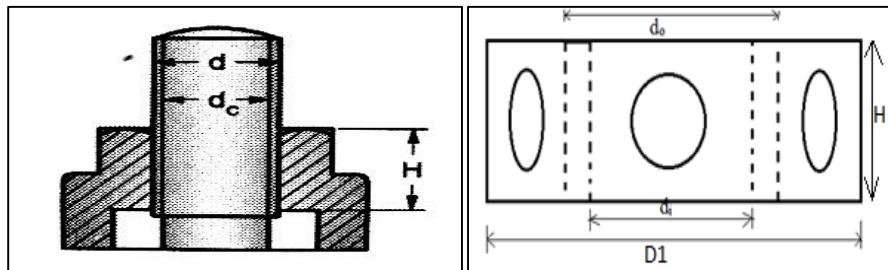


Fig. 2.34 Nut

Material Selection, Let C-25 with design stresses as,

$$[\sigma_t] = 80 \text{ N/mm}^2, [\tau] = 50 \text{ N/mm}^2, [\sigma_{cr}] = 120 \text{ N/mm}^2$$

Proportions:

$$H = D = 68 \text{ mm}; D_1 = 2D = 136 \text{ mm} \quad \dots \dots \text{PSG 9.11}$$

Load, $W_D = 110 \text{ KN}$

Failures: a) Shearing failure, b) Crushing of the thread

$$(\tau) = \frac{[W_d]}{A_s} = \frac{110 \times 10^3}{\pi \times D \times H} = \frac{110 \times 10^3}{\pi \times 68 \times 68} = 7.572 \text{ N/mm}^2 < [\tau]$$

Hence, safe in shear failure

$$(\sigma_{cr}) = \frac{W_D}{A_{cr}} = \frac{W_D}{\frac{\pi}{4} \times (d_o^2 - d_i^2) \times n}$$

Where, $n = \text{no. of threads} = \frac{H}{\text{Pitch}} = \frac{68}{5} = 14$

$$d_o = 68 \text{ mm}; d_i = 63 \text{ mm}$$

$$(\sigma_{cr}) = \frac{110 \times 10^3}{\frac{\pi}{4} \times (68^2 - 63^2) \times 14} = 15.273 \text{ N/mm}^2 < [\sigma_{cr}]$$

Hence, safe in crushing failure.

Step 10.7: Selection of bearing for hook

Selecting single thrust ball bearing directly based on the dimensions and static load carrying capacity C_0

Selecting Bearing No- 51314, from series 513 PSG 9.11

$C_0 = 27700 \text{ kgf}$, $d = 70 \text{ mm}$, $d_2 = 72 \text{ mm}$, $r = 2 \text{ mm}$, $D = 125 \text{ mm}$, $H = 40 \text{ mm}$ PSG 4.28

Static load required, $P_a = V \times F_a \times 1.2 = 1 \times 110 \times 10^3 \times 1.2 = 13200 \text{ kgf} < C_0$

Hence, Bearing is safe.

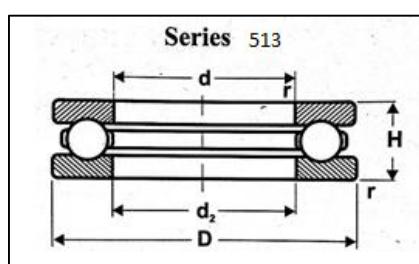


Fig. 2.35 Thrust Bearing

Step 11: Design of Cross piece

Cross piece is secured in cross plate and casing with fastener. The main body is rectangular while the ends are modified in cylindrical form called trunion. The trunion provides swinging effect; a provision is made to house the thrust bearing.

Material selected, Plain Carbon steel

Design stresses are,

$$[\sigma_t] = 100 \text{ N/mm}^2, [\sigma_{br}] = k \times [\sigma_t] = 0.75 \times 100 = 75 \text{ N/mm}^2$$

$$[\tau] = 60 \text{ N/mm}^2$$

(Constant K = 0.75 suggested by Rudenko for cross piece, K = 0.1 to 1.5, depends on relative motion)

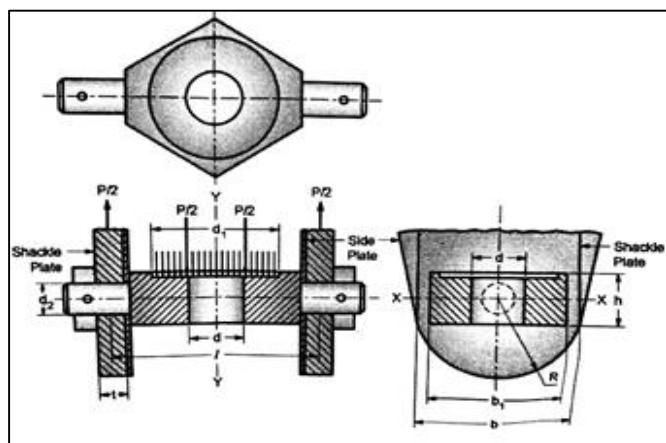


Fig. 2.36 Cross Piece

Calculate diameter of Trunion of cross piece by considering

11.1 Shear failure at trunion,

$$[\tau] = \frac{W_d/2}{\frac{\pi}{4}xd^2} = 60, \quad d = 34\text{mm}$$

Hence diameter, d= 34 mm, Assuming d= 50 mm

Let height of cross piece h = 1.5d = 75mm

11.2 Checking trunion in Bending,

$$(\sigma_b) = \frac{M}{Z} = \frac{\frac{55 \times 10^3 \times 8}{\pi \times 50^3}}{32} = 35.85 \frac{\text{N}}{\text{mm}^2} < [\sigma_b],$$

Therefore, safe in bending.

11.3 Check in bearing failure,

$$(\sigma_{\text{br}}) = \frac{W/2}{d \times L} = \frac{55 \times 10^3}{50 \times 16} = 68.75 \frac{\text{N}}{\text{mm}^2} < [\sigma_{\text{br}}] \text{ i.e } 75 \frac{\text{N}}{\text{mm}^2}$$

Therefore, safe in bearing failure.

B = Width of the cross piece = Size of the bearing or nut + clearance+2 x Flange thickness + End margin

Therefore, $B = 136 + 4 + 2 \times 12 + 46 = 170 \text{ mm}$

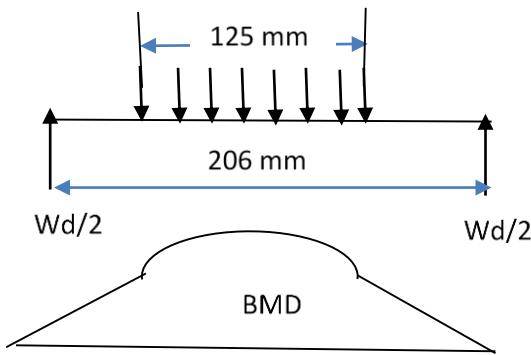


Fig. 2.37 Force diagram and BMD

Figure show the force diagram and BMD for cross piece.

$$BM_{\max} = \frac{W_d}{2} \times \frac{1}{2} - \frac{W_d}{2} \times \frac{D}{4}$$

$$BM_{\max} = \frac{110 \times 10^3}{2} \times \frac{206}{2} - 55 \times 10^3 \times \frac{125}{4}$$

$$BM_{\max} = 3946250 \text{ N.mm}$$

$$Z = \frac{1}{6} (B - d) \times H^2$$

$$Z = \frac{1}{6} (170 - 50) \times 75^2 = 112500 \text{ mm}^3$$

$$(\sigma_b) = \frac{M_{\max}}{Z} = \frac{3946250}{112500} = 35.08 \text{ N/mm}^2 < [\sigma_b]$$

Therefore, safe in bending as the induced stress is less than the design stress.

Figure 2.38 shows the cross section for section modulus Z.

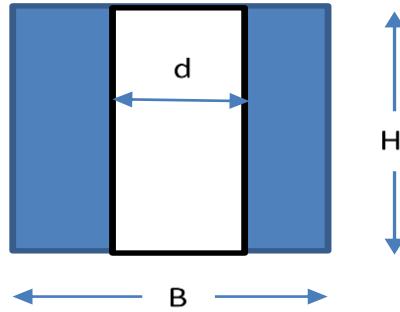


Fig 2.38 Cross Section for section modulus

Step 12: Shackle Plate

As the crosspiece is secured in the side plates which are strengthened with shackles or straps. Only shackles are checked for strength neglecting the plates in view of their relatively small thickness.

Material: Plane Carbon Steel C-20

$$[\sigma_t] = 100 \text{ N/mm}^2, [\tau] = 60 \text{ N/mm}^2, [\sigma_{cr}] = 150 \text{ N/mm}^2$$

d_1 = Axle size in shackle plate = 70mm,

d_2 = Trunion size of cross piece = 50mm,

t = Thickness of plate = 16mm,

Let $h_1 = d_1 = 70\text{mm}$, $h_3 = d_2 = 50\text{mm}$ and B = width of the plate = $3d_1 = 210\text{mm}$,

$h_2 = (\text{Sheave diameter}/2) + \text{Height of cross piece above centre line} + \text{thrust bearing/nut height} + \text{margin}$

$$h_2 = 715/2 + 75/2 + 68 + 10 = 473\text{mm} = 480\text{mm approximately.}$$

Now checking for tensile failure at section 1-1,

$$(\sigma_t) = \frac{w/2}{(B - d_1)t} = \frac{55 \times 10^3}{(310 - 70) \times 16}$$

$$(\sigma_t) = 14.32 \frac{N}{mm^2} < [\sigma_t] \text{, hence safe in tension}$$

Checking for double shear stress at section 2 -2,

$$(\tau) = \frac{w/2}{2 \times h_1 \times t} = 24.55 \frac{N}{mm^2} < [\tau] \text{, hence safe in shear}$$

Similarly checking for crushing failure at section 3-3,

$$(\sigma_{cr}) = \frac{w/2}{d_1 \times t} = \frac{55 \times 10^3}{70 \times 16} = 49.10 \frac{N}{mm^2} < [\sigma_{cr}]$$

Hence, Safe in crushing failure.

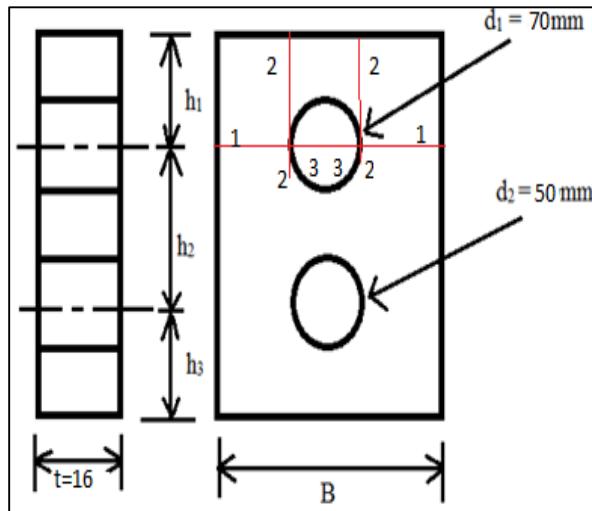


Fig. 2.39 Shackle Plate

Step 13: Design of Drum

The rope drum should be made of seamless pipe machined & grooved accurately to ensure proper seating of wire rope in a proper layer. The drum should be fitted with two heavy duty Ball / Roller bearings of reputed make for smooth operation & longer life.

Diameter of Compensating pulley = $0.6 \times$ diameter of movable pulley = 345 mm

Un-grooved length or unrolled length = $0.6 \times$ Diameter of compensating pulley = 200 mm

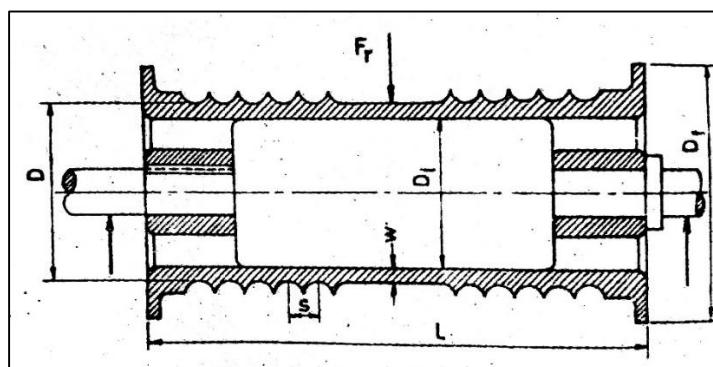


Fig. 2.40 Cross Section of Rope Drum

From PSG 9.2, length of drum for two rope is given as,

$$L_D = \left(\frac{2 \times H \times i}{\pi \times D} + 12 \right) \times S + l_1$$

Where, H = Hoisting lift = 8 m

$$i = \text{Ratio of pulley system} = \frac{n_p}{2} = \frac{4}{2} = 2;$$

l_1 = Length of ungrooved length = 200 mm

D = Drum Diameter,

D_{\min} = 575 mm,

Therefore,

$$D = 575 + 25 = 600 \text{ mm}$$

From PSG 9.9 and using interpolation method,

For $d=25 \text{ mm}$; $S=28 \text{ mm}$

$$L_d = \left(\frac{2 \times 8000 \times 2}{\pi \times 600} + 12 \right) \times 28 + 200 = 1011.34 \text{ mm}$$

Approximately, $L_D = 1050 \text{ mm}$

Note- Keeping 50 mm out of 1050 mm on either side. Hence, mounting is at 950 mm. Depending on the position of the load, the rope will be located at minimum distance of 200 mm and maximum distance of 950 mm. As the bending moment depends on the distance of the load from the support, the maximum bending moment will occur when load is at distance L_1 .

Selecting material for drum as C-40 with $[\sigma_c] = 140 \text{ MPa}$ also neglecting the weight of drum and rope.

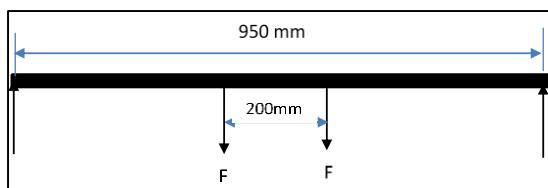


Fig. 2.41 Force Diagram

$$\text{B.M}_{\max} = F \times \left(\frac{L_D}{2} - \frac{L_1}{2} \right) = 9.87 \text{ kN.m}$$

Drum is subjected to direct compression over the area = $(S \times \text{thickness})$

$$[\sigma_c] = \frac{F_{\max}}{S \times t} \quad \text{hence} \quad 140 = \frac{26.2 \times 10^3}{28 \times t}$$

$$t = 6.71\text{mm}, \text{ approx. } t = 8\text{mm}$$

Thickness is calculated and depth of groove C_1 can be calculated based on diameter of 25 mm,
depth of groove, $C_1 = 6\text{mm}$ and $t = 8\text{mm}$ below the groove PSG 9.9

Hence, actual thickness of the plate is $6 + 8 = 14\text{ mm}$.

Now actual compressive stress is given by,

$$(\sigma_c) = \frac{F_{\max}}{S \times t} = \frac{26.32 \times 10^3}{28 \times 14} = 67.13 \text{ N/mm}^2 \quad \dots \dots \text{PSG 9.3}$$

And stress due to bending is given by,

$$(\sigma_b) = \frac{M_{\max}}{Z} = \frac{9.87 \times 10^6}{\frac{\pi}{32} \times \left(\frac{575^4 - 547^4}{575} \right)} = 2.921 \frac{\text{N}}{\text{mm}^2}$$

It is observed that compressive stress due to direct compression is the predominant stress, hence for quicker analysis directly design based on the direct compressive stress.

13.1 Total compressive stress,

$$\sigma_{c\text{total}} = \sigma_c + \sigma_b = 67.13 + 2.9213 = 70.05 \text{ N/mm}^2$$

Drum is subjected to torsion also,

The maximum torque acting on the rope drum = $2 \times F_{\max} \times \frac{D_o}{2}$

$$T = 2 \times 26.32 \times 10^3 \times \frac{600}{2} = 15.792 \text{ KN.m}$$

Torsional shear stress induced is given by,

$$(\tau) = \frac{T}{J} = \frac{T}{\pi \times \left(\frac{D^4 - D_i^4}{16 \times D} \right)} = \frac{15.792 \times 10^6}{\frac{\pi}{16} \times \left(\frac{575^4 - 547^4}{575} \right)} = 2.33 \frac{\text{N}}{\text{mm}^2}$$

13.2 Principal Stresses,

$$\sigma_1 = \frac{\sigma_{c\text{total}}}{2} + \sqrt{\left(\frac{\sigma_{c\text{total}}}{2} \right)^2 + \tau^2}$$

$$\sigma_1 = \frac{70.05}{2} + \sqrt{\left(\frac{70.05}{2} \right)^2 + 2.33^2} = 70.127 \frac{\text{N}}{\text{mm}^2} < \left[140 \frac{\text{N}}{\text{mm}^2} \right]$$

Hence, safe in design.

Step 14: Selection of motor

Given, Hoisting Speed = 8 m/min

Output Power = Design load * Hoisting Speed

$$W_d \times V = 110 \times 10^3 \times \frac{8}{60} = 14666.6 \text{ watt}$$

Considering transmission efficiency as 0.85.

$$\text{Input Power} = \frac{\text{Output Power}}{\text{transmission efficiency}} = \frac{14666.6}{0.85} = 17.254 \text{ kW}$$

Selecting Standard Motor of 18 KW

..... PSG 5.124

Step 15: Design of Drum Shaft

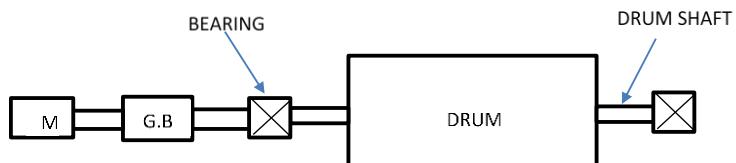


Fig. 2.42 Drum Shaft

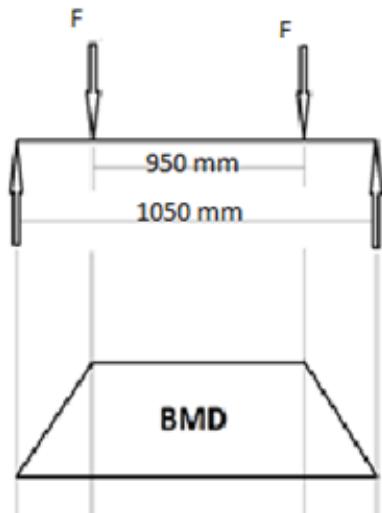


Fig. 2.43 Force and BM Diagram

Drum shaft is subjected to Torque and Bending Moment. SFD and BMD is as shown in figure 2.42 and 2.43.

$$BM_{max} = 26.32 \times 10^3 \times 50 = 1.316 \text{ KN.m}$$

$$\text{Torque } T = 2F \times \frac{D_{drum}}{2}$$

$$T = 52.64 \times 10^3 \times \frac{600}{2} = 15.793 \text{ KN.m}$$

Equivalent torque, $T = \sqrt{M^2 + T^2} = 15.85 \text{ KN.m}$

From PSG 1.13, Let material be 40cr1 with

$$S_{yt} = 540 \frac{\text{N}}{\text{mm}^2}; \text{ FOS} = 4$$

$$[\sigma_t] = 135 \frac{\text{N}}{\text{mm}^2}; [\tau] = 80 \text{ N/mm}^2$$

$$\text{Now, } T_{eq} = \frac{\pi}{16} x d^3 x [\tau], 15.85 \times 10^6 = \frac{\pi}{16} x d^3 x 80$$

$$d = 100 \text{ mm}$$

Selecting shaft with diameter as 100mm.

Step 16: Design of bearing for drum shaft

For drum shaft let's select cylindrical roller bearing,

Forces acting on each bearing are, $F_r = 26.32 \text{ KN}$, $F_a = 0$

Assuming life in hours, $L_{hr} = 10,000 \text{ hrs}$

Linear speed of the drum = 2x linear speed of the load = $2 \times 6 = 12 \text{ m/min}$

$$V = \pi D N, 12 = \pi \times 0.6 \times N$$

$$N = 6.36619 \text{ rpm}$$

Life in millions of revolutions is given by,

$$L_{mr} = \frac{L_{hr} \times N \times 60}{10^6} = \frac{10000 \times 6.37 \times 60}{10^6}$$

$$L_{mr} = 3.81971 \text{ mr}$$

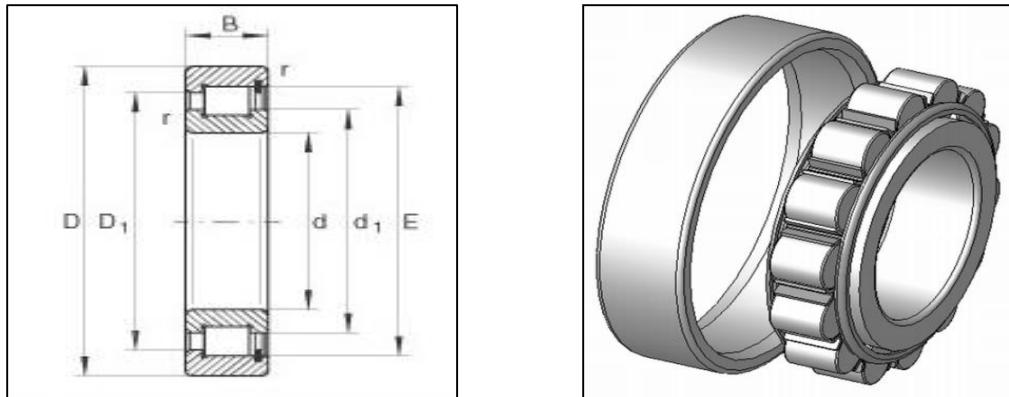


Fig. 2.44 Cylindrical roller Bearing

Equivalent load acting on the bearing,

$$P_{eq} = (X \cdot V \cdot F_r + Y \cdot F_a)S$$

Assuming X=1, V = 1 and S = 1.2

$$P_{eq} = 31.584 \text{ KN}$$

Dynamic load carrying capacity is given by,

$$C = P \cdot (L_{mr})^{\frac{1}{k}} = 52.64 \times (3.822)^{\frac{3}{10}} = 47.2119 \text{ KN}$$

Where k = 10/3 for roller bearing,

Therefore, $C = 4720 \text{ kg.f}$

Therefore, selecting from PSG 4.21,

Standard bearing as SKF NU 2220 OR SKF 6220 for $d=100\text{mm}$

with $d = 100 \text{ mm}; D = 180 \text{ mm};$

$$C_0 = 19000 \text{ kg.f}; \quad C = 20000 \text{ kg.f}$$

Hence, bearing is safe for design.

Numerical 2.2. Design of Hoisting Mechanism for the following specifications.

Load to be lifted = 300 kN

Hoisting Speed = 5m/min

Lift = 10 m

Solution -

Design includes Rope, Sheave, Bearing, Axle, Hook, Thrust Bearing, Nut, Cross piece, Shackle Plate, Drum, Hoisting motor, drum shaft, bearing for drum shaft.

Note: PSG design data book is referred and suitable data is assumed

Step 1: Selection of number of falls and pulley system

The given load to be lifted = 300KN

η_p – Efficiency of pulley system = 0.90

Therefore, selecting 8 fall system & efficiency of pulley system as

η_p – Efficiency of pulley system = 90 %

Minimum no. of bends = 7

$$\frac{D_{min}}{d} = 30 \quad \dots\dots \text{PSG 9.1}$$

For no. of bends = 7,

Selecting cross lay type of rope and 6×37 type rope for greater flexibility.

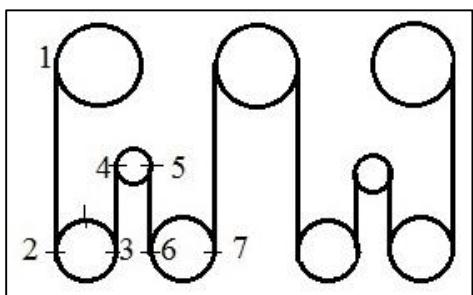


Fig. 2.45 No. of falls

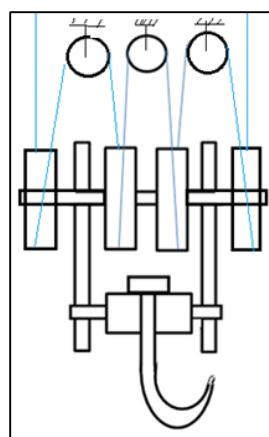


Fig. 2.46 Snatch Block for 8 fall system

Selecting material for rope as IPS with $\sigma_{ut} = 1800\text{N/mm}^2$.

Step 3: Maximum tension in the rope

$$T_{max} = F = \frac{Q}{\eta_f \times \eta_p} = \frac{300}{8 \times 0.9}$$

$$F = 41.667 \text{ kN}$$

Step 4: Rope Selection

Stress factor $n = n' \times S = 5 \times 1.2 = 6$

Where n' = factor of safety (for class-2) $n' = 5$) PSG 9.1

S = duty factor based on strength ($S = 1.2$) PSG 9.2

Cross Section Area, $A = \frac{F}{\frac{\sigma_{ut}}{n} - 3600 \times \frac{d}{D_{min}}}$ PSG 9.1

$$A = \frac{41.667 \times 1000}{\frac{1800}{6} - 3600 \times \frac{1}{30}}$$

$$A = 231.48 \text{ mm}^2$$

$$\text{Actual Area} = 0.4 \times \frac{\pi}{4} \times d^2 = 231.48$$

Therefore, $d = 27.14 \text{ mm}$

By taking standard value of rope from PSG 9.4, $d = 29 \text{ mm}$

Hence $D_{min} = 30 \times d = 870 \text{ mm}$.

Step 5: Life of the rope

By Empirical Formula, $D \geq e_1 \times e_2 \times d$

Where, $e_1 = 25$ (for medium power hoisting mechanism)

$e_2 = 1$ (for cross laid), $d = 29 \text{ mm}$

Therefore, $D = 25 \times 29 \times 1 = 725 \text{ mm}$

Selecting $D_{min} = 870\text{mm}$, $D = 870 + 29 \approx 900\text{mm}$

$$\text{Now, } \sigma = \frac{10F}{\pi d^2} = \frac{10 \times 41.667 \times 10^3 \text{ N}}{\pi \times 29^2} = 14.7 \text{ kgf/mm}^2$$

$$\frac{D}{d} = m \sigma C_1 C_2 C + 8 \quad \dots \dots \dots \text{PSG 9.7}$$

$$C_1 = 1.09 \text{ rope size factor for } d=29\text{mm} \quad \dots \dots \dots \text{PSG 9.8}$$

$$C_2 = 1 \text{ material and surface finish factor (varies from 0.63 to 1.15)} \quad \dots \dots \dots \text{PSG 9.7}$$

$$C = 1.02 \text{ strength and rope construction factor.} \quad \dots \dots \dots \text{PSG 9.8}$$

$$\frac{725}{29} = (m \times 14.7 \times 1.09 \times 1 \times 1.02) + 8$$

$$m = 1.2642, \quad m = 126.42 \text{ (in hundred)}$$

From PSG 9.8,

Z	170	X	190
M	118	126.42	129

Therefore, x = 154.69 (by interpolation)

Z in thousands, Z = 154.69 × 10³

$$N = \frac{0.4 Z}{\alpha \beta Z_2} \quad \dots \dots \dots \text{PSG 9.7}$$

From PSG 9.8, Table 2, considering medium duty 16hrs/day

α = 3400 working cycles/month

β = 0.4 Endurance factor

Z_2 = 3 no. of repeated bend/cycle

$$N = \frac{0.4 \times 154.69 \times 1000}{3400 \times 3 \times 0.4} = 15.656 \text{ months}$$

Life of Rope in months is 15. 66 months.

Step 6: Selection of pulley/ sheave

Selecting std. sheave dimensions for wire dia.

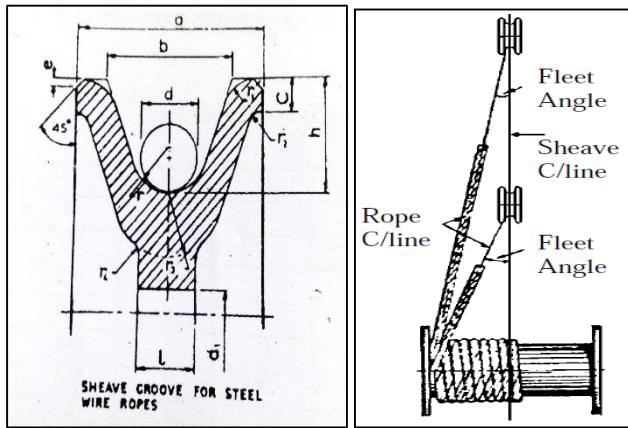


Fig. 2.47 Pulley cross section and fleet angle

$$d = 34.5\text{mm from PSG 9.10}$$

$$\text{Where, } a = 90\text{mm, } b = 70\text{mm}$$

$$C = 15\text{mm, } h = 55\text{mm, } L = 22\text{mm}$$

Checking fleet angle α ,

$$\tan \alpha = \frac{\frac{b-d}{2}}{D_o \times 0.5}$$

$$D_o = D_{\min} + 2h = 870 + 2 \times 55 = 980 \text{ mm}$$

Therefore, $\alpha = 2.26^\circ < 5^\circ$, Hence, acceptable.

Step 7: Speed of sheave

$$\text{Velocity of sheave} = \pi D N$$

$$V = \text{Velocity of hoisting} \times \frac{n}{2}, \text{ Where, } n - \text{no. of fall}$$

$$N = \frac{V \times \frac{n}{2}}{\pi D}$$

$$N = \frac{6 \times \frac{7}{2}}{\pi \times 725 \times 10^{-3}} = 9.25 \text{ rpm}$$

Step 8: Selection of bearing

Selection of two bearing per pulley so that the radial load acting on bearing is shared and small bearing can be used also the balancing of load will be automatically taken care.

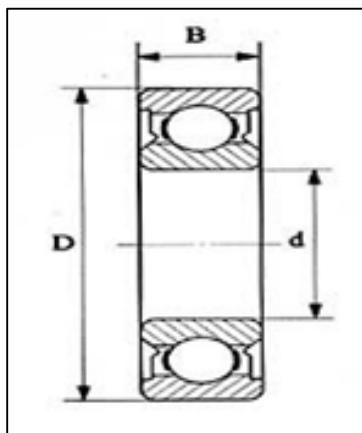


Fig. 2.48 Deep Groove Ball Bearing

Now forces acting on each bearings are,

$$F_r = 41667 \text{ N}, F_a = 0 \text{ N}$$

$$P_{eq} = (X \cdot F_r \cdot V + Y \cdot F_a) S$$

Assuming, Radial load factor $X = 1$, Race rotation factor $V = 1.2$

And Service factor $S = 1$

Equivalent load,

$$P_{eq} = 50000.4 \text{ N}$$

Assuming life of the bearing = 10000 hrs.

$$L_{mr} = \frac{10000 \times 7.07 \times 60}{10^6} = 4.24 \text{ mr}$$

Dynamic Capacity, $C = P_{eq} \times (L_{mr})^{\frac{1}{K}}$ where $K = 1/3$ for ball bearing

$$\text{Dynamic capacity } C = 50000 \times 4.24^{\frac{1}{3}} = 80926.72 \text{ N} = 8092.67 \text{ kgf}$$

From PSG 4.13, Selecting DGBB SKF 6219.PSG 4.13

Having, $d = 95 \text{ mm}$, $D = 170 \text{ mm}$, $B = 32 \text{ mm}$, $C = 8500 \text{ kgf}$.

Step 9: Design of axle

(Assuming two pulleys on axle are in between the shackle plate and two pulleys are at two ends of axle)

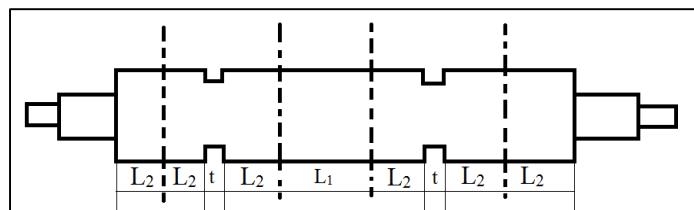


Fig. 2.49 Axe span length

Let, Design stresses for the axle material be $[\sigma_t] = 100 \text{ N/mm}^2$, $[\tau] = 60 \text{ N/mm}^2$

$$L_1 = a + \text{margin} = 90 + 10 = 100 \text{ mm}$$

$$L_2 = a/2 + \text{margin} = 45 + 10 = 55 \text{ mm}$$

$$t = \text{casing thickness} + \text{Shackle plate thickness} = 4 + 12 \text{ mm} = 16 \text{ mm}$$

$$\text{Support Span, } L = L_1 + 2L_2 + t$$

$$\text{Support Span, } L = 100 + 110 + 16 = 226 \text{ mm}$$

$$\text{Total length of Axle} = L + t + 2L_2 + \text{Margin}$$

From Force and Bending Moment diagram,

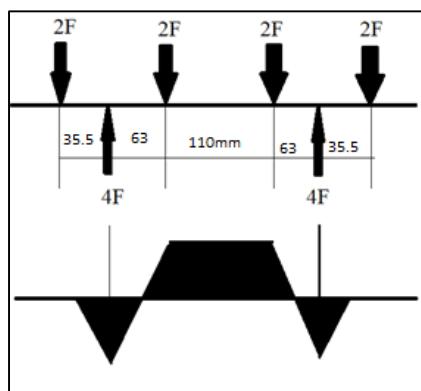


Fig. 2.50 Force Diagram and Bending Moment Diagram

$$B.M_{\max} = 41.667 \times 10^3 \times 2 \times 63$$

$$B.M_{\max} = 5250.042 \text{ KN-mm}$$

$$[\sigma_b] = \frac{B.M_{\max}}{Z} = \frac{5250.042 \times 1000}{\frac{\pi}{32}d^3} = 100$$

Therefore $d^3 = 534764.88$

$$d = 81.16 \text{ mm}$$

Let, $d = 95 \text{ mm}$ (considering DGBB SKF 6219)

And $d_1 = 85 \text{ mm}$

Step 10: Design of hook

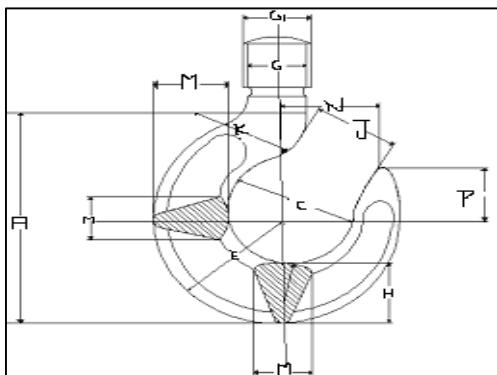


Fig. 2.51 Cross Sectional diagram of hook

10.1 Design load

Assuming extra load, $[W_d] = 320 \text{ kN}$

10.2 Selection of Material

Selecting Mild Steel, assuming design stresses as,

$[\sigma_t] = 200 \text{ N/mm}^2$ for shank and saddle part of a hook

$[\sigma_t] = 100 \text{ N/mm}^2$ for threaded part of a hook

From PSG 9.11 for 32 tonne load,

$$C = 207 \text{ mm}$$

$$G = 110 \text{ mm}$$

$$G_1 = M100$$

$$A = 2.75C = 569.25 \text{ mm}^2$$

$$B = 1.31C = 271.17 \text{ mm}$$

$$D = 1.44C = 298.08 \text{ mm} \quad , H = 0.93C = 192.51 \text{ mm}$$

$$Z = 0.12C = 24.84 \text{ mm} \quad , M = 0.6C = 124.2 \text{ mm}$$

$$b_o = 2Z = 49.68 \text{ mm}$$

10.3 Theoretical Area of cross section,

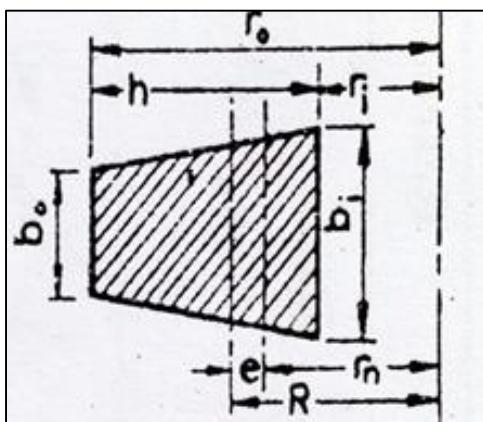


Fig. 2.52 Cross Section of hook

$$(A) = \frac{1}{2}(b_i + b_o)H = \frac{1}{2}(124.2 + 49.68) \times 192.51$$

$$A_{th} = 16736.81 \text{ mm}^2$$

$$\text{Actual area } a' = 0.9 A_{th} = 15063.13 \text{ mm}^2$$

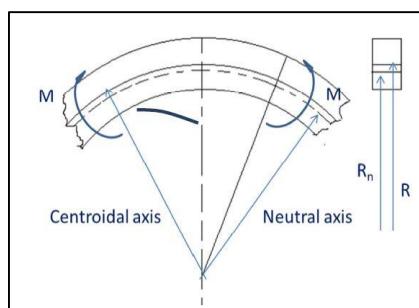


Fig. 2.53 Beam Diagram

10.4 Location parameters.....PSG 6.2/6.4

$$r_i = 0.5C = 103.5 \text{ mm}$$

$$r_o = r_i + H = 296.01 \text{ mm}$$

From PSG 6.3,

$$R = r_i + \frac{H}{3} \left[\frac{b_i + 2b_o}{b_i + b_o} \right] = 186 \text{ mm}$$

$$R_N = \frac{0.5(b_i + b_o)H}{\left(\frac{b_i r_o - b_o r_i}{H} \right) \ln \left(\frac{r_o}{r_i} \right) - 74.51} = \frac{16736.81}{164.26 \times 1.05082 - 74.51} = 170.63 \text{ mm}$$

$$h_i = R_N - r_i = 170.35 - 103.5 = 66.86 \text{ mm}$$

$$h_o = H - h_i = 192.51 - 66.86 = 125.65 \text{ mm}$$

$$e = R - R_N = 15.37 \text{ mm}$$

10.5 Checking for different Failures:

1) At section 1-1 (tension only)

$$(\sigma_t) = \frac{Wd}{\frac{\pi}{4} \times d \times d} = \frac{320000}{\frac{\pi}{4} \times 100 \times 100} = 40.74 \text{ N/mm}^2 < [\sigma_t] \text{ Hence Safe}$$

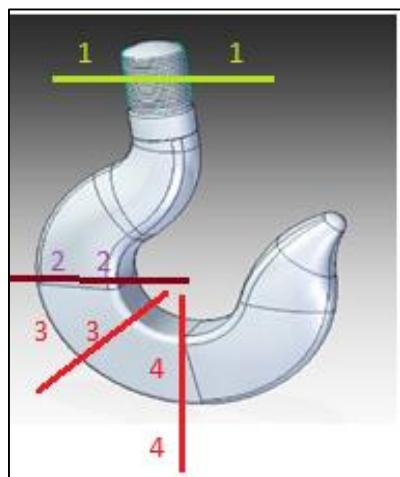


Fig. 2.54 Sections of hook

2) At Section 2-2

(a) At inner fibre (tension and bending)

$$(\sigma_t)_{\text{total}} = (\sigma_{t1}) + (\sigma_{b1})$$

$$(\sigma_{t1}) = \frac{[Wd]}{a'} = \frac{320000}{15063.13} = 21.25 \text{ N/mm}^2$$

$$(\sigma_{b1}) = \frac{Mb \times hi}{a' \times e \times ri} = \frac{320000 \times 186 \times 66.86}{15063.13 \times 15.37 \times 103.5} = 166.07 \text{ N/mm}^2$$

$$(\sigma_t)_{\text{total}} = 21.25 + 166.07 = 187.32 \text{ N/mm}^2$$

$(\sigma_t)_{\text{total}} < [\sigma_t]$ Hence, Safe in tensile stresses.

(b) At outer fibre [Compressive stress only]

$$(\sigma_c)_{\text{total}} = (\sigma_{b2}) + (\sigma_{t1})$$

$$(\sigma_c)_{\text{total}} = \left[\frac{Mb \cdot ho}{a' \cdot e \cdot ro} \right] - \left[\frac{Wd}{a'} \right]$$

$$(\sigma_c)_{\text{total}} = \frac{320000 \times 186 \times 125.65}{15063.13 \times 15.37 \times 296.01} - \frac{320000}{15063.13}$$

$$(\sigma_c)_{\text{total}} = 109.12 - 21.25 = 87.87 \text{ MPa},$$

$(\sigma_c)_{\text{total}} = 87.87 \text{ N/mm}^2 < [\sigma_c]$ Hence, Safe in compressive stress.

3) At Section 3-3 (tension, bending and shear)

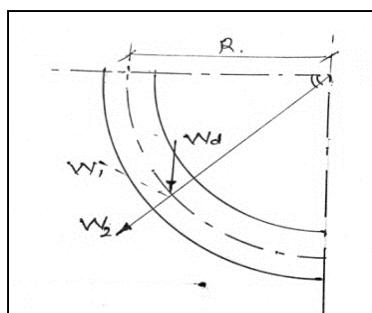


Fig. 2.55 Hook section

a) Tensile Stress induced :

$$(\sigma_{t1}) = \frac{Wd \cos 45}{a'} = \frac{320000 \cos 45}{15063.13} = 15.02 \text{ N/mm}^2$$

b) Shear Stress induced:

$$(\tau) = \frac{Wd \cos 45}{a'} = \frac{320000 \sin 45}{15063.13} = 15.02 \text{ N/mm}^2$$

c) Bending Stress induced:

$$(\sigma_b) = \frac{Mb hi}{a' e ri} = \frac{Wd \cos 45 \times R \times hi}{a' e ri}$$

$$(\sigma_b) = \frac{320000 \times \cos 45 \times 186 \times 66.86}{15063.13 \times 15.37 \times 103.5} = 117.43 \text{ N/mm}^2$$

Net stress at inner fibre at section 3-3 –by Maximum Shear Stress Theory

$$(\tau) = \sqrt{\left(\frac{\sigma_t + \sigma_b}{2}\right)^2 + (\tau)^2} = \sqrt{\left(\frac{15.02 + 117.43}{2}\right)^2 + 15.02^2}$$

$(\tau) = 67.9 \text{ N/mm}^2 < [\tau] = 90 \text{ N/mm}^2$ Therefore, Safe in shear stress.

By Maximum Principal Stress Theory,

$$(\sigma_1) = \frac{(\sigma_b + \sigma_t)}{2} + \sqrt{\left(\frac{\sigma_t - \sigma_b}{2}\right)^2 + (\tau)^2}$$

$$(\sigma_1) = \frac{15.02 + 117.43}{2} + \sqrt{\left(\frac{15.02 + 117.43}{2}\right)^2 + 15.02^2}$$

$(\sigma_1) = 134.125 \text{ N/mm}^2 < [\sigma_t]$ Therefore, Safe for tensile stress.

4) At Section 4-4 (shear stress)

$$(\tau) = \frac{[W_d]}{a'} = \frac{320 \times 1000}{15063.13} = 21.24 \text{ N/mm}^2 < [\tau] = 90 \text{ N/mm}^2$$

Hence, safe in shear failure.

Step 10.6: Design of nut

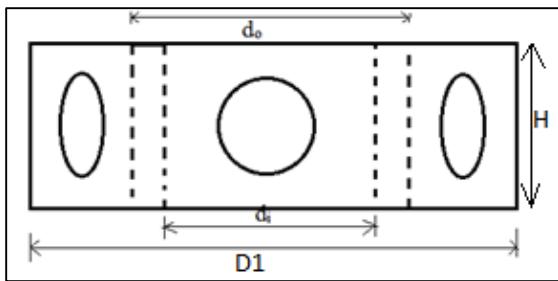


Fig. 2.56 Nut

Material Selection, Let C-25 with design stresses as,

$$[\sigma_t] = 80 \text{ N/mm}^2, [\tau] = 50 \text{ N/mm}^2, [\sigma_{cr}] = 120 \text{ N/mm}^2$$

Proportions – Cylindrical Nut,

$$H = D = 82 \text{ mm}, D_1 = 2D = 164 \text{ mm}$$

Failures - (a) Shearing of thread

(b) Crushing of thread

$$\text{Shear stress induced } (\tau) = \frac{[Wd]}{As} = \frac{[Wd]}{\pi D H} = \frac{320000}{\pi \times 82 \times 82} = 15.14 \text{ N/mm}^2 < 50 \text{ N/mm}^2$$

Therefore, Safe for shear stress.

$$\text{Crushing stress induced } (\sigma_{cr}) = \frac{[Wd]}{A} = \frac{[Wd]}{0.25\pi(d_o^2 - d_i^2)n}$$

$$\text{Where, } n = \frac{H}{pitch} = \frac{82}{6} = 13.66 = 14, d_o = 82 \text{ mm}, d_i = 76 \text{ mm}$$

$$(\sigma_{cr}) = \frac{320 \times 1000}{0.25\pi(82^2 - 76^2)14} = 330.7 \text{ N/mm}^2 < 120 \text{ N/mm}^2$$

Therefore, Safe for crushing stress.

Step 10.7: Selection of bearing for hook

Selecting bearing based on the dimensions.....PSG 9.11

Series = 513

Bearing no. = 51319

d	H	C _o	D
88 mm	49 mm	40800 kgf	150 mm

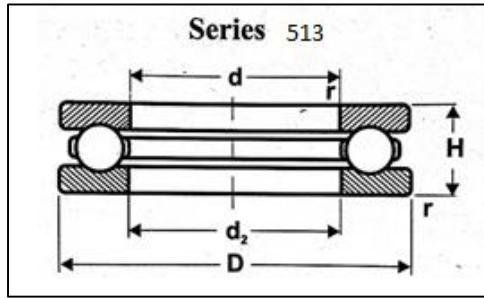


Fig. 2.57 Thrust Bearing

Static load, P_{eq} = Y. F_a. S = 320 x 10³ x 1.2 = 384000 N

C_o = 38400 kgf < [C_o = 40800 kgf]

Bearing is safe

Step 11: Design of Cross Piece

Cross piece is secured in cross plate and casing with fastener. The main body is rectangular while the ends are modified in cylindrical form called trunion. The trunion provides swinging effect; a provision is made to house the thrust bearing.

Material – Plain carbon steel

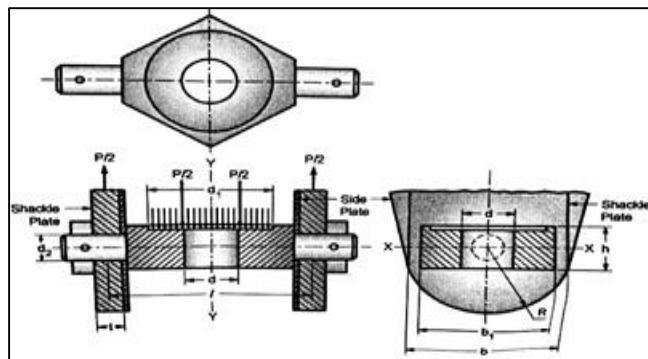


Fig. 2.58 Cross Piece for hook

$$[\sigma_t] = [\sigma_b] = 100 \text{ N/mm}^2, [\tau] = 60 \text{ N/mm}^2, [\sigma_{br}] = k [\sigma_t]$$

(Constant K = 0.75 suggested by Rudenko for cross piece, K = 0.1 to 1.5, depends on relative motion)

$$[\sigma_{br}] = 75 \text{ N/mm}^2$$

d_1 = shank dia. + clearance

$$d_1 = 100 + 2 = 102 \text{ mm}$$

D = outer dia. of bearing, D = 150 mm

L₁ = 210 mm, L₂ = 16 mm, H = 1.5d

11.1: Considering shear failure for trunion:

$$[\tau] = \frac{[Wd]/2}{\frac{\pi}{4} d \times d}$$

$$60 = \frac{165 \times 10^3}{\frac{\pi}{4} d \times d}$$

Trunion diameter d = 59.17 mm, let d = 60 mm, Height of cross piece H = 1.5 x 60 = 90 mm

11.2: Checking trunion in bending failure:

$$(\sigma_b) = \frac{M}{Z}$$

$$(\sigma_b) = \frac{165 \times 10^3 \times 8}{\frac{\pi}{32} \times 60^3} = 62.24 \text{ N/mm}^2 < [\sigma_b]$$

11.3: Checking under bearing failure:

$$(\sigma_{br}) = \frac{Wd/2}{d \times L2} = \frac{165 \times 1000}{60 \times 16} = 171.87 \text{ N/mm}^2 > [\sigma_{br}] \text{ hence fail in bearing failure.}$$

Now changing d to 80mm and thickness of shackle plate to 20mm, required bearing stress for material is, $[\sigma_{br}] = \frac{165 \times 1000}{80 \times 20} = 103.125 \text{ MPa.}$

Hence changing material such that $[\sigma_{br}]$ is 110MPa. Hence H = 1.5d = 120mm

Width of cross piece B = size of bearing + clearance + 2(flange thickness) + margin

$$B = 164 + 4 + 2(12) + 8 = 200 \text{ mm}$$

Checking cross piece for bending failure, $(\sigma_b) = \frac{M}{Z}$ where,

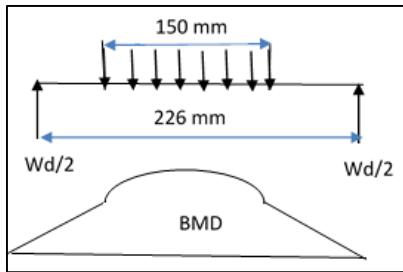


Fig. 2.59 Force and BMD

$$BM_{max} = \frac{Wd}{2} \left[\frac{l}{2} - \frac{D}{4} \right]$$

$$BM_{max} = 165 \times 10^3 [226/2 - 150/4]$$

$$BM_{max} = 12.45 \times 10^6 \text{ N-mm}$$

Section modulus (Z)

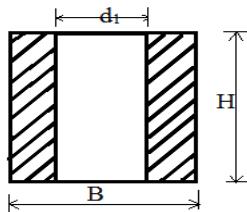


Fig. 2.60 Cross Section of Cross Piece

$$Z = (1/6) (B - d_1) H^2 = (1/6) (200 - 102) 120^2 = 235.2 \times 10^3 \text{ mm}^3$$

$$(\sigma_b) = \frac{M}{Z} = \frac{12450 \times 1000}{235200} = 52.93 \text{ N/mm}^2 < [\sigma_b] \text{ Hence Safe}$$

Step 12: Shackle Plate Design

As the crosspiece is secured in the side plates which are strengthened with shackles or straps. Only shackle plates are checked for strength neglecting the side plates in view of their relatively small thickness.

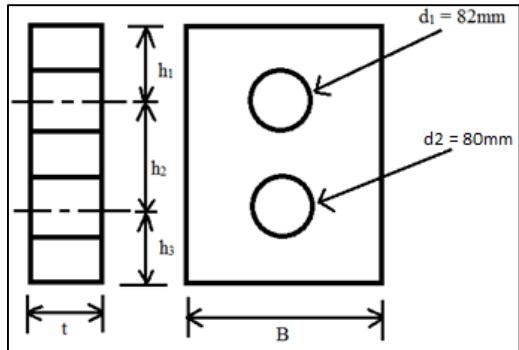


Fig. 2.61 Cross Section of Shackle Plate

Material: Plane Carbon steel C-20

$$[\sigma_t] = 100 \text{ N/mm}^2,$$

$$[\tau] = 60 \text{ N/mm}^2,$$

$$[\sigma_{cr}] = 150 \text{ N/mm}^2$$

d_1 = Axle size in shackle plate = 70mm,

d_2 = Trunion size of cross piece = 50mm,

t = Thickness of plate = 20mm,

Let $h_1 = d_1 = 82\text{mm}$, $h_3 = d_2 = 80\text{mm}$

B = width of the plate = $3d_1 = 246\text{mm}$,

$h_2 = (\text{Sheave diameter}/2) + \text{Height of cross piece above centre line} + \text{thrust bearing/nut height} + \text{margin}$

$$h_2 = 900/2 + 120/2 + 68 + 20 = 598\text{mm}$$

$h_2 = 600\text{mm}$ approximately.

Checking for tensile failure,

$$(\sigma_t) = \frac{0.5Wd}{(B-d_1)t} = \frac{165000}{(246-82) \times 20} = 50.30 \text{ N/mm}^2 < [\sigma_t] \text{ hence safe.}$$

Checking for Double shear,

$$(\tau) = \frac{0.5Wd}{2 \times h_1 \times t} = \frac{165000}{2 \times 82 \times 20} = 50.30 \text{ N/mm}^2 < [\tau] \text{ hence safe.}$$

Checking for Crushing,

$$(\sigma_{cr}) = \frac{0.5Wd}{d_1 \times t} = \frac{165000}{82 \times 20} = 100.61 \text{ N/mm}^2 < [\sigma_{cr}] \text{ hence safe}$$

Step 13: Drum design

The rope drum should be made of seamless pipe machined & grooved accurately to ensure proper seating of wire rope in a proper layer. The drum should be fitted with two heavy duty Ball / Roller bearings of reputed make for smooth operation & longer life.

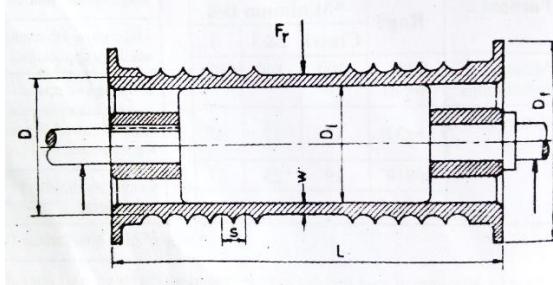


Fig. 2.62 Rope Drum Cross Section

$$\text{Compensating pulley diameter} = 0.6 \times \text{diameter of movable pulley} = 0.6 \times 870 = 540 \text{ mm}$$

$$\text{Un-grooved length or unrolled length} = 0.6 \times \text{Diameter of compensating pulley} = l_1 = 0.6 \times 540 = 324 \text{ mm}$$

$$\text{Length of drum , } L = \left(\frac{2Hi}{\pi D} + 12 \right) S + l_1 \quad \dots \dots \dots \text{PSG 9.2}$$

Where, H = hoisting lift

$$i = \text{velocity ratio} = \text{no. of falls} \times 0.5 = 0.5 \times n_f$$

$$D = D_{\min} + d = 870 + 29 = 899 \text{ mm, Let } D = 900 \text{ mm}$$

$$H = 10 \text{ m} = 10000 \text{ mm, } i = 4, S = 33 \quad \dots \dots \text{PSG 9.9}$$

$$L = \left(\frac{2 \times 10000 \times 4}{\pi \times 900} \right) \times 33 + 330 = 1233.59 \text{ mm,}$$

$$\text{Let } L = 1300 \text{ mm}$$

Keeping 50 mm out of 1300 mm on either side. Hence, mounting is at 1200 mm. Depending on the position of the load, the rope will be located at minimum distance of 324 mm and maximum distance of 1200 mm. As the bending moment depends on the distance of the load from the support, the maximum bending moment will occur when load is at distance L_1 .

Selecting material for drum as C-40 with $[\sigma_c] = 140 MPa$ also neglecting the weight of drum and rope

Drum is subjected to direct compressive stress over the area ($s \times t$)

$$\sigma_c = F_{max} / (S \times t)$$

$$[\sigma_c] = \frac{41.66 \times 1000}{33 t}, \quad 140 = \frac{41666}{33 t}$$

Therefore, $t = 9\text{mm}$

For $d = 29\text{ mm}$, $C_1 = 9$ PSG 9.9

Therefore total thickness $= t + C_1 = 9 + 9 = 18\text{mm}$

$$(\sigma_c) = 41666 / (33 \times 18) = 70.13 \text{ N/mm}^2$$

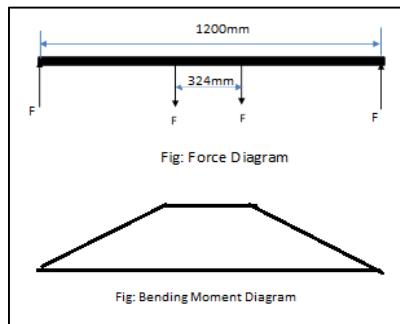


Fig. 2.63 Bending Moment Diagram

$$B.M_{max} = F \times \left(\frac{L_D}{2} - \frac{L_1}{2} \right) = 41667 \times \left(\frac{1200}{2} - \frac{324}{2} \right) = 18.25 \text{ kN.m}$$

$$(\sigma_b) = \frac{M_{max}}{Z} = \frac{18.25 \times 10^6}{\frac{\pi}{32} \times \left(\frac{900^4 - 882^4}{900} \right)} = 3.285 \text{ N/mm}^2$$

It is observed that compressive stress due to direct compression is the predominant stress, hence for quicker analysis directly design based on the direct compressive stress.

$$\text{Total compressive stress, } (\sigma_c)_{total} = (\sigma_c) + (\sigma_b) = 70.13 + 3.285 = 73.41 \text{ N/mm}^2$$

Drum is subjected to torsion also, The maximum torque acting on the rope drum = $2 \times F_{max} \times \frac{D_o}{2}$

$$T = 2 \times 41.67 \times 10^3 \times \frac{900}{2} = 37.5 \text{ kN.m}$$

$$(\tau) = \frac{T}{J} = \frac{T}{\frac{\pi}{16} \times \left(\frac{D^4 - D_i^4}{D} \right)} = \frac{37.5 \times 10^6}{\frac{\pi}{16} \times \left(\frac{900^4 - 882^4}{900} \right)} = 3.37 \text{ N/mm}^2$$

Principal Stress,

$$\sigma_1 = \frac{\sigma_{ctotal}}{2} + \sqrt{\left(\frac{\sigma_{ctotal}}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \frac{73.41}{2} + \sqrt{\left(\frac{73.41}{2}\right)^2 + 3.37^2}$$

$$\sigma_1 = 73.56 \frac{\text{N}}{\text{mm}^2} < \left[140 \frac{\text{N}}{\text{mm}^2} \right], \quad \text{hence safe}$$

Step 14: Selection of motor

Given, Hoisting Speed = 5 m/min

Output Power = Design load * Hoisting Speed

$$W_d \times V = 330 \times 10^3 \times \frac{5}{60} = 27.5 \text{ Kw},$$

Considering transmission efficiency as 0.85

$$\text{Input Power} = \frac{\text{Output Power}}{\text{transmission efficiency}}$$

$$\text{IP} = \frac{27500}{0.85} = 32.35 \text{ kW}$$

Selecting std. motor with 32.5KW power

Step 15: Design of drum shaft

Drum shaft is subjected to Torque and Bending Moment.

Assuming bearing span is 1300mm,

$$\text{B.M}_{\max} = M = 83.32 \times 1000 \times (650-600)$$

$$M = 4.166 \times 10^6 \text{ N-mm}$$

$$\text{Torque } T = 37.5 \times 10^6 \text{ N-mm}$$

$$\text{Equivalent torque} = \sqrt{M^2 + T^2} = 37.73 \times 10^6 \text{ N-mm}$$

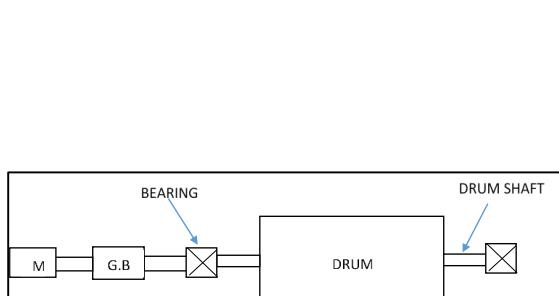


Fig. 2.64 Drum Shaft

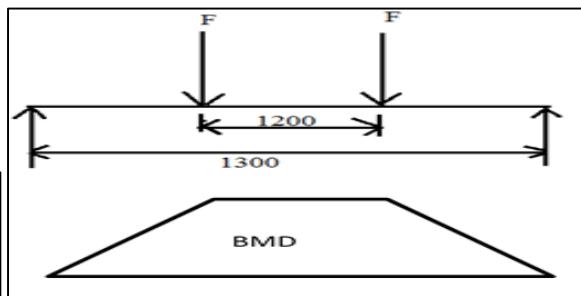


Fig. 2.65 Loading Diagram & BMD

Let shaft material be 40Cr1 and FOS = 4

Design stresses are $[\tau] = 80 \text{ N/mm}^2$,

$$[\sigma_t] = 135 \text{ N/mm}^2$$

$$T_{eq} = \frac{\pi}{16} x d^3 x [\tau],$$

$$37.73 \times 10^6 = \frac{\pi}{16} x d^3 x 80$$

Therefore, $d = 133.92 \text{ mm}$,

Let $d = 140 \text{ mm}$.

Step 16: Design of bearing (cylindrical roller bearing)

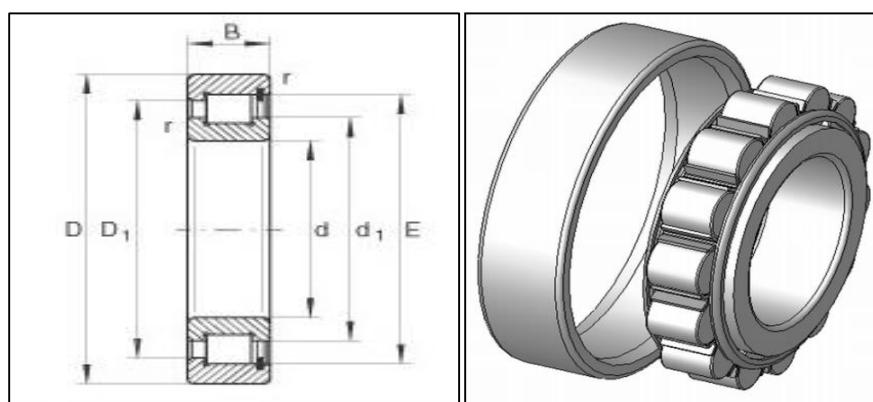


Fig. 2.66 Cylindrical Roller Bearing

For drum shaft lets select cylindrical roller bearing,

Forces acting on each bearing are, $F_r = 41.67\text{KN}$, $F_a = 0$

Assuming life in hours, $L_{hr} = 10,000 \text{ hrs}$

Linear speed of the drum = 2x linear speed of the load

$$V = 2 \times 5 = 10 \text{ m/min}$$

$$V = \pi DN$$

$$10 = \pi \times 0.9 \times N$$

$$N = 3.54 \text{ rpm}$$

Life in millions of revolutions is given by,

$$L_{mr} = \frac{L_{hr} \times N \times 60}{10^6} = \frac{10000 \times 3.54 \times 60}{10^6}$$

$$L_{mr} = 2.122 \text{ mr}$$

Equivalent load acting on the bearing,

$$P_{eq} = (X \cdot V \cdot F_r + Y \cdot F_a)S$$

Assuming $X=1$, $V = 1$ and $S = 1.2$

$$P_{eq} = 50 \text{ KN}$$

Dynamic load carrying capacity is given by,

$$C = P \cdot (L_{mr})^{\frac{1}{k}} = 50 \times (2.122)^{\frac{3}{10}} = 62.66 \text{ kN}$$

Where $k = 10/3$ for roller bearing,

Therefore required Dynamic load carrying capacity, $C = 6266 \text{ kgf}$

Selecting NU2228PSG 4.21

With $d = 140 \text{ mm}$, $D = 250 \text{ mm}$, $C_0 = 38000 \text{ kgf}$, $C = 36500 \text{ kgf}$

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Module 3

Design of Belt Conveyor System

3.1 Introduction

Belt Conveyors are traction type conveyors and a powerful material handling tool. Using conveyor systems is a good way to reduce the risks of musculoskeletal damage in tasks or processes that involve manual handling, as they reduce the need for cyclic lifting and carrying. They offer the opportunity to improve productivity, reduce product handling and damage, and minimize labor content in a manufacturing or distribution facility. Belt conveyor can be more economical than railways and automotive transport over a distance of 25km and even 100km when the tonnage to be hauled amounts to 25 millions tons per year.

Conveyors are generally classified as either Unit Load Conveyors that are designed to handle specific uniform units such as cartons or pallets, or Process Conveyors that are designed to handle loose product such as sand, gravel, coffee, cookies, etc. which are fed to machinery for further operations or mixing. It is quite common for manufacturing plants to combine both Process and Unit Load conveyors in its operations. Roller conveyor is not subjected to complex state of loading even though we found that it is designed with higher factor of safety. If we redesigned critical parts e.g. Roller (Idler), Bearing & Frame etc then it is possible to minimize the overall weight of the assembly.

Idler is the supporting device for belt and cargo of a belt conveyor. Idlers move as the belt moves so as to reduce the running resistance of the conveyor. Idlers' qualities depend on the usage of the belt conveyor, particularly the life span of the belt. However, the maintenance costs of idlers have become the major part of the conveyor's operating costs. Hence, idlers need to have reasonable structure, durability in use, small ratio of steering resistance, reliability, and dust or coal dust cannot get in bearing, due to which the conveyor has a small running resistance, saves energy and prolongs the service life.

Major Components of Conveyor

- Driving traction element i.e. Long Continuous belt
- Idlers or rollers held by frame
- Drive pulleys and take up unit at ends
- Pulleys coupled to gears and motors
- Peripheral Devices to drop on belt, direct around corner, clean, discharge etc. (Loading and unloading devices, snub pulley, belt cleaners etc.)

Conveyor Belting Parts

- Carcass - woven fabric or material for tensile strength
- Skims - rubber layers between carcass plies

- Braker - fabric coat above carcass to break impact of load
- Top Cover - A rubber that resists cutting abrasion and sometimes chemical action

3.2 List of conveyor categories and sub categories

1. **Gravity Conveyor** – gravity, non-powered conveyor, is typically used in truck off loading, package sorting, and assembly or kitting areas. Gravity is the cheapest form of conveyor but lacks in product control.
2. **Belt Conveyor** – typically used in package handling, raw material handling, and small part handling. It is effectively used for elevation change or incline/decline applications. A more common application of belt conveyor is in the check-out line at your local grocery store.
3. **Powered Roller Conveyor**
 - a) Live Roller Conveyor – typically used in general transport when product accumulation is not required. It is also used in package handling applications and is ideal for light- to medium-product loads.
 - b) Minimum Pressure Conveyor – used in short sections of accumulation, general transport of product, and is ideal for medium to light loads such as package handling applications.
 - c) Zero Pressure Conveyor – commonly found in distribution centers where there is a wide variety of product width and weight. Applications include buffering of product prior to sortation, packaging, kitting, or shipping areas. It is also ideal for picking areas and palletizing areas. This conveyor is used in high throughput systems.

4. Pallet Conveyor

- a) Drag Chain – is typically used for handling extremely heavy loads, special pallet configurations, and extremely low- to high-temperature areas.
- b) Roller – Roller pallet conveyor is typically used for handling extremely heavy loads and is ideal for accumulation zones in pack out areas.

5. **Overhead Conveyor** – typically used in paint and finishing lines, trash removal, food packing, and assembly lines. These are some of the oldest conveyors still used in the industry today because they are very reliable and require lesser maintenance.

Table Top Chain is used in accumulation, package handling, filling, labeling, and wash-down applications. Magnetic Slide Conveyor is used in metal stamping, chip removal, and small part transport application.

3.3 Conveyor Belting Types

- a) Multi-Ply - multi-ply carcass separated by skims - traditional - trade-off between stiffness and strength.
- b) Reduced Ply - complex interwoven carcass not dependent on separate plies thinner less stiff for same strength.
- c) Steel - carcass lengthwise steel belts - high tensile strength - heavy ores long runs.
- d) Solid Woven - Carcass impregnated with elastomer.

3.3.1 Requirements of Conveyer Belt

- a) It should have adequate strength
- b) Good longitudinal and lateral flexibility
- c) High wear resistance
- d) Freedom from ply separation due to repeated bending
- e) Low elastic and permanent elongation
- f) Low water absorption
- g) Effect of moisture on strength and life of belt should be negligible.

3.3.2 Typical Belt Conveyor Paths

Conveyor transportation are available in various layout as per requirement. Some layouts are shown in figure 1.

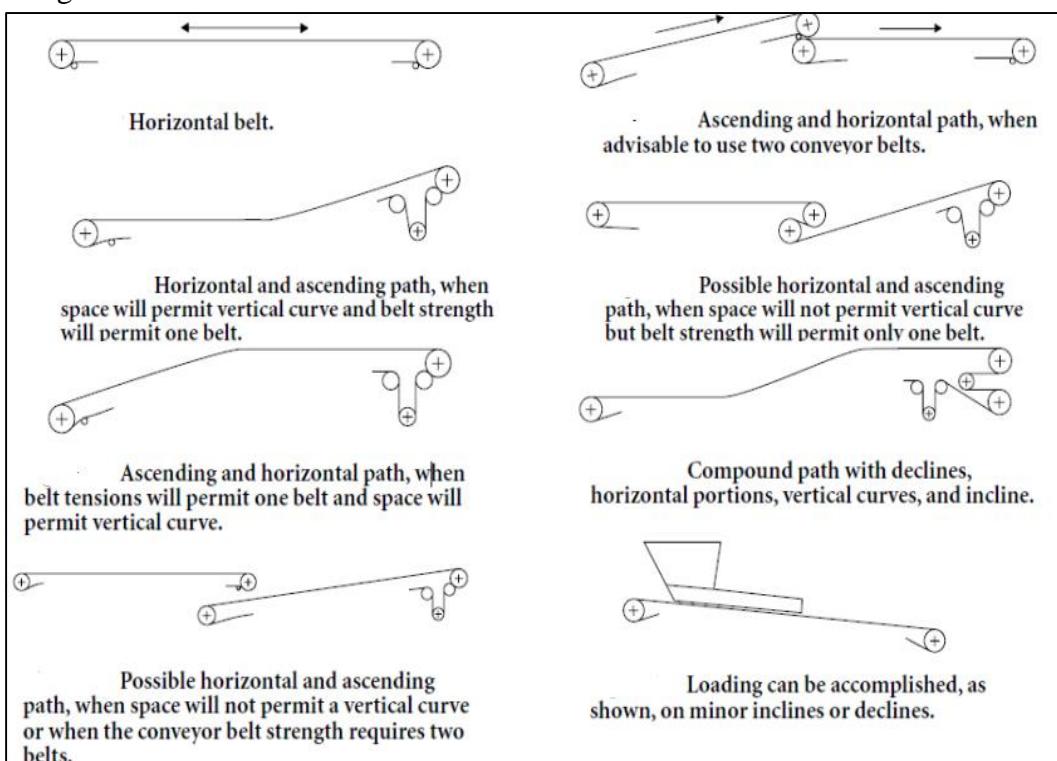


Fig.3.1: Typical Belt Conveyor Paths

3.3.3 Advantages of Conveyer Belt

- i. Noiseless operation.
- ii. Large length of conveying path.
- iii. Lower power consumption.
- iv. Long life.
- v. Adaptability to different types of goods.
- vi. Ability to carry almost any bulk material.
- vii. High reliability of operation.

- viii. Can transport material in any direction.
- ix. Reduction in numbers of personnel.
- x. Reduction in energy consumption.
- xi. Long periods between maintenance.
- xii. Independence of the system to its surrounds.
- xiii. Reduced business costs.

3.3.4 Disadvantages of Conveyer Belt

- i. Accumulation is difficult.
- ii. Complicated marshalling/ assembly.
- iii. The loss of light weight bulk material carried away as dust or spilled from the belt along its path is another objectionable feature.
- iv. Continuous or periodic monitoring of belt is necessary.
- v. Heat affects the material of belt.
- vi. The normal design of a belt conveyor is opened. If your product needs to be contained, covers and or drip pans can become expensive and cumbersome.
- vii. If the material is sticky, belt cleaning can be difficult and generally not very successful.
- viii. There is almost always material carry over from the belt discharge and this becomes a house keeping problem.
- ix. If the material being conveyed is sticky it will ultimately get transferred to the return side of the belt and then to the rolls, idlers and pulleys, then belt tracking can be an ongoing issue.

3.3.5 Applications of Conveyer Belt

- i. Moving loads and bulk material in foundry, mining, cement industry,
- ii. In line production,
- iii. Coal handling plant,
- iv. Automotive sector,
- v. Agricultural,
- vi. Food processing,
- vii. Pharmaceutical,
- viii. Chemical, bottling and canning,
- ix. Print finishing and packaging.

3.4 Belt speed

Speed of conveyer depends on following factors.

3.4.1 Size of the lump and nature of the material to be conveyed.

Fine material transported at low speed to avoid spillage due to fanning effect (Fanning Effect is loss of material from the conveyor due to flowing of air). Heavy lump transported at low speed to avoid the damage of the belt while passing over the idlers and pulleys. Coke and coal are transported at low speed to avoid degradation through reduction in size.

3.4.2 Belt Width

Wider belt width can be operated at higher speed allowing to better distribution of the material and aligning of the belt in travel.

Very high speeds have meant a large increase in the volumes conveyed. Compared with the load in total there is a reduction in the weight of conveyed material per linear meter of conveyor and therefore there is a reduction in the costs of the structure in the troughing set frames and in the belt itself. The physical characteristics of the conveyed material are the determining factor in calculating the belt speed. With the increase of material lump size, or its abrasiveness, or that of its specific weight, it is necessary to reduce the conveyor belt speed. Considering the factors that limit the maximum conveyor speed we may conclude. When one considers the inclination of the belt leaving the load point; the greater the inclination, the increase in the amount of turbulence as the material rotates on the belt. This phenomenon is a limiting factor in calculating the maximum belt speed in that its effect is to prematurely wear out the belt surface. The repeated action of abrasion on the belt material, given by numerous loadings onto a particular section of the belt under the load hopper, is directly proportional to the belt speed and inversely proportional to its length.

Table 3.1 Dimensions of belt

Lump size Maximum Dimensions		Belt Minimum Width	Maximum Speed (m/s)			
Uniform upto mm	Mixed upto mm	mm	A	B	C	D
50	100	400	2.5	2.3	2	1.65
75	150	500				
125	200	650	3	2.75	2.38	2
170	300	800	3.5	3.2	2.75	2.35
250	400	1000	4	3.65	3.15	2.65
350	500	1200				
400	600	1400	4.5	4	3.5	3
450	650	1600				
500	700	1800	5	4.5	3.5	3
550	750	2000				
600	800	2200	6	5	4.5	4

3.5 Basic components of a typical belt conveyor

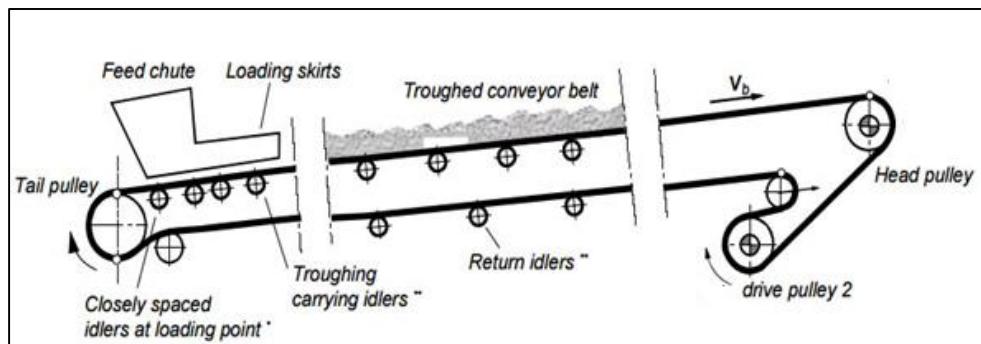


Fig.3.2 Components of Belt Conveyor Assembly

3.5.1 Drive

May be of traditional design or with motorised drum unit:

3.5.1.1 Traditional

Comprises a drive group consisting of a drive drum of a diameter appropriately sized to the load on the belt, and an idler drum at the opposing end. The power is supplied by a motor which is directly coupled to a gearbox or by a direct or parallel shaft drive driving the drive drum through a suitably sized couple.

3.5.1.2 Motorised Drum

In this arrangement the motor, gearbox and bearings form a complete designed unit inside and protected by the drum shell which directly powers the belt. This eliminates all the external complication of external drive, couples etc. as described above in the traditional design. Now a day motorised drums are produced in diameters up to 800mm with power in the order of 130 KW and with a drive efficiency which may reach 97 %.

3.5.2 Drive pulley

The shell face of the conventional drive pulley or the motorised drum may be left as normal finish or clad in rubber of a thickness calculated knowing the power to be transmitted. The cladding may be grooved as herringbone design; or horizontal grooves to the direction of travel; or diamond grooves; all designed to increase the coefficient of friction and to facilitate the release of water from the drum surface.

3.5.3 Return pulleys

The shell face does not necessarily need to be clad except in certain cases, and the diameter is normally less than that designed for the drive pulley.

3.5.4 Snub pulleys

These are used to increase the angle of wrap of the belt and does necessary changes in belt direction in the areas of counterweight tensioner, mobile unloader etc. The choice of the optimum conveyor system and its project design and rationalisation depends on full knowledge of the construction characteristics and the forces involved that apply themselves to all the system components.

3.5.5 Rollers

These are used to support the belt and are guaranteed to rotate freely and easily under load. They are the most important components of the conveyor and represent a considerable value of the whole cost. The correct sizing of the roller is fundamental to the guarantee of the plant efficiency and economy in use.

3.5.5.1 Upper carrying troughing and return sets

The carrying rollers are in general positioned in brackets welded to a cross member or frame. The angle of the side roller varies from 20° to 45° . It is also possible to arrive at angles of up to 60° using the “garland” suspension design. The return roller set may be designed incorporating one single width roller or two rollers operating in a “V” formation at angles of 10° . Depending on various types of material being conveyed the upper carrying sets may be designed symmetrically.

3.5.6 Tension units

The necessary force to maintain the belt contact to the drive pulley is provided by a tension unit which may be a screw type unit, a counterweight or a motorised winch unit.

The counterweight provides a constant tensional force to the belt independent of the conditions. Its weight is designed according to the minimum limits necessary to guarantee the belt pull and to avoid unnecessary belt stretch.

The designed movement of the counterweight tension unit is derived from the elasticity of the belt during its various phases of operation as a conveyor. The minimum movement of a tension unit must not be less than 2% of the distance between the centres of the conveyor using textile woven belts, or 0.5% of the conveyor using steel corded belts.

3.5.7 Hopper

The hopper is designed to allow easy loading and sliding of the material in a way to absorb the shocks of the load and avoids blockage and damage to the belt. It caters for instantaneous charging of load and its eventual accumulation. The hopper slide should relate to the way the material falls and its trajectory and is designed according to the speed of the conveyor. Lump size and the specific gravity of the charge and its physical properties such as humidity, corrosiveness etc. are all very relevant to the design.

3.5.8 Cleaning devices

The system of cleaning the belt today must be considered with particular attention to reduce the need for frequent maintenance especially when the belt is conveying wet or sticky materials. Efficient cleaning allows the conveyor to obtain maximum productivity. There are many types and designs of belt cleaners. The most straight forward simple design is that of a straight scraper blade mounted on rubber supports.

3.5.9 Conveyor covers

Covers over the conveyor are of fundamental importance when it is necessary to protect the conveyed material from the atmosphere and to guarantee efficient plant function.

3.5.10 Conveyed material

The correct project design of the belt conveyor must begin with an evaluation of the characteristics of the conveyed material and the angle of repose and the angle of surcharge. The angle of repose of a material, also known as the “angle of natural friction” is the angle at which the material, when heaped freely onto a horizontal surface takes up to the horizontal plane.

3.6 Belt Width Selection:

The width of belt is predominantly governed by two factors, the lump size of the material conveyed and the capacity requirements of the conveyor. For individual loads, free margins between 50 and 100mm wide must be provided on either side of the belt. The width of the belt conveyor moving bulk material cannot be less than a certain minimum width decided by the size of parts.

For example, Minimum width $B = 2 \times a_{max} + 200$ mm for ungraded material and $B = 3.3 \times a_{max} + 200$ mm for graded material.

The following three factors primarily determine the capacity of a belt conveyor:

- a) Cross section of load on the belt:

The crosssectional load on the belt will vary with the width of belt, the type of carrying idlers used which determines the amount of toughing given to the belt and the nature of the material being handled, which determines the quantity of material that can be safely loaded on to a given cross-section

- b) Speed of belt
- c) Slope factor

The standard width of belts in millimetres as specified in

IS 1891 (Part I) are as follows:

300, 400, 500, 600, 650, 800, 1000, 1200, 1400, 1600, 1800 and 2000.

3.7 Belt Construction

A conveyor belt consists of two elements, the carcase and the cover. The fabric ply- constructed belting is the most widely used kind. Its carcase is made up of number of layers or plies of cotton fabric.

The carcase is the reinforcing member and may be of either and may be of either textile reinforcements steel cords and supplies the tensile strength and the body to the belt to hold the sh

ape. Carcase absorbs longitudinal tensile stresses and elastic protective filler protect carcase against moisture, mechanical damage, and abrasion due to conveying material.

In case of textile reinforcement, the carcass is normally built up of plies of textile fabric. The strength of fabric and the number of plies in the carcass of the belt may be varied together to suit the strength requirements. However, the strength of carcass has a practical limit. If the belt is too tough, toughing and training the belt will be very difficult. Therefore, the belt with lesser number of plies with stronger fabric is generally preferred because it is more flexible both in toughing and going around the terminal pulleys. The steel cord belting is used to meet the condition of small elongation and good troughbility in conjunction with higher operating tensile forces. PVC belting is generally selected for underground mining applications where fire hazard exists.

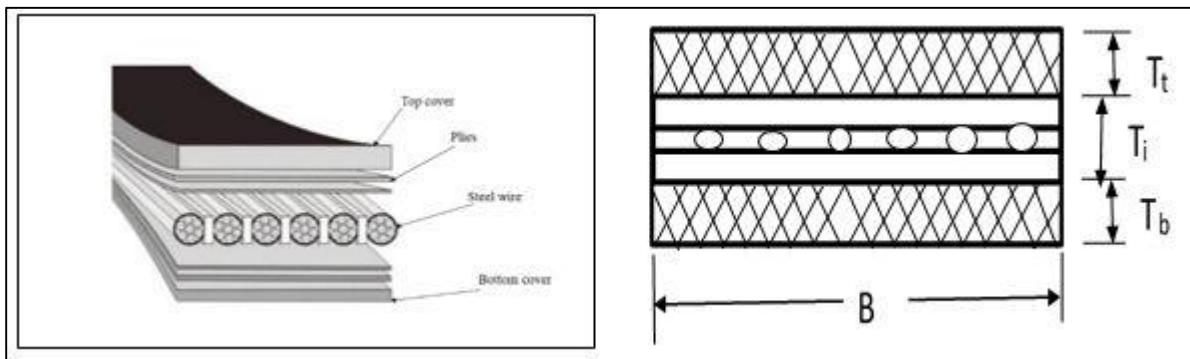


Fig. 3.3 Belt Construction

3.8 Belt Protective Cover Thickness

The properties needed for the cover of belt include resistance for cutting, gauging, tearing, abrasion, aging, moisture absorption and in some conditions to oils, chemical and heat. The grade and thickness of top cover of belt depend upon number of conditions, the most important of which are:

- Abrasive qualities of the material being handled.
- Loading cycle, that is, the frequency with which the belt receives the load.
- Lump size of the material.
- Loading and unloading conditions.
- Temperature of the material to be handled.
- Chemical activity of the material.
- Contamination of the material with oils.
- Fire resistant cover needed or not.

The back-cover thickness of a belt is generally 1.0 to 3.0 mm for textile rubber belts and 0.8 to 1.2 mm for PVC belts. In case of steel cord belts, back cover thickness is minimum 4.0 mm and range up to full thickness of face cover.

Care shall be taken for the determination of back cover thickness for belts on tandem drives and other special applications, where there is considerable wear and tear in the back side of the belt. In such cases the back-cover thickness may be increased to 3 mm and above if necessary.

The cover grade is determined by characteristics of the material handled. Belt protective cover thickness ratio is the ratio of upper cover thickness to the thickness of lower cover thickness. For different cases the ratio is as below.

- a) Low abrasive bulk material 3:1
- b) Moderately abrasive fine lump material 4.5:1
- c) Medium size lump material (4.5-8): 2
- d) Strongly abrasive, medium and large size lump material (4.5 – 10): (2-3)
- e) For unit loads (2-3): (1-2)

3.9 End Joining

Various methods are used for the joining of the ends of conveying belt which includes, Hot or cold cementing using special adhesive between pressure plates (Vulcanization), Metallic staples, Hinges, Rivets in lapped joints, leather straps etc.

The two options are available for end joining are explained as follows

- 1. Vulcanization or welding (the process of joining belt ends using heat and/or chemicals)
- 2. Mechanical splicing (the process of joining belt ends using metal or plastic hinges or plates)

3.9.1 Vulcanization or welding

In heat welding, the two ends of a polyurethane belt can be joined in the factory, in a specialized belt shop, or in the field. The belt ends are typically joined using either a finger splice or a butt splice.

3.9.2 Mechanical Fastening

3.9.2.1 Wire Hooks

The wire hooks were designed to penetrate and grab onto the fabric plies of the belt carcass. Wire hooks offer a low profile fastening system that is relatively simple to install. The tooling is inexpensive. Hooks are available in a wide variety of wire diameters, materials, leg and point lengths, and strip lengths. There are various methods of installation, including a rolling device and a hydraulic device.

Hooks and connecting pins are available in stainless steel for food processing operations where sanitation is a prime concern. The key benefit to this fastening system is ease of installation and the ability to take the belt on and off. The risk of the hooks breaking and contaminating the food stream is a factor to consider before employing this fastening method. Metal staples are well suited for light and medium duty fastener applications on synthetic carcass belts. The staples can be pre-inserted into a one-piece fastener strip which is placed over the ends of the belt and installed using a lightweight tool. The staples are then driven into place with a hammer. Metal staples are available in stainless steel alloys for food grade applications. They can be used to repair a belt for temporary use, or as a permanent splice.

3.9.2.2 Metal Lacing

Metal lacing gives the appearance of a piano hinge. The laces are provided in a continuous strip to match the width of the belt. They are placed over the ends of the belt and the teeth are embedded into the belt carcass with a hammer. Metal lacing creates a low-profile splice that is economical to install. It can operate over pulleys as small as 1" in diameter. Both fasteners and hinge pins are available in stainless steel for food grade applications. The hinge pins are removable so the belt can be separated for cleaning.

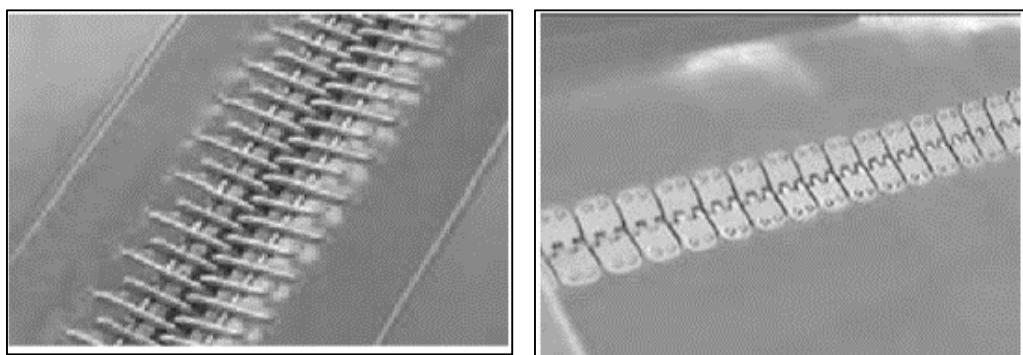


Fig. 3.4 Wire hook fastener and Metal staples

3.9.2.3 Plastic Rivets

Plastic rivets are a non-metallic fastener that can pass through metal detectors. This non-scratching, non-magnetic fastening system has rivets with bevelled front edges that are moulded into the carcass to present a flat surface. They travel over conveyor components more easily and quietly than metal systems.

Plastic rivet fasteners have hinge pins that can be removed for belt disassembly and cleaning. This fastening system requires a special tool for assembly, and offers a low-cost alternative to vulcanization. Plastic rivet fasteners have mechanical fastener ratings up to 65 PIW, and a minimum pulley size of 1 ½".

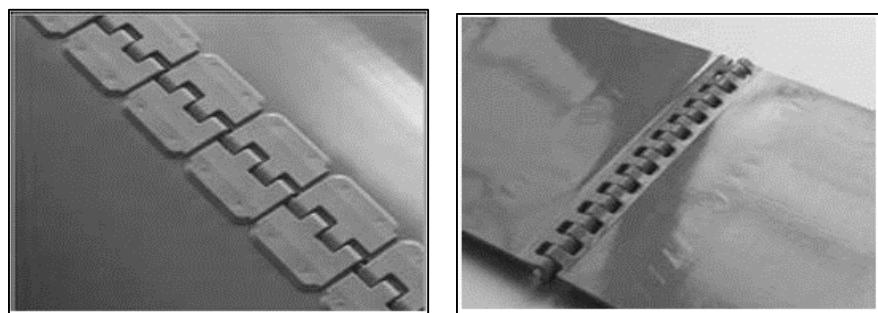


Fig. 3.5 Plastic rivets and Pin Splice

3.9.2.4 Hybrid Joining System: Pin Splice

Pin splice joining system combines vulcanization with mechanical fastening. Designed for light to medium weight loads, the fastening system has no metal parts to set off metal detectors. The pin splice is designed for fibre-reinforced food grade urethane belts. The vulcanization process takes

place at the factory, where urethane is welded to the belt in the pin area. An end cap is welded to the belt ends to seal off the tensile cords and maintain the integrity of the reinforcement. This process also prevents exposing the cords to bacteria.

A plastic pin is inserted through the splice to join the belt ends in the field. No special tooling or equipment is needed. The splice is easy to clean and sanitize.

3.10 Belt Supports

The rollers are the supporting members of the belt. There are 2 types of rollers upper and lower sets.

The upper rollers carry the load of the belt. They are straight for straight belt and sometimes trough for 2-3 m wide belt conveying bulk loads. The spacing between the rollers is less, for standard conveyors it is 1.5 m.

The lower rollers are mostly straight, single roller cylindrical, some are trough with angle of roller inclined at 10° for better centering of moving belts if width is more than 1 m. The spacing is more for return idlers about 3 m.



Fig.3.6 Straight Lower roller



Fig.3.7 Trough Upper roller

3.11 Belt Drive

Types of Belt Drive are as follows,

3.11.1 Single, un-snubbed

The simplest drive arrangement consists of one steel pulley connected to the source of power, having belt wrapped around it on an arc of 180° to 240° . It is suitable for low capacity, short centre conveyors handling non-abrasive material. The pulley may be lagged to increase the coefficient of friction and avoid pulley wear for abrasive materials. This is simple design with high reliability, smaller dimension.

3.11.2 Bare/lagged pulley drive

The ratio of maximum belt tension to effective belt tension for the drive is decreased by snubbing the belt at head pulley which may be bare or lagged. The arc of contact is increased from 180° to

210° and can further be increased to 260° by providing snub/drive pulley. In majority of normal medium to large capacity belt conveyors, handling mild abrasive to abrasive materials, 210° snub pulley drive with head pulley lagged with hard rubber is adopted.

3.11.3 Tandem drive

Where the belt tensile forces are very high and it is necessary to increase the angle of arc of contact, tandem drives are used. The tandem pulleys are both driven and share the load resulting in a lower effective tension for a given power transmitted. The tandem drive with arc of contact from 300° to 440° or more can function with one or two motors. The location of such drive is usually determined by the physical requirements of the plant and its accessibility. These have greater dimension and more intricate design, lower reliability of operation, higher cost. Because of multiple bending belt life is shorter.

3.12 Belt Take Up Arrangement

Main functions of take up are:

1. Ensuring adequate tension of the belt leaving the drive pulley so as to avoid any slipping of the belt.
2. Permanently ensuring adequate belt tension at the loading point and at any other point of the conveyor to keep the troughed belt in shape and limit belt sag between carrying idler
3. Compensating for operating belt length variation due to physical factors (instantaneous tensions, permanent elongation, outside temperature, temperature of conveyed material, dampness, etc.)
4. Making available, if needed, an adequate extra length of belt to enable re-joining without having to add an extra piece of belt.

Two types of belt take up devices are generally used:

1. Fixed take-up device s that are adjusted periodically
2. Automatic take-up devices (constant load type)

3.12.1 Fixed take up devices:

In this type of take-up devices, the take-up pulley remains fixed between successive periodic adjustments. Take-ups of this type generally used are:

1. Screw take-up — in this system the adjustment is manually effected by means of two screws acting upon the pulley bearings and which are tightened simultaneously or successively. The screw is normally of non-extendable type and sliding surfaces are suitably protected against ingress of dirt. In this system, the applied tension is not fully determinable. This generally leads to excessive tension of belt (when tension is insufficient, belt slips and quickly deteriorates). This excessive tension is unavoidable and shall be taken into account when determining the size of the belt, designing the mechanical components and calculating the adjustments. For this reason, these devices are used only in case of short conveyors of up to 60 m length s and under light duty cycle condition.

2. Winch take-up — in this system, the tension is adjusted by means of a mechanical motorized device which does not automatically compensate for belt length variations. A tension indicator may be included between winch and pulley. This system also requires careful checking of tension and leads to excessive belt tension in order to avoid too frequent take-ups. However, it may be used for long conveyors and under heavy duty conditions provided that these conveyors are equipped with belts having very low elongation coefficient under the effect of load and over a long period, for example, steel cord belts which are used almost exclusively.

3.12.2 Automatic take up devices:

In this system, take-up pulley is mounted on slides or on a trolley and travels freely while a constant tension is automatically maintained to ensure normal conveyor operation in all cases. The most frequently used type is gravity weight operated take-up device. Hydraulic, pneumatic or electrical take-up devices of various types are also used. All types of automatic take up devices shall include a system for adjusting belt tension. Automatic take-up has following features:

- a) It is self-adjusting and automatic.
- b) Greater take-up movement is possible.
- c) It is suitable for horizontal or vertical installation.
- d) It is preferred for long centre conveyors.
- e) It can be located at drive end (preferred for low tensions).
- f) In case of underground mines, provision of loop at drive end may be made to cater for take-up and small extension of belt conveyor lengths.

3.12.3 Winch take up (automatic):

Winch take-up device can also be used as automatic take-up arrangement when automatic tension regulation (ATR, by employing load cells, electronic sensing devices etc.) is provided to signal for the winch motor to run in one direction or reverse for specific number of turns or to stop as governed by predetermined values of belt tensions for any installation. This is highly recommended for long centres high capacity belt conveyors since it fetches less space (horizontal/vertical) and do not unnecessarily put the belt always in heavy tension as imparted by the constant counter weights necessary for operation at maximum design load in a gravity take-up. The heavy tension in gravity type-up arrangements continues to exist in the belt even when it is not running.

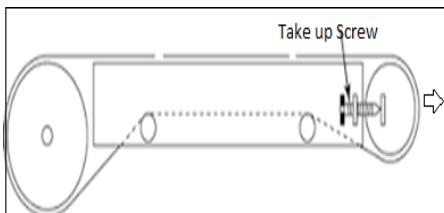


Fig.3.8 Screw take up



Fig.3.9 Gravity take-up

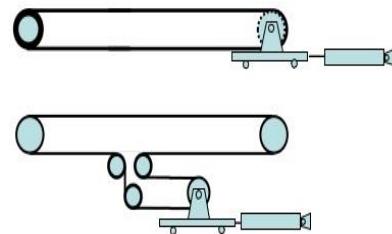


Fig.3.10 Gear take up

3.13 Cleaning Devices

3.13.1 Requirements

It is necessary to clean the belt when snub pulleys or tandem drive pulleys contact the dirty side of the belt on the return run. This is especially the case when handling materials which are likely to pack on the belt, such as sticky and wet materials, or those that are likely to have damp, greasy or oily patches which eventually rot the belt.

3.14 Types of Cleaners

There are three main types of external automatic belt cleaners:

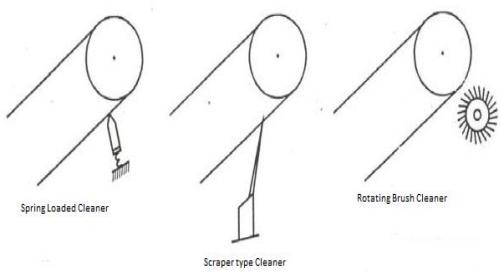


Fig.3.11 Belt Cleaners type

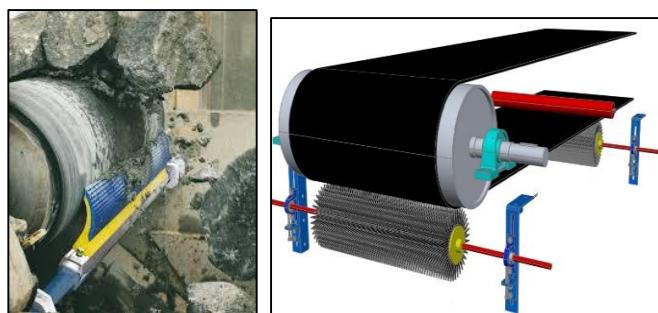


Fig.3.12 Wiper and Rotary Brush

In addition, internal belt cleaners of V-type shall be used near tail pulley to prevent material from getting between the pulley and belt as it wraps round the tail pulley. These V-type plough cleaners shall be adjusted to float on the return belt without exerting undue load on belt surface. These can also be made of adjustable counterweighted type. Their use is recommended even on short conveyor lengths and where full length decking is provided from the feed to the discharge end as a safety measure.

The rotary brush is fitted with a drive pulley and is rotated in opposite direction from it by a short centre roller chain drive. A cantilevered weighted arm is attached to the brush for facing it to the belt, whilst at the same time, preventing jamming and unnecessary wear on the bristles. The speed of the tip of the bristles for brushes 200 to 300 mm in diameter shall generally be:

Dry materials: 2.0 to 3.0 m/s

Damp materials: 5.0 to 7.5 m/s

Wet and sticky materials: 6.0 to 7.5 m/s

The brush is mounted so that it can be adjusted towards the belt to compensate for wear on the tips of bristles and in such a way that the drive to the brush is not affected.

Because of high speed, brushes are short-lived. The most effective as well as most economic type of belt cleaning apparatus is the spring-loaded type belt wiper. Dry or very wet materials are easiest to remove.

When particularly gummy materials with tendency to cling to the belt and solidify, are encountered, it is recommended to use water spray directed against the return belt about 500 mm

ahead of the cleaner. It is also beneficial to have a very fine spray directed against the belt just before it passes under the loading point, as this will tend to keep the material from adhering. Usually a small amount of water will suffice. As the cleaner is especially effective in removing water, there is little danger of objectionable dribble.

Where cleaning is of primary importance, two cleaners may be provided so that one can be removed at any time for servicing. In any case, pulleys are arranged at the discharge end to allow ample room for cleaning equipment including dribble chute or flume. The blades of the spring type cleaner shall engage the belt only where it is straight, never on a pulley. One cleaner shall generally have at least 300 mm of straight belt, two cleaners not less than 750 mm.

It is sometimes necessary to use steel scrapers on the rims of snub pulleys. Tripper pulleys and deflector pulleys that engage the carrying side of belt, to prevent accumulation of material, may hurt the surface of the belt or cause it to run out of line. The blade shall be located so that the scrapings can be disposed of.

3.14 Causes of Belt Failure

Conveyor belts are subject to three primary failure mechanisms: Yield, Fatigue and Wear.

Yield Failure occurs when an object has been loaded to the point where it no longer can return to its original shape. If a conveyor belt is deformed to the point where it no longer fulfills the function for which it was designed, it has failed. While a belt usually breaks shortly after the start of yield, it does not, however, should break to be considered ‘failed’.

Fatigue Failure occurs when an object is subject to a time varying load, called cyclic loading, which includes some component of tension.

Wear Failure is an erosion of material which occurs whenever there is a relative motion between two contacting surfaces.

3.15 Belt Sag

Belt sag is the vertical sag of the belt between adjacent idlers. Often belt sag doesn’t allow for effective sealing of the belt, which increases dust and spillage. Belt sag encourages material spillage which increases the risk of trips, slips, falls and entanglement. Fugitive materials like spillage and dust increase maintenance costs and reduce efficiency, plant safety, and product quality. Fugitive material may bury idlers, conveyor components, or structural supports, requiring costly clean-up labour and replacement parts. Belt sag can be eliminated by correctly supporting the belt. The spacing between the rolling components has a dramatic effect on the idler’s support. Idlers should be placed close enough to support a belt preventing the belt from sagging excessively between the rollers. If the belt is allowed too much sag, the load shifts as it is carried up and over each idler and down into the valley between. This shifting of the load increases wear in the belt and power consumption. The sag also encourages material spillage.

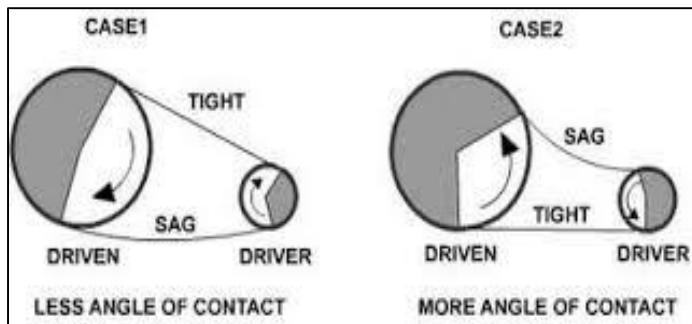


Fig.3.13 Belt sag

3.16 Types of Belt Conveyor

3.16.1 Horizontal Belt Conveyor

This type of belt conveyor consists of a centre drive, gear motor, and take-up. Based on the drive of the conveyor, it can come with one or two pulleys at the end. The belt of the conveyor is flexible and the entire system has floor supports along its length.

3.16.2 Incline & Decline Convenor

This type of conveyor is similar to a horizontal belt conveyor, but has an additional component. It comes with a single or double nose over and sometimes it also has a feeder portion. Typically, this type of conveyor has a rough surface on the belt during incline or decline rather than making use of a smooth-surfaced belt. This offers more traction to the items placed on the conveyor and prevents them from rolling backwards or forwards.

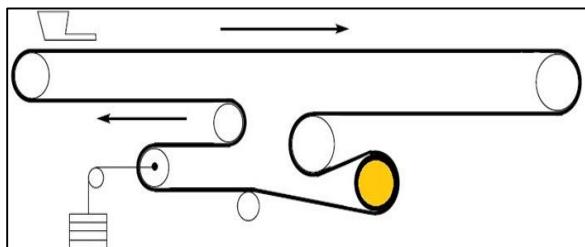


Fig. 3.14 Horizontal Belt Conveyor System



Fig. 3.15 Inclined Belt Conveyor

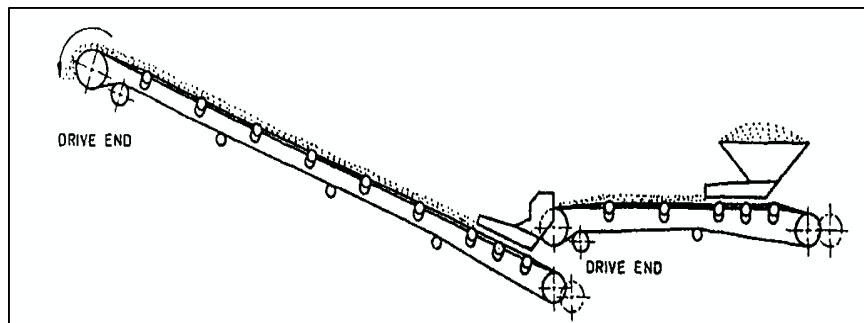


Fig. 3.16 Combination of inclined and horizontal conveyor

3.17 Drive Pulley Consideration

- a) Pulley should be large enough to avoid hard bend at point of tension application.
- b) Pulley should have grab - risk of slippage is function of grab and tension - more grab - less tension - could mean a cheaper belt.
- c) Place Drive in practical location to minimize the highest tension in the belt - a head pulley location often good on a belt up a slope.

3.17.1 Idlers

The efficiency of belt conveyor is mainly dependent on idlers. For higher efficiency of conveying systems, the idlers must be accurately made and provide a rigid framework. This will maintain a permanent, well balanced smooth running alignment.

There are, in general, three kinds of belt carrying idlers used in handling of bulk materials. The type of idlers affects the cross-sectional load on the belt.

1. Flat belt idlers: are used for granular materials having an angle of repose of not less than 35° . Flat belt idlers are preferred for low capacity requirements.
2. Toughing idlers with 20° through: are used for conveying all kinds of bulk materials.
3. Troughing idlers with 35° and 45° through: are used for transportation of small particles and lightweight materials like grain, cotton seed etc.

There are two basic type of belt conveyor idlers,

1. Carrying idlers which support the loaded run of the conveyor belt; and
2. Return idlers which support empty return run of the conveyor belt.

3.17.2 Carrying idlers

- a) Usually troughed with 3 equal size rollers on a frame in mining applications
- b) Some suspended catenary systems have 5 rollers
- c) Toughing angles are usually $20, 35, 45$ degrees
- d) Deeper trough more volume
- e) Requires thinner belt to lay in trough which limits strength.

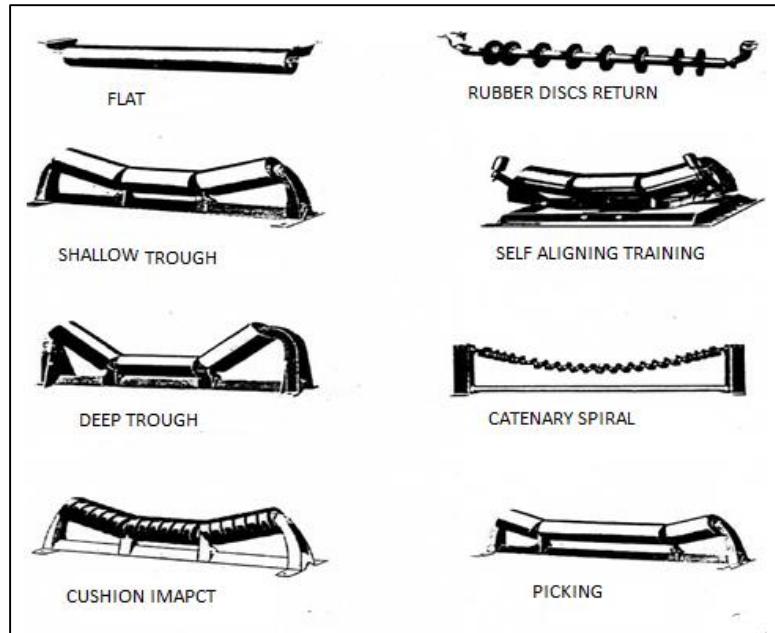


Fig.3.17 Types of Idlers

3.17.3 Returning Idlers

- a) Usually Flat and one piece.
- b) Sometimes two-piece V for belt training.
- c) Spiral roll to self and belt clean.

3.17.4 Rollers

A wide range of rollers, from standard belt-conveyor rollers to shaft-free, rubber-disk return, rubber impact, and aluminium rollers.

3.18 Loading the conveyor

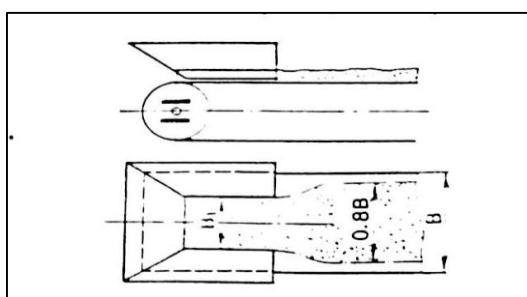


Fig.3.14 Feeding Hopper of Belt conveyor

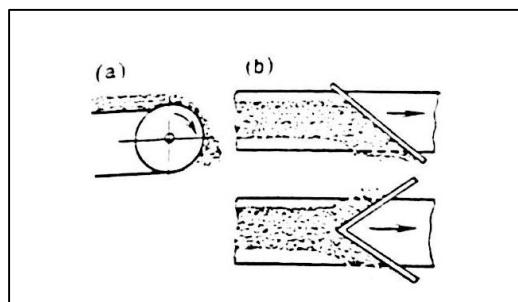


Fig.3.15 Belt Unloading Arrangement

Loading can be done at any point along the path, yet the preferred location of the loading station is at the foot pulley. Conveyors handling unit loads can be loaded by means of a gravity or manually.

Those intended to move bulk material are provided with a chute. To feed the material on the belt along its centre line in a uniform way, the outlet from the chute has to be of a width $B = 0.6$ or $0.7 B$. The size of chute must be inclined at an angle exceeding the angle of internal friction of the material by 10 to 15 degrees or this will tend to accumulate in the chute. Perforated chutes offer a means of reducing belt wear due to the impact from heavy lumps at loading station.

3.18.1 Unloading the Convenor

The simplest and convenient way of doing this is to dump the material over drive pulley. Yet process requirement may call for removing materials from conveyor at more than one stations along the path as this may be the case in the foundry where number of moulding machines are supplied with sand by a conveyor running down the shop. There are various kind of unloading devices. The simplest among which is the plough – a blade extending across the belt for one side discharge or a V-shaped blade for two side discharge.

3.19 Power Requirement of Belt Conveyors:

The power Requirement of Belt conveyor consist of two parts. One for lifting the load to given height and second for moving the load horizontally. The required power in KW is given by

$$N = N_{\text{lifting}} + N_{\text{Horizontal}} = Q \cdot H / 367 + C_o \cdot Q \cdot L / 367$$

Where, Q - Capacity in TPH (Tonnes per hour), H – Height in meter, L – Belt Length, C_o – specific friction factor (i.e. The Frictional resistance to be overcome in moving 1ton through 1 m).

Frictional resistance is of two different kind. 1) Distributed frictional resistance over the length of the path and 2) concentrated at given point e.g. Pulley, loading, unloading and cleaning devices points. Distributed frictional resistance calculation is explain as below.

3.19.1 Distributed Frictional Resistance of the Belt Conveyor

3.19.1.1 Frictional resistance of the idlers supporting horizontal belt section

Let's Idlers are spaced a distance ' t ' apart and each sustain load ' K '. In carrying run $K = q_0 \cdot t$ and in return run $K = (q + q_0) \cdot t$. where q – load per unit length due to weight of material and q_0 – load per unit length due to belt weight.

The resistance moment set up at the idler as made up of resistance moment at the bearing and that due to rolling over the rubber belt can be determined from

$$M_r = M_1 + M_2 = (K + G_i) \frac{d}{2} f + \mu K$$

Where,

K = Load sustained, G_i = Dead weight of idler

d = diameter of idler pin journal in cm,

f = Coefficient of friction for idler bearing

μ = Rolling friction coefficient

The resistance moment set up by number of idlers within a section of conveyor will be given by

$$M'_r = (\Sigma K + \Sigma G_i) \frac{d}{2} f + \Sigma \mu K$$

Since the values of f and μ are influenced by numerous factors and can be assessed with difficulty hence frictional factor c is considered, covering all components of the frictional resistance to motion within every section of the conveyor. Then the resisting force exerted by a group of idlers within a horizontal section of the carrying run can be expressed as-

$$F_{c.r.} = \left(q + q_o + \frac{\Sigma G_i}{l} \right) lc \text{ and of the return run as } F_{r.r.} = \left(q_o + \frac{\Sigma G_i}{l} \right) lc$$

Where, $\frac{\Sigma G_i}{l} = q_i$ is the distributed load due to dead weight of group of idlers.

3.19.1.2 Frictional resistance of the idlers supporting an inclined belt section

The loading sustained by an idler in this case is quite different from the previous one. An inclined conveyor with a belt of length l_o raising material through a vertical height h carries a load,

$$\Sigma K = (q + q_o) l_o$$

Some conveying installations consisting of number of horizontal sections make use of idler-supported convex or concave transition curves as shown below.

The total resisting force coming into play in convex transition curve is,

$$F = [T_1 - (q_o + q + q_i)R](e^{\beta c} - 1)$$

Where, T_1 - Belt tension, $q_o = G_g$ = weight of conveyed material per meter length, $q = G_b$ = weight of belt per meter length, q_i is the distributed load, R – Radius of curve, β –angle between the roller as shown in figure, c - frictional factor.

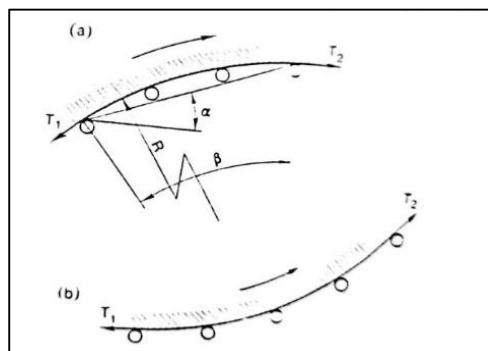


Fig.3.20 Frictional Resistance of Idlers at transition curves

To ensure that the belt will not lift off the idlers supporting a curved section, the minimum allowable radius of curvature is given by $R_{min1} = \frac{T_1}{q_o}$ for empty belt and $R_{min2} = \frac{T_1}{q+q_o}$ for the loaded belt.

Practically the transition curves are designed as a part of circle with radius R which is determined from the relationship $\frac{T_1}{q+q_o} < R < \frac{T_1}{q_o}$.

3.20 Nomenclature

ρ = density

δ = angle of inclination

a_{max} = maximum lump size

C = factor for type of idler,

V = belt speed

B = belt width

w_o = resistance of belt on top run

w_u = resistance of belt on bottom run

G_g = weight of conveyed material per meter length

G_b = weight of belt per meter length

G_{ro} = Weight of straight idlers on bottom run

f = fraction factor between idler and belt

L = conveyor length

T_1 = tension in tight side

T_2 = tension in slack side

θ = angle of contact between pulley and belt

i = No. of ply

L_b = Bearing span,

d_s = shaft diameter

d_h = hub diameter

F_a = axial load

F_r = radial load

3.21 Design procedure for Belt Conveyor System

The design of a belt conveyor system consist of the following steps:

- a) Finding the dimension, capacity and speed of the Belt.
- b) Determining the maximum load carrying capacity of the Belt.
- c) Determine the roller dimensions.
- d) Finding the Belt power and tension.
- e) Select Idler spacing.
- f) Determine the Pulley diameter.
- g) Selection of Motor.
- h) Decide the type of drive unit.
- i) Finding the location and arrangement of pulley.

3.22 Design Steps

3.22.1 Selection of Belt Width and Velocity

The selection of belt width and velocity is probably the most complicated problem facing the designer. There are a variety of factors being used, factors such as the belt width must be three times the maximum lump size, the belt width must be such that the system can cater for 66% excess capacity, and if a tripper is used the factors must be increased by a further 30% etc.

Belt width is calculated using general formula,

$$B = (X \cdot a_{\max} + 200)$$

Where $X = 2$ factor for graded and $X = 3.3$ for ungraded material, a_{\max} is maximum lump size. Then velocity of belt is assumed from material and the minimum width is to be calculated using formula

$$B_{\min} = 1.11[\{Q/(\rho \cdot c \cdot v)\}^{1/2} + 0.05] \quad \dots \dots \text{PSG 9.18},$$

Then for higher value, standard belt width is to be selected and from this width the actual velocity is to be found out.

3.22.2 Design of Drive Unit

The standardization of drives is the key to most successful conveyor systems. The problem is however that some drives should be drastically oversized to obtain some degree of conformity.

The Resistance to the belt on the top run and returning motion is need to calculate as.

$$W_o = C \cdot f \cdot l [(G_g + G_b) \cos \delta + G_{ro}] \pm H(G_g + G_b) \quad \dots \dots \text{PSG 9.18},$$

W_o = The resistance to the top run which is depends on many factor like

C = secondary resistance factor corresponding to length of conveyor,

L in metre	10	20	32	40	50	80	90	100	140	200	300	500	2000
C	4.5	3.0	2.6	2.4	2.2	1.92	1.86	1.78	1.63	1.45	1.27	1.20	1

f: Coefficient of friction between idler and belt depend on types of conveyors, L: length of conveyor, G_g : weight of conveyed material per meter length, G_b : Weight of belt per meter length, δ :Angle of inclination, G_{ro} : Weight of troughing idler on the top run per meter length, H: Height of the conveyor system and the material conveying direction. (Note: + for material conveying upward and – for material conveying downward direction)

The resistance for the bottom run is given by,

$$W_u = C. f. l[(G_b \cos \delta + G_{ru})] \pm H. G_b$$

G_{ru} : Weight of straight idler on the bottom run per meter length, (Note: – for material conveying upward and + for material conveying downward direction)

Then the effective load on the belt is calculated by adding both the resistances. From this data and velocity, Motor power required for the whole system is to be calculated and standard motor is to be selected.

3.22.3 Force analysis

Tension on tight side and slack side are to be calculate from power and belt tension equations.

3.22.4 Traction element calculation

No. of plies required are to be calculated in this step by using equation, No. of ply: $i = \frac{T_1}{B*f}$

Where, T_1 is maximum tension in the belt, B is Width of the belt and f is working tension for the belt depend on joint type, take up arrangement and std. types of belt to be selected from PSG 9.21.

Now by assuming thickness of each ply, top cover thickness and bottom cover thickness, total thickness for a belt are to be calculated.

3.22.5 Idler Design

The choice of type and spacing for Idlers should be on a more scientific basis. The types of Idler to be used on conveyors are; transition, troughing, impact and return idlers. Design of troughing idler and returning idler are need to carry out and checked for failure in this step.

3.22.6 Pulley, Shaft and Bearing design

Pulley and shaft diameters should be kept to a minimum of two per conveyor, with as much standardization as possible being employed on the whole conveyor system. In this step dimensions of the pulleys are decided, shaft diameter based on equivalent torque and bearing design is to be done. Also, snub pulley dimensions are to be decided.

3.22.7 Checking for a need of an arresting mechanism

In this step, Resistance of the belt on the top run W_o is calculated for reverse motion and the need for an arresting mechanism is checked by observing positive or negative value of resistance. If the resistance is negative then there is need for the arresting mechanism.

3.22.8 Design of tension take up unit.

The weight required to be attached to belt on the slack side to maintain the required tensions is to be calculated in this step.

NUMERICALS

3.1. Design of Belt Conveyor System for Specification.

Troughing angle = 20 degrees

Material to be conveyed – Coal

Capacity = 200 TPH (Tonnes Per Hour)

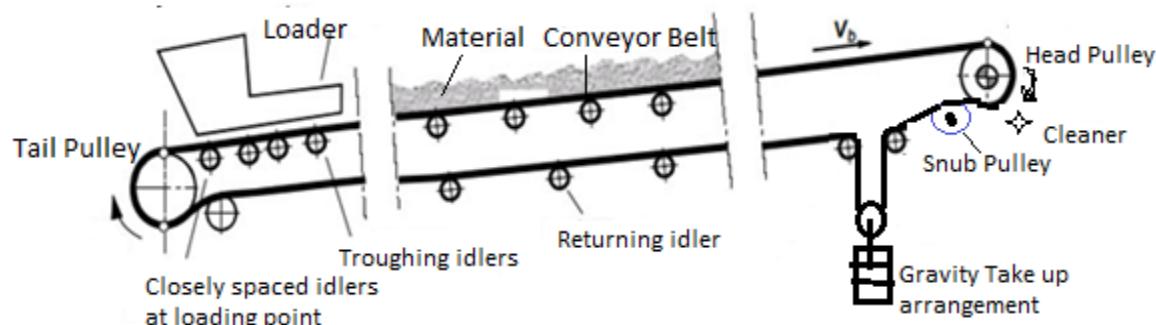
Lump Size = 125mm

Inclination = 15 degrees

Centre to Centre Distance = 90m

Solution:

Assumption: It is assumed that loading is taking place at tail pulley and unloading at head pulley.



Given Data:

Material = Coal,

$Q = 200 \text{TPH}$, $a_{\max} = 125 \text{ mm}$, $\delta = 15 \text{ degree}$, $L = 90 \text{ m}$, troughing angle = 20,

Design Capacity and Material Properties

Assuming 10% Extra,

Design capacity = $1.1 * Q = 1.1 * 200 = 220 \text{ TPH}$

Material properties:

For Coal,

$$\text{Density} = 0.65 \sim 0.78 = 0.7 \text{ Ton/m}^3 \quad \dots \dots \text{ (PSG 9.23)}$$

Step 1: Belt Width

Using empirical relation with lump size, Assuming graded material hence $x= 3.3$

$$\text{Belt Width, } B = (x \cdot a_{\max} + 200) = 3.3 \times 125 + 200 = 612.5 \text{ mm}$$

Also,

$$B_{\min} = 1.11[\{Q/(\rho \cdot c \cdot v)\}^{1/2} + 0.05] \quad \dots \dots \text{ (PSG 9.18)}$$

Where, c = Factor for type of idler (460 for troughing angle 20)

$$V = \text{Belt speed} = 2 \text{ m/s} \quad \dots \dots \text{ (Assumption)}$$

$$B_{\min} = 1.11 \left[\left\{ \frac{220}{(0.7 * 460 * 2)} \right\}^{1/2} + 0.05 \right] = 0.704 \text{ m}$$

Selecting standard width for belt as $B= 800\text{mm} \dots \dots \text{ (PSG 9.19)}$

Substituting $B= 800\text{mm}$ in above formula

$$800 = 1.11[\{220/(0.7 \times 460 \times v)\}^{1/2} + 0.05]$$

$$v_{\text{actual}} = 1.52 \text{ m/s}$$

Step 2: Drive Unit

Providing driving traction required for moving the belt and load. A drive unit consist of motor, gear reduction unit, a drive pulley and couplings. The drive should be capable of moving the belt at constant or variable speed according to the requirement.

Resistance of the belt on the top run w_o for conveying up is given by,

$$w_o = C \cdot f \cdot l [(G_g + G_b) \cos \delta + G_{ro}] + H(G_g + G_b) \quad \dots \dots \text{ (PSG 9.18)}$$

Where, C = secondary resistance factor = 1.8 for $L= 90\text{m}$ from graph,

f = friction between idler and belt =0.02 for std. conveyors,

$$l = 90 \text{ m}$$

G_g = weight of conveyed material per meter length

$$G_g = \frac{Q}{3.6 v} = \frac{220}{3.6 * 1.52} = \frac{220}{3.6 * v} = 40.21 \text{ kgf/m}$$

$$G_b = \text{Weight of belt per meter length} = 12 \text{ kgf} \quad \dots \dots \text{ (Table 1: PSG 9.19)}$$

G_{ro} = Weight of troughing idler on the top run per meter length

G_{ru} = Weight of straight idler on the bottom run per meter length

Tube diameter = 140mm and bearing diameter = 25 mm (Assumption) (PSG 9.19/T2)

Maximum spacing for troughing idler = 1.5m and

Maximum spacing for straight idler = 3 m... (PSG 9.19/T2)

$$G_{ro} = \frac{27.4}{1.5} = 18.26 \text{ kgf/m}, G_{ru} = \frac{18.8}{3} = 6.26 \text{ kgf/m}$$

Height through which material is to be lifted (H)

$$H = L \sin \delta = 90 \sin (15) = 23.29 \text{ m}$$

Resistance of the belt on the top run,

$$w_o = C_f l [(G_g + G_b) \cos \delta + G_{ro}] + H(G_g + G_b)$$

$$w_o = 1.8 * 0.02 * 90 [(40.21 + 12) \cos 15 + 18.26] + 23.29 (40.21 + 12) = 1438.53 \text{ kgf}$$

Resistance of the belt on the bottom run w_u is given by,

$$w_u = C. f. l [(G_b \cos \delta + G_{ru})] - H. G_b \quad \dots\dots \text{ (PSG9.18)}$$

$$w_u = 1.8 * 0.02 * 90 [40.21 \cos 15 + 6.26] - (23.29 * 12)$$

$$w_u = -221.64 \text{ kgf}$$

Effective load on belt,

$$P = w_o + w_u = 1438.53 - 221.64 = 1216.89 \text{ kgf}$$

$$\text{Motor Power} = P * V = 1216.89 * 1.52 * 10 = 18496 \text{ W}$$

Selecting a standard motor Power = 18.5 kW (PSG 5.124)

Step 3: Force Analysis

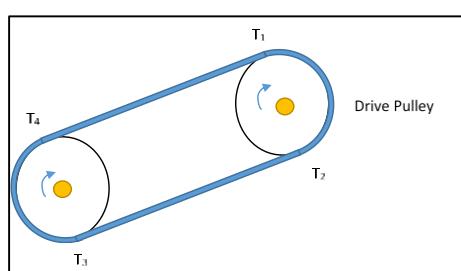


Fig.3.22 Belt drive

Assume coefficient of friction between belt and pulley, $\mu=0.25$ and $\theta = 200^\circ$

$$\frac{T_1}{T_2} = e^{\mu\theta}, \quad \frac{T_1}{T_2} = e^{(0.25 \times 200 \times \frac{\pi}{180})}, \\ T_1 = 2.39 T_2 \quad \dots\dots\dots (1)$$

Also, $P = (T_1 - T_2) V,$

$$(T_1 - T_2) = 1217.11 \text{kgf} \quad \dots\dots\dots (2)$$

From (1) & (2),

$$T_2 = 875.6 \text{kgf}, \quad T_1 = 2092.7 \text{ kgf}$$

$$T_3 = T_1 + W_u = 2092.7 - 221.64 = 1871.11 \text{kgf}$$

Step 4: Traction Element

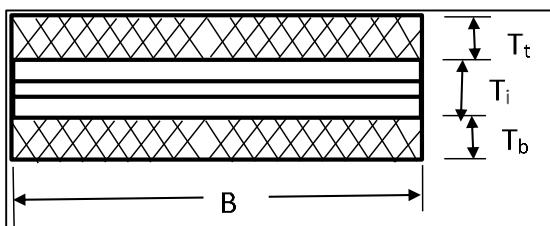


Fig. 3.23 Belt Cross Section

$$\text{No. of ply: } i = \frac{T_1}{B \times f} \dots\dots\dots (\text{PSG 9.18})$$

For density 0.7 Ton/m³ and belt width 800mm,

Working tension $f = 0.62$ for 32 OZ vulcanised joint and gravity take up..... (PSG 9.21/T5)

$$i = \frac{2092.7}{0.62 \times 800} = 4.21 = 5$$

Also, corresponding to density of the material and belt width, Maximum no. of ply = 7

And Minimum no. of ply = 4 (PSG 9.21/T4)

$$i = 5$$

Let, thickness of each ply = 2mm for fabric strength of 400N/mm²

t_i = Total thickness of ply = $6 \times 2 = 12 \text{mm},$

Top thickness = $t_t = 2 \text{mm}$

Bottom thickness = $t_b = 1 \text{mm},$

Total thickness of belt, $T = t_i + t_t + t_b = 15 \text{mm}$

Step 5: Idler Design

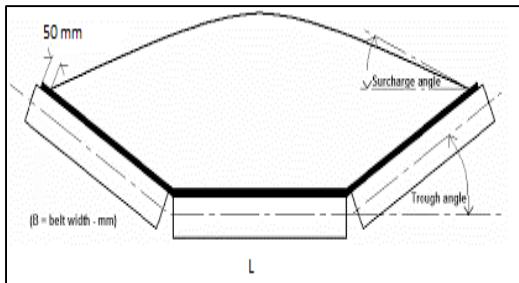


Fig. 3.24 Troughing Idler Assembly

To prevent the belt from sagging due to gravity and under the load, a number of idler rolls between the terminal pulleys. The most common configuration for idler rollers is 3 rollers of equal length.

Step 5.1 Troughing Idler

Let the material for idler be C-25, with $[\sigma_b] = 60 \text{ MPa}$,

Shape is tubular and provision for bearing block

Belt Width is given by,

$$B = L + (2L - 100)$$

$$800 = 3L - 100$$

Length of troughing idler, $L = 300 \text{ mm}$,

Assuming Troughing Idler Size: $\phi 140 * 300 \text{ mm}$

Total no. of troughing idler systems = $L / \text{spacing} = 90/1.5 = 60$

Total no. of troughing idlers = $3 * 60 = 180$

Total weight acting on troughing idler,

$$W' = (G_g + G_b + G_{ro}) * \text{spacing} = (40.21 + 12 + 18.25) * 1.5 = 105.7 \text{ kgf}$$

Middle idlers generally take 70% of total load, $W = 0.7 * 105.7 = 74 \text{ kgf} = 740 \text{ N}$

$$W_o = (40.21 + 12 + 18.25) = 70.46 \text{ kgf/m} = 0.7046 \text{ N/mm}$$

$L_1 = 300 \text{ mm}$, $L_2 = 200 \text{ mm}$ (Assuming margin for bearing block)

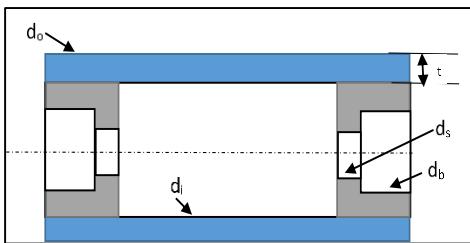


Fig. 3.25 Idler Cross Section

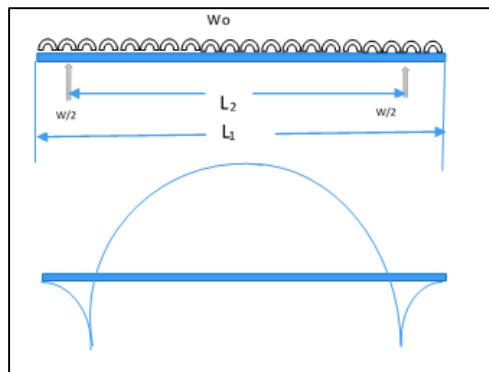


Fig. 3.26 Force and BMD for Idler

Calculation for maximum bending moment at the centre,

$$BM_{max} = W_o \times \frac{L_1^2}{8} - \frac{W}{2} \times \frac{L_2}{2} = 0.7046 \times \frac{300^2}{8} - \frac{740}{2} \times \frac{200}{2}$$

$$BM_{max} = \text{mod}(-29073.25) = 29073.25 \text{ N.mm}$$

Assuming, thickness of idler as 2.5mm,

$d_o = 140\text{mm}$, $d_i = 135\text{mm}$, $d_s = \text{Axele diameter} = 30\text{mm}$, $d_b = \text{bearing outer diameter} = 47\text{ mm}$

(Select any bearing from data book having diameter 30 mm)

$$(\sigma_b) = \frac{M}{Z} = \frac{29073.25 \times 32}{\pi * \left(\frac{d_o^4 - d_i^4}{d_o} \right)} = \frac{29073.25 \times 32}{\pi * \left(\frac{140^4 - 135^4}{140} \right)} = 0.797 \text{ N/mm}^2$$

$(\sigma_b) < [\sigma_b]$, Hence, design is safe.

Step 5.2 Returning Idler

(For returning idler, material load is not present hence assumed as safe under different failure, the bearing and the bearing selected for troughing idler can be used for returning idler also.)

Length of returning idler,

$$L_2 = B + \text{Margin} = 800 + 100 = 900\text{mm}$$

Returning idler $\varphi 140 * 900$ mm

Total no. of returning idlers = $L / \text{spacing} = 90/3 = 30$

Step 5.3 Design of bearing for idler

Bearing inner diameter = 25mm (Assumed)

Forces acting on bearing are,

$$F_r = \frac{W}{2} = \frac{740}{2} = 370 \text{ N}, \quad F_a = 0$$

Assuming working at room temp, nature of load as medium shock and life in hours as 15000Hrs,
Equivalent load,

$$P_{eq} = (X \cdot V \cdot F_r + Y \cdot F_a) S$$

Where,

$$X = 1, V = 1.2, S = 1.2,$$

$$P_{eq} = (1.2 * 370 * 1.2) = 532.8 \text{ N}$$

$$N = \frac{60 v}{\pi d_o} = \frac{60 \times 1.52}{\pi \times 0.140} = 207.35 \text{ rpm}$$

$$L_{mr} = \frac{L_{hr} \times N \times 60}{10^6} = \frac{15000 \times 207.35 \times 60}{10^6} = 186.62 \text{ mr, also } k=3 \text{ for Ball bearing}$$

Dynamic load carrying capacity,

$$C = P_{eq} (L_{mr})^{\frac{1}{k}} = 532.8 * (186.62)^{(1/3)}$$

$$C = 3044.74 \text{ N, } C = 304.47 \text{ Kgf}$$

Selecting Deep Groove Ball Bearing, No. 6005 with $d = 25 \text{ mm}$, $D = 47 \text{ mm}$,

Width = 12mm and $C = 780 \text{ Kgf}$ (PSG 4.12)

Step 5.4 Design axle for idler

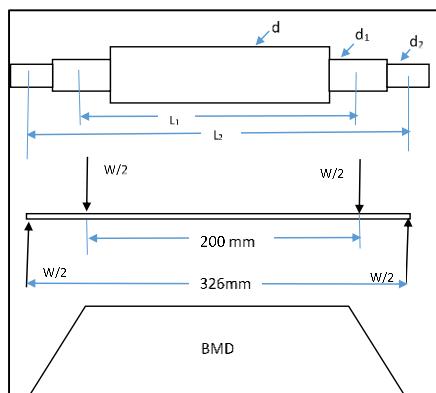


Fig. 3.27 Idler Shaft, Force and BMD

Let the material for axle be C-25, with

$$[\sigma_b] = 60 \text{ N/mm}^2, \quad [\sigma_{cr}] = 90 \text{ N/mm}^2$$

Axle diameter, $d = 30 \text{ mm}$, $d_1 = \text{bearing inner diameter} = 25 \text{ mm}$,

$d_2 = 20\text{mm}$ (assumed)

$L_1 = 200\text{mm}$ (from idler design)

$L_2 = \text{Rigid support span} = \text{Idler length} + \text{margin} + \text{frame width}$

$L_2 = 300 + 12 + 14 = 326\text{mm}$ (Assuming frame thickness = 14mm)

Load acting $W/2 = 370\text{N}$

Checking for bending failure,

$$(\sigma_b) = \frac{\text{BM max}}{Z} = \frac{\left(\frac{W}{2}\right) \times (L_2 - L_1)/2}{\frac{\pi \times d^3}{32}} = \frac{370 \times 63}{2650.7} = 8.79 \frac{\text{N}}{\text{mm}^2} < [\sigma_b]$$

Hence, safe in bending.

Checking for crushing failure,

$$(\sigma_{cr}) = \frac{W/2}{d_2 \times \text{Frame thickness}} = \frac{370}{20 \times 14} = 1.32 \frac{\text{N}}{\text{mm}^2} < [\sigma_{cr}]$$

Hence, safe in crushing failure.

Step 6: Pulley Design

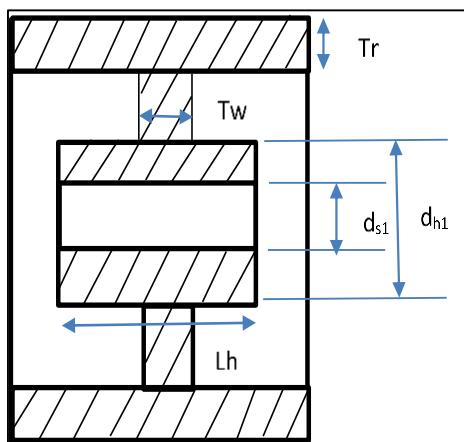


Fig. 3.28 Cross Section of pulley

Pulleys are necessary in conveyors to change the direction of belt in vertical planes and to form endless loop for continuous motion.

$$D_1 = k_1 k_2 i$$

Where, k_1 = factor for pulley material = 1.4 (for high tensile Anide)

k_2 = factor depends upon angle of contact and belt tension

$k_2 = 100$ (for $\theta > 180^\circ$)

$$D_1 = 1.4 \times 100 \times 5 = 700 \text{ mm}$$

$$t_r = 6 \sim 8 = 7 \text{ mm}, t_w = 10 \text{ mm}$$

Width of the pulley,

$$B_1 = B + \text{margin} = 800 + 100 = 900 \text{ mm}$$

$$V = \frac{\pi DN}{60}, 1.52 = \frac{\pi \times 0.7 \times N}{60}, N = 34.56 \text{ rpm}$$

Step 6.1 Shaft Diameter

Let shaft material be C-25, Due to shock loads taking more FOS,

$$[\tau] = 25 \text{ N/mm}^2, [\sigma_b] = 60 \text{ N/mm}^2$$

$$T = \frac{P \times 60}{2\pi N} = \frac{18500 \times 60}{2 \times \pi \times 34.56} = 5111.75 \text{ N-m}$$

Bearing span can be take larger for easy maintenance but to control lateral deflection,

$$\text{Assuming bearing span} = 1.25B_1 = 1.25 \times 900 = 1125 \text{ mm}$$

$$W = T_1 + T_2$$

$$BM_{\max} = \frac{WL}{4} = \frac{(2092.7 + 875.6) \times 10 \times 1125}{4 \times 1000} = 8348.34 \text{ N-m}$$

$$\text{Equivalent torque} = T_{eq} = \sqrt{T^2 + M^2} = \sqrt{(5111.75)^2 + (8348.34)^2} = 9789 \text{ N-m}$$

$$[\sigma_s] = \frac{T_{eq} 16}{\pi (d_s)^3}$$

$$25 = \frac{9789 \times 1000 \times 16}{\pi \times (d_s)^3}$$

$$\text{Therefore, } d_s = 125.87 \approx 126 \text{ mm,}$$

$$\text{Let, } d_{s1} = 1.1d_s = 138.6 \approx 140 \text{ mm,}$$

$$\text{Let, } h_1 = d_{h1} = 1.5d_{s1} = 1.5 \times 140 = 210 \text{ mm}$$

Step 6.2 Bearing Selection for pulley

A bearing is a machine element that constrains relative motion to only the desired motion, and reduces friction between moving parts. Bearing in the conveyor belt system is used between pulley and shaft.

$$F_r = \frac{W}{2} = \frac{T_1 + T_2}{2} = \frac{(2092.7 + 875.6)10}{2} = 14841.5 \text{ N, } F_a = 0$$

$$P_{eq} = (X.V.F_r + Y.F_a) S = (1.2 \times 14841.5) \times 1.2 = 21371.76 \text{ N}$$

$$\text{Assuming } L_{hr} = 10000 \text{ hrs}$$

$$L_{mr} = \frac{L_{hr} \times N \times 60}{10^6} = \frac{10000 \times 34.56 \times 60}{10^6} = 20.72 \text{ mr}$$

$$C = P_{eq}(L_{mr})^{\frac{1}{k}} = 21371.76 * (20.72)^{(1/3)}$$

$$C = 5869.98 \text{ kgf}$$

For d= 140 & C > 5869.98kgf,

Selecting bearing no. SKF6028 [d=140mm, C=8600kgf].. PSG 4.12

Step 7: Snub Pulley

Snub rollers are used to

- Increase the angle of contact between the belt and the driving pulley and improve the torque transmitted.
- Decrease the distance between the belt carrying side and return side.
- Improve belt tracking with adjustable snub rollers.

It is located at the slack side of the head pulley. The construction features and material is same as head pulley.

Diameter of Snub pulley = $0.6 \times$ head pulley diameter = 420mm

Step 8: Checking for arresting mechanism

Resistance of the belt on the top run W_o for conveying down, is given by ...PSG 9.18

$$w_o = Cfl [(G_g + G_b) \cos\delta + G_{ro}] - H(G_g + G_b)$$

$$w_o = 1.8 \times 0.02 \times 90 [(40.21+12) \cos 15 + 18.26] - 23.29(40.21+12) = -993.412 \text{ kgf}$$

Since W_o is negative, if motor stops working, the belt will move in opposite direction. Hence there is a need of arresting mechanism.

Step 9: Tension take up unit

The tension take-up unit is required to maintain the tension in belt. The simplest form of it is gravity loaded tension take-up unit. To determine the tensioning weight required (T_R) with 180 degree arc of contact, using the relation,

$$T_R = 2T_2 - T_{TR}$$

Where, T_2 – Slack side tension, T_{TR} – Weight of tension roller

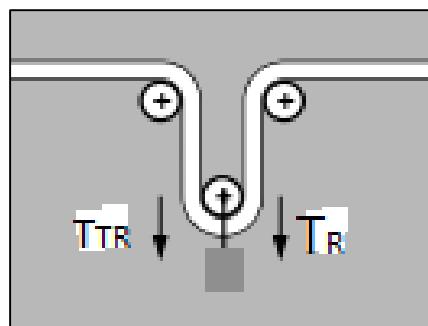


Fig. 3.29 Tension take up system

$$T_R = 2 \times 875.6 - 50 \quad (\text{Assuming weight of tension roller as } 50\text{Kgf})$$

$$\text{The tensioning weight required (} T_R \text{)} = 1701\text{Kgf}$$

Numerical 3.2 Design of belt conveyor system for following specification

Material to be conveyed = stone (lime)

Capacity = 300 TPH

Troughing angle= 25^0

Lump size= 80mm

Inclination (δ) = 12^0

CTC distance= 50m

Solution:

Assumption: It is assumed that loading is taking place at tail pulley and unloading at head pulley.

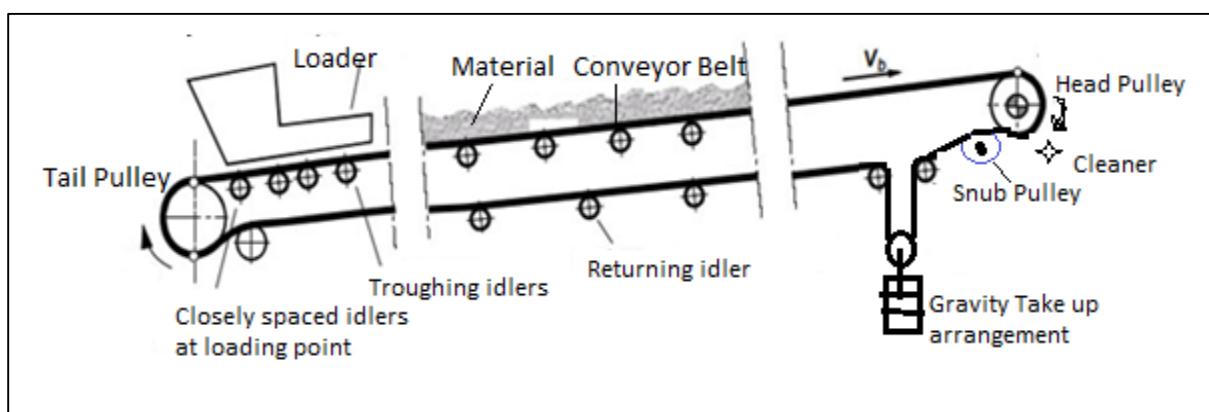


Fig. 3.30 Belt conveyer system

Given Data:

Drive selection: AC motor, head pulley is driven, belt supports, over run troughing idler assembly and

Under run idler assembly is single straight idler assembly.

$$\text{Design capacity} = [Q] = \frac{300}{0.9} = 333.33 = 334 \text{ TPH}$$

Material properties for Stone (lime) (PSG 9.22)

Density (ρ) = 1.2 to 1.5, let 1.4 Tonne/m³

Max inclination = 20⁰ and Angle of repose= 45⁰

Step 1: Belt Width

Let, speed of the belt is (v) = 1.75 m/s (PSG 9.20 table 3)

Using relation,

$$B = X \cdot a_{\max} + 200$$

Where, X = 3.3 for ungraded material and X = 2.5 for graded material

$$B = (3.3 \times 80) + 200 = 464 \text{ mm}$$

Also based on capacity, the minimum width of pulley is given by,

$$B_{\min} = 1.11 \left[\left(\frac{Q}{\rho c v} \right)^{1/2} + 0.05 \right]$$

Where,

Density ρ = 1.4 T/m³

Q = 334 TPH and factor for types of idler

C = 510 for troughing angle of 25⁰

$$B_{\min} = 1.11 \left[\left(\frac{334}{1.4 \times 510 \times 1.75} \right)^{1/2} + 0.05 \right] = 0.659 \text{ m} = 629.38 \text{ mm}$$

Selecting next standard width of belt B = 650mm

Calculating Actual velocity, V_{act} = 1.63 m/s, Acceptable.

Step 2: Drive unit

f = friction between idler and belt (PSG 9.18)

let f = 0.02 for standard conveyer

Length L = 50m, C =2.2 from graph

Now, Weight of conveyed material per meter length

$$G_g = \frac{\text{Wt.of conveyer material}}{\text{metre lenght}} = \frac{Q}{3.6 V}$$

$$G_g = \frac{334}{3.6 \times 1.63} = 56.92 \text{ kgf/m}$$

Weight of Belt per meter length,

$$G_b = \frac{\text{Weight of belt}}{\text{meter length}} = 9 \text{ kgf/m (PSG 9.19)}$$

Weight of troughing idler on the top run per meter length ,

$$G_{ro} = \frac{\text{Wt.of troughing idler on the top run}}{\text{metre length}}$$

Let tube diameter be 140 mm and Bearing diameter = 25mm

Weight of troughing idler pulley = 24.6 kgf/idler assembly

Maximum spacing for idler

For density = 1.2~ 2 Tonne/m³ and Belt Width B= 650mm (PSG 9.19 table 2)

Max spacing = 1.5 m, Spacing for return idler = 3 m,

$$G_{ro} = 24.6/1.5 = 16.4 \text{ Kgf/ m} - \text{for troughing idler,}$$

$$G_{ru} = 15.9 /3 = 5.3 \text{ Kgf/ m} - \text{for returning idler,}$$

Now, Height through which material is conveyed H is given by

$$H = L * \sin \delta = 50 \times \sin 12^\circ = 10.396 \text{ m}$$

Resistance of the belt on the top run and bottom run (PSG 9.18)

$$W_o = C. f. L [(G_g + G_b) \cos \delta + G_{ro}] + H (G_g + G_b)$$

$$W_o = 2.2 \times 0.02 \times 50 [(56.92 + 9) \cos 12^\circ + 16.4] + 10.4 (56.92 + 9) = 863.5 \text{ kgf}$$

And, $W_u = C.f.L [G_b \times \cos \delta + G_{ru}] - H \times G_b$

$$W_u = 2.2 \times 0.02 \times 50 [9 \cos 12^\circ + 5.3] - 10.4 \times 9 = -62.57 \text{ kgf}$$

Effective tension is given by,

$$P = W_o + W_u \dots \dots \dots \text{ (PSG 9.18)}$$

$$P = 863.5 - 62.57 = 800.9 \text{ kgf}$$

Motor power = $P \times V = 800.9 \times 1.63 = 1305.5 \text{ kgf.m/s} = 13.06 \text{ kW}$

Selecting 15 kW standard motor (PSG 5.124)

Step 3: Force Analysis

$$\text{Power } P = (T_1 - T_2) V$$

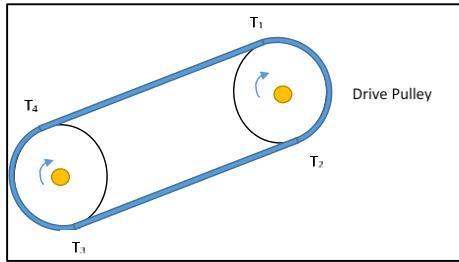


Fig. 3.31 Belt Drive

$$\text{No. of plies } i = T_1 / B.f \quad (\text{PSG 9.18})$$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Assume coefficient of friction between belt and pulley, $\mu=0.25$ and $\theta = 200$ degree

$$\frac{T_1}{T_2} = e^{(0.25 \times 200 \times \frac{\pi}{180})}$$

$$T_1 = 2.39T_2 \dots\dots (1)$$

Also,

$$P = (T_1 - T_2) V$$

$$(T_1 - T_2) \times 1.63 = 1500 \text{ kgf m/s}$$

$$(T_1 - T_2) = 920.25 \dots\dots (2)$$

From (1) & (2), $T_2 = 662.05 \text{ kgf}$, $T_1 = 1582.3 \text{ kgf}$

$$T_3 = T_1 + W_u = 1582.3 - 62.57 = 1519.73 \text{ kgf} \quad \text{and } T_4 = T_3$$

Step 4: Traction element

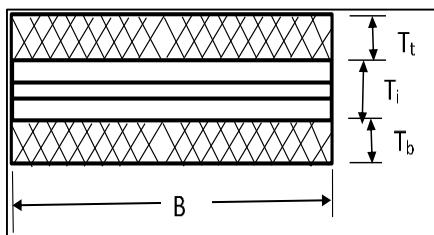


Fig. 3.32 Belt Cross Section

Now No. of plies,

$$i = T_1 / B \cdot f$$

where f is working tension in conveyor belts, from PSG 9.21, corresponding to density assuming 32OZ designation of belt fabric and assuming mechanical joint and gravity take up,

$$f = 0.57 \dots \text{Table 5, PSG 9.21}$$

$$i = 1582.3 / (650 \times 0.57) = 4.27$$

For Belt width 650 mm, and 32oz designation,

Maximum and Minimum No. of plies are 5 and 6...Table 4, PSG 9.21

Selecting no. of plies as $i = 5$

Proportions:

Thickness of ply = $i \times$ thickness of ply = $5 \times 2 = 10\text{mm}$

Assuming, thickness of ply = 2mm,

Top cover thickness (t_t) = 2mm and

Bottom cover thickness (t_b) = 1mm

Total thickness= $t_t + t_b + t_i = 13\text{mm}$

Step 5: Idler Design

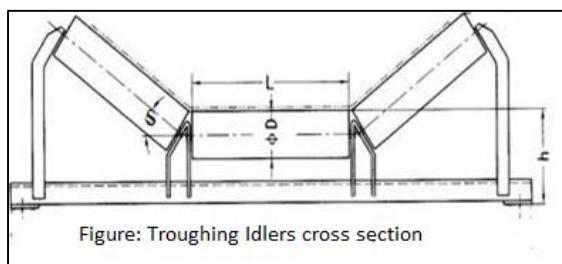


Fig. 3.33 Troughing Idler Assembly

The troughing idler angle for the current problem as a troughing angle of 25^0 and the material to be transported is stone i:e a bulk material

Step 5.1 Troughing Idler

Let, the material for idler be C-25, with $[\sigma_b] = 60\text{MPa}$,

Shape is tubular and provision for bearing block

$$\text{Belt Width} = B = L + (2L - 100)$$

$$650 = 3L - 100$$

Length of troughing idler, $L = 250\text{mm}$,

Troughing Idler Size: $\phi 140 * 250\text{mm}$

Total no. of troughing idler systems = $L / \text{spacing} = 50/1.5 = 33.33 = 34$

Total no. of troughing idlers = $3 * 34 = 102$

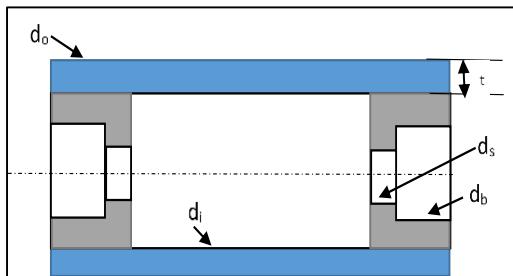


Fig. 3.34 Idler Cross Section

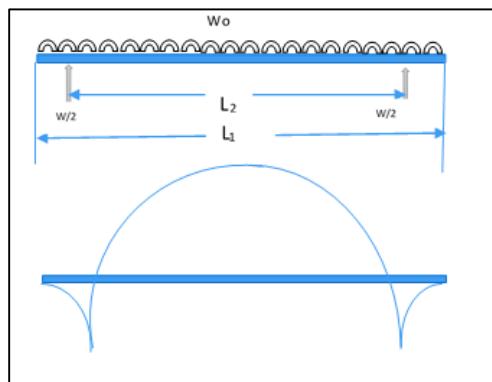


Fig. 3.35 Force and BMD for idler

Total weight acting on troughing idler,

$$W' = (G_g + G_b + G_{ro}) * \text{spacing}$$

$$W' = (56.92 + 9 + 16.4) \times 1.5 = 123.48 \text{ kgf}$$

Middle idlers generally take 70% of total load,

$$W = 0.7 * 123.48 = 86.44 \text{ kgf} = 864 \text{ N}$$

$$\text{UDL, } W_o = (56.92 + 9 + 16.4) = 82.32 \text{ kgf/m} = 0.8232 \text{ N/mm}$$

$L_1 = 250\text{mm}$, $L_2 = 150\text{mm}$ (Assuming margin for bearing block)

Calculation for maximum bending moment at the centre,

$$BM_{max} = W_o \times \frac{L_1^2}{8} - \frac{W}{2} \times \frac{L_2}{2} = 0.8232 \times \frac{250^2}{8} - \frac{864}{2} \times \frac{150}{2}$$

$$BM_{max} = \text{mod}(-25968.75) = 25968.75 \text{ N.mm}$$

Assuming thickness of idler as 2.5mm, $d_o=140\text{mm}$, $d_i=135\text{mm}$,

$$d_s = \text{Axe diameter} = 30\text{mm},$$

$$d_b = \text{bearing outer diameter} = 47 \text{ mm (from bearing design)}$$

$$(\sigma_b) = \frac{M}{Z} = \frac{25968.75 \times 32}{\pi \times \left(\frac{d_o^4 - d_i^4}{d_o} \right)} = \frac{29073.25 \times 32}{\pi \times \left(\frac{140^4 - 135^4}{140} \right)} = 0.712 \text{ N/mm}^2,$$

$(\sigma_b) < [\sigma_b]$, Hence, design is safe.

Step 5.2 Returning Idler

(For returning idler, material load is not present hence assumed as safe under different failure, the bearing and the bearing selected for troughing idler can be used for returning idler also.)

Length of returning idler,

$$L_2 = B + \text{Margin} = 650 + 100 = 750 \text{ mm}$$

Returning idler $\varphi 140 * 750 \text{ mm}$

Total no. of returning idlers = $L / \text{spacing} = 50/3 = 17$

5.3 Design of bearing for idler

Bearing inner diameter = 25mm (Assumed)

Forces acting on bearing are,

$$F_r = \frac{W}{2} = \frac{864}{2} = 432 \text{ N}, \quad F_a = 0$$

Assuming working at room temp, nature of load as medium shock and life in hours as 15000Hrs,

Equivalent load is given by,

$$P_{eq} = (X \cdot V \cdot F_r + Y \cdot F_a) S$$

Where, $X = 1$, $V = 1.2$, $S = 1.2$

$$P_{eq} = (1.2 * 432 * 1.2) = 622 \text{ N}$$

$$N = \frac{V \times 60}{\pi d_o} = \frac{1.63 \times 60}{\pi \times 0.140} = 222.36 \text{ rpm}$$

Assuming life of bearing as 15000 Hrs,

$$L_{mr} = \frac{L_{hr} \times N \times 60}{10^6} = \frac{15000 \times 222.36 \times 60}{10^6} = 200.13 \text{ mr},$$

Also $k=3$ for Ball bearing, Dynamic load carrying capacity,

$$C = P_{eq} (L_{mr})^{\frac{1}{k}} = 622 \times (200.13)^{(1/3)}$$

$$C = 3638.24 \text{ N}, C = 363.8 \text{ Kgf},$$

Selecting Deep Groove Ball Bearing, No. 6005 with $d= 25\text{mm}$, $D= 47\text{mm}$, width =12mm and

$$C = 780 \text{ kgf (PSG 4.12)}$$

Step 5.4 Design of axle for idler

Let the material for axle be C-25, with $[\sigma_b] = 60 \text{ N/mm}^2$, $[\sigma_{cr}] = 90 \text{ N/mm}^2$

Axle diameter, $d = 30\text{mm}$, $d_1 = \text{bearing inner diameter} = 25\text{mm}$,

$d_2 = 20\text{mm}$ (assumed)

$L_1 = 150\text{mm}$ (from idler design)

$L_2 = \text{Rigid support span} = \text{Idler length} + \text{margin} + \text{frame width}$

$L_2 = 250 + 10 + 10 = 270\text{mm}$ (Assuming frame thickness = 10mm)

Load acting $W/2 = 432\text{N}$

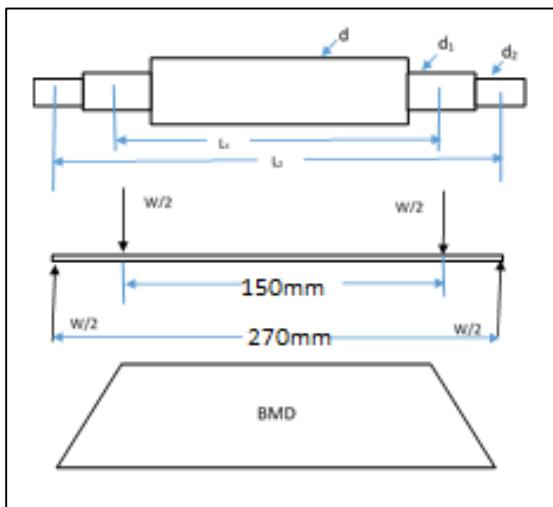


Fig. 3.36 Idler Shaft, Force and BMD

Checking for bending failure,

$$(\sigma_b) = \frac{\text{BM max}}{Z} = \frac{\left(\frac{W}{2}\right) \times \frac{L_2 - L_1}{2}}{\frac{\pi \times d^3}{32}} = \frac{432 \times 60}{2650.7} = 9.778 \frac{\text{N}}{\text{mm}^2}$$

$$(\sigma_b) < [\sigma_b] \text{ Hence, safe.}$$

Checking for crushing failure,

$$(\sigma_{cr}) = \frac{W/2}{d_2 \times \text{Frame thickness}} = \frac{432}{20 \times 10} = 2.16 \frac{\text{N}}{\text{mm}^2} < [\sigma_{cr}] \text{ Hence, safe.}$$

Step 6: Pulley Design

Head/Drive Pulley is located at the discharge terminus of the conveyor. It provides the driving force for the conveyor. In order to increase pulley life and traction, it has often larger diameter than other pulleys.

Drive Pulley Considerations

- Pulley should be large enough to avoid bending at point of tension application.
- Pulley should have grab, risk of slippage is function of grab and tension, more grab - less tension - could mean a cheaper belt
- Place Drive in practical location to minimize the highest tension in the belt - a head pulley location often good on a belt up a slope.

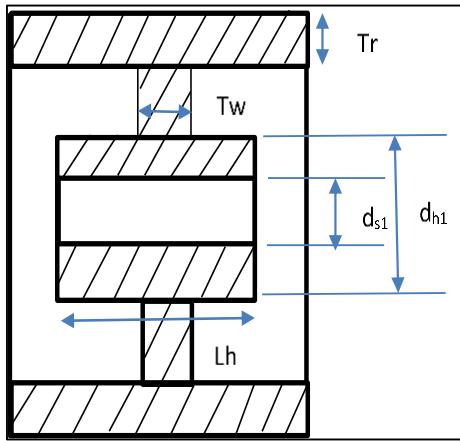


Fig. 3.37 Cross Section of pulley

Diameter of Pulley is given by,

$$D = K_1 \cdot K_2 \cdot I$$

For fabric ply construction,

K_1 = factor for ply material (1.25 to 2.5) = 1.4

K_2 = 100 Factor depends on angle of contact $\Theta > 180^\circ$,

$$D = 1.4 \times 100 \times 5$$

$$D = 700 \text{ mm}$$

Now,

$$V = \pi D N / 60$$

$$N = (1.63 \times 60) / (\pi \times 0.7) = 44.47 \text{ rpm}$$

Assuming, $t_w=10\text{mm}$, $t_r = 6$ to 8 mm

$$B_1 = B + \text{margin} = 650 + 100 = 750\text{mm}$$

6.1 Shaft Diameter

Let, material for shaft be C-25/C30, Permissible stress $[\tau] = 25 \text{ N/mm}^2$

Torque is given by,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 1000 \times 60}{2\pi \times 44.47} = 3.22 \times 10^6 \text{ N.mm}$$

Let bearing span, $L_{b1} = 1.25 \times B1 = 1.25 \times 750 = 937.5 \text{ mm}$

Total load

$$F = T_1 + T_2 = 15823 + 6620.5 = 22443.5 \text{ N},$$

i.e. Total load acting $F = 22.44 \text{ kN}$

Bending Moment is given by,

$$M = F \cdot L_{b1}/4 = (22443 \times 937.5) / 4$$

$$M = 5.26 \times 10^6 \text{ N.mm}$$

$$T_{eq} = \sqrt{M^2 + T^2} = \sqrt{(5.26 \times 10^6)^2 + (3.22 \times 10^6)^2}$$

$$T_{eq} = 6.167 \times 10^6 \text{ N.mm}$$

Shear Stress can be calculated by,

$$[\tau] = T_{eq} / (\pi/16 \cdot d_s^3)$$

$$25 = (6.167 \times 10^6 \times 16) / (\pi d_s^3)$$

$$d_s = 107.9 \text{ mm}$$

To account for key effect, $d_s = 1.1 \times 107.9 = 118.69 \text{ mm}$

Selecting $d_s = 120 \text{ mm}$

6.2 Bearing Design

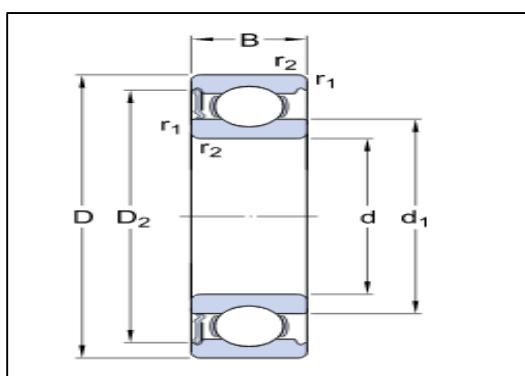


Fig. 3.38 Cross Section of bearing

Load acting on the bearing are,

Radial Load is given by,

$$F_r = W/2 = (T_1 + T_2)/2 = 22.44/2 = 11.22 \text{ kN}$$

Axial Load, $F_a = 0 \text{ N}$

Equivalent Load,

$$P_{eq} = (X \cdot V \cdot F_r + Y \cdot F_a) \times 1.2$$

Assuming service factor as 1.2, $X=1$, $V=1$

$$P_{eq} = 13.464 \text{ kN} = 1346.4 \text{ kgf}$$

Now, $N = 44.47 \text{ rpm}$

Assuming life of bearing as $L_{hr} = 5000 \text{ hrs.}$

Life in millions of revolution,

$$L_{mr} = (L_{hr} \times 60 \times N) / 10^6 = (5000 \times 60 \times 44.47) / 10^6 = 13.34 \text{ mr}$$

Dynamic Load carrying capacity,

$$C = (L_{mr})^{1/k} \cdot P_{eq} \quad k=3 \text{ for ball bearing,}$$

$$C = (13.34)^{1/3} \times 1346.4 = 3193.3 \text{ kgf}$$

Selecting standard bearing as SKF 6024 (PSG 4.12)

$d = 120\text{mm}$ and $C = 6700\text{Kgf}$

Step 7: Snub Pulley

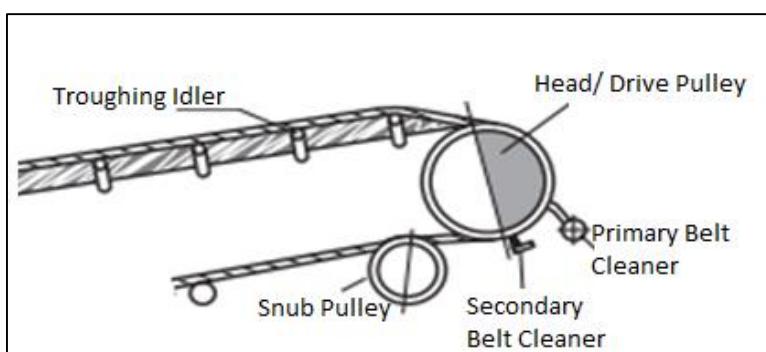


Fig. 3.39 Snub Pulley

Snub pulley is provided to increase/decrease and to warp/contact angle of belt on nearby pulley. Mounted close to the drive pulley on the return side of the belt, the snub pulley's primary job is to increase the angle of wrap around the drive pulley, thereby increasing traction. Its secondary

purpose is reducing belt tension, which is important in maximizing conveyor component life. May be lagged for longer wear life.

Location: slack side of Head pulley

$$\text{Size of snub pulley} = 0.6 \times \text{Diameter of head pulley} = 0.6 \times 700 = 420 \text{ mm}$$

Step 8: Check for arresting mechanism

Resistance to belt on the top run for conveying down is given by

$$W_o = C.f.L [(G_g + G_b) \cos\delta + G_{ro}] - H(G_g + G_b)$$

$$W_o = 2.2 \times 0.02 \times 50 [(56.92 + 9) \cos 12 + 16.4] - 10.4 (56.92 + 9)$$

$$W_o = -507.63 \text{ kgf}$$

Negative sign shows that under power failure the material flow reverses, so there is a need of arresting mechanism.

Step 9: Tension take up unit

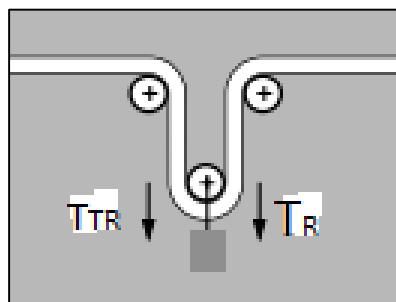


Fig. 3.40 tension take up system

The tension take-up unit is required to maintain the tension in belt. The simplest form of it is gravity loaded tension take-up unit.

To determine the tensioning weight required (T_R) with 180 degree arc of contact, using the relation,

$$T_R = 2T_2 - T_{TR}$$

Where,

T_2 – Slack side tension, T_{TR} – Weight of tension roller

$$T_R = 2 \times 662.5 - 50 \text{ (Assuming weight of tension roller as 50Kgf)}$$

The tensioning weight required (T_R) = 1275 kgf

Numerical 3.3. Design a belt conveyer system for the following specifications,

Material to be conveyed: Coal

Troughing angle: 20°

Capacity: 100 TPH

Lump size: 150 mm

Inclination: 15°

Centre to centre distance: 150 m

Solution:

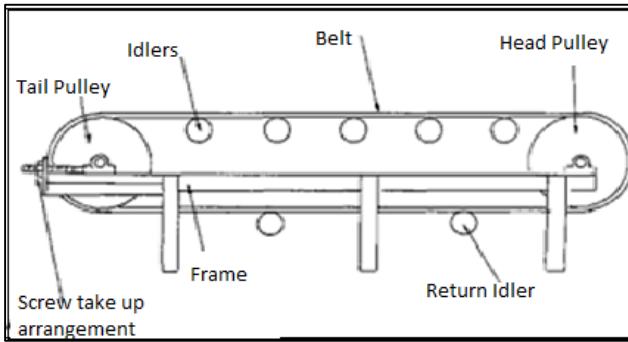


Fig. 3.41 Conveyer System

Design Capacity considering overload,

$$[Q] = \frac{Q}{0.9} = \frac{100}{0.9} = 112 \text{ TPH}$$

Material Properties: For Coal

Density (ρ) = 0.65 – 0.78 tonne/m³

Let $\rho = 0.7$ tonne/ m³

Max. Inclination, $\delta = 18^\circ$

Angle of Repose = 50° (PSG 9.22)

Step 1: Belt Width

$$B = X \cdot a_{\max} + 200 \dots \text{empirical relation}$$

Let, $X = 2$, for ungraded material

$$B = (150 \times 2) + 200 = 500 \text{ mm}$$

$$B_{\min} = 1.11 \left[\left\{ \frac{Q}{\rho_{CV}} \right\}^{0.5} + 0.05 \right] \quad (\text{PSG 9.18})$$

Let, C = 460 for 20 degree troughing idler,

Assuming, V = 2m/s

$$B_{\min} = 1.11 \left[\left\{ \frac{112}{0.7 \times 460 \times 2} \right\}^{0.5} + 0.05 \right]$$

$$B_{\min} = 0.518 \text{ m} = 518 \text{ mm}$$

Selecting standard belt width, B = 650mm (PSG 9.20)

Actual velocity,

$$0.65 = 1.11 \left[\left\{ \frac{112}{0.7 \times 460 \times V} \right\}^{0.5} + 0.05 \right]$$

$$V_{act} = 1.21 \text{ m/s}$$

Step 2: Drive Unit

Resistance of belt on top run

$$W_o = CfL [(G_g + G_b)\cos\delta + G_{ro}] + H(G_g + G_b) \quad (\text{PSG 9.18})$$

For L = 150, C = 1.6 from graph (PSG 9.18)

$$G_g = \frac{\text{Weight of Conveyed material}}{\text{Meter length}} = \frac{112}{3.6 \times 1.21} = 25.71 \text{ kgf/m}$$

G_b = Weight of belt per meter length = 9 kgf/m

G_{ro} = Weight of straight idlers due to bottom run / meter length

Let, Tube diameter = 140 mm, Bearing diameter = 25 mm

Weight of troughing idler assembly = 24.6 kgf/idler

Max spacing for idler = 1.6m... (For B= 650mm & ρ = 0.7 tonne/m³)

$$G_{ro} = \frac{24.6}{1.6} = 15.375 \text{ m} \quad \dots\dots\dots \text{PSG 9.19}$$

Let spacing for returning idler = 3m

$$G_{ru} = \frac{15.9}{3} = 5.3 \text{ kgf/m}$$

Now, H = L sinδ = 150 sin15° = 38.82m

$$W_o = CfL [(G_g + G_b)\cos\delta + G_{ro}] + H(G_g + G_b)$$

Where, C = 1.6

f = 0.02 for std. conveyors,

$$L = 150\text{m}$$

Substituting all values,

$$W_o = 1.6 \times 0.02 \times 150 [(25.71 + 9)\cos 15 + 15.375] + 38.82 (25.71 + 9)$$

$$W_o = 1582.17 \text{ kgf}$$

Resistance of belt on bottom run,

$$W_u = C.f.L \left[(G_b \cos\delta + G_{ru}) - H(G_b) \right]$$

$$W_u = 1.6 \times 0.02 \times 150 (9 \cos 15 + 5.3) - 38.82 \times 9$$

$$W_u = -282.21 \text{ kgf}$$

Effective tensions.

$$P = W_o + W_u = 1582.17 - 282.21 = 1300 \text{ kgf (PSG 9.18)}$$

$$\text{Motor power} = P \times V = 1300 \times 1.21 = 15730 \text{ W}$$

Selecting Standard motor of 18.5 kW (PSG 5.124)

Step 3: Force Analysis

Using relations,

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Let $\mu = 0.25$, $\theta = 200^\circ$

Also

$$P \equiv (T_1 - T_2) V$$

$$(T_1 = T_2) \times 1.63 \equiv 1850 \text{ kgf/m/s}$$

$$T_1 - T_2 \equiv 1134.9 \quad \text{.....(2)}$$

Solving, $T_1 = 1951.42 \text{ kgf} = 19514.2 \text{ N}$, and $T_2 = 816.52 \text{ kgf} = 8165.2 \text{ N}$

Step 4: traction element

No of plies.

$$i = \frac{T_1}{Bf} \quad (\text{PSG 9.18})$$

Material Designation: For density $\rho = 0.7$ tonne/ m^3 and belt width 650mm

Selecting 32 OZ

(Table 4, PSG 9.21, $i_{\max} = 6$, $i_{\min} = 4$)

For 32 OZ Belt and Vulcanized & joint gravity take up,

Working tension, Kgf/mm width per ply, $f = 0.62$ (Table 5, PSG 9.21)

No of plies,

$$i = \frac{T_1}{Bf}$$

$$i = \frac{1951.42}{650 \times 0.62} = 4.84 \approx 5$$

$$i = 5$$

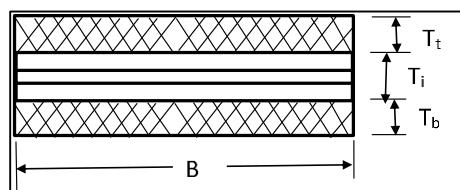


Fig. 3.42 Traction element

Proportions of traction element is as shown in figure.

Thickness of ply = $i \times$ thickness of 1 ply = $5 \times 2 = 10$ mm

Assuming, Top thickness, $t_t = 4$ mm

Bottom thickness, $t_b = 2$ mm

Total thickness,

$$T = 10 + 4 + 2 = 16 \text{ mm}$$

Step 5: Idler Design

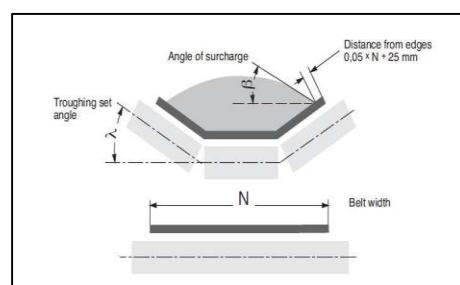


Fig. 3.43 Troughing idler

Step 5.1 Troughing Idler

Assuming 50 mm margin on both sides,

$$B = 3L - 100$$

$$650 = 3L - 100$$

$$L = 250 \text{ mm}$$

Diameter of roller = $\varphi 140 \times 250 \text{ mm}$

$$\text{Total no. of troughing idlers} = \frac{L}{\text{Spacing}} = \frac{150}{1.6} = 94$$

Total idlers = $100 \times 3 = 300$

For returning idlers,

$$L = B + \text{margin} = 650 + 100 = 750 \text{ mm}$$

Step 5.2 Returning Idler

Dimensions of returning idlers = $\varphi 140 \times 750 \text{ mm}$

$$\text{Total no. of returning idlers} = \frac{150}{3} = 50$$

Total weight acting on troughing idler = $(G_g + G_b + G_{ro}) \times \text{spacing}$

$$W' = (25.71 + 9 + 15.375) \times 1.6 = 80.136 \text{ kgf}$$

Middle idlers generally take 70% of total load,

$$W = 0.7 \times 80.136 = 56.09 \text{ kgf} = 561 \text{ N}$$

$$\text{UDL, } W_o = (25.71 + 9 + 15.375) = 50.085 \text{ kgf/m} = 0.5 \text{ N/mm}$$

$L_1 = 250 \text{ mm}$, $L_2 = 150 \text{ mm}$ (Assuming margin for bearing block)

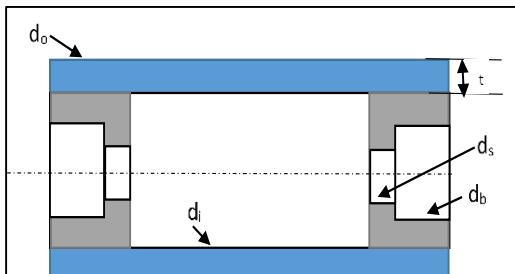


Fig. 3.44 Idler cross section

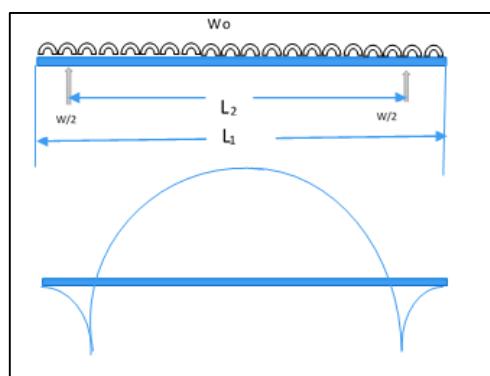


Fig. 3.45 Force and BMD for idler

Calculation for maximum bending moment at the centre,

$$BM_{max} = W_o \times \frac{L_1^2}{8} - \frac{W}{2} \times \frac{L_2}{2} = 0.5 \times \frac{250^2}{8} - \frac{561}{2} \times \frac{150}{2}$$

$$BM_{max} = \text{mod}(-17131.25) = 17131.25 \text{ N.mm}$$

Assuming thickness of idler as 2 mm, $d_o=140$ mm, $d_i=136$ mm,

Let, d_s = Axle diameter = 25mm,

$$(\sigma_b) = \frac{M}{Z} = \frac{M \times 32}{\pi \times \left(\frac{d_o^4 - d_i^4}{d_o} \right)} = \frac{17131.25 \times 32}{\pi \times \left(\frac{140^4 - 136^4}{140} \right)} = 0.5808 \text{ N/mm}^2,$$

$(\sigma_b) < [\sigma_b]$, Hence, design is safe.

Step 5.3 Design of bearing for idler

Bearing inner diameter = 25mm (Assumed)

Forces acting on bearing are,

$$F_r = \frac{W}{2} = \frac{561}{2} = 280 \text{ N}, \quad F_a = 0$$

Assuming working at room temp, nature of load as medium shock and life in hours as 15000Hrs,

Equivalent load,

$$P_{eq} = (X \cdot V \cdot F_r + Y \cdot F_a) S$$

Where, $X=1$, $V=1.2$, $S=1.2$,

$$P_{eq} = (1.2 \times 280 \times 1.2) = 403.2 \text{ N}$$

$$N = \frac{\nu \times 60}{\pi d_o} = \frac{1.21 \times 60}{\pi \times 0.140} = 165.07 \text{ rpm}$$

Assuming life of bearing as 15000 Hrs,

$$L_{mr} = \frac{L_{hr} \times N \times 60}{10^6} = \frac{15000 \times 165.07 \times 60}{10^6} = 148.56 \text{ mr}$$

Also $k=3$ for Ball bearing,

Dynamic load carrying capacity,

$$C = P_{eq} (L_{mr})^{\frac{1}{k}} = 403.2 * (148.56)^{(1/3)}$$

$$C = 2135.44 \text{ N}, C = 213.5 \text{ Kgf},$$

Selecting Deep Groove Ball Bearing, **No. 6005** with $d=25$ mm, $D=47$ mm, width =12mm and $C=780$ Kgf

Step 5.4 Design axle for idler

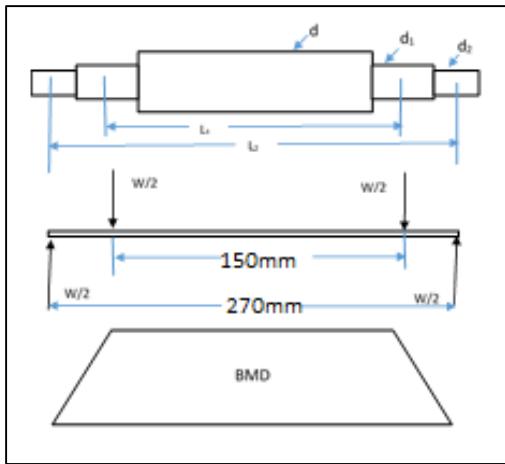


Fig. 3.47 Idler Shaft, force and BMD

Let the material for axle be C-25, with $[\sigma_b] = 60 \text{ N/mm}^2$, $[\sigma_{cr}] = 90 \text{ N/mm}^2$

Axle diameter, $d = 25\text{mm}$, $d_1 = \text{bearing inner diameter} = 25\text{mm}$,

$d_2 = 20\text{mm}$ (assumed)

$L_1 = 150\text{mm}$ (from idler design)

$L_2 = \text{Rigid support span} = \text{Idler length} + \text{margin} + \text{frame width}$

$L_2 = 250 + 10 + 10 = 270\text{mm}$ (Assuming frame thickness = 10mm)

Load acting $W/2 = 280 \text{ N}$

$$\text{Checking for bending failure, } (\sigma_b) = \frac{BM \max}{Z} = \frac{\left(\frac{W}{2}\right) \times \frac{L_2 - L_1}{2}}{\frac{\pi * d^3}{32}} = \frac{280 \times 60}{2650.7} = 6.338 \frac{N}{mm^2}$$

$(\sigma_b) < [\sigma_b]$ Hence, safe

Checking for crushing failure,

$$(\sigma_{cr}) = \frac{W/2}{d_2 \times \text{Frame thickness}} = \frac{280}{20 \times 10} = 1.4 \frac{N}{mm^2} < [\sigma_{cr}] , \text{ Hence, safe.}$$

Step 6: Pulley Design

$$D_1 = k_1 \cdot k_2 \cdot i$$

Let $k_1 = 1.4$ (factor for pulley material),

$k_2 = 100$, $\theta > 180^\circ$, $i = 5$

Diameter of pulley = $1.4 \times 100 \times 5 = 700 \text{ mm}$

$$V = \frac{\pi D N}{60}$$

$$N = \frac{60 \times V}{\pi D} = \frac{60 \times 1.21}{\pi \times 0.7} = 33 \text{ rpm}$$

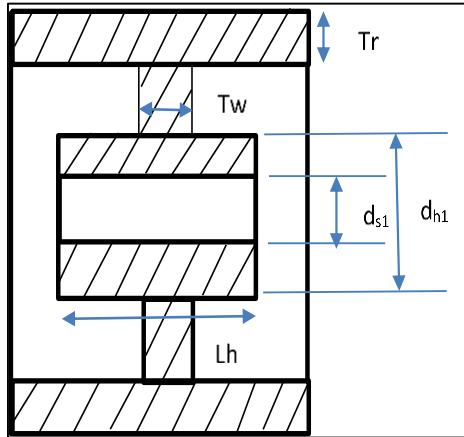


Fig. 3.48 Section of pulley

Assuming, $t_w=10\text{mm}$, $t_r = 6 \text{ to } 8 \text{ mm}$

$$B_1 = B + \text{margin} = 650 + 100 = 750\text{mm}$$

Shaft 6.1 Shaft Diameter

Let's design Shaft for pulley,

Let material for shaft be C-25/C30, Permissible stress $[\tau] = 30 \text{ N/ mm}^2$

$$\text{Torque } T = \frac{P \times 60}{2\pi N} = \frac{18.5 \times 1000 \times 60}{2\pi \times 33} = 5.354 \times 10^6 \text{ N.mm}$$

Let bearing span,

$$L_{b1} = 1.25 \times B_1 = 1.25 \times 750 = 937.5 \approx 938 \text{ mm}$$

Total load,

$$F = T_1 + T_2 = 19514.2 + 8165.2 = 27679.4 \text{ N}$$

Total load acting,

$$F = 27.68 \text{ kN}$$

Bending Moment,

$$M = F \cdot L_{b1}/4 = (27679.4 \times 938) / 4$$

$$M = 6.5 \times 10^6 \text{ N.mm}$$

$$T_{eq} = \sqrt{T^2 + M^2} = \sqrt{(6.5 \times 10^6)^2 + (5.35 \times 10^6)^2}$$

$$T_{eq} = 8.422 \times 10^6 \text{ N.mm}$$

$$[\tau] = T_{eq} / (\pi/16 \cdot d_s^3)$$

$$30 = (8.422 \times 10^6 \times 16) / (\pi d_s^3)$$

Solving $d_s = 112.65$ mm,

To account for key effect, $d_s = 1.1 \times 112.65 = 123.9$ mm, Selecting $d_s = 130$ mm

Diameter of hub = Length of hub = $1.5 d_s = 195$ mm

Step 6.2 Bearing Selection of pulley

Loads acting on the bearing are, Radial load F_r

$$F_r = W/2 = (T_1 + T_2)/2 = (27679.4)/2 = 13839.7\text{N}$$

Axial Load, $F_a = 0$

Equivalent Load is given by,

$$P_{eq} = (XVF_r + YF_a) S \text{ (PSG 4.4)}$$

Assuming Race rotation factor $V = 1$ (inner race rotates)

Radial Load factor $X = 1.2$ and Service factor $S = 1.2$

$$P_{eq} = 1 \times 1.2 \times 13.84 \times 1.2 = 19.93 \text{kN}$$

Assuming life of bearing as $L_{hr} = 8000$ hrs

$$L_{mr} = \frac{L_h \times N \times 60}{10^6} = \frac{8000 \times 33 \times 60}{10^6} = 14.4 \text{ millions of revolutions,}$$

Dynamic Load Capacity required,

$$C = (L_{mr})^{1/k} \times P_{eq} = (14.4)^{1/3} \times 19.93 = 48.487 \text{kN} = 4848.7 \text{ kgf}$$

Selecting suitable bearings as SKF 6026,

$d = 130$ mm, $D=200$ mm, $B = 33$ mm, $C= 8300$ kgf

Step 7: Snub Pulley

Need: To improve lap angle

Location: Slack side of head pulley

Size of snub pulley = $0.6 \times$ Diameter of head pulley = $0.6 \times 700 = 420$ mm

Step 8: Checking for arresting mechanism

Resistance to belt on the top run for conveying down is given by,

$$W_o = CPL [(G_g + G_b) \cos\delta + G_{ro}] - H(G_g + G_b)$$

$$W_o = 1.6 \times 0.02 \times 150 [(25.71 + 9) \cos 15 + 15.375] - 38.82[25.71 + 9]$$

$$W_o = -1112.71 \text{ kgf}$$

The value obtained is negative, an arresting mechanism will be required for the setup.

Step 9: Tension take up unit

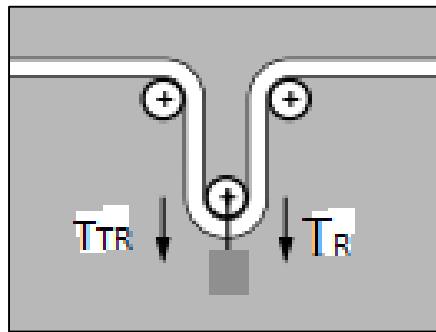


Fig. 3.48 Tension take up system

The tension take-up unit is required to maintain the tension in belt. The simplest form of it is gravity loaded tension take-up unit. To determine the tensioning weight required (T_R) with 180 degree arc of contact, using the relation,

$$T_R = 2T_2 - T_{TR}$$

Where, T_2 – Slack side tension, T_{TR} – Weight of tension roller

$$T_R = 2 \times 816.52 - 50 \quad (\text{Assuming weight of tension roller as } 60\text{Kgf})$$

The tensioning weight required (T_R) = 1583Kgf

References:

10. ‘Material Handling Equipment’ by Rudenko - M.I.R. publishers, Moscow.
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12. Bhandari V.B., “Design of Machine Elements”, Tata McGraw Hill Pub.Co.Ltd.
13. “Design Data Book ”, P.S.G. College of Technology, Coimbatore

Module 4

Design of Internal Combustion Engines

4.1 Introduction

An internal combustion engine (ICE) is a heat engine where the combustion of a fuel occurs with an oxidizer (usually air) in a combustion chamber that is an integral part of the working fluid flow circuit. In an internal combustion engine the expansion of the high-temperature and high pressure gases produced by combustion apply direct force to some component of the engine. The force is applied typically to pistons, turbine blades, rotor or a nozzle. This force moves the component over a distance, transforming chemical energy into useful mechanical energy. The first commercially successful internal combustion engine was created by Étienne Lenoir around 1859 and the first modern internal combustion engine was created in 1864 by Siegfried Marcus. Working fluids can be air, hot water, pressurized water or even liquid sodium, heated in a boiler. ICEs are usually powered by energy-dense fuels such as gasoline or diesel, liquids derived from fossil fuels. While there are many stationary applications, most ICEs are used in mobile applications and are the dominant power supply for vehicles such as cars, aircraft, and boats.

4.2 Classification of IC Engines

There are different types of IC engines that can be classified on the following basis:

4.2.1 According to the Thermodynamic cycle

- Otto cycle engine or constant volume heat supplied cycle.
- Diesel cycle engine or constant pressure heat supplied cycle.
- Dual combustion cycle Engine.

4.2.2 According to Fuel Used

- Petrol Engine
- Diesel engine
- Gas engine

4.2.3 According to the number of strokes

- Two stroke Engines
- Four stroke Engines

4.2.4 According to the method of Ignition

- SI engines (Spark Ignition)
- CI Engines (Compression Ignition)

4.2.5 According to the number of cylinders

- Single cylinder
- Multi-cylinder

4.2.7 According to the cooling system

- Air cooled
- Water cooled
- Oil cooled

4.3 Difference between SI And CI Engines

1. **Basic Cycle:** SI engine works on Otto cycle whereas CI engine works on Diesel cycle.
2. **Fuel Used:** In SI engines Fuel used is petrol also called as Gasoline and in CI engines it is Diesel.
3. **Introduction of fuel:** In SI engines fuel and Air Introduced as a gaseous mixture in the suction stroke. A Carburetor is necessary to provide Mixture throttle control and quantity of mixture introduced. In CI engines Fuel is directly injected in combustion chamber at high pressure at the end of compression stroke, Carburetor is eliminated but fuel pump and injector is necessary.
4. **Ignition system:** In SI engines Spark plug is required whereas in CI engines Ignition takes place due to high temperature rise because of compression.
5. **Compression ratio:** 6 to 10 for SI engines where as it is 14 to 22 for CI engines.
6. **Speed:** In SI engines one get higher RPM due to less weight, in CI engines are low speed engine.
7. **Thermal Efficiency:** Thermal Efficiency of SI is lower than that of CI engines.
8. **Weight:** SI engines are lighter whereas CI engines are heavies due to higher pressures required for compression.
9. **Noise:** Operation of SI engines is silent whereas for CI it is noisy.
10. **Initial cost:** Initial cost of SI engines is low and for CI engines it is high.
11. **Time of knocking:** Knocking takes place at the end of combustion in SI engines whereas in case of CI engines it takes place at start of combustion.
12. **Pressures Generated:** Pressures are high in case of CI engines and low in case of SI engines.

4.4 Different Parameters Of IC Engines

4.4.1 Indicator power

This is defined as the rate of work done by the gas on the piston as evaluated from an indicator diagram obtained from engine. Indicator power represents the maximum power from the engine under Ideal and perfect conditions. I.P is calculated on the basis of engine size displacement, operational speed and pressure developed theoretically in the cylinder. I.P will always be more than B.P.

4.4.2 Brake power

Brake power is known as Indicated power output. B.P is the power developed inside the engine cylinder by the combustion of charge. It is also called Brake power because Brake is used to slow down the shaft inside a dynamometer. Brake horsepower is also used to compare the engine

characteristics. Automotive manufacturers show brake horsepower to show the differences between the engines.

4.4.3 Frictional power

It is the horsepower being used to overcome the internal friction. Anytime two objects touch each other while moving, friction is produced. Friction has the tendency to slow down the engine. Frictional power is the difference between input power and the brake power.

4.4.4 Brake Thermal efficiency

The power output of the engine is obtained by the chemical energy of the fuel supplied. The overall efficiency is given by the Brake thermal efficiency.

$$\eta_{bth} = \frac{\text{Brake power}}{\text{input fuel energy}}$$

4.4.5 Mechanical Efficiency

It is the ratio of brake power to indicated power. It is also defined as ratio of brake thermal efficiency to the indicated thermal efficiency. For petrol engine mechanical efficiency is 70 – 90%, for diesel engine 70 – 85%.

4.4.6 Volumetric efficiency

The power output of an IC engine depends directly upon the charge which is directly introduced in the cylinder. This is referred to as breathing capacity of an engine and is expressed quantitatively as volumetric efficiency. It is defined as ratio of volume of air induced at free air conditions to the swept volume of cylinder.

$$\eta_v = v/v_s$$

The range of volumetric efficiency at full throttle for SI engine is 80 to 85 % whereas for CI engine it is 85 to 90%.

Volumetric efficiency of an engine is affected by various variables such as:

- a) Compression ratio
- b) Valve timing
- c) Induction and port design
- d) Mixture strength
- e) Latent heat of evaporation of fuel
- f) heating of the induced charge
- g) Cylinder temperature
- h) Atmospheric conditions

4.4.7 Specific Fuel Consumption (SFC)

SFC is the mass of Fuel consumed per kW developed per hour and is the criterion of economic power production.

$$S.F.C. = \frac{\text{Fuel consumption per unit time}}{\text{Power}}$$

The specific fuel consumptions on the basis of Brake power and indicated power are denoted by BSFC and ISFC respectively.

4.4.8 Mean effective pressure

Mean effective pressure is an important parameter for comparing the performance of different engines. It is defined as the average pressure acting over piston throughout a power stroke. It is given by the following relation,

$$P_i = (P_{im} \times 60 / L \times A \times N_1 \times n_c)$$

Where,

P_i = Indicated power in kW

P_{im} = actual mean effective pressure (N/mm^2)

A = Bore Area (m^2)

L = stroke length (m)

N_1 = Rotational speed (rpm) ($N_1 = N/2$ for four stroke and $N_1 = N$ for two stroke)

n_c = No of cylinders.

4.4.9 L/D ratio

This is also known as Stroke to bore ratio. It is also denoted by S/D. This is an important parameter in classifying the size of the engine. If $d < L$ it is called under-square engine, If $d = L$ it is called square engine and If $d > L$ it is called as over square engine. Here d = the nominal inner diameter of the working cylinder is called as cylinder bore. It is denoted by the letter 'd' and is usually expressed in mm. L is the nominal distance through which piston moves between successive reversals of its direction of motion is called the stroke. It is denoted by the letter 'L' and is usually expressed in mm.

4.4.10 Mean Piston Speed (V_{pm})

The mean piston speed is the average speed of the piston in a reciprocating engine. It is a function of stroke and RPM.

$$MPS = 2 \times \text{Stroke} \times \text{rpm} / 60$$

According to MPS different classes are as below.

Low speed diesels ~8.5 m/s for marine and electric power generation applications

Medium speed diesels ~11 m/s for trains or trucks

High speed diesel ~14 m/s for automobile engines

Medium speed petrol ~16 m/s for automobile engines

High speed petrol ~20–25 m/s for sport automobile engines or motorcycles

4.5 Major components of IC Engines

Some of the important IC Engine components are as follows. Figure 4.1 shows cross-section of single cylinder SI engine with major components.

1. Valves: Piston engine valves and Control valves
2. Exhaust systems
3. Cooling systems
4. Piston
5. Propelling nozzle
6. Connecting Rod
7. Crankshaft
8. Flywheels
9. Starter systems
10. Heat shielding systems
11. Lubrication systems
12. Control systems
13. Diagnostic systems

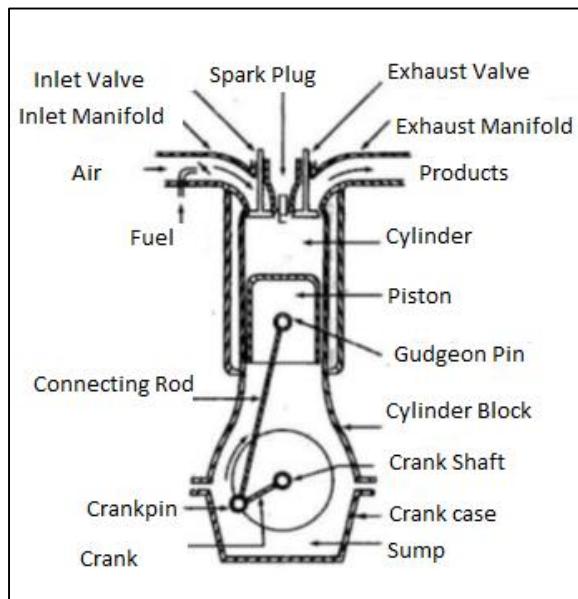


Fig. 4.1 cross-section of single cylinder SI engine

The major components and their functions are briefly discussed below.

4.5.1 Cylinder block

Cylinder block is the main supporting structure of various elements. A cylinder of a multi cylinder engine are cast as a one block known as cylinder block. The cylinder head is mounted on the

cylinder block. The cylinder head and cylinder block are provided with water jackets for water cooling or cooling fins in case of Air cooling. Cylinder head gasket is incorporated between cylinder block and cylinder head. The cylinder head is held tightly to the cylinder block with the help of bolts or studs. The bottom portion of cylinder block is called as crankcase. A cover called crankcase which becomes a sump for lubricating oil is fastened to the bottom of the crankcase. The inner surface of the cylindrical block which is machines accurately and finished is called as bore or face. Figure 4.2 shows a cylinder block for four cylinder engine.



Fig. 4.2 Cylinder block for four cylinder engine

4.5.2 Cylinder

It is a cylindrical vessel or space in which the piston makes the reciprocating motion. The varying volume created in the cylinder during operation of the engine is filled with the working fluid and subjected to different thermodynamic processes. The cylinder is supported in the cylinder block.

4.5.3 Piston

It is the cylindrical component fitted into the cylinder forming the moving boundary of the combustion system. It perfectly fits into the cylinder providing a gas tight space with the piston rings and the lubricant. It forms the first link in transmitting the gas forces to the output shaft.

4.5.4 Combustion Chamber

The space enclosed in the upper part of the cylinder by the cylinder head and the piston top during the combustion process is called the combustion chamber. The combustion of the fuel and the consequent release of thermal energy results in the building up of pressure in this part of the cylinder.

4.5.5 Inlet Manifold

The pipe which connects the intake system to the inlet valve of the engine and through which air or air fuel mixture is drawn into the cylinder is called as inlet manifold.

4.5.6 Exhaust Manifold

The pipe which connects the exhaust system to the exhaust valve of the engine and through which the products of combustion escape into the atmosphere is called as exhaust manifold.

4.5.7 Inlet and exhaust valves

Valves are commonly mushroom shaped poppet type. They are provided either on the cylinder head or on the side of the cylinder for regulating the charge coming into the cylinder and for discharging the products of combustion from the cylinder.

4.5.8 Spark Plug

It is a component to initiate the combustion process in SI engines and is usually located on the cylinder head.

4.5.9 Connecting Rod

It interconnects the piston and the crankshaft and transmits the gas forces from the piston to the crankshaft. The two ends of the connecting rod are called as small end and the big end. Small end connected to the piston by gudgeon pin and the big end is connected to the crank shaft by the crank pin.

4.5.10 Crankshaft

It converts the reciprocating motion of the piston into useful rotary motion of the output shaft. In the crankshaft of single cylinder engine there are pair of crank arms and balance weights. The balance weights are provided for static and dynamic balancing of the rotating system. The crankshaft is enclosed in crankcase.

4.5.11 Piston rings

Piston rings fitted into the slots around the piston, provides tight seal between the piston and the cylinder walls thus preventing leakages of combustion gases.

4.5.12 Gudgeon Pin

It forms the link between the small end of the connecting rod and the piston.

4.5.13 Cam shaft

The camshaft and its associated parts control the opening and closing of the two valves. The associated parts are push rods, rocker arms, valve springs and tappets. This shaft also provides the drive to the ignition system. The camshaft is driven by the crankshaft through timing gears.

4.5.14 Cams

These are made as an integral parts of the camshaft and are designed in such a way to open the valves at correct timing and to keep them open for the necessary duration.

4.5.14 Flywheel

The net torque imparted to the crankshaft during the one complete cycle of operation of the engine fluctuates causing a change in angular velocity of the shaft. In order to achieve a uniform torque and inertia mass in the form of a wheel is attached to the output shaft which is called as flywheel.

4.6 Design procedure of cylinder

Cylinder is one of the most important part of the engine in which the piston moves to and fro to develop power. Generally the engine cylinder has to withstand higher pressures more than 50 bar and higher temperatures more than 2000 degrees. Thus the materials for an IC engine cylinder should be such that they retain their strength at such high temperatures and pressures. For ordinary cylinders the material used is cast iron whereas for high duty engines materials made up of aluminum alloys are used.

Design Procedure:

- a) At first one needs to know the specification of the engine to be designed. For example whether the engine is 2 stroke or 4 stroke, whether it is petrol or diesel, air cooled or water cooled, single cylinder or multi cylinder, vertical or horizontal etc.
- b) Then one needs stroke to bore ratio, MEP, mechanical efficiency, brake power, cycles per min. Indicated power is calculated on the basis of mechanical efficiency.
- c) The standard bore diameter to be selected from the Design data book.

- d) For liner design one must first select the material and note down the stresses for that material and calculate the design stresses based on that liner thickness required can be obtained. Total stresses check is done which has pressure criteria and thermal criteria.
- e) Then for designing the cylinder head, one needs to know whether the engine is air cooled or water cooled and cover thickness is to be decided and the bolting design is to be done according to that. Then calculation for the number of bolts and loads acting on it to be done which helps to find the size of bolts.

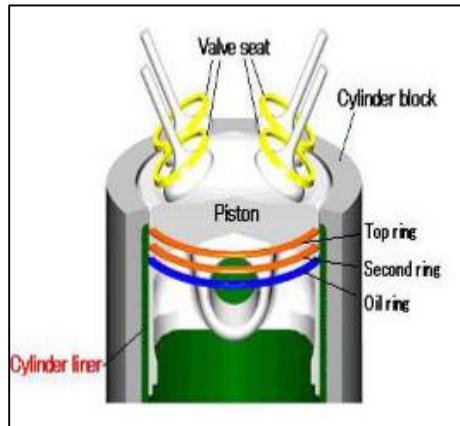


Fig. 4.3 Cross Section of the cylinder on an IC engine

4.6.1 Cylinder Liner

A cylinder liner is a cylindrical part to be fitted into an engine block to form a cylinder. It is one of the most important functional parts to make up the interior of an engine. This is also called cylinder sleeve.

4.6.1.1 Formation of sliding surface

The cylinder liner, serving as the inner wall of a cylinder, forms a sliding surface for the piston rings while retaining the lubricant within. The most important property of cylinder liners is the excellent characteristic as sliding surface and following necessary properties

- a. High anti-galling properties
- b. Less wear on the cylinder liner itself
- c. Less wear on the partner piston ring
- d. Less consumption of lubricant

The cylinder liner receives combustion heat through the piston and piston rings and transmits the heat to the coolant.

4.6.1.2 Compression gas sealing

The cylinder liner prevents the compressed gas and combustion gas from escaping outside. It is necessary that a cylinder liner which is hard to transform by high pressure and high temperature in the cylinder.

4.6.2 Liner material and its requirements

A cylinder wall in an engine is under high temperature and high pressure, with the piston and piston rings sliding at high speeds. In particular, since longer service life is required of engines for trucks and buses, cast iron cylinders that have excellent wear-resistant properties are only used for cylinder parts. Also, with the recent trend of lighter engines, materials for engine blocks have been shifting from cast iron to aluminum alloys. However, as the sliding surface for the inner cylinder, the direct sliding motion of aluminum alloys has drawbacks in deformation during operation and wear-resistance. For that reason, cast iron cylinder liners are used in most cases. So major requirements of liner material is as follows:

- a) Strength to resist the gas pressure.
- b) Good wear resistance
- c) Strength to resist thermal stresses
- d) Corrosion resistance
- e) Good Surface structure
- f) It should be symmetrical in shape to avoid unequal deflection and expansion
- g) No distortion of the inner surface.

4.6.3 Materials

It should be strong, hard, and corrosion resistant and produce a good bearing surface.

- 1. Good Grade grey CI with homogenous and closed structure.
- 2. Nickel CI and Nickel Chromium CI
- 3. Nickel Chromium Cast steel with molybdenum.
- 4. Surface treatment for hardening the liner surface must be done.

4.6.4 Liner distortion

Recent emission norms have forced the engine manufacturers to reduce the Particulate Matter (PM) emissions along with other emissions substantially. In order to achieve low PM emissions the lubrication oil consumption need to be controlled by optimizing piston group design with low liner bore distortion. Bore Distortion is the deviation of actual profile from perfect circular profile at any plane perpendicular to axis of cylinder. Liner bore distortion in engine causes number of problems like deterioration of piston ring performance, liner-ring conformability issues, high

lubricating oil consumption and emissions. Besides traditional prediction of stresses, fatigue life and verification by mechanical testing, the prediction of liner bore distortion is one of most important topics in crankcase structure development. Low bore distortion opens up potential for optimized piston group design.

4.6.5 Stresses on liners

Basically two types of stresses occurs in case of liners

1. Mechanical stresses
2. Thermal stresses

4.6.5.1 Mechanical Stresses

In super charged engines maximum firing pressure is about 90 to 100 bars and for non-super charged it's about 75-85 bar. This pressure produces following two types of stresses:

1. Circumferential (hoop) stress
2. Longitudinal stress

But hoop stress is twice than that of longitudinal stress so only hoop stress is to be considered.

4.6.5.2 Thermal Stresses

Resistance to heat flow through liner material produces temperature gradient across the liner hence inner wall expands more relative to the outer wall. Thicker liner will increase the temperature gradient and hence increasing the thermal stresses but thicker liner has good resistance to mechanical stresses this is the reason why liner design becomes complex. Also liner inner surface temperature must be sufficiently low to retain oil film and also high enough to avoid acid dew.

4.7 Connecting rod

The connecting rod is the intermediate member between the piston and the crank shaft. Its primary function is to transfer push and pull from the piston pin to the crank pin and thus converts the reciprocating motion of the piston into rotary motion of the crank. The connecting rod is under tremendous stress from the reciprocating load represented by the piston, actually stretching and being compressed with every rotation, and the load increases to the third power with increasing engine speed.

4.7.1 Material of connecting rod

Steel is normally used for construction of automobile connecting rods because of its strength, durability, and lower cost. However, steel with its high mass density exerts excessive stresses on

the crankshaft of a high speed engine. This in turn requires a heavier crankshaft for carrying the loads and, therefore, the maximum RPM of the engine is limited. Additionally, higher inertia loads, such as those caused by steel connecting rods and heavier crankshafts reduces the acceleration or declaration rates of engine speed. The automobile engine connecting rod is a high volume production, critical component. It connects reciprocating piston to rotating crankshaft, transmitting the thrust of the piston to the crankshaft. Every vehicle that uses an internal combustion engine requires at least one connecting rod depending upon the number of cylinders in the engine. With steel forging, the material is inexpensive and the rough part manufacturing process is cost effective.

Commonly used material are

- Mild steel (SAE 1000 – SAE 1040) with ultimate tensile strength of (550-670 MPa)
- Ni,Cr,Mo steel (SAE 4340) with $S_{ut} = (940 - 1200 \text{ MPa})$
- Aluminum alloy (SAE 39) with $S_{ut} = (200-300 \text{ MPa})$

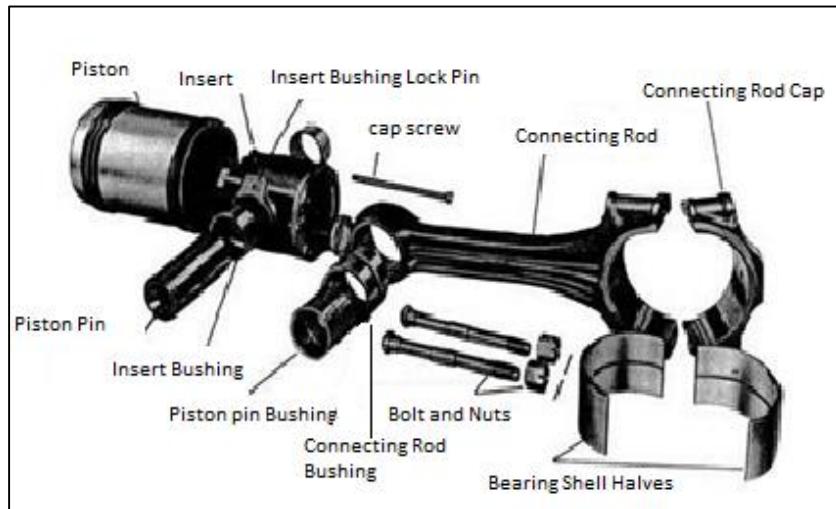


Fig.4.4 Connecting Rod Assembly

4.7.2 Manufacturing methods of connecting rod

- Wrought forged process
- Fiberglass spray lay-up process
- Hand layup method for composite material

4.7.3 Connecting rod force analysis

The stresses set up in connecting rod are due to following forces:

Gas force: The direct force due to gas pressure acting on the piston is transferred to the connecting rod which is given by the following formulae.

$$F_{\text{gas}} = P_{\max} \times \text{area of cross section}$$

4.7.4 Inertia force due to reciprocating parts

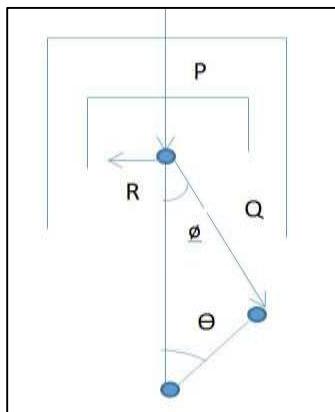


Fig. 4.5 Forces on connecting rod

The inertia force due to reciprocating parts is calculated by the following relation:

$$F_{\text{inertia}} = -mr\omega^2 (\cos\theta + \cos 2\theta / n)$$

The net force is given by $F = F_{\text{gas}} + F_{\text{inertia}}$,

The direct force acting on connecting rod is shown as below.

$$Q = P / \cos \phi$$

$$\sin \phi = \sin \theta / n$$

The small end of connecting rod has pure translation while the big end has pure rotation. The intermediate points move in elliptical orbit. Lateral oscillation of rod results in bending forces along the length of rod, this being due to linearly varying centrifugal forces of rotating mass. The forces act opposite to each other and the action is termed as whipping and the stress induced is whipping stress.

Connecting rod bearings provide rotating motion of the crank pin within the connecting rod, which transmits cycling loads applied to the piston. Connecting rod bearings are mounted in the Big end of the connecting rod. A bearing consists of two parts (commonly interchangeable).

4.7.4 Small end bushes

Small end bushes provide relative motion of the piston relatively to the connecting rod joined to the piston by the piston pin (gudgeon pin). End bushes are mounted in the small end of the connecting rod. Small end bushes are cycling loaded by the piston pushed by the alternating pressure of the combustion gases.

The dimension of small end can be found by empirical relation:

1. Inner diameter: $d_{si} = (1.1 - 1.25) d_{ps}$
2. Outer diameter: $d_{so} = (1.25 - 1.65) d_{ps}$
3. Length of small end: $l_s = (0.3 - 0.45) D$

Where,

d_{ps} = diameter of piston pin, D = diameter of bore

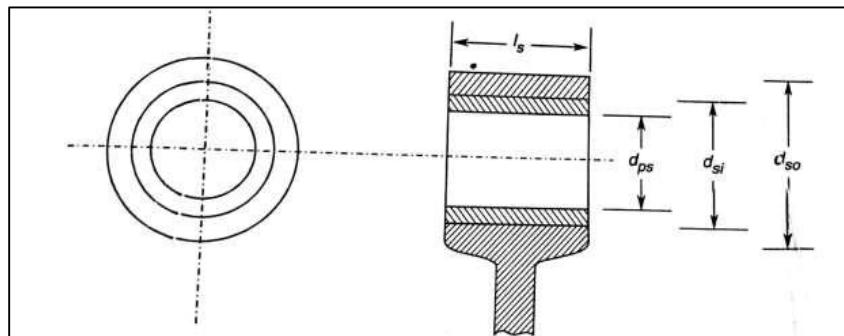


Fig. 4.6 Small end of connecting rod

Small end is designed and checked for following conditions:

1. Bearing failure of pin: The limiting bearing pressure is calculated by equation

$$P_{br} = \frac{P}{l_c \times d_{pc}} \leq P_{allowable}$$

2. Bending of pin: bending due to gas force should be checked as follows

$$\sigma_b = \frac{M}{Z} \leq \sigma_{allowable}$$

3. Tensile load:

$$\sigma_t = \frac{F_{inertia}}{(d_{so} - d_{si})l_s} \leq \sigma_{allowable}$$

4.7.7 Big end

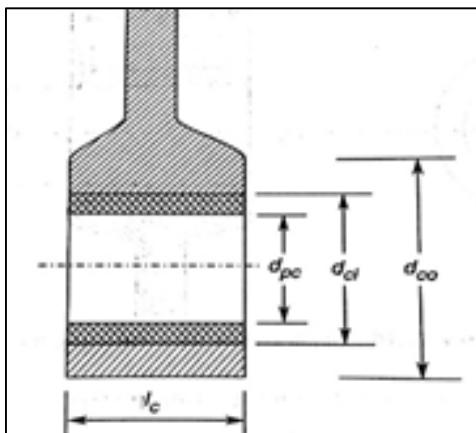


Fig. 4.7 Big end of connecting rod

The dimension for the big end can be found empirically:

$$\text{Crank pin Diameter: } d_{pc} = (0.55 - 0.75) D$$

$$\text{Length of big end: } L_c = (0.45 - 1) d_{pc}$$

$$\text{Bush thickness: } t_{bush} = (0.03 - 0.1) d_{pc}$$

$$\text{Distance between centre of bolts} = (1.3 - 1.75) d_{pc}$$

The big end of connecting rod is checked for bearing and bending considerations.

Bearing consideration:

At the crank pin end the bearing pressure varies between 7.5 and 15 N/mm² depending upon bearing material and lubrication.

$$P_{br} = \frac{P}{l_c \times d_{pc}}$$

4.7.8 Bolts for big end

The big end of connecting rod is usually made of two halves which are fastened by bolts. Maximum force on the bolt is the inertia force at TDC of suction stroke. The initial tightening force must be 2 – 3 times the inertia force per bolt. Thus total force on bolt

$$P_{bolt} = F_{initial} + K \times \frac{P_{inertia}}{N}$$

Where, K = Gasket Factor and N = No. of Bolts

4.7.9 Cap of the big end

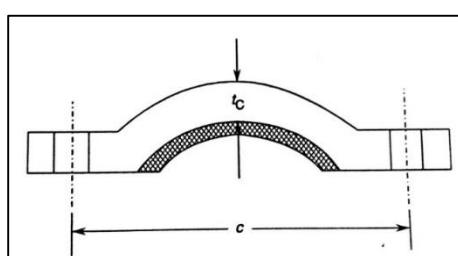


Fig. 4.8 Cap of big end

The cap of big end is designed as a beam supported at the bolt centre and may be assumed to be loaded with concentrated load. The thickness of cap is computed by relation

$$t_c = \left(\frac{P_{inertia} \times C}{l_c \times \sigma_y} \right)$$

Where, C – Distance between Bolts centres

4.7.10 Reciprocating weight

The first step in the process of designing a connecting rod bolt is to determine the load that it must carry. This is accomplished by calculating the dynamic force caused by the oscillating piston and connecting rod. This force is determined from the classical concept that force equals mass times acceleration. The mass includes the mass of the piston plus a portion of the mass of the rod. This mass undergoes oscillating motion as the crankshaft rotates. The resulting acceleration, which is at its maximum value when the piston is at top dead centre and bottom dead centre, is proportional to the stroke and the square of the engine speed. The oscillating force is sometimes called the reciprocating weight. Its numerical value is proportional to:

$$\left(\text{Piston Weight} + \frac{\text{Rod Weight}}{3} \right) \times \text{Stroke} \times \text{rpm}^2$$

It is seen that the design load, the reciprocating weight, depends on the square of the RPM speed. This means that if the speed is doubled, for example, the design load is increased by a factor of 4. This relationship is shown graphically below for one particular rod and piston.

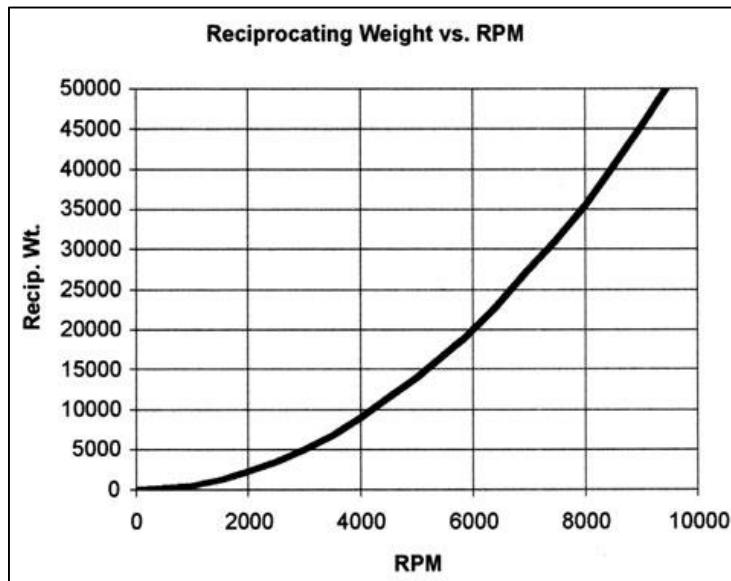


Fig. 4.9 Graph of reciprocating weight Vs. rpm

A typical value for this reciprocating weight is in the vicinity of 20,000 lbs. For purposes of bolt design, a “rule of thumb” is to size the bolts and select the material for this application such that each of the 2 rod bolts has a strength of approximately 20,000 lbs. (corresponding to the total reciprocating weight). This essentially builds in a nominal safety factor of 2. The stress is calculated according to the following formula.

$$\left(\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{\text{Reciprocating Weight}}{\frac{\pi}{4} \times d^2} \right)$$

This formula shows that the thread size can be smaller if a stronger material is used or for a given thread size, a stronger material will permit a greater reciprocating weight.

4.7.11 Buckling of connecting rod



Fig. 4.10 Buckled connecting rod

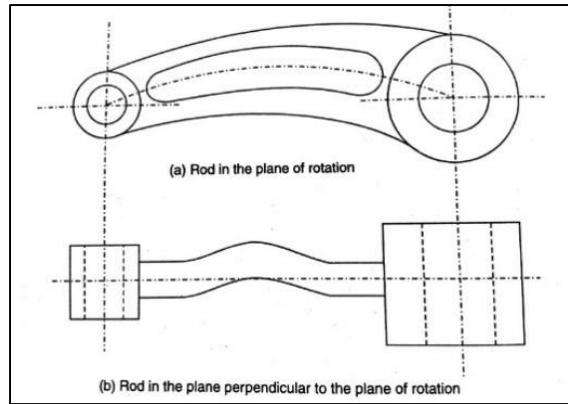


Fig. 4.11 Crippling of Connecting rod

Buckling is a mode of failure characterized by an unstable lateral material deflection caused by compressive stresses. When designing a connecting rod, one needs to pay attention to the buckling strength of the rod. Light weight and low fuel consumption engines require optimizing the connecting rod section geometries to be progressively changing from the small end to the big end. The buckling of the connecting rod in the plane of motion and the connecting rod in a plane perpendicular to the plane of motion is shown in Fig. 4.11. In the plane of motion, the ends of connecting rod are hinged in the crank pin and piston pin. Therefore, for buckling about the XX-axis, the end fixity coefficient (n) is one. In the plane perpendicular to the plane of motion the ends of the connecting rod are fixed due to constraining effect of bearings at the crank pin and piston pin. Therefore, for buckling about the YY-axis, the end fixity coefficient (n) is four. The connecting rod is four times stronger for buckling about the YY-axis as compared to buckling about the XX-axis. If a connecting rod is designed in such a way that it is equally resistant to buckling in either plane then $4I_{yy} = I_{xx}$.

The crippling stress induced in connecting Rod can be computed by Rankine formula

$$\sigma_{cr} = \frac{P}{A} \left[1 + a \left(\frac{l}{k} \right)^2 \right]$$

Where, a is the Rankine constant (a=1/ 7500 for steel, 1/1600 for CI, 1/1700 for Aluminium.)

4.7.12 Connecting rod cross section

The axial stresses are produced due to cylinder gas pressure (compressive only) and the inertia force rising in account of reciprocating action (both tensile as well as compressive), whereas bending stresses are caused due to the centrifugal effects. It consists of a long shank, a small end and a big end. The cross-section of the shank may be rectangular, circular, tubular, I-section or H-section. Generally circular section is used for low speed engines while I-section is preferred for high speed engines.

I section is used for adequate strength, stiffness and minimum weight of the connecting rod. Twice every revolution, the connecting rod comes to a stop. This exerts a massive pressure on the big-end bearing and on the little-end bearing. By reducing the mass of the rod, this pressure is reduced. The typical "I" section reduces the mass without weakening the rod.

When gas force acts on connecting rod, it behaves like a strut with both ends hinged in plane of rotation and both ends fixed in the perpendicular plane. The connecting rod is equally strong if satisfies the condition:

$$I_{xx} = 4 \cdot I_{yy} \text{ where, } I - \text{moment of inertia}$$

In the plane of rotation connecting rod is considered as a strut or column subjected to maximum compressive load Q_{max} and pin joints at both the ends. Hence equivalent length L_e is equal to length between the supports in the plane perpendicular to the plane of rotation. If the ends are fixed the equivalent length L_e is $L/2$.

As rotation is very important, the design of connecting rod is based on buckling.

By Rankine Formula,

$$F_1 = \frac{A\sigma_c}{1 + \frac{1}{a}(\frac{L_{exx}}{K_{xx}})^2} \text{ for plane of rotation}$$

And $F_2 = \frac{A\sigma_c}{1 + \frac{1}{a}(\frac{L_{eyy}}{K_{yy}})^2} \text{ for plane perpendicular to plane of rotation}$

σ_c and $\frac{1}{a}$ are constant for different material,

For safe stress value, $F_1 \leq F_2$

$$\frac{A * \sigma_c}{1 + \frac{1}{a}(\frac{L_{exx}}{K_{xx}})^2} \leq \frac{A * \sigma_c}{1 + \frac{1}{a}(\frac{L_{eyy}}{K_{yy}})^2}$$

$$1 + \frac{1}{a} \left(\frac{L_{exx}}{K_{xx}} \right)^2 \geq 1 + \frac{1}{a} \left(\frac{L_{eyy}}{K_{yy}} \right)^2$$

$$\left(\frac{L_{exx}}{K_{xx}} \right)^2 \geq \left(\frac{L_{eyy}}{K_{yy}} \right)^2, \text{ now } L_{exx} \text{ is 1 and } L_{eyy} \text{ is } 1/2 \text{ hence,}$$

$$\frac{l^2}{K_{xx}^2} \geq \frac{l^2}{4K_{yy}^2}, \text{ now } I_{xx} = A \cdot K_{xx}^2 \text{ and } I_{yy} = A \cdot K_{yy}^2$$

$$K_{xx}^2 \leq 4 K_{yy}^2$$

$$\frac{I_{xx}}{A} \leq \frac{I_{yy}}{A}$$

$$I_{xx} \leq 4 I_{yy}$$

This relation is satisfied by I section shown in figure 4.12.

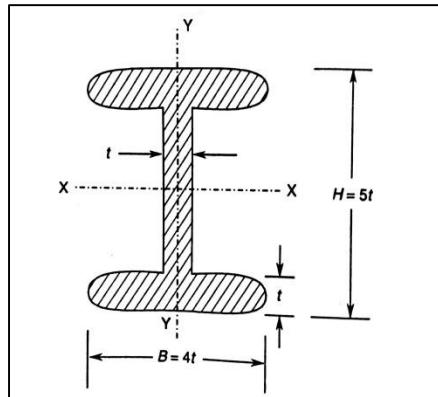


Fig. 4.12 Section of connecting rod

Hence it is considered to be most economical. If circular or other symmetrical section is used i.e. if $I_{xx} = I_{yy}$, more material is wasted and section does not become economical.

4.8 Piston



Fig. 4.13 Piston and connecting rod

A piston is a component of reciprocating engines, reciprocating pumps, gas compressors and pneumatic cylinders, among other similar mechanisms. It is the moving component that is contained by a cylinder and is made gas-tight by piston rings. In an engine, its purpose is to transfer

force from expanding gas in the cylinder to the crankshaft via a piston rod and/or connecting rod. In a pump, the function is reversed and force is transferred from the crankshaft to the piston for the purpose of compressing or ejecting the fluid in the cylinder. In some engines, the piston also acts as a valve by covering and uncovering ports in the cylinder wall.

4.8.1 Piston ring

A ring groove is a recessed area located around the perimeter of the piston that is used to retain a piston ring. Ring lands are the two parallel surfaces of the ring groove which function as the sealing surface for the piston ring. A piston ring is an expandable split ring used to provide a seal between the piston and the cylinder wall. Piston rings are commonly made from cast iron. Cast iron retains the integrity of its original shape under heat, load, and other dynamic forces. Piston rings seal the combustion chamber, conduct heat from the piston to the cylinder wall, and return oil to the crankcase. Piston ring size and configuration vary depending on engine design and cylinder material.

Piston rings commonly used on small engines include the compression ring, wiper ring, and oil ring. A compression ring is the piston ring located in the ring groove closest to the piston head. The compression ring seals the combustion chamber from any leakage during the combustion process. When the air-fuel mixture is ignited, pressure from combustion gases is applied to the piston head, forcing the piston toward the crankshaft. The pressurized gases travel through the gap between the cylinder wall and the piston and into the piston ring groove. Combustion gas pressure forces the piston ring against the cylinder wall to form a seal. Pressure applied to the piston ring is approximately proportional to the combustion gas pressure. A wiper ring is the piston ring with a tapered face located in the ring groove between the compression ring and the oil ring. The wiper ring is used to further seal the combustion chamber and to wipe the cylinder wall clean of excess oil. Combustion gases that pass by the compression ring are stopped by the wiper ring.

An oil ring is the piston ring located in the ring groove closest to the crankcase. The oil ring is used to wipe excess oil from the cylinder wall during piston movement. Excess oil is returned through ring openings to the oil reservoir in the engine block. Two-stroke cycle engines do not require oil rings because lubrication is supplied by mixing oil in the gasoline, and an oil reservoir is not required.

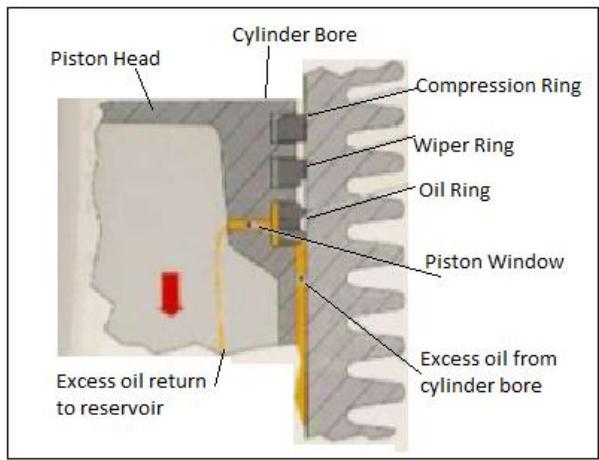


Fig.4.14 Piston Rings

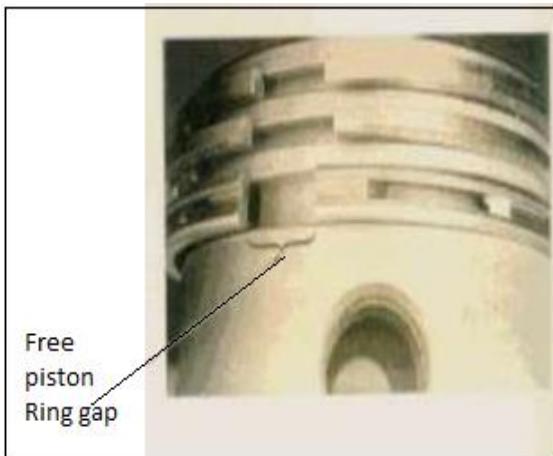


Fig.4.15 Piston Ring Gap

Piston rings seal the combustion chamber, transferring heat to the cylinder wall and controlling oil consumption. A piston ring seals the combustion chamber through inherent and applied pressure. Inherent pressure is the internal spring force that expands a piston ring based on the design and properties of the material used. Inherent pressure requires a significant force needed to compress a piston ring to a smaller diameter. Inherent pressure is determined by the uncompressed or free piston ring gap. Free piston ring gap is the distance between the two ends of a piston ring in an uncompressed state. Typically, the greater the free piston ring gap, the more force the piston ring applies when compressed in the cylinder bore.

A piston ring must provide a predictable and positive radial fit between the cylinder wall and the running surface of the piston ring for an efficient seal. The radial fit is achieved by the inherent pressure of the piston ring. The piston ring must also maintain a seal on the piston ring lands. In addition to inherent pressure, a piston ring seals the combustion chamber through applied pressure. Applied pressure is pressure applied from combustion gases to the piston ring, causing it to expand. Some piston rings have a chamfered edge opposite the running surface. This chamfered edge causes the piston ring to twist when not affected by combustion gas pressures. Another piston ring design consideration is cylinder wall contact pressure. This pressure is usually dependent on the elasticity of the piston ring material, free piston ring gap, and exposure to combustion gases. Widely used material for piston ring is cast iron as it is easily conforms to the cylinder wall. In addition, cast iron is easily coated with other materials to enhance its durability. Care must be exercised when handling piston rings, as cast iron is easily distorted. Piston rings commonly used on small engines include the compression ring, wiper ring, and oil ring.

4.8.2 Compression ring

The compression ring is the top or closest ring to combustion gases and is exposed to the greatest amount of chemical corrosion and the highest operating temperature. The compression ring transfers 70% of the combustion chamber heat from the piston to the cylinder wall. Most engines

use either taper-faced or barrel-faced compression rings. A taper faced compression ring is a piston ring that has approximately a 1° taper angle on the running surface. This taper provides a mild wiping action to prevent any excess oil from reaching the combustion chamber. A barrel faced compression ring is a piston ring that has a curved running surface to provide consistent lubrication of the piston ring and cylinder wall. This also provides a wedge effect to optimize oil distribution throughout the full stroke of the piston. In addition, the curved running surface reduced the possibility of an oil film breakdown due to excess pressure at the ring edge or excessive piston tilt during operation.

4.8.3 Wiper ring

The wiper ring, sometimes called the scraper ring, Napier ring, or back-up compression ring, is the next ring away from the cylinder head on the piston. The wiper ring provides a consistent thickness of oil film to lubricate the running surface of the compression ring. Most wiper rings in engines have a taper angle face. The tapered angle is positioned toward the oil reservoir and provides a wiping action as the piston moves toward the crankshaft. The taper angle provides contact that routes excess oil on the cylinder wall to the oil ring for return to the oil reservoir. A wiper ring incorrectly installed with the tapered angle closest to the compression ring results in excessive oil consumption. This is caused by the wiper ring wiping excess oil toward the combustion chamber.

4.8.4 Oil ring

An oil ring includes two thin rails or running surfaces. Holes or slots cut into the radial center of the ring allow the flow of excess oil back to the oil reservoir. Oil rings are commonly one piece, incorporating all of these features. Some on-piece oil rings utilize a spring expander to apply additional radial pressure to the piston ring. This increases the unit (measured amount of force and running surface size) pressure applied at the cylinder wall. The oil ring has the highest inherent pressure of the three rings on the piston. Some engines use a tree-piece oil ring consisting of two rails and an expander. The oil rings are located on each side of the expander. The expander usually contains multiple slots or windows to return oil to the piston ring groove. The oil ring uses inherent piston ring pressure, expander pressure, and the high unit pressure provided by the small running surface of the thin rails.

The piston acts as the movable end of the combustion chamber and must withstand pressure fluctuations, thermal stress, and mechanical load. Piston material and design contribute to the overall durability and performance of an engine. Most pistons are made from die- or gravity-cast aluminum alloy. Cast aluminum alloy is lightweight and has good structural integrity and low manufacturing costs. The light weight of aluminum reduces the overall mass and force necessary to initiate and maintain acceleration of the piston. This allows the piston to utilize more of the

force produced by combustion to power the application. Piston designs are based on benefits and compromises for optimum overall engine performance.

4.9 Crankshaft

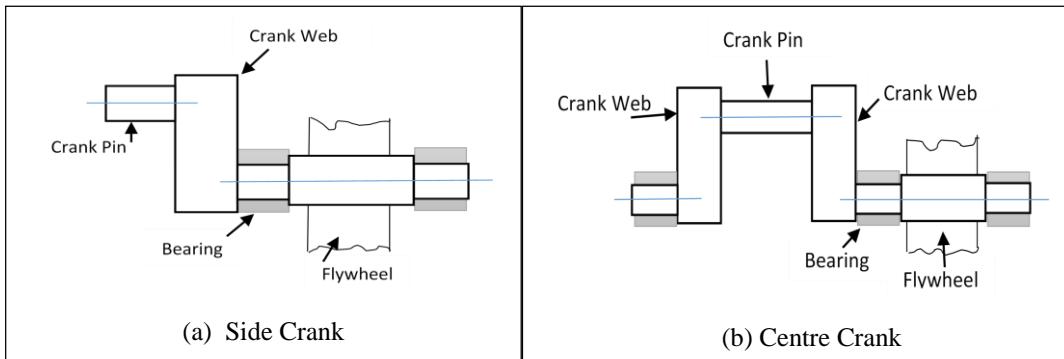


Fig.4.16 Crank shaft types

Crankshaft converts the reciprocating motion of piston into rotary motion through the connecting rod. The crankshaft consists of three portions – crankpin, crank web and shaft. The big end of the connecting rod is attached to the crankpin. The crank web connects the crankpin to the shaft portion. The shaft portion rotates in the main bearings and transmits power to the outside source through the belt drive, gear or chain drive.

There are two types of crankshafts, side crankshaft and center crankshaft as shown in figure 4.16. The side crankshaft is also called ‘overhung crankshaft’. It has only one crank web and requires only two bearings for support. The Centre crankshaft has two webs and three bearings for support. Crankshafts are also classified as single-throw and multi-throw crankshafts depending on the number of crankpins used in the assembly. Crankshafts used in multi-cylinder engines have more than one crankpin. They are called multi-throw crankshafts.

4.9.1 Material requirement of crankshaft

- Material should be readily shaped, machined and heat-treated.
- It should have adequate strength, toughness, hardness.
- It should have high fatigue strength.
- It should be machinable to high finish bearing surface.

4.9.2 Manufacturing methods for crankshaft

The crankshaft are manufactured from steel by following methods.

- Forging
- Casting.

The main bearing and connecting rod bearing liners are made of Babbitt, a tin and lead alloy. Forged crankshafts are stronger than the cast crankshafts, but are more expensive. Forged

crankshafts are made from SAE 1045 or similar type steel. Forging makes a very dense, tough shaft with a grain running parallel to the principal stress direction. Crankshafts are cast in steel, modular iron or malleable iron. The major advantage of the casting process is that crankshaft material and machining costs are reduced because the crankshaft may be made close to the required shape and size including counterweights. Cast crankshafts can handle loads from all directions as the metal grain structure is uniform and random throughout. Great care must be observed in the manufacture of crankshafts since it is the most important part of the engine. Small crankshafts are drop forged. Larger shafts are forged and machined to shape. Casting of the crankshafts allows a theoretically desirable but complicated shape with a minimum amount of machining and at the smallest cost. These are cast in permanent molds for maximum accuracy and a minimum of machining. While machining, the shaft must be properly supported between centers and special precautions should be taken to avoid springing. The journals and crankpins are ground to exact size after turning. After this, the crankshaft is balanced. Large shafts of low speed engines are balanced statically; Crankshafts of high-speed engines are balanced dynamically on special balancing machines. Most crankshafts are ground at the journals and crankpins. In some cases grounding is followed by hand lapping with emery cloth.

4.9.3 Design procedure for crankshaft

1. Determine the magnitudes of the various loads acting on the crankshaft.
2. Determine the distance between supports. The distances will depend upon the lengths of the bearing. The lengths & diameters of the bearings are determined on the basis of maximum permissible bearing pressures, l/d ratios and the acting loads
3. For the sake of simplicity and safety, the shaft is considered to be supported at the centers of the bearings.
4. The thickness of the crank webs is assumed, about $0.5d$ to $0.6d$, where d is the shaft diameter or from $0.22D$ to $0.32D$, where D is the cylinder bore.
5. Calculate the distance between supports.
6. Assume allowable bending and shearing stresses.
7. Compute the necessary dimensions of the crankshaft.

The above procedure is general design procedure. It may change as per the requirements and definition of the given problem. Note: All the forces and reactions are assumed to be acting at the centers of the bearings.

4.9.4 Crank pin

In a reciprocating engine, the crankpins, also known as crank journals are the journals of the big end bearings, at the ends of the connecting rods opposite to the pistons. If the engine has a crankshaft, then the crank pins are the journals of the off-center bearings of the crankshaft.

4.9.5 Crank web

The portion of a crank between the crankpin and the shaft or between adjacent crankpins called also crank arm, crank throw.

4.9.6 Main bearing

In a piston engine, the main bearings are the bearings on which the crankshaft rotates, usually plain or journal bearings. The bearings hold the crankshaft in place and prevent the forces created by the piston and transmitted to the crankshaft by the connecting rods from dislodging the crankshaft, instead forcing the crank to convert the reciprocating movement into rotation. When describing a crankshaft design, the number of main bearings is generally quoted, as the number of crank pins is determined by the engine configuration. For example, a crankshaft for an inline six engine will be described as three bearing or four bearing depending on its number of main bearings. In a crankshaft, the journals are the main bearing journals only. The crank pins are not normally called journals although they form the center shafts of the big end bearings and are therefore journals in the more general sense.

NUMERICALS

Numerical 4.1 Design a 4-stroke, single cylinder, water cooled, vertical diesel engine with following specifications:

Brake power= 15 kW

Speed = 1200 rpm

Compression Ratio = 14

Solution:

Design Data book referred for this design: Design data book by Kale and Khandare (KK/Page no)

Given:

Diesel Engine,

Stroke 4,

Cylinder = 1

BP = 15 kW

N_{engine} = 1200 rpm

Compression ratio = 14

Let, Stroke to bore ratio (S/D)

Step 1: Calculation for maximum pressure

Assuming,

$$\frac{S}{D} = 1 \quad \dots \quad (\text{KK/6})$$

Assuming Mechanical Efficiency,

$$\eta_m = 0.8$$

Assuming mean effective pressure,

$$P_{im} = 1 \text{ MPa} \quad \dots \quad (\text{KK/5})$$

$$N_i = N_{\text{engine}}/2 = 1200/2 = 600 \text{ rpm}$$

Indicated power,

$$IP = BP / \eta_m = 15 / 0.8 = 18.75 = 19 \text{ kW}$$

$$IP = P \times L \times A \times N_i \times n$$

$$19000 = 10^6 \times D \times \frac{\pi}{4} \times D^2 \times \frac{600}{60} \times 1$$

$$D = 134.2 \text{ mm} = 135 \text{ mm}$$

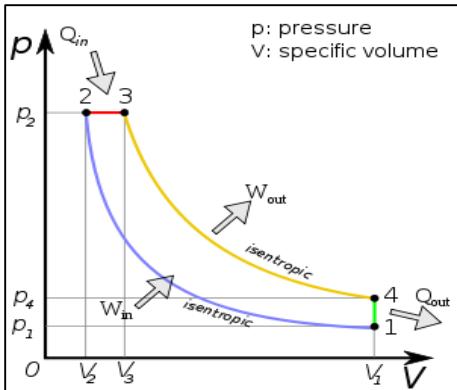


Fig. 4.17 Diesel Cycle

Selecting standard bore diameter = D = 140 mm, L = 140 mm

Radius of crank,

$$r = \frac{L}{2} = \frac{140}{2} = 70 \text{ mm}$$

Length of connecting rod = 2L = 280 mm

Assuming suction pressure at point 1 is less than 1 bar.

$$P_i = 0.9 \text{ bar} = 0.09 \text{ MPa}$$

Assuming adiabatic index , n = 1.35

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^n = 0.09 \times 14^{1.35} = 3.173 \frac{\text{N}}{\text{mm}^2}$$

$$P_{\max} = 3.173 \text{ N/mm}^2$$

Step 2: Liner design

$$\text{Minimum Liner Thickness} = t_{\min} = \frac{D}{2} \left[\left(\frac{S_d + 0.4P_{\max}}{S_d - 1.3P_{\max}} \right)^{0.5} - 1 \right] \dots \dots \text{(KK/12)}$$

Where,

Sd = Design Stress = 50~60 MPa for CI

Assuming Sd = 55 Mpa

P_{max} = 3.173 Mpa, D = 140 mm

t_{min} = 3.6174 mm

Considering constant for circulation C = 5 (range 3~5)

$$t = 3.6174 + 5 = 9 \text{ mm}$$

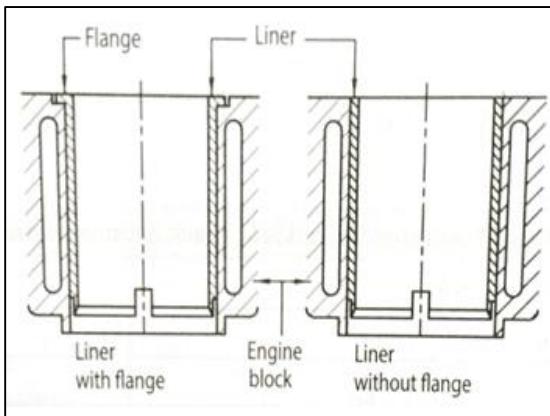


Fig. 4.18 Dry liners

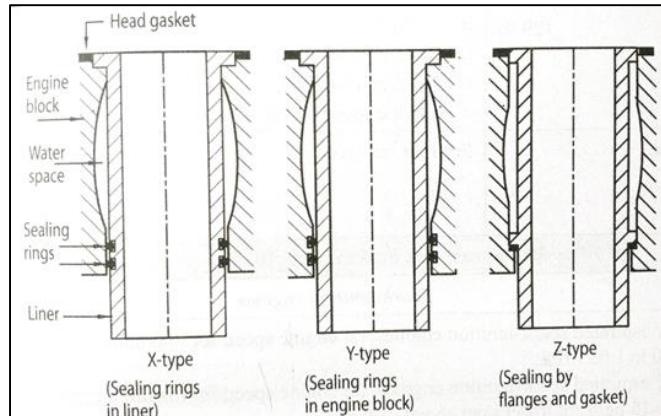


Fig. 4.19 Wet liners

Step 3: Check for Pressure criteria

$$S_x = \frac{P_{max} \times D}{2 \times t_1} \dots \dots \text{ (KK/12)}$$

$$S_x = \frac{3.173 \times 140}{2 \times 9} = 24.678 \text{ MPa} < 30 \sim 60 \text{ MPa}$$

Step 4: Check for Thermal criteria

$$St = \frac{E \times \alpha \times \Delta T}{2 \times (1 - \mu)} \dots \dots \text{ (KK/12)}$$

$$E = 10^5 \frac{\text{N}}{\text{mm}^2} \dots \dots \text{ (KK/1)}$$

$$\alpha = 10.6 \times 10^{-6} \frac{\text{mm}}{\text{mm}^\circ\text{C}}$$

$$\Delta t = 100 \sim 150 = 100^\circ\text{C}$$

$$S_t = \frac{10^5 \times 10.6 \times 10^{-6} \times 100^\circ\text{C}}{2 \times (1 - 0.271)}$$

$$S_t = 72.7 \text{ MPa}$$

$$S_T = S_x + S_t = 97.378 < 130 \text{ MPa}, \text{ Therefore, the design is safe.}$$

Step 5: Cylinder head design for water cooled engine

Step 5.1 Cover Thickness of cylinder head

$$tc = K_2 \times D_p \times \left(\frac{P_{max}}{S_d} \right)^{0.5} \dots \dots \text{ (KK/13)}$$

$$K_2 = 0.54 \text{ for CI}$$

Assuming M 20 bolt,

$$t = 1.25db = 25 \text{ mm}$$

$$D_p = D + 2l$$

$$D_p = 140 + (2 \times 25) = 190 \text{ mm}$$

$$t_c = 0.54 \times 190 \times \left(\frac{3.173}{55}\right)^{0.5}$$

$$t_c = 24.643 \text{ mm} \approx 25 \text{ mm}$$

Step 5.2 Thickness of water jacket for water cooled engine,

$$t_w = 0.03D + 2.2$$

$$t_w = (0.03 \times 140) + 2.2$$

$$t_w = 6.4 \text{ mm} \approx 7 \text{ mm}$$

Step 6: Stud or bolt design

Number of bolt,

$$n = 0.015D + 4 = 6.1 \text{ mm} \approx 8 \text{ mm} \quad \dots \dots \dots \text{(KK/15)}$$

Total load on bolt,

$$F_t = x \cdot F_{max} + F_{pl}$$

$$F_{max} = \frac{P_{max} \times A}{n_g} = \frac{3.173 \times \pi \times 140^2}{4 \times 8} = 6.105 \text{ kN}$$

$$F_{pl} = m \cdot (1 - x) \cdot F_{max}$$

$$m = 1.8 \text{ without gasket} \quad \dots \dots \text{(KK/15)}$$

$$x = 0.15 \sim 0.25, \text{ Let } x = 0.2 \quad \dots \dots \text{(KK/15)}$$

$$F_{pl} = 1.8 \times (1 - 0.2) \times 6.105 = 8.7912 \text{ kN}$$

$$F_t = x \cdot F_{max} + F_{pl}$$

$$F_t = (0.2 \times 6.105) + 8.7912$$

$$\text{Total load on bolt, } F_t = 10.012 \text{ kN}$$

Design stress for bolt material,

SAE 1120 Steel with design stress $S_d = 110 \text{ MPa}$ (KK/16)

$$S_d = \frac{F_t}{A_c}$$

$$A_c = \frac{F_t}{S_d} = \frac{10.012 \times 1000}{110} = 91.018 \text{ mm}^2$$

Selecting standard bolt M14 with fine area $A_c = 111 \text{ mm}^2$ (KK/17)

Step 7: Piston Design (KK/8)

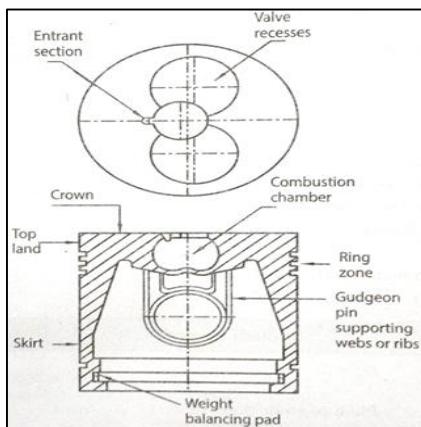


Fig. 4.20 Solid skirt piston

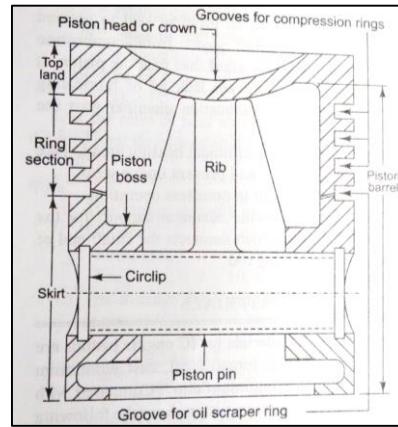


Fig. 4.21 Piston

Piston material: - Aluminium

Design Stress $S_d = 80 \text{ MPa}$

Crown thickness t_c

Step 7.1 Using pressure criteria

$$t_c = D \left[\frac{3 \times P_{max}}{16 \times S_d} \right]^{0.5} = 140 \left[\frac{3 \times 3.173}{16 \times 80} \right]^{0.5}$$

$$t_c = 12.07 \text{ mm} \approx 13 \text{ mm}$$

Step 7.2 Thermal stress criteria

$$t_c = \frac{q \times D^2 \times 10^{-2}}{16 \times C_t \times \Delta T} \quad \dots \dots \quad (\text{KK/8})$$

$$q = k \times \text{BSFC} \times C_v \times P_b \times n_c / A$$

$$C_t = 1.986 \text{ for Al}$$

$$C_v = 44000 \frac{\text{kJ}}{\text{kg}} \text{ for diesel fuel}$$

$$\Delta T = 75^\circ\text{C} \text{ for Aluminium} \quad \dots \dots \quad (\text{KK/8})$$

$$K = 0.1 \quad \dots \dots \quad (\text{KK/8})$$

$$\text{From KK/5, ISFC} = 0.2 \frac{\text{kg}}{\text{hr.kW}}$$

$$\text{BSFC} = \eta_m \times \text{ISFC} = 0.2 \times 0.8 = 0.16 \frac{\text{kg}}{\text{hr. kW}}$$

$$q = 0.1 \times 0.16 \times 44000 \times 1 \times \frac{15}{\frac{\pi}{4} \times 14^2}$$

$$q = 68.6 \frac{\text{kJ}}{\text{hr. cm}^2}$$

$$t_c = \frac{68.6 \times 140^2 \times 10^{-2}}{16 \times 1.986 \times 75}$$

$$t_c = 5.641 \text{ mm} \approx 6 \text{ mm}$$

Step 7.3 Empirical Relation

$$t_c = 0.032D + 1.5, t_c = (0.032 \times 140) + 1.5$$

$$t_c = 5.98 \text{ mm} \approx 6 \text{ mm}$$

Taking higher value of crown thickness $t_c = 13 \text{ mm}$

Step 8 Design of piston ring

Number of piston rings,

$$n = 0.4\sqrt{D} = 4.73 \approx 5$$

Radial Thickness of Piston ring,

$$t_r = (0.038 \sim 0.043) \times D = 5.6 \text{ mm} \approx 6 \text{ mm}$$

Axial Thickness of Piston ring,

$$t_a = (0.6 \sim 1.0) \times t_r = 0.8 \times t_r = 0.8 \times 6 = 4.8 \approx 5 \text{ mm}$$

Proportionate Dimensions of Piston

Length of piston $L = D = 140 \text{ mm}$

Height of Piston top part $= 0.7D = 98 \text{ mm}$

Thickness of skirt wall $t = (2 \sim 5) = 4 \text{ mm}$

Thickness of piston crown wall, $t_c = 13 \text{ mm}$

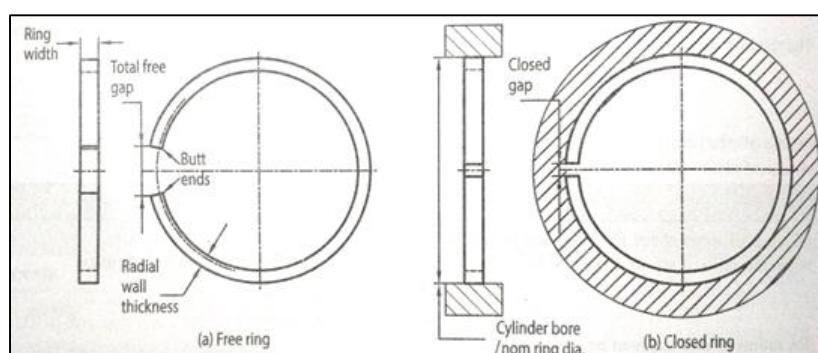


Fig. 4.22 Piston rings

Step 9: Piston pin design

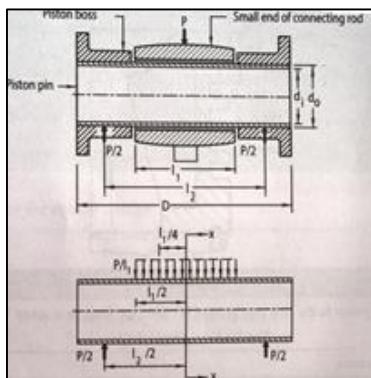


Fig. 4.23 Piston pin and force diagram

Using Bearing Stress criteria

$$F_{max} = S_b \times A_b$$

$$A_b = \text{Bearing Area} = L_p \times d_p$$

Where,

L_p = Length of Pin

D_p = Diameter of Pin

$S_b = 15 \sim 20 \text{ MPa}$

Let, $S_b = 18 \text{ MPa}$ and $L_p = 1.5d_p$

$D = 140\text{mm}$, $P_{max} = 3.173\text{MPa}$

$$\frac{\pi}{4} \times D^2 \times P_{max} = 18 \times 1.5 \times d_p^2$$

$$d_p = 42.533 \text{ mm} \approx 43 \text{ mm}$$

$$L_p = 1.5d_p = 1.5 \times 43 = 64.5 \text{ mm} \approx 65 \text{ mm}$$

Step 10: Checking for bending stress

$$BM_{max} = \frac{F_{max} \times L_p}{8} = \frac{48.82 \times 65}{8} = 396.825 \text{ KNmm}$$

$$(S_x) = \frac{BM}{Z} = \frac{396.825 \times 1000 \times 32}{\pi \times d_p^3} = 50.838 \text{ N/mm}^2$$

Safe Bending stress

$$[S_d] = 110 \text{ N/mm}^2 \text{ for carbon steel SAE 1020 (KK/10/T4.4A)}$$

As $S_d < [S_d]$ Hence the design is safe.

Step 11: Connecting rod design (CR) (KK/19)

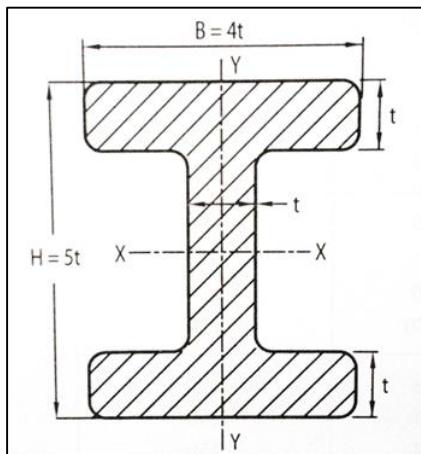


Fig. 4.24 Cross section of connecting rod

$$\text{Normal load on CR} = F_N = F_g - F_i$$

$$F_g = \frac{\pi}{4} \times D^2 \times P_{\max}$$

$$F_g = \frac{\pi}{4} \times 140^2 \times 3.123$$

$$F_g = 48.844 \text{ kN}$$

$$W_c = (W_1 + W_2 + W_3)A_p$$

$$W_1 = \text{Piston weight} = 200 \text{ kg/m}^2 \quad (\text{KK/20})$$

$$W_2 = \text{Weight of CR} = 300 \text{ kg/m}^2$$

$$W_3 = \text{Unbalanced mass of crankshaft} = 200 \text{ kg/m}^2$$

$$W_c = (200 + 300 + 200) \times \frac{\pi}{4} \times D^2 = 700 \times \left(\frac{\pi}{4} \times 0.140^2 \right) = 10.78 \text{ kg} =$$

$$\omega = 2\pi \times \frac{1200}{60} = 125.663 \text{ rad/s}$$

$$F_i = W_c \times \omega^2 \times r_c \times \left(1 + \frac{1}{n}\right) \times 10^{-3}$$

$$F_i = 10.78 \times 125.663^2 \times 70 \times 10^{-3} \times \left(1 + \frac{1}{4}\right)$$

$$F_i = 14.89 \text{ kN}$$

$$F_N = F_g - F_i = 48.844 - 14.89 = 33.95 \text{ KN}$$

Now, critical load for buckling on CR

$$F_{CR} = F_c \times F_s = \frac{F_N}{\sqrt{1 - \frac{\sin^2 \theta}{n^2}}} \times F_s = \frac{33.95}{\sqrt{1 - \frac{\sin^2 8}{4^2}}} \times 1.2$$

$$F_{CR} = 40.76 \text{ KN}$$

Step 11.1 Based on buckling strength

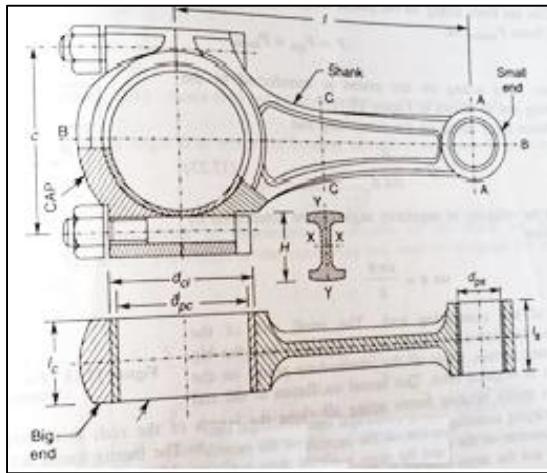


Fig. 4.25 Connecting rod

$$F_{CR} = \frac{S_d \times A_s}{\left[1 + \left(\frac{1}{a} \right) \left(\frac{l_c}{k} \right)^2 \right]}$$

$S_d = 300$ for SAE 4340

$a = 7500$ for steel, $A_s = 11t^2$

Since $I_{xx} \leq 4I_{yy}$; Radius of gyration, $k = \sqrt{\frac{I_{xx}}{A}} = 1.78t$

$$40760 = \frac{300 \times 11t^2}{\left[1 + \left(\frac{1}{7500} \right) \left(\frac{280}{1.78t} \right)^2 \right]}$$

$$40760t^2 + 134477.29 = 3300t^4$$

$$t^2 = 15.05$$

$$t = 3.88 = 4 \text{ mm}$$

$$A_m = 11t^2 = 176 \text{ mm}^2$$

Step 11.2 Check for bending stress in CR

$$BM_{max} = \frac{\rho \times A_m \times r_e \times \omega^2 \times l_c^2 \times 10^{-12}}{9\sqrt{3}}$$

$$BM_{max} = \frac{7840 \times 176 \times 70 \times 125.663^2 \times 280^2 \times 10^{-12}}{9\sqrt{3}}$$

$$BM_{max} = 7671.045 \text{ Nmm}$$

$$B = 4t = 4 \times 4 = 16 \text{ mm}$$

$$H = 5t = 5 \times 4 = 20 \text{ mm}$$

$$b = h = 3t = 3 \times 4 = 12 \text{ mm}$$

$$\text{Section modulus, } z = \frac{BH^3 - bh^3}{6H} = 893.82 \text{ mm}^3$$

$$(s_b) = \frac{M}{z} = \frac{7671.045}{893.82} = 8.58 \text{ Nmm}^2 < [s_b] \text{ Hence, Safe in bending}$$

Step 11.3 Design of small end of Connecting Rod

Inside diameter of small end,

$$d_{pi} = 1.1d_p = 71.5 \text{ mm} \quad (\text{KK/20})$$

Outside diameter of small end,

$$d_{po} = 1.2d_p = 78 \text{ mm}$$

Length of small end,

$$R_i = 0.3D = 42 \text{ mm}$$

Radial thickness of wall bushing,

$$t_{rb} = 4.55 \text{ mm}$$

Bearing pressure is given by,

$$P_b = \frac{\pi D^2 P_{max}}{4l_s \times d_p} = 17.89 \frac{N}{\text{mm}^2}, P_b < [P_b] = 120 \sim 200 \frac{N}{\text{mm}^2}, \text{ Hence, safe}$$

Step 12: Design of crankshaft

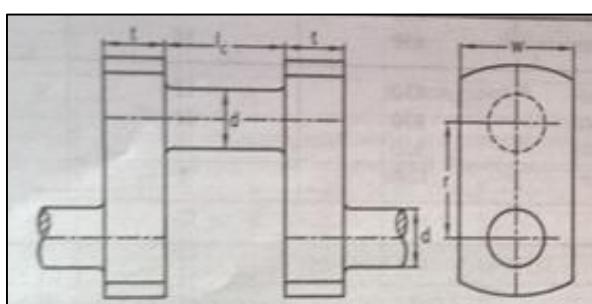


Fig. 4.26 Central crankshaft

$$\text{Distance between centre crank} = L_p = 4d_p = 4 \times 0.6D = 336 \text{ mm} \quad (\text{KK/22})$$

Selecting $K = \frac{1}{4}$ for centre crank,

Crankpin design based on bending moment criteria

$$M_{\max} = K \cdot F_{\max} \cdot L_p = \frac{\pi \times 140^2 \times 3.173 \times 336}{4 \times 4}$$

$$Z = \frac{\pi \times D^3}{32}$$

Selecting material mild steel SAE 1000 – SAE 1040

$$[S_d] = 80 \sim 120 \text{ N/mm}^2$$

$$\text{Let } [S_d] = 100 \text{ N/mm}^2$$

$$100 = \frac{M}{Z} = \frac{4.1029 \times 10^6 \times 32}{\pi \times d^3}$$

$$d_c = 74.765 \text{ mm} \approx 75 \text{ mm}$$

Step 12.1 Checking for bearing pressure criteria,

Maximum gas force,

$$F_{\max} = P_b \times l_{cp} \times d_c$$

$$l_{cp} = 0.8d_c = 60 \text{ mm}$$

$$(P_b) = \frac{F_{\max}}{l_{cp} \times d_c} = \frac{48.84 \times 1000}{60 \times 75} = 10.853 \frac{\text{N}}{\text{mm}^2} < [P_b] = 16 \sim 35 \text{ N/mm}^2$$

Thus design is safe.

$$\text{Width of the crank web} = 1.2d_c = 90 \text{ mm}$$

$$\text{Thickness of crank web } h = 0.25d_c = 18.75 \text{ mm}$$

Numerical 4.2 Design a 4 stroke single cylinder, water cooled, and vertical petrol engine with following specification

Brake Power = 40 kW,

Speed = 2500 rpm,

Compression ratio = 7.5

Solution:

Step 1: Calculation for maximum pressure

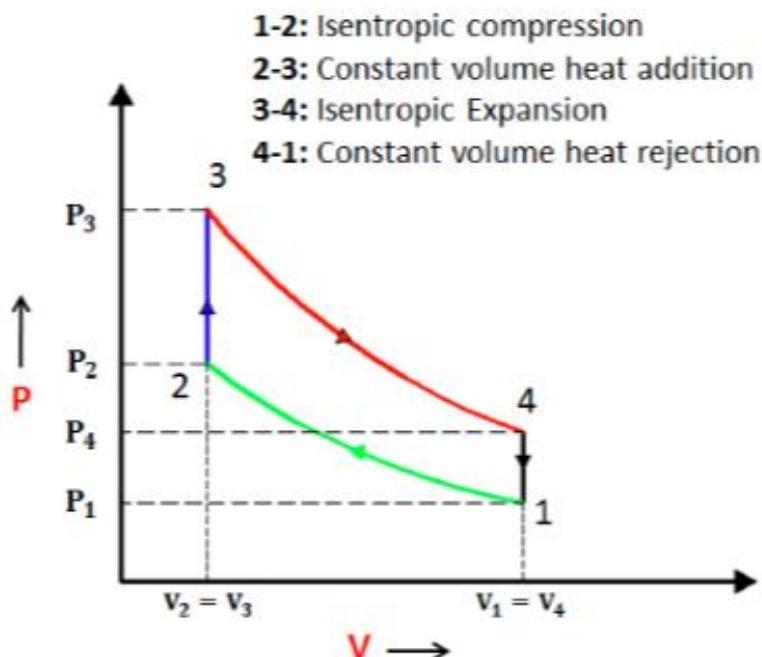


Fig. 4.27 PV diagram for Otto cycle

Stroke to bore ratio (S/D) = 1 (0.8-1.2) (KK-6)

$\eta_m = 0.8$ (For Petrol Engine)

Mean effective pressure (0.7 – 1.1) (KK-5)

$P_{im} = 1 \text{ MPa}$

$N = N_{\text{engine}}/2 = 2500/2 = 1250 \text{ rpm}$

$$IP = BP/\eta_m = 40/0.8 = 50 \text{ kW}$$

$$IP = P_{im} L A N \eta_c \quad (\text{KK-7})$$

$$50 \times 103 = 106 \times D \times (\pi/4) \times D^2 \times (1250/60) \times 1$$

$$D = 0.145 \text{ m}$$

$$D = 146 \text{ mm}$$

Standard bore diameter, (D) = 146 mm

$$L = D = 146\text{mm}$$

Radius of crank,

$$r = L/2 = 146/2 = 73 \text{ mm}$$

Length of connecting rod = $2L = 292 \text{ mm}$

Assuming suction pressure at point 1 is less than 1 bar

$$P_1 = 0.9 \text{ bar} = 0.09 \text{ N/mm}^2$$

Assuming adiabatic index = $n = 1.35$

$$P_2 = P_1 (V_1/V_2)^n$$

$$P_2 = 0.09 (7.5)^{1.35}$$

$$P_2 = 1.36639 \text{ N/mm}^2$$

For a petrol engine,

$$\text{Explosion ratio } Re = 3 - 3.5$$

$$R_e = (P_3/P_2) = 3.25$$

$$P_3 = 3.25 \times 1.36639 = 4.44 \text{ N/mm}^2$$

$$\text{Design Pressure} = P_d = \text{Safety Factor} \times P_3$$

Assuming Safety Factor = 1.2

$$P_d = 1.2 \times 4.44$$

$$P_d = 5.328 \text{ N/mm}^2$$

Step 2: Liner Design

Minimum thickness of liner is given by,

$$T_{1\min} = \frac{D}{2} \left(\frac{(S_d + 0.4 P_{max})^{1/2}}{S_d - 1.3 P_{max}} - 1 \right)$$

$$\text{Where, } P_{max} = P_3 \quad \dots \dots \dots \text{ KK-12}$$

$$S_d = \text{Design Stress} = 50-60 \text{ MPa for CI} \quad \dots \dots \dots \text{ KK-12}$$

$$\text{Assume } S_d = 55 \text{ MPa}$$

$$t_{min} = 6.580 \text{ mm}$$

Considering constant for circulation $C = 3 \sim 5$

$$t_1 = 6.58 + 5 = 11.58 \text{ mm}$$

Step 3: Check for pressure criteria

$$S_x = \frac{P_{max} \cdot D}{2 t_1} = \frac{5.328 \times 146}{2 \times 11.58} = 33.58 < 60 \text{ MPa, Hence safe.}$$

Step 4: Checking for thermal criteria

$$S_t = \frac{E \alpha \Delta T}{2(1-\mu)} \quad \dots \dots \text{ (KK-12)}$$

Where,

$$E = 10 \times 10^4 \text{ N/mm}^2, \mu = 0.271, \alpha = 10.6 \times 10^{-6}/^\circ\text{C}, \Delta t = 100$$

$$S_t = \frac{10 \times 10^4 \times 10.6 \times 10^{-6} \times 100}{2(1-0.271)}$$

$$S_t = 72.7 \text{ MPa}$$

Total stress,

$$S_t = S_x + S_t = 32.41 + 72.7$$

$$S_t = 105.11 \text{ MPa} < 130 \text{ MPa}, \text{ Therefore, Safe.}$$

Step 5: Cylinder head design for water cooled engine

Step 5.1 Cover Thickness of cylinder head

Cover thickness is given by,

$$t_c = K_2 \cdot D_p \left(\frac{P_{max}}{S_2} \right)^{1/2} \quad \dots \dots \dots \text{KK-13}$$

$K_2 = 0.54$ for CI, assuming M20 bolt

$$t = 1.25 d_b = 25$$

Pitch circle diameter of the bolt,

$$D_p = D + 2t = 146 + 2t = 146 + 50 = 196$$

Let's $D_p = 200$

$$t_c = 0.54 \times 200 \times \left(\frac{5.328}{55} \right)^{1/2}$$

$$t_c = 33.61 \text{ mm}$$

Step 5.2 Thickness of water jacket

.....(KK-13)

$$t_w = 0.03D + 2.2$$

$$t_w = (0.03 \times 146) + 2.2$$

$$t_w = 6.58 \text{ mm}$$

$$t_w \approx 7 \text{ mm}$$

Step 6 Stud or bolt design

$$\text{Number of bolt, } n = 0.015 D + 4 \quad \dots \dots \dots \text{ (KK-15)}$$

$$n = (0.015 \times 146) + 4$$

$$n = 6.19 \approx 7$$

Total load on bolt,

$$F_t = x \cdot F_{max} + F_{pl}$$

$$\text{Where, } F_{max} = \frac{P_{max} \cdot A}{n_b} = \frac{5.328 \times \pi}{7 \times 4} \times 146^2 = 12.74 \text{ KN}$$

Preloading force of stud, $F_{pl} = M(1-x)$

$$F_{max} = 1.8 \times (1 - 0.2) \times 12.74 , F_{max} = 18.34 \text{ kN}$$

Where,

M = tightening coefficient for bolt,

M = 1.8, assuming without gasket

And coefficient of main load on threaded joint, x= 0.2 (Range is 0.15 to 0.25) (KK-15)

$$F_{pl} = 18.34 \text{ kN}$$

Total load on stud,

$$F_t = 0.2 \times 12.74 + 18.34$$

$$F_t = 20.88 \text{ kN}$$

Design stress for bolt with SAE 1120 steel,

$$S_d = 110 \text{ MPa} \quad \dots\dots \text{T 4.8A, KK-16}$$

$$S_d = \frac{F_t}{A_c}$$

$$A_c = \frac{20.88 \times 10^3}{110} = 189.81 \text{ mm}^2$$

Selecting std. bolt M20 with area 190 mm^2 (KK-17)

Step 7: Piston Design (KK/8)

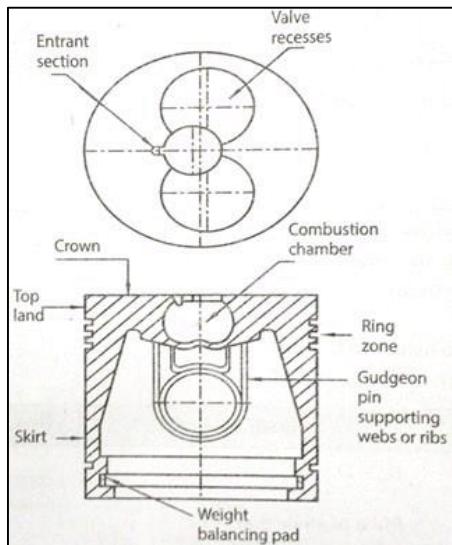


Fig. 4.28 Piston

Assuming Piston material: Aluminum with

$$S_d = 80 \text{ MPa} \quad \dots\dots \text{(T 4.1 KK-8)}$$

Step 7.1 Using pressure criteria

Thickness of crown,

$$t_c = D \left(\frac{3}{16} \frac{P_{\max}}{S_d} \right)^{1/2}$$

$$t_c = 146 \left(\frac{3}{16} \frac{5.328}{80} \right)^{1/2}$$

$$t_c = 16.31 \approx 17 \text{ mm}$$

Step 7.2 Thermal stress criteria

$$t_c = \frac{qD^2 \times 10^{-2}}{16c_1 \times \Delta t}$$

Where, c_1 is heat conduction factor = 1.986 for Aluminum

Temperature difference between center and edge of piston head, $\Delta t = 75^\circ\text{C}$ for Aluminum

Heat flux flowing through crown, $q = K \times \text{BSFC} \times C_v \times (P_B/A) \times \eta_m$

Fraction of heat evolved reaching to crown surface, $K = 0.05$

Calorific value of fuel, $C_v = 42,000 \text{ KJ/Kg}$ for petrol

Brake specific fuel consumption

$$\text{BSFC} = \eta_m \times \text{ISFC} = 0.8 \times 0.3 = 0.24$$

$$q = 100.26 \text{ kJ/hr-cm}^2$$

$$t_c = \frac{100.26 \times 146^2 \times 10^{-2}}{16 \times 1.956 \times 75} = 9.10 \approx 10 \text{ mm}$$

Step 7.3 Empirical Relation

$$t_c = 0.032 D + 1.5, t_c$$

$$t_c = (0.032 \times 146) + 1.5$$

$$t_c = 6.172 \text{ mm}$$

Taking higher value = $t_c = 7 \text{ mm}$

Step 8 Design of piston ring

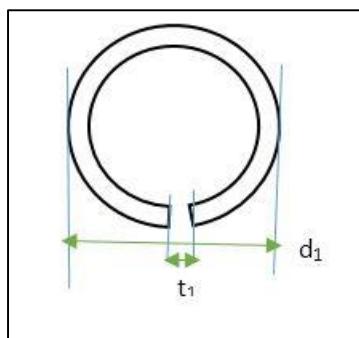


Fig. 4.29 Piston ring

No. of piston ring = 3-4 for automobile engine and 5-7 for stationary engine
 $n = 0.4 D^{1/2} = 5 \dots\dots\text{ T4.2, KK-9}$

Radial thickness of piston ring,

$$t_r = (0.038 \times 0.043)D = 0.04 \times 146 = 6 \text{ mm}$$

Axial thickness of piston ring = $0.7t_r = 4.2 \text{ mm}$

Proportionate dimension of piston (KK-11)

Length of piston, $L = (0.8 \text{ to } 1.3) D$

$$L = D = 146 \text{ mm}$$

Height of piston top part = $(0.45 \text{ to } 0.75) D$

$$\text{Height} = 0.4D = 60 \text{ mm}$$

$$\text{Skirt length} = (0.7) D = 102.2 \text{ mm}$$

Thickness of skirt wall (t) = $(1.4 \text{ to } 4.5)$, Let $t = 4 \text{ mm}$

Thickness of crown piston wall = $t_c = 11 \text{ mm}$

Step 9: Piston pin design

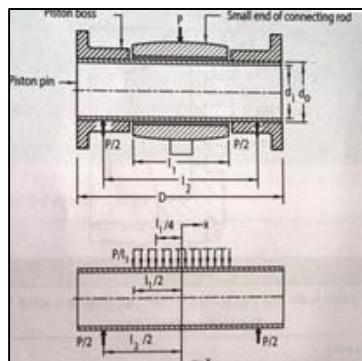


Fig. 4.30 Piston pin and force diagram

Bearing stress criteria;

$$F_{\max} = S_b \times A_b$$

Where, F_{\max} is maximum combustion force

$$F_{\max} = \frac{\pi}{4} D^2 \times P_{\max} = 89.19 \text{ kN}$$

A_b = bearing area = $L_p \cdot d_p$

S_b is safe bearing stress (15 – 20 MPa)

Let, $S_b = 18 \text{ MPa}$

$$\frac{\pi}{4} D^2 \times P_{\max} = 18 \times 1.5 d_p^2$$

$$d_p = 57.47 \text{ mm}, \text{ let } d_p = 58 \text{ mm}$$

$$L_p = 1.5 d_p, L_p = 87 \text{ mm}$$

Step 10: Checking for bending stress

$$BM_{max} = \frac{F_{max} \times L_p}{8} = \frac{89.19 \times 87}{8} = 969941.25 \text{ N.mm}$$

$$(S_b) = \frac{BM_{max}}{Z} = \frac{969941.25}{\frac{\pi}{32}d_p^3} = 50.63 \text{ N/mm}^2$$

Safe bending shear stress $[S_d] = 110 \text{ N/mm}^2$, Carbon Steel SAE 1020 (KK-T4.4A)
 $S_b < [S_d]$ Therefore, safe.

Step 11: Connecting rod design (CR) (KK/19)

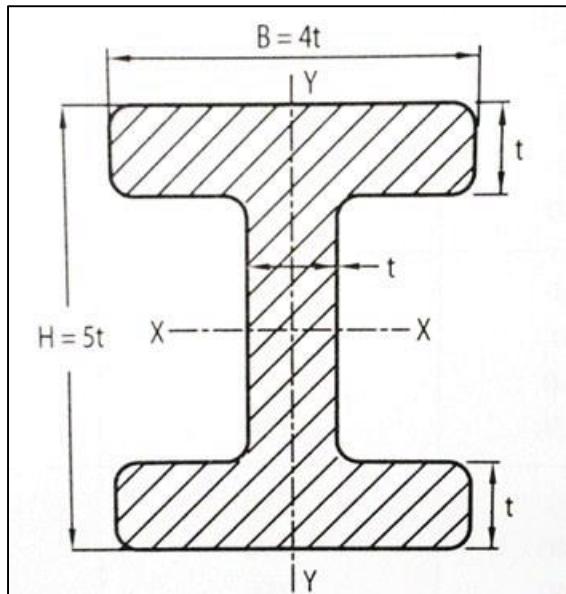


Fig. 4.31 Cross section of connecting rod

Normal load on connecting rod,

$$F_N = F_g - F_i$$

Where,

Gas Force,

$$F_g = \frac{\pi}{4} D^2 \times P_{max} = \frac{\pi}{4} 146^2 \times 5.328 = 89.19 \text{ kN}$$

Inertia force,

$$F_i = W_e \omega^2 r_c \left(1 + \frac{1}{n}\right)$$

W_e – weight of reciprocating part,

$$W_e = (W_1 + W_2 + W_3) A_p, \text{ where, } A_p \text{ – area of piston in m}^2$$

W_1 = piston weight = 80 to 150 kg/m² for Aluminum alloy

Let $w_1 = 150 \text{ kg/m}^2$

W_2 = weight of Connecting Rod = 100 to 200 kg/m²

Let $w_2 = 150 \text{ kg/m}^2$

W_3 = unbalanced part of crankshaft = 100 to 200 kg/m²

Let $w_3 = 150 \text{ kg/m}^2$

$$W_e = (W_1 + W_2 + W_3) A_p = 450 \times \frac{\pi}{4} D^2 = 7.534 \text{ kg}$$

Angular velocity,

$$\omega = \frac{2\pi N}{60} = 261.8 \text{ rad/s}$$

r_c = Crank radius = 73 mm

$$n = l_c/r_c = 4$$

$$F_i = 7.534 \times (261.8)^2 \times 73/1000 \times (1+1/4) = 47.117 \text{ kN}$$

$$F_N = F_g - F_i = 89.19 - 47.117 = 42.07 \text{ kN}$$

Step 11.1 Based on buckling strength

Now, critical load for buckling,

$$F_{cr} = F_C \times F_S$$

$$F_{cr} = \frac{F_N \times F_S}{\sqrt{1 - \frac{(\sin \theta)^2}{n^2}}}$$

$$F_{cr} = \frac{42.07 \times 1000 \times 1.2}{\sqrt{1 - \frac{(\sin 8)^2}{4^2}}}$$

$$F_{cr} = 50514.58 \text{ N}$$

Where,

Factor of Safety = FOS = 1.2

$\theta = 8^\circ$ (5 to 10°) KK-19/ T4.10

$$F_{cr} = 50.514 \text{ kN}$$

Based on buckling strength F_{cr}

$$F_{cr} = \frac{S_d \times A_s}{\left[1 + \left(\frac{1}{a} \right) \left(\frac{l_c}{k} \right)^2 \right]}$$

Safe stress in connecting rod,

$S_d = 300 \text{ MPa}$ for (SAE 4340) (T4.13, KK-21)

$a = 7500$ for steel,

Area of section at small end $A_s = 11t^2$

Since $I_{xx} \leq 4I_{yy}$

For I cross section Radius of gyration,

$$K = \sqrt{\frac{BH^3 - bh^3}{12(BH - bh)}} \quad (\text{KK-3})$$

Where,

$$B = 4t, H = 5t, b = 3t, h = 3t$$

$$K = \sqrt{\frac{500t^4 - 81t^4}{12(20t^2 - 9t^2)}} = 1.78t \text{ in mm}$$

Now,

$$50514 = \frac{300 \times 11t^2}{\left[1 + \left(\frac{1}{7500}\right) \left(\frac{292}{1.78t}\right)^2\right]}$$

$$50514t^2 + 181249.24 = 3300t^4$$

$$t^2 = 18.3073$$

$$t = 4.278 \text{ mm}$$

$$t = 5 \text{ mm}$$

$$A_m = 11t^2 = 275 \text{ mm}^2$$

Step 11.2 Check for bending stress in CR

Check for bending stress in CR

$$BM_{max} = \frac{\rho \times A_m \times r_e \times \omega^2 \times l_c^2 \times 10^{-12}}{9\sqrt{3}}$$

$$BM_{max} = \frac{7840 \times 275 \times 63 \times 261.8^2 \times 292^2 \times 10^{-12}}{9\sqrt{3}}$$

$$BM_{max} = 50920.32 \text{ Nmm}$$

$$B = 4t = 4 \times 4 = 16 \text{ mm}$$

$$H = 5t = 5 \times 4 = 20 \text{ mm}$$

$$b = h = 3t = 3 \times 4 = 12 \text{ mm}$$

$$z = \frac{BH^3 - bh^3}{6H} = 893.867 \text{ mm}^3$$

$$(s_b) = \frac{M}{z} = \frac{50920.32}{893.867} = 56.96 \text{ N/mm}^2 < [s_b] , \text{ Hence safe.}$$

Step 11.3 Design of small end of Connecting Rod

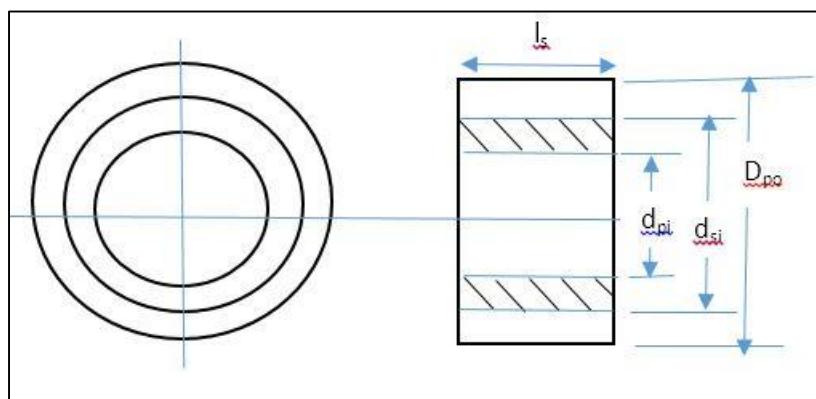


Fig. 4.32 Small end of connecting rod

From..... T 4.12, KK-20

Inside dia. of small end,

$$d_{pi} = d_p \times 1.1 = 32 \times 1.1 = 36 \text{ mm (with bush)}$$

($d_p = 32\text{mm}$ from piston pin design.)

Outside dia. of small end, $d_{po} = d_p \times 1.2 = 32 \times 1.2 = 40 \text{ mm}$

Length of small end, with retaining pin, $l_s = 0.3D = 0.3 \times 146 = 43.8 \text{ mm}$

Radial thickness of wall bushing = 0.07 $d_p = 2.5 \text{ mm}$

Checking for bearing pressure,

$$\text{Bearing pressure, } P_b = \frac{\pi \times D^2 \times P_{max}}{4 \times l_s \times d_p} = \frac{\pi \times 146^2 \times 5.328}{4 \times 43.8 \times 32} = 63.64 \text{ N/mm}^2$$

$(P_b) < [P_{bs}] = 80 \text{ MPa}$, Hence safe, $[P_{bs}] = (60 \text{ to } 200 \text{ MPa}) \dots \text{ (KK/10)}$

Step 12: Design of crankshaft

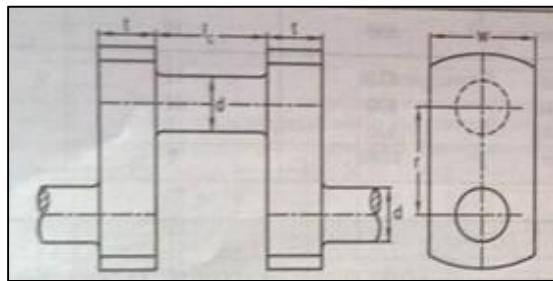


Fig. 4.33 Central crankshaft

Crank pin design with bending moment criteria (KK/22)

$$M_{max} = k F_{max} L_D$$

where, $k = 0.25$ for center crank

$$F_{max} = \text{Maximum gas force} = \frac{\pi}{4} \times D^2 \times P_{max}$$

$$L_p = \text{Length between crank pin} = 4 \times d_c \dots \text{ (KK/24)}$$

Assume,

$$d_c = \text{Crank pin diameter} = 0.6 D = 88\text{mm}$$

$$L_p = 4 \times 88 = 352 \text{ mm}$$

$$M_{max} = \frac{1}{4} \times \frac{\pi}{4} \times 146^2 \times 5.328 \times 352$$

$$M_{max} = 7.849 \times 10^6 \text{ N-mm}$$

$$Z = \frac{\pi d^3}{32}$$

Selecting material mild steel SAE 1000 ~ SAE 1040

$$[S_d] = 80 \sim 120 \text{ N/mm}^2$$

$$\text{Let } [S_d] = 100 \text{ N/mm}^2$$

$$100 = \frac{M}{Z} = \frac{7.849 \times 10^6 \times 32}{\pi d^3}$$

$$d = 92.81 \text{ mm}$$

Let modify Crank pin diameter from 88 mm to 94 mm, $d_c = 94 \text{ mm}$

Step 12.1 Checking for bearing pressure criteria

$$F_{\max} = P_b \times L_{cp} \times d_c$$

L_{cp} = length of crank pin,

let $L_{cp} = 0.6 D = 87.6 \text{ mm}$ (T4.16A, KK-24)

$$(P_b) = \frac{F_{\max}}{L_{cp} \times d_c} = \frac{\frac{\pi}{4} \times 146^2 \times 5.328}{87.6 \times 94}$$

$$(P_b) = 10.83 \text{ N/mm}^2$$

$$(P_b) < [P_b] = 35 \text{ N/mm}^2$$

Safe in bearing failure

Now, Width of crank web = $1.2 d_c = 1.2 \times 94 = 112.8 \text{ mm}$

Thickness of crank web = $h = 0.25 d_c = 23.5 \text{ mm}$

Numerical 4.3: Design a 4 stroke single cylinder, water cooled, and vertical Petrol engine with following specification:

Brake Power: 50 kW,

Speed: 2000 rpm,

Compression Ratio: 7

Solution:

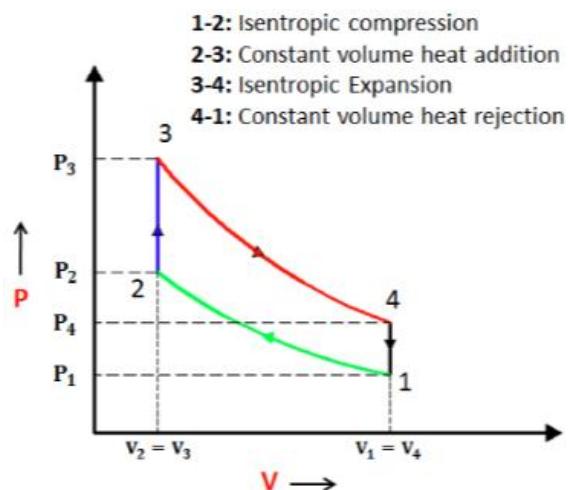


Fig. 4.34 PV diagram for Otto Cycle

Assuming L/D = 1

Assuming mechanical Efficiency, $\eta_{\text{mech}} = 0.8$ (KK-6)

Assuming mean Effective pressure, $P_{\text{imp}} = 1 \text{ MPa}$ (0.6 – 1.4)..... (KK-5)

Speed, $N_1 = N_{\text{engine}}/2 = 2000/2 = 1000 \text{ rpm}$

Now, Indicated Power = Brake Power / mechanical Efficiency (η_m)

$$P_i = 50/0.8 = 62.5 \text{ kW}$$

Also, $P_i = P_{\text{im}} \times L \times A \times N_1 \times N_c$ (KK-7)

$N_c = \text{No. of Cylinder} = 1$

$$P_i = 62.5 \times 10^3 = 10^6 \times D \times \left(\frac{\pi}{4}\right) \times D^2 \times \frac{1000}{60} \times 1$$

$$D = 0.168 \text{ m}, D = 168 \text{ mm}$$

Standard Bore Diameter = 180 mm, L= D = 180mm

Radius of crank , $r = L/2 = 90 \text{ mm}$

Length of connecting rod = $2L = 360 \text{ mm}$

Assuming suction pressure at point 1 is less than 1bar, $P_1 = 0.9 \text{ bar} = 0.09 \text{ N/mm}^2$

Assuming adiabatic index = $n = 1.35$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^n = 0.09(7)^{1.35} = 1.2448 \text{ N/mm}^2$$

$$P_2 = 1.2448 \text{ N/mm}^2$$

For petrol engine explosion ratio $R_e = (3-3.5)$

Assuming $R_e = 3.25$, $R_e = (P_3/P_2)$

$$P_3 = 3.25 \times 1.2448 = 4.17 \text{ N/mm}^2$$

Assuming FOS = 1.2, Design pressure, $P_d = \text{FOS} \times P_3 = 1.2 \times 4.17 = 5 \text{ N/mm}^2$

Step 2: Liner design

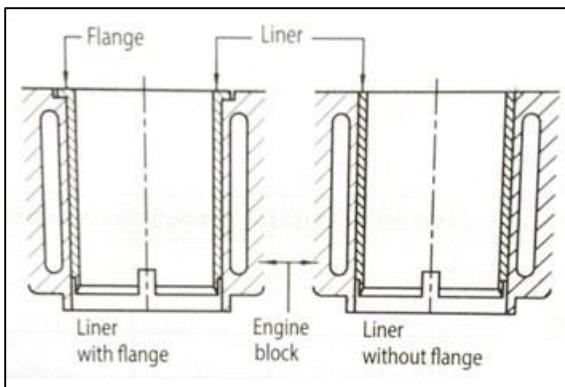


Fig. 4.35 Dry liners

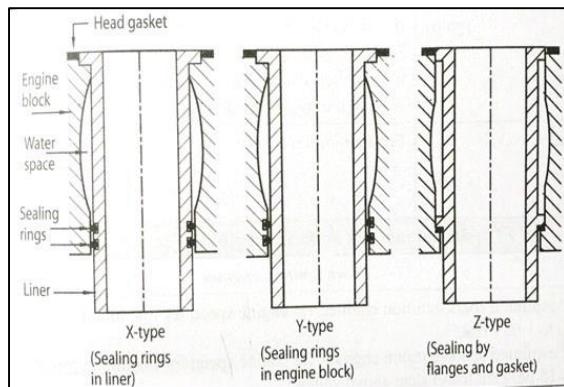


Fig. 4.36 Wet liners

Minimum thickness of liner is given by,

$$t_{1\min} = \frac{D}{2} \left(\left[\frac{S_d + 0.4P_{\max}}{S_d - 1.3P_{\max}} \right]^{\frac{1}{2}} - 1 \right) \quad \dots\dots\dots \text{(KK-12)}$$

$$P_{\max} = P_3$$

$$S_d = \text{Design Stress} = 50 - 60 \text{ MPa For CI}$$

$$\text{Assume } S_d = 55 \text{ MPa} \quad \dots\dots\dots \text{(KK-12)}$$

$$t_{1\min} = 7.56 \text{ mm}$$

Considering constant for circulation $c = 5$

$$t_1 = 7.56 + 5 = 12.56 \text{ mm}$$

Step 3: Check for Pressure criteria

$$S_x = \frac{P_{\max} \times D}{2t_1}$$

$$S_x = \frac{5 \times 180}{2 \times 12.56}$$

$$S_x = 35.82 \text{ MPa} < 60 \text{ MPa}, \text{ Hence safe.}$$

Step 4: Check for Thermal criteria

$$S_t = \frac{E\alpha\Delta t}{2(1-\mu)}$$

Where, $E = 10 \times 10^4 \frac{\text{N}}{\text{mm}^2}$, $\alpha = 10.6 \times 10^{-6} \frac{\text{mm}}{\text{mm}^\circ\text{C}}$, $\mu = 0.271$, $\Delta t = 100^\circ\text{C}$ (Assume)

$$S_t = \frac{10 \times 10^4 \times 10.6 \times 10^{-6} \times 100}{2 \times (1 - 0.271)}$$

$$S_t = 72.7 \frac{N}{mm^2}$$

Total Stress = $S_x + S_t = 35.82 + 72.7 = 108.52 \text{ MPa} < 130 \text{ MPa}$, Hence safe.

Step 5: Cylinder head design for water cooled engine

Step 5.1 Cover Thickness of cylinder head

$$\text{Cover thickness, } t_c = K_2 D_p \left[\frac{P_{\max}}{S_d} \right]^{\frac{1}{2}} \quad \dots \dots \dots \text{ (KK-13)}$$

Where, D_p = Pitch circle diameter of bolts = $D + 2l$

$K_2 = 0.54$ for CI, Assuming M20 Belt,

$S_d = 55 \text{ MPa}$ for CI

Assuming M20 Bolt, $l = 1.25db = 25 \text{ mm}$

$D_p = D + 2l = 180 + 2(25) = 230 \text{ mm}$

$$t_c = 0.54 \times 230 \times \left[\frac{5}{55} \right]^{\frac{1}{2}}$$

$$t_c = 37.44 \text{ mm}$$

Thickness of the water pockets for water cooled

$$t_w = 0.03D + 2.2 = 0.03 \times 180 + 2.2 = 7.6$$

$$t_w = 8 \text{ mm}$$

Step 6: Stud or bolt design (KK-15)

Number of Studs, $n = 0.015D + 4$

$$n = (0.015 \times 180) + 4, \quad n = 6.7$$

Let no. of bolts selected = 8

Total Load on Stud,

$$F_t = x \cdot F_{\max} + F_{pl}$$

Where, $F_{\max} = P_{\max} \times \frac{A}{n_b}$, $F_{\max} = \frac{\text{Total Gas Force}}{\text{Number of Bolts}}$

$$F_{\max} = 5 \times \frac{\frac{\pi}{4} \times 180^2}{8} = 15.904 \text{ kN}$$

$$F_{pl} = \text{Preloading force of Stud} = m(1 - x)F_{max}$$

$m = 1.5$ Without Gasket, Coefficient of threaded joint $x = 0.2$

$$F_{pl} = 1.5(1 - 0.2) \times 15.904 = 19.09 \text{ kN}$$

Total Load on Stud

$$F_t = x \cdot F_{max} + F_{pl}$$

$$F_t = 0.2 \times 15.904 + 19.09$$

$$F_t = 22.27 \text{ kN}$$

$$S_d = \frac{F_t}{A_c} \quad (\text{Design Stress for SAE 1120 } S_d = 110 \text{ MPa})$$

$$A_c = \frac{F_t}{S_d} = \frac{22.27 \times 10^3}{110} = 202.46 \text{ mm}^2$$

Selecting Standard Bolt **M20** with $A_c = 251 \text{ mm}^2 \dots \dots \dots (\text{KK} - 17)$

Step 7: Piston Design (KK/8)

Piston material: Aluminum, Design Stress $S_d = 70 \text{ MPa}$

Step 7.1 Using pressure criteria

Crown Thickness using Max. prssure criteria(t_c)

$$t_c = D \left[\frac{3}{16} \times \frac{P_{max}}{S_d} \right]^{\frac{1}{2}} = 180 \times \left[\frac{3}{16} \times \frac{5}{70} \right]^{\frac{1}{2}} \dots \dots \dots (\text{KK-8})$$

$$t_c = 20.83 \text{ mm} ,$$

Let $t_c = 22 \text{ mm}$

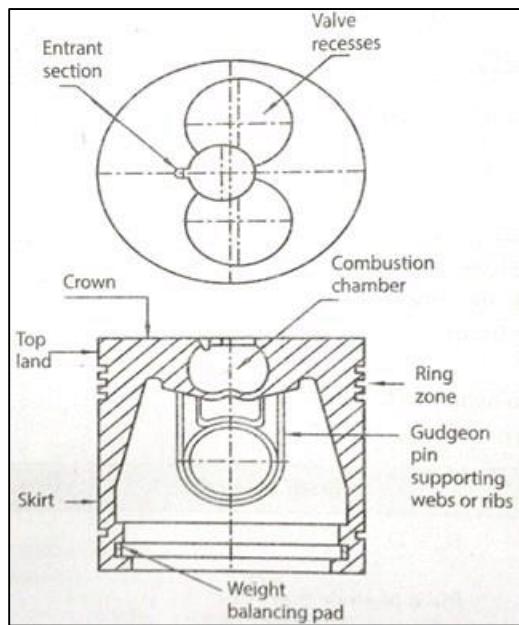


Fig. 4.37 Solid skirt piston

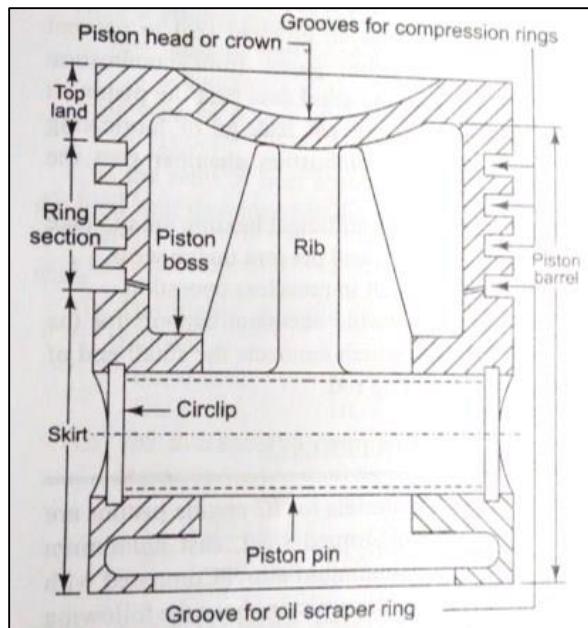


Fig. 4.38 Piston

Step 7.2 Thermal stress criteria

Crown thickness using thermal criteria

$$t_c = \frac{q \times D^2 \times 10^{-2}}{16 \times C_1 \times \Delta T}$$

For Aluminium, $C_1 = 1.986$, $\Delta T = 75^\circ\text{C}$

q = Heat Flux Flowing through Crown

$$q = K \times \text{BSFC} \times C_v \times \frac{P_b}{A} \times n_c$$

Where, $K = 0.05$, $P_b = \text{Brake power} = 50 \text{ kW}$

Brake specific Fuel Consumption, $\text{BSFC} = \eta_{\text{mechanical}} \times \text{ISFC}$

Assume $\text{ISFC} = 0.30 \text{ Kg/hr - Kw}$

$$\text{BSFC} = 0.8 \times 0.3$$

$$\text{BSFC} = 0.24 \text{ Kg/hr - Kw}$$

$$\text{Area of Piston in cm}^2 = \frac{\pi}{4} D^2 = 254.469 \text{ cm}^2$$

C_v = Calorific value of fuel = 42000 KJ/Kg

$$q = 0.05 \times 0.24 \times 42000 \times \frac{50}{254.469} \times 1$$

$$q = 99.03 \text{ kJ/hr - cm}^2$$

$$t_c = \frac{99.03 \times 180^2 \times 10^{-2}}{16 \times 1.986 \times 75} = 13.46 \text{ mm}$$

Step 7.3 Empirical Relation

$$t_c = 0.032D + 1.5$$

$$t_c = 7.26 \text{ mm}$$

Selecting Crown Thickness $t_c = 22 \text{ mm}$ (highest value from pressure, thermal criteria and empirical relation)

Step 8 Design of piston ring

Radial thickness of piston ring From bearing pressure criteria

$$t_r = \left[\frac{3P_w}{S_d} \right]^{\frac{1}{2}}$$

Where, P_w Radial Pressure = $0.03 \frac{\text{N}}{\text{mm}^2}$ and $S_d = 100 \text{ MPa}$ (KK-9)

$$t_r = \left[\frac{3 \times 0.03}{100} \right]^{\frac{1}{2}} = 5 \text{ mm}$$

Using Empirical relation,

For Compression Ring, $t_r = 0.04D = 0.04 \times 180 = 8 \text{ mm}$ (KK-9)

For oil control ring, $t_r = (0.038 \text{ to } 0.043)D = 0.04 \times 180 = 8 \text{ mm}$ (KK-9)

Number of piston Ring, $n = 0.4\sqrt{D}$, $n = 6$

Axial Thickness of piston Ring, $t_a = 0.7t_r = 0.7 \times 8 = 6 \text{ mm}$

Proportionate dimensions of Piston (KK-11)

Length of Piston = $1.1D = 200 \text{ mm}$,

Height of piston top part = $0.6D = 110 \text{ mm}$,

Skirt length = $0.7D = 126 \text{ mm}$

Thickness of Skirt wall = 3 mm

Thickness of piston crown wall = $t_c = 15\text{mm}$

Step 9: Piston pin design

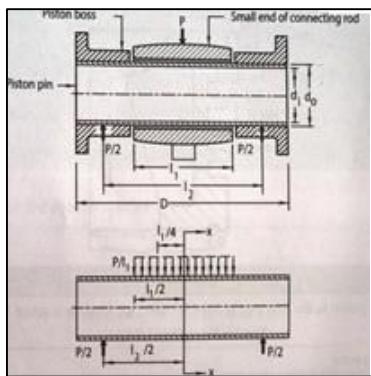


Fig. 4.39 Piston pin and force diagram

Bearing Stress Criteria,

$$F_{\max} = S_b \times A_b, \quad \text{Let } S_b = 18 \text{ MPa},$$

$$A_b = \text{Bearing Area} = L_p \times D_p$$

$$L_p = 1.5d_p$$

$$\text{Also, } F_{\max} = \frac{\pi}{4} D^2 \times P_{\max} = 127.23 \text{ KN}$$

$$127.23 \times 10^3 = 18 \times 1.5 d_p^2$$

$$d_p = 68.62 \text{ mm} = 70 \text{ mm}, \quad L_p = 105 \text{ mm}$$

Step 10: Checking for bending stress

Checking piston pin under bearing stress criteria

Design Stress [$S_d = 110\text{MPa}$] for SAE 1020 Carbon steel material

$$S_d = \frac{M}{Z}, \quad \text{Where, } M = F_{\max} \times \frac{L_p}{8}$$

$$M = \frac{127.17 \times 10^3 \times 105}{8}$$

$$M = 1.6699 \times 10^6 \text{ Nmm}$$

$$Z = \frac{\pi}{32} d_p^3 = 33673.95 \text{ mm}^3$$

$$S_d = 49.59 \frac{N}{\text{mm}^2} < [S_d] \quad \dots\dots(\text{KK-10, T4.4A})$$

Hence safe in bending.

Step 11: Connecting rod design (CR)

..... (KK/19)

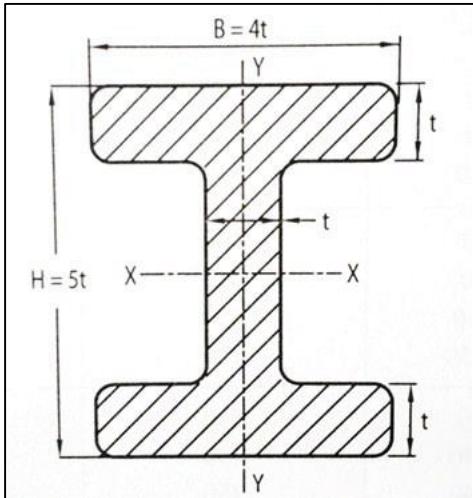


Fig. 4.40 Cross section of connecting rod

Net force = max gas pressure (F_g) – Inertia force (F_i)

$$F_N = \frac{\pi}{4} D^2 P_{max} - W_e \omega^2 r_c \left(1 + \frac{1}{n}\right)$$

Where, $D = 180\text{mm}$, $P_{max} = 5 \frac{\text{N}}{\text{mm}^2}$

W_e = Weight of reciprocating parts = $(W_1 + W_2 + W_3)A_p$

W_1 = Piston Weight for Aluminium = $150 \frac{\text{Kg}}{\text{m}^2}$

W_2 = Weight of connecting Rod = $250 \frac{\text{Kg}}{\text{m}^2}$

W_3 = Unbalance part of crankshaft = 100 Kg/m^2

$$W_e = 500 \times \frac{\pi}{4} 0.18^2 = 12.73 \text{ Kg}$$

$$\omega = \text{Angular Velocity} = \frac{2\pi N}{60} = 209.439 \text{ rad/s}^2$$

$$r_c = \text{Crank radius} = \frac{L}{2} = \frac{180}{2} = 90 \text{ mm} = 0.09 \text{ m}$$

L_c = Length of connecting rod = $4r_c = 2L = 360\text{mm}$

$$n = \frac{L_c}{r_c} = 4$$

Normal load on Connecting Rod,

$$(F_N) = 127234.50 - 62819.725$$

$$(F_N) = 64414.77 \text{ N}$$

$$(F_N) = 64.41 \text{ kN}$$

Now Critical load for buckling on connecting rod,

$$F_{cr} = F_c \times F_s$$

$$\text{Where, } F_c = \text{Load on connecting rod} = \frac{F_N}{\left(1 - \frac{\sin^2 \theta}{n^2}\right)^{\frac{1}{2}}}$$

Where, θ = crank angle at max. Gas force for IC engine, $\theta = 5$ to 10

$$\text{Let, } \theta = 8 \text{ and } F_s = 1.2, n = 4, \quad F_{cr} = 64.449 \times 1.2 = 77.33 \text{ KN}$$

Now, based on buckling strength F_{cr} minimum section dimension can be calculated as follows

$$F_{cr} = \frac{S_d A_s}{1 + \frac{1}{a} \left(\frac{l_c}{k}\right)^2} \quad \dots\dots\dots (\text{KK-19})$$

$$\text{Let, } S_d = 300 \text{ MPa}$$

$$A_s = 11t^2, 1/a = 1/7500 \text{ For steel}$$

$$L_c = 360 \text{ mm}$$

$$K = \sqrt{\left(\frac{I_{xx}}{A}\right)} = 1.78t$$

$$F_{cr} = 77.33 \times 10^3 = \frac{300 \times 11t^2}{1 + \frac{1}{7500} \left(\frac{360}{1.78t}\right)^2}$$

$$t = 2.429 \text{ mm}, \quad \text{Let } t = 3 \text{ mm}$$

Step 11.1 Based on buckling strength

$$BM_{max} = \frac{\rho A r_c \omega^2 l_c^2}{9\sqrt{3}} \times 10^{-12}$$

$$\rho = 7840 \frac{\text{kg}}{\text{mm}^3}, A = 99 \text{ mm}^2, l_c = 360\text{mm}, \omega = 209.439 \frac{\text{rad}}{\text{s}}, r_c = 90\text{mm}$$

$$BM_{\max} = 25474.79 \text{ Nmm}$$

Now, $B = 4t = 12 \text{ mm}, H = 5t = 15 \text{ mm}$

$$b = h = 3t = 9\text{mm}$$

Section modulus,

$$Z = \frac{BH^3 - bh^3}{6H}$$

$$Z = \frac{12 \times 15^3 - 9 \times 9^3}{6 \times 15} = 377.1 \text{ N/mm}^2$$

Step 11.2 Check for bending stress in CR

Induced Bending stress,

$$(S_b) = \frac{M}{Z} = \frac{25474.79}{377.1} = 67.554 \frac{\text{N}}{\text{mm}^2}$$

$(S_b) < [S_d]$, Hence Safe in bending

Step 11.3 Design of small end of Connecting Rod

Inside diameter of small end $d_{pi} = dp = 70 \text{ mm}$

Inside diameter of small end with bush $d_{pi} = 1.1dp = 77 \text{ mm}$

outside diameter of small end $d_{po} = 1.25dp = 87.5 \text{ mm}$

length of small end $l_s = 0.3D = 54 \text{ mm}$

Radial thickness of wall bushing

$$T_{rb} = 0.06dp = 4.2 \text{ mm}$$

$$\text{Bearing pressure, } P_b = \frac{\pi D^2 P_{\max}}{4l_s d_p}$$

$$P_b = 33.66 \frac{\text{N}}{\text{mm}^2} < [P_{bs} = 140 \text{ N/mm}^2], \text{ Hence Safe.}$$

Step 12: Design of crankshaft

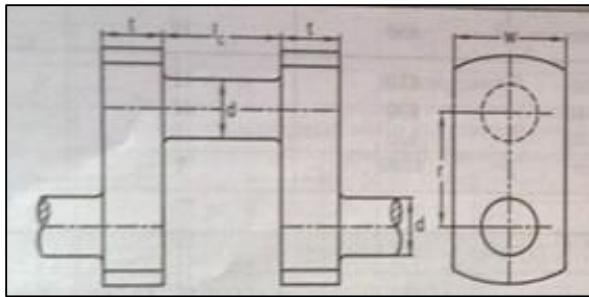


Fig. 4.41 Central crankshaft

Using bending moment criteria

$$M_{\max} = K \times F_{\max} \times L_p$$

$$K = \frac{1}{4} \text{ For centre crank}$$

$$d_c = 0.6D = 108 \text{ mm}$$

$$L_p = 432 \text{ mm}$$

$$F_{\max} = 31.676 \text{ kN}$$

$$M_{\max} = \frac{1}{4} \times 31.676 \times 10^3 \times 432 ,$$

$$M_{\max} = 3421008 \text{ Nmm}$$

Selecting Material MS,

$$[S_d] = 100 \frac{\text{N}}{\text{mm}^2}$$

$$Z = \frac{M_{\max}}{S_d} = 34210.08$$

$$\frac{\pi}{32} d^3 = 34210.08$$

$$d = 75 \text{ mm}$$

Step 12.1 Checking for bearing pressure criteria

$$F_{\max} = P_b \times l_{cp} \times d_c$$

$$(P_b) = \frac{F_{\max}}{l_{cp} \times d_c}$$

$$l_{cp} = 0.8d_c = 85 \text{ m}$$

$$(P_b) = \frac{31.676 \times 10^3}{85 \times 75} = 5 \frac{\text{N}}{\text{mm}^2} < \left[P_b = 20 \frac{\text{N}}{\text{mm}^2} \right], \text{ Hence safe.}$$

Width of crank web (b) = $1.2dc = 1.2 \times 75 = 90$ mm

Thickness of crank web (h)

$$h = 0.25d_c = 19 \text{ mm}$$

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Module 5

Design of Pump

A pump is a mechanical device which converts the mechanical energy supplied to it from external source to hydraulic energy and transfer the same to the liquid, hence increasing the pressure energy of the liquid which is converted into potential energy, as the liquid is lifted from lower to higher level.

5.1 Pump Classification

Pumps classification is shown in figure 5.1

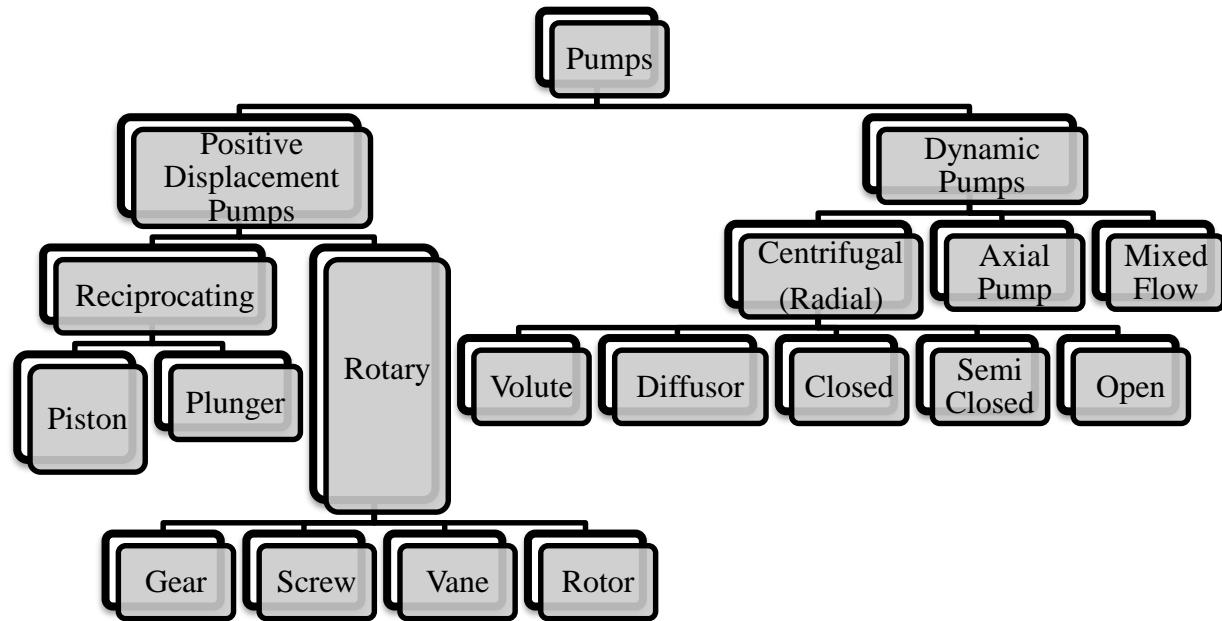


Fig. 5.1 Pump Classification

5.2 Centrifugal Pump

Centrifugal pumps are a sub-class of dynamic axisymmetric work-absorbing turbo machinery. Centrifugal pumps are used to transport fluids by the conversion of rotational kinetic energy to the hydrodynamic energy of the fluid flow. The rotational energy typically comes from an engine or electric motor. The fluid enters the pump impeller along or near to the rotating axis and is accelerated by the impeller, flowing radially outward into a diffuser or volute chamber (casing), from where it exits. Common uses include water, sewage, petroleum and petrochemical pumping.

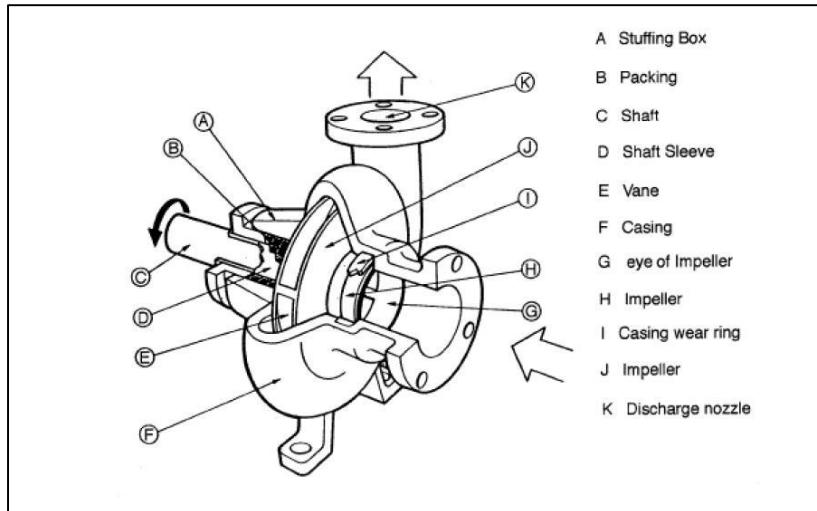


Fig. 5.2 Centrifugal pump components

5.2.1 Components and working

Various components of Centrifugal pump is shown in figure 5.2.

5.2.1.1 Impeller

The impeller is the main rotating part that provides the centrifugal acceleration to the fluid that is to impart kinetic energy to liquid.

Basic requirements of impeller are,

1. It should have adequate mechanical strength.
2. It should have anti-wear and anti-corrosion material properties.
3. It should have low density, good cast ability, weld ability, cheap.

They are often classified in many ways.

Based on major direction of flow in reference to the axis of rotation

- Radial flow , Axial flow and Mixed flow

Based on suction type,

- Single-suction: Liquid inlet on one side.
- Double-suction: Liquid inlet to the impeller symmetrically from both sides.

Based on mechanical construction:

- Closed: Shrouds or sidewall enclosing the vanes.
- Open: No shrouds or wall to enclose the vanes.
- Semi-open or vortex type.

Closed type is used for less viscous fluid, semi-closed is used for semi viscous fluid and open type impeller is used for high viscous fluid. The material for the impeller is generally cast iron or Steel. Steel is used for controlling the size of the impeller but widely used material is cast iron as it is easily available and cheap.

The number of impellers determines the number of stages of the pump. A single stage pump has one impeller only and is best for low head service. A two-stage pump has two impellers in series for medium head service. A multi-stage pump has three or more impellers in series for high head service.

5.2.1.2 Impeller nut

Function of Impeller nut is to lock the impeller in its proper axial position and to prevent axial movement due to hydraulic thrust.

5.2.1.3 Blades

Blades of the impeller should have adequate mechanical strength it should have less fluidic friction and shockless entry and exit.

There are three types of blades shown in figure 5.3.

1. Forward type
2. Radial type
3. Backward types.

Forward type of blades are used for compressor and fans and its efficiency is 75%. Radial types of blades are used for blowers and fan, its efficiency is 80 to 85% and backward type blades are used for Centrifugal Pump its efficiency is 85 to 90%.

Now theoretically the number of blades required are Infinity for preventing the back circulation flow but increasing the number of blades frictional losses increases, so the compromisation between two is required. Normally the number of blades are 2 to 3 for open type of impeller and 6 to 12 for closed type of impeller. As per the pump manufacturers the number of blades (Z) are given by the following formula.

$$Z = 6.5 \times \frac{(D_2 - D_1)}{(D_2 + D_1)} \sin\left(\frac{\beta_2 + \beta_1}{2}\right)$$

Where, D_1 , D_2 are inlet and outlet diameter of the impeller and β_1 , β_2 are entry and exit angle of blades.

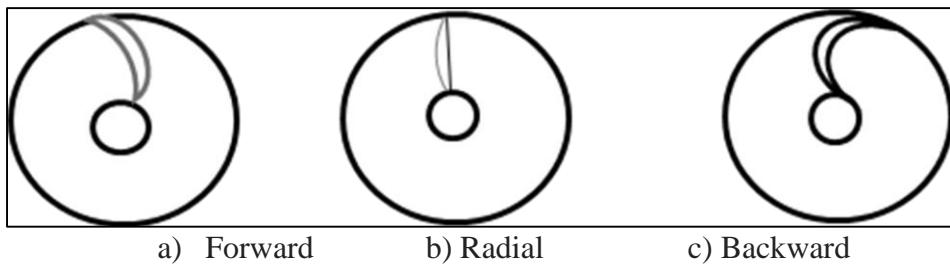


Fig. 5.3 Types of blades

5.2.1.4 Casing

The function of casing is to accommodate the incoming liquid from the impeller, to maintain the uniform velocity of the liquid and to convert the partly kinetic energy into Pressure energy. Casings are generally of two types, volute and circular. The impellers are fitted inside the casings. The casing contains the liquid and acts as a pressure containment vessel that directs the flow of liquid in and out of the pump. In most cases the casing includes the suction and discharge nozzles of the

pump that connect it to the external piping. In some vertical pumps the casing may be referred to as bowl, and in some small pumps it called as housing. Casing incorporates nozzles to connect suction & discharge piping, it directs flow into & out of the impeller and provides support to the bearing bracket also.

Casing should have gradual increase of area, friction losses in casing should be minimum and it should be easy in assembly so longitudinally split casing is widely used.

For construction of volute casing throat velocity is assumed as thirty percent of exit velocity of the impeller as per manufacturing catalogue.

5.2.1.5 Wear rings

Wear ring provides an easily and economically renewable leakage joint between the impeller and the casing. Clearance becomes too large the pump efficiency will be lowered causing heat and vibration problems. Most manufacturers require that you disassemble the pump to check the wear ring clearance and replace the rings when this clearance doubles. Functions of wear ring are,

- a) To control the leakage losses across the annular path between impeller and wear ring
- b) To protect the rotating impeller from rubbing with the stationary casing.
- c) To provide a replaceable wear joint.

5.2.1.6 Shaft

The shaft is usually the longest part of a pump and is made of one piece. Its function is to transmit the input power from the driver into the impeller. In a close-coupled pump, the motor has an extended length of shaft that acts as the pump shaft. In some vertical pumps, the so-called line shaft pump, the shaft may be supplied in more than one piece of ten Feet sections depending on the pump bowl setting below the ground surface. The basic function of a centrifugal pump shaft is to transmit the torques encountered when starting and during operation while supporting the impeller and other rotating parts.

Failure of the shaft may take place due to bending of shaft by Impeller weight (W_i) and Dynamic force (F_d), also by tensile or compression due to Axial thrust.

Following assumptions can be made for easy design of the shaft,

- a) Weight of shaft, coupling can be neglected.
- b) Dynamic load calculation based on average radius.
- c) Axial thrust due to change in direction can be neglected.
- d) Inlet can be considered as axial and exit as radial direction.

5.2.1.7 Shaft sleeve

The functions of shaft sleeve are:

- a) To protect the shaft from erosion & corrosion
- b) To enhance the stiffness of the rotating element
- c) To protect the shaft from abrasion wear at packed stuffing box or at leakage joints

5.2.1.8 Coupling

The function of a coupling is to connect the pump shaft and the driver shaft, and to transmit the input power from the driver into the pump.

5.2.1.9 Bearings

The functions of the bearings are to support the weight of the shaft (rotor) assembly, to carry the hydraulic loads acting on the shaft, and to keep the pump shaft aligned to the shaft of the driver. Taper roller bearing of deep groove ball bearing can be used for the impeller shaft as radial and axial forces are exist. The axial thrust can be considered as 30 % to axial load as thrust holes are provided on the base plate in high pressure side near the hub level.

5.2.1.10 Back plate

The back plate is of pressed steel manufacture, which together with the pump casing forms the actual fluid chamber in which the fluid is transferred by means of the impeller.

5.2.1.11 Bearing housing

The bearing housing is used to enclose and protect the shaft bearings, ensuring proper alignment. The housing will also include some type of method for lubricating the bearings and cooling the pump.

5.2.2 Advantages of centrifugal pump:

1. Small size, space saving and less capital costs.
2. Maintenance is easy.
3. No danger creates if discharge valve is closed while starting.
4. Large volume of fluid can be handled.
5. Ability to work at medium to low head.
6. Ability to work with medium to low viscous fluid.

5.2.3 Disadvantages of centrifugal pump:

1. Cavitation may occur.
2. Wear of the impeller can be worsened by suspended solids.
3. Corrosion inside the pump caused by the fluid properties.
4. Overheating due to low flow.
5. Leakage along rotating shaft.
6. Priming is required, that is a centrifugal pumps must be filled (with the fluid to be pumped) in order to operate.

5.2.4 Applications

Centrifugal pump are widely used in almost all kind of industries. Some of the applications are mentioned as below.

1. Textile - Bleaching of fabrics and silks.
2. Food - Sugar refining, bleaching, disinfecting.
3. Waste Water/Chemicals - Purifying water, deodorizing sewage, industrial effluents.
4. Pulp & Paper - Bleaching pulp, fireproofing and wood preserving.
5. Agriculture - Fertilizer, fungicide, and insecticide manufacturing.
6. Electronics - Chip and metal cleaning, acid waste transfer.
7. Steel - Steel and Stainless steel picking, acid recovery and acid transfer.
8. Oil & Gas - Well acidizing, petroleum purification.
9. Chemical - Catalyst transfer, acid transfer and neutralizing.
10. Metal Treating - Anodizing, electroplating, plating.

5.3 Heads

Head is the height to which the liquid is raised. The suction head (H_s) is the height of the centre of the impeller above the liquid surface in the sump. The delivery head (H_d) is the height to which the liquid is raised above the centre line of the impeller. Suction head and delivery head together ($H_s + H_d$) is called as static head, which is net total vertical height through which the liquid is lifted by the pump.

Low pressure at the suction side of a pump may cause the fluid to evaporate with reduced efficiency, cavitation and damage of the pump as a result. Vaporization starts when the pressure in the liquid is reduced to the vapour pressure of the fluid at the actual temperature. To characterize the potential for boiling and cavitation the difference between the total head on the suction side of the pump close to the impeller, and the liquid vapour pressure at the actual temperature can be used.

5.3.1 Suction Head

Based on the Energy Equation - the suction head in the fluid close to the impeller can be expressed as the sum of the static and velocity head:

$$h_s = (P_s / \gamma_{\text{liquid}}) + (V_s^2 / 2 g)$$

Where, h_s = suction head close to the impeller (m, in)

P_s = static pressure in the fluid close to the impeller (Pa (N/m²))

γ_{liquid} = specific weight of the liquid (N/m³)

V_s = velocity of fluid (m/s)

$$g = \text{acceleration of gravity (9.81 m/s}^2)$$

One cannot measure the suction head "close to the impeller". In practice one can measure the head at the pump suction flange. Depending of the design of the pump - the contribution to the NPSH value from the suction flange to the impeller can be substantial.

5.3.2 Liquids Vapour Head

The liquids vapour head at the actual temperature can be expressed as:

$$h_v = P_v / \gamma_{\text{vapour}}$$

Where,

$$h_v = \text{vapour head (m, in)}$$

$$P_v = \text{vapour pressure (m, in)}$$

$$\gamma_{\text{vapour}} = \text{specific weight of the vapour (N/m}^3)$$

5.3.3 Net Positive Suction Head - NPSH

The Net Positive Suction Head - NPSH - can be defined as the difference between the Suction Head, and the Liquids Vapour Head and can be expressed as,

$$\text{NPSH} = h_s - h_v$$

$$\text{NPSH} = (P_s / \gamma) + (V_s^2 / 2 g) - (P_v / \gamma)$$

5.3.4 Available NPSH – NPSH a or NPSHA

The Net Positive Suction Head available from the application to the suction side of a pump is often named NPSHa. The NPSHa can be estimated during the design and the construction of the system, or determined experimentally by testing the actual physical system. The available NPSHa can be estimated with the Energy Equation.

For a common application - where the pump lifts a fluid from an open tank at one level to another, the energy or head at the surface of the tank is the same as the energy or head before the pump impeller and can be expressed as:

$$h_0 = h_s + h_l$$

Where, h_0 = head at surface (m, in), h_s = head before the impeller (m, in)

h_l = head loss from the surface to impeller - major and minor loss in the suction pipe (m, in)

In an open tank the head at the surface can be expressed as:

$$h_0 = (P_0 / \gamma) = (P_{\text{atm}} / \gamma)$$

For a closed pressurized tank the absolute static pressure inside the tank must be used.

The head before the impeller can be expressed as:

$$hs = (Ps / \gamma) + (Vs^2 / 2 g) + he$$

Where,

h_e = Elevation from surface to pump

(Positive if pump is above the tank, negative if the pump is below the tank (m, in))

$$(P_{atm} / \gamma) = (Ps / \gamma) + (Vs^2 / 2 g) + he + hl$$

The head available before the impeller can be expressed as:

$$(Ps / \gamma) + (Vs^2 / 2 g) = (P_{atm} / \gamma) - he - hl$$

Or as the available NPSHa:

$$NPSH_a = (P_{atm} / \gamma) - he - hl - (P_v / \gamma)$$

Where, $NPSH_a$ = Available Net Positive Suction Head (m, in)

5.3.5 Required NPSH - $NPSH_r$ or $NPSH_{r,s}$

The $NPSH_r$, called as the Net Suction Head as required by the pump in order to prevent cavitation for safe and reliable operation of the pump. The required $NPSH_r$ for a particular pump is in general determined experimentally by the pump manufacturer and a part of the documentation of the pump. The available $NPSH_a$ of the system should always exceed the required $NPSH_r$ of the pump to avoid vaporization and cavitation of the impellers eye. The available $NPSH_a$ should in general be significant higher than the required $NPSH_r$ to avoid the head loss in the suction pipe and in the pump casing, as the local velocity, acceleration and pressure decreases also the fluid start boiling on the impellers surface. The required $NPSH_r$ increases with the square of capacity. Pumps with double-suction impellers has lower $NPSH_r$ than pumps with single-suction impellers. A pump with a double-suction impeller is considered hydraulically balanced but is susceptible to an uneven flow on both sides with improper pipe-work.

5.3.6 Monomeric head

The monomeric head is defined as the head against which a centrifugal pump has to work. It is denoted by H_m . This is defined by British Standards as the sum of the actual lift, the friction losses in the pipes and the discharge velocity head. However, for special pumps allowance must also be made for the velocity of flow towards the suction intake and any pressure differences at the water surfaces in the supply and receiving tanks.

Thus: $H_m = h + h_f + (V_d^2/2g) = (P_2 - P_1)/w + (V_2^2 - V_1^2/2g)$

Commonly the suction and delivery pipes are of equal diameter. In which case:

$$H_m = [(P_2 - P_1)/w]$$

If the two pressures are registered on different gauges. A correction must be made for any difference in the datum heights of the gauges.

5.4 Water hammer

In a piping installation, energy necessary to move the water through the pipeline is achieved by the pump. If a valve is suddenly closed at the end of the discharge line, the moving water column is brought to a stop. The kinetic energy given to the water column must be dissipated. Since flow is stopped, the water column gets compressed, the pressure rises and some of the kinetic energy is transformed to internal energy. If the pressure raise in the pipe is sufficient it may rupture the pipe. The pressure wave which started at the closed valve travels back upstream continuing its dissipation of energy within the water and pipe wall. This will continue until the kinetic energy is fully converted to internal energy.

When there is sudden power failure, the kinetic energy of the rotating parts of the pump becomes so small that it cannot maintain flow against the discharge head. The pump speed reduces and the flow through the pump reverses. The pump starts immediately behaving like a turbine.

The above phenomenon is perceived as a series of shocks sounding like hammer blows and known as water hammer. The water hammer effects are obtained from the equations below:

For instantaneous valve closing the maximum pressure rise above normal pressure in meters is given by h_{max} and calculated as

$$h_{max} = \frac{aV}{g}$$

For partial valve closure, the pressure rise and fall is given by

$$h = \frac{a}{g} (V_1 - V_2) = \frac{a}{g} \cdot \Delta V$$

Where,

a = Velocity of pressure wave in m/s

V = Velocity of water just before valve closure in m/s

V_1, V_2 = Initial and final velocity of water in m/s during valve closure

$g = 9.81$ m/s²

The magnitude of the velocity of pressure valve is determined by the following formula

$$a = \frac{1}{\sqrt{\frac{\gamma}{g} \left(\frac{1}{K} + \frac{d}{t \cdot E} \right)}}$$

Where, γ = Specific weight of liquid,

K = Bulk modulus of liquid,

d = Pipe diameter,

t = Pipe wall thickness,

E = Modulus of elasticity of pipe material,

For cold water, K = 21000 kg/cm² for pressure up to 35 atm gauge.

Considering E = 2.1 x 10⁶ kg/cm² for steel, the above equation becomes,

$$a = \frac{1440}{\sqrt{1 + \frac{d}{100t}}}$$

5.4.1 Protective measures against water hammer

To reduce the effect of water hammer the rise in pressure has to be checked and that is possible by reducing the flow velocity. The following means can help in accomplishing the reduction of velocity.

1. Pipe systems designed with low velocity.
2. Automatically controlled quick closing valve.
3. Pressure relief valves.
4. Accumulators / surge tanks / air vessels.
5. Flywheels.
6. Automatically controlled bypasses.

5.5 Churning effect

In centrifugal pump, there is no pressure development if delivery valve is closed, there is no harm only churning will be there. Delivery valve is only safety valve in case of centrifugal pump but in case of gear pump if relief valve is closed, pressure will continuously built up and may cause bursting. When a pump is running with its discharge valve closed for longer time, the temperature inside pump rises continuously as the same liquid is continuously rotating inside pump. Due to rise in temperature of liquid, metal component's temperature also begins to rise. As the mass of impeller is much less than the pump casing. The expansion due to heat on impeller is higher than the casing. A time comes when impeller starts rubbing against the casing which further increases its temperature and the impeller seizes with casing. This is the effect of churning.

5.6 Cavitation

Cavitation is the formation of bubbles or cavities in liquid, developed in areas of relatively low pressure around an impeller. The collapsing of these bubbles generates the intense shockwaves inside the pump, causing significant damage to the impeller and the pump housing. Pump cavitation may cause:

1. Failure of pump housing
2. Destruction of impeller
3. Excessive vibration - leading to premature seal and bearing failure
4. Higher than necessary power consumption
5. Decreased flow and/or pressure

There are two types of pump cavitation: suction and discharge.

If a pump is experiencing cavitation, following things can be done to troubleshoot the problem:

- a) Checking filters and strainers - clogs on the suction, or discharge side can cause an imbalance of pressure inside the pump
- b) Reference the pump's curve - Use a pressure gauge and/or a flow meter to understand where a pump is operating on the curve. Make sure it is running at its best efficiency point.
- c) Re-evaluating pipe design and ensuring the path the liquid takes to get to and from a pump is ideal for the pump's operating conditions.

5.7 Stuffing box

Stuffing box is used to seal off the passage around the rotating shaft and thereby to prevent leakage of liquid along the shaft. The stuffing box is entirely filled with packing which is pressed tight by means of glands.

Stuffing box refers to a chamber, either integral with or separate from the pump case housing that forms the region between the shaft and casing where sealing media are installed. When the sealing is achieved by means of a mechanical seal, the chamber is commonly referred to as a Seal Chamber. When the sealing is achieved by means of packing, the chamber is referred to as a Stuffing Box. Both the seal chamber and the stuffing box have the primary function of protecting the pump against leakage at the point where the shaft passes out through the pump pressure casing. When the pressure at the bottom of the chamber is below atmospheric, it prevents air leakage into the pump. When the pressure is above atmospheric, the chambers prevent liquid leakage out of the pump. The seal chambers and stuffing boxes are also provided with cooling or heating arrangement for proper temperature control.

5.8 Operation of centrifugal pump

Centrifugal pumps are used to induce flow or raise pressure of a liquid. Its working is simple. Fig. 5.4 shows a layout of Centrifugal pump. At the heart of the system impeller is present. It has a series of curved vanes fitted inside the shroud plates. The impeller is always immersed in the water. When the impeller is made to rotate, it makes the fluid surrounding to rotate. Since the rotational mechanical energy is transferred to the fluid, at the discharge side of the impeller, both the pressure and kinetic energy of the liquid will rise. At the suction side, liquid is getting displaced, so a negative pressure will be induced at the eye. Such a low pressure helps to pull fresh liquid stream into the system again, and this process continues.

The negative pressure at the eye of the impeller helps to maintain the flow in the system. If no liquid is present initially, the negative pressure developed by the rotating air, at the eye will be negligibly small to pull fresh stream of liquid. As a result the impeller will rotate without charging and discharging any liquid content. So the pump should be initially filled with liquid before starting it. This process is called as priming. The impeller is fitted inside a casing. As a result the liquid moves out will be collected inside it, and will move in the same direction of rotation of the impeller, to the discharge nozzle.

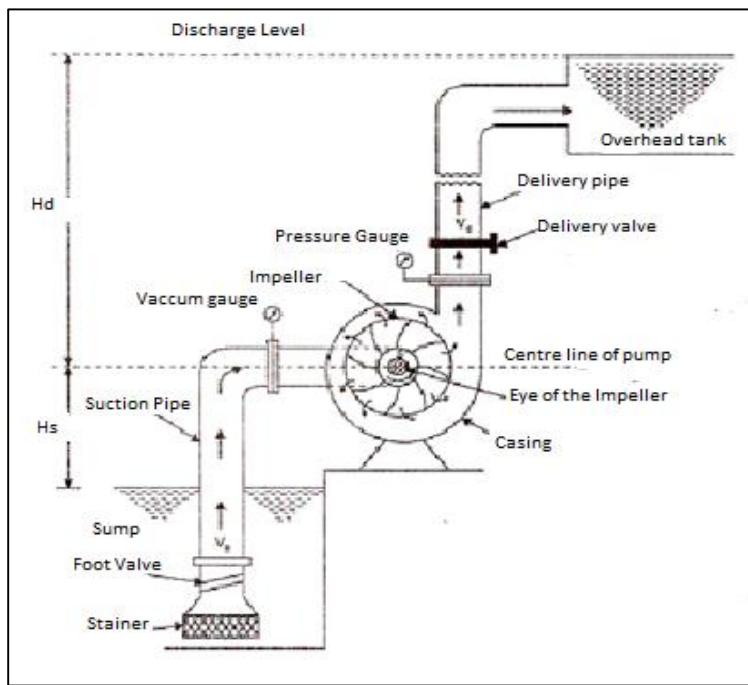


Fig. 5.4 Layout of Centrifugal pump

5.9 Specific Speed

It is the outcome of dimensional analysis of the pump operation in geometrically similar condition. Specific speed is defined as the speed of rotation of a geometrically similar pump of such a size that it ensures a delivery of 75 litres per second discharge at a height of 1 metre. If Discharge Q, Speed N and Head H are known for the pump, specific speed for the pump is given by,

$$N_{s1} = 3.65 \frac{N * \sqrt{Q}}{H^{3/4}}$$

Many designer often use,

$$Ns = \frac{N * \sqrt{Q}}{H^{3/4}}$$

Where pump characteristics are reduced by 3.65 times. Specific speed is not affected by the specific weight of the liquid being handled. Specific speed plays important role in the selection of the pump. By knowing the specific speed of the pump, the performance of the pump can be predicted. Generally single stage centrifugal pump is design for the specific speed range 10 m/s to 30m/s. For $N_s < 10$ m/s multistage centrifugal pump or reciprocating pump is designed.

5.10 Pump Characteristics Curves

The performance of a centrifugal pump can be shown graphically on a characteristic curve. A typical characteristic curve shows the total dynamic head, brake horsepower, efficiency, and net positive Suction head all plotted over the capacity range of the pump. Figures below shows non-dimensional curves which indicate the general shape of the characteristic curves for the various types of pumps. They show the head, brake horsepower, and efficiency plotted as a percent of their values at the design or best efficiency point of the pump. Fig. 5.5 below shows that the head curve for a radial flow pump is relatively flat and that the head decreases gradually as the flow increases. Note that the brake horsepower increases gradually over the flow range with the maximum normally at the point of maximum flow.

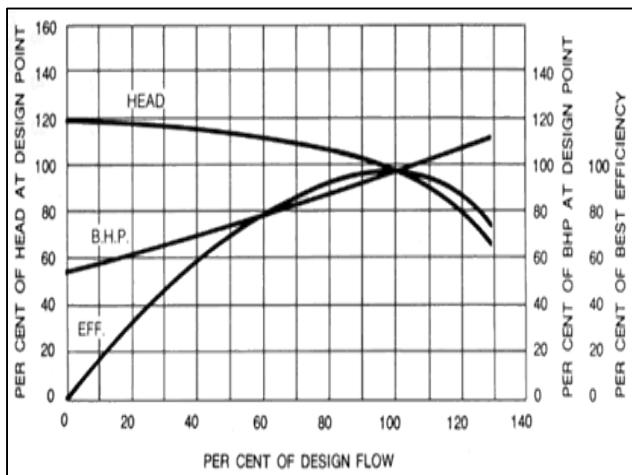


Fig 5.5 Radial Flow Pump

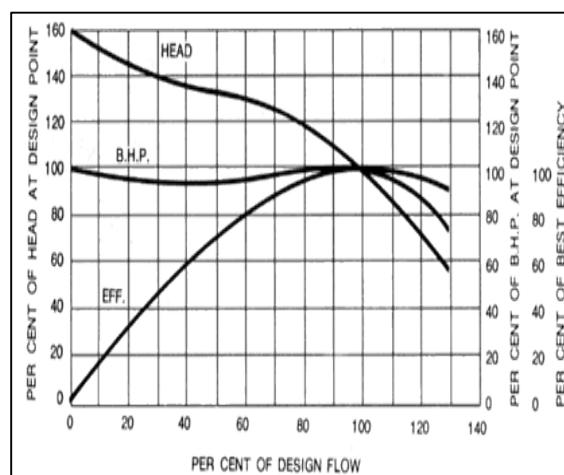


Fig 5.6 Mixed Flow Pump

Mixed flow centrifugal pumps and axial flow or propeller pumps have considerably different characteristics as shown in Figure 5.6 and 5.7 respectively. The head curve for a mixed flow pump is steeper than for a radial flow pump. The shut-off head is usually 150% to 200% of the design head, the brake horsepower remains fairly constant over the flow range. For a typical axial flow pump, the head and brake horsepower both increase drastically near shutoff as shown in Fig. 5.7. The distinction between the above three classes is not absolute and there are many pumps with characteristics falling somewhere between the three. For instance, the Francis vane impeller would have a characteristic between the radial and mixed flow classes.

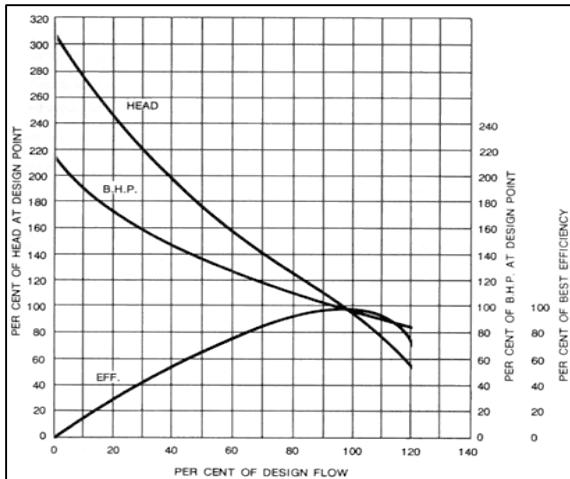


Fig 5.7 Axial Flow Pump

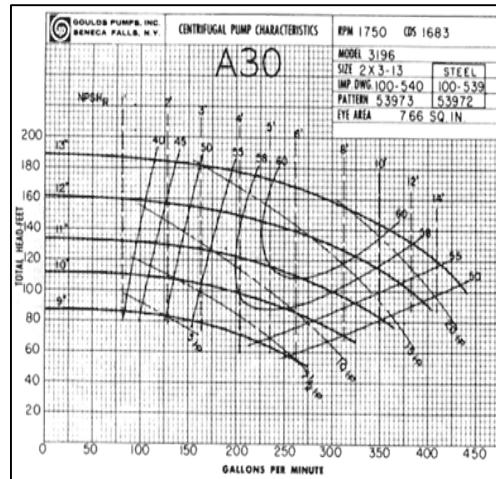


Fig 5.8 Composite Performance Curve

Fig. 5.8 shows a typical pump curve as furnished by a manufacturer. It is a composite curve which tells at a glance what the pump will do at a given speed with various impeller diameters from maximum to minimum. Constant horsepower, efficiency, and $NPSH_R$ lines are superimposed over the various head curves. It is made up from individual test curves at various diameters.

5.11 Effect of Discharge Angle

Due to the vast application it is very important that centrifugal pump should work efficiently. There have been continuous efforts to improve the performance of centrifugal pumps. There are still many unknown issues associated with increasing the efficiency in these pumps, which need to be investigated. Some of the key studies are based on the modification of pump geometry, especially impeller and diffuser. Since the impeller is an active part that adds energy to the fluid, its geometry plays a major role in the centrifugal pump performance. Any change in the impeller geometry would have an impact on the impeller inlet or exit velocity triangles, which may result in significant performance change. The blade exit angle have very important role in the performance of the centrifugal pump.

- The blade exit angle has significant and equal effect on the head and the efficiency.
- With the increase in blade exit angle the performance of the centrifugal pump increases.
- There may be some inaccuracy due to the complication of the geometrical dimensions.

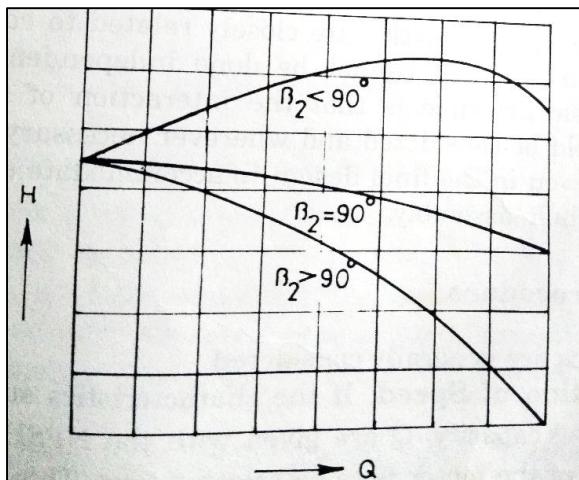


Fig. 5.9 Effect of discharge angle on performance characteristics

A pump having a rising characteristics with $\beta_2 < 90$ is generally used when the actual lift is small, volume is less and the friction head high, varying with the rate of discharge. A pump having a flat characteristics with $\beta_2 = 90$ is desirable where a little variation in total head is encountered and the discharge rate fluctuates. A pump with steeply falling characteristics where $\beta_2 > 90$ would be desirable in pumping out dry docks where the head constantly increases. With such a pump the rate of discharge would decrease as the head increased and there would be no danger of over loading the driving motor.

5.12 Basic requirement for Trouble Free Operation

In general there are two basic requirements that have to be met at all the times for a trouble free operating and longer service life of centrifugal pumps. The first requirement is that no cavitation of the pump occurs throughout the broad operating range. The second requirement is that a certain minimum continuous flow is always maintained during operation. A clear understanding of the concept of cavitation, its symptoms, its causes, and its consequences is very much essential in effective analyses and troubleshooting of the cavitation problem. There are many forms of cavitation, each demanding a unique solution, there are a number of unfavourable conditions which may occur separately or simultaneously when the pump is operated at reduced flows.

Some unfavourable conditions include:

- a) Cases of heavy leakages from the casing, seal, and stuffing box.
- b) Deflection and shearing of shafts.
- c) Seizure of pump internals.
- d) Close tolerances erosion.
- e) Separation and cavitation.
- f) Product quality degradation.
- g) Excessive hydraulic thrust.
- h) Premature bearing failures.

Each condition may dictate a different minimum flow low requirement. The final decision on recommended minimum flow is taken after careful analysis by the pump user and the manufacturer. The consequences of prolonged conditions of cavitation and low flow operation can be disastrous for both the pump and the process. Such failures in services have often caused damaging, resulting in loss of machine, production, and worst of all, human life. Thus, such situations must be avoided at all cost whether involving modifications in the pump and its piping or altering the operating conditions. Proper selection and sizing of pump and its associated piping can not only eliminate the chances of cavitation and low flow operation but also significantly decrease their harmful effects.

5.13 Vibration and noise in centrifugal pump

Mechanical problems that exist in a pump are vibration and sound. Vibrations are caused by manufacturing defects or improper installation. The possible sources of vibration are mechanical or hydraulic. Up to certain limit vibration is normal and when it exceeds acceptable limit it becomes a problem. A vibration analyser determines the source and causes of the vibration, thus becoming an effective tool for preventive maintenance. It measures the amplitude, frequency and phase of vibration. Also when vibration offers at several frequencies, it filters one frequency from another so that each individual vibration characteristic can be measured and probable cause of the vibration can be identified.

Vibration amplitude is an important parameter which indicates the approximate severity of the dynamics stress level in the Pump. It is measured in terms of peak to peak displacement and is indicated on vibration meter. Vibration reading must be recorded regularly to have maintenance reliability program with pumps. Readings at the pump suction and discharge flanges and at each bearing in three planes, horizontal, vertical and axial may be recorded and analysed to determine the most likely cause of vibration. Abrupt change in the readings are a sign of impending failure. A gradual increase in the vibration indicates the deteriorating condition of the pump and helps in taking a corrective measures before it reaches a dangerous level.

Mechanical vibrations in certain cases are sufficient to result in fatigue failure, if timely rectification work is not taken care. Other undesirable effects observed in pumping units are flow pulsations leading to noisy operation and increased flow turbulence associated with higher pressure drops, erosions, damage of seals, leakage at critical joints etc. An unbalanced impeller or other unbalanced rotating mass is a source of excitation of periodic perturbing force which causes forced vibration. Unless the rotating element is very carefully balanced or supported on an elastic foundation for the purpose of vibration isolation, forced vibration with a frequency equal to the rotating speed may occur in nearby structures leading to possible failures. A piping system vibrates with frequency equal to the speed of rotating element or the pulses of reciprocating piston clearly indicating the source of excitation which can be corrected by balancing of impeller, changing the speed of rotating elements, inclusion of vibration dampeners, pulsation snubbers etc.

To control the noise at its source designers provides shielding of a noise source by a partial enclosure combined with sound absorption treatment or total enclosure depending upon the

working of a pump. This noise control measures also protects the operator from the noise of his own machine.

In centrifugal pumps the reasons of noise are,

- 1) Imbalanced impeller
- 2) Speed nearer to critical speed
- 3) Worn out bearings, seals
- 4) Shaft seating eccentric
- 5) Misaligned couplings.

Liquid noise sources like turbulence, flow separation, cavitation and water hammer are the sources of noise which generally excite either the piping or the pump itself into mechanical vibration.

The noise control measures are done in following ways,

- 1) Installation of Acoustic hoods.
- 2) Installation of enclosure with insulation cladding.
- 3) Provision of silencers in intake and exhaust system.
- 4) Provision of acoustic filter element in silencer.
- 5) Stopping of equipment vibrations by providing anti vibration mountings.
- 6) Adequate lubrication of moving parts on machines.
- 7) Balancing of rotating components of machines.
- 8) Proper tightening of bolts at locations.
- 9) Confining the noisy mercenary in separate enclosure.

To control the noise at design stage fallowing points are considered.

- 1) Pump speed should not be same as system resonance speed.
- 2) Liquid pressure should be more to avoid cavitation.
- 3) Suction lifts should be less.
- 4) Rotating or oscillation elements should be balanced.
- 5) Drive should be smooth.
- 6) Pump clearances at different locations are to be maintained as per design requirement.
- 7) Impellers flow passages should be smooth.
- 8) Pump should be run at designed capacity since noise develops sometimes due to running of the pump at lower capacity.

5.14 General Design Considerations:

The casing itself represents only losses and does not add anything to the total energy developed by the pump. In designing of pump casings it is important to utilize all available means of minimizing casing losses. However, commercial considerations dictate some deviations from this approach,

and experience has shown that these do not have a significant effect on casing losses. The following design rules are applicable to all casing designs.

1. Constant angles on the volute sidewalls should be used rather than different angles at each volute section. These two approaches give as good results and the use of constant wall angles reduces pattern costs and saves manufacturing time.
2. Chemical compatibility: Pump parts in contact with the pumped media and addition additives (cleaners, thinning solutions) should be made of chemically compatible materials that will not result in excessive corrosion or contamination. Take help of metallurgist for proper metal selection when dealing with corrosive media.
3. Explosion proof: Non-sparking materials are required for operating environments or media with particular susceptibility to catching fire or explosion.
4. Sanitation: Pumps in the food and beverage industries require high density seals or seal-less pumps that are easy to clean and sterilize.
5. Wear: Pumps which handle abrasives require materials with good wearing capabilities. Hard surfaces and chemically resistant materials are often incompatible. The base and housing materials should be of adequate strength and also be able to hold up against the conditions of its operating environment.
6. A shaft has to be sufficiently stiff to absorb without excessive deflection any radial load that might develop in operation. If this is not the case, the shaft will break because of endurance failure or it will produce maintenance troubles at the stuffing box and bearings.
7. Circular volutes should be considered for pumps below a specific speed of 600. Circular volutes should not be considered for multistage pumps.
8. The total divergence angle of the diffusion chamber should be between 7° and 13° . The final kinetic energy conversion is obtained in the discharge nozzle in a single-stage pump and in both the discharge nozzle and crossover in a multi-stage pump.
9. In designing a volute, be liberal with the space surrounding the impeller. In multi-stage pumps in particular, enough space should be provided between the volute walls and the impeller shroud to allow end float and casting variations. A volute that is tight in this area will create axial thrust.

5.15 Different Efficiency Considerations

5.15.1 Mechanical efficiency of a pump (η_m)

Mechanical efficiency of a pump (η_m) is the ratio of theoretical power that must be supplied to operate the pump to the actual power delivered to the pump.

Mechanical efficiency can be used to determine the power loss in bearings and other moving parts of a pump. It determines the actual power that must be supplied to a pump for desired result. It is in the range of $0.9 \sim 0.95$.

$$\text{Mechanical Efficiency } (\eta_m) = \frac{\text{Theoretical power that must be delivered to a pump}}{\text{Actual power delivered to a pump}} \times 100$$

5.15.2 Hydraulic efficiency of a pump (η_h)

Hydraulic efficiency of a pump (η_h) is defined as the ratio of the useful hydrodynamic energy in fluid to Mechanical energy supplied to rotor. It is in the range of 0.9 ~ 0.99.

$$\text{Hydraulic Efficiency } (\eta_h) = \frac{\text{Useful hydrodynamic energy in fluid}}{\text{Mechanical energy supplied to rotor}} \times 100$$

5.15.3 Volumetric efficiency of a centrifugal pump (η_v)

Volumetric efficiency of a centrifugal pump (η_v) is defined as the ratio of the actual flow rate delivered by the pump to the theoretical discharge flow rate (flow rate without any leakage) that must be produced by the pump. Volumetric efficiency can be used to determine the amount of loss of liquid due to leakage in a pump during the flow. It is in the range of 0.9 ~ 0.98.

$$\text{Volumetric Efficiency } (\eta_v) = \frac{\text{Actual flow rate produced by a pump}}{\text{Th. flow rate that must be produced by the pump}} \times 100$$

5.15.4 Overall efficiency of a centrifugal pump (η_o)

Overall efficiency of a centrifugal pump (η_o) is the ratio of the actual power output of a pump to the actual power input to the pump. It is the efficiency that determines the overall energy loss in a centrifugal pump.

$$\text{Overall Efficiency } (\eta_o) = \frac{\text{Actual power output of a pump}}{\text{Actual power input of a pump}} \times 100$$

Overall efficiency of a centrifugal pump is the product of the volumetric efficiency and mechanical efficiency of a centrifugal pump. It is usually in the range of 0.72 ~ 0.92.

$$\text{Overall efficiency of a centrifugal pump } (\eta_o) = \text{Mech. efficiency } (\eta_m) \times \text{Vol. efficiency } (\eta_v).$$

5.16 Parameters Affecting Volumetric Efficiency

Pump operation and performance can best be described by a few fundamental parameters; flow rate, pressure, head, power, and efficiency.

5.16.1 Volume flow rate (Q), also referred as capacity, is the volume of liquid that travels through the pump in a given time (measured in gallons per minute or gpm). It defines the rate at which a pump can push fluid through the system. In some cases, the mass flow rate (m) is also used, which describes the mass through the pump over time. When selecting pumps, the flow rate or rated capacity of the pump must be matched to the flow rate required by the application or system.

5.16.2 Pressure is a measure of resistance: the force per unit area of resistance in the system. The pressure rating of a pump defines how much resistance it can handle or overcome. It is usually given in bar or psi (pounds per square inch). Pressure, in conjunction with flow rate and power, is used to describe pump performance. When selecting pumps, the rated operating or discharge pressure of the pump must be equal to or more than the required pressure for the system at the desired flow rate.

5.16.3 Head is the height above the suction inlet that a pump can lift a fluid. It is a shortcut measurement of system resistance (pressure) which is independent of the fluid's specific gravity. It is defined as the mechanical energy of the flow per unit weight. It is expressed as a column height of water given in feet (ft) or meters (m). In other words, if water was pumped straight up, the pump head is equivalent to the height it reaches.

5.17 Procedure to draw Blade Profile

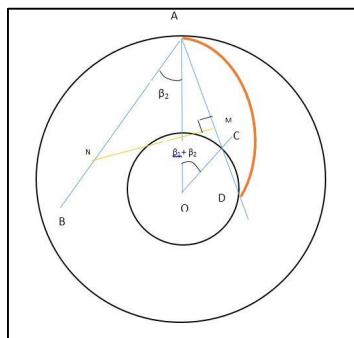


Fig. 5.10 Blade Profile

Following data is required to draw centrifugal pump blade profile.

Impeller diameters D_1 and D_2 , Inlet and Exit Blade angles β_1 and β_2 and Number of blades Z . Step by step procedure to draw blade profile is as follows.

Step 1: draw two concentric circles with diameters D_1 and D_2 and centre O.

Step 2: Draw a point A on the bigger circle and join OA.

Step 3: Draw lines AB and OC such that angle OAB is β_2 and angle OAC is $(\beta_1 + \beta_2)$

Step 4: Name a point C where line OC intersect smaller circle.

Step 5: Draw a line AC and extend it till point D where it intersect smaller circle.

Step 6: Find midpoint M of a line segment AD and draw perpendicular line MN at M which intersect line AB at N.

Step 7: Taking point N as a centre and NA as a radius draw arc AD which is a blade profile.

Step 8: Now constructing circle through Point N, locate the no. of centres equal to no. of blades and taking radius equal to NA draw other blade profiles.

Numerical

Numerical 5.1 Design a centrifugal pump for following specification

1. Static suction head = $h_s = 4 \text{ m}$
2. Static delivery head = $h_d = 10 \text{ m}$
3. Length of suction pipe = 5 m
4. Length of delivery pipe = 20 m
5. Discharge = $Q = 800 \text{ LPM}$

(Subscript 's' denotes the suction side whereas the subscript 'd' denotes the delivery side)

(Design should include following components of centrifugal pump Piping unit, Power unit, Pump Unit)

Solution:

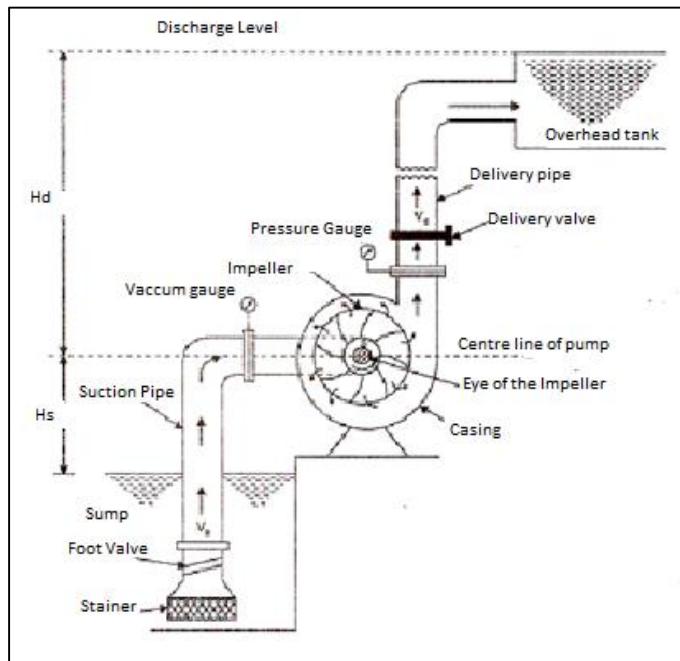


Fig. 5.11 Layout of centrifugal pump

Layout of the centrifugal pump is as shown in figure 5.11.

Step 1: Basic elements (Discharge Q , Manometric head H_{mano} & specific speed N_s)

Theoretical discharge is given as $Q_{act} = 800 \text{ LPM}$

Assuming Volumetric efficiency as $(\eta_v) = 0.95$

Actual discharge is given by,

$$Q_{th} = \frac{Q_{act}}{\eta_v} = \frac{800}{0.95} = \frac{800}{0.95}$$

$$Q_{th} = 842.105 \text{ lpm i.e. } Q_{th} = 0.0140351 \text{ m}^3/\text{s}$$

Let's, there be 15% head loss due to friction,

Therefore, $H_{mano} = H_{static} \times 1.15$

Where, H_{mano} = Manometric head (m)

H_{static} = Static head (m)

$$H_{mano} = 1.15 \times (h_s + h_d) = 1.15 \times (4 + 10)$$

Therefore, $H_{mano} = 16.1 \text{ m}$

Now, specific speed,

$$N_s = \frac{N \sqrt{Q}}{H^{3/4}}$$

Where,

N = Speed of pump (rpm)

N_s = Specific speed (rpm)

Minimum speed of the pump for single stage pump is given by,

$$N = 10 \frac{(H)^{3/4}}{\sqrt{Q}}$$

Assuming $N = 1440 \text{ rpm}$,

$$N_s = \frac{1440 \times \sqrt{0.0140351}}{(16.1)^{3/4}}$$

$$N_s = 21.2252 \text{ rpm}$$

Here, $10 < N_s < 25$, Hence, single stage pump can be used.

Therefore, lets pump type is 1. Radial flow 2. Horizontal axis 3. Single stage

Step 2: Piping unit design

Assuming velocity in suction pipe = $V_s = 1.5$ to 3 m/s

and velocity in delivery pipe = $V_d = 3$ to 6 m/s , (From manufacturer's catalogue)

Let, Velocity in suction pipe $V_s = 2 \text{ m/s}$

Discharge is given by,

$$Q = A \times V$$

Where, A = cross sectional area of the suction pipe

Therefore,

$$0.0140351 = \frac{\pi}{4} \times d_s^2 \times 2$$

Where, d = Diameter of pipe

$$d_s^2 = 8.935 \times 10^{-3}$$

$$d_s = 0.094525 \text{ m} = 94.525 \text{ mm} = 3.72"$$

Therefore, selecting 4" diameter suction pipe $d_s = 4"$

Now actual suction velocity,

$$V_s = \frac{Q}{A}$$

$$V_s = \frac{0.0140351 \times 4}{\pi \times 0.1016 \times 0.1016} = 1.7312 \text{ m/s}$$

Let, Velocity in delivery pipe

$$V_d = 4 \text{ m/s}$$

$$Q = A \times V$$

Therefore,

$$0.0140351 = \frac{\pi}{4} \times d_d^2 \times 4$$

$$d_d^2 = 4.4675 \times 10^{-3}$$

$$d_d = 0.066839 = 66.839 \text{ mm} = 2.6315"$$

Therefore, selecting 3" diameter suction pipe $d_s = 3"$

Now actual velocity in delivery pipe,

$$V_d = \frac{Q}{A}$$

$$V_d = \frac{0.0140351 \times 4}{\pi \times 0.0762 \times 0.0762}$$

$$V_d = 3.0776 \text{ m/s}$$

Step 3: Drive Unit

Power can be given by,

$$P = \frac{W Q H}{\eta_o}$$

Here,

η_o = Overall efficiency = Volumetric efficiency \times Hydraulic efficiency \times Mechanical efficiency

$\eta_o = \eta_v \eta_h \eta_m$

Assuming all the efficiencies to be equal to 0.95, $\eta_v = \eta_h = \eta_m = 0.95$

Therefore, $\eta_o = 0.8574$, also

W = Specific weight of water = $\rho \times g = 1000 \times 9.81 = 9810 \text{ N/m}^3$

Where, ρ = Density of water = 1000 kg/m^3 and g = Acceleration due to gravity = 9.81 m/s^2

Head loss:

$$H_{losses} = H_{suction} + H_{delivery}$$

$$H_{losses} = \frac{4fL_s V_s^2}{d_s \cdot 2g} + \frac{4fL_d V_d^2}{d_d \cdot 2g} + \frac{V_d^2}{2g} + H_{bend} + H_{valves}$$

(Head loss/bend = 0.035 m and Head loss valve = 0.2 m , also $4f = 0.01$)

$$H_{losses} = \frac{0.01 \times 5 \times 1.73^2}{0.1016 \times 2 \times 9.81} + \frac{0.01 \times 20 \times 3.08^2}{0.0762 \times 2 \times 9.81} + \frac{3.08^2}{2 \times 9.81} + (2 \times 0.035) + (2 \times 0.2)$$

$$H_{losses} = 2.297 \text{ m}$$

$$H_{mano} = H_s + H_d + H_{losses} = 4 + 10 + 2.297 = 16.29 \text{ m}$$

$$H_{mano} = 16.29 \text{ m}$$

Therefore,

$$\text{Power} = \frac{W Q H}{\eta_o} = \frac{9810 \times 0.0140351 \times 16.29}{0.8574} = 2615.9 \text{ W}$$

Hence, selecting standard motor, Power = 3.7 kW (PSG 5.124)

Step 4: Pump Unit

(Pump unit consist of Impeller, Blade, Casing, Impeller shaft and Bearing)

Step 4.1 Impeller Design

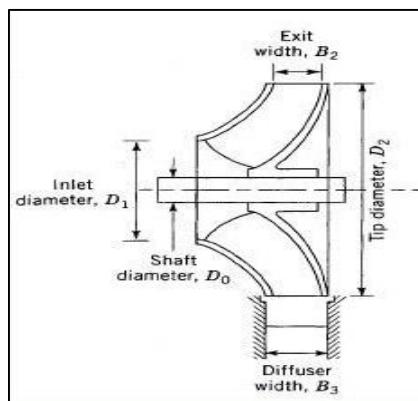


Fig. 5.12 Impeller design

Elements of impeller are 1. Blade, 2. Hub, 3. Shroud

Selecting closed type impeller and Let material used for impeller be GCI 20 with (σ_u) = 200 N/mm²

Let, Factor of safety = 8,

Therefore, $[\sigma_t] = (\sigma_u) / \text{FOS} = 200 / 8 = 25 \text{ N/mm}^2$

Basic dimensions are D_1 and D_2 ,

Now, exit velocity is given by,

$$u_2 = k \sqrt{g H}$$

Where,

k = Velocity constant (1.3 to 1.8)

Let $k=1.6$,

Therefore,

$$u_2 = 1.6 \sqrt{9.81 \times 16.29}$$

$$u_2 = 20.22 \text{ m/s}$$

$$\text{But, } u_2 = \frac{\pi \times D_2 \times N}{60},$$

$$20.22 = \frac{\pi \times D_2 \times 1440}{60}$$

$$\text{Hence, } D_2 = 0.2682 \text{ m} = 268.2 \text{ mm,}$$

Also, for successful working of pump and $D_2 / D_1 = 2$,

$$D_{2\min} = 97.7 \frac{\sqrt{H}}{N} = 97.7 \frac{\sqrt{16.29}}{1440} = 0.273 \text{ m}$$

Selecting $D_2 = 280 \text{ mm}$

$$D_2 / D_1 = 2, D_1 = 140 \text{ mm}$$

$$u_1 = \frac{\pi \times D_1 \times N}{60} = \frac{\pi \times 140 \times 1440}{60 * 1000} = 10.56 \text{ m/s}$$

$$u_1 = 10.56 \text{ m/s}$$

And,

$$u_2 = \frac{\pi * D_2 * N}{60} = \frac{\pi * 280 * 1440}{60 * 1000} = 21.11 \text{ m/s}$$

$$u_2 = 21.11 \text{ m/s}$$

Now,

$$(u_2^2 - u_1^2) / (2g) = (21.11^2 - 10.56^2) / (2 \times 9.81) = 17.03 \text{ m}$$

Here, $(u_2^2 - u_1^2) / (2g) > H_{mano}$, Hence, pump will work successfully.

Step 4.2 Blade design

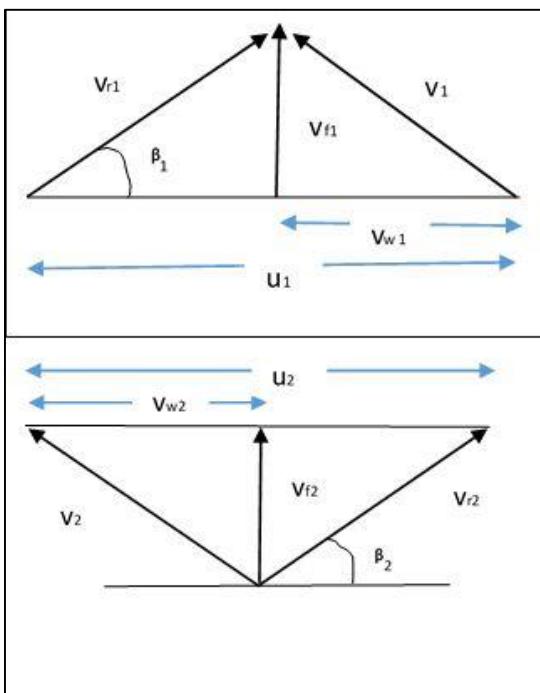


Fig. 5.13 Inlet & Outlet triangle

Selecting backward types blades.

Using inlet and outlet triangles as shown,

$$\tan \beta_1 = V_{f1} / (u_1 - V_{w1}),$$

$$\text{Let, } V_{f1} = (0.125 \text{ to } 0.25) \sqrt{2gH}$$

$$\text{Let, } V_{f1} = V_{f2} = 3 \text{ m/s}$$

Assuming whirl velocity at inlet $V_{w1} = 0.4 u_1$

$$\text{Therefore, } V_{w1} = 0.4 \times 10.556 = 4.2224 \text{ m/s}$$

$$\tan \beta_1 = \frac{3}{10.556 - 4.2224} = 0.47366$$

$$\beta_1 = 25.345^\circ$$

But recommended $\beta_1 = 15^\circ$ to 25° ,

Let, $\beta_1 = 25^\circ$

Now, $\tan \beta_2 = V_{f2} / (u_2 - V_{w2})$

V_{w2} is calculated from hydraulic efficiency,

$$\eta_h = \frac{H \text{ (manometric)}}{H \text{ (theoretical)}}$$

$$\eta_h = \frac{H}{\frac{V_{w2} * u_2}{g} - \frac{V_{w1} * u_1}{g}}$$

$$0.95 = \frac{16.29 \times 9.81}{V_{w2} \times 21.11 - 4.22 \times 10.556}$$

$$V_{w2} = 10.08 \text{ m/s}$$

$$\tan \beta_2 = \frac{3}{21.11 - 10.08} = 0.31174$$

$$\beta_2 = 15.22^\circ$$

But, $\beta_2 = 17^\circ$ to 27.5°

Let, $\beta_2 = 17^\circ$

Hence, $V_{f2} = 3.37 \text{ m/s}$

Now, number of blades (Z),

Compromising between the back circulation and friction losses,

Z = 2 to 3 for open impeller and

Z = 6 to 12 for closed impeller.

No. of blades, $Z = 6.5 \times \frac{D_2 + D_1}{D_2 - D_1} \times \sin \left(\frac{\beta_1 + \beta_2}{2} \right)$ (from manufacturing catalogue)

$$Z = 6.5 \times \left(\frac{280 + 140}{280 - 140} \right) \times \sin \left(\frac{25 + 17}{2} \right)$$

$$Z = 6.988$$

$$\text{Let, } Z = 8$$

Now, blade width (b)

Let, blade width at inlet is b_1 and blade width at outlet is b_2 .

The thickness of blade (t) at the inlet and outlet be, $t_1 = 8 \text{ mm}$, $t_2 = 6 \text{ mm}$

Now,

$$Q_{th} = (\pi D_1 - \frac{Z t_1}{\sin \beta_1}) b_1 V_{f1}$$

$$0.014 = (\pi \times 0.14 - \frac{8 \times 0.008}{\sin 25}) \times b_1 \times 3$$

$$\text{Therefore, } b_1 = 0.01618 \text{ m} = 16.18 \text{ mm}$$

$$\text{Let, } b_1 = 20 \text{ mm}$$

Similarly,

$$Q_{th} = (\pi D_2 - \frac{Z t_2}{\sin \beta_2}) b_2 V_{f2}$$

$$0.014 = (\pi \times 0.28 - \frac{8 \times 0.006}{\sin 17}) \times b_2 \times 3.37$$

$$\text{Therefore, } b_2 = 0.0058 \text{ m} = 5.8 \text{ mm}$$

$$b_2 = 6 \text{ mm}$$

Blade profile is as shown in figure

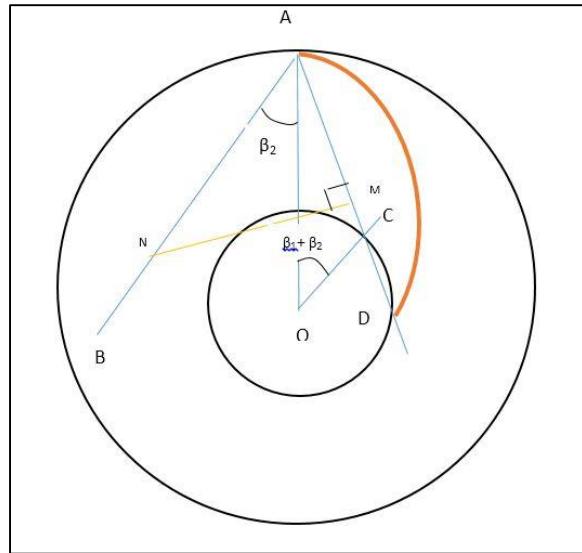


Fig. 5.14 Blade profile

Step 4.3 Impeller shaft

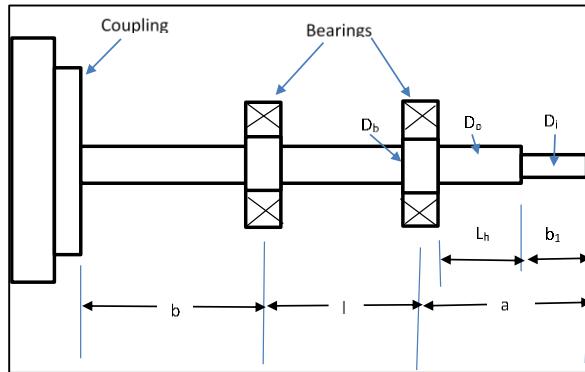


Fig. 5.15 Impeller shaft

Based on torsional shear failure,

$$T = P / \omega$$

Where, T = Torque, P = Power and ω = Angular velocity = $\frac{2\pi N}{60}$

Torque is given by, $T = \frac{3.7 \times 1000 \times 60}{2\pi \times 1440}$

$$T = 24.54 \text{ N.m}$$

Let, the material for the shaft be C30 with $[\tau] = 60 \text{ N/mm}^2$

$$[\tau] = T / \left(\frac{\pi d^3}{60} \right)$$

$$60 = \frac{24.54 \times 1000 \times 16}{\pi d^3}$$

$$\text{Therefore, } d^3 = 2083.01 \text{ mm}^3$$

$$d_s = 12.77 \text{ mm}$$

Let, $d_s = 24 \text{ mm}$ (Shaft diameter)

Assuming the proportions of the shaft as shown in figure 5.15,

Hub dimensions (diameter and length) can be given as,

$$d_h = 1.5 \times d_s = 36 \text{ mm}, \quad l_h = 1.5 \times d_s = 36 \text{ mm}$$

Let the shroud thickness be as follows,

$t = 4 \text{ mm}$ for low pressure side (base plate)

$t = 6 \text{ mm}$ for high pressure side (crown plate)

Now, let the circular dimensions be as follows,

$$d_i = 24 \text{ mm}, \quad d_b = 40 \text{ mm} \text{ and } d_p = 30 \text{ mm}$$

Now, linear dimensions can be given as,

Overhanging length (a), $a = l_h + b_1 + \text{Margin}$

$$a = 36 + 20 + 14, \quad a = 70 \text{ mm}$$

$$l = 1.5 \times a = 1.5 \times 70 = 105 \text{ mm}$$

(To control bearing load and overall size)

$$b = 3 \times d_i = 3 \times 24 = 72 \text{ mm}$$

Failure of the shaft is due to 1) Bending of shaft by W_I and F_d , 2) tensile or compression due to Axial thrust.

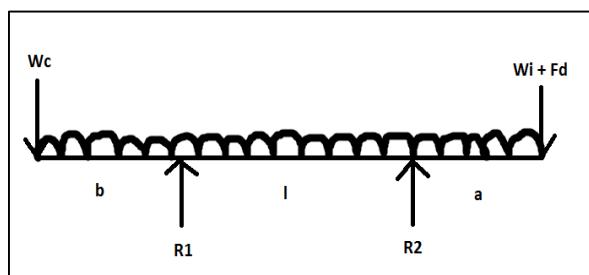


Fig. 5.16 force diagram of impeller shaft

Now, from force diagram given above,

W_I = Impeller weight,

W_c = coupling weight,

W_b = Blade weight,

W_h = Hub weight,

W_s = Shroud weight,

F_d = Dynamic force,

R_1, R_2 = Reactions at the bearings

Following assumption are made,

- 1) Weight of shaft, coupling is neglected.
- 2) Dynamic load is based on average radius
- 3) Axial thrust due to change in direction is neglected
- 4) Inlet considered as axial and exit as radial.

Weight of impeller,

$$W_I = W_b + W_s + W_h$$

Let, density of material be, $\rho = 7200 \text{ kg/m}^3$

$$W_s = \frac{\pi}{4} \times D_2^2 \times t \times \rho \times 2g$$

(Assuming thickness of shroud as 5 mm)

$$W_s = \frac{\pi}{4} \times 0.28^2 \times 0.005 \times 7200 \times 2 \times 9.81,$$

$$W_s = 43.492 \text{ N}$$

Considering 15% excess for blades and hub weight.

$$W_I = 1.15 W_s = 1.15 \times 43.492 = 50.156 \text{ N}$$

Now, $m_i = W_I / g$, Therefore, $m_i = 5.098 \text{ kg}$

Now, dynamic load, $F_d = m \times r \times \omega^2$ (Unbalanced mass is assumed as 10%)

Total unbalance mass, $m = 0.1 \times m_i = 0.5098 \text{ kg}$

Average radius, $r = (D_1 + D_2) / 2 = 105 \text{ mm}$

$$\text{Therefore, Dynamic Load } F_d = 0.5098 \times 0.105 \times \left(\frac{2\pi \times 1440}{60} \right)^2$$

$$F_d = 1217.23 \text{ N}$$

$$W_{\text{total}} = W_I + F_d = 50.156 + 1217.23$$

$$W_{\text{total}} = 1267.38 \text{ N}$$

Now, Bending Moment, $M_b = W_{\text{total}} \times a = 1267.38 \times 70$

$$M_b = 8.87168 \times 10^4 \text{ N.mm}$$

Now, Equivalent torque

$$T_{\text{eq}} = \sqrt{T^2 + Mb^2} = \sqrt{8.87168^2 + 2.454^2} \times 10^4$$

$$T_{\text{eq}} = 9.2048 \times 10^4 \text{ N.mm}$$

$$(\tau) = \frac{T_{\text{eq}}}{\left(\frac{\pi d^3}{16} \right)} = \frac{9.2048 \times 10000 \times 16}{\pi \times 24^3} = 33.912 \text{ N/mm}^2 < [\tau], \text{ Hence safe design.}$$

Step 4.4 Bearing Design

Reactions at the bearing can be given as,

$$R_2 = \frac{W_{\text{total}} (a+l)}{l} = \frac{1267.38 \times (105+70)}{105} = 2112.3 \text{ N}$$

$$R_1 = - \frac{W_{\text{total}} \times a}{l} = - \frac{1267.38 \times 70}{105} = - 844.92 \text{ N}$$

Axial force, $F_a = A \times \Delta P$

Here, Pressure difference is given by,

$$\Delta P = P_d - P_s = w \times (H_d - H_s) = 9810 \times (10 - 4) = 5.886 \times 10^4 \text{ Pa}$$

$$A = \frac{\pi}{4} \times (D_2^2 - D_1^2) = \frac{\pi}{4} \times (0.28^2 - 0.14^2) = 0.04618 \text{ m}^2$$

$$\text{Therefore, } F_a = 5.886 \times 10^4 \times 0.04618 = 2718.24 \text{ N}$$

But, axial thrust is reduced to 30% by providing holes on base plate (high pressure side) near the hub level. Therefore, axial thrust = $0.3 \times 2718.24 = 815.47 \text{ N}$

$$\text{Radial force} = \max(R_1, R_2) = 2112.3 \text{ N}$$

Selecting taper roller bearing (since both radial and axial load are acting).

From impeller shaft design,

$$F_R = 2112.3 \text{ N}, F_a = 815.47 \text{ N}$$

Assuming life of bearing as 10000 hours,

$$L_{MR} = \frac{60 \times N \times Lhr}{1000000} = \frac{60 \times 1440 \times 10000}{1000000} = 864 \text{ mr}$$

Let the bearing be 32208, as $d_b = 40\text{mm}$, $e = 0.37$ (PSG 4.25)

$$F_a / F_R = 0.69 > e, \text{ Therefore, } X = 0.4; Y = 1.6 \text{ (PSG 4.4)}$$

Let, service factor, $s = 1.1$, Also, $V = 1$ (inner recess rotating)

Therefore,

$$P_{eq} = (XVF_R + YF_a) \times S = (0.4 \times 2112.3 + 1.6 \times 815.47) = 2149.67 \text{ N}$$

$$C = P_{eq} \times L^{1/k}, \text{ Here, } k = 10/3$$

$$C = 2149.67 \times 864^{3/10} = 16342.8113 \text{ N} = 1665.9 \text{ kgf}$$

For bearing 32208, $d = 40 \text{ mm}$, $C = 6350 \text{ kgf}$ (PSG 4.25) Hence, bearing selection is suitable.

Step 4.5 Casing Design

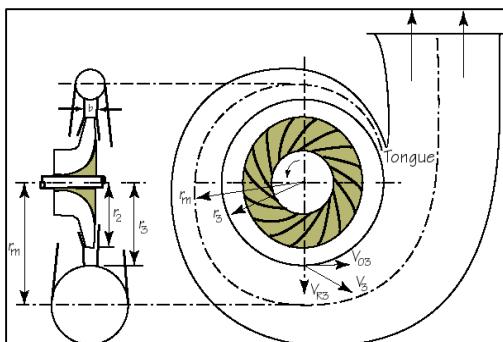


Fig. 5.17 Volute Casing

Let material for casing be GCI 20 with $(\sigma_u) = 200 \text{ N/mm}^2$ and FOS = 8

$$\text{Therefore, } [\sigma_t] = (\sigma_u) / \text{FOS} = 200 / 8 = 25 \text{ N/mm}^2$$

Let, the thickness of casing be $t_c = 8 \text{ mm}$

Casing failure may takes place by hoop stresses under fluidic pressure,

Tensile stress induced is given by $(\sigma_t) = (P \times D_2) / (2 \times t_c)$

$$\text{Where, } P = w \times H_{mano} = 9810 \times 16.29 = 159804 \text{ N/m}^2$$

$$\text{Therefore, } (\sigma_t) = \frac{159804 \times 0.28}{2 \times 0.008 \times 1000000} = 2.796 \text{ N/mm}^2 < [\sigma_t]$$

Hence, safe in tensile failure.

Now, throat velocity (V_{th}),

$$\text{Assuming as } V_{th} = 0.3 \times u_2 = 0.3 \times 21.11 = 6.33 \text{ m/s}$$

$$\text{Now, } Q_{th} = V_{th} \times \pi / 4 \times d_{th}^2$$

Where, d_{th} = Throat diameter of casing

$$0.014 = 6.33 \times \pi / 4 \times d_{th}^2$$

$$d_{th}^2 = 2.816 \times 10^{-3}$$

$$d_{th} = 0.053066 \text{ m} = 53.06 \text{ mm}$$

Let, $d_{th} = 55 \text{ mm}$

Now, diameter of casing at an angle θ is given by,

$$d_\theta = d_{th} \times \sqrt{\frac{\theta}{360 - \theta_{exit}}}$$

Let exit angle $\theta_{exit} = 0^\circ$ Therefore, $d_\theta = 55 \times \sqrt{\theta/360}$

$D_2 = 280\text{mm}$ and Let the clearance between impeller and casing, $c = 5 \text{ mm}$

Also, the distance of centreline of the volute casing from the eye of the impeller can be given as,

$$R_\theta = (D_2/2) + d_\theta + c$$

θ	$d_\theta (\text{mm})$	$R_\theta (\text{mm})$
30	5.88	160.88
60	22.45	167.45
90	27.5	172.5
180	38.89	183.89
270	47.63	192.63
360	55	200

Numerical 2:

Design of centrifugal pump for following specification:

Total Manometric head: 25 m

Discharge: 500 LPM

Material to be handled is Water

Solution:

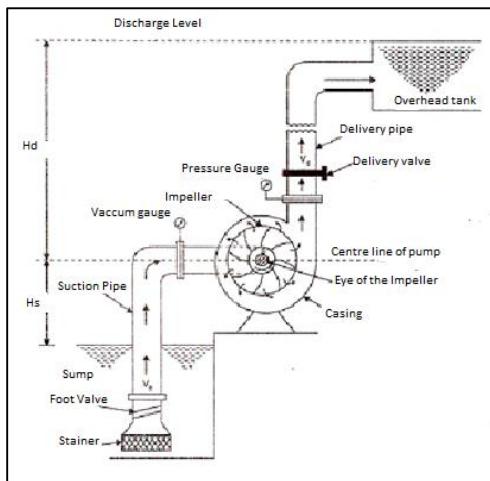


Fig. 5.18 Layout of centrifugal pump

Step 1: Basic elements (Q , H_{mano} & N_s)

Theoretical Discharge is given by,

$$Q_{th} = \frac{500 \times 10^3}{60} = 8.33 \times 10^{-3} \text{ m}^3/\text{sec}$$

Assuming about 5% Leakage, then Volumetric efficiency = 0.95

Therefore, Actual Discharge

$$Q_{th} = \frac{Q_{act}}{0.95} = \frac{8.33 \times 10^{-3}}{0.95} = 0.00876 \text{ m}^3/\text{sec}$$

Manometric Head (H_{mano}) = 25 m

Assuming, Speed of pump (N) = 1440 rpm.

$$\text{Specific Speed } (N_s) = \frac{N \times \sqrt{Q_{act}}}{(H)^{3/4}} = \frac{1440 \times \sqrt{0.00876}}{(25)^{3/4}} = 12.06 \text{ rpm}$$

Here, $10 < N_s < 25$ Hence, single stage pump can be used.

Let pump type is 1. Radial pump 2. Horizontal axis 3. Single stage

Step 2: Piping Unit Design

From manufacturing Catalogue,

V_s (velocity at suction) = 1.5 to 3 m/s and V_d (velocity at delivery) = 3 to 6 m/s

Assuming velocity at suction V_s = 2m/s

At Suction, $Q_{act} = \text{Area (A)} \times \text{Velocity (V}_s\text{)}$

$$0.00876 = \frac{\pi}{4} d_s^2 \times 2$$

(d_s) Diameter of pipe at suction end = 0.0746 m = 74 mm \approx 75 mm

Selecting the Pipe of 3" diameter at suction end.

$$V_{s(act)} = 1.92 \text{ m/s for 3" pipe.}$$

Assuming velocity at delivery, V_d = 4m/s

At Delivery end, $Q_{act} = \text{Area (A)} \times \text{Velocity (V}_d\text{)}$

$$0.00876 = \frac{\pi}{4} d_d^2 \times 4$$

(d_d) Diameter of pipe at delivery end = 0.052 m = 52 mm

Selecting the Pipe of 2" diameter at delivery end.

$$V_{d(act)} = 4.322 \text{ m/s for 2" pipe.}$$

Step 3: Drive Unit

Assuming, η_v = Volumetric efficiency = 0.95

η_{mech} = Mechanical efficiency = 0.95

η_H = Hydraulic efficiency = 0.95

Overall efficiency is given by,

$$\eta = \eta_v \times \eta_{mech} \times \eta_H = 0.95 \times 0.95 \times 0.95 = 0.857$$

Therefore, (P) Power of motor = $\frac{w Q_{act} H}{\eta}$

Where, w is specific weight of water as 9810 Kg/ m³

$$P = \frac{9810 \times 0.00875 \times 25}{0.857} = 2.506 \text{ kW}$$

So, selecting standard motor (Foot mounted motor) of 3.7 Kw rating. (PSG 5.124)

Step 4: Pump Unit

It consists of Impeller, Blade, Casing, Impeller shaft and Bearing.

Step 4.1 Impeller Design

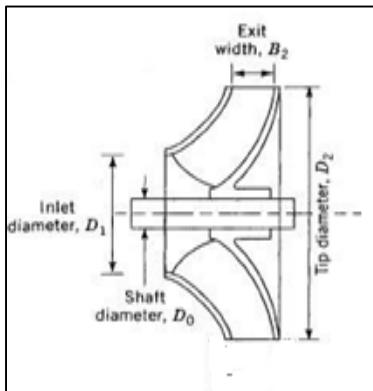


Fig. 5.19 Impeller

Elements of impeller: 1. Blade, 2. Hub, 3. Shrouds

Material – GCI 20 (PSG 1.4)

$[\sigma_u] = 200 \text{ N/mm}^2$, Taking Factor of safety = 8

$$\therefore [\sigma_u] = \frac{200}{8} = 25 \text{ N/mm}^2$$

U_2 = velocity of water at exit of impeller

$$U_2 = K \sqrt{g H} \quad (K = 1.3 \text{ to } 1.8)$$

$$\text{Let, } K = 1.6, \quad U_2 = 1.6 \sqrt{9.81 \times 25} = 25.056 \text{ m/s}$$

$$\text{As, } U_2 = \frac{\pi \times D_2 \times N}{60}$$

$$\therefore 25.056 = \frac{\pi \times D_2 \times 1440}{60}$$

$$\therefore D_2 = 332 \text{ mm... Diameter of impeller at exit}$$

Also, for successful working of pump and $D_2 / D_1 = 2$,

$$D_{2\min} = 97.7 \frac{\sqrt{H}}{N} = 97.7 \frac{\sqrt{25}}{1440} = 0.3392 \text{ m}$$

$$\text{Let, } D_2 = 340 \text{ mm}$$

$$U_2 = \frac{\pi \times D_2 \times N}{60} = \frac{\pi \times 0.340 \times 1440}{60} = 25.63 \text{ m/s}$$

$$U_2 = 25.63 \text{ m/s}$$

Also, $\frac{D_2}{D_1} = 2$,

$$D_1 = \frac{340}{2} = 170 \text{ mm}$$

Diameter of impeller at entry D_1 is 170 mm

Now, U_1 is velocity of water at entry of impeller

$$U_1 = \frac{\pi \times D_1 \times N}{60} = \frac{\pi \times 0.17 \times 1440}{60} = 12.82 \text{ m/s}$$

$$U_1 = 12.82 \text{ m/s}$$

$$\text{Now, Velocity Head } \frac{U_2^2 - U_1^2}{2g} = \frac{25.63^2 - 12.82^2}{2 \times 9.81} = 25.10 > H_{\text{mano}}$$

Here, $\frac{U_2^2 - U_1^2}{2g} > H_{\text{mano}}$, Hence pump will work successfully.

Step 4.2 Blade Design

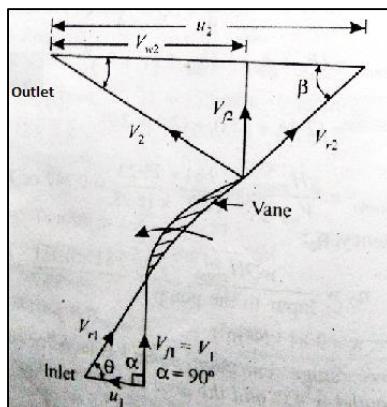


Fig. 5.20 Inlet and Outlet angle

Selecting Backward Blades;

Assuming radial entry for easy design, velocity triangles are shown below.

Now, β_1 and β_2 is the impeller blade angle at inlet and outlet.

$$\tan \beta_1 = \frac{V_{f1}}{U_1} = \frac{1.92}{12.82} = 0.149$$

$$\beta_1 = 8.52^\circ$$

The range of 17° to 27° for good design,

$$\text{Let } \beta_1 = 17^\circ$$

Now, β_2 is the impeller blade angle at exit,

$$\tan \beta_2 = \frac{V_{f2}}{U_2 - V_{w2}}$$

Now Velocity of whirl at outlet,

V_{w2} can be calculated from Hydraulic efficiency as below.

$$\eta_H = \text{Hydraulic efficiency} = \frac{H}{\frac{V_{w2} \cdot U_2 - V_{w1} \cdot U_1}{g}}$$

$$\therefore 0.95 = \frac{25}{\frac{V_{w2} \cdot 25.63}{9.81}}$$

$$V_{w2} = 10.07 \text{ m/s}$$

Assuming Flow velocity,

$$V_{f2} = \left(\frac{1}{4} \text{ to } \frac{1}{8} \right) \sqrt{2gH_{mano}}$$

That is Flow velocity = 2.76 to 5.53 m/s

Assuming flow velocity, $V_{f2} = 3 \text{ m/s}$

$$\tan \beta_2 = \frac{V_{f2}}{U_2 - V_{w2}} = \frac{3}{25.63 - 10.07}$$

$\therefore \beta_2 = 10.91^\circ$. But β_2 should be in the range of 20° to 40° .

\therefore Let, $\beta_2 = 20^\circ$, Hence, at $\beta_2 = 20^\circ$ the value of $V_{f2} = 5.66 \text{ m/s}$

No. Of Blades can be given by,

Compromising between the back circulation and friction losses,

$Z = 2$ to 3 for open impeller and $Z = 6$ to 12 for closed impeller.

No. Of blades (from manufacturing catalogue)

$$Z = 6.5 \times \frac{D_2 + D_1}{D_2 - D_1} \times \sin\left(\frac{\beta_1 + \beta_2}{2}\right)$$

$$Z = 6.5 \times \frac{(D_2 + D_1)}{(D_2 - D_1)} \sin\left(\frac{\beta_1 + \beta_2}{2}\right) = 6.5 \times \frac{(0.34 + 0.17)}{(0.34 - 0.17)} \sin\left(\frac{17 + 20}{2}\right)$$

$Z = 6.187$, lets selecting 8 Blades.

Let blade width be , Inlet width = b_1 , Outlet width = b_2

Inlet thickness of blade $t_1 = 8 \text{ mm}$

Outlet thickness of blade $t_2 = 6\text{mm}$

$$Q_{th} = \left(\pi D_1 - \frac{zt_1}{\sin \beta_1} \right) b_1 \cdot V_{f1}$$

$$0.00876 = \left(\pi \times 0.17 - \frac{8 \times 8}{1000 \sin 17} \right) \times b_1 \times 1.92$$

$$b_1 = 9.5 \text{ mm}, \text{ Let, } b_1 = 10\text{mm}$$

$$\text{Also, } Q_{th} = \left(\pi D_2 - \frac{zt_2}{\sin \beta_2} \right) b_2 \cdot V_{f2}$$

$$0.00876 = \left(\pi \times 0.34 - \frac{8 \times 6}{1000 \sin 20} \right) b_2 \times 5.66$$

$$b_2 = 1.668 \text{ mm}$$

$$b_2 = 2 \text{ mm}$$

Shroud thickness is given by,

For low pressure side $t = 4\text{mm}$ for base plate

For High pressure side $t = 6\text{mm}$ for crown plate

Blade Profile is shown in figure

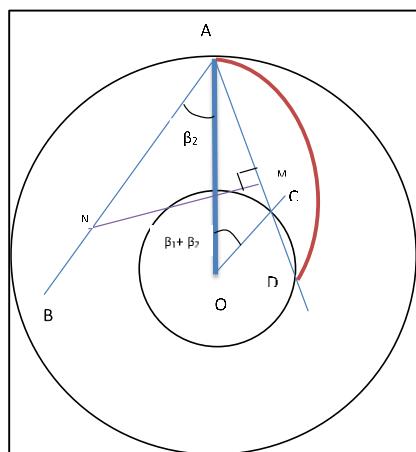


Fig. 5.21 Blade Profile

Step 4.3 Impeller Shaft

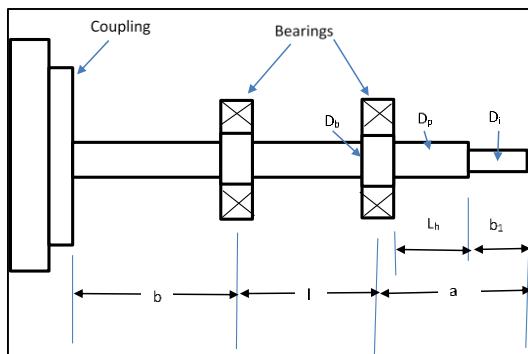


Fig. 5.22 Impeller Shafts

Based on torsional shear failure,

$$T = P / \omega$$

Where, T = Torque, P = Power and ω = Angular velocity = $\frac{2\pi N}{60}$

Therefore, Torque is given by,

$$T = \frac{3.7 \times 1000 \times 60}{2\pi \times 1440}$$

$$T = 24.54 \text{ N.m}$$

Let, the material for the shaft be C30 with $[\tau] = 40 \text{ N/mm}^2$

$$[\tau] = T / \left(\frac{\pi d^3}{60} \right)$$

$$40 = \frac{24.54 \times 1000 \times 16}{\pi d^3}$$

$$d^3 = 3124.52 \text{ mm}^3$$

$$d_s = 14.622.77 \text{ mm}$$

$$d_s = 20 \text{ mm (Shaft diameter)}$$

Hub dimensions (diameter and length) can be given as,

$$\therefore d_h = L_h = 1.5 \times d_s = 1.5 \times 20 = 30 \text{ mm}$$

Now, Let Circular dimensions as follows,

$$d_i = 20 \text{ mm}, \quad d_b = d_i + 10 = 30 \text{ mm}, \quad d_p = 25 \text{ mm}$$

Now, linear dimension can be given as,

Overhanging Length (a) = $L_h + b_i + \text{margin}$

$$a = 30 + 10 + 16 = 56 \text{ mm}$$

Bearing span,

$$L = 1.5 a = 1.5 \times 56 = 84 \text{ mm}$$

$$b = 3 \times d_i = 3 \times 20 = 60 \text{ mm}$$

Following assumption are made,

- 1) Weight of shaft, coupling is neglected
- 2) Dynamic load is based on average radius
- 3) Axial thrust due to change in direction is neglected
- 4) Inlet considered as axial and exit as radial.

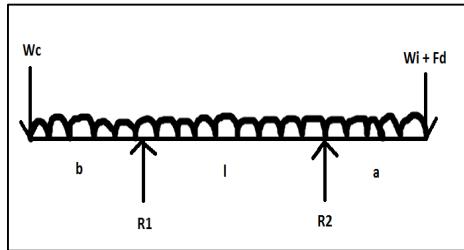


Fig. 5.23 Net force acting on the shaft

Let,

W_i = Impeller weight

W_b = Blade weight

W_s = Shroud weight

W_h = Hub weight

$$W_i = W_b + W_s + W_h$$

Let, density of material (ρ) = 7200 kg/m^3

$$W_s = \left(\frac{\pi}{4} \times D_2^2 \times t \times \rho \right) \times 2$$

$$W_s = \left(\frac{\pi}{4} \times 0.340^2 \times 0.004 \times 7200 \right) \times 2 \times 9.81 = 51.30 \text{ N}$$

Considering 15% excess for blades and hub weight.

$$W_i = 1.15 W_s = 1.15 \times 51.30 = 58.997 \text{ N}$$

Hence mass of the impeller $M_i = 6.013 \text{ Kg}$

Let, $F_d = \text{Dynamic force}$, $r = \text{Average Radius}$

Assuming Unbalanced mass $m = 0.1 M_i = 0.1 \times 6.013 = 0.6013 \text{ kg}$

$$F_d = m r \omega^2$$

$$F_d = 0.6013 \times \left(\frac{0.170+0.340}{4} \right) \times \left(2\pi \times \frac{1440}{60} \right)^2$$

$$F_d = 1743.34 \text{ N}$$

$$W_{\text{total}} = W_i + F_d$$

$$W_{\text{total}} = 58.98 + 1743.34 = 1802.3 \text{ N}$$

$$M_b = W_{\text{total}} \times a = 1802.3 \times 56 = 100.93 \times 10^3 \text{ N.mm}$$

Equivalent Torque is given by,

$$(T_{\text{eq}}) = \sqrt{24.54^2 + 100.93^2} \times 10^3 = 103.87 \times 10^3 \text{ N.mm}$$

Torsional stress

$$(\tau) = \frac{T_{\text{eq}}}{\frac{\pi}{16} d^3} = \frac{103.87 \times 10^3}{\frac{\pi}{16} 20^3} = 66.126 > [\tau] \dots \text{It is not safe}$$

Modifying d to 25mm,

$$(\tau) = 33.85 \text{ MPa} < [\tau]$$

Hence safe, $d = 25 \text{ mm}$

Let R_1 and R_2 are reaction on the bearing

$$R_2 = (1802.3 \times \frac{84+56}{84}) = 3003 \text{ N}$$

$$R_1 = F - R_2 = 3003 - 1802.3 = -1201 \text{ N} = 1201 \text{ N}$$

Axial force can be given as,

$$F_a = A \cdot \Delta P$$

where, $\Delta P = w \times H_{\text{difference}}$, assuming the $H_d - H_s = 10 \text{ m}$

$$F_a = 9810 \times 10 = 98100 \text{ Pa}$$

$$F_a = \frac{\pi}{4} (0.34^2 - 0.17^2) \times 98100 = 6680 \text{ N}$$

Actual axial thrust is reduced to 30 % by using thrust hole,

$$(F_a) = 0.3 F_a = 0.3 \times 6680 = 2004 \text{ N}$$

And Radial Load (Fr) = Max between R_2 and R_1 , $R_2 = 3003 \text{ N}$

Step 4.4 Bearing Design

Forces acting on bearing are,

$$F_r = 3003 \text{ N} \text{ and } F_a = 2004 \text{ N}$$

Assume life of the bearing as $L_{hr} = 8000 \text{ Hrs.}$

$$L_{mr} = (60 \times N \times L_{hr})/10^6 = \frac{60 \times 1440 \times 8000}{10^6} = 691.2 \text{ mr}$$

Selecting Taper roller bearing of type 32206A (PSG 4.25)

$$\text{Where, } e = 0.37 \text{ and } \frac{F_a}{F_r} = 0.67 > e$$

$$X = 0.4, Y = 1.6, S = 1.1, V = 1 \text{ (PSG 4.2)}$$

Equivalent load is given by,

$$P_{eq} = (XVF_r + YF_a)S$$

$$P_{eq} = (0.4 \times 3003 + 1.6 \times 2004) \times 1.1 = 4848.36 \text{ N}$$

Dynamic Load Rating

$$C = P_{eq} (L)^{1/k} \text{ (k= 10/3, for roller bearing) (PSG 4.2)}$$

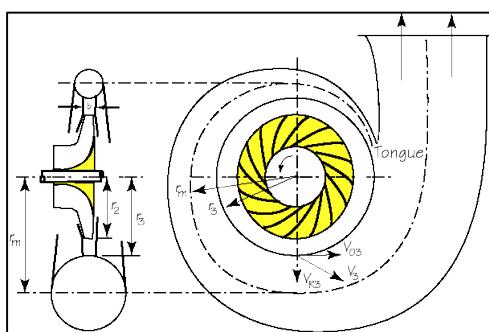
$$C = 4848.36 \times 691.2^{3/10} = 34472.8 \text{ N} = 3447.3 \text{ Kgf}$$

For taper roller bearing 32206A, value of $C = 4380 \text{ Kgf}$ (PSG 4.25)

$d = 30 \text{ mm}$, $D = 62 \text{ mm}$, $B = 20 \text{ mm}$ and $C = 4380 \text{ kgf}$

Hence, selected taper roller bearing can be used as it is safe.

Step 4.5 Casing Design



Let Material for casing is GCI -20 (PSG 1.11)

$$[\sigma_u] = 200 \text{ N/mm}^2 \quad [\sigma_t] = 25 \text{ N/mm}^2$$

Let thickness of casing be $t_c = 8\text{mm}$

Total pressure $P = 98100 \text{ N/m}^2$

Tensile stress induced is given by $(\sigma_t) = \frac{PD}{2t}$

$$(\sigma_t) = 98100 \times 0.34 \times \frac{10^{-6}}{2 \times 0.008} = 20.84 \text{ N/mm}^2 < [\sigma_t], \text{ Safe under hoop stress failure.}$$

Assuming Throat velocity (V_{th}) = 0.3 $U_2 = 0.3 \times 25.63 = 7.69 \text{ m/s}$

$$\text{Theoretical discharge, } Q_{th} = V_{th} \times \frac{\pi}{4} D_{th}^2$$

$$0.00876 = 7.69 \times \frac{\pi}{4} D_{th}^2 \quad \text{Where, } D_{th} = \text{throat diameter of casing,}$$

$$D_{th} = 38.08 \text{ mm} \approx 40 \text{ mm}$$

Assuming, Exit Angle = 0° and Clearance between impeller and casing is 5 mm

Diameter of the casing at any angle θ is given by,

$$d_\theta = d_{th} \sqrt{\frac{\theta}{360 - \text{exit angle}}}$$

And the distance of centreline of the volute casing from the eye of the impeller in mm

$$\text{can be given as, } R_\theta = (D_2/2) + d_\theta + c$$

θ	d_θ	$R_\theta = \frac{D_2}{2} + d_\theta + c$
30°	11.54	186.5
60°	16.32	191.32
90°	20	195
180°	28.28	203.28
270°	34.64	209.64
360°	40	215

Where,

d_θ = Diameter at an angle θ

C = clearance

R_θ = The distance of centerline of the volute casing from the eye of the impeller in mm.

5.18 Introduction to Gear Pump

A gear pump uses the meshing of gears to pump fluid by displacement. They are one of the most common types of pumps for hydraulic fluid power applications. Gear pumps are also widely used in chemical industries to pump high viscosity fluids. Gear pumps are positive displacement or fixed displacement, meaning they pump a constant amount of fluid for each revolution. Gear pumps can be found on the following machines:

1. In the car's oil pump
2. Hydraulically driven lawn care equipment
3. Some hydraulically driven log splitters
4. Hydraulic power units on trucks and construction equipment
5. Metering applications (gear pumps are good at controlling volume flow rate)

Gear pump is a robust and simple positive displacement pump. It has two meshed gears revolving about their respective axes. These gears are the only moving parts in the pump. They are compact, relatively inexpensive and have few moving parts. The rigid design of the gears and houses allow for very high pressures and the ability to pump highly viscous fluids. They are suitable for a wide range of fluids and offer self-priming performance. These pump includes helical and herringbone gear sets (instead of spur gears), lobe shaped rotors similar to Roots blowers (commonly used as superchargers), and mechanical designs that allow the stacking of pumps. A gear pump uses the meshing of gears to pump fluid by displacement. They are one of the most common types of pumps for hydraulic fluid power applications.

Gear pumps are two main variations; external gear pumps which use two external spur gears, and internal gear pumps which use an external and an internal spur gears. Some gear pumps are designed to function as either a motor or a pump. Gear Pump cross section for external gear pump is shown in figure 5.25.

As the gears rotate they separate on the intake side of the pump, creating a void and suction which is filled by fluid. The fluid is carried by the gears to the discharge side of the pump, where the meshing of the gears displaces the fluid. The mechanical clearances are small of the order of 10 μm . The tight clearances, along with the speed of rotation, effectively prevent the fluid from leaking backwards. The rigid design of the gears and houses allow for very high pressures and the ability to pump highly viscous fluids.

5.18.1 Tooth form

Stub gear form is widely used although various form can be used. Single and double helical forms are also used to achieve smooth and noise free continuous drive. To avoid excessive manufacturing cost and complexity in design, complex tooth form are undesirable, therefore the simple stub tooth

form with less number of teeth are used. Reduction in teeth number helps in increasing the displacement per revolution.

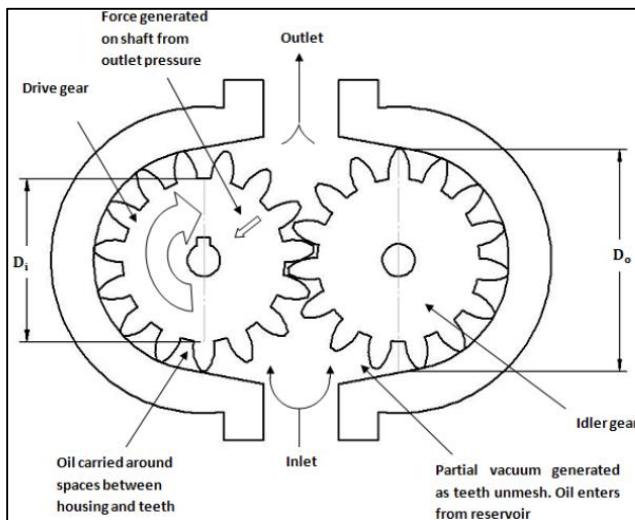


Fig. 5.25 Gear Pump cross section

5.18.2 Difference between Gear Pump and Centrifugal Pump

GEAR PUMPS	CENTRIFUGAL PUMPS
It is Positive displacement Pump. For every revolution, it discharges constant quantity of liquid.	It is a rotodynamic pump and does not necessarily deliver fixed quantity of liquid in every revolution.
During working of pump, if the delivery valve is closed, the casing may burst as a result. Hence safety valves are provided in gear pumps.	During working of centrifugal pump, if the delivery valve is closed, No risk of excessive pressure development.
This is Low discharge and high pressure pump.	This is High discharge pump.
This pump is used for high viscosity liquid, may work with low viscosity fluid too.	This pump is used for low viscosity fluids.
Priming is not required as the pump will create negative pressure itself.	Priming is required in Centrifugal Pump

i. Constructional Details

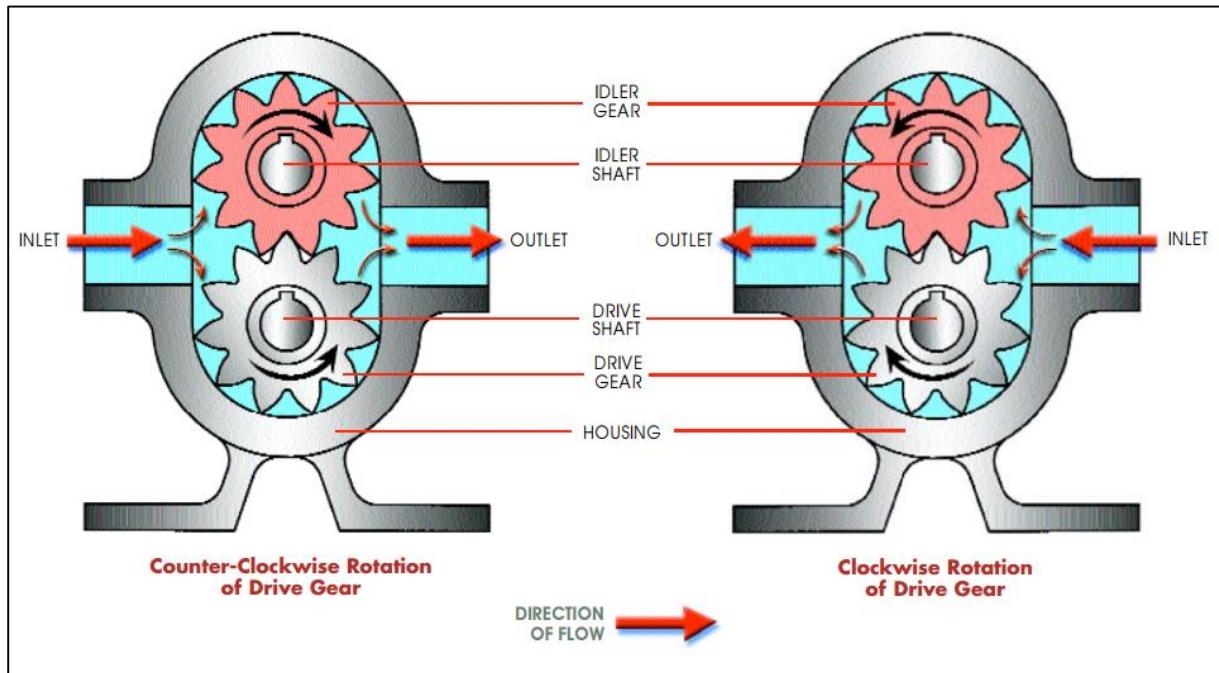


Fig. 5.26 External Gear Pump

1. One shaft is driven by a motor or some other means.
2. The gear mounted to this shaft (driving gear) engages the other gear (driven gear).
3. Fluid on the inlet side flows into and is trapped between the rotating gear teeth and the housing.
4. The fluid is carried around the outside of the gears to the outlet side of the pump.
5. As the fluid cannot seep back along the path it came nor between the engaged gear teeth must it exit from the outlet port.

5.18.9 Working of Gear Pump

The external gear pump consists of externally meshed two gears housed in a pump case as shown in figure 5.26. One of the gears is coupled with a prime mover and is called as driving gear and another is called as driven gear. The rotating gear carries the fluid from the tank to the outlet pipe. The suction side is towards the portion where the gear teeth come out of the mesh. When the gears rotate, volume of the chamber expands leading to pressure drop below atmospheric value. Therefore the vacuum is created and the fluid is pushed into the void due to atmospheric pressure. The fluid is trapped between housing and rotating teeth of the gears. The discharge side of pump is towards the portion where the gear teeth run into the mesh and the volume decreases between meshing teeth. The pump has a positive internal seal against leakage; therefore, the fluid is forced into the outlet port. The gear pumps are often equipped with the side wear plate to avoid the

leakage. The clearance between gear teeth and housing and between side plate and gear face is very important and plays an important role in preventing leakage. In general, the gap distance is less than 10 micro meters. The amount of fluid discharge is determined by the number of gear teeth, the volume of fluid between each pair of teeth and the speed of rotation. The important drawback of external gear pump is the unbalanced side load on its bearings. It is caused due to high pressure at the outlet and low pressure at the inlet which results in slower speeds and lower pressure ratings in addition to reducing the bearing life. Gear pumps are most commonly used for the hydraulic fluid power applications and are widely used in chemical installations to pump fluid with a certain viscosity. Working of External Gear Pump is shown in figure 5.27.

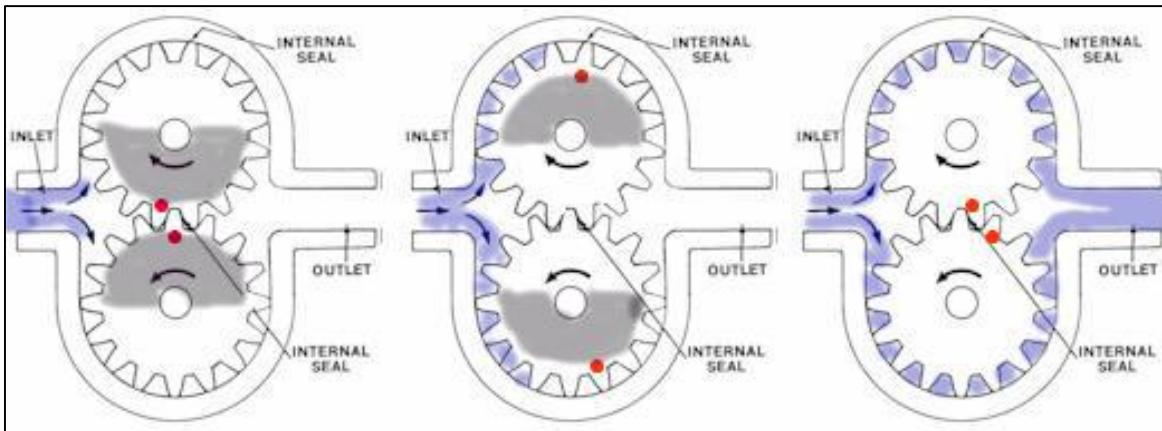


Fig. 5.27 Working of External Gear Pump

5.19 Internal Gear pump

In internal gear pump design, one inner gear is located within an outer gear ring mounted in a housing. The inner gear, whose teeth are in contact with the inner surface of the outer gear ring, is driven and takes the outer ring with it in the same direction. The rotary movement produces a series of contacting and expanding pockets allowing the fluid to run from suction side to delivery side. To prevent backflow from the outlet to the inlet side crescent shaped partition can be fitted to the casing between the gears. Internal gear pumps are exceptionally versatile. While they are often used on thin liquids such as solvents and fuel oil, they excel at efficiently pumping thick liquids such as asphalt, chocolate, and adhesives. The useful viscosity range of an internal gear pump is from 1CP to over 1,000,000 CP (Centi Poise).

In addition to their wide viscosity range, the pump has a wide temperature range as well, handling liquids up to 750°F / 400°C. This is due to the single point of end clearance (the distance between the ends of the rotor gear teeth and the head of the pump). This clearance is adjustable to accommodate high temperature, maximize efficiency for handling high viscosity liquids, and to accommodate for wear.

The internal gear pump is non-pulsing, self-priming, and can run dry for short periods. They are also bi-rotational, meaning that the same pump can be used to load and unload vessels. Because internal gear pumps have only two moving parts, they are reliable, simple to operate and easy to maintain.



Fig. 5.28 Internal Gear Pump

5.19.1 Working of Internal Gear Pumps

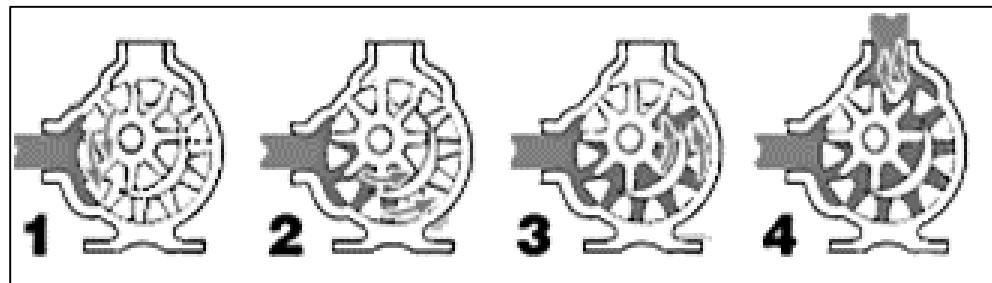


Fig. 5.29 Working steps of Internal Gear Pump

1. Liquid enters the suction port between the rotor (large exterior gear) and idler (small interior gear) teeth. The arrows indicate the direction of the pump and liquid.
2. Liquid travels through the pump between the teeth of the "gear-within-a-gear" principle. The crescent shape divides the liquid and acts as a seal between the suction and discharge ports.
3. The pump head is now nearly flooded, just prior to forcing the liquid out of the discharge port. Intermeshing gears of the idler and rotor form locked pockets for the liquid which assures volume control.
4. Rotor and idler teeth mesh completely to form a seal equidistant from the discharge and suction ports. This seal forces the liquid out of the discharge port.

5.19.2 Advantages & Disadvantages

Advantages	Disadvantages
<ul style="list-style-type: none">• Only two moving parts• Only one stuffing box• Non-pulsating discharge• Excellent for high-viscosity liquids• Constant and even discharge regardless of pressure conditions• Operates well in either direction• Can be made to operate with one direction of flow with either rotation• Low NPSH required• Single adjustable end clearance• Easy to maintain• Flexible design offers application customization	<ul style="list-style-type: none">• Usually requires moderate speeds• Medium pressure limitations• One bearing runs in the product pumped• Overhung load on shaft bearing

5.19.3 Applications

Common internal gear pump applications include, but are not limited to following : All varieties of fuel oil and lube oil , Resins and Polymers ,Alcohols and solvents ,Asphalt, Bitumen, and Tar, Polyurethane foam (Isocyanine),Food products such as corn syrup, chocolate, and peanut butter ,Paint, inks, and pigments, Soaps and surfactants ,Glycol.

5.19.4 Requirement of gear pump

1. Optimum clearance between the two gears to avoid clashing between the two gears
2. The gears used should be of anti-corrosive material since they are in continuous contact with water.
3. They should have low discharge and high pressure.
4. They should even work with low viscosity fluid.
5. The gears should have no backlash.
6. The gears should have accurate tooth spacing.
7. The gears should be noiseless in operation.

5.20 Effect of parameters on Volumetric efficiency

1. Pressure : As the pressure increases, there will be more losses and the volumetric efficiency will reduce
2. Viscosity : As viscosity increases the volumetric efficiency increases
3. Clearance : As clearance increases, volumetric efficiency decreases
4. Speed : With increase in speed, theoretical discharge increases. Hence volumetric efficiency increases.

5.21 Gear Selection criteria

1. For uniform discharge, two gears of equal size can be used. Because of equal size there will be control on the bearing load and better loading on input and output shaft.
2. Gear profile selection can be done as involute or Stub. Stub profile is widely used. The parameters like pressure angle, type of gear (spur or helical) etc. are assumed as per need. For pressure exceeding 200 bar helical gears are used whereas spur gears can be used for pressure below 200 bar. Precision cut gears can be selected with stub profile and S_n gearing type of mesh.
3. Gears are precision cut, Accuracy of gear manufacture, noise level and bearing type determines the allowable maximum speed.
4. Loss in inlet pressure due to centrifugal action is also considered when designing gears for gear pump.
5. Gears should have provision for the trapped liquid to go to the delivery side.

5.22 Design Procedure

The following steps have been followed in order to design the gear pump for the given specifications.

5.22.1 Drive Unit

1. To design the drive unit, the standard motor speeds should be considered to choose from 2880/1440/960/720 RPM. More speed is used for controlling the size but difficult to select bearing.
2. Mechanical, volumetric, hydraulic efficiencies are required to assume from the standard range of efficiencies.
3. Motor power can be find out by using discharge, pressure and efficiency values. By assuming service factor standard motor selection can be made.

5.22.2 Transmission unit

- A flexible bush pin coupling is used for transmission. The purpose of fixing the flexible coupling is to transmit torque from one shaft to another, where misalignments may occur

and also to absorb shock loads, etc. Very simple in construction, the Pin Bush coupling is so designed that it transmits high powers and maximum speed. Though normally the flanges are of cast iron, but for higher speeds flanges can be provided in good quality steel. This type of coupling will permit drive on either direction, does not require any lubrication nor adjustment after fitting. These flexible bearings remain unaffected either by dust, water or atmospheric conditions.

- Selection of flexible bush pin coupling specification for transmission unit using PSG reference. It is based on the maximum kilowatt rating as mentioned in the foot note of the specific page of the data book.

5.22.3. Pump Unit

Pump unit consist of Gear set, Bearings, shaft, casing and fasteners. Design of pump unit is very important. Following steps to be considered for pump unit design.

1. Module selection is based on discharge and checking under the bending, dynamic loading and pitting failure can be done.
2. Tooth proportions are determined using the obtained specification of the gears.
3. Shaft design requires force analysis for both the gear and heavily loaded shaft can be designed.
4. Needle bearing or Roller bearing can be selected. Selecting any one for radial load only, life of the bearing is assumed or as required. However the outer diameter of the bearing is constrained. Thus the bearing selected is checked for its dynamic capacity
5. Selecting casing material and thick cylinder theory can be applied to the gear pump casing design.
6. Bolt design based on tensile failure can be done.

5.22.4 Piping Unit

Piping unit basically consists of a suction pipe and a delivery pipe. Both the pipes need to be according to the specification so as to suit the operating parameters of the gear pump. The material of the pipes is decided on the basis of the type of fluid to be conveyed through the pump i.e. corrosive nature, viscosity or adhesive properties. Thus piping unit affects the head losses directly and hence should be designed carefully considering all the requirements.

1. Size of suction and delivery pipe is to be determined in the design of piping unit.
2. Continuity equation is applied to suction and delivery pipes in order to find the diameters of the pipes as per the assumed velocities for flow.

5.23 Material of Components

For Casing of the external pumps, commonly used material is cast iron, newer materials are allowing the pumps to handle liquids such as sulphuric acid, sodium hypochlorite, ferric chloride, sodium hydroxide, and hundreds of other corrosive liquids.

1. Externals (head, casing, bracket) - Iron, ductile iron, steel, stainless steel, high alloys, composites
2. Internals (shafts) - Steel, stainless steel, high alloys, alumina ceramic
3. Internals (gears) - Steel, stainless steel, PTFE, composites
4. Bushing - Carbon, bronze, silicon carbide, needle bearings
5. Shaft Seal - Packing, lip seal, component mechanical seal, magnetically-driven pump

5.24 Derivation of hydraulic force

Let P_{max} is the maximum pressure that can be attained in a gear pump. The pressure increases from 0 to P_{max} in 180^0 and remain constant on delivery side at P_{max} .

Consider, an elementary strip subtending angle $d\theta$ at angle θ . Let R be the external radius of the gear or internal radius of the casing. The force developed on the elementary strip will be,

$$dF = R \times d\theta \times b \times P_\theta$$

Force acting on the elementary strip can be used to find the total force.

$$dF_x = dF \cos\theta = P_\theta \times R \times b \times d\theta \times \cos\theta$$

And

$$dF_y = dF \sin\theta = P_\theta \times R \times b \times d\theta \times \sin\theta$$

Where,

$$P_\theta = P_{max} \times \frac{\theta}{\pi} \quad \dots \dots \dots \quad 0 < \theta < \pi$$

$$P_\theta = P_{max} \quad \dots \dots \dots \quad \pi < \theta < \frac{3\pi}{2}$$

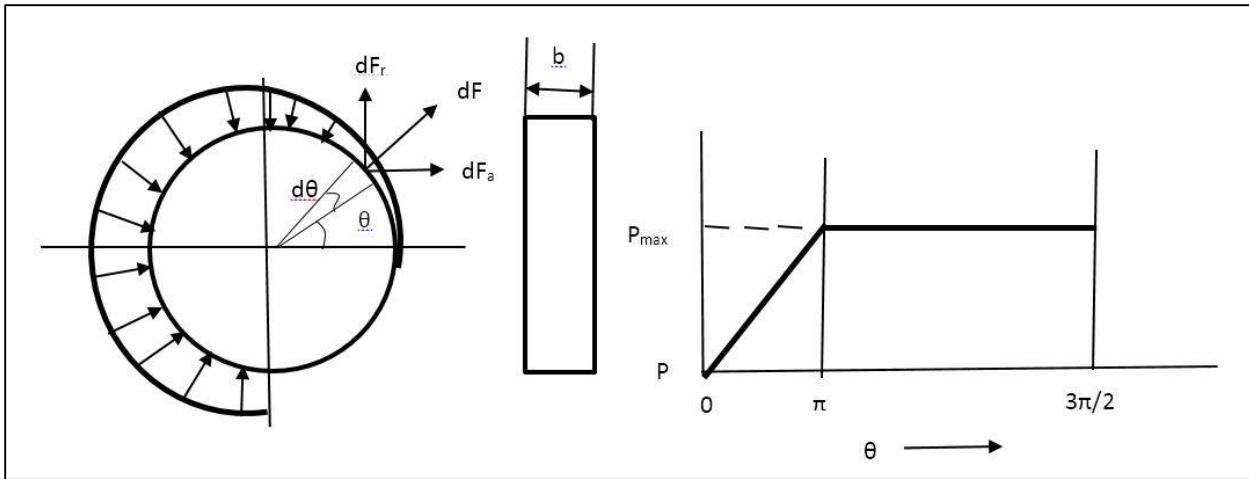


Fig. 5.30 Hydraulic force acting on gear and pressure

Total force acting in x direction is given as,

$$F_x = \int_0^\pi P_{max} \times \frac{\theta}{\pi} \times R \times d\theta \times b \times \cos\theta + \int_{\pi}^{\frac{3\pi}{2}} P_{max} \times R \times d\theta \times b \times \cos\theta$$

Solving integration,

$$F_x = \left(\frac{-2 \times P_{max} \times R \times b}{\pi} \right) - (P_{max} \times R \times b)$$

$$F_x = - P_{max} \times R \times b \times \left(\frac{2}{\pi} + 1 \right),$$

$$F_x = - 1.6366 \times P_{max} \times b \times R$$

Negative sign indicates that net force is towards the negative x-axis,

$$F_y = \int_0^\pi P_{max} \times \frac{\theta}{\pi} \times R \times b \times \sin\theta \times d\theta + \int_{\pi}^{\frac{3\pi}{2}} P_{max} \times R \times b \times \sin\theta \times d\theta = 0$$

So the hydrostatic force developed is only in x-direction

$$\mathbf{F_h = 1.6366 \times P_{max} \times b \times R}$$

5.25 Derivation of module equation based on discharge

In gear pump, the criteria for the gears design is based on discharge. Further, the gears are checked for bending, dynamic load and pitting failure.

Let,

Q = Actual Discharge per min,

N = Speed of the motor (rpm)

η_{vol} = Volumetric efficiency,

z = Number of teeth on the gear,

m = module in metre

D = Pitch diameter = $m.z$

D_o = Outer diameter = $m.z + 2m$

D_f = Root diameter = $m.z - 2m$

b = Width = $\emptyset.m$

Discharge area = $\pi/4 [D_o^2 - D_f^2]$

Discharge volume = $\pi/4 [D_o^2 - D_f^2] \times b$

Discharge per min = $\pi/4 [D_o^2 - D_f^2] \times b \times N$

Now,

$$Q = \eta_{vol} \times \pi/4 [D_o^2 - D_f^2] \times b \times N$$

Substituting D_o , D_f and b in terms of m and rearranging terms,

$$Q = \eta_{vol} \times \pi/4 [(m.z + 2m)^2 - (m.z - 2m)^2] \times b \times N$$

$$m = \sqrt[3]{\frac{Q}{2\pi \cdot \emptyset \cdot z \cdot \eta_{vol} \cdot N}}$$

NUMERICALS

Numerical 5.3 Design a Gear Pump for following Specifications.

Discharge- 40 LPM (Litres per Minute) & Pressure 50 bar

SOLUTION:

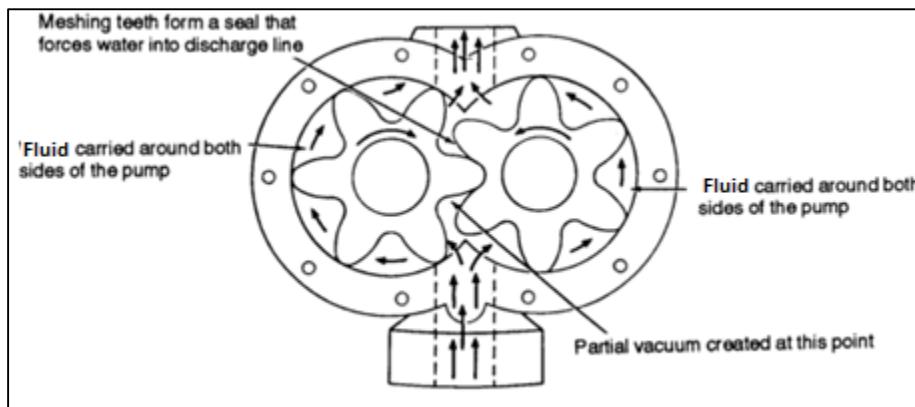


Fig. 5.31 Basic layout of gear pump

Components are:

1. Drive Unit
2. Transmission Unit
3. Pump Unit
4. Piping Unit

Step 1: Drive Unit

The Drive Unit consists of a motor that converts mechanical energy into hydraulic energy: a rotating shaft. It uses electricity flow to generate torque and rotation.

Assuming Mechanical Efficiency = (η_{mech}) = 0.93

Volumetric Efficiency = (η_{vol}) = 0.97

Discharge can be given as,

$$(Q) = 40 \text{ LPM} = \frac{40 \times 10^{-3}}{60} \text{ m}^3/\text{s}$$

$$\text{Pressure (P)} = 50 \text{ bar} = 50 \times 10^5 \text{ N/m}^2$$

$$\text{Motor power} = \frac{Q \times P}{\eta_{\text{mech}} \times \eta_{\text{vol}}} = 3695.08 \text{ W}$$

Service factor = 1.5 for rotary pump (PSG 7.109)

Therefore,

$$\text{Power} = 3695.08 \times 1.5 = 5542.62 \text{ W}$$

Thus, selecting a standard motor of 5.5 kW and speed (N) = 960 rpm (PSG 5.124)

Step 2: Transmission Unit

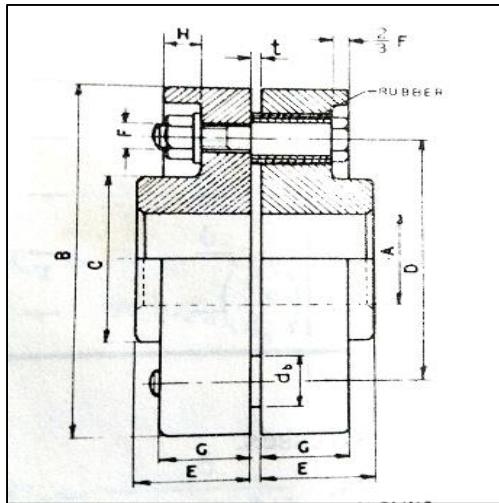


Fig. 5.32 Bush type flexible coupling

A flexible bush pin coupling is used for transmission.

Selecting Flexible bush type coupling.

$$\text{Max. Rating at 100 rpm} = \frac{\text{Power} \times 100}{900} = 0.57$$

Selecting Coupling No. 2 can be selected where the

$$\text{Max Rating at 100 rpm} = 0.6 \text{ (PSG 7.108)}$$

Thus, $d = A = 16 \sim 22 \text{ mm}$, $B = 100$, $C = 30 \text{ mm}$, $E = 30$, $G = 20 \text{ mm}$

Step 3: Pump Unit

Assumptions:

Gear Pair: Profile Selection – Involute, Pressure angle (α) = 20° .

Quality of gear: Precision cut

Type of meshing: S_n gearing ($X_1 = X_2 = 0, f_o = 1$)

Type of gear selection: Spur Gear (For $P < 200$ bar)

Thus, selecting 2 gears of equal size ($i=1$) for uniform discharge, better loading on input/output shafts and better control on bearing load.

Step 3.1 Design of gears

Gears are toothed wheels meshed together inside the casing of a hydraulic gear pump. The gears are precision cut so that no fluid leaks resulting into a loss of pressure. The design specification of the gears affects the performance parameters of a gear pump viz. pressure head, discharge or flow rate.

Material Selection

Selecting material for gear 40 Ni 2 Cr 1 Mo 28 (PSG 8.5)

Design Stresses $[\sigma_b] = 400 \text{ N/mm}^2, [\sigma_c] = 1100 \text{ N/mm}^2$

Module Calculation

Design Criteria- Module selection is based on discharge. Further, checking for bending, dynamic load and pitting.

$$Q = \frac{40 \times 10^{-3}}{60} \text{ m}^3/\text{s} = 6.67 \times 10^{-4} \text{ m}^3/\text{s}$$

Assuming volumetric efficiency as,

$$\eta_{vol} = 0.97$$

Let,

$$Z = \text{Number of teeth} = 14$$

$$m = \text{module}$$

$$D = \text{Pitch diameter} = m.z$$

$$D_o = \text{Outer diameter} = m.z + 2m = 14m + 2m = 16m$$

$$D_f = \text{Root diameter} = m.z - 2m = 14m - 2m = 12m$$

$$b = \text{Width} = 7m$$

$$Q = \eta_{vol} \times \pi/4 [(D_o^2 - D_f^2)] \times b \times N$$

$$40 \times 10^{-3} = 0.97 \times \pi/4 [(16m)^2 - (12m)^2] \times 7m \times 960$$

Thus,

$$m^3 = 5.479 \times 10^{-8}$$

$$m = 3.798 \times 10^{-3} \text{ metre} = 3.798 \text{ mm}$$

Selecting standard module, $m = 4 \text{ mm}$ (PSG 8.2)

Therefore,

$$D_o = 16m = 64 \text{ mm},$$

$$D = m.z = 14m = 56 \text{ mm}$$

$$D_f = 12m = 48 \text{ mm}$$

$$\text{Width } b = 7m = 28 \text{ mm}$$

Also,

$$m = \sqrt[3]{\frac{Q}{2\pi \cdot \emptyset_m \cdot z \cdot \eta_{vol} \cdot N}}$$

Where,

$$Q (\text{mm}^3/\text{min}) = (40 \times 10^{-3}) \times 10^9$$

$$\emptyset_m = b/m = b/4$$

$$z = 14, \eta_{vol} = 0.97, N = 960 \text{ rpm}$$

Thus,

$$(4)^3 = \frac{40 \times 10^6}{2\pi \times \left(\frac{b}{4}\right) \times 14 \times 0.97 \times 960}$$

$$b = 30.52 \text{ mm}$$

Considering higher value of width and taking $b = 30 \text{ mm}$

Check for Bending

For spur gear,

$$m = 1.26 \sqrt[3]{\frac{[M_{t1}]}{(\sigma b_1) \cdot \emptyset_m \cdot z_1 \cdot Y_1}} \quad (\text{PSG 8.13 A})$$

$$[M_{t1}] = [M_t]/2$$

(Since both gears are subjected to external load, torque on each gear is subjected to $\frac{1}{2}$ of the total torque)

$$[M_{t1}] = \frac{[P]}{2 \times 2\pi N/60} = \frac{5.5 \times 10^3}{2 \times 2\pi \times 960/60} = 27.355 \text{ N m} = 27.355 \times 10^3 \text{ N.mm}$$

Lewis form factor, for $\alpha = 20^\circ$ involute stub tooth (PSG 8.50)

$$Y_1 = \pi (0.175 - \frac{0.95}{z})$$

Now, $z = 14$, $Y_1 = 0.3366$

$$\varnothing_m = b/m = 30/4 = 7.5$$

Therefore,

$$4 = 1.26 \sqrt[3]{\frac{27.355 \times 10^3}{(\sigma_{b1}) \times 0.3366 \times 14 \times 7.5}}$$

$$(\sigma_{b1}) = 24.19 \text{ N/mm}^2 < [\sigma_{b1}] = 400 \text{ N/mm}^2$$

The gear is Safe in Bending.

Check for Dynamic Load

Static strength

$$F_s = [\sigma_{b1}] b Y_1 m \text{ (PSG 8.50)}$$

$$F_s = 400 \times 30 \times 0.3366 \times 4 = 16.16 \text{ kN}$$

Lewis Dynamic Load,

$$F_d = F_t \times C_v \quad \text{(PSG 8.51)}$$

For precision cut gear, Barth Velocity Factor,

$$C_v = \frac{5.5 + \sqrt{V}}{5.5}$$

$$v = \pi D N / 60 = \pi (0.056) (960/60) = 2.815 \text{ m/s}$$

Therefore,

$$C_v = \frac{5.5 + \sqrt{2.815}}{5.5} = 1.305$$

And Tangential Load,

$$F_t = \frac{[P]}{v} = \frac{5.5 \times 1000}{2.815} = 1.954 \text{ kN}$$

$$F_d = 1.954 \times 1.305 = 2.55 \text{ kN} < F_s = 13.392 \text{ kN},$$

The gear is Safe in Dynamic Load.

Check for Pitting

Induced contact stress is given by,

$$(\sigma_c) = 0.74 (i + 1)/a \sqrt{\frac{i+1}{i.b} E [Mt_1]} \quad (\text{PSG 8.13})$$

Where,

$$i = 1$$

$$a = D = 56 \text{ mm}$$

$$b = 30 \text{ mm}$$

$$E = \text{Modulus of Elasticity} = 2.1 \times 10^5 \text{ N/mm}^2$$

$$[Mt_1] = 27.355 \times 10^3 \text{ N.mm}$$

$$(\sigma_c) = 0.74 (1 + 1)/56 \sqrt{\frac{1+1}{30} \times 2.1 \times 10^5 \times 27355}$$

$$\text{Thus, } (\sigma_c) = 517.2 \text{ N/mm}^2 < [\sigma_c] = 1100 \text{ N/mm}^2$$

The gear is Safe in Pitting.

Tooth Proportions:

$$a = m = 4 \text{ mm}$$

$$D = m.z = 56 \text{ mm}$$

$$c = 0.25 m = 1 \text{ mm}$$

$$D_o = 16m = 64 \text{ mm}$$

$$D_f = 12m = 48 \text{ mm}$$

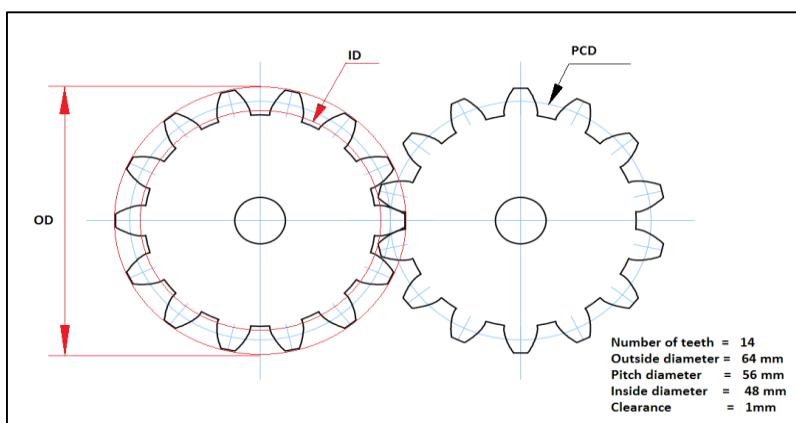


Fig. 5.33 Gear Proportions

Force Analysis

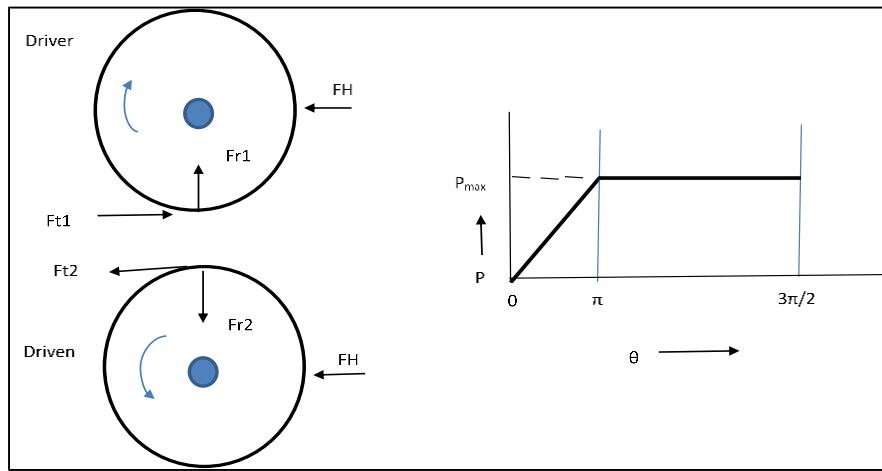


Fig. 5.34 force acting on the gear and pressure distribution

Tangential Load is,

$$F_t = 1.954 \text{ kN}$$

Radial Load,

$$F_r = F_t \tan \alpha = 1.954 \tan (20) = 0.711 \text{ kN}$$

Hydraulic Force,

$$F_H = 1.635 R_o b P_{max}$$

Where,

$$R_o = D_o / 2 = 64 / 2 = 32 \text{ mm}$$

$$b = 30 \text{ mm}$$

$$\text{Let, } P_{max} = 1.2P = 1.2 \times 5 = 6 \text{ N/mm}^2$$

Therefore,

$$F_H = 1.6366 R_o b P_{max} = 1.6366 \times 32 \times 30 \times 6 = 9.427 \text{ kN}$$

The Resultant force acting on the shaft,

$$F = \sqrt{(F_h + F_t)^2 + Fr^2} = \sqrt{(9.427 + 1.954)^2 + 0.711^2} = 11.40 \text{ kN}$$

Step 3.2 Bearing Design

Radial Force

$$F_r = F/2 = 11.40/2 = 5.7 \text{ kN}$$

Axial Force, $F_a = 0 \text{ kN}$

Speed, $N = 960 \text{ rpm}$

Assume Life of bearing in hours (L_{hr}) = 5000 hrs

Life in millions of revolutions

$$(L_{mr}) = L_{hr} \times 60 \times N / 10^6 = 288 \text{ mr}$$

The Equivalent load is given by,

$$P_{eq} = (X V F_r) S = (5.7) \times 1.1 = 6.27 \text{ kN} \text{ (PSG 4.2)}$$

Dynamic Load capacity,

$$C = P_{eq} L_{mr}^{1/k} \quad (k = 10/3 \text{ for needle bearing})$$

$$C = 6.27 (288)^{3/10} = 34.2835 \text{ KN} = 3428.35 \text{ kgf}$$

Constraint on bearing outer diameter (D_o): $(D_o - D_f) \leq 10 \text{ mm}$

$D_o - 48 \leq 10 \text{ mm}$, $D_o \leq 58 \text{ mm}$

Selecting Needle bearing of RNA 69 series (without inner race) (PSG 4.35)

For $D_o = 55 \text{ mm}$,

Dynamic capacity of bearing: $C = 4200 \text{ kgf} > 3428.35 \text{ kgf}$

Therefore, selecting RNA 6907 ($d_r = 42 \text{ mm}$, $B = 36 \text{ mm}$, $D = 55 \text{ mm}$)

Step 3.3 Shaft Design

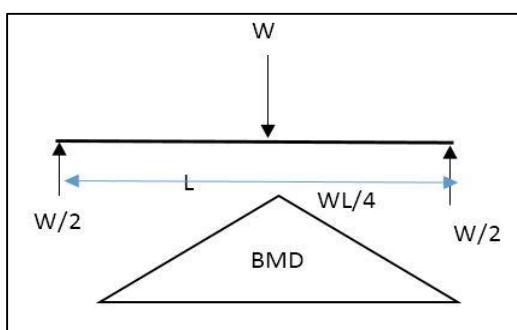


Fig. 5.35 SFD & Bending moment diagram

Assuming Shaft Material as C-40 with $[\tau] = 45 \text{ N/mm}^2$

Span length, $L = B + b + \text{Clearance}$

$$L = 36 + 30 + 10 = 76 \text{ mm}$$

Maximum bending moment is given by,

$$(BM_{\max}) = W \times L / 4$$

$$(BM_{\max}) = 11.39 \times 10^3 \times 76 / 4 = 216.41 \times 10^3 \text{ N.mm}$$

$$[M_{t1}] = T_{\max} = 27.355 \times 10^3 \text{ N.mm}$$

$$T_{eq} = \sqrt{T^2 + M^2} = 218.132 \times 10^3 \text{ N.mm}$$

$$(\tau) = \frac{T_{eq}}{\frac{\pi}{16}(d)^3} = \frac{218.132 \times 10^3}{\frac{\pi}{16}(42)^3} = 14.99 \text{ N/mm}^2$$

$$(\tau) < [\tau] = 45 \text{ N/mm}^2, d = 42 \text{ mm safe.}$$

Reselecting the Flexible Bush Coupling as No. 4, with $d = 30.$

Step 3.4 Casing

The casing houses both the gears and has provisions for occupying the shafts. The casing should be designed to withstand high pressures as it may burst if designed improperly. Improper casing design may also result into leaks and hence reduced efficiency. The inside surface of the casing should be precision machined so that the clearance between gear teeth and the surface remains constant.

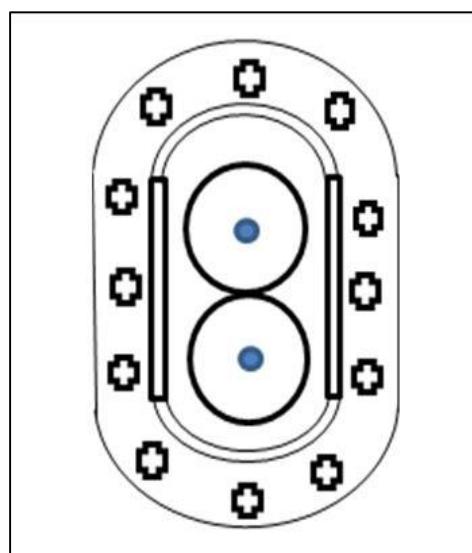


Fig. 5.36 Casing

Selecting Material: GCI -20, with $\sigma_u = 200 \text{ N/mm}^2$

Taking FOS = 6, $[\sigma_t] = 200 / 6 = 30 \text{ N/mm}^2$

$$P_{\max} = 1.2 P = 1.2 \times 5 = 6 \text{ N/mm}^2$$

Tip diameter of gear, $D_o = 64 \text{ mm}$,

By thick cylinder theory,

Considering hoop stress failure

$$t = \frac{D_o}{2} \left[\sqrt{\frac{[\sigma_t] + P_{\max}}{[\sigma_t] - P_{\max}}} - 1 \right]$$

$$t = \frac{64}{2} \left[\sqrt{\frac{30+6}{30-6}} - 1 \right] = 7.19 \text{ mm}$$

Taking thickness, $t = 8 \text{ mm}$

From Figure, $C = D + D_o = 56 + 64 = 120 \text{ mm}$

PCD for casing = $C + 3 \times d_{\text{bolt}}$

Outer diameter of casing = PCD of casing + $3 \times d_{\text{bolt}}$

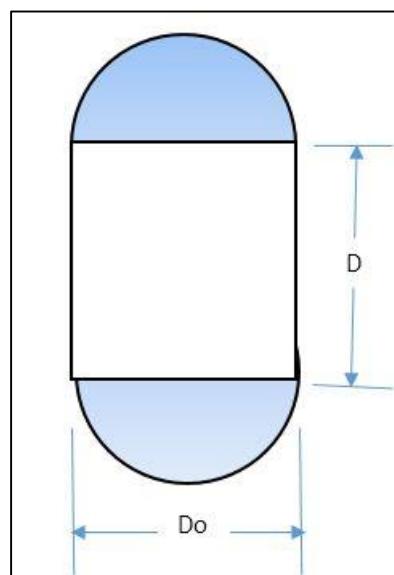


Fig. 5.37 Projected area

Step 3.5 Fasteners / Bolts

Let, Fastener Material – C20 with $[\sigma_t] = 80 \text{ N/mm}^2$

Operating Pressure, $P = 5 \text{ N/mm}^2$

Design Pressure, $P_{\max} = 1.2 \times 5 = 6 \text{ N/mm}^2$

Opening Pressure, $P_o = 1.5 \times 5 = 7.5 \text{ N/mm}^2$

$$\text{Projected Area} = \frac{\pi}{4} D_o^2 + D_o \times D = \frac{\pi}{4} \times 64^2 + 64 \times 56 = 6801 \text{ mm}^2$$

External Force, $F_e = P_{\max} \cdot A_o = 6 \times 6801 = 40.806 \text{ kN}$

Opening Force, $F_o = P_o \cdot A_o = 7.5 \times 6801 = 51.008 \text{ kN}$

Let the Stiffness factors for leak proof joints

$C_c = 0.33$ and $C_b = 0.67$

Initial tightening Load on bolt,

$$F_i = F_o \times C_c = 51.008 \times 0.33 = 16.832 \text{ kN}$$

Net force on bolt, $F_b = F_i + F_e \cdot C_b$

$$F_b = 16.832 + 40.806 \times 0.67 = 44.1725 \text{ kN}$$

Now, $F_b = n \times A_c \times (\sigma_t)$

Where, n is no. of bolts, A_c is core area.

Taking standard bolt of M12 and number of bolts (n) = 8

Core Area, $A_c = 84.3 \text{ mm}^2$ (PSG 5.42)

$$\text{Therefore, } (\sigma_t) = \frac{44172.5}{8 \times 84.3} = 65.5 \text{ N/mm}^2 < [\sigma_t] = 80 \text{ N/mm}^2$$

Hence, M12 size, 8 bolts are selected.

Now PCD for casing = $C + 3 \times d_{\text{bolt}} = 120 + 3 \times 12 = 156 \text{ mm}$

Outer diameter of casing = PCD of casing + $3 \times d_{\text{bolt}} = 156 + 3 \times 12 = 192 \text{ mm}$

Step 4: Piping unit

Piping unit basically consists of a suction pipe and a delivery pipe. Both the pipes need to be according to the specification so as to suit the operating parameters of the gear pump. The material of the pipes is decided on the basis of the type of fluid to be conveyed through the

pump i.e. corrosive nature, viscosity or adhesive properties. Thus piping unit affects the head losses directly and hence should be designed carefully considering all the requirements.

Step 4.1 Suction Side

Range of velocity in suction pipe, $v_s = 1 \sim 1.5 \text{ m/s}$, Taking $v_s = 1 \text{ m/s}$

Discharge

$$Q = A V = \frac{40 \times 10^{-3}}{60} = \frac{\pi}{4} d_s^2 \times 1$$

$$d_s = 29.13 \text{ mm}$$

Selecting 1" diameter (25.4 mm) pipe for suction side. Actual velocity, $v_s = 1.32 \text{ m/s}$.

Step 4.2 Delivery Side

Velocity in delivery pipe, $v_d = 1.5 \sim 3 \text{ m/s}$, Taking $v_d = 2 \text{ m/s}$

Discharge,

$$Q = A V = \frac{40 \times 10^{-3}}{60} = \frac{\pi}{4} d_d^2 \times 2$$

$$d_d = 20.6 \text{ mm}$$

Selecting 3/4" diameter (19.05 mm) pipe for delivery side.

Actual velocity, $v_d = 2.34 \text{ m/s}$.

Numerical 5.4 Design a Gear Pump for following Specifications

Pressure – 80 bar

Discharge 100 LPM

SOLUTION:

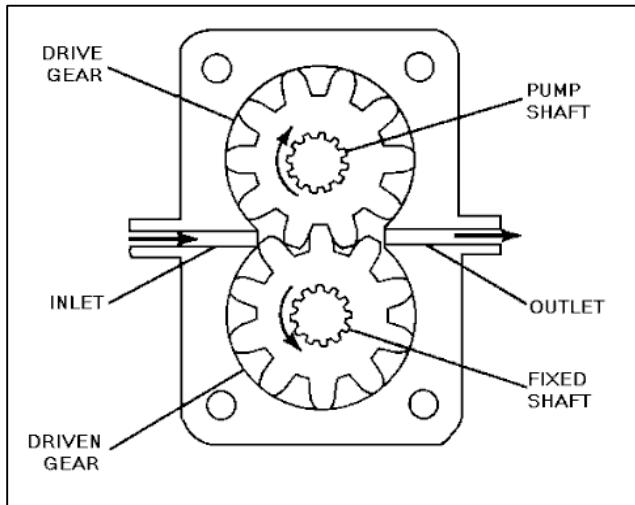


Fig. 5.38 Gear pump layout

There are four units in the gear pump.

1. Drive Unit
2. Pump Unit
3. Transmission Unit
4. Piping Unit

Step 1 Drive Unit

There are different standard speed of the motor available like 2880/1440/960/720

Selecting Speed = 960 rpm

Standard range of efficiencies η are:

$\eta_{\text{mech}} = 0.9 \text{ to } 0.95$

$\eta_v = 0.9 \text{ to } 0.98$,

Where,

η_{mech} = Mechanical Efficiency

η_v = Volumetric Efficiency

Selecting efficiency as $\eta_m = 0.95$, $\eta_v = 0.98$

$$\text{Power input} = \frac{Q \cdot P}{\eta_m \times \eta_v}$$

Where,

$$Q = \text{discharge} = 0.1 \text{ m}^3/\text{min}$$

$$P = \text{pressure} = 80 \times 10^5 \text{ N/m}^2$$

$$\text{Power input} = 80 \times 10^5 \frac{0.1 \times 1}{60 \times 0.95 \times 0.98} = 14.322 \text{ kW}$$

Motor Power = Power input \times Service Factor (S.F) (S.F from PSG 7.109)

$$\text{Motor Power} = 14.322 \times 1.5 = 21.482 \text{ kW}$$

Selecting Standard motor from PSG 5.124, P = 22 kW

Step 2: Transmission unit

Selecting flexible bush pin coupling form PSG 7.108

$$\text{Maximum rating at 100 rpm} = \text{Power} \times \text{S.F} \times 100 / 960 = \frac{22 \times 100}{960} = 2.29$$

Selecting Coupling No. 5 having diameter range of 45 – 56 mm.

Step 3 Pump Unit

There are different component in the pump unit and they are as follows,

1. Gear Pair, 2.Shaft, 3.Bearing, 4.Casing, 5.Fasteners

Step 3.1 Design of Gear pair

Assumptions

Profile Selection – Involute, Pressure angle - 20°

Quality of gear – Precision cut, Type of meshing – SN gearing ($X_1 = X_2 = 0, F_o = 1$)

Since the given pressure is $P < 200$ bar so selecting Spur Gear

Both the gears are of equal size for uniform discharge, better loading on input and output shaft and control of bearing load.

Gear material selection (PSG 8.5)

Let the gear material selected be 40Ni2Cr1Mo

$$[\sigma_b] = 400 \text{ N/mm}^2 \text{ for, } m < 6 \text{ and } [\sigma_c] = 1100 \text{ N/mm}^2$$

Design criteria

Module selection is based on discharge and checking is done for Bending, Dynamic load and pitting failure,

Now Discharge,

$$Q \text{ (m}^3/\text{min}) = \eta_v \times \frac{\pi}{4} (d_a^2 - d_f^2) \times b \times N$$

Where, Q = Discharge

η_v = Volumetric Efficiency

d_a = Addendum Diameter

d_f = Root Diameter

b = Width of the gear

N = Speed in rpm

Let, z = number of teeth = 14 and

$\Phi_m = b/m = 6$ to 8

Where, m = module of the gear

Let, $b/m = 7$

$d_a = Z \times m + 2 \times m = 14m + 2m = 16m$

$d_f = Z \times m - 2 \times m = 14m - 2m = 12m$

$$Q = \eta_v \times \frac{\pi}{4} (d_a^2 - d_f^2) b \times N$$

$$0.1 = 0.98 \times \frac{\pi}{4} [16^2 - 12^2] \times m^2 \times 7m \times 960$$

$$m = 5.567 \times 10^{-3} \text{ metre}$$

$$m = 5.5 \text{ mm}$$

Selecting Standard value of module (PSG 8.1) $m = 6$

$$b = 7 \times 6 = 42 \text{ mm}$$

$$b = 42 \text{ mm}$$

Checking for bending failure

There is a bending of gear tooth due to the force acting on the tooth when in contact with each other in gear pump. So checking for bending failure.

$$m = 1.26 \sqrt[3]{\frac{[Mt_1]}{(\sigma_b) Y Z \phi}} \quad \text{PSG 8.13)$$

Where,

M_t = bending moment of the teeth

σ_b = bending stress

b = width of the gear,

Y = Lewis form factor

Z = number of teeth

$$[M_t] = \frac{P}{W} = \frac{P}{2\pi N/60} = \frac{22000}{2\pi \times 960/60} = 218.83 \text{ Nm}$$

$$[M_t] = 2.183 \times 10^5 \text{ N.mm}$$

$$[M_{t1}] = [M_t]/2$$

Since both gears are subjected to external load, torque on each gear is subjected to half of the total torque

$$[M_{t1}] = 2.183 \times 10^5 / 2 = 1.09 \times 10^5 \text{ N.mm}$$

Y = Lewis form factor

For Full Depth 20° Involute tooth profile from PSG 8.50/E 2.1

$$Y = \pi \times [0.154 - \frac{0.912}{Z}]$$

$$Y = \pi \times [0.154 - \frac{0.912}{14}] = 0.2792$$

Therefore putting all the values in the equation,

$$m = 1.26 \sqrt{\frac{[M_{t1}]}{(\sigma b) Y Z \phi}}$$

$$6 = 1.26 \sqrt{\frac{1.09 \times 10^5}{(\sigma b) \times 0.2792 \times 14 \times 7}}$$

$$(\sigma_b) = 175.68 \text{ N/mm}^2$$

$$(\sigma_b) = 175.68 \text{ N/mm}^2 < [\sigma_b] = 400 \text{ N/mm}^2$$

Where $[\sigma_b]$ = design bending stress , So the gear is safe in bending.

Checking for dynamic load

Static strength (PSG 8.50)

$$F_s = [\sigma_{b1}] \times b \times Y_1 \times m$$

Where,

F_s = Static strength of the gear tooth

$$F_s = 400 \times 42 \times 0.279 \times 6 = 28.143 \text{ KN}$$

Lewis dynamic force (PSG 8.50)

$$F_d = F_t \times C_v$$

Where, F_d = dynamic force acting on the gear

F_t = tangential force acting on the gear

C_v = Barth Velocity Factor

$$C_v = \frac{5.5 + \sqrt{V}}{5.5}$$

Where, V = velocity with which the gear operates

$$V = \frac{\pi D N}{60}$$

Where, D = pitch diameter of the gear

N = speed in rpm

$$V = \frac{\pi \times 6 \times 14 \times 960}{60} = 4.22 \text{ m/s}$$

$$C_v = \frac{5.5 + \sqrt{4.22}}{5.5} = 1.3736$$

Tangential Load is given by, $F_t = \frac{P}{V} = \frac{22000}{4.22} = 5.213 \text{ KN}$

Therefore, Dynamic force is given by,

$$F_d = F_t \times C_v = 5.213 \times 1.3736 = 7.160 \text{ kN}$$

Here, $F_d < F_s$

Since the static strength of gear is greater than the dynamic load so the gear is safe in dynamic load.

Check for Pitting

Contact stress is given by (PSG 8.51),

$$\sigma_{c1} = 0.74 \frac{i+1}{a} \sqrt{\frac{i+1}{i b} E Mt}$$

Where,

i = speed ratio = 1

a = centre distance between two gears = $m \times z = 6 \times 14 = 64 \text{ mm}$

b = width of the gear = 42mm

E = young's modulus = $2.1 \times 10^5 \text{ N/mm}^2$

$$\sigma_{c1} = 0.74 \frac{1+1}{64} \sqrt{\frac{1+1}{42} * 2.1e5 * 1.09e5}$$

$$\sigma_{c1} = 763.47 \text{ N/mm}^2 < [\sigma_{c1}] = 1100 \text{ N/mm}^2$$

Since the induced contact stress is less than the design contact stress so the gear is safe in pitting failure.

Tooth Proportion

Centre distance $a = m = 6 \text{ mm}$

Addendum diameter $d_a = 16 \times m = 16 \times 6 = 96 \text{ mm}$

Clearance $C = 0.25 \times m = 0.25 \times 6 = 1.5 \text{ mm}$

Root diameter $d_f = 12 \times m = 12 \times 6 = 72 \text{ mm}$

Pitch diameter of gear $d = m \times z = 6 \times 14 = 84 \text{ mm}$

Step 3.2 Bearing Design

The suitable types of bearing used in the gear pump are needle type bearing

For Needle Type Bearing, forces acting on bearing are,

$$F_r = \frac{F}{2} = \frac{36.93}{2} = 18.465 \text{ kN}$$

Where F = Net Force acting on the gear and the axial force, $F_a = 0$

Assuming, Life $L_{hr} = 5000 \text{ hrs}$

$$\text{Life in millions of revolution, } L_{mr} = \frac{L_{hr} \times 60 \times N}{10^6}$$

Where, L_{hr} = life of the bearing in hour

$$L_{mr} = \frac{5000 \times 60 \times 960}{10^6} = 288 \text{ mr}$$

$$P_{eq} = X \times V \times F_r \times S \text{ (PSG 4.2)}$$

Where, P_{eq} – equivalent force acting on bearing

$$P_{eq} = 1 \times 1 \times 18.45 \times 1.1$$

$$P_{eq} = 20.31 \text{ kN}$$

$$C = P_{eq} (L_{mr})^{1/k}$$

Where, C – dynamic load carrying capacity of the bearing

$$C = 20.31 (288)^{3/10}$$

$$C = 111.05 \text{ KN} = 11105 \text{ kgf}$$

Constrain on bearing diameter,

$$D_o - D_f \leq 10 \text{ mm}$$

Where D_o - outer diameter of bearing

$$D_o - 72 \leq 10$$

$$D_o \leq 82 \text{ mm}$$

Hence, from PSG 4.33, checking for Needle type bearing.

There is no suitable needle bearing available for required dynamic load capacity and outer bearing diameter constrain. Hence selecting sliding contact bearing.

Let outer diameter of bearing, $D_o = 80 \text{ mm}$

Journal diameter $D = 76 \text{ mm}$ (Assuming thickness of brass as 2mm)

Now Radial load is given by,

$$F_r = P \times \text{Projected area}$$

$$18.465 \times 1000 = \text{Pressure} \times L \times D$$

Assuming pressure $p = 1.4 \text{ N/mm}^2$ (0.7 to 1.4 N/mm²) (PSG 7.31)

$$L = 18465 / (1.4 \times 76) = 173.54 \text{ mm} \approx 180 \text{ mm}$$

Bearing span, $l = \text{Bearing length} = L + \text{Gear width } b = 180 + 42 = 222 \text{ mm}$

For Shaft design,

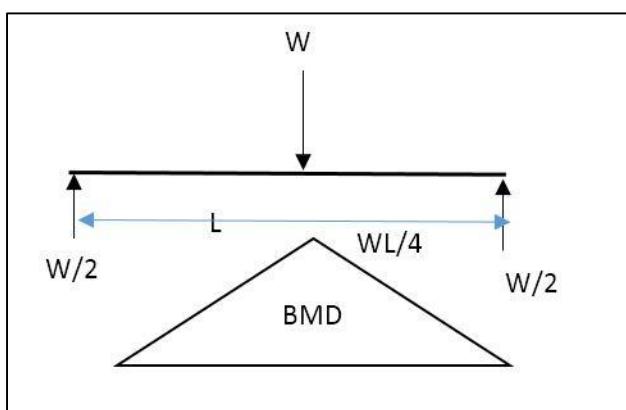


Fig. 5.40 SFD and Bending Moment Diagram

Maximum bending moment,

$$\text{B.M. max} = F.l / 4$$

Where, $l = \text{span between the two bearing}$

$$\text{B.M. max} = (36.93 \times 10^3 \times 222) / 4$$

$$B.M. \max = 2049615 \text{ N-mm}$$

$$T_{\max} = [M_{t1}] = 1.09 \times 10^5 \text{ N.mm}$$

Equivalent torque acting on the shaft is given by,

$$T_{eq} = \sqrt{T^2 + M^2} = 2052511.3 \text{ N.mm}$$

$$(\tau) = \frac{T_{eq}}{\frac{\pi}{16}(d)^3} = \frac{2052511.3}{\frac{\pi}{16}(70)^3} = 30.47 \text{ N/mm}^2$$

As $(\tau) < [\tau]$ Hence Safe.

Shaft diameter can be taken as 70 mm as root diameter is 72 mm.

Step 3.3 Shaft Design

Assuming Shaft Material same as gear as integral type construction can be used

Hence, assuming $[\tau] = 120 \text{ N/mm}^2$,

Tangential force $F_t = 5.213 \text{ kN}$ from the dynamic load

Radial force, $F_r = F_t \tan\alpha = 1.8973 \text{ kN}$,

Where, F_r = radial force acting on the gear,

Now, $F_h = 1.6366 \times R_a \times b \times P_{\max}$

Where, F_h = hydraulic force

R_a = Addendum radius = $96/2 = 48 \text{ mm}$

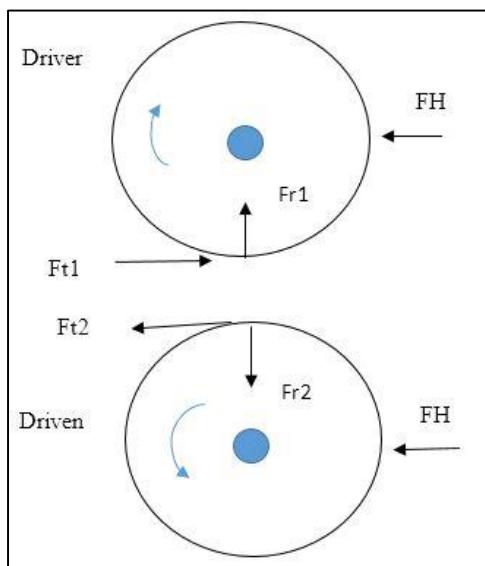


Fig. 5.39 force acting on the gear

b = Width of the gear = 42mm

P_{max} = Maximum pressure acting on the gear = 1.2×P

$$F_h = 1.6366 \times \frac{96}{2} \times 42 \times 1.2 \times 8$$

$$F_h = 31.674 \text{ kN}$$

So the Net force (F_{net}) acting on the gear is given as,

For output shaft

$$F_{\text{net}} = \sqrt{(F_h + F_t)^2 + F_r^2} = \sqrt{(31.674 + 5.213)^2 + 1.8972^2} = 36.93 \text{ kN}$$

But to calculate Bending moment on the shaft, length of the shaft is not known hence shaft diameter will be decided after bearing selection.

Step 3.4 Casing Design

Selecting the material of the casing to be GCI-20 with σ_u = 180 N/mm²

Where σ_u – ultimate strength of the GCI

[σ_t] = σ_u / FOS, where σ_t – tensile stress of GCI and FOS = factor of safety

$$[\sigma_t] = 180/6 = 30 \text{ N/mm}^2$$

$$P_{\text{max}} = 1.2 \times 80 = 96 \text{ bar} = 9.6 \text{ N/mm}^2$$

By Thick Cylinder theory,

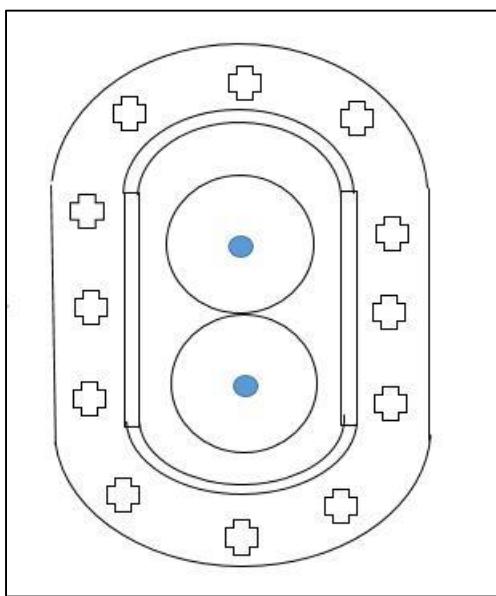


Fig. 5.40 Casing

Considering Hoop stress failure,

$$t = \frac{D_a}{2} \left[\frac{\sqrt{[\sigma_t] + P_{max}}}{\sqrt{[\sigma_t] + P_{max}}} - 1 \right]$$

$$t = \frac{96}{2} \left[\frac{\sqrt{30+9.6}}{\sqrt{30-9.6}} - 1 \right]$$

$$t = 18.87 \text{ mm}$$

Let $t = 20 \text{ mm}$, where t = thickness of casing

$$C = D_f + D_o = 72 + 96 = 168 \text{ mm}$$

$$\text{PCD for casing} = C + 3.d_b = 168 + 3 \times 16 = 216 \text{ mm}$$

Where, PCD = pitch diameter of casing and d_b = diameter of bolt let's use M16 bolts

$$D_e = \text{PCD} + 3.d_b = 216 + 3 \times 16 = 264 \text{ mm}$$

Where D_e = Outer diameter of casing.

Step 3.5 Fasteners

Selecting the material of the fastener to be material C- 20 $[\sigma_t] = 80 \text{ N/mm}^2$, $P = 8 \text{ N/mm}^2$

Where, P = operating discharge pressure

$$P_{max} = 1.2 \times 8 = 9.6 \text{ N/mm}^2$$

Where, P_{max} = design pressure

$$P_o = 1.5 \times P = 1.5 \times 8 = 12 \text{ N/mm}^2$$

Where, P_o - opening pressure

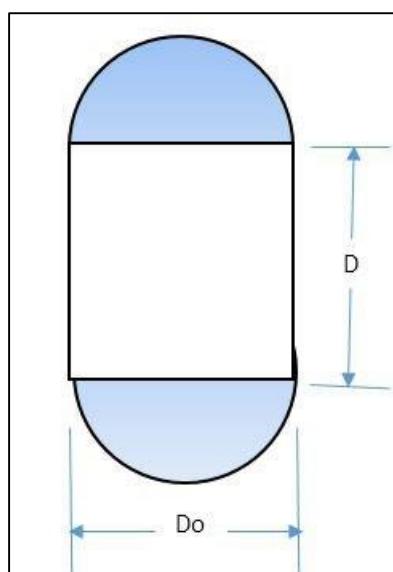


Fig. 5.41 Projected area

Projected Area, from above figure

$$A_o = \frac{\pi}{4} D a^2 + D a \times D = \frac{\pi}{4} 96^2 + (96 \times 84)$$

$$A_o = 15302.23 \text{ mm}^2$$

External force,

$$F_e = P_{\max} \times A_o = 9.6 \times 15302.23 = 146.9 \text{ kN}$$

Opening Force,

$$F_o = P_o \times A_o = 12 \times 15302.23$$

$$F_o = 183.63 \text{ kN}$$

Assuming stiffness factor as $C_c = 0.33$ and $C_b = 0.67$

Net Force (F) acting on Bolt,

$$F = F_o \times C_c + F_i \times C_b = 183.63 \times 0.33 + 146.9 \times 0.67$$

$$F = 159.02 \text{ kN}$$

Where, C_c and C_b are Stiffness Factor for Leak Proof Point

Diameter of Bolt can be calculated from

$$[\sigma_t] = \frac{F}{nA_c}$$

Assuming no. of bolts $n = 12$

$$A_c = \frac{F}{n[\sigma_t]} = \frac{159020}{12 \times 80} = 165.64 \text{ mm}^2$$

Now $A_c = 157 \text{ mm}^2$ for M16 bolts hence increasing the no of bolts to 16,

$$\text{Core area required} = A_c = \frac{F}{n[\sigma_t]} = \frac{159020}{16 \times 80} = 124.23 \text{ mm}^2$$

Selecting 16 no. of M16 Bolt.

Step 4: Piping unit

For the piping unit in the gear pump there are two types of pipes used i.e. suction and delivery pipe.

Step 4.1 Suction Pipe

Range of velocity in suction pipe is 1 to 1.5 m/s

$$Q = \frac{0.1}{60} = 1.667 \times 10^{-3} \text{ m}^3/\text{s}, \text{ Where } Q = \text{discharge},$$

$$Q = A \times V$$

Let,

$V_s = 1.25 \text{ m/s}$, where V_s = Velocity in suction pipe

$$A = \frac{1.667 \times 10^{-3}}{1.25} = 1.333 \times 10^{-3} \text{ m}^2$$

$$A = \frac{\pi \times d^2}{4}$$

$$d_s = 0.0412 \text{ m}$$

Where, d_s = diameter of suction pipe

$$d_s = 41.2 \text{ mm}$$

Therefore, selecting 1.5 " pipe diameter i.e. $d_s = 50.8 \text{ mm}$

$$V_{\text{actual}} = Q/A = 1.667 \times 10^{-3} \times 4/\pi \times (38.1/1000)^2$$

$$V_{\text{actual}} = 1.462 \text{ m/s}$$

Step 4.2 Delivery pipe

Range of velocity in suction pipe is 1.5 to 3 m/s

$$Q = (100 \times 10^{-3})/60 = 1.667 \times 10^{-3} \text{ m}^3/\text{s}$$

Now,

$$Q = A \times V$$

Let, $V_d = 2.5 \text{ m/s}$, where V_d = Velocity in delivery pipe

$$A = \frac{1.667 \times 10^{-3}}{2.5} = 0.6668 \times 10^{-3} \text{ m}^2$$

$$A = \frac{\pi \times d^2}{4}$$

$$d_d = 0.02913 \text{ m}$$

Where, d_d = diameter of delivery pipe

$$d_d = 29.2 \text{ mm}$$

Therefore selecting 1" pipe diameter.

$$V_{d \text{ actual}} = Q/A$$

$$V_{d \text{ actual}} = 1.667 \times 10^{-3} \times 4/\pi \times (25.4/1000)^2$$

$$V_{d \text{ actual}} = 2.5838 \text{ m/s}$$

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11. Pump Handbook

Module 6

Design of Machine Tool Gear Box

6.1 Introduction

Gearbox is referred as transmission unit that uses gears and gear trains to provide speed and torque conversions from a rotating power source to another device. Gearboxes are employed to convert input from a high speed power sources to low speed (examples are Lift, Cranes, and Crushing Machine etc.) or into a many of speeds (examples are Lathe, Milling Machine and Automobiles). A gearbox that converts a high speed input into a single output it is called a single stage gearbox. It usually has two gears and shafts. A gearbox that gives more than one output is called multispeed gearbox.

6.2 Importance of gearbox

A gearbox is actually a speed variable device which changes the speed of power transmission device according to the requirements of torque. Normally various power transmission devices are used for machines like lathe machines, turning machines, milling machines and other machine tool which requires power at different torque or speed as per the will of operator or based on operation performed. High torque operations requires low speed and low torque operation requires high speed. For example, when vehicle is to be drive at steeper road, the speed has to be low to have increased torque. The power and torque are directly proportional but speed and torque are inversely proportional ($P = 2\pi NT/60$). Hence to perform different operations that require different torque or power, one can fulfil the requirements by changing the speed. The speed variation can be done by using different gears having different teeth on it on various shaft at proper distance to have perfect meshing between the two gears. In case of lathe, milling machine the speed are fixed by the required cutting feed or cutting speed for range of materials. Low strength material can be cut at highest speed and high strength material required to cut at low speed compared to that of high strength material. In automobiles maximum and minimum speed of the vehicle determine the size of the gearbox.

Machine tools are characterized by their large number of spindle speeds and feeds to cope with the requirements of machining parts of different materials and dimensions using different types of cutting tool materials and geometries. The cutting speed is determined on the basis of the cutting ability of the tool used, surface finish required and economic considerations. A wide variety of gearboxes utilize sliding gears or friction or jaw coupling. The selection of a particular mechanism depends on the purpose of the machine tool, the frequency of speed change, and the duration of the working movement. The advantage of a sliding gear transmission is that it is capable of transmitting higher torque and is small in radial dimensions and the disadvantages of these gearboxes is the impossibility of changing speeds during running. Clutch-type gearboxes require

small axial displacement needed for speed changing, less engagement force compared with sliding gear mechanisms and therefore can employ helical gears.

The extreme spindle speeds of a machine tool main gearbox N_{\max} and N_{\min} can be determined by

$$N_{\max} = \frac{V_{\max}}{\pi d_{\min}} \text{ and } N_{\min} = \frac{V_{\min}}{\pi d_{\max}} \quad \dots\dots\dots (1)$$

Where,

V_{\max} = maximum cutting speed (m/min) used for machining the most soft and Machinable material with a cutting tool of the best cutting property.

V_{\min} = minimum cutting speed (m/min) used for machining the hardest material using a cutting tool of the lowest cutting property or the necessary speed for thread cutting.

d_{\max}, d_{\min} = maximum and minimum diameters (m) of work piece to be machined.

The speed range R_n becomes,

$$R_n = \frac{N_{\max}}{N_{\min}} = \frac{V_{\max}}{V_{\min}} \frac{d_{\max}}{d_{\min}} = R_v \times R_d \quad \dots\dots\dots (2)$$

Where,

R_v = cutting speed range

R_d = diameter range

In case of machine tools having rectilinear main motion (planers and shapers), the speed range R_n is dependent only on R_v . For other machine tools, R_n is a function of R_v and R_d , large cutting speeds and diameter ranges are required. Generally, when selecting a machine tool, the speed range R_n is increased by 25% for future developments in the cutting tool materials. Table 1 shows the maximum speed ranges in modern machine tools.

Speed Range for Different Machine Tools Machine are as below:

Numerically controlled lathes $R_n = 250$,

Boring $R_n = 100$,

Milling $R_n = 50$,

Drilling $R_n = 10$,

Surface grinding $R_n = 4$.

6.3 Determination of variable speed range

Various law of speed regulation of rpm of main drive are used for designing gear boxes. In step regulation of speed only certain discrete values of RPM are available on machine tool .Different criteria are used for choosing the discrete steps. Between two extreme available values of spindle rpm the same number of steps may be placed in number of ways. The various steps of RPM values will have different operating characteristic. There are 4 speed range distribution

- 1) RPM Values with arithmetic progression

- 2) RPM Values with geometric progression
- 3) RPM Values with harmonic progression
- 4) RPM Values with logarithmic progression

Despite of many shortcoming, GP is commonly used in machine tool drive because of its constant loss of economy cutting speed in whole rpm range, constant loss of productivity in whole rpm range & better design features.

6.4 Aim of speed and feed rate regulation

A machining operation should be conducted at such values of cutting parameters (speed, feed, depth of cut etc.) that ensures the minimum cost price of the machine component. The machining cost is expressed by the equation,

$$P = P_{mt} + P_{npt} + P_{tc} + P_t \quad \dots\dots (3)$$

Where,

P_{mt} - The cost of machining time,

P_{npt} - The cost of non-productive time,

P_{tc} - The tool changing cost per component,

P_t - The cost of the tool per component.

The cost of machining time is given by,

$$P_{mt} = (P_w + P_o) t_{m/c} \quad \dots\dots (4)$$

Where, P_w is the wage rate, P_o is the cost of operating the machine tool per unit time and $t_{m/c}$ is the machining time.

The cost of non-productive time is given by,

$$P_{npt} = (P_w + P_o) t_{npt} \quad \dots\dots (5)$$

Where t_{npt} is the total time of nonproductive operation.

The tool changing cost per component is given by,

$$P_{tc} = (P_w + P_o) * t_r / Q \quad \dots\dots (6)$$

Where, t_r is the time required for replacing a blunt tool and setting the new one and Q the no. of components machined during the period of tool life.

The cost of the tool per component is given by,

$$P_t = T_t / Q \quad \dots\dots (7)$$

Where, T_t is the cost of the tool for a period equal to the tool life and can be determined as the tool cost divided by the number of permissible regrinding.

In order to machine a part of arbitrary diameter, the spindle rpm must be set as i.e. There must be a step less regulation of velocity V so that any desired value of the spindle rpm may be set

corresponding to the optimum cutting speed. By a similar logic the machine tool should have provision for step less variation of the feed rate. The provision for regulating the spindle rpm and feed rate is an essential requirement of machine tools to ensure economic machining of work pieces of different materials and sizes by cutting tools of different shapes and composition.

6.5 Laws of step regulation

For choosing the discrete values of spindle rpm, there are four cases from which the most suitable law of speed range distribution is chosen [1]. For the same number of stages and same extreme values of speed in rpm, there are number of ways to take intermittent steps. The various steps of rpm values will have different operational characteristics. The following laws can be employed to decide the rpm value of different steps.

1. Rpm values in Arithmetic progression
2. Rpm values in Geometric progression
3. Rpm values in Harmonic progression
4. Rpm values in logarithmic progression

Analysis of these laws are given below.

6.5.1 Arithmetic progression

Arithmetic progression based upon the idea that the difference between the adjacent RPM values is constant.

The speed at Z^{th} step is given by, $N_Z = N_1 + (Z - 1)a$ OR $N_{x+1} = N_x + a$ (8)

Where, a – common difference of arithmetic progression, and can be calculated from, $a = \frac{N_Z - N_1}{z-1}$

For Example:

If $N_1 = 100$ rpm, $N_z = 1000$ rpm and speed step $Z = 10$, Then, $a = \frac{1000 - 100}{10 - 1} = 100$

Hence,

$$\begin{aligned}N_2 &= 200 \text{ rpm} \\N_3 &= 300 \text{ rpm} \\N_4 &= 400 \text{ rpm} \\N_5 &= 500 \text{ rpm} \\N_6 &= 600 \text{ rpm} \\N_7 &= 700 \text{ rpm} \\N_8 &= 800 \text{ rpm} \\N_9 &= 900 \text{ rpm} \\N_{10} &= N_z = 1000 \text{ rpm.}\end{aligned}$$

For a particular cutting speed which is the maximum permissible under the selected cutting conditions, the diameter range of work pieces that can be machined by a particular spindle rpm value N_x can be determine as follows.

Upper limit of the range:

$$d_x = \frac{1000V}{\pi N_x} \quad \dots \dots (9)$$

Lower limit of the range:

$$d_{x+1} = \frac{1000V}{\pi N_{x+1}} \quad \dots \dots (10)$$

Hence the diameter range served by this particular RPM is,

$$\Delta d_x = \frac{1000V}{\pi} \left(\frac{1}{N_x} - \frac{1}{N_{x+1}} \right) \quad \dots \dots (11)$$

To find the diameter range for different rpm values in Arithmetic progression, consider a following example, where, $N_1 = 30$ rpm, $N_z = 375$ rpm, speed step $Z = 12$, $V = 20$ m/min and diameter of work piece $d = 212$ mm. Common difference of arithmetic progression can be calculated from, $375 = 30 + (12-1)a$, hence $a = 31.4$

Suppose the machining of the work piece of diameter 212 mm is started. For a cutting speed of $V = 20$ m/min, the value $N_1 = 30$ rpm will correspond to the optimum cutting. Before speed changes to next rpm i.e., $N_2 = 61.4$ rpm one must remove 108.3mm of metal from the work piece diameter. This required 11 passes assuming a permissible depth of cut 5mm. During all passes except the first one, the machining is uneconomical as the actual cutting speed would be less than the permissible value. On the other hand in changing over an rpm value of $N_{11} = 344$ rpm to $N_{12} = 375$ rpm one has to reduce the work piece diameter only by 1.6mm though one can actually remove more than this allowance in one pass.

Table 6.1 Arithmetic progression

N_x in rpm	AP ratio , $\varphi_x = \frac{N_{x+1}}{N_x}$	$d_x = (d_{x-1}) - (\Delta d_{x-1})$	$\Delta d_x = \frac{1000 \times 20}{\pi} \left(\frac{1}{N_x} - \frac{1}{N_{x+1}} \right)$
$N_1 = 30$	2.04	212	108.3
$N_2 = 61.4$	1.51	103.7	35.1
$N_3 = 92.8$	1.33	68.6	17.3
$N_4 = 124.2$	1.25	51.3	10.4
$N_5 = 155.6$	1.20	40.9	6.9
$N_6 = 187$	1.17	34.0	4.84
$N_7 = 218.4$	1.14	29.16	3.66
$N_8 = 249.8$	1.12	25.5	2.90
$N_9 = 281.2$	1.11	22.6	2.20

$N_{10} = 312.6$	1.10	20.4	1.9
$N_{11} = 344$	1.09	18.5	1.6
$N_{12} = 375$	---	16.9	---

From this analysis it is observed that in high rpm range some values of speed steps are redundant where as in low rpm range there is clearly a need to add more steps between the calculated values.

6.5.2 Geometric progression

For Geometric progression the speed at Zth step is given by,

$$N_Z = N_{(Z-1)}\phi = N\phi^{Z-1} \text{ Or } N_{x+1} = N_x\phi \quad \dots\dots (12)$$

Where, ϕ = geometric progression ratio.

$$\phi = \left(\frac{N_Z}{N_1}\right)^{\frac{1}{Z-1}} \quad \dots\dots (13)$$

Analyzing the same problem for Geometric progression law,

Before the change over from a speed $N_1 = 30$ rpm to $N_2 = 37.5$ rpm an allowance of 42mm must be machined from work piece diameter. For the maximum depth of cut 5mm, this allowance may be removed in four to five passes. In changing over from $N_{11} = 300$ rpm to $N_{12} = 375$ rpm one has to take the depth of cut of 2.1mm which can be easily accomplished in one pass. It is seen that in order to make the machine performance equally feasible in the whole range, the lower rpm values of rpm should be brought still closer while high rpm values can be widen a little.

Table 6.2 Geometric progression

N_x in rpm	GP Ratio $\phi_x = \frac{N_{x+1}}{N_x}$	$d_x = (d_{x-1}) - (\Delta d_{x-1})$ in mm	$\Delta d_x = \frac{1000 \times 20}{\pi} \left(\frac{1}{N_x} - \frac{1}{N_{x+1}} \right)$ in mm
$N_1 = 30$	1.26	212	42
$N_2 = 37.5$	1.26	170	36
$N_3 = 47.5$	1.26	134	28
$N_4 = 60$	1.26	106	21
$N_5 = 75$	1.26	85	18
$N_6 = 95$	1.26	67	13

$N_7 = 118$	1.26	54	11.5
$N_8 = 150$	1.26	42.5	9
$N_9 = 190$	1.26	33.5	6.5
$N_{10} = 235$	1.26	27	5.8
$N_{11} = 300$	1.26	21.2	4.2
$N_{12} = 375$	---	17	---

6.5.3 Harmonic progression

Harmonic progression is developed from the idea that diameter range served by each rpm of the progression is equal.

$$\Delta d_x = \frac{1000 \times V}{\pi} \left(\frac{1}{N_x} - \frac{1}{N_{x+1}} \right) = \text{constant} \quad \dots \dots (14)$$

$$N_z = \frac{N_1}{1-(z-1)CN_1} \quad \dots \dots (15)$$

$$\text{Where, } C \text{ constant} = \frac{1}{N_x} - \frac{1}{N_{x+1}}$$

Constant C can be calculated from the above equation if N_{\min} , N_{\max} and speed steps Z are known. Now analyzing the same problem for Harmonic progression law.

It is observed from the table below that in order to change the speed from $N_{11} = 183$ rpm to $N_{12} = 375$ rpm the work piece diameter must be reduced from 35 mm to 17 mm. on a slender work piece of 35mm, a large depth of cut cannot be taken as this would lead to deformation of the work piece. If 2 mm assumed as a permissible depth, the total allowance will be machined in four to five passes. It may be concluded that in Harmonic progression the RPM values in high range are too wide apart making this range uneconomical.

Table 6. 3 Harmonic Progression

N_x in rpm	Harmonic progression ratio, $\Phi_x = \frac{N_{x+1}}{N_x}$	$d_x = (d_{x-1}) - (\Delta d_{x-1})$ in mm	$\Delta d_x = \text{constant in mm}$
$N_1 = 30$	1.09	212	18

$N_2 = 32.7$	1.11	194	18
$N_3 = 36.2$	1.11	176	18
$N_4 = 40$	1.13	158	18
$N_5 = 45.1$	1.13	140	18
$N_6 = 51$	1.18	122	18
$N_7 = 60.1$	1.205	105	18
$N_8 = 72.4$	1.25	89	18
$N_9 = 90.6$	1.34	71	18
$N_{10} = 121.2$	1.51	53	18
$N_{11} = 183$	2.1	35	18
$N_{12} = 375$	---	17	---

6.5.4 Logarithmic progression

In Logarithmic progression the diameter range is function of diameter.

$$\Delta d_x = 2M \cdot (d_x)^P \quad \dots \dots (16)$$

Where M: location coefficient, generally P = 0.5

$$d_x - d_{x+1} = 2M \cdot (d_x)^P \quad \dots \dots (17)$$

$$d_{x+1} = d_x \left[1 - \frac{2M}{(d_x)^{1-P}} \right] \quad \dots \dots (18)$$

The progression can be written as,

$$N_z = N_{z-1} \times \phi_{z-1} \quad \text{OR} \quad N_{x+1} = N_x \times \phi_{x-1}, \quad \dots \dots (19)$$

$$\text{Where, } \phi_{x-1} = 1 - 2M \left(\frac{\pi \times N_{x-1}}{1000V} \right)^{1-P} \quad \dots \dots (20)$$

It is rather difficult to develop the logarithmic progression for given values of N_{\min} , N_{\max} and speed steps Z as this has to be done by successive trials with different values of location coefficient M. For example,

$$d_2 = (d_1) - 2M \sqrt{d_1} \quad , \quad \text{Assuming } M=1,$$

$$d_2 = (d_1) - 2 \sqrt{d_1}$$

Now knowing d_2 , next diameter can be calculated as

$$d_3 = (d_2) - 2 \sqrt{d_2} \quad \dots \dots \text{ and so on.}$$

In this way d_{12} can be found for two values of M ($M = 1$ and $M = 0.5$). As it is known that $d_{12} = 17\text{mm}$, the correct value of M is found out by linear interpolation. For $M= 0.89$ following table shows diameter, progression ratios and Δd_x for all rpm range.

Table 6.4 Logarithmic Progression

N_x in rpm	LP Ratio, $\varphi_x = \frac{N_{x+1}}{N_x}$	$d_x = (d_{x-1}) - (\Delta d_{x-1})$ in mm	$\Delta d_x = 2M \sqrt{d_i}$ in mm
$N_1 = 30$	1.14	212	26
$N_2 = 34.2$	1.15	186	24
$N_3 = 39.4$	1.16	162	22
$N_4 = 45.8$	1.18	140	21
$N_5 = 54.1$	1.20	119	19
$N_6 = 64$	1.22	100	18
$N_7 = 78.1$	1.25	82	17
$N_8 = 97.5$	1.28	65	15
$N_9 = 125$	1.34	50	12
$N_{10} = 167.5$	1.41	38	10
$N_{11} = 236$	1.55	28	9
$N_{12} = 361$	---	17	---

It is observed from above table that the values of the logarithmic progression lies between the geometric and harmonic progression for low as well as high rpm range. It may be concluded that as far as the operational efficiency of the machine tool is concern, logarithmic progression is most suitable. The efficiency of the geometric progression is poorer in low rpm range while harmonic progression is poorer in high rpm range.

Despite major shortcomings, the geometric progression is commonly used in machine tool drives because of the advantages mentioned inn next section.

6.6 Importance of Geometric Progression

6.6.1 Constant loss of economic cutting speed in whole range of rpm values

The optimum cutting speed for material should be maintained for every cut, as the diameter is reduced in each cut, speed has to increase .In a stepped regulation drive, rpm values are provided such that for one cut, all other cuts will not be operated at the optimum cutting speed. They will

operate at lower cutting speed. The difference between these two is the loss of cutting speed. In geometric progression rpm value drives, this loss is constant over the entire speed range. As it is only a function of the ratio between two consecutive speeds i.e.

$$\frac{N_{x+1}}{N_x} = \phi^x. \quad \dots \dots (21)$$

The loss in each range is given as,

$$\text{Loss} = \Delta v = \frac{\phi^{x-1}}{\phi^{x+1}} \times V_{\text{optimum}} \quad \dots \dots (22)$$

6.2.2 Constant loss of productivity in the whole rpm range

Productivity of the machining operation is defined as the surface area of the metal removed in time. In a stepped drive, only one cut will give optimum machining of the work piece and during all other cuts until changeover, there will be loss of productivity. Since the loss of cutting speed is constant, the loss of productivity is also constant over the entire speed range.

6.2.3 Better design features

To optimize the dimension of the gear box, proper material selection for each of the gears, as well as provision of multi speed or multi stage arrangements are most essential. It is difficult to establish the gear ratio in multi speed arrangements. It is therefore suggested to select the ratio and the no. of stages to meet the requirement by trial and error.

This difficulty is overcome by using geometric progression ratio due to its following properties:

1. In a geometric progression with progression ratio, every x^{th} value is also in geometric progression with ratio ϕ^x .
2. Progression ratio multiplied with a constant value also gives values in G.P. with each value constant times greater.
3. If a geometric progression having ratio ϕ is multiplied by a factor ϕ^x , the resulting series is also a geometric progression which is shifted by x numbers.

6.7 Guidelines for Selecting Proper Geometric Progression Ratio

Standard values of GP Ratios are established from the following two main considerations:

1. If there is spindle rpm value n_x , then after a certain number of steps s_1 , there must occur a spindle rpm value $= 2 n_x$ i.e. $n_x \cdot \phi^{s_1} = 2 \times n_x$.
2. The geometric progression should be developed by keeping the standards of preferred numbers and preferred series in mind. The geometric progression should satisfy the condition $n_x \cdot \phi^{s_2} = 10 \times n_x$

The numbers obtained by adopting geometric progression are known as progression preferred number. By adopting different progression ratios, several different series of preferred numbers are developed. There are basically six series to give six different progression ratios.

$$\begin{aligned}
 R_{20} & \quad \phi = \sqrt[20]{10} = 1.12 \\
 R_{10} & \quad \phi = \sqrt[10]{10} = 1.26 \\
 R_{20/3} & \quad \phi = \sqrt[20/3]{10} = 1.41 \\
 R_5 & \quad \phi = \sqrt[5]{10} = 1.58 \\
 R_4 & \quad \phi = \sqrt[4]{10} = 1.78 \\
 R_{\frac{10}{3}} & \quad \phi = \sqrt[\frac{10}{3}]{10} = 2
 \end{aligned}$$

6.8 Breakup of Speed steps

The values of the number of speed steps z are rounded off to the next whole number, preference being given to the number which can be broken into multiples of 2 and 3.

For example:

Numbers between 8.5 and 10 are rounded off to $z=9$

Numbers between 11 and 13 are rounded off to $z=12$ and so on.

Suppose the Z is no. of speed steps are obtained in u number of transmission. If p_1 is the number of speed steps in the first transmission group and p_2 the number of speed steps in the second transmission group, then after the second stage one will have $p_1 \times p_2$ speed steps. Similarly, after the third stage one will have $p_1 \times p_2 \times p_3$ speed steps, and finally Z speed steps after u number of transmissions. i.e. $Z=p_1 \times p_2 \times p_3 \times \dots \times p_u$. For obtaining a particular number of speed steps using minimum no. of gears, it is necessary that $p_1 = p_2 = p_3 = \dots = p_u$, Hence the number of speed steps in each transmission group can be found from the expression $p = (Z)^{1/u}$, For example, if $z=27$ speed steps are to be obtained in 3 stages, then at each stage one must have $p = (Z)^{1/u}=3$ transmissions. However, if the quantity $(Z)^{1/u}$ is not a whole number, then the number of speed steps should be divided in such a manner that $z = 2^{E1} \times 3^{E2}$, where, E_1 and E_2 are whole numbers, , example $12^{1/3} = 2.3$, then $12= 2^2 \times 3^1$

6.8 Structural Diagrams and selecting best possible version

Suppose a speed on one shaft yields two speed values on the next shaft, i.e., the number of speed steps of the particular transmission group is $p = 2$. If the transmission is through gears, the transmission ratios that provide the two new speed values must lie in the following range.

For $\phi = 1.41$, $i_{max} = 2$ and $i_{min} = 1/4$.

The maximum reduction of speed is limited to 4 times to keep the radial dimensions of the speed box within reasonable limits, while the maximum increase of speed is restricted to two times due to limitations of the pitch line velocity. The transmission range of the group is given by

$$i_g = \frac{i_{max}}{i_{min}} = 8$$

For z speed steps $n_1, n_2, n_3 \dots n_z$ in a particular transmission group such that $\frac{n_2}{n_1} = \frac{n_3}{n_2} = \dots = \frac{n_z}{n_{z-1}} = k$

Since the speeds on the last shaft of the speed box must constitute a geometric progression, the following relationship must be satisfied: $K = \phi^x$

Here, X is known as the characteristic of the transmission group and denotes the number of steps of the spindle rpm geometric progression by which two adjacent rpm values of the particular group are separated. A transmission group may thus be conveniently denoted by $p(X)$, p is the number of speed steps in transmission group and X is characteristic. The transmission ratio of m^{th} group can be expressed as $i_m = \phi^{(p_{m-1})X_m}$

Where p_m is the number of speed steps in the m^{th} group and x^m its characteristics.

The number of speed steps are represented by

$$Z = p_1 \times p_2 \times p_3 \times \dots \times p_u \quad \dots \dots \dots (23)$$

Elaborated expression for z may be written as

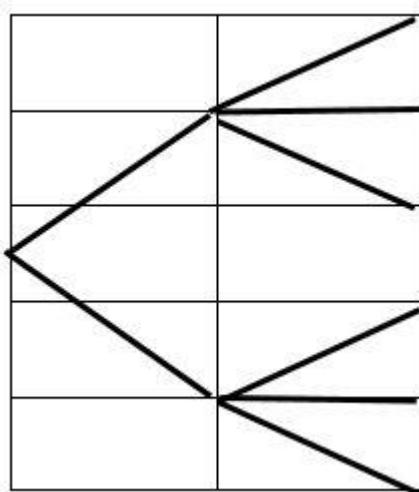
$$Z = p_1(X_1) \times p_2(X_2) \times p_3(X_3) \times \dots \times p_u(X_u), \quad \dots \dots \dots (24)$$

Where $X_1=1, X_2=p_1, X_3=p_1 \times p_2$

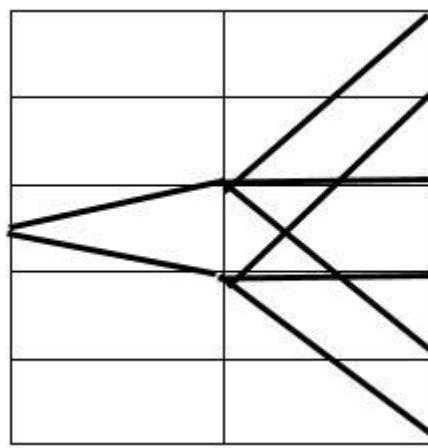
$$X_u = p_1 \times p_2 \times p_3 \times \dots \times p_{u-1} \quad \dots \dots \dots (25)$$

Above equation is known as structural formula of the speed box.

There are two types of structure or ray diagram: - open and crossed. The types are based on distributive connections between input and output points. When the paths do not cross each other, it is called 'open' type of distribution pattern. If the paths cross each other, it is called 'crossed' type of distribution pattern.



2(3) 3 (1)



2 (1) 3 (2)

Fig. 6.1 Ray Diagrams open

Fig. 6.2 Ray Diagrams crossed

From all the possible structural diagrams, the selection of the best version is guided by following two main factors:

1. Transmission ratio relation in a group should have transmission range of less than $i = i_{\max}/i_{\min}$. For example for $\phi = 1.41$, the step down speed ratio should be less than 4 and step up ratio should be less than 2 in one transmission group, similarly for $\phi = 1.26$, step down speed ratio should be less than 6 and step up ratio should be less than 3 in one transmission group as $i_g < 18$.
2. $X_1 < X_2 < X_3 < \dots < X_{u-1} < X_u$ to keep the shaft dimensions minimum.

Apart from the above factors, following points also should be considered:

1. Number of gears on last shaft should be minimum possible.
2. Number of gear pairs in one transmission groups should not be more than 3 in Exceptional cases, 4 gear pairs may be adopted.
3. For least radial dimensions of gear box $i_{\max} \times i_{\min} = 1$.

Limiting values of transmission intervals for different ϕ :

Transmission Ratio ϕ	1.06	1.12	1.26	1.41	1.58	1.78	2.0
Speed Reduction	24	12	6	4	3	2	2
Speed Increase	12	6	3	2	1	1	1

6.10 Steps for the design of machine tool gearbox

1. Determine the maximum and minimum speeds of the output shaft. Calculate the number of steps or Speed reduction stages for this range. This depends on the application as well as space optimisation. Higher reduction stages require more space because of more number of gears and shafts requirements.
2. Select the type of speed reducer or gearbox based on power transmission requirements, gear ratio, position of Axis, space available and make sure that for low gear ratio, single speed reduction is required.
3. Determine the progression ratio that is the ratio of maximum speed and minimum speed. Nearest standard progression ratio can be selected.
4. Write the structural formulae and draw the structural diagram for all possible combinations depending upon the number of steps, stages and their specifications.

5. Select the best structural formula using the following criteria.
 - a. Criteria 1: $i_g < \text{constant}$
 - b. $I = I_{\max}/I_{\min}$, Constant depends upon reduction ratio. (i.e. 8 for $\varphi = 1.41$)
 - c. Criteria 2: $X_1 < X_2 < X_3 < X_4 \dots$ (minimum summation of diameter of shaft)
6. Draw Ray diagram and speed chart.
7. Find no. of gears and no. of teeth on each gear.
8. Draw kinematic diagram
9. Select suitable material and design gear based on strength criteria and checked for dynamic load and pitting failure.
10. Determine the shaft dimensions based on torque and bending moment consideration.
11. Select suitable bearings based on loading and operating condition and the dynamic load carrying capacity required.
12. Design the casing for the gear box.

6.11 Ray Diagram

A ray diagram is a representation of structural formula. Ray diagram is constructed to find the actual speed structures from input speed to output speeds. It provides information such as speed in each stage and its exact location with input and output speed point, the transmission ratio in each stage, the total number of speeds and its values.

Procedure for plotting Ray-Diagram:

1. Draw No. of stages +1 No. of vertical lines i.e. one greater than in the structure diagram.
(e.g. Draw 4 vertical lines at convenient distance.)
2. Draw $Z+1$ No. of horizontal lines. (e.g. $12+1=13$).
3. Draw the rays depicting transmission between shaft and the shaft preceding it. The rays are drawn from the lowest rpm of last shaft keeping in mind the transmission ratio restriction condition of i_{\max} and i_{\min} , (e.g. $i_{\max} \leq 2$ and $i_{\min} \geq 1/4$ for $\varphi = 1.41$)

Ex: For $Z=2(1)3(2)2(6)$, $(i_g)_{\max} = \varphi^6 = (1.41)^6 = 7.85 < 8$; Also, $X_1 \leq X_2 \leq X_3$, Therefore, ray diagram is:

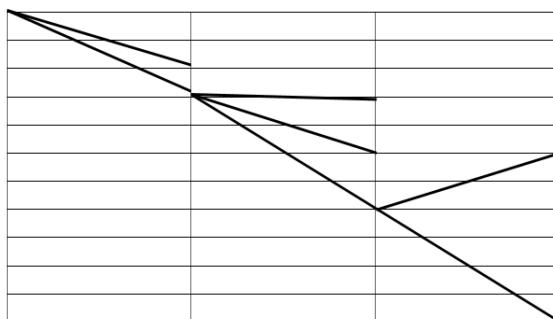


Fig. 6.3 Ray Diagram

6.12 Speed Chart

The speed chart construction is necessary to determine the transmission ratio. Therefore, the following remarks must be considered.

1. The horizontal ray in the speed chart means that there is no speed change i.e. the transmission ratio $i = 1$
2. The upward inclination ray represents speed increasing, i.e. $i > 1$
3. The downward inclination ray means speed reduction, i.e. $i < 1$.

From the speed chart the minimum transmission ratio, i.e. maximum speed reduction can be calculated.

For the example considered in ray diagram mentioned above, i.e., for $Z=2(1)3(2)2(6)$, the speed chart is as shown below:

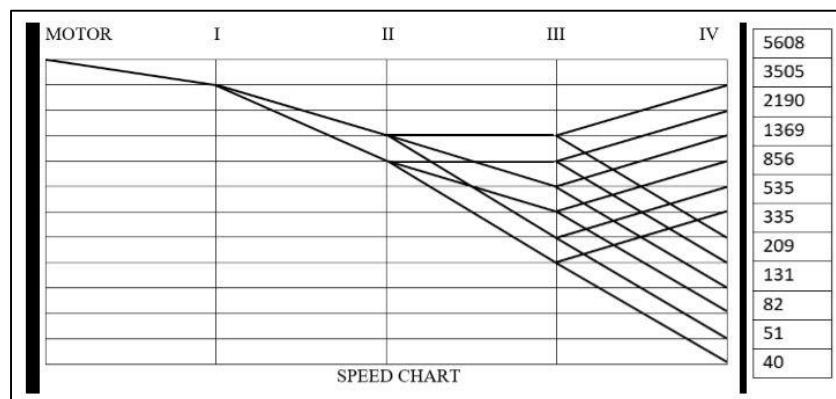


Fig. 6.4 Speed Chart

6.13 Feed Gearbox

Feed gearboxes are designed to provide the feed rates required for the machining operation. The values of feed rates are determined by the specified surface finish, tool life, and the rate of material removal. The classification of feed gearboxes according to the type of mechanism used to change the rate of feed is as follows:

6.13.1 Feed gearboxes with pick-off gears

Used in batch-production machine tools with infrequent change over from job to job, such as automatic, semiautomatic, single-purpose, and special purpose machine tools. These gearboxes are simple in design and are similar to those used for speed changing

6.13.2 Feed gearboxes with sliding gears

These gearboxes are widely used in general-purpose machine tools, transmit high torques, and operate at high speeds. Figure shows a typical gearbox that provides four different ratios. Accordingly, gears Z2, Z4, Z6, and Z8 are keyed to the drive shaft and mesh, respectively, with

gears Z₁, Z₃, Z₅, and Z₇, which are mounted freely on the driven key shaft. The sliding key engages any gear on the driven shaft. The engaged gear transmits the motion to the driven shaft while the rest of the gears remain idle. The main drawbacks of such feed boxes are the power loss and wear occurring due to the rotation of idle gears and insufficient rigidity of the sliding key shaft. Feed boxes with sliding gears are used in small- and medium-size drilling machines and turret lathes.

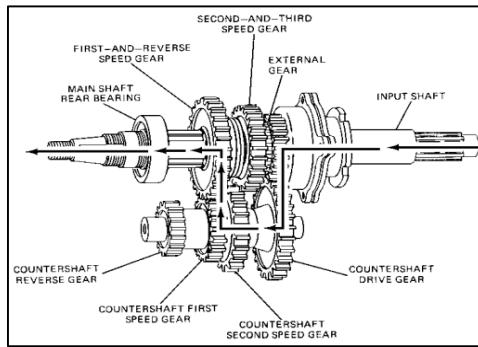


Fig. 6.5 Sliding Gear box

6.13 Norton gearboxes

These gearboxes provide an arithmetic series of feed steps that is suitable for cutting threads and so are widely used in engine lathe feed gearboxes as shown in Fig.

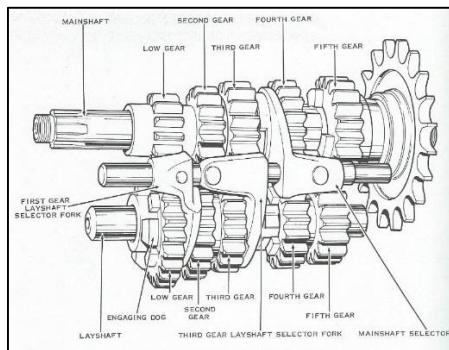


Fig. 6.6 Norton Gear box

6.14 Shaft

Shafts are the members of the gearbox that transmit the rotary motion of the gears to subsequent stages and also transmit power from one stage to the other. They are also the members on which the gears are mounted. The shafts are coupled to the bearings to enable the shafts to rotate without much friction. In a gearbox, two types of shafts are primarily used, keyed shafts and Splined shafts.

6.14.1 Keyed Shaft

They are the shafts in which a keyway is machined so as to enable a gear to be mounted to the said shaft rigidly with the help of a key. In the case of such shafts the gears are rigidly coupled with the shaft and cannot move relative to the shaft.

6.14.2 Splined Shaft

These are the shafts in which splines are cut to enable the gears which have an opposite mating spline cut into them to transmit rotational motion from the gear through the shaft without causing slip. The splines are cut to enable the axial movement of sliding of the gears on the shaft while executing rotational motion without slip.

6.14.3 Properties of shaft material

1. Should have high strength.
2. Should have good machinability.
3. Should have great heat treatment properties.
4. Should have high wear resistant properties.
5. The material used for ordinary shaft is carbon steel of grades 40C8, 45C8, 50C4 and 50C12.

6.15 Minimization of shaft size

In the design of a machine tool gear box, minimization of total sum of shaft diameters is an important criteria. As the shaft diameters are depends on material used and total torque. The torque is ratio of power and speed. If speeds are selected as per GP series, the torques on different shafts forms opposite series of speeds for the same power. For example, if $N_2 = \phi N_1$, then $T_1 = \phi T_2$, similarly shaft diameters will also form GP series with Progression ratio of $\phi^{\frac{1}{3}}$. The two adjacent shaft diameters will give relation as $D_1 = \phi^{\frac{1}{3}} D_2$. The series of diameter can be written as

$$D_1, \quad D_1/\phi^{\frac{1}{3}}, \quad D_1/\phi^{\frac{2}{3}}, \quad D_1/\phi^{\frac{3}{3}}, \quad D_1/\phi^{\frac{4}{3}} \dots \dots$$

6.16 General Recommendation for developing the gearing diagram

- (A) All requirements that are essential for the proper functioning of gear transmission must be satisfied and gear-box dimensions kept minimum.

These requirements are:

1. The no. of teeth on the smallest gear of transmission should be such that there is no undercutting of gear teeth, i.e., $Z_{\min} \geq 17$.
2. If gear pairs on parallel shafts have same modules, the sum of no. of teeth of mating gear pairs must be same.
3. The spacing between adjacent gears on a shaft should be such that one gear pair gets completely disengaged before the next begins to mesh.

4. The minimum difference between the no. of teeth on adjacent gears must differ by at least 4.
5. The axial gap between two adjacent gears must be equal to at least twice the face width of the gear.

(B) Specific features specific to the functioning of machine tools for which the gear box is designed should be taken into account. Some of the features are given below:

1. In machine tools with large inertia of driven member, friction clutch and brakes should be provided on input shaft.
2. Reversing devices with friction clutches should be provided with turret lathes, thread cutting lathes, etc. so that after cutting the threads, the tool can be returned to its initial position.
3. If spindle head traverses during working operations the electric motor should be mounted on the gear box and transmission from motor shaft to the input shaft of radial drilling machines.
4. If the spindle is kinematically linked to the feed mechanism the transmission from spindle to feed train must be shown on gearing diagram.

6.17 Determination of shaft and gear dimensions

6.17.1 Shaft

Shaft dimension is based on equivalent shear stress. The shear stress is due to the driving torque and the bending moment due the gears mounted on the shaft.

Procedure:

1. Torque is required to calculate from motor power and rpm value.
2. Bending moment is required to calculate from SFD & BMD diagrams.
3. The equivalent shear stress can be find out.
4. The equivalent shear stress can be equated with the safe value of shear stress of the material and the diameter can be obtained.

6.17.2 Gear

Gear dimensions are based on the value of module (m) of the gear. Module of the gear depends upon the torque value. The gear is designed based on strength criteria and checked for dynamic load and pitting failure. Based on module circular dimensions can be calculated.

For spur gear

- a) Pitch circle diameter, $d = mz$; where, z = no. of teeth
- b) Root diameter, $d_f = mz - 2m$
- c) Tip diameter, $d_a = mz + 2m$
- d) Width = $m\varphi$; where $\varphi \sim 6$ to 10

6.18 Deviation diagram

In machine tool gear box, the geometric progression ratio ϕ is lies between 1 and 2. ($1 < \phi < 2$)

- a) If $\phi=1$, proportional speed loss = 0 as $\frac{\phi-1}{2} = 0$
- b) If ϕ tends to 1 system becomes step less.
- c) If $\phi = 2$, it is 50%.

In A.C.motor the usual speed ratio is approximately equal to 2. Standard values of ϕ are 1.06, 1.12, 1.26, 1.58, 1.78 and 2. But in design of machine tool gear box 1.26, 1.41, 1.58, 1.78 are widely used. Lesser the value of ϕ , more complicated and expensive the gear box is bound to be. If the difference between the calculated and actual rpm is denoted by Δn , as a rule, $\Delta n \leq 10(\phi-1)$ i, usually this value should not be more than 4i. As mentioned above the actual speeds available on machine will be deviated from calculated speed by an amount Δn which can have either positive or negative value. The speeds should be so designed by choosing suitable gear train such that $\sum \Delta n = 0$. Such deviation while plotted give the deviation diagram as shown in figure 6.7.

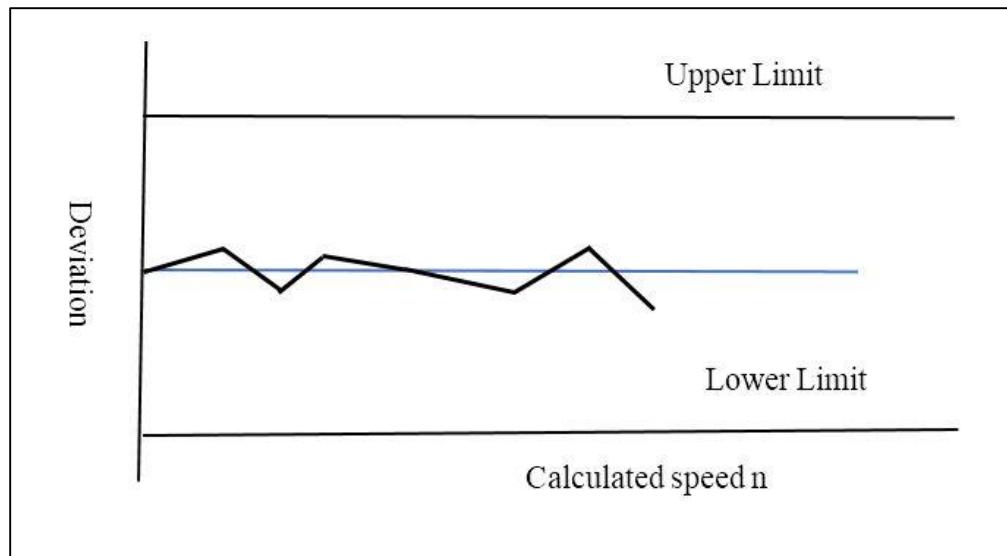


Fig. 6.7 Deviation Diagram

6.19 Layout of Gear Box or Gearing Diagram

Gearing diagram shows all the arrangement of gears and shaft. Total number of shaft and number of gears on each shaft are shown in this diagram and to obtain the specific speed the gear meshing can be shown from input shaft to output shaft.

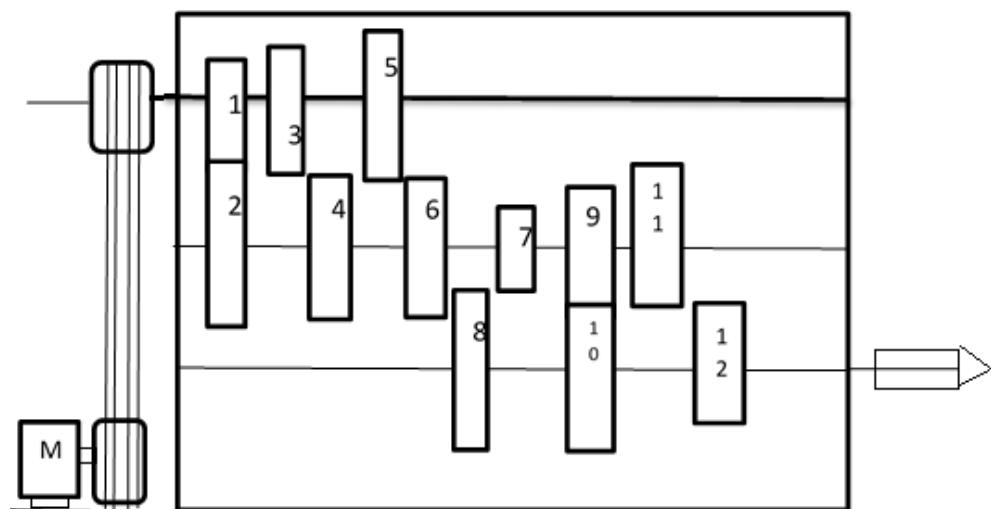
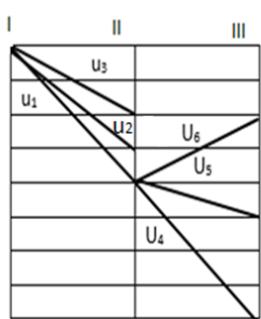


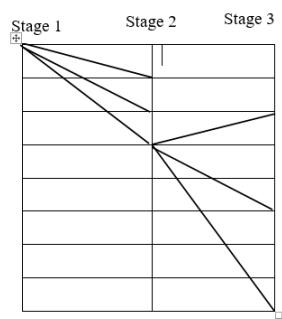
Fig. 6.8 Gear Box Layout

NUMERICALS

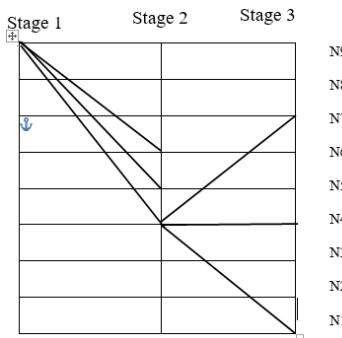
Numerical 6.1 A 3×3 Gear box is transmitting a power of 10KW. Choose the best ray diagram based on minimum summation of shaft diameters made of same material with permissible shear stress of 36 N/mm^2 . Use GP ratio of 1.26 and Lowest speed $N_1 = 100 \text{ RPM}$.



Option (a)



Option (b)



Option (c)

Solution:

$$P = 10 \text{ KW}$$

$$N_1 = 100 \text{ RPM},$$

$$\emptyset = 1.26,$$

$$[\tau] = 36 \text{ N/mm}^2,$$

For GP ratio \emptyset , the series of speed is $N_1, \emptyset N_1, \emptyset^2 N_1, \dots$

Similarly the series of torque is $T_1, T_1/\emptyset, T_1/\emptyset^2, \dots$

The series of diameter is $D_1, \frac{D_1}{\emptyset^{(1)}}, \frac{D_1}{\emptyset^{(2)}}, \frac{D_1}{\emptyset^{(3)}}, \dots \dots$

Calculating Torque and diameter for first speed,

$$T_1 = \frac{P}{2\pi N_1} = \frac{10000 \times 60}{2\pi \times 100} = 954.929 \text{ Nm}$$

For this torque and allowable shear stress $[\tau] = 36 \text{ N/mm}^2$, the shaft diameter is given by,

$$D_1 = \left(\frac{16T_1}{\pi[\tau]} \right)^{\frac{1}{3}} = 51.31 \text{ mm}$$

Now from series of speed and diameters,

Speed (RPM)	100	126	158.76	200.04	252.05	317.58	400.15	504.19	635.28
Diameter (mm)	51.31	47.52	44	37.72	34.92	32.33	29.93	27.72	25.66

Now from ray diagram given,

For option (a)

$$\text{Sum of diameters is } \sum D = D_1 + D_5 + D_9 = 51.31 + 34.92 + 25.66 = 111.09\text{mm}$$

For option (b)

$$\text{Sum of diameters is } \sum D = D_1 + D_6 + D_9 = 51.31 + 32.33 + 25.66 = 109.3\text{mm}$$

For option (c)

$$\text{Sum of diameters is } \sum D = D_1 + D_4 + D_9 = 51.31 + 37.72 + 25.66 = 114.69\text{mm}$$

Minimum sum of diameter is for option (b), hence based on minimum summation of shaft diameters, the best ray diagram is option (b).

Numerical 6.2 Design a machine tool gear box for following Specification:

$Z = 12$ Speed, Minimum speed $N_{\min} = 40$ rpm, Progression ratio $\Phi = 1.41$

Solution:

Step 1: Speed calculation

The geometric progression is ratio is given as 1.41. The specifications state that there are to be 12 speeds and the minimum speed is 40 rpm.

Now different speeds in RPM can be calculated as,

$$N_1 = 40 \text{ rpm}$$

$$N_2 = 40 \times 1.41 = 56.4 \text{ rpm}$$

$$N_3 = 56.4 \times 1.41 = 79.524 \text{ rpm}$$

$$N_4 = 79.524 \times 1.41 = 112.13 \text{ rpm}$$

$$N_5 = 112.13 \times 1.41 = 158.1 \text{ rpm}$$

$$N_6 = 158.1 \times 1.41 = 222.92 \text{ rpm}$$

$$N_7 = 222.92 \times 1.41 = 314.32 \text{ rpm}$$

$$N_8 = 314.32 \times 1.41 = 443.19 \text{ rpm}$$

$$N_9 = 443.19 \times 1.41 = 624.9 \text{ rpm}$$

$$N_{10} = 624.9 \times 1.41 = 881.11 \text{ rpm}$$

$$N_{11} = 881.11 \times 1.41 = 1242.37 \text{ rpm}$$

$$N_{12} = 1242.37 \times 1.41 = 1751.74 \text{ rpm}$$

Step 2: Structural Formula and structure diagram

$$Z = P_1(X_1).P_2(X_2).P_3(X_3)$$

Where,

$$Z = \text{No. of speed range} \quad X_1 = 1$$

$$P_i = \text{No. of stages} \quad X_2 = \text{Co-efficient of } X_1$$

$$X_i = \text{Stage characteristic} \quad X_3 = (\text{Co-efficient of } X_1) (\text{Co-efficient of } X_2)$$

$$\text{As } Z = 12, 12 = 2 \times 3 \times 2 = P_1.P_2.P_3$$

There are 6 kinds of possible combinations or Structural Formulae are,

a) $Z = P_1(X_1).P_2(X_2).P_3(X_3), \quad 12 = 2(1).3(2).2(6)$

b) $Z = P_1(X_1).P_2(X_3).P_3(X_2), \quad 12 = 2(1).3(4).2(2)$

c) $Z = P_1(X_2).P_2(X_1).P_3(X_3), \quad 12 = 2(3).3(1).2(6)$

d) $Z = P_1(X_2).P_2(X_3).P_3(X_1), \quad 12 = 2(3).3(4).2(1)$

e) $Z = P_1(X_3).P_2(X_1).P_3(X_2), \quad 12 = 2(6).3(1).2(3)$

f) $Z = P_1(X_3).P_2(X_2).P_3(X_1), \quad 12 = 2(6).3(2).2(1)$

From all 6 Structural diagrams, Selecting best structural diagram:

Selecting best structural diagram,

Criteria 1: for $\Phi = 1.41$,

$$i_g \leq 8 = \frac{I_{\max}}{I_{\min}} = \frac{2}{1/4}$$

a) For first combination, (a), 2(1).3(2).2(6)

Between Shaft (I) and (II),

$$I_g = \phi^{(P_1-1)*X_1} = \phi^{(2-1)*1} = \phi^1 = 1.41 < 8$$

Between Shaft (II) and (III),

$$I_g = \phi^{(P_2-1)*X_2} = \phi^{(3-1)*2} = \phi^4 = 4 < 8$$

Now Between Shaft (III) and (IV),

$$I_g = \phi^{(P_3-1)*X_3} = \phi^{(2-1)*6} = \phi^6 = 7.85 < 8$$

Criteria 1 is satisfied.

b) For second combination (b), 12 = 2(1).3(4).2(2)

Between Shaft (I) and (II),

$$I_g = \phi^{(P_1-1)*X_1} = \phi^{(2-1)*1} = \phi^1 = 1.41 < 8$$

Between Shaft (II) and (III),

$$I_g = \phi^{(P_2-1)*X_2} = \phi^{(3-1)*4} = \phi^8 = 15.62 > 8$$

Criteria-1 is not satisfied.

Similarly checking for other combinations, It is observed that criteria 1 is not satisfied by (b) and (d)

Criteria 2: $X_1 < X_2 < X_3$

After observing, It is seen that, only (a) is satisfying Criteria-2

Therefore selecting [a] as the best combination.

Following diagram shows, structure diagrams for Cases (a), (b) and (f) similarly structure diagrams for other case can be drawn.

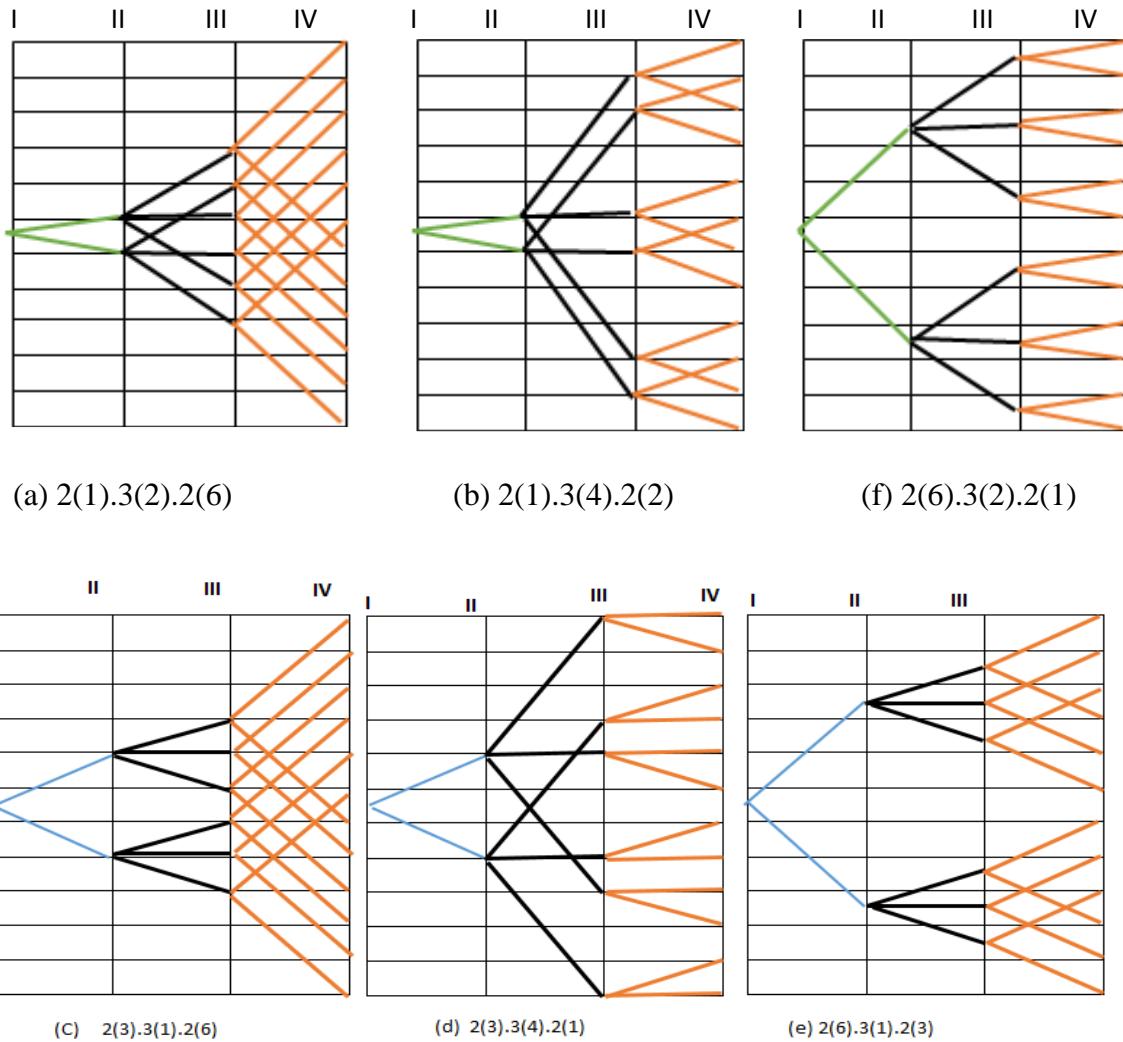


Fig. 6.9 structure diagrams for Cases (a), (b), (f), (c), (d) and (e) respectively.

Step 3: Ray Diagram and Speed Chart: Ray diagram and Speed Chart are as shown in figure.

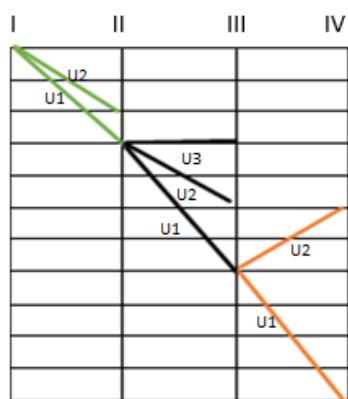


Fig. 6.10 Ray Diagram

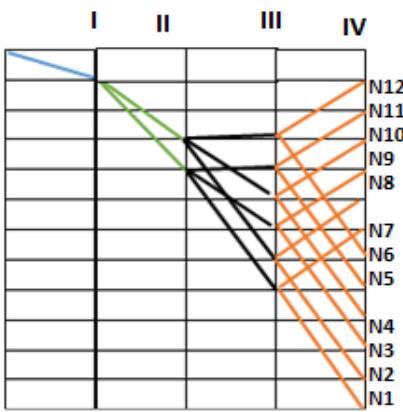


Fig.6.11 Speed Chart

Step 4: Determination of Number of Teeth

Here,

$$Z = 12 = 2 \times 3 \times 2$$

Hence No. of gears in gear train = $2(2+3+2) = 14$

From shaft I to II

			$\frac{F_i}{G_i}$	$F_i + G_i$
U1	$\frac{1}{\varphi 3}$	$\frac{1}{1.43^3}$	$0.3567 = \frac{5}{14}$	19
U2	$\frac{1}{\varphi 2}$	$\frac{1}{1.41^2}$	$0.5 = \frac{1}{2}$	3

LCM of 19 & 3 = 57

U_1 is minimum

$$E \times 57 \times \frac{5}{19} \geq 17, \quad E \geq 1.13$$

$$C = E \times LCM = 1.13 \times 57 = 64.4$$

$$Z_1 = C \times \frac{F_i}{F_i + G_i} = 64.4 \times \frac{5}{19} = 17$$

$$Z_2 = C \times \frac{G_i}{F_i + G_i} = 64.4 \times \frac{14}{19} = 48$$

$$Z_3 = C \times \frac{F_i}{G_i + F_i} = 64.4 \times \frac{1}{3} = 22$$

$$Z_4 = C \times \frac{G_i}{F_i + G_i} = 64.4 \times \frac{2}{3} = 43$$

From shaft II to III

		$F_i/(G_i)$	$F_i + G_i$
U1	$\frac{1}{\varphi 4}$	$0.253 \approx \frac{1}{4}$	5
U2	$\frac{1}{\varphi 2}$	$0.502 \approx \frac{1}{2}$	3
U3	1	$\frac{1}{1}$	2

LCM OF 5, 3, 2 = 30

$$E \times LCM \times \frac{F_i}{F_i + G_i} \geq 17$$

U1 is minimum

$$E \times LCM \times \frac{F_i}{F_i + G_i} \geq 17$$

$$E \times 30 \times \frac{1}{5} \geq 17, E = 2.84$$

$$C = E \times LCM = 2.84 \times 30 = 85.2$$

$$Z_5 = C \times \frac{1}{5} = 85.2 \times \frac{1}{5} = 17$$

$$Z_6 = C \times \frac{4}{5} = 85.2 \times \frac{4}{5} = 69$$

$$Z_7 = C \times \frac{1}{3} = 85.2 \times \frac{1}{3} = 29$$

$$Z_8 = C \times \frac{2}{3} = 85.2 \times \frac{2}{3} = 57$$

$$Z_9 = Z_{10} = 85.2 \times \frac{1}{2} = 43$$

$$\text{Checking: } Z_5 + Z_6 = Z_7 + Z_8 = Z_9 + Z_{10} = 86$$

Note: No. of teeth are modified according to centre distance between the shaft.

From shaft III to IV

		$\frac{F_i}{G_i}$	$F_i + G_i$
U_1	$\frac{1}{\varphi 4}$	$0.253 \approx \frac{1}{4}$	5
U_2	$\varphi 2$	$1.988 = 2/1$	3

LCM of 3 and 5 is 15,

U_1 is minimum

$$E \times LCM \times \frac{F_i}{F_i+G_i} \geq 17,$$

$$E \times 15 \times \frac{1}{5} \geq 17, E = 5.67$$

$$C = E \times LCM = 5.67 \times 15 = 85$$

$$Z_{11} = C \times \frac{1}{5} = 85 \times \frac{1}{5} = 17$$

$$Z_{12} = C \times \frac{4}{5} = 85 \times \frac{4}{5} = 68$$

$$Z_{13} = C \times \frac{2}{3} = 85 \times \frac{2}{3} = 57$$

$$Z_{14} = C \times \frac{1}{3} = 85 \times \frac{1}{3} = 28$$

Step 5: Gearing diagram

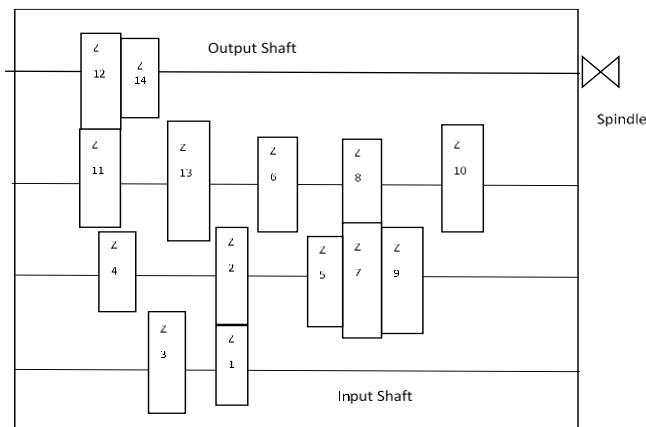


Fig. 6.12 Gearing diagram

Step 6: Deviation diagram

Finding the actual values of speeds from no. of teeth calculated. Let the available speed on shaft 1 is 1750 rpm.

Speeds on shaft 2 are

$$1750 \times (Z1/Z2) = 1750 \times (17/48) = 619.79 \text{ rpm}$$

$$1750 \times (Z3/Z4) = 1750 \times (23/43) = 936.04 \text{ rpm}$$

Speeds on shaft 3 are

$$619.79 \times (Z5/Z6) = 152.70 \text{ rpm}$$

$$619.79 \times (Z7/Z8) = 315.33 \text{ rpm}$$

$$619.79 \times (Z9/Z10) = 619.79 \text{ rpm}$$

$$936.04 \times (Z5/Z6) = 230.62 \text{ rpm}$$

$$936.04 \times (Z7/Z8) = 476.23 \text{ rpm}$$

$$936.04 \times (Z9/Z10) = 936.04 \text{ rpm}$$

Speeds on shaft 4 are

$$N1 = 152.70 \times (Z11/Z12) = 38.175 \text{ rpm}$$

$$N7 = 152.70 \times (Z13/Z14) = 310.85 \text{ rpm}$$

$$N3 = 315.33 \times (Z11/Z12) = 78.83 \text{ rpm}$$

$$N9 = 315.33 \times (Z13/Z14) = 641.92 \text{ rpm}$$

$$N5 = 619.79 \times (Z11/Z12) = 154.95 \text{ rpm}$$

$$N11 = 619.79 \times (Z13/Z14) = 1261.72 \text{ rpm}$$

$$N2 = 230.62 \times (Z11/Z12) = 57.655 \text{ rpm}$$

$$N8 = 230.62 \times (Z13/Z14) = 469.48 \text{ rpm}$$

$$N4 = 476.23 \times (Z11/Z12) = 119.06 \text{ rpm}$$

$$N10 = 476.23 \times (Z13/Z14) = 969.47 \text{ rpm}$$

$$N6 = 936.04 \times (Z11/Z12) = 234.01 \text{ rpm}$$

$$N12 = 936.04 \times (Z13/Z14) = 1905.51 \text{ rpm}$$

Calculating the percentage deviation value as, $d = \frac{N_a - N_t}{N_a} \times 100$,

where, N_a – Actual Speed, N_t – Theoretical speed

speed	Actual speed, Na	Theoretical speed, Nt	Deviation $d= 100 \times (Na - Nt)/Na$
N12	1905.21 rpm	1751.74	8.06
N11	1261.72	1242.37	1.53
N10	969.47	881.11	9.11
N9	641.92	624.9	2.65
N8	469.48	443.19	5.60
N7	310.85	314.32	-1.12
N6	234.01	222.92	4.74
N5	154.95	158.1	-2.03
N4	119.06	112.13	5.82
N3	78.83	79.524	-0.88
N2	57.655	56.4	2.18
N1	38.175	40	-4.78

Plotting the percentage deviation for all speed deviation chart is obtained as below.

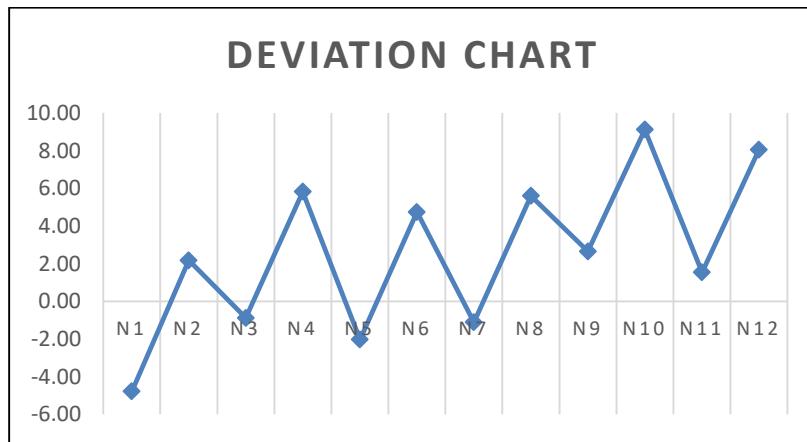


Fig 6.13 : Speed Deviation chart for 12 speed Gear Box

Numerical 6.3 Design a nine speed gear box having $N_{min} = 100$ rpm and $N_{max} = 630$ rpm. Assume motor speed 1400 rpm. The design should include structural diagram, ray diagram, speed chart, gearing diagram and number of teeth of the gear.

Solution:

Given Data:

No. of speeds, $Z = 9$

$$N_{min} = 100 \text{ rpm}$$

$$N_{max} = 630 \text{ rpm},$$

$$n_m = 1400 \text{ rpm}$$

Step1: Speed Calculation

The first step is to select the progression for the speed values. The geometric progression is selected. The specifications state that there are to be 9 speeds. Then, the progression ratio is:

$$\phi = \left(\frac{N_{max}}{N_{min}} \right)^{1/Z-1} = \left(\frac{630}{100} \right)^{1/9-1} = 1.2586 = 1.26$$

With the preferred numbers, the nearest value of ϕ is 1.26 and hence it is selected.

Now different speeds in RPM can be calculated as,

$$N_1 = 100 \text{ RPM}$$

$$N_2 = 100 \times 1.26 = 126 \text{ rpm}$$

$$N_3 = 126 \times 1.26 = 158 \text{ rpm}$$

$$N_4 = 158 \times 1.26 = 200 \text{ rpm}$$

$$N_5 = 200 \times 1.26 = 252 \text{ rpm}$$

$$N_6 = 252 \times 1.26 = 317 \text{ rpm}$$

$$N_7 = 317 \times 1.26 = 400 \text{ rpm}$$

$$N_8 = 400 \times 1.26 = 504 \text{ rpm}$$

$$N_9 = 504 \times 1.26 = 630 \text{ rpm}$$

Number of transmission groups Z is to be factorized with 2 and 3. Hence,

$$Z = 9 = 3 \times 3$$

Step 2: Structural formula & structural diagram

$Z = P_1(X_1) P_2(X_2)$, Here, $X_1 = 1$, $X_2 = P_1$

[A] $Z = P_1(X_1) P_2(X_2)$, $Z = 3(1) 3(3)$

[B] $Z = P_1(X_2) P_2(X_1)$, $Z = 3(3) 3(1)$

Structural Diagram

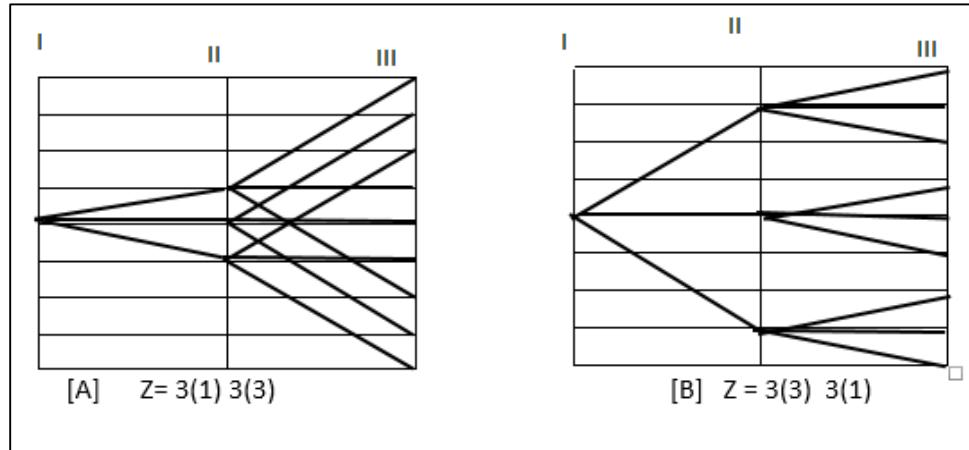


Fig. 6.14 Structural Diagram

Selecting best structural diagram,

Criteria 1: $I_g \leq 18 = \frac{I_{max}}{I_{min}} = \frac{3}{1/6}$, for $\phi = 1.26$, $i_{max} = 3$ and $i_{min} = 1/6$

For, [A] $z = 3(1) 3(3)$

Between Shaft (I) and (II),

$$I_g = \phi^{(P_1-1)*X_1} = \phi^{(3-1)*1} = \phi^2 = 1.5876 < 18$$

Between Shaft (II) and (III),

$$I_g = \phi^{(P_2-1)*X_2} = \phi^{(3-1)*3} = \phi^6 = 4 < 18$$

Criteria-1 is satisfied.

For, [B] $z = 3(3) 3(1)$

Between Shaft (I) and (II),

$$I_g = \phi^{(P_1-1)*X_1} = \phi^{(3-1)*3} = \phi^6 = 4 < 18$$

Between Shaft (II) and (III),

$$I_g = \phi^{(P_2-1)*X_2} = \phi^{(3-1)*1} = \phi^2 = 1.5876 < 18$$

Criteria-1 is satisfied.

Criteria 2: $X_1 < X_2$

After observing, [A] $z = 3(1) 3(3)$ and [B] $z = 3(3) 3(1)$

It is seen that, only [A] is satisfying Criteria-2

Therefore selecting [A] as the best combination.

Step 3: Ray Diagram & Speed Chart

For $\phi = 1.26$, $i_{\max} = 3$ and $i_{\min} = 1/6$ hence from one shaft to other shaft maximum increase in speed by steps and maximum decrease in speed is 6 step possible.

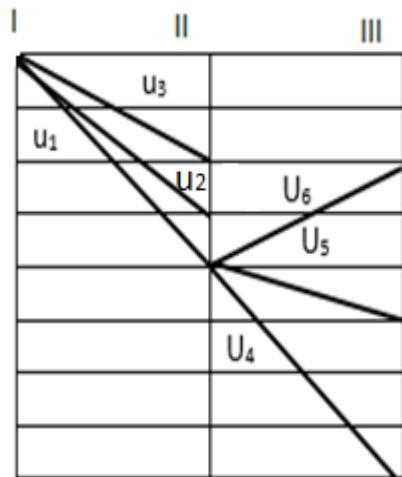


Fig. 6.15 Ray Diagram

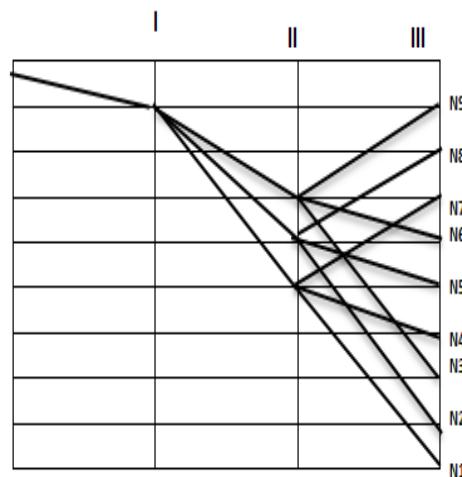


Fig. 6.16 Speed Chart

$$Z = 3 \times 3$$

Therefore, Number of gears required = $2 \times (3 + 3) = 12$

Step 4: Number of teeth on gears

For Stage 1, between shafts (I) and (II),

$$U_1 = \frac{1}{\phi^4} = \frac{1}{1.26^4} = 0.3967 = \frac{2}{5} = \frac{fi}{gi}$$

Therefore, $f_i + g_i = 2+5 = 7$

$$U_2 = \frac{1}{\phi^3} = \frac{1}{1.26^3} = 0.4999 = \frac{1}{2}$$

Therefore, $f_i + g_i = 1+2 = 3$

$$U_3 = \frac{1}{\phi^2} = \frac{1}{1.26^2} = 0.6298 = \frac{5}{8}$$

Therefore, $f_i + g_i = 5+8 = 13$

Therefore, LCM of 3, 7, 13 is 273 = K

Here, U_1 is minimum = $\frac{2}{5}$

$$E \times \text{LCM} \times \left(\frac{f_i}{f_i+g_i}\right) \geq 17$$

$$E \times 273 \times \frac{2}{7} \geq 17$$

$$E = 0.218$$

Therefore, $C = E \times \text{LCM} = 0.218 \times 273 = 59.5$

$$\text{Now, } Z_1 = C \times \frac{f_i}{f_i+g_i} = 59.5 \times \frac{2}{7},$$

$$Z_1 = 17, \quad Z_2 = C \times \frac{g_i}{f_i+g_i} = 59.5 \times \frac{5}{7}$$

$$Z_2 = 42.5 = 43$$

Here, $\frac{Z_1}{Z_2} = \frac{17}{43} = 0.39 \approx U_1$, Therefore Accepted.

$$Z_3 = 59.5 \times \frac{1}{3} = 19.83 = 20$$

$$Z_4 = 59.5 \times \frac{2}{3} = 39.66 = 40$$

$$\frac{Z_3}{Z_4} = \frac{20}{40} = 0.5 = U_2, \quad \text{Therefore Ok.}$$

$$Z_5 = 59.5 \times \frac{5}{13} = 22.88 = 23$$

$$Z_6 = 59.5 \times \frac{8}{13} = 36.6 = 37$$

$$\frac{Z_5}{Z_6} = \frac{23}{37} = 0.62 = U_3, \quad \text{Therefore Ok.}$$

For Stage-2, between shafts (II) and (III),

$$U_4 = \frac{1}{\phi^4} = \frac{1}{1.26^4} = 0.3967 = \frac{2}{5}$$

Therefore, $f_i + g_i = 2+5 = 7$

$$U_5 = \frac{1}{\phi} = \frac{1}{1.26} = 0.7936 = \frac{4}{5}$$

$$f_i + g_i = 4 + 5 = 9$$

$$U_6 = \phi^2 = 1.5876 = \frac{8}{5}$$

$$f_i + g_i = 5 + 8 = 13$$

Therefore, LCM of 7,9,13 is 819 = K

$$\text{Here, } U_4 \text{ is minimum} = \frac{2}{5}$$

$$E \times \text{LCM} \times \left(\frac{f_i}{f_i + g_i} \right) \geq 17$$

$$E \times 819 \times \frac{2}{7} \geq 1$$

$$E = 0.0726$$

Therefore, $C = E \times \text{LCM} = 0.0726 \times 819 = 59.5$

$$\text{Now, } Z_7 = C \times \frac{f_i}{f_i + g_i} = 59.5 \times \frac{2}{7}$$

$$Z_7 = 16.98 = 17$$

$$Z_8 = C \times \frac{g_i}{f_i + g_i} = 59.5 \times \frac{5}{7}, Z_8 = 42.5 = 43$$

$$\text{Here, } \frac{Z_7}{Z_8} = \frac{17}{43} = 0.4, \text{ Therefore Ok.}$$

$$Z_9 = 59.5 \times \frac{4}{9} = 26.4 = 27$$

$$Z_{10} = 59.5 \times \frac{5}{9} = 33$$

$$\frac{Z_9}{Z_{10}} = \frac{27}{33} = 0.8 \text{ Therefore Ok.}$$

$$Z_{11} = 59.5 \times \frac{8}{13} = 36.6 = 37$$

$$Z_{12} = 81.9 \times \frac{5}{13} = 22.88 = 23$$

$$\frac{Z_{11}}{Z_{12}} = \frac{37}{23} = 1.59 = U_6, \text{ Therefore Accepted.}$$

Step 5: Gearing Diagram

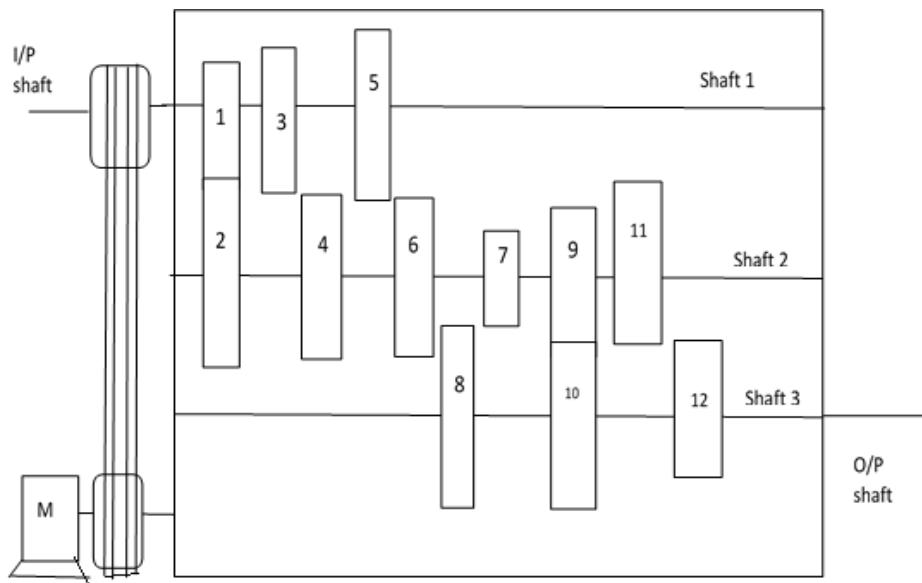


Figure. Layout of Gear Box

Fig 6.17: Gearing Diagram

Step 6: Deviation diagram

Finding the actual values of speeds from no. of teeth calculated. Let the motor speed 1400 rpm reduced to available speed on shaft 1 as 630 rpm.

Speeds on shaft 2 are

$$630 \times (Z1/Z2) = 630 \times (17/43) = 249.07 \text{ rpm}$$

$$630 \times (Z3/Z4) = 630 \times (20/40) = 315 \text{ rpm}$$

$$630 \times (Z5/Z6) = 630 \times (23/37) = 391.62 \text{ rpm}$$

Speeds on shaft 3 are

$$N1 = 249.07 \times (Z7/Z8) = 249.07 \times (17/43) = 98.47 \text{ rpm}$$

$$N4 = 249.07 \times (Z9/Z10) = 249.07 \times (27/33) = 203.78 \text{ rpm}$$

$$N7 = 249.07 \times (Z11/Z12) = 249.07 \times (37/23) = 400.68 \text{ rpm}$$

$$N2 = 315 \times (Z7/Z8) = 315 \times (17/43) = 124.54 \text{ rpm}$$

$$N5 = 315 \times (Z9/Z10) = 315 \times (27/33) = 257.73 \text{ rpm}$$

$$N8 = 315 \times (Z11/Z12) = 315 \times (37/23) = 506.74 \text{ rpm}$$

$$N3 = 391.62 \times (Z7/Z8) = 391.62 \times (17/43) = 154.83 \text{ rpm}$$

$$N6 = 391.62 \times (Z9/Z10) = 391.62 \times (27/33) = 320.42 \text{ rpm}$$

$$N_9 = 391.62 \times (Z_{11}/Z_{12}) = 391.62 \times (37/23) = 629.997 \text{ rpm}$$

Calculating the percentage deviation value as, $d = \frac{N_a - N_t}{N_a} \times 100$,

where, N_a – Actual Speed, N_t – Theoretical speed

speed	Actual speed, N_a	Theoretical speed, N_t	Deviation $d = 100 \times (N_a - N_t)/N_a$
N1	98.47	100	-1.55
N2	124.54	126	-1.17
N3	154.83	158	-2.05
N4	203.78	200	1.85
N5	257.73	252	2.22
N6	320.42	317	1.07
N7	400.68	400	0.17
N8	506.74	504	0.54
N9	629.997	630	0.00

Plotting the percentage deviation for all speed deviation chart is obtained as below.

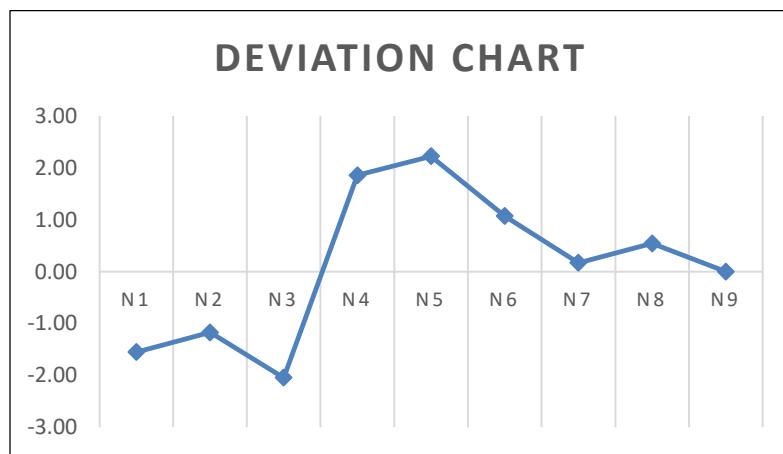


Fig 6.18 : Speed Deviation chart for 9 speed gear box

Summary:

Number of gears = 12

No. of teeth on each gears are

$Z_1 = 17, Z_2 = 43, Z_3 = 20, Z_4 = 40, Z_5 = 23, Z_6 = 37, Z_7 = 17,$

$Z_8 = 43, Z_9 = 27, Z_{10} = 33, Z_{11} = 37, Z_{12} = 23$

The different speed the combination of gear pairs can be obtained from speed chart as below.

Speed Number	Gear Pair of shaft I and II	Gear Pair of shaft II and III
N1	Gear 1 - Gear 2	Gear 7 – Gear 8
N2	Gear 3 - Gear 4	Gear 7 – Gear 8
N3	Gear 5 - Gear 6	Gear 7 – Gear 8
N4	Gear 1 - Gear 2	Gear 9 – Gear 10
N5	Gear 3 - Gear 4	Gear 9 – Gear 10
N6	Gear 5 - Gear 6	Gear 9 – Gear 10
N7	Gear 1 - Gear 2	Gear 11 – Gear 12
N8	Gear 3 - Gear 4	Gear 11 – Gear 12
N9	Gear 5 - Gear 6	Gear 11 – Gear 12

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