Combinatorial Game Theory:

#. Combi games are games without chance. with no hidden information,
Traditional recreational rulesets.

#. One of Earliest Combi Game in Theory:

Heap of beans: Start position is four heap of sizes. (for Example)
(2, 3, 4, 5)
Player can remove at least one bean from the heap. Last player to play a move wins.
(and at most the whole heap).

Mr. Bouton found the winning pattern

Nim addition: Write heap sizes in binary, and add without carry, that is Each column adds to zero, if and only if it contains even number of 1s.

Example Game:

1	0	1
1	0	0
0	1	1
0	1	0
0	0	0

The nim-sum is zero, => Player who does not start wins

Space for Explanation

Space for Explanation

So, Given any starting position, & given best play by both.

#. Exactly one of the players can play 0-position in Every move.

Proving it for case of two heaps :→

Take $n > m$, $\underset{\text{Pos } 0?}{(m,n)}$ winning first move : (m, m)

$\Rightarrow$ Pos 1 → (m, k) for some $0 \le k < m$.

{Heaps are of diff. sizes}

Every player can give the two heaps size. ⇒ nim-sum is zero.
{Special case of general idea}.

#. Notation : $n \in \mathbb{N} = \{1, 2, \ldots\}$ [$*n$ (Heap of size n).]

* Wythoff's variation of NIM (wythoff nim).
[Corner the lady / Corner the Queen].

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