GENERAL TOPOLOGY, SPRING 2024, HW2

You are highly encouraged to discuss and collaborate with your friends. However, you must write the solutions in your own words to get points.

Due date : Jan 31, 2024 in tutorial session.

1. List of problems

(1) The following problems are from the book "Topology" (Updated second edition) by James Munkres.
  (A) Page 111 : problems 3-a, b, $5, 6$.
  (B) Page 112 : problems $9, 13$
(2) For each of the following functions describe the topology on the domain generated by f, and determine if it is a Hausdorff topology.
  (A) $f : \mathbb{R} \to (\mathbb{R}, \text{standard topology})$ and $f(x) := 1$;
  (B) $f : \mathbb{R} \to (\mathbb{R}, \text{standard topology})$ and $f(x) := x$;
  (C) $f : \mathbb{R} \to (\mathbb{R}, \text{standard topology})$ and $f(x) := x^2$;
  (D) $f : \mathbb{R} \to (\mathbb{R}, \text{standard topology})$ and $f(x) :=$ greatest integer $\leq x$;
  (E) $f : \mathbb{R} \to (\mathbb{R}, \text{standard topology})$
$$f(x) := \begin{cases} 1/x, & x \neq 0 \\ 1, & x = 0; \end{cases}$$
  (F) $f : \mathbb{R} \to (\mathbb{R}, \text{standard topology})$
$$f(x) := \begin{cases} 1/x^2, & x \neq 0 \\ 1, & x = 0; \end{cases}$$
  (G) Suppose d is a metric on X, and fix a point $p_0 \in X$. Consider $f : X \to (\mathbb{R}, \text{standard topology})$ defined by
$$f(x) := d(x, p_0).$$
  (Look at, for example, $X = \mathbb{R}^2$ and $p_0 = (0, 0)$, and d is the Euclidean distance)
(3) Optional : Give an example of a topology τ on $[0, 1]$, if there is any, where $1/n \to 0$ is the "only" convergent sequence : More precisely, a sequence $\{x_n\}$ converges in τ if and only if one of the following holds.
  (i) It is eventually constant, $x_N = x_{N+1} = \ldots = x_0$ for some N;
  (ii) It is a eventually "almost a subsequence" of $\{1/n\}$ i.e. the set $\{x_N, x_{N+1}, \ldots\} \subset \{0, 1, 1/2, 1/3, \ldots\}$ for some N, and the set $\{n : x_n = 1/m\}$ is finite for each fixed m (Think of the sequence, $\{1, 0, 1/2, 0, 1/3, 0, \ldots\}$ or $\{1, 1/2, 1/2, 1/3, 1/3, 1/3, 1/4, \ldots\}$).
  How about two distinct topologies satisfying the above? How about infinitely many such topologies on $[0, 1]$?

Date: January 24, 2024.