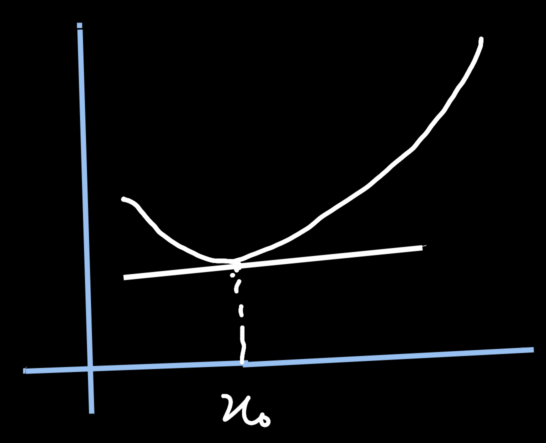
Multi-Variable Calculus

Lecture 1st.

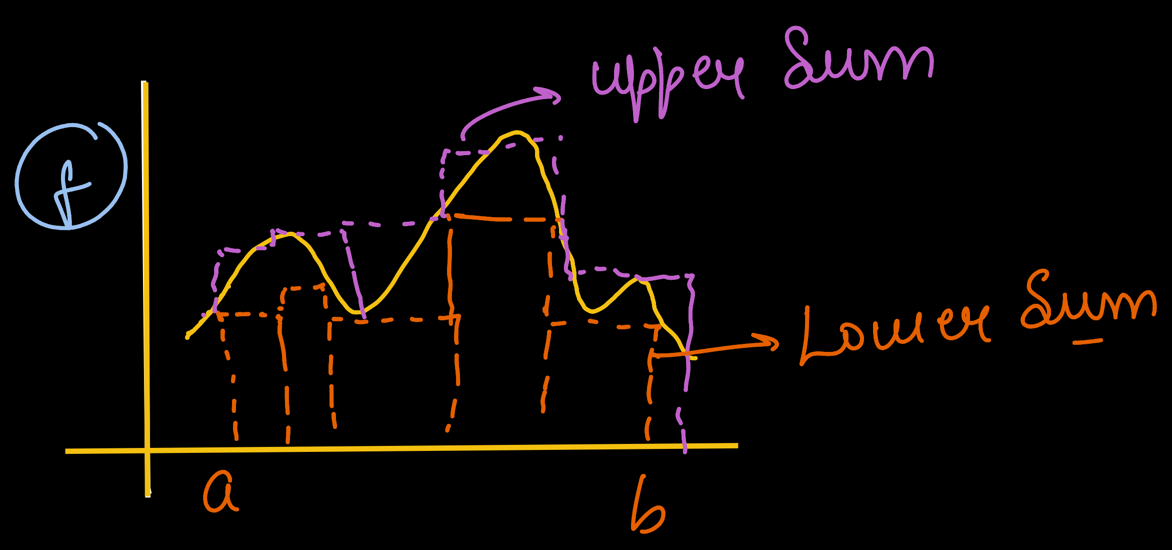
#.



Derivatives.

$$\left\{ f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h) - f(x)}{h} \right\}$$

Integration : Riemann Integral
↳ lower sum = upper sum.



Partition of $[(a, b)]$

$U(P, f)$, $L(P, f)$

#. Say f is Riemann Integral if lower & upper sum converges.

#. If $\forall\ \varepsilon > 0$, $\exists\ P$ such that,

$$U(P, f) - L(P, f) < \varepsilon.$$

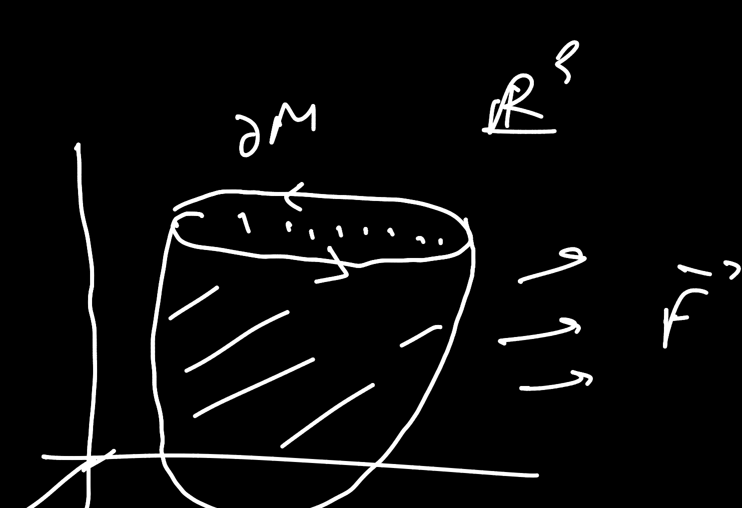
$$\{ \sup L(P, f) - \inf (P, f). \}$$

#> Green's theorem,

$$\iint_M \left(\frac{\partial \beta}{\partial x} - \frac{\partial \alpha}{\partial y} \right) dA = \int_{\partial M} \alpha\, dx + \beta\, dy$$

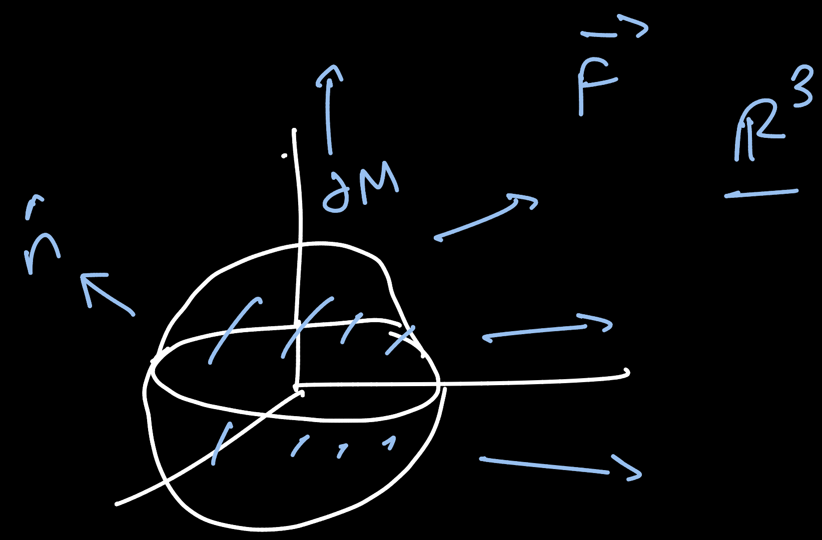
#. Stokes theorem:

$$\iint (\vec{\nabla} \times \vec{F})\, dA = \oint_{\partial M} \vec{F} \cdot d\vec{s}.$$



Divergence theorem

$$\iiint_M \operatorname{div} \vec{F}\, dV = \iint_{\partial M} \vec{F} \cdot \hat{n}\, dA$$



Stokes theorem:

{main Idea's skeleton}

$$\int_M d\omega = \int_{\partial M} \omega$$

M → n-dimensional manifold.

Derivatives again:

$$0 = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0) - f'(x_0) \cdot h}{h}$$

#. $g : (-\varepsilon, \varepsilon) \longrightarrow \mathbb{R}$

$h \longmapsto f(x_0+h) - f(x_0)$

#. $T : (-\varepsilon, \varepsilon) \longmapsto \mathbb{R}$.

T is a good approximation to g for small values of h.

T is a linear approximation to g.

for, $f : \mathbb{R}^n \to \mathbb{R}^n$

Say f is differentiable at x_0, if $\exists$ $T : \mathbb{R}^n \to \mathbb{R}^m$ (linear transformation), such that,

$$\lim_{h \to 0} \frac{\| f(x_0+h) - f(x_0) - T(h) \|}{\|h\|} = 0.$$

If this happens, Say $Df(x_0) = T$.