

# Introduction

Stack Shove is a combinatorial game played with stacks of counters, where players take turns making one of two possible moves:

- **Removal Move:** Remove any positive number of counters from a single stack.
- **Shoving Move:** Transfer some counters from one stack into an adjacent stack, provided the target stack has fewer counters.

In its original form, Stack Shove is a partisan game because Left and Right have different shoving directions—Left can only shove leftward, and Right can only shove rightward. However, I explored two variations of the game: one that remains partisan but is not all-small, and another that ensures the game is all-small by adjusting removal rules.

## Creating a Partizan Variation: Asymmetric Stack Shove

To introduce asymmetry while keeping the game partizan, I created "Asymmetric Stack Shove." This variation modifies removal rules to ensure that some positions favor one player over the other, preventing the game from being all-small.

### Rules:

- Players take turns making one of two possible moves:
  - **Removal Move:** Remove counters from a single stack.
  - **Shoving Move:** Transfer counters from one stack into an adjacent stack, provided the target stack has fewer counters.
- **Left can only shove leftward, and Right can only shove rightward.**
- **Left can remove any number of counters, while Right can only remove an even number of counters.**
- The player who makes the last move wins.

This variation is not all-small because there exist positions where only one player has a move. For example, a single counter is a winning move for Left but not for Right.

### Analysis of Asymmetric Stack Shove

1. **Single Counter:** Left wins immediately by removing it, making it a first-player win position.
2. **Stacks with multiple counters:** Left has more flexibility in removals, while Right's removal options are limited to even numbers, creating an inherent asymmetry.

3. **Shoving as a strategic tool:** Since Right can only remove even numbers, shoving may force unfavorable moves for Right, giving Left a significant advantage.

## Creating an All-Small Variation: Balanced Stack Shove

To transform Stack Shove into an all-small game, I introduced a balanced removal rule while keeping the shoving mechanics unchanged. This ensures that every position always allows both players to make a move.

### Rules:

- Players take turns making one of two possible moves:
  - **Removal Move:** Remove counters from a single stack.
  - **Shoving Move:** Transfer counters from one stack into an adjacent stack, provided the target stack has fewer counters.
- **Left can only shove leftward, and Right can only shove rightward.**
- **Left can only remove an odd number of counters, while Right can only remove an even number of counters.**
- The player who makes the last move wins.

### Analysis of Balanced Stack Shove

1. **Single Counter:** Left removes it, ensuring at least one player always has a move.
2. **Stacks of different sizes:** The game remains balanced because Left and Right have specific removal constraints that complement each other.
3. **Strategic Positioning:** Since removals follow strict parity rules, shoving plays a more critical role in creating favorable positions for either player.

## Conclusion

Through these modifications, I successfully created a partizan variation (Asymmetric Stack Shove) that is not all-small and an all-small variation (Balanced Stack Shove). These adjustments illustrate how minor rule changes can significantly impact the nature of a combinatorial game, influencing both fairness and strategy.