# Lemma 41

$$G+x = \{G^L + x,\ G + x^L \mid G^R + x,\ G + x^R\}$$

For showing left has a winning strategy, $\longrightarrow$ $G^L + x \geq G + x^L$ for any $x^L$

Lemma 39 says $G^L - G + x - x^L \geq 0$

Explicit Explanation: $Rs(A+B) \geq Rs(A) + Rs(B)$ {We know it}.

applying Same Idea to $G^L - G$:

$Rs(G^L - G) \geq Rs(G^L) + Rs(-G)$. [and $Rs(-G) = -Ls(G)$]

$\Rightarrow Rs(G^L - G) \geq Rs(G^L) - Ls(G)$

#. $G$ is not a number, It has at least one left option $G^L$. So we choose $G^L$ s.t. $LS(G) = Rs(G^L)$

$\downarrow$

strategic choice on the structure of $G$.

$\odot$ Now substituted $Rs(G^L) = Ls(G)$ into Inequality

$Rs(G^L - G) \geq LS(G) - LS(G) = 0$ —— $\circledast$

{$G + x^L$ dominated by $G^L + x$} By using $\circledast$ $\exists$ $G^L$ s.t. $Rs(G^L - G) \geq 0$.

$\Rightarrow G^L - G + x - x^L > 0$

Exactly where argument needs to be strong.