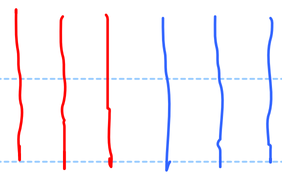
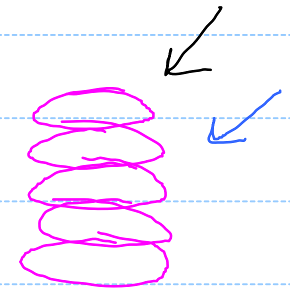
Left

$\{2 \mid -2\}$  Right $+$ 

Toppling Domino — Nim

2 moves for Left
if Left starts.

Right is earning 2 moves if Right starts.

Normal Play : Last move wins.

Misére play : Last move loses.

Urgency is estimated by 'something' called temperature.

$\{2 \mid -2\}$

$\{2-p \mid -2+p\}$

$p=1$ $\{1 \mid -1\}$

$\rightarrow$

$\boxed{p=2}$ $\{2-p \mid -2+p\} = \{0 \mid 0\} = *$

$+$

Temperature: estimating urgency

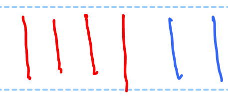
$\downarrow$

penalizing players to make the game no more urgent

$\downarrow$

the minimum penalty which makes the game no more urgent

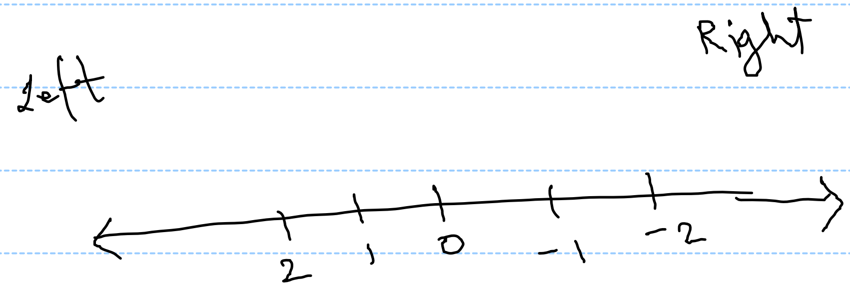
urgent $\{3 \mid -1\}$



urgent $\{3 \mid 1\}$

not urgent $\{3 \mid 4\} = 0$

$$G = \{ 3 \mid -1 \}$$



$$G_p = \{ 3 - p \mid -1 + p \}$$

$p = 1 \qquad G_p = \{ 2 \mid 0 \} \qquad \frac{3}{2} > \frac{1}{2}$

$p = 1\frac{1}{2} \qquad G_p = \{ \frac{3}{2} \mid \frac{1}{2} \}$

$p = \boxed{\frac{5}{3}} \qquad G_p = \{ \frac{4}{3} \mid +\frac{2}{3} \} \qquad \times$

p can only take values from $\mathbb{D}$.

$$\boxed{\mathbb{D} = \left\{ \frac{n}{2^k} \;\middle|\; n \in \mathbb{Z},\ k \geq 0 \right\}}$$

$p = 2$ $\quad \{3-2 \mid -1+2\} = \{1 \mid 1\}$

temperature of $G = t(G) = 2$

$G = \{3 \mid 1\}$

$G_p = \{3-p \mid 1+p\}$

$G_1 = \{2 \mid 2\}$ $\quad t(G) = 1$

$G_2 = \{1 \mid 3\} \overset{?}{=} 2$

=

$$G = \{ x \mid y \} \qquad x > y$$

$$t(G) = \frac{x-y}{2} \qquad G_p = \{ \underbrace{x-p}_{} \mid \underbrace{y+p}_{} \}$$

$$x - p = y + p$$

$$p = \frac{x-y}{2}$$

$$G = \{ 12 \mid \{ 11 \mid -11 \} \}$$

$$G = \{ 12 \mid \{ 11 \mid -11 \} \}$$

$$\begin{tikzcd}[column sep=small]
 & \{ 12 \mid \{ 11 \mid -11 \} \} \arrow[dl] \arrow[dr] & \\
12 & & \{ 11 \mid -11 \} \arrow[d] \\
 & & 11
\end{tikzcd}$$

$$\rightarrow \quad G_p = \{ 12 - p \mid \{ 11 - p \mid -11 + p \} + p \}$$

$$\boxed{G = \{ \{ \underset{-p}{12} \mid \underset{+p}{11} \} - p \mid \{ 11 - p \mid \{ \underset{-p}{7} \mid \underset{+p}{-7} \} + p \} + p \}}$$

$p = \frac{1}{4}$

$p = \frac{1}{8}$ $\qquad$ $p = \frac{1}{2}$ $\qquad$ $G_p = \left\{ 11 + \frac{1}{2} \;\middle|\; \underbrace{\left\{ 10 + \frac{1}{2} \;\middle|\; -11 + \frac{1}{2} \right\}}_{H} + \frac{1}{2} \right\}$

$p = \frac{3}{4}$ $\qquad\qquad\qquad\qquad$ $H + \frac{1}{2}$

$p = \frac{3}{8}$ $\qquad\qquad\qquad\qquad$ $\left\{ H^L + \frac{1}{2} \;\middle|\; \underset{-11+\frac{1}{2}+\frac{1}{2}}{H^R + \frac{1}{2}} \right\}$

$$\boxed{\underset{\substack{\uparrow \\ \text{Number}}}{G + x} = \left\{ \underset{\cancel{G + x^L}}{G^L + x} \;\middle|\; \underset{\cancel{G + x^R}}{G^R + x} \right\}}$$

Number Avoidance Theorem

$$G_p = \left\{ \frac{23}{2} \;\middle|\; \left\{ \underline{11} \;\middle|\; \underline{-10} \right\} \right\}$$

$\boxed{\frac{23}{2}}$ $\qquad\qquad$ $\{11 \mid -10\}$

$\boxed{11}$

$p = 1$

$$G_{1p} = \{\, 11 \mid \{\, 11 - 1 \mid -11 + 1 \,\} + 1 \,\}$$

$$= \{\, 11 \mid \{\, 11 \mid -9 \,\} \,\}$$

$$\boxed{11} \qquad \{\, 11 \mid -9 \,\}$$

$$\boxed{11}$$

$t(G) = 1$

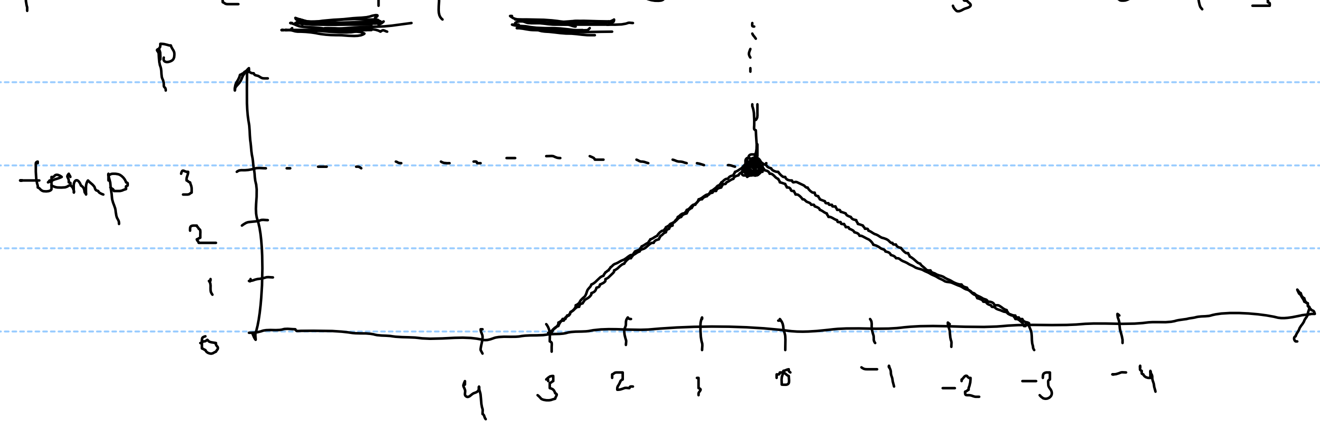
$$G = \{ 6, \{10 \mid \{5 \mid 3\}\} \mid -5 \}$$

$$G_p = \{ 6-p, \{10-p \mid \{5-p \mid 3+p\} + p\} - p \mid -5+p \}$$

Thermograph:

$$G = \{ 3 \mid -3 \}$$

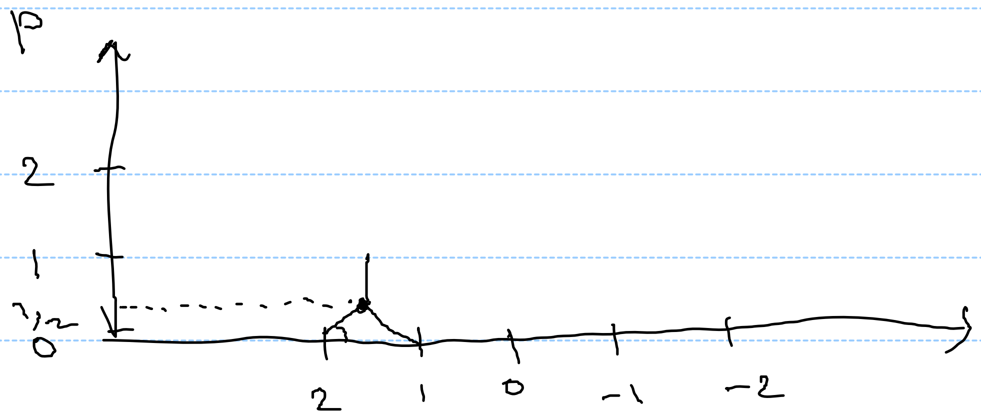
$$G_p = \{ \underline{3-p} \mid \underline{-3+p} \} \qquad G_3 = \{0 \mid 0\}$$



Thermograph: Graph of Left & Right stops of G_p v/s penalty p.

$G = \{2 \mid 1\}$ $\qquad$ $G_p = \{\underline{2 - p} \mid \underline{1 + p}\}$

Temp $= \frac{1}{2}$



Definition 3.4 (Stops)**.** For a game G, the Left stop $\ell(G)$ and the Right stop $r(G)$ are defined as:

$$\ell(G) = \begin{cases} x & \text{if } G \text{ equals a dyadic } x; \\ \max_{G^L} \left(r(G^L)\right) & \text{otherwise;} \end{cases}$$

$$r(G) = \begin{cases} x & \text{if } G \text{ equals a dyadic } x; \\ \min_{G^R} \left(\ell(G^R)\right) & \text{otherwise.} \end{cases}$$

The stops of a game G is the ordered pair $s(G) = (\ell(G), r(G))$.

Defini[illegible]**enalized Position**[illegible] Let G be a short game in canonical form and let [illegible] Then, G *penali*[illegible]*by* p, de[illegible]ed by G_p, is [illegible] defined as

- $G_p =$ [illegible] $p \mid$ [illegible] $p + p\}$ for all $0 \le p \le t$ where t is the minimum p for which the Left and [illegible]ht stops of G_p are equal to a dyadic, say x,
- $G_p = x$ for all $p > t$.

Here $G^{\mathcal{L}}{}_p$ denotes the set of games of the form $G^L{}_p$, and similarly for Right.

Let $\mathbb{D}^+$ denote the set of non-negative dyadics. [illegible]

Definition 3.11 (Temperature)**.** The temperature of a dyadic $G = k/2^n$ is $t(G) = -1/2^n$, where $k \in \mathbb{Z}$ and $n \in \mathbb{N} \cup \{0\}$ and if $n > 0$, k is an odd integer. The temperature $t(G)$ of a non-dyadic G is the smallest $p \in \mathbb{D}^+$ such that $\ell(G_p) = r(G_p)$.

L(G) , R(G)

$$G = \{\, \underline{6} \mid \boxed{\underline{\{3 \mid -3\}}} \,\}$$

$$G_p = \{\, \underline{6 - p} \mid \underline{\{3 - p \mid -3 + p\} + p} \,\}$$

$$G_p = \{\, 6 - p \mid \{3 - p \mid -3 + p\} + p \,\} \qquad \{3 \mid -3 + 2p\}$$

$$p = 1 \qquad G_1 = \{\, 5 \mid \{2 \mid -2\} + 1 \,\} \qquad \{3 - p \mid -3 + p\}$$

$p = 1\frac{1}{2}$ $\quad G_p = \left\{ 4 + \frac{1}{2} \;\middle|\; \left\{ 1\frac{1}{2} \middle| -1\frac{1}{2} \right\} + 1\frac{1}{2} \right\}$

$p = 2$ $\quad G_p = \left\{ 4 \;\middle|\; \{ 1 \mid -1 \} + 2 \right\}$

$\underline{p = 3}$ $\quad G_p = \left\{ 3 \;\middle|\; \boxed{\{ 0 \mid 0 \}} + 3 \right\}$ $\quad \to 0$

$\quad = \left\{ 3 \;\middle|\; \{ 3 \mid 3 \} \right\}$

$p = 4$ $\quad G_p = \left\{ 7 - p \mid 0 + p \right\}$

$\{ 0 \mid 0 \} = * \quad = * + 0$

$3 = \underline{\{ 3 \mid 3 \}} = \underline{3 + *} = 3 + \{ 0 \mid 0 \}$

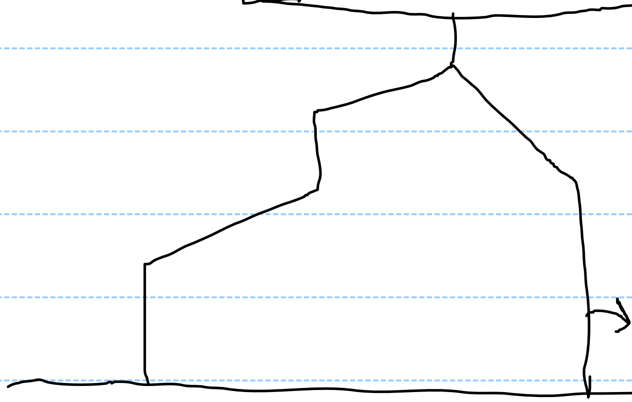
$= \{ 3 + 0 \mid 3 + 0 \}$

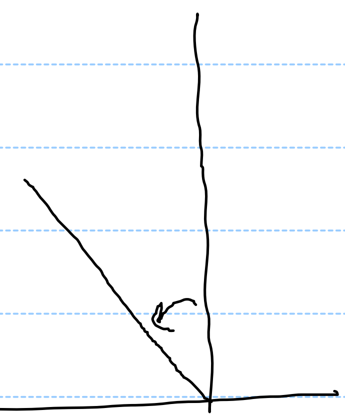
Thermograph (G) Temperature (G)

Quiz: Draw the thermographs.

$G = \{ 10 \mid \{5 \mid 3\} \}$

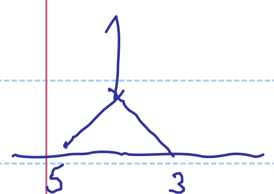
$H = \left\{ \boxed{\{10 \mid \{5 \mid 3\}\}} \,\middle\|\, \underline{\underline{-5}} \right\}$

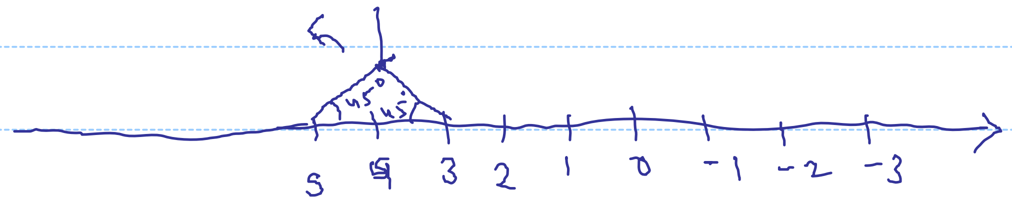




$g \quad \{10 \mid \{5 \mid 3\}\}$

$G^R = \{5 \mid 3\}$, $G^R_p = \{5 - p \mid 3 + p\}$ $\quad p \geq 0$





$G = \{10 \mid \boxed{\{5 \mid 3\}}\}$

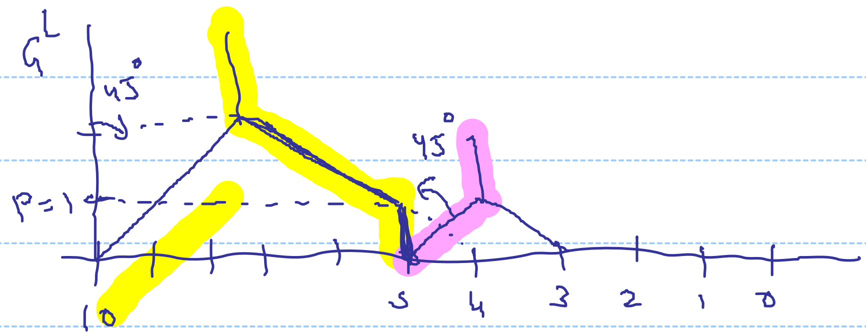
$G_p = \{\underline{10 - p} \mid \{5 - p \mid 3 + p\} + p\}$

$\underline{L(G_p) = 10 - p}$

$\boxed{R(G_p)} = L(\{5 - p \mid 3 + p\} + p)$

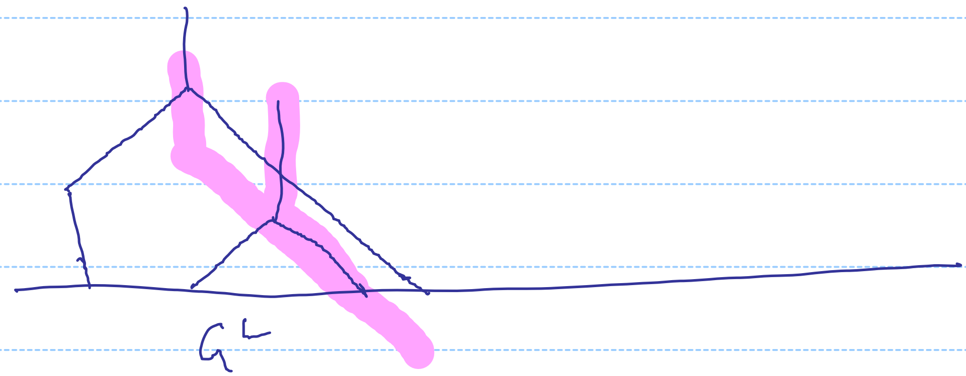
$R(G_p) = \underline{5 - p + p}$ till

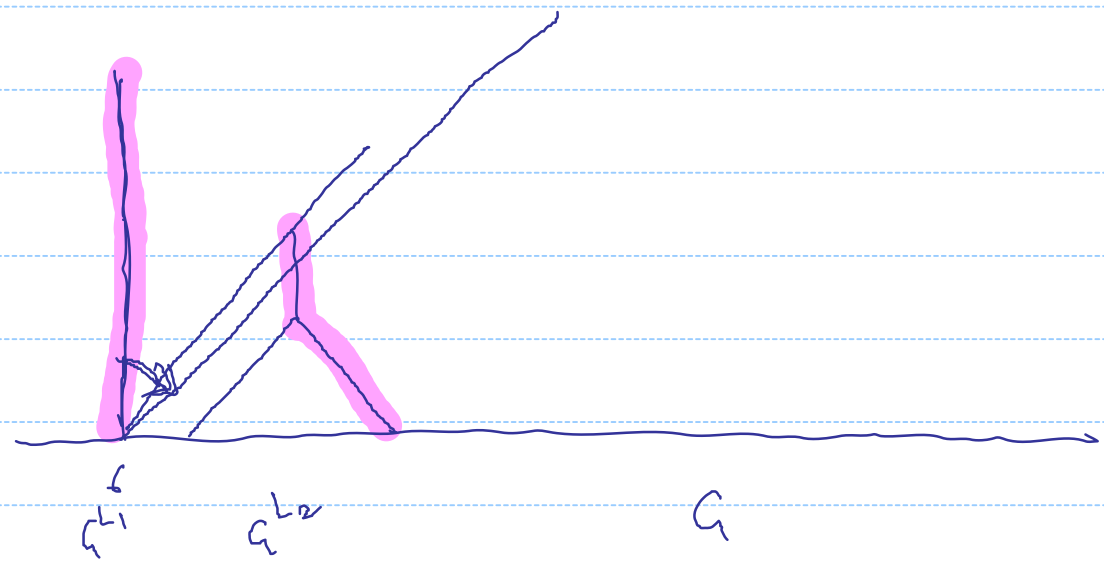
$4+p$



$$G_p = \{10 - p \mid \{5-p \mid 3+p\} + p\} \qquad p<1$$

$$\{10-p \mid 4+p \qquad p=1$$


G^L


ℓ
G^{L_1}
G^{L_2}
G