#. $F(x,y,z) = 2x^2y^1z^1 + x^1y^5z^4$ $\qquad F \in T^3(R^5)$ $\qquad G(x,y) = x^1y^3 + x^3y^1$

B_1: $T^3(R^5)$ consisting of tensor products of basis covectors.

B_2: $T^2(R^5)$ consisting of tensor products of two basis covectors.

B_3: $\Lambda^3(R^5)$ consisting of wedge products of three basis covectors

B_4: $\Lambda^2(R^5)$ consisting of wedge products of two basis covectors.

(a) $\left[F = 2\varphi^2 \times \varphi^1 \times \varphi^1 + \varphi^1 \times \varphi^5 \times \varphi^4\right]$

(b) $\left[G = \varphi^1 \times \varphi^3 + \varphi^3 \times \varphi^1\right]$

(c) $\mathrm{Alt}(f) = \frac{1}{3!} \sum_{\sigma \in S_3} \mathrm{sgn}(\sigma) F_\sigma(i_1, i_2, i_3)$

The terms $\varphi_2 \times \varphi_2 \times \varphi_1$ are not already antisymmetric, thier altr. gives zero.

only relevant term,

$$\left[\mathrm{alt}(f) = \frac{1}{6} \sum_{\sigma \in S_3} \mathrm{sgn}(\sigma)\left(\varphi_{\sigma(1)} \wedge \varphi_{\sigma(5)} \wedge \varphi_{\sigma(4)}\right)\right]$$

$$\mathrm{Alt}(f) = \varphi_1 \wedge \varphi_5 \wedge \varphi_4.$$

#. $\omega \wedge \xi \wedge \eta = -\eta \wedge \xi \wedge \omega$

r $\qquad t$

Both being r, t odd.

$\alpha \wedge \beta = (-1)^{pq} \beta \wedge \alpha$

$p \qquad q$

$(-1)^{r} \xi \wedge \omega \wedge \eta \rightarrow \xi \wedge \eta \wedge \omega$

$(-1)^{r} (-1)^{pq} \quad \eta \times \xi \times \omega$

$(-1)^{r} (-1)^{rt} (-1)^{t}$

$\cancel{(-1)^{2(rt)}} \cdot \cancel{(-1)^{2t}} \quad (-1)^{rt + [2(r+t)]}$ ↳ Even

$(-1) \leftarrow$ odd $\leftarrow (-1)^{rt}$

(d) Express $\mathrm{Alt}(G) \rightarrow B_4$.

$\left\{\mathrm{Alt}.(G) = \frac{1}{2}(G - G^T)\right\}$

Which means,

$$\mathrm{Alt}(G) = \frac{1}{2}\left(\varphi_1 \times \varphi_3 + \varphi_3 \times \varphi_1 - \varphi_3 \times \varphi_1 - \varphi_1 \times \varphi_3\right)$$

$[\mathrm{Alt}(G) = 0]$

We need to show that

#. $\varphi_{i_1} \wedge \dots \wedge \varphi_{i_k}(e_{i_1}, \dots, e_{i_k}) = 1$

→ wedge product $\varphi_{i_1} \wedge \dots \wedge \varphi_{i_k}$ → alternating K-form

$$\det \begin{bmatrix} \varphi_{i_1}(e_1) & \cdots & \varphi_{i_1}(e_{i_k}) \\ \vdots & \ddots & \vdots \end{bmatrix}$$

$$\det \begin{bmatrix} \vdots & & \vdots \\ \varphi_{i_k}(e_i) & \cdots & \varphi_{i_k}(e_{i_k}) \end{bmatrix}$$

since φ_i, dual basis vector satisfying

$$\varphi_{i_j}(e_{i_m}) = \delta_{i_j i_m}.$$

$\rightarrow$ Kronecker ~~data~~ delta.

Simply $k \times k$ Identity matrix.

3) b.) $v_1, \dots, v_k \in \mathbb{R}^n$.

$$\varphi_{i_1} \wedge \dots \wedge \varphi_{i_k}(v_1, \dots, v_k)$$

$\rightarrow$ $v_j = (v_j^1, \dots, v_j^n)$. wedge product evaluates

as, $\varphi_{i_1} \wedge \dots \wedge \varphi_{i_k}(v_1, \dots, v_k)$

$$= \det \begin{bmatrix} \varphi_{i_1}(v_1) & \cdots & \varphi_{i_1}(v_k) \\ \vdots & \ddots & \vdots \\ \varphi_{i_k}(v_1) & \cdots & \varphi_{i_k}(v_k) \end{bmatrix}$$

$\rightarrow$ $\varphi_{i_j}(v_l) = v_l^{i_j}$, this is the determinant of the matrix.

$$\det \begin{bmatrix} v_1^{i_1} & & v_k^{i_1} \\ \vdots & & \vdots \\ v_1^{i_k} & & v_k^{i_k} \end{bmatrix}$$

~~det~~.

$$\begin{bmatrix} v_1^{i_k} & \cdots & v_k^{i_k} \end{bmatrix}$$

3. This is precisely the determinant of the $k \times k$ minor formed by selecting rows $i_1, \dots, i_k$ from the matrix $[v_1 \cdots v_k]$.

Answer for (b):

$$\boxed{\varphi_{i_1} \wedge \cdots \wedge \varphi_{i_k}(v_1, \dots, v_k) \text{ is the determinant of the } k \times k \text{ minor with rows } i_1, \dots, i_k.}$$