Q.1) $f: R^2 \to R$ $D_2 f = 0$ $\forall x \in R$ $f(x, y_1) = f(x, y_2) \; \forall \; y_1, y_2 \in R$

we effectively need to prove $f(x,y) = g(x)$. $f(x,y) = g(x)$

By definition $D_2 f = \lim_{K \to 0} \dfrac{f(x, y+K) - f(x,y)}{|K|} = 0$

we get: $f(x, y+K) = f(x,y)$

#. Similary f does not change at all if $D_1 f = 0$ and $D_2 f = 0$ (x & y both Constant]
[Both Classes going zero].

Q.2) $f: R \to R$ is defined by $f(x) = e^{\frac{-1}{(x-1)^2}} \cdot e^{\frac{-1}{(x+1)^2}}$ $x \in (-1, 1)$

So, $C^\infty \to$ Infinite derivatives.

Sol) We already know that e^{-1/x^2} is C^∞ and $f^{(i)}(0) = 0$. $\forall i$

Now, $e^{\frac{-1}{(x-1)^2} - \frac{1}{(x+1)^2}} = \dfrac{-(x+1)^2 - (x-1)^2}{(x-1)^2 (x+1)^2} = - \dfrac{\cancel{x^2} + 2x + \cancel{1} - \cancel{x^2} + 2x - \cancel{1}}{(x-1)^2 (x+1)^2} =$

$= \dfrac{-4x}{(x-1)^2 (x+1)^2}$

Here, we can say and reduce the function as,

$e^{\theta(x)} \Rightarrow \theta(x)$ is polynomial of type $\to \left(\frac{-1}{x^3}\right)$

thus, $\left\{ e^{\theta(x)} = 1 + \dfrac{\theta(x)}{1} + \dfrac{\theta(x)}{2!} + \text{-----} \right\}$ Thus By def^n of derivative of any function at x.

#. $\lim_{h \to 0} \dfrac{e^{(\theta(x+h))} - e^{\theta(x)}}{h} = \dfrac{e^{\theta(h)} - \cancel{e^{\theta(0)}}}{h} = \lim_{h \to 0} \dfrac{e^{\theta(h)}}{h}$

putting $h \to \left(\frac{1}{t}\right)$ $\lim t e^{\theta\left(\frac{1}{t}\right)}$ $\left\{ \lim_{t \to \infty} \dfrac{4(1/t)}{\left(\frac{1}{t} - 1\right)^2 \left(\frac{1}{t} + 1\right)^2} = \dfrac{\left(\frac{4}{t}\right)}{(1-t)^2 (1+t)^2} \right.$

$t \to \infty$ $\qquad t^4 3$

$= \infty(1) = \infty$

$$\lim_{t\to\infty} \frac{4t^3}{(1-t)^2(1+t)^2} = 0$$

$$\frac{-4x}{(x-1)^2(x+1)^2} = \frac{-4(x+h)}{(x-1+h)^2(x+1+h)^2} = \frac{-4h}{(h-1)^2(h+1)^2} = \frac{-\frac{4}{t}}{\frac{(1-t)^2(1+t)^2}{t^4}}$$

$$= \frac{-4t^3}{(1-t)^2(1+t)^2}$$

$(1+t^2-2t)(1+t^2+2t)$

$= 1 + t^2 + \cancel{2t} + t^2 - \cancel{2t} + t^4 + \cancel{2t^3} - \cancel{2t^3}$

$-4t^2$

$1 - 2t^2 + t^4$

$$= \frac{-4t^3}{t^4-2t^2+1} = \frac{-4t^3}{(t^2-1)^2}$$

$$= \frac{-4}{t - \frac{2}{t} + \frac{1}{t^3}} = \frac{-4}{\infty} = 0$$

$$e^{\frac{-1}{(x+1)^2}} \cdot e^{\frac{-1}{(x+1)^2}} = e^{-1} \cdot e^{-1} = \frac{1}{e^2}$$

Q) Let f : R2 → R2 be given by f (x, y) = (ex cos y, ex sin y).
(a) Show that f is one-to-one on the set A = {(x, y) : 0 < y < 2π}.
(b) What is the set B = f (A)?
(c) If g is the inverse function, g = f −1, find g′((0, 1)).

$f: R^2 \to R^2$

$f(x,y) = (e^x \cos y, e^x \sin y)$

$A = \{(x,y) : 0 < y < 2\pi\}$

a) assume $f(x,y)$ is not one-one on set A. Then $f(x_1,y_1) = f(x_2,y_2)$

$(e^{x_1}\cos x_1, e^{x_1}\sin y_1) = (e^{x_2}\cos x_2, e^{x_2}\sin y_1)$

checking $e^{x_1}\cos x_1 = e^{x_2}\cos x_2$

Maybe It is the wrong way, let's do the mono tonicity stuff.

#. $f(x,y) = (e^x \cos y, e^x \sin y)$

$\frac{\partial f}{\partial x} = (e^x \cos y, e^x \sin y)$ $\qquad \frac{\partial f}{\partial y} = (-e^x \sin y, e^x \cos y)$

Both Exists and thus proves the function is continous.

#. $\frac{\partial^2 f}{\partial x^2} = (-e^x \sin y, e^x \cos y)$, $\frac{\partial^2 f}{\partial y^2} = (-e^x \cos y, -e^x \sin y)$

always -ve in $[0, 2\pi]$ always +ve in $[0, 2\pi]$

Thus double derivative, says derivative → monotonous ✓

#. {set (B) = f(A)}

Thus, as It is one-one, $0 \le x \le 2\pi$ $(e^x \cos y,\ e^x \sin y)$

$e^0 \cos(0) = 1$

$e^0 \sin(0) = 0$

$e^{2\pi} \cos 2\pi = 2\pi$

$e^{2\pi} \sin 2\pi = 0$

#. $f(x,y) = (xy,\ x^2+y^2+e^{(x-2)(y-1)})$, $\exists\ r>0$ such that for every $(a,b) \in B_r((2,6))$,

$\exists\ (x,y)$ such that $f(x,y) = (a,b)$

Question says there is this some function, for every point around the ball, output set contains $B_r((2,6))$.

Sol^n $f(2,6) = (12,\ 40+e^5)$

#. $f(x,y) = (xy,\ x^2+y^2+e^{(x-2)(y-1)})$.

$(2,6)$

$3xy = x^2+y^2+e^{(x-2)(y-1)}$