PUBV(S-I) — BCA
(GE – 1) Math

2023

Time : 3 hours

Full Marks : 70

Pass Marks : 32

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from all the Groups as directed.

Group – A

(Objective Type Questions)

1. Choose the correct alternative in each of the following :   1×5 = 5

   (a) The number of distinct equivalence classes coresponding to the relation

XE – 12/3   (Turn over)

'congruence modulo 3' on the set of integers are :

(i) 0 $\equiv (mod\ 3)$

(ii) 1

(iii) 2

(iv) 3

(b) If $f : R \to R$ and $g : R \to R$ are defined as $f(x) = x + 1$ and $g(x) = x^2 - 1$ then $gof(x)$ equals :

$x^2 + 1 + 2x$

(i) $2x^2 + x$

$x^2 + 2x$

(ii) $x^2 + 2x$

(iii) $x^2 + x$

(iv) None of these

(c) The symmetric difference of $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5, 7\}$ is :

(i) $\{1, 3, 5\}$

(ii) $\{7\}$

(d)

(iii) {2, 4}

(iv) None of these

(d) The remainder when 3^{201} is divided by 9 is :

(i) 0

(ii) 1

(iii) 2

(iv) None of these

(e) If $y = \log(\cos e^x)$ then $e^{-x} \dfrac{dy}{dx}$ is :

(i) $-\cos e^x$

(ii) $-\sin e^x$

(iii) $-\tan e^x$

(iv) None of these

$$e^x \cdot \frac{-\sin e^x}{\cos e^x}$$

$$= -\tan e^x$$

2. Fill in the blanks in each of the following :

$1 \times 5 = 5$

(a) A singular square matrix must have a determinant equal to _____.

[5]

(b) The order of the element '2' in z_5 is

_____.

(c) If A is a finite set having 11 elements, then the number of elements in the power set of A×A is _____.

(d) If $y = x^n$, then y_n is equal to _____.

(e) The gcd (12378, 3054) is _____.

Group – B

(Short-answer Type Questions)

Answer any **four** questions of the following :

$$5×4 = 20$$

3. Show that $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$

4. Define an equivalence relation on a non-empty set A. If R is a relation on N×N defined by (a, b) R (c, d) iff $a + d = b + c$, then prove that R is an equivalence relation.

5. In a group G prove that the identity element is unique.

6. Find a solution to the Diophantine equation $172x + 20y = 1000$.

7. If A, B, and C are three sets then prove that :

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

8. Prove that every square matrix can be espressed as the sum of a symmetric and a skew-symmetric matrix.

Group – C

(Long-answer Type Questions)

Answer any **four** questions of the following :

$$10 \times 4 = 40$$

9. (a) If $u = e^{xyz}$, show that $\dfrac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$.

(b) If $u = f(y/x)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial x} = 0$.

10. (a) State and prove Mclaurin's theorem.

(b) Expand the function $\cos(x)$ in the powers of x with the help of Mclaurin's series.

11. If $y = e^{a\sin^{-1}x}$, then prove that :

(a) $(1 - x^2)\, y_2 - xy_1 - a^2 y = 0$

(b) $(1 - x^2)\, y_{n+2} - (2x + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$

12. Prove that $\left\{ \displaystyle\bigcup_{i=1}^{n} A_i \right\}' = \displaystyle\bigcap_{i=1}^{n} A_i'$

13. Test the consistency and solve the following system of simultaneous linear equations by matrix method :

$$x - 3y - 2z = 6,$$

$$2x - 4y - 3z = 8,$$

$$-3x + 6y + 8z = -5.$$

14. Let $f : x \rightarrow y$ be a mapping and let $A \subseteq X$, $B \subseteq X$, then show that :

(a) $f(A \cap B) = f(A) \cup f(B)$

(b) $f(A \cap B) \subseteq f(A) \cap f(B)$

— ❖ —