2021

Full Marks : 70

Time : 3 hours

The figures in the right-hand margin indicate marks

Answer **all** Groups as directed

Group—A

(Objective Type Questions)

1. *(a)* Choose the correct alternative in each of the following : 5×1

(i) For two sets A and B, $A \cap (A \cup B) =$ ———.

(1) A

(2) B

(3) ϕ

(4) None of these

(ii) In the group $(z, *)$, $*$ is defined as $a * b = a + b + 1$, then the identity element in z is

(1) -1

(2) 0

(3) 1

(4) None of these

(iii) If a matrix $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then $(A-2I)(A-3I) =$ ______.

(1) A

(2) I

(3) 0

(4) None of these

(iv) If $y = e^{2x}$ and $y_5 = ke^{2x}$, then $k =$ ______.

(1) -5

(2) 5

(3) 2

(4) 32

(v) If $n(>1)$ be always odd, then $n^3 - 2n^2 + n$ is always divisible by

(1) 3

(2) 4

(3) 5

(4) None of these

(b) Fill in the blanks in each of the following : 5×1

(i) If A and B be two sets, such that $n(A) = 2$ and $n(B) = 3$, then the number of relations from A to B is 64.

(ii) In a group $(G, *)$, if $a * a = a, \forall a \in G$, then $a =$ identity element

(iii) The inverse of a square matrix exists only if determinant of a matrix is non zero value. $D \neq 0$

(iv) $\lim\limits_{x \to \infty} \frac{x^8}{e^x} =$ 0.

(v) Every natural number other than 1 admits of a set of prime factor.

Group—B

(Short Answer Type Questions)

Answer *any four* questions of the following : 4×5

2. For two sets A and B, prove that—

(i) $A - B = A \cap B'$

(ii) $A - B = B' - A'$

3. Prove that the intersection of two subgroups, of a group G, is also a subgroup of G.

4. State and prove Euler's theorem on homogeneous function of two variables, in Differential Calculus.

5. Using Maclaurin's theorem, expand $\log(1+e^x)$ as far as the term x^3.

6. If A and B be two non-singular matrices of the same order, then prove that $(AB)^{-1} = B^{-1}A^{-1}$.

7. If $N = p_1 p_2 p_3$ where p_1, p_2, p_3 are distinct primes and the sum of the divisors of N be $3N$, then prove that the sum of the reciprocal of the divisors of N is 3.

Group—C

(Long Answer Type Questions)

Answer *any four* questions of the following :

4×10

8. (a) Define equivalence relation. Prove that the relation $a \equiv b \pmod{m}$ on the set of integers is an equivalence relation.

(b) Define partition of a set. Write down all the partitions of the set $A = \{1, 2, 3, 4\}$.

9\. (a) Define Abelian group. Prove that the set whose elements are the cube roots of unity forms a group under usual multiplication.

(b) If $(G, *)$ be a group and $a, b, c \in G$, then prove that—

(i) $a * b = a * c \Rightarrow b = c$

(ii) $b * a = c * a \Rightarrow b = c$

10. If $y = \sin(m \sin^{-1} x)$, then prove that—

(i) $(1 - x^2) y_2 - x y_1 + m^2 y = 0$

(ii) $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} + (m^2 - n^2) y_n = 0$

(iii) $\lim_{x \to 0} \dfrac{y_{n+2}}{y_n} = n^2 - m^2$

11\. (a) If $u = e^{xyz}$, then prove that—

$$\frac{1}{u} \frac{\partial^3 u}{\partial x \, \partial y \, \partial z} = 1 + 3xyz + x^2 y^2 z^2$$

$a^2 - 7a + 3 = 0$

$\sqrt{b^2 - 4ac}$

$\sqrt{49 - 12}$

$\frac{-b \pm \sqrt{37}}{2a}$

(b) Find the points on the curve $y = x^2 + 7x + 4$, the tangents at which pass through the origin.

12. Test the consistency of the following system of simultaneous linear equations and if consistent, solve them by the matrix method :

$$x + y + z = 9,\ 2x + 5y + 7z = 52,\ 2x + y - z = 0$$

13. (a) If $a \equiv b \pmod{m}$, $c \equiv d \pmod{m}$, then prove that $ac \equiv bd \pmod{m}$.

(b) Find the remainder when $2^{100} + 3^{100} + 4^{100} + 5^{100}$ is divided by 7.

★★★

$\frac{+7 \pm \sqrt{37}}{2}$