2021
(Voc)

Full Marks : 70

Time : 3 hours

The figures in the right-hand margin indicate marks

Answer from **all** the Groups as directed

Group—A

(Objective Type Questions)

1. Choose the correct alternative in each of the following : 5×1

(a) If $A = \{1, 2, 3, 4\}$, the relation $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (2, 3), (3, 4), (2, 4)\}$ defined on set A, then R is

(i) reflexive, symmetric and transitive

(ii) reflexive, transitive but not symmetric

(iii) symmetric, transitive but not reflexive

(iv) symmetric but neither reflexive nor transitive

(b) The total numbers of generators of cyclic group $(96\mathbb{Z}, +)$ are

(i) 1 (ii) 2

(iii) 95 (iv) 96

(c) If

$$\sin u = \frac{x^3 + y^2}{x - y}$$

then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$$

equal to

(i) 0 (ii) $2\cot u$

(iii) $2\tan u$ (iv) nu

(d) $\underset{x \to 0}{\text{Lt}} \dfrac{e^x + e^{-x} - 2}{x^2}$ is equal to

(i) 0 (ii) 1

(iii) 2 (iv) -3

(e) When 5^{11} is divided by 7, then remainder is

(i) -3 (ii) 3

(iii) 4 (iv) 7

2. Fill in the blanks in each of the following : 5×1

(a) If sets A and B have n elements in common, how many elements do $A \times B$ and $B \times A$ have in common ——.

(b) If each element, except the identity, of a group be of order 2, then group is ——.

(c) The diagonal elements of a real or complex skew-symmetric matrix are all ——.

(d) The g.c.d. $(-272, -1479)$ is ——.

(e) The nth differential coefficient of $\sin 2x \cdot \cos 3x$ is ——.

Group—B

(Short Answer Type Questions)

Answer *any four* questions : 4×5

3. If A, B, C be three sets, the prove that $(A - B) \times C = (A \times C) - (B \times C)$.

4. Show that a cyclic group is necessarily Abelian. Show by an example that the converse may not be true.

5. Evaluate : 5

$$\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x} \right]$$

6. Expand $e^x \sin^2 x$ by Maclaurian's theorem up to x^4. 5

7. If A be symmetric matrix of order m and P be an $m \times n$ matrix, then prove that $P^T AP$ is a symmetric matrix. 5

8. Find g.c.d. of 256 and 1166 and express g.c.d. as linear combination of 256 and 1166.

Group—C

(Long Answer Type Questions)

Answer *any four* questions : 4×10

9. *(a)* Define equivalence relation. Prove that the intersection of two equivalence relation on a set is an equivalence relation.

(b) Define partition of a set. Write down all the partitions of the set $A = \{1, 2, 3\}$.

10. *(a)* Show that the set $\{5, 15, 25, 35\}$ forms a group under multiplication modulo 40.

(b) Let R be a commutative ring. Then R is an integral domain if and only if $ab = ac \Rightarrow b = c$, where $a, b, c \in R$ and $a \neq 0$.

11. *(a)* If $y = x^n \log x$, then prove that—

$$y_n = n!\left[\log x + 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right]$$

(b) If $y = e^{a \sin^{-1} x}$, then prove that—

$$(1 - x^2) y_{n+2} - (2n + 1)\, x y_{n+1} - (n^2 + a^2)\, y_n = 0$$

12. *(a)* If

$$u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$$

then show that—

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

(b) Find the equation of the tangent at (a, b) to the curve

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$$

13. Test the consistency of the following set of simultaneous equations and solve them by matrix method :

$$x + 2y + 3z = 14,$$
$$3x + y + 2z = 11,$$
$$-2x + 3y + z = 11$$

14. Define linear diophantine equation and find the general solution of $311x - 112y = 73$.

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