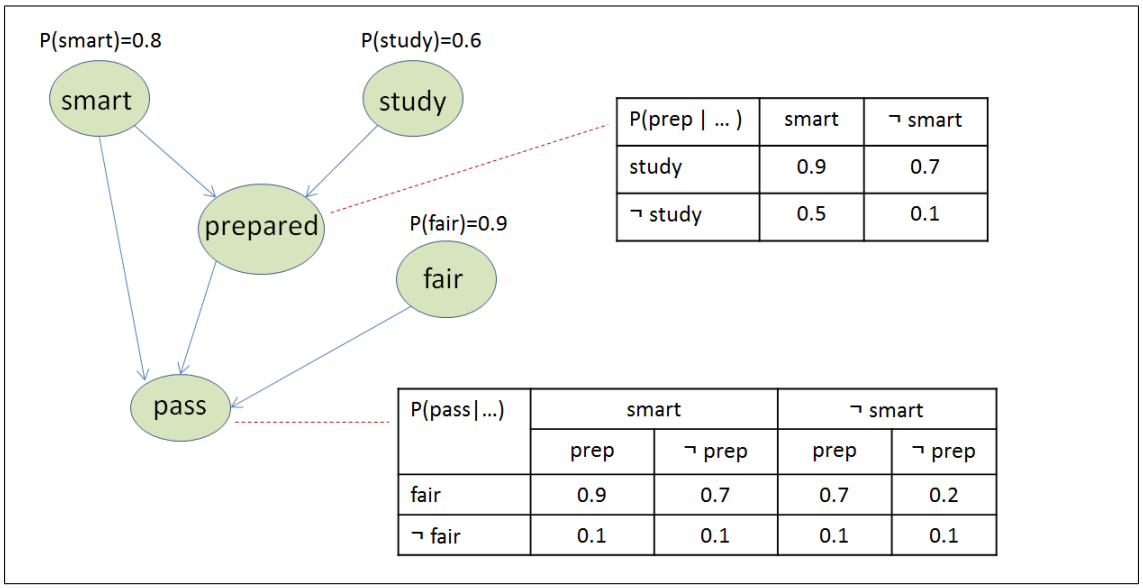
Bayesian and Decision Networks

Aim: the aim of this lab is to use Bayesian Networks and Decision Networks to model reasoning under uncertainty.

About the lab: this lab consists of three tasks, and it can be done by at most two persons.

Lab examination: demonstrate the tasks you solved to Pierangelo during any scheduled lab session.

Grading: Task A → 3 / Tasks A, B → 4 / Tasks A, B, C → 5



P(prep | ...)	smart	¬ smart
study	0.9	0.7
¬ study	0.5	0.1

P(pass|...)	smart		¬ smart	
	prep	¬ prep	prep	¬ prep
fair	0.9	0.7	0.7	0.2
¬ fair	0.1	0.1	0.1	0.1

Figure 1: Passing an exam

Preparation

- Download the tool JBayes from the course website.
- Look at the JBayes tutorials; Tutorials Four and Six.
- A few examples are available in the JBayes tool; Load Sample Problem in the File tab.
- Run JBayes and implement the scenario given in Figure 1.
- Calculate P(pass | study) with JBayes; the probability that a student passes an exam given that the student has studied.

Task A - Diagnostic Reasoning

- Calculate the following probability with JBayes:

  If a student passes an exam, what is the probability that the student has studied for the exam?

- Give a hand-written proof of it by using probability theory.

Task B - Predictive Reasoning

- Calculate the following probability with JBayes:

  If a smart student has not studied for an exam, what is the probability that the student passes the exam?

- Give a hand-written proof of it by using probability theory.

Task C - Decision Making

Mary's Dilemma
Mary, a famous and well-known anthropologist, recently returned from a long trip in Central America. One day, she feels like 'burning up' and immediately suspects a fever. The reason, she thought, could be the flu or perhaps a more serious disease caught during her trip.

Part A
You have the following information.

- The prior probability of having the flu is 0.2 and the one of having the serious disease is 0.1. Assume that the flu and the disease influence the body temperature as follows. If Mary has the disease as well as the flu, then she has strong fever. On the contrary, if Mary is healthy (no flu, no disease), then she has no fever. If Mary has the flu and not the disease, then she has the probability of 0.3 of no fever and 0.7 of fever. Vice versa, if she has the disease and no flu, then she has 0.1 probability of having fever and 0.9 probability of having strong fever.
- Mary has an old thermometer that detects body temperatures with a certain error. If Mary has a normal body temperature, then the thermometer correctly detects no fever, but in case of fever, the thermometer detects no fever with probability 0.1 and fever with probability 0.9. In case of high body temperature (strong fever), the thermometer detects strong fever with probability 0.9 and fever with probability 0.1.

Figure 2: Prior and conditional probabilities for Mary's Dilemma

1. Construct a Bayesian Network for the Mary's Dilemma. Your network should contain exactly four chance nodes corresponding to the following random variables:

   - Flu - Mary has caught the flu.
     Domain={T,F}
   - Disease - Mary has the disease.
     Domain={T,F}
   - Fever - Mary has fever.
     Domain={noFever, Fever, strongFever}
   - Therm - Mary's old thermometer.
     Domain={noFever, Fever, strongFever}

   The arcs defining your Bayesian network should accurately capture the probabilistic dependencies between these variables as defined in Figure 2.

2. Calculate the probabilities below in JBayes.

- If Mary has caught the flu, what is the probability that she has fever?

- If Mary has the disease, what is the probability that she has fever?

Part B

Extend your Bayesian network (developed in Part A) to become a decision network. The aim is to create a support system for helping Mary in taking a decision. Extend your Bayesian network by adding the following four nodes:

- FeverLater - chance node; Mary has fever afterwards.
  Domain={T,F}
- Reaction - chance node; reaction due to medication.
  Domain={T,F}
- Action - decision node; possible actions: rest, aspirin, medication.
  Domain={rest, aspirin, medication}
- U - utility node.

The arcs into the decision node should encode informational dependencies and the arcs into the utility node should encode utility dependencies as defined in Figure 3.

Compute the following with JBayes.

1 Which action should Mary take if she has the flu? what is the expected utility in such a case?

2 Which action should Mary take if she has the disease? what is the expected utility in such a case?

- Mary has three possible actions; have rest, take aspirin, and take medication.
- Mary must decide which action to take according to the following probabilities.
  - If Mary has rest:
    If Mary is healthy, she will have no fever afterwards. On the contrary, if she has the disease, then she will certainly have fever afterwards (independently on whether or not she has the flu). If she has the flu, but not the disease, then she has probability 0.5 of having fever afterwards.
  - If Mary takes an aspirin:
    If Mary is healthy, then she will have no fever afterwards. On the contrary, if she has the flu and the disease, then she will have fever. If she has the flu, but not the disease, then she has the probability of 0.1 of having fever. If instead she has the disease but not the flu, then the probability of fever afterwards increases to 0.9.
  - If Mary takes medication:
    If Mary is healthy, then she will have no fever afterwards. On the contrary, if she has the flu and the disease, then she will have probability 0.6 of having fever afterwards. While if she has the flu, but not the disease, then she will have no fever afterwards. If instead she has the disease but not the flu, then the probability of fever afterwards is 0.1.
- When taking medicines there is the possibility of having a body reaction. If Mary takes aspirin with strong fever, then the probability of a body reaction is 0.1; while if she has either no fever or fever the probability decreases to 0.05. If Mary takes medication and she has strong fever, the probability of a reaction is 0.4; while if she has either no fever or fever then the probability is 0.2.
- Mary's aim is to have no fever and no reaction. In this case the utility is 20. If Mary has fever afterwards and no reaction, the utility is -10. A bit worse is the case in which she does not have fever afterwards while having a reaction. The utility in this case is -15. The worst case is when she has both fever and reaction. The utility in this case is -20.

Figure 3: Probabilities, actions and utility values for Mary's Dilemma