Jacobian & Hessian matrix:

Consider function: $f(x,y,z) = x^2yz$

Jacobian: → 1st order derivative : $\partial f/\partial x$

$$J = \frac{\partial}{\partial x}(x^2yz), \frac{\partial}{\partial y}(x^2yz), \frac{\partial}{\partial z}(x^2yz)$$

(constant)

$$= [2xyz, x^2z, x^2y]$$

Hessian: → 2nd order derivative: $\frac{\partial^2 f}{\partial^2 x}$

$$H = \begin{bmatrix} 2yz & 2xz & 2xy \\ 2xz & 0 & x^2 \\ 2xy & x^2 & 0 \end{bmatrix} \begin{matrix} \to \partial/\partial x \\ \to \partial/\partial y \\ \to \partial/\partial z \end{matrix}$$

Example:

$$f(x,y) = x^2 + y^2$$

$$\therefore J = [2x, 2y], \quad H = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$|H| = 2.2 - 0.0 = 4$

$|H|$ positive mean either max or min.

if this is +ve means we got minimum

if this is -ve means we got maximum