

Statistics: a study of uncertainty
measured by probability

3 Frameworks:

→ Classical: equally likely outcome

(eg: rolling dice:

→ Frequentist: relative frequency

(how many times it occurs / freq)

→ Bayesian:

personal perspective

↓
based on my own info/belief

n → win \$3

+ → lose \$4

$$E = 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right) \\ = \frac{3}{2} - 2 = 1.5 - 2 = -0.5$$

Conditional Probability:

Probability of A under condition B:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

If probability of B doesn't depend on A:

$$P(A|B) = P(A)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

Bayes Theorem:

conditional prob. $\rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$

in other way $\rightarrow P(B|A) = \frac{P(B \cap A)}{P(A)}$

$$\Rightarrow P(B|A) \cdot P(A) = P(B \cap A) = P(A \cap B)$$

Bayes theorem \Rightarrow

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Problem: Let's say there 2 children & 1 is surely girl
what's the probability of both are girls

$$P(2 \text{ Girls} | 1 \text{ Girl}) = \frac{P(1G | 2G) \cdot P(2G)}{P(1G)}$$

$$= \frac{1 \cdot 1/4}{1/2}$$

$$= \frac{1}{2} \cdot \frac{3}{4}$$

$$= \frac{1}{3}$$

Probability of 1G among 2 girls sure

GG GB BG

G = Girl

B = Boy

Same equations:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad [\text{conditional}]$$

$$= \frac{P(B|A) \cdot P(A)}{P(B)} \quad [\text{bayes}]$$

$$= \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c)} \quad [\text{bayes}]$$

$$P(A \cap B) = P(B \cap A) \Rightarrow P(B|A) \cdot P(A) = P(A|B) \cdot P(B)$$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$