Bivariate Gaussian + Linear Transformation of Gaussians

1. Explain in your own words what effect does the choice of Covariance matrix have on the Bivariate Gaussian (compare spherical, elliptical). What does it mean when the covariance matrix is not diagonal?

Linear Transformation of Multivariate Normal Distribution

① $f(\bar{x}) = \bar{y} = A\bar{x} + b$ is from normal distribution since x and b are from distribution. So $\bar{y}$ has also two parameters:

① Mean

② Variance

$$\mu_y = E[\bar{y}] = E[A\bar{x} + b] = AE[x] + E[b] = \underline{A\mu_x}$$

0

$$\Sigma_y = Var(A\bar{x} + b) = Var(Ax) + Var(b)$$

$$= \underline{A\Sigma_x A^T + \Sigma_b}$$

$$\therefore \rightarrow \bar{y} \sim N(A\mu_x, A\Sigma_x A^T + \Sigma_b)$$

② $[\bar{x}, \bar{y}]^T$ => normal distributions since $\bar{x}, b, \bar{y}$ are normal distribution

$$\mu_{xy} = \begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix} = \begin{bmatrix} \mu_x \\ A\mu_x \end{bmatrix}$$

here we have bivariate Normal distribution & we will have Covariance matrix instead of variance,

$$Cov(x, y) = E[xy] - E[x]E[y]$$

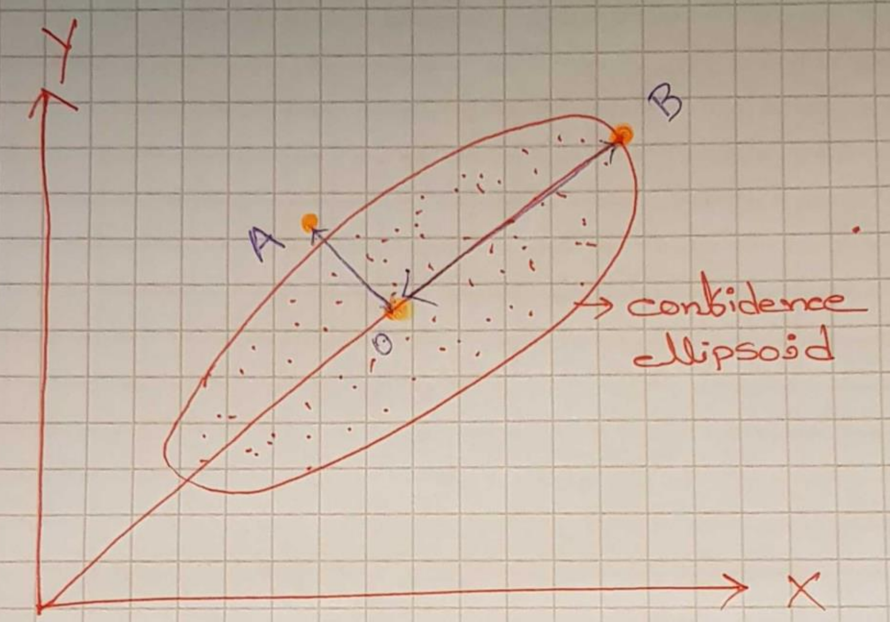
$$= \Sigma[xy]$$

$$= \Sigma[xx^T A^T + xb^T] - \mu_x \mu_y A^T$$

$$= \underline{\Sigma_x A^T}$$

2. What is the meaning of Mahalanobis distance? What is the relation of this to the eigenvalues of the Covariance matrix? Draw a sketch either in Python or by hand for the Bivariate case (K=2)

→ mahalanobis distance:



Euclidean distance:

$|OB| > |OA|$

Mahalanobis distance: (MD)

$MD(A) > MD(B)$

why?

→ point A is out of data set

→ Covariance shows how the data fit in.

Relation of Mahalanobis distance
and Covariance matrix:

$$D^2 = (x - m)^T c^{-1} (x - m)$$

Here,

D^2 = Mahalanobis distance

x = vector

m = mean values

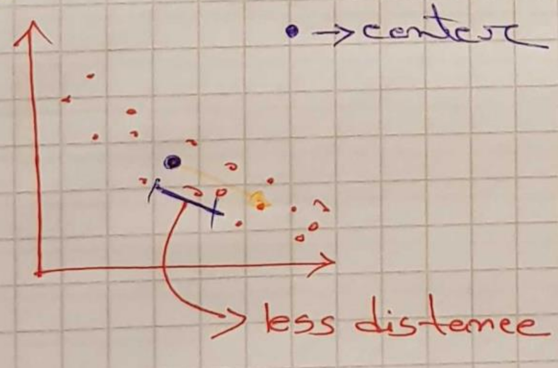
c = Covariance.

$$\Rightarrow \quad D^2 \propto c^{-1}$$

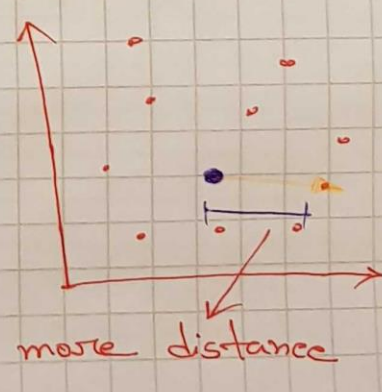
→ more covariance meas less distance

→ less covariance means more distance

High covariance:



Low covariance:



Eigenvalues: → rescale/reshape values
to reduce covariance