CS 571: Programming Practicum Date: 13/09/22

Assignment No# 4

Random walks: A class of random processes where the trajectory of an object moving in an appropriate (mathematical) space is determined by a succession of random steps.

In this assignment, we shall simulate random walks in bounded and unbounded one-dimensional (1D) spaces with steps sampled from different distributions.

- *1D random walk on integers:* Let $S[n]$ denote the position of an object at nth time instant, where
$$S[n] = S[n-1] + X, \tag{1}$$
and X is a random variable that takes 1 and -1 with probability p and $(1-p)$, respectively. Assume, $S[0] = 0$.

- *1D random walk on real line:* $S[n]$ is expressed in the same way it was earlier. However, X is now a continuous random variable, e.g., Gaussian, Laplace, etc.

- *1D random walk with absorbing boundaries:* In this case, in addition to (1), the following equations need to be included
$$S[n] = \begin{cases} N, & S[m] \geq N, \\ -N, & S[m] \leq -N, \end{cases} \tag{2}$$
$\forall n \geq m$ and $N \in \mathbb{Z}$ or $\mathbb{R}$ represents the boundary of the random walk.

- *1D random walk with reflecting boundaries:* In this case, in addition to (1), the following equations need to be considered
$$S[n] = \begin{cases} 2N - S[n], & S[n] \geq N \\ -2N - S[n], & S[n] \leq -N \end{cases} \tag{3}$$

- *1D random walk with periodic boundaries:* In this case, in addition to (1), the following equations need to be considered
$$S[n] = \begin{cases} -2N + S[n], & S[n] \geq N \\ 2N + S[n], & S[n] \leq -N \end{cases} \tag{4}$$

Write programs to simulate all the five different types of random walks mentioned above, and then plot the following:

1. $S[n]$ with respect to n (limit n to 100) for a single sample path.
2. Histogram of $S[n]$ for a single sample path.

3. $S[n]$ with respect to n for 5 sample paths (limit n to 100). Use different colors to denote different paths.

4. Histogram of $S[10]$, $S[20]$, $S[50]$, $S[100]$ for $100, 1000$ and 10000 sample paths.

5. Histogram of first passage time to 0 (T_0) for $100, 1000$ and 10000 sample paths of 1D random walk on integers, where the first passage time to a state $m \in \mathbb{Z}$ is defined as

$$T_m := \min\{n \geq 0 : S[i] \neq m, i = 1, 2, \ldots, n-1; S[n] = m\}$$

Note: Your plots must include (i) x and y labels, (ii) legends, (iii) x and y-ticks.