Programming Practicum (CS571)

Date: 23 Aug 22

Assignment-1

Q.1 **Taylor series expansion and convergence error:** Recall the Taylor series expansion of a single variable, real-valued, infinitely differentiable function $f(x)$ about a point a.

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots \frac{f^{(N)}(a)}{N!}(x-a)^N + \cdots.$$

If $a = 0$, then this expansion is known as a Maclaurin series.

In this assignment, we will consider the Maclaurin series expansion of some well-known functions and calculate the convergence error by varying the number of terms in the series expansion.

(1) Limit the number of terms in the expansion to 5 and calculate the convergence error (up to 8 decimal digits) for e^x , where $x = 0.1, 0.3, 0.45, 0.75, 0.95$

(2) Now vary the number of terms and calculate the convergence error again. Take $N = 3, 5, 7, 8, 9$

(3) Now find the largest value of N for which error is in the range of $\epsilon = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}$ when $x = 0.1, 0.3, 0.45, 0.75, 0.95$. If we denote the largest N by N_{max}, then

$$N_{\max} = \max\left\{ \mathrm{n} \in \mathbb{N}: \left| \mathrm{f}(x) - \sum_{i=0}^{n} \frac{f^{(i)}(0)x^i}{i!} \right| \geq \epsilon \right\}$$

Q.2 **Johnson counter:** A Johnson counter, also known as ring counter, is a shift register whose memory units (flip-flops) are interconnected to form a circular structure. The counting pattern depends on the connection between the right-most and the left-most memory units.

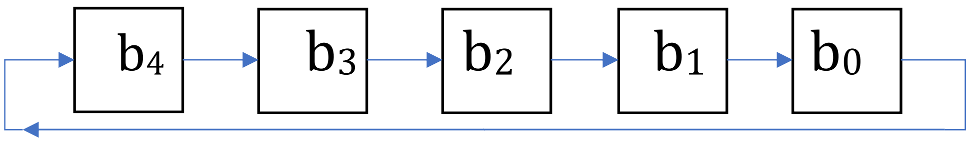

Fig.1: a 5-bit straight Johnson counter

(1) Write a program to print the output of a 5-bit Johnson counter (refer to the Fig. 1) in binary and decimal format when the counter is initialized with “10000” and (a) the right-most memory unit is connected to the left-most register unit directly, (b) the connection is through a logic inverter (twisted counter).

(2) Modify your program to add an option to change the direction of the shifting operation. Note that the counter shown in Fig.1 supports only the right-shifting operation.

(3) Modify your program further to add an option to change the direction of the shifting operating based on a one-bit random input and the current state of the counter. This one-bit random input appears after every third count, and if the bit is 0, then $b_0 \oplus b_2 \oplus b_4$ is used for deciding the direction. In case the bit is 1, $b_1 \oplus b_2 \oplus b_3$ is considered.