Signals and Systems: Module 9

Suggested Reading: S&S 4.4, 4.5

Now that we have an understanding of the CTFT and some of its basic properties – time shifting, duality, and so forth – we will study two other properties that are particularly important in the analysis of LTI systems: the convolution property and the multiplication property. We begin here by looking at the convolution property and several of its important applications.

The Convolution Property

We have already seen how an LTI system responds to a periodic signal $x(t)$ represented as a Fourier series $x(t) = \sum_k a_k e^{jk\omega_0 t}$: it multiplies each coefficient a_k by the frequency response $H(jk\omega_0)$, giving the output $y(t) = \sum_k H(jk\omega_0) a_k e^{jk\omega_0 t}$, which itself is also a Fourier series. We will now extend this result to aperiodic signals using the Fourier transform. Together with the corresponding convolution property we will study later for discrete-time systems, these may be the most important results we study in this course.

Derivation. Consider an LTI system with impulse response $h(t)$. We know that the output for an input $x(t)$ will be

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Now, suppose we apply the Fourier transform equation to the output:

$$Y(j\omega) = \mathcal{F}\{y(t)\} = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau\right] e^{-j\omega t} dt$$

Then, we can interchange the order of integration, noting that $x(\tau)$ is independent of t:

$$Y(j\omega) = \int_{-\infty}^{\infty} x(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau)e^{-j\omega t} dt\right] d\tau$$

The term in the inner brackets is the Fourier transform of $h(t-\tau)$: $\mathcal{F}\{h(t-\tau)\} = \int_{-\infty}^{\infty} h(t-\tau)e^{-j\omega t}dt$. With $\mathcal{F}\{h(t)\} = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt = H(j\omega)$, by the time delay property, we have $\mathcal{F}\{h(t-\tau)\} = H(j\omega)e^{-j\omega\tau}$. Noting that $H(j\omega)$ is independent of τ, it follows that

$$Y(j\omega) = \int_{-\infty}^{\infty} x(\tau)H(j\omega)e^{-j\omega\tau}d\tau = H(j\omega)\int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}d\tau$$

This last integral is just the Fourier transform of $x(t)$, i.e.,

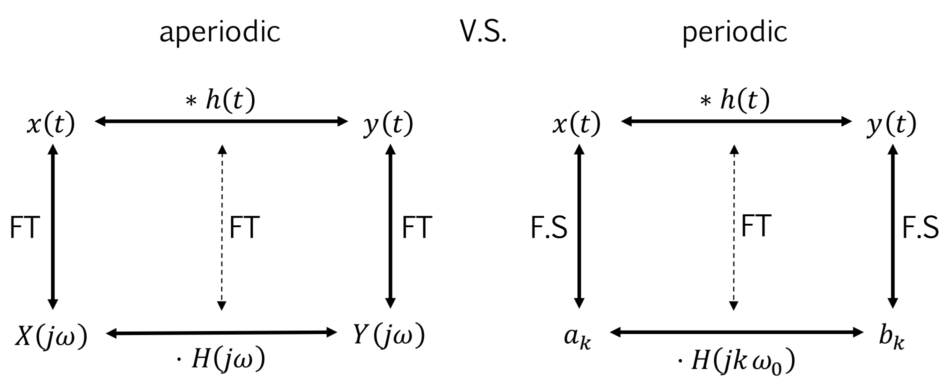
$$Y(j\omega) = H(j\omega)X(j\omega)$$

The property. What we have shown is that

$$y(t) = h(t) * x(t) \quad \overset{F}{\longleftrightarrow} \quad Y(j\omega) = H(j\omega)X(j\omega)$$

What this says is that the convolution operation associated with finding the output of an LTI system in the time domain can be done through multiplication in the frequency domain. The importance of this property to signal and system analysis cannot be overstated!

This property generalizes what we had in the case of periodic input signals to LTI systems. The following visualizes the difference between the convolution property for the aperiodic and periodic cases: Therefore, we have two



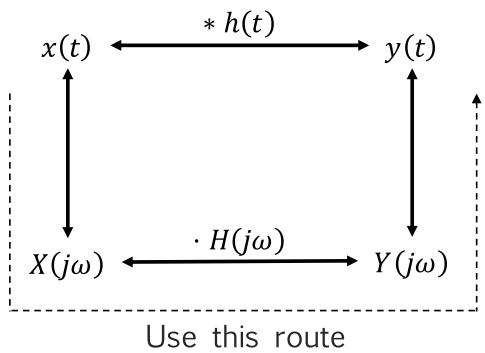
ways of finding the output $y(t)$: through convolution in the time domain, or multiplication in the frequency domain. Multiplication is usually much easier, but the caveat is we need to be able to find the inverse transform back to the time domain, which is sometimes very complicated.

More generally – beyond LTI system analysis – this property tells us that the convolution of two signals maps to the product of their Fourier transforms.

Example 1. *For a signal* $x(t) = \frac{\sin(3t)}{\pi t}$*, answer the following questions.*

Q: Find the output $y(t)$ *when* $x(t)$ *is input to an LTI system with impulse response* $h(t) = \frac{\sin(3t)}{\pi t}$*.*

Ans: This problem will be much easier to answer in the frequency domain.

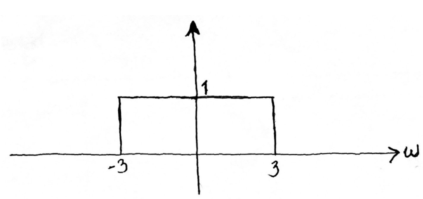


Specifically, from our transform pairs, we know that

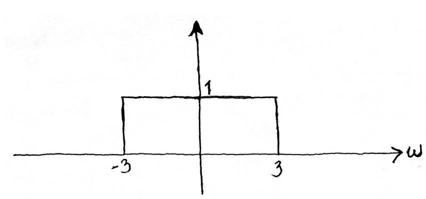
$$x(t) = \frac{\sin(3t)}{\pi t} \overset{F}{\longleftrightarrow} X(j\omega) = \begin{cases} 1 & |\omega| < 3 \\ 0 & otherwise \end{cases}$$

Also, since $x(t) = h(t)$ *in this problem,* $X(j\omega) = H(j\omega)$*. By the convolution property,* $Y(j\omega) = X(j\omega) \cdot H(j\omega)$*. Therefore we can find* $Y(j\omega)$ *easily by just taking the product of two rectangles:*

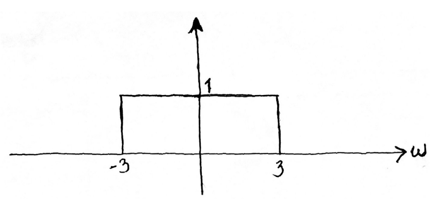
$X(j\omega)$



$H(j\omega)$



$Y(j\omega) = X(j\omega) \cdot H(j\omega)$



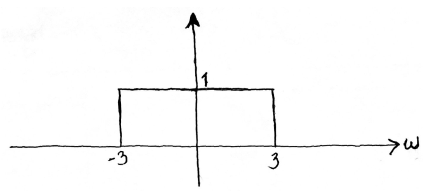
By the inverse transform, then,

$$Y(j\omega) \overset{F}{\longleftrightarrow} y(t) = \frac{\sin(3t)}{\pi t}$$

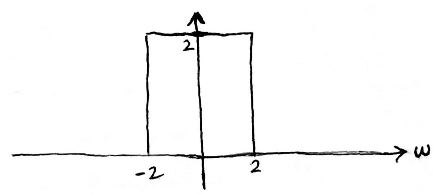
Q: If the impulse response of an LTI system is $h_2(t) = \frac{2\sin(2t)}{\pi t}$*, find* $y_2(t) = x(t) * h_2(t)$*.*

Ans: We use the exact same procedure, the only difference being that $X(j\omega)$ *and* $H_2(j\omega)$ *are no longer equivalent:*

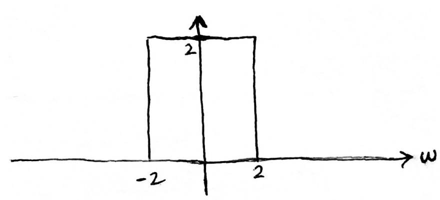
$X(j\omega)$



$H_2(j\omega)$



$Y_2(j\omega)$



Thus,

$$y_2(t) = \frac{2\sin(2t)}{\pi t}$$

Example 1 illustrates a case where the Fourier transform makes convolution extremely easy. While it will not always be so simple, it will oftentimes make the computation easier than carrying out the convolution operation. At the very least, it will give us insight into the frequency characteristics of the output.

In what follows, we will build upon the convolution property in considering four important applications.

1. Characterizing/Identifying LTI Systems

$H(j\omega)$, the Fourier transform of the impulse response $h(t)$, is the frequency response of the system. As does $h(t)$, $H(j\omega)$ completely characterizes an LTI system. By the equation $Y(j\omega) = H(j\omega)X(j\omega)$, it is clear that the frequency response of the system captures the change in complex amplitude of the Fourier transform of the input at each frequency ω.

Example 2. *What is the frequency response of the delay system $y(t) = x(t - t_0)$?*

Ans: Using Fourier transforms and the delay property, $Y(j\omega) = X(j\omega)e^{-j\omega t_0}$. Therefore,

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = e^{-j\omega t_0}$$

A delay system, then, will not affect the magnitude of the input, and will introduce a phase shift of $-\omega t_0$ that is linear in ω.

Previously, we had determined the impulse response $h(t)$ of an LTI system by measuring the output when we input $x(t) = \delta(t)$. In practice, $\delta(t)$ is hard to generate, given its infinite amplitude. The CTFT gives us another, often more practical way of finding $h(t)$: we can determine the frequency response $H(j\omega)$ by measuring how the system responds to different frequencies, and then finding the inverse transform $h(t) = \mathcal{F}^{-1}\{H(j\omega)\}$.

Thus, we need not choose $x(t) = \delta(t)$ to find $h(t)$ in an LTI system. More generally, we can do the following:

1. For a given $x(t)$ inputted to the system, measure $y(t)$.
2. Calculate the Fourier transforms $X(j\omega)$ and $Y(j\omega)$.
3. Find the frequency response $H(j\omega) = Y(j\omega)/X(j\omega)$.
4. Find the impulse response as $h(t) = \mathcal{F}^{-1}\{H(j\omega)\}$.

There is one caveat to this procedure, however: $X(j\omega)$ must be non-zero at all frequencies ω (otherwise we would have to divide by 0 in step 2). The implication is that in order to exactly determine $h(t)$, our "test signal" $x(t)$ must have an infinite bandwidth. We have already studied many such signals: the decaying exponential $e^{-at}u(t)$, the rectangular pulse $u(t+T_0) - u(t - T_0)$, and of course, the unit impulse $\delta(t)$.

Example 3. *Suppose an input $x(t) = e^{-bt}u(t)$ is applied to an LTI system with impulse response $h(t) = e^{-at}u(t)$, where $a, b > 0$. What will be the output $y(t)$?*

Ans: Rather than computing $y(t) = x(t) * h(t)$ directly, let's transform it to the frequency domain. We know that

$$X(j\omega) = \frac{1}{b + j\omega} \qquad H(j\omega) = \frac{1}{a + j\omega}$$

And thus

$$Y(j\omega) = H(j\omega)X(j\omega) = \frac{1}{(b + j\omega)(a + j\omega)}$$

*Now comes the caveat of the frequency domain: we need to inverse-transform $Y(j\omega)$ back to the time domain $y(t)$. For problems like this with product terms in the denominator, we resort to **partial-fraction expansion** where we wish to express*

$$\frac{1}{(b + j\omega)(a + j\omega)} = \frac{A}{a + j\omega} + \frac{B}{b + j\omega}$$

for constants A and B. Multiplying both sides by $(a + j\omega)(b + j\omega)$,

$$1 = A(b + j\omega) + B(a + j\omega)$$

Then by setting $j\omega = -b$, it follows that $B = 1/(a - b)$, and by setting $j\omega = -a$, $A = 1/(b - a)$. Thus

$$Y(j\omega) = \frac{1/(b - a)}{a + j\omega} + \frac{1/(a - b)}{b + j\omega}$$

And now we can take the inverse transform of each term separately:

$$y(t) = \mathcal{F}^{-1}\left\{\frac{1/(b - a)}{a + j\omega}\right\} + \mathcal{F}^{-1}\left\{\frac{1/(a - b)}{b + j\omega}\right\} = \frac{1}{b - a}\left[e^{-at} - e^{-bt}\right]u(t), \quad a \neq b$$

The partial fraction expansion we just found, however, is only valid for $a \neq b$. When $a = b$, we resort back to the original $Y(j\omega)$:

$$Y(j\omega) = \frac{1}{(a + j\omega)^2}$$

Note that

$$\frac{d}{d\omega}\left[\frac{1}{a + j\omega}\right] = \frac{-j}{(a + j\omega)^2} \quad \rightarrow \quad Y(j\omega) = j\frac{d}{d\omega}\left[\frac{1}{a + j\omega}\right]$$

The CTFT's frequency differentiation property tells us that

$$tz(t) \overset{F}{\longleftrightarrow} j\frac{d}{d\omega}Z(j\omega)$$

With $Z(j\omega) = 1/(a + j\omega)$ *in this case, it follows that*

$$y(t) = te^{-at}u(t), \quad a = b$$

We can also use the convolution property to characterize the power in the output waveforms of LTI systems.

Example 4. *Suppose a signal* $x(t)$ *with Fourier transform* $X(j\omega) = \omega^2, \ |\omega| < 3$ *and* $X(j\omega) = 0$ *otherwise is put through the integrator system* $y(t) = \int_{-\infty}^{t} x(s)ds$*. What will be the total energy in the output* $y(t)$*?*

Solving this problem in the time domain would clearly be difficult. But the integration property of the CTFT tells us that

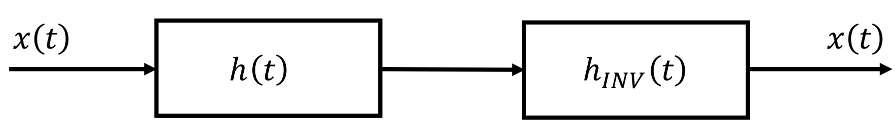
$$Y(j\omega) = \frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega) = \frac{1}{j\omega}\omega^2 + \pi \cdot 0 \cdot \delta(\omega) = \frac{\omega}{j}$$

By Parseval's relation, we can obtain the total energy as

$$E_\infty = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(j\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-3}^{3} \left|\frac{\omega}{j}\right|^2 d\omega = \frac{1}{2\pi}\int_{-3}^{3} \omega^2 d\omega = \frac{1}{6\pi}\omega^3\Big|_{-3}^{3} = \frac{27}{3\pi}$$

2. Invertibility and Stability of LTI Systems

We can use the convolution property to come up with an explicit rule for telling whether an LTI system is invertible or not. Recall our rule for invertibility: for an LTI system with impulse response $h(t)$ to be invertible, we must be able to construct the inverse $h_{inv}(t)$ such that



for an input $x(t)$. Specifically, the impulse response of the concatenated system must be

$$h(t) * h_{inv}(t) = \delta(t)$$

Noting that $\delta(t) \overset{F}{\longleftrightarrow} 1$, then by the convolution property,

$$H(j\omega) \cdot H_{inv}(j\omega) = 1 \rightarrow H_{inv}(j\omega) = \frac{1}{H(j\omega)}$$

so long as $H(j\omega) \neq 0$. In summary, given $h(t)$, we can try to find $h_{inv}(t)$ through the following sequence of steps:

$$h(t) \rightarrow H(j\omega) \rightarrow H_{inv}(j\omega) = \frac{1}{H(j\omega)} \rightarrow h_{inv}(t) = \mathcal{F}^{-1}(H_{inv}(j\omega))$$

If $H_{inv}(j\omega)$ exists, then we can conclude that the system is invertible. On the other hand, if $H_{inv}(j\omega)$ does not exist, then the system is not invertible. We can thus conclude that *an LTI system is invertible if and only if* $H(j\omega) \neq 0$ *for all* ω. If there are certain frequencies which become nullified through the system, then there is no way to recover the input.

Example 5. *Consider an LTI system with impulse response* $h(t) = e^{-t}u(t)$. *Is the system invertible? If so, find the impulse response of the inverse system, and its corresponding input-output relationship.*

Ans: We can find the frequency response and its inverse as

$$H(j\omega) = \frac{1}{1 + j\omega} \Rightarrow H_{inv}(j\omega) = 1 + j\omega$$

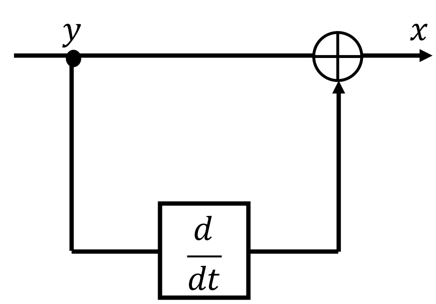
$H_{inv}(j\omega)$ *is defined for all* ω, *so the system is invertible. Since*

$$X(j\omega) = (1 + j\omega)Y(j\omega)$$

We can take the inverse transform with the differentiation property to find

$$x(t) = y(t) + \frac{d}{dt}y(t)$$

The inverse system for recovering $x(t)$ *can thus be represented as the following block diagram:*



The impulse response $h_{inv}(t)$ is a bit tricky to express mathematically, because it involves a derivative of $\delta(t)$. We can write it concisely as

$$h_{inv}(t) = \mathcal{F}^{-1}[1] + \mathcal{F}^{-1}[j\omega \cdot 1] = \delta(t) + \frac{d}{dt}\delta(t)$$

Example 6. *Is the integrator system $y(t) = \int_{t-T_1}^{t+T_1} x(s)ds$ invertible? If so, find the impulse response of the inverse system.*

Ans: We find the impulse response as

$$h(t) = \int_{t-T_1}^{t+T_1} \delta(s)ds = u(t + T_1) - u(t - T_1)$$

This rectangular pulse is one of our known transform pairs, giving a sinc function in frequency:

$$H(j\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

Can we find $H_{inv}(j\omega) = 1/H(j\omega)$? No. Due to the sinusoid, values of $\omega = k\pi$ for $k \neq 0$ make $H(j\omega) = 0$, and therefore $H_{inv}(j\omega)$ does not exist. The system is <u>not invertible</u>.

Example 7. *Is the "dual" system of Example 6, an LTI system with impulse response $h(t) = \frac{\sin Wt}{\pi t}$, invertible?*

Ans: We know the frequency response is

$$H(j\omega) = u(\omega + W) - u(\omega - W)$$

Can we find $H_{inv}(j\omega)$? No, because $H(j\omega) = 0$ for $-W < \omega < W$. The system is <u>not invertible</u>.

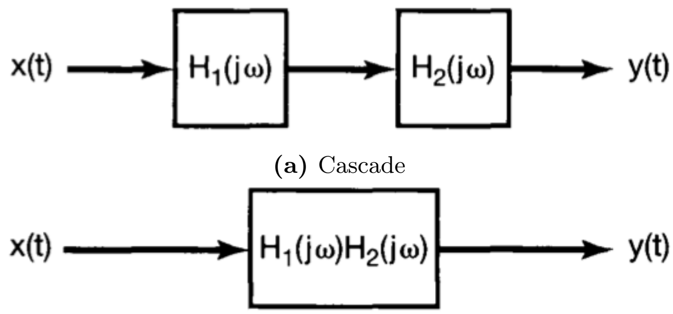
We can also use the convolution property to reason about stability of LTI systems. Recall that the existence of the Fourier transform, like the Fourier series, is guaranteed only under certain conditions. Consequently, the frequency response $H(j\omega)$ cannot be defined for *every* LTI system. However, two of these conditions are satisfied for virtually all $h(t)$ of practical significance. The other condition is absolute integrability: $\int_{-\infty}^{\infty} |h(t)|dt < \infty$.

Since absolute integrability of $h(t)$ is equivalent to stability, we conclude that, for all practical purposes, *an LTI system is stable if and only if its frequency response $H(j\omega)$ is defined.* The Laplace transform is a more general version of the Fourier transform that allow us to examine unstable LTI systems, but we will not be covering that in this course.

3. Interconnection of LTI Systems

We can also apply the Fourier transform to simplify the analysis of interconnected LTI systems.

For example, recall that when two LTI systems S_1 and S_2 are placed in cascade, the overall impulse response $h(t)$ is the convolution of the individual impulse responses, i.e., $h(t) = h_1(t) * h_2(t)$. In the frequency domain, then, the overall response $H(j\omega)$ is the multiplication of the individual frequency responses, i.e., $H(j\omega) = H_1(j\omega)H_2(j\omega)$, as shown in Figure 3.

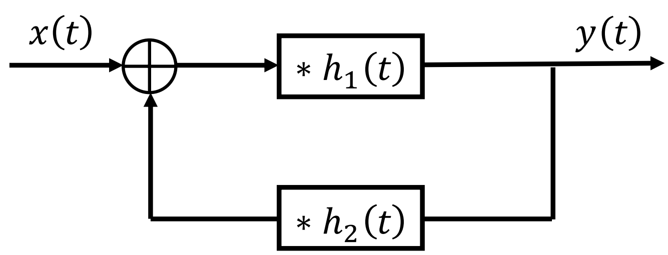


(a) Cascade

(b) Product

Figure 3: The frequency response of the cascade of two LTI systems is the product of their individual frequency responses.

Moreover, consider the feedback system:



Since convolution in the time domain becomes multiplication in the frequency domain, we can conduct analysis here very similarly to what you probably learned with the Laplace transform. In the forward loop, the input to $h_1(t)$ is $x(t) + y(t) * h_2(t)$. Thus:

$$Y(j\omega) = H_1(j\omega)\left(X(j\omega) + H_2(j\omega)Y(j\omega)\right)$$
$$\Rightarrow Y(j\omega) = \underbrace{\frac{H_1(j\omega)}{1 - H_1(j\omega)H_2(j\omega)}}_{\text{Composite } H(j\omega) \text{ frequency response}} X(j\omega)$$

This composite frequency response $H(j\omega)$ is equivalent to the feedback system.

4. Frequency-based Manipulation of Signals

The above applications have also suggested that the convolution property allows us to design techniques for manipulating the frequency components of signals. A key use-case of this is **filtering**, were we aim to remove certain frequency components.

Let's compare the impulse and frequency responses of an **ideal lowpass filter** and an **RC lowpass filter**. An ideal, "brick wall" lowpass filter will have a frequency response

$$H_I(j\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

where ω_c is the cutoff frequency. The impulse response corresponding to this is the inverse transform, which we already know from one of our transform pairs:

$$h_I(t) = \frac{\sin \omega_c t}{\pi t}$$

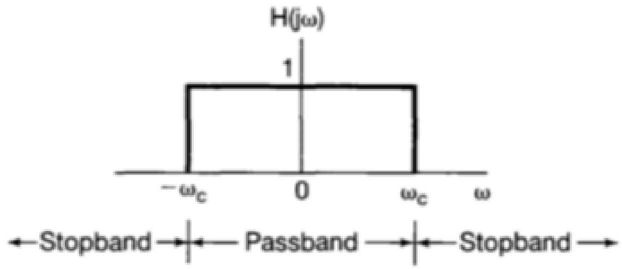
An RC lowpass filter (created from a resistor and capacitor connected in series), on the other hand, will have an impulse response of the form

$$h_{RC}(t) = e^{-t/\tau}u(t)$$

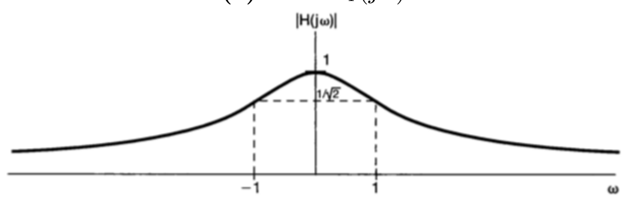
where $\tau = RC$ is the time constant of the circuit. The frequency response $H_{RC}(j\omega)$ is the Fourier transform of $h_{RC}(t)$, which we also know how to find:

$$H_{RC}(j\omega) = \frac{1}{1/\tau + j\omega}$$

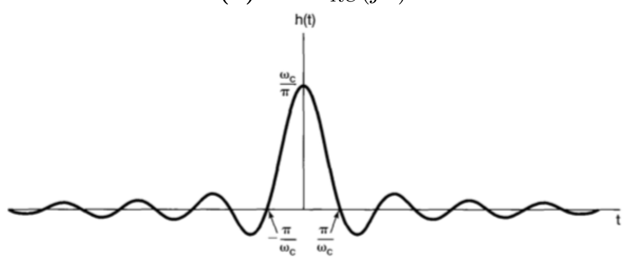
Out of the two frequency responses, in theory, H_I would be preferred to H_{RC} if the objective is to block frequencies higher than ω_c. But the impulse response $h_I(t)$ is difficult to even approximate in practice. Even if it could be implemented, it has several undesirable characteristics: it is not causal ($h(t) \neq 0$ for $t < 0$), and it has oscillatory behavior.



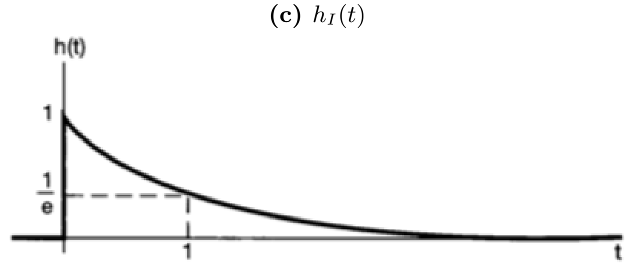
(a) Ideal $H_I(j\omega)$



(b) RC $H_{RC}(j\omega)$



(c) $h_I(t)$



(d) $h_{RC}(t)$

Figure 4: Comparison of the impulse response $h(t)$ and the frequency response $H(j\omega)$ of (a)&(c) an ideal lowpass filter and (b)&(d) an RC lowpass filter ($\tau = 1$).

On the other hand, while H_{RC} does not have the strong frequency selectivity of an ideal lowpass filter, it is easy to implement (you built many of these in circuits lab). Moreover, its impulse response $h_{RC}(t)$ is causal and decays naturally without oscillations. Sometimes we have to sacrifice better behavior in the frequency domain to get better behavior in the time domain, and vice versa. Plots of the impulse responses and frequency responses of these lowpass filters are given in Figure 4.

The Multiplication Property

The convolution property, in an nutshell, says that convolution in the time domain becomes multiplication in the frequency domain. The multiplication property arises from applying duality to this: multiplication in the time domain corresponds to convolution in the frequency domain. Specifically, if $s(t)$ and $p(t)$ are two signals, then

$$r(t) = s(t)p(t) \stackrel{F}{\longleftrightarrow} R(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(j\theta)P(j(\omega-\theta))d\theta$$

where θ is the change of variables in the frequency domain as τ is in the time domain.

Multiplying one signal by another can be thought of as using one signal to scale or modulate the amplitude of another. For this reason, the multiplication of two signals is often referred to as **amplitude modulation**, and the multiplication property is sometimes called the **modulation property**. Amplitude modulation (and modulation in general) has very important applications to communication systems: AM Radio gets its name because of the fact that it employs amplitude modulation, for instance.

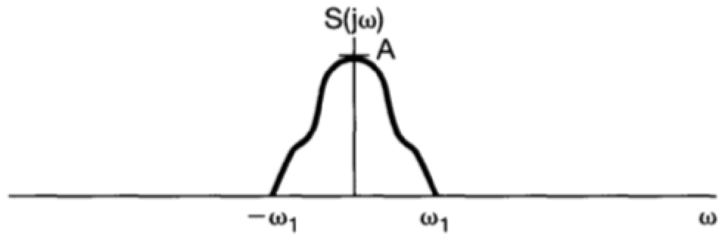
Consider the generic signal $s(t)$ with spectrum $S(j\omega)$ given in Figure 5(a): it has a bandwidth between $-\omega_1$ and ω_1 and a DC magnitude of $S(0) = A$. If we multiply $s(t)$ by the sinusoid $p(t) = \cos\omega_0 t$ to obtain $r(t) = s(t)p(t)$, what will be the resulting frequency spectrum?

First, we obtain the spectrum $P(j\omega)$ as the Fourier transform of $p(t)$. Since $\cos(\omega_0 t)$ is two complex exponentials at $\pm\omega_0$, the transform is

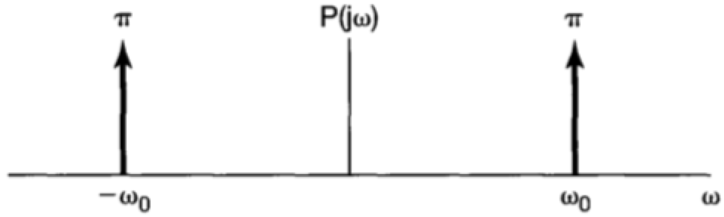
$$P(j\omega) = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$$

which is plotted in Figure 5(b). Now, by the multiplication property, the transform of $r(t) = s(t)p(t)$ is

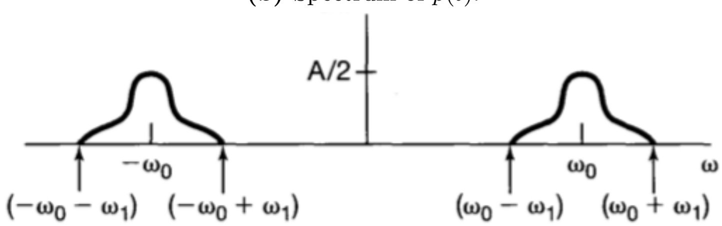
$$R(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} P(j\theta)S(j(\omega-\theta))d\theta$$



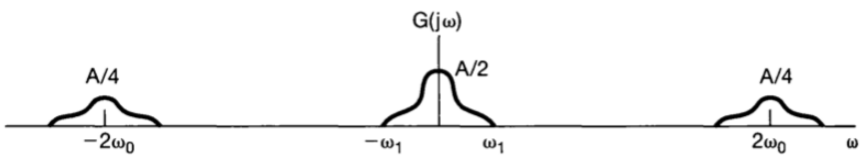
(a) Spectrum of $s(t)$.



(b) Spectrum of $p(t)$.



(c) Spectrum of $r(t) = s(t)p(t)$.



(d) Spectrum of $g(t) = r(t)p(t)$.

Figure 5: Multiplying $s(t)$ by the sinusoid $p(t)$ at frequency $\pm\omega_0$ causes the spectrum of $s(t)$ to shift to $\pm\omega_0$. Multiplying by $p(t)$ again shifts up to $\pm 2\omega_0$, but also replicates the original components of $S(j\omega)$.

$$
\begin{aligned}
&= \frac{1}{2\pi}\left[\pi\delta(\omega-\omega_0) * S(j\omega) + \pi\delta(\omega+\omega_0) * S(j\omega)\right] \\
&= \frac{1}{2}S(j(\omega-\omega_0)) + \frac{1}{2}S(j(\omega+\omega_0))
\end{aligned}
$$

In other words, $R(j\omega)$ is $S(j\omega)$ cut in half and replicated at two different frequencies: $\pm\omega_0$, corresponding to the sinusoid. This is plotted in Figure 5(c), where we have assumed that $\omega_0 > \omega_1$. All of the information in $s(t)$

is preserved after the multiplication: it has simply been shifted to higher frequencies. This fact forms the basis for sinusoidal amplitude modulation communication systems.

Now, what happens if we repeat this process to obtain $g(t) = r(t)p(t)$? $g(t)$ is $r(t)$ multiplied by $p(t)$ again, so its spectrum is the convolution of $R(j\omega)$ and $P(j\omega)$:

$$\begin{aligned}
G(j\omega) &= \frac{1}{2\pi} R(j\omega) * P(j\omega) \\
&= \frac{1}{2\pi}\left[\frac{1}{2}S(j(\omega-\omega_0)) + \frac{1}{2}S(j(\omega+\omega_0))\right] * \left[\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)\right] \\
&= \frac{1}{4}S(j(\omega-\omega_0)) * \delta(\omega-\omega_0) + \frac{1}{4}S(j(\omega-\omega_0)) * \delta(\omega+\omega_0) + \cdots \\
&= \frac{1}{4}S(j(\omega-2\omega_0)) + \frac{1}{4}S(j\omega) + \frac{1}{4}S(j\omega) + \frac{1}{4}S(j(\omega+2\omega_0))
\end{aligned}$$

Which finally simplifies to

$$G(j\omega) = \frac{1}{4}S(j(\omega+2\omega_0)) + \frac{1}{2}S(j\omega) + \frac{1}{4}S(j(\omega-2\omega_0))$$

$g(t)$, then, is the sum of $(1/2)s(t)$ and another signal that is non-zero only at much higher frequencies centered around $\pm 2\omega_0$. This is visualized in Figure 5(d). If we then applied $g(t)$ as the input to a lowpass filter that has a cutoff frequency between ω_1 and $2\omega_0 - \omega_1$, the output would (in theory) be the original signal $s(t)$. This illustrates a key idea in amplitude modulation: have a transmitter multiply its message by a sinusoid at some high frequency ω_0, and then have the receiver multiply by this sinusoid again and apply a lowpass filter. By assigning different frequencies ω_0 to different transmitter-receiver pairs, we can transmit many messages simultaneously.

We are now ready to discuss our amplitude modulation application more formally, which we will do in the next module.