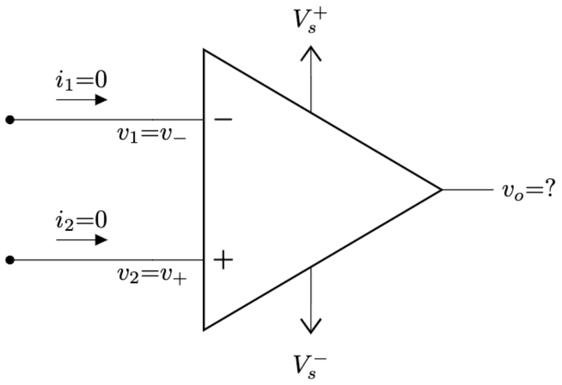
Lecture 5

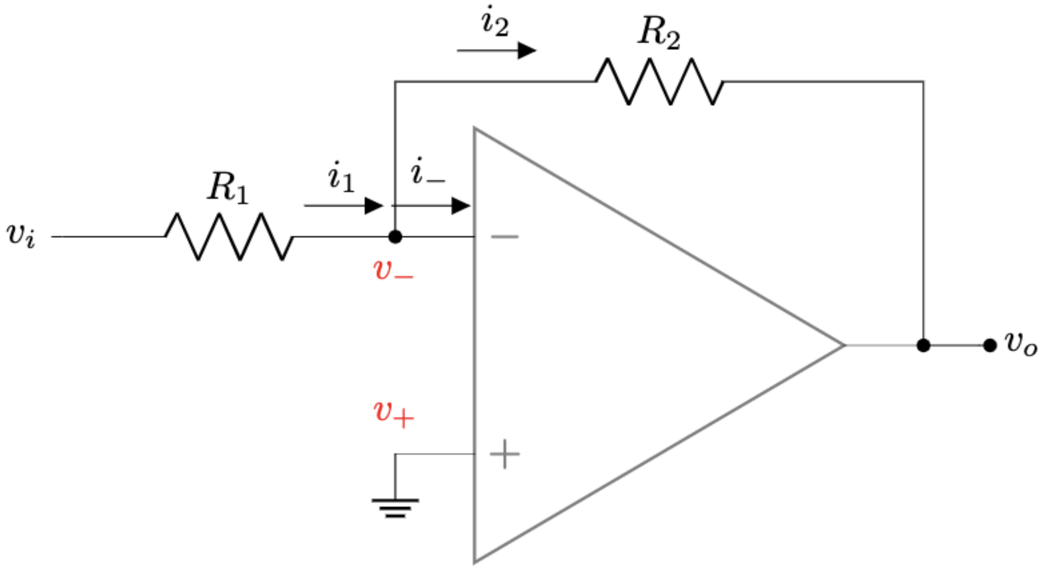
Op Amp – Part 3

Solving Circuit with Ideal Op Amp + NF

- For ideal op-amp
  - Infinite input resistance, $R_i = \infty = \text{open circuit}$
  - Zero output resistance, $R_o = 0 = \text{short circuit}$
  - $\boldsymbol{i_i = 0}$ **and** $\boldsymbol{i_+ = 0}$
- ***<u>When there is negative feedback</u>***, For ideal A as is infinitely high, for a finite output voltage v_o, $\frac{v_o}{A} = v_d = 0 \Rightarrow \boldsymbol{v_+ = v_-}$. This is called **virtual short circuit**
- Because of these, solving ideal op-amp circuit with negative feedback is very simple



Review – Inverting Amplifier



Since v_+ is connected to ground, $v_+ = 0V$

Since there is negative feedback, from virtual short, $v_- = v_+ = 0V$

From Ohm's law for $R_1 \Rightarrow i_1 = \frac{v_i - 0V}{R_1} = v_i/R_1$

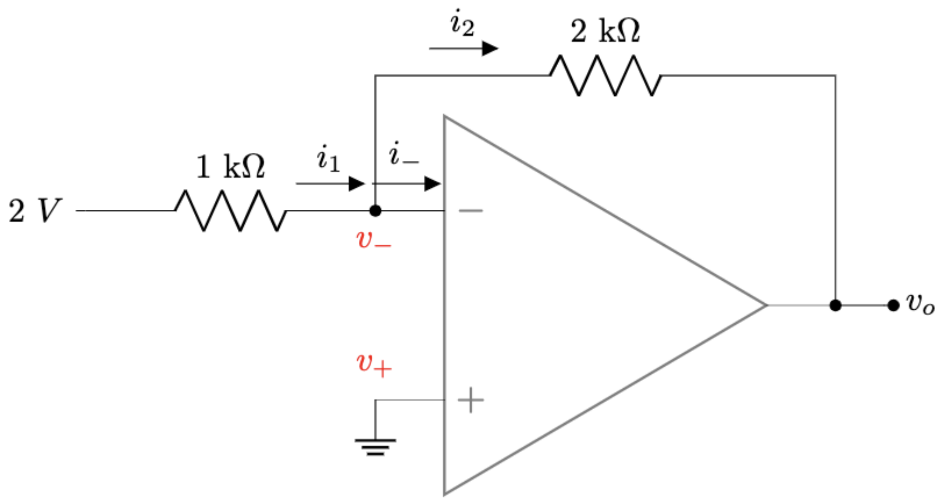
Since ideal op-amp, $i_- = i_+ = 0$

From KCL at v_-, $i_1 = i_- + i_2 \Rightarrow i_1 = i_2 = v_i/R_1$

From Ohm's law for $R_2 \Rightarrow i_2 = \frac{v_- - v_0}{R_2} = \frac{v_i}{R_1} \Rightarrow v_o = -i_2 \times R_2 \Rightarrow v_o = -\frac{R_2}{R_1} v_i$ [ANS]

Gain = $-\frac{R_2}{R_1}$

Example – Inverting Amplifier



Since v_+ is connected to ground, $v_+ = 0V$

Since there is negative feedback, from virtual short, $v_- = v_+ = 0V$

From Ohm's law for $1\ k\Omega \Rightarrow i_1 = \frac{2V - 0V}{1\ k\Omega} = 2mA$

Since ideal op-amp, $i_- = i_+ = 0$

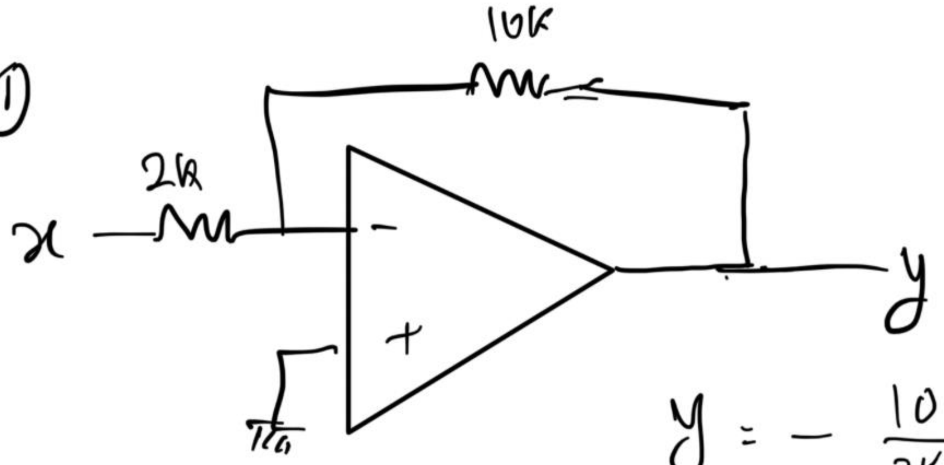
From KCL at v_-, $i_1 = i_- + i_2 \Rightarrow i_1 = i_2 = 2\ mA$

From Ohm's law for $2\ k\Omega \Rightarrow i_2 = \frac{v_- - v_0}{2\ k\Omega} = 2mA \Rightarrow v_o = -i_2 \times 2 = -4V$ [ANS]

Gain = $-\frac{4V}{2V} = -2$ (hence **inverting**)

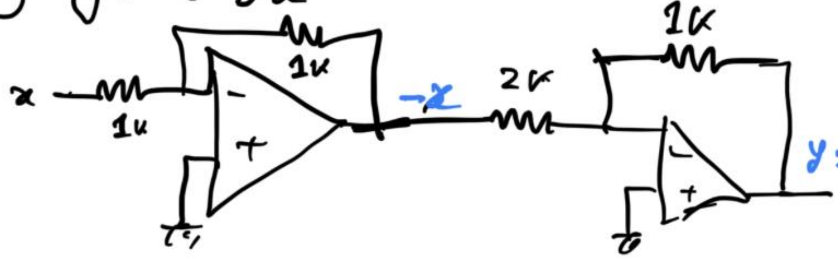
Example

1



$$y = -\frac{10K}{2K}x \Rightarrow \boxed{y = -5x}$$

2 $y = 0.5x$



$y = \left(-\frac{1}{2}\right)(-x) = 0.5x$

Inverting Adder

Consider v_1 first, and deactivate other (v_2, v_3, v_4) sources.

It is nothing but a non-inverting amplifier.

So, $v_{o1} = -\frac{R_f}{R_1} v_1$

Similarly, if we active one source and deactivate others, we will get:

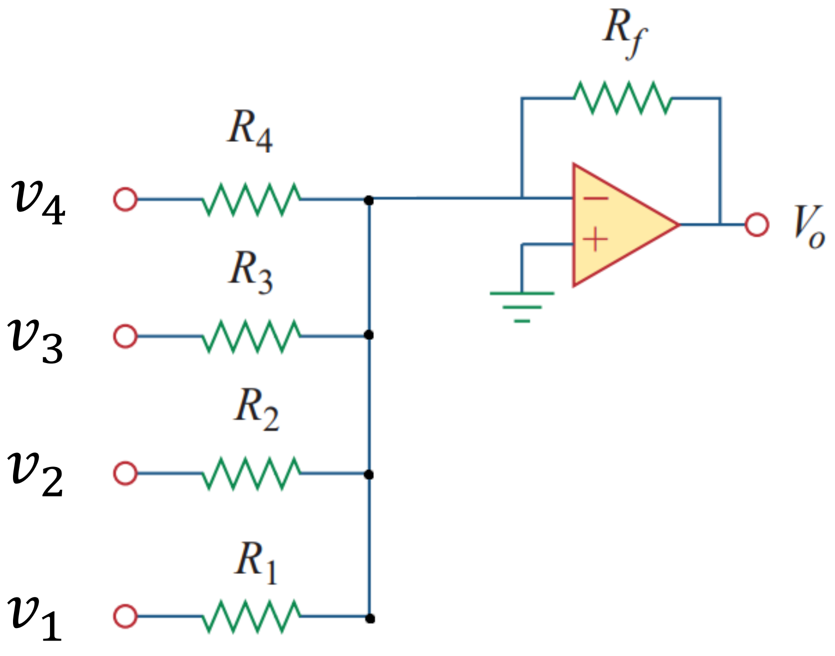
$v_{o2} = -\frac{R_f}{R_2} v_2$, $v_{o3} = -\frac{R_f}{R_3} v_3$, $v_{o4} = -\frac{R_f}{R_4} v_4$

Now, using **superposition principle**,

v_o=v_{o1}+v_{o2}+v_{o3}+v_{o4}

So, $v_o = -\frac{R_f}{R_1} v_1 - \frac{R_f}{R_2} v_2 - \frac{R_f}{R_3} v_3 - \frac{R_f}{R_4} v_4$

Or, $v_o = -(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 + \frac{R_f}{R_4} v_4)$



We can use this circuit to add any 'n' number of inputs!

Example

Implement the following function using op-amps:

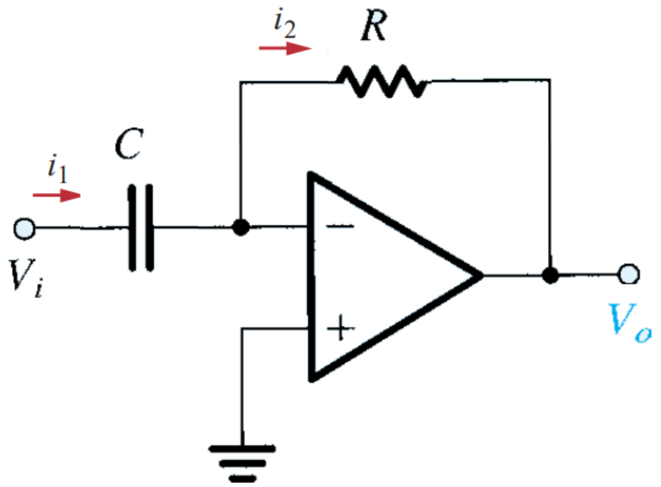
$$v_0 = -(v_1 + 0 \cdot 5v_2 + \boldsymbol{v_3})$$

Solution:

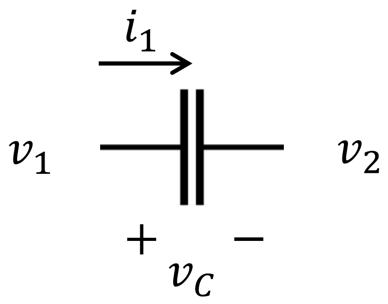
Here, $R_f/R_1 = 1$, $R_f/R_2 = 0.5$, $R_f/R_3 = 1$

If $R_f = 1\ \mathrm{k\Omega}$, $R_2 = 2\ \mathrm{k\Omega}$, $R_3 = 1\ \mathrm{k\Omega}$

Op Amp as Differentiator



Review – Capacitor



$$i_1 = \mathrm{C}\frac{dv_C}{dt} = C\frac{d(v_1 - v_2)}{dt}$$

Since v_+ is connected to ground, $v_+ = 0V$

Since there is negative feedback, from virtual short, $v_- = v_+ = 0V$

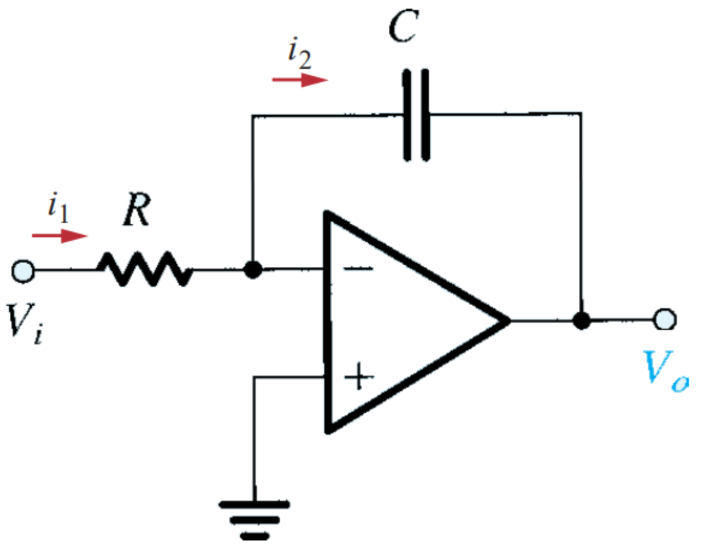
For the capacitor C, $\Rightarrow i_1 = \mathrm{C}\frac{dv_C}{dt} = C\frac{d(v_i - v_-)}{dt} = C\frac{dv_i}{dt}$

From Ohm's law for $R \Rightarrow i_2 = \frac{v_- - v_O}{R} = -\frac{v_O}{R}$

Since ideal op-amp, $i_- = i_+ = 0$, so $i_1 = i_2$

$$\Rightarrow -\frac{v_o}{R} = C\frac{dv_i}{dt} \Rightarrow v_o = -RC\frac{dv_i}{dt}$$ [Ans.]

Op Amp as Integrator



Since v_+ is connected to ground, $v_+ = 0V$

Since there is negative feedback, from virtual short, $v_- = v_+ = 0V$

From Ohm's law for $R \Rightarrow i_1 = \frac{v_i - v_-}{R} = \frac{v_i}{R}$

For the capacitor C, $\Rightarrow i_2 = C\frac{dv_C}{dt} = C\frac{d(v_- - v_o)}{dt} = -C\frac{dv_o}{dt}$

Since ideal op-amp, $i_- = i_+ = 0$, so $i_1 = i_2$

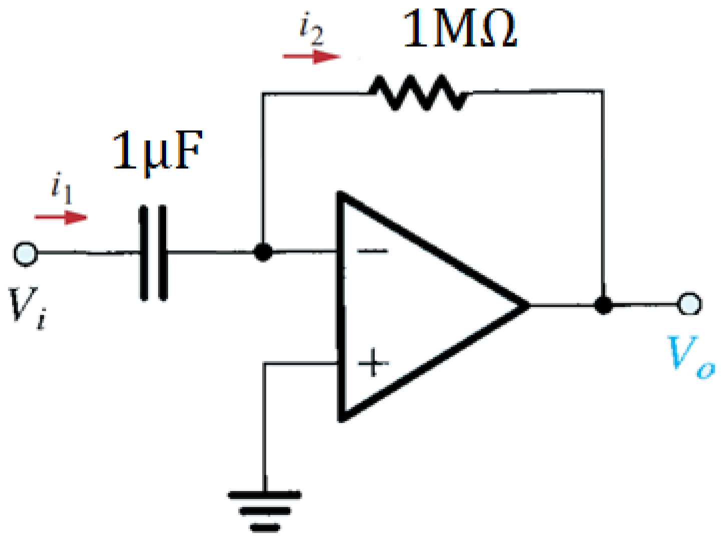
$$\Rightarrow \frac{v_i}{R} = -C\frac{dv_o}{dt}$$

$$= -RC\frac{dv_o}{dt}$$

$$\Rightarrow v_o = -\frac{1}{RC}\int v_i \, dt$$

Example

Observe the following Figure. If v_i = 5sin6t, Find the value of $\boldsymbol{v_0}$.



Solution:

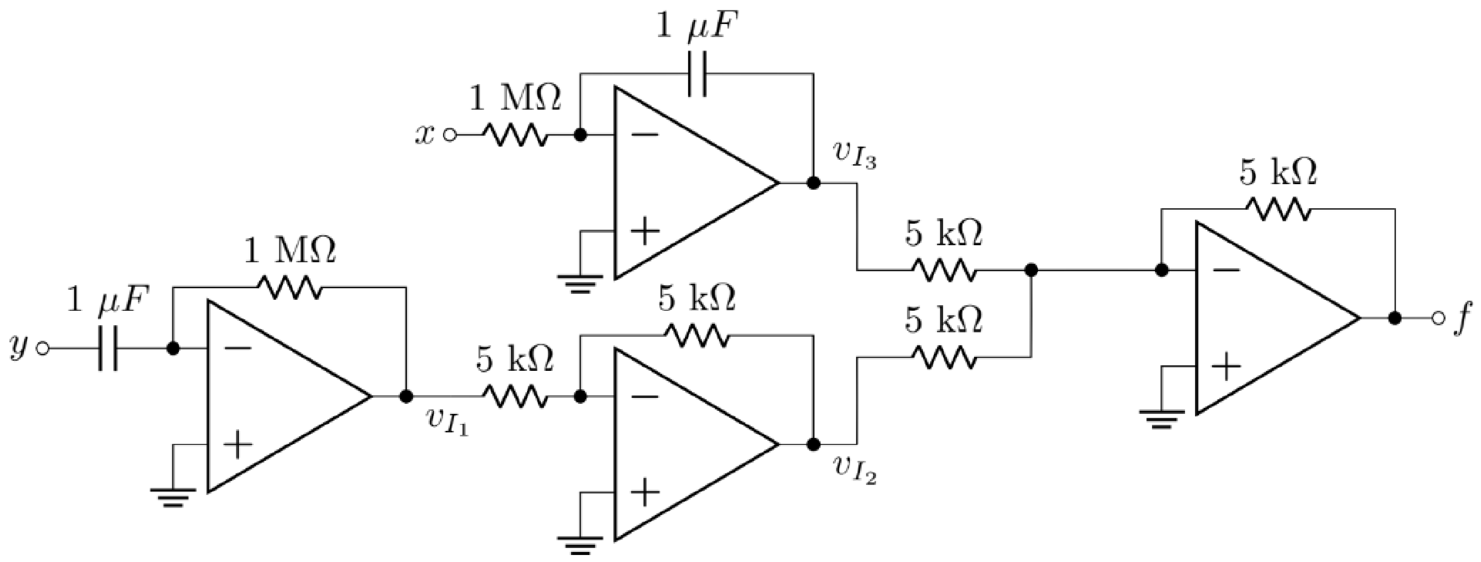
This is a **differentiator**.

$$So, v_o = -RC\frac{dv_i}{dt} = -1\times10^6\times1\times10^{-6}\times\frac{d(5\sin 6t)}{dt}$$

$$\Rightarrow v_o = -1\times(5\times 6cos6t) = -30cos6t \text{ [Ans.]}$$

Example

Analyze the circuit below to **find** an expression of f in terms of inputs x and y.



Solution:

$$v_{f1} = -\frac{dy}{dt}; v_{f2} = -\frac{1}{RC}\int x\, dt; v_{f3} = -v_{f1} = \frac{dy}{dt};\ v_o = -(v_{f2}+v_{f3})$$

Op Amp as Comparator

- A comparator compares two voltages to determine which is larger.

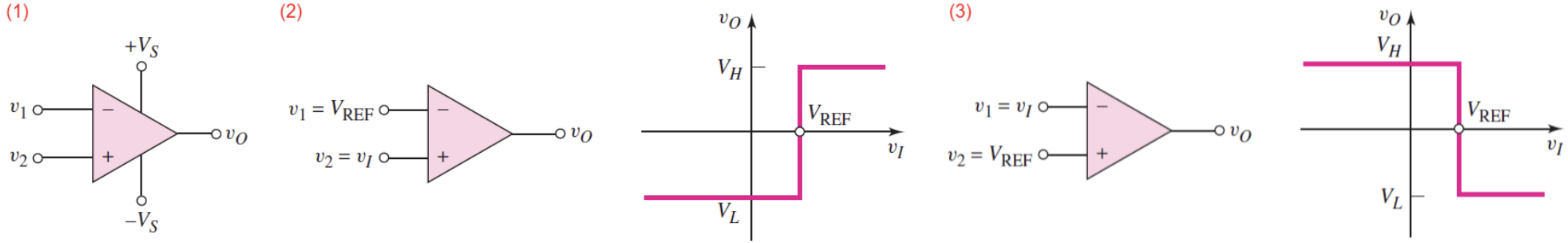
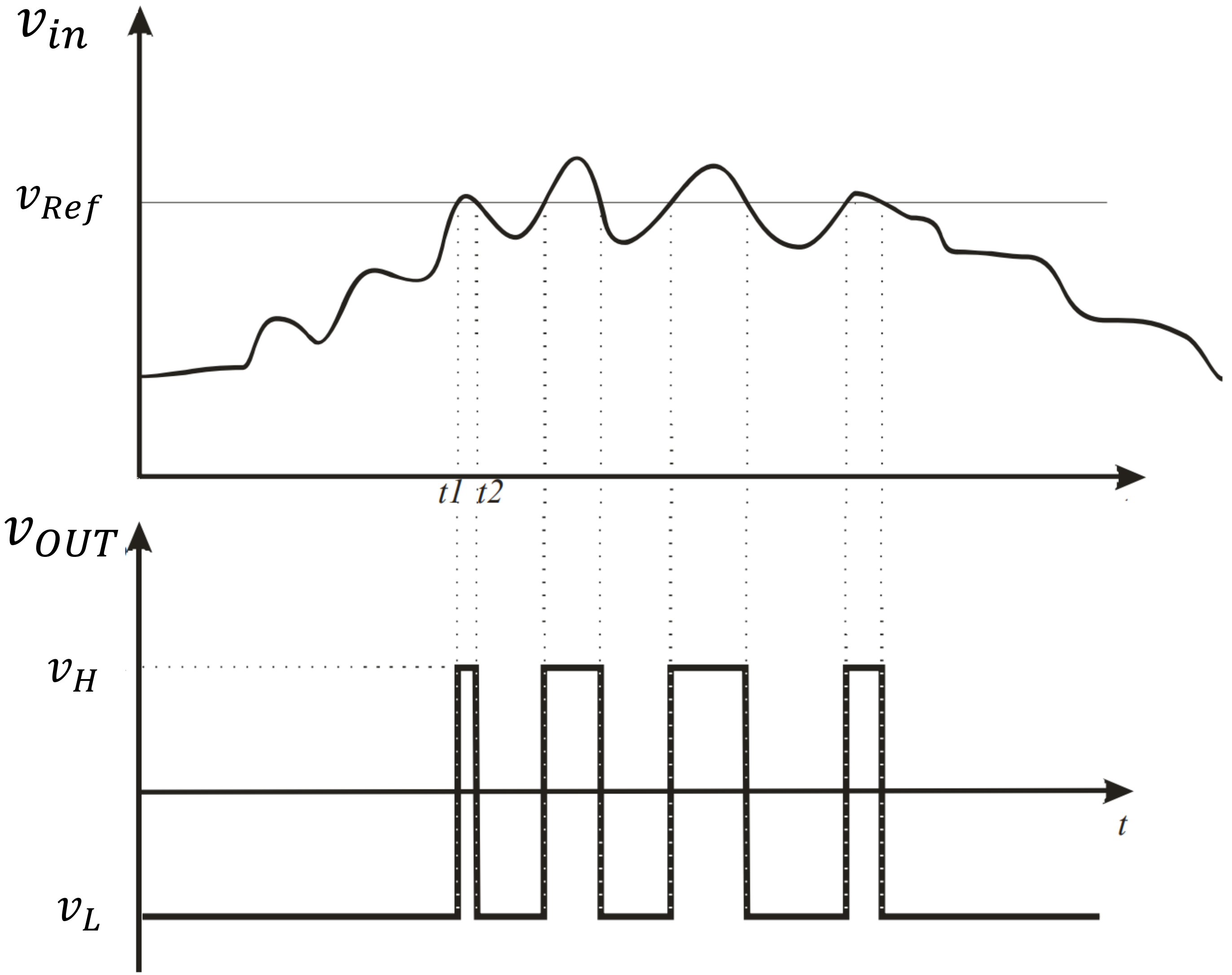


Figure 2: (1) Op-Amp Comparator (2) Noninverting Circuit (3) Inverting Circuit

What if noise enters the system?

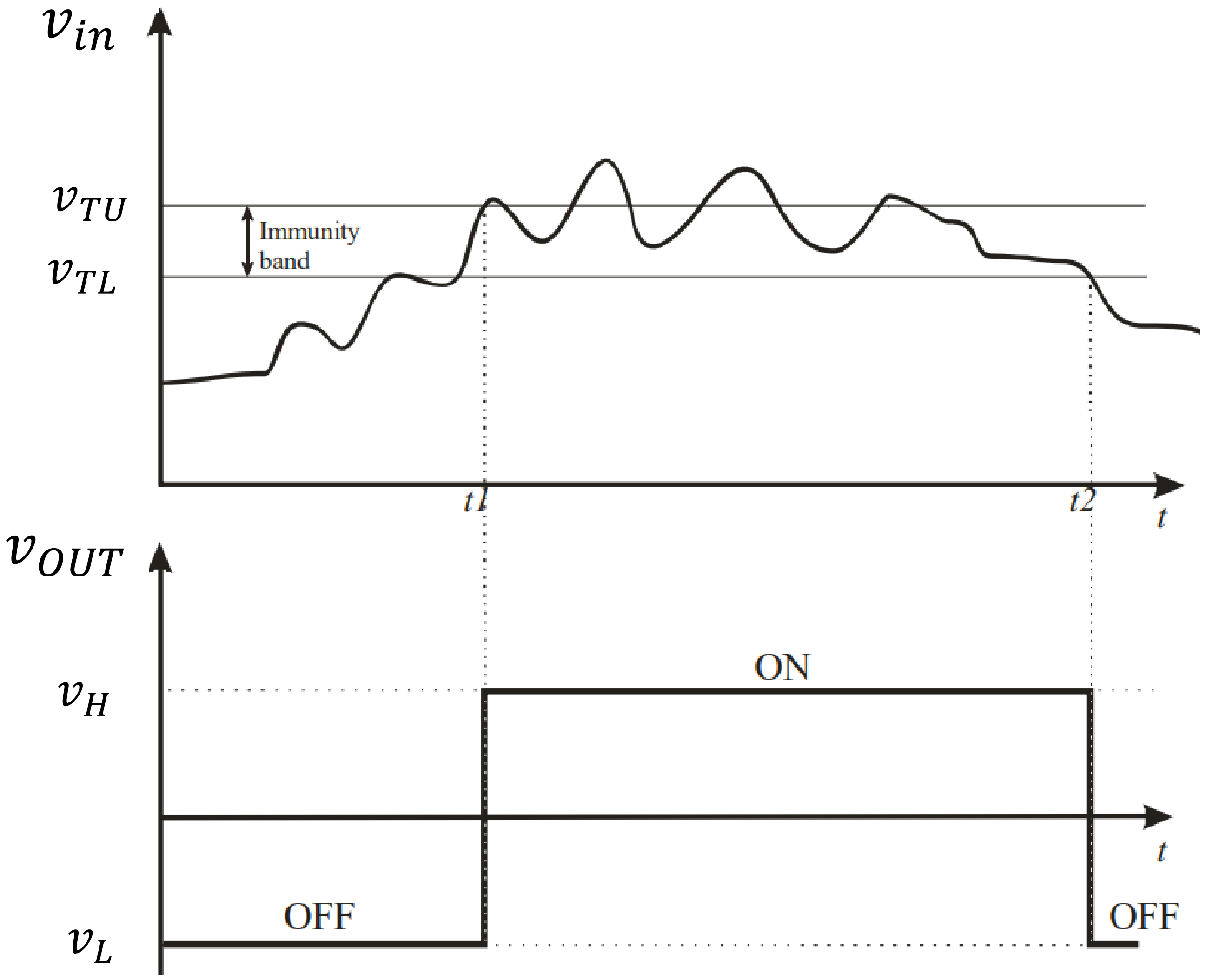
- As v_{in} exceeds v_{Ref}, the voltage at the output of the comparator (v_{OUT}) switches from v_L to v_H

- We want to make the system immune to noise.



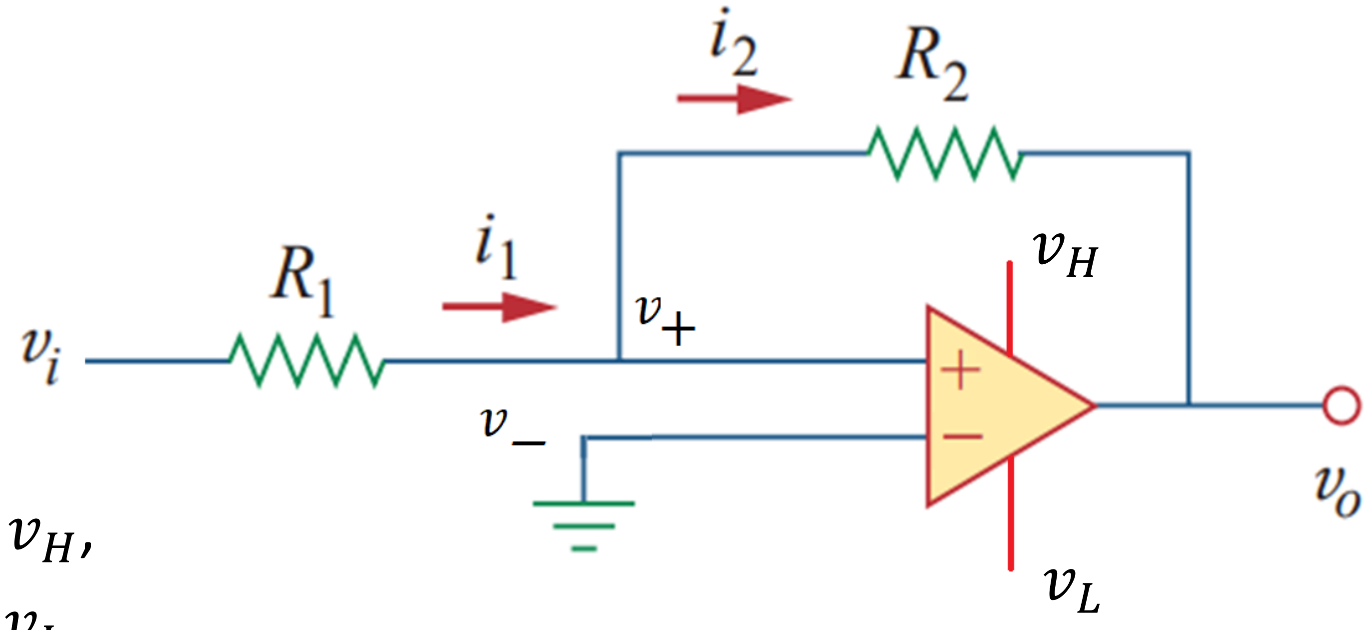
Introducing double thresholds (v_{TH} and v_{TL})

- As v_{in} exceeds v_{TU}, the voltage at the output of the comparator (v_{OUT}) switches from v_L to v_H
- As v_{in} goes below v_{TL}, the voltage at the output of the comparator (v_{OUT}) switches from v_H to v_L



Positive feedback: Schmitt Trigger (Non-inverting)

- It is not negative feedback, v_+ and v_- are not always equal.
- $i_i = 0$ and $i_+ = 0$



(1) If $v_+ > v_-$, i.e., if $v_+ > 0, v_o = v_H$,

(2) If $v_+ < v_-$, i.e., if $v_+ < 0, v_o = v_L$

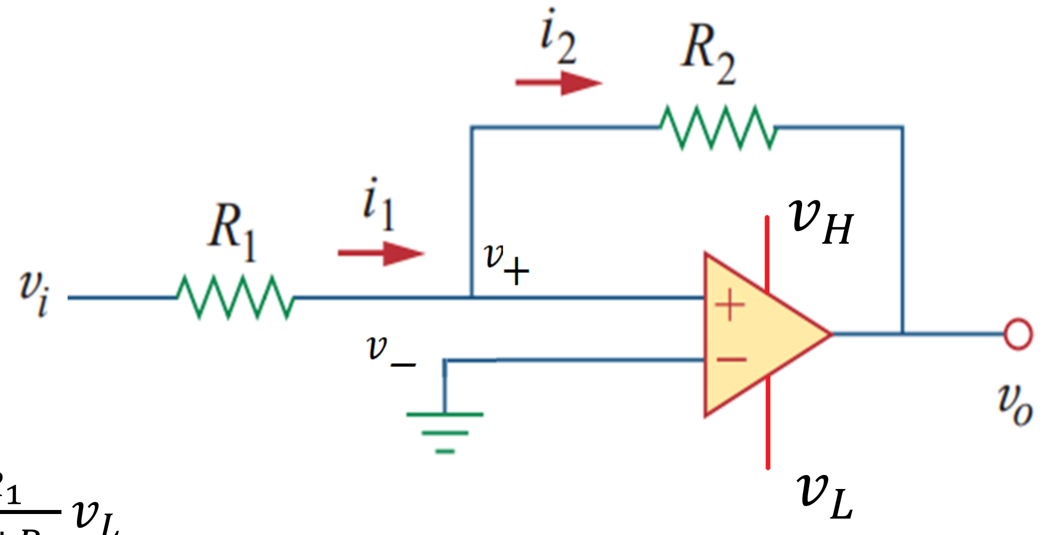
- Now v_+ is connected to v_o [positive feedback], resulting in **double threshold**

Positive feedback: Schmitt Trigger (Non-inverting)

$$i_1 = \frac{v_i - v_+}{R_1} = i_2 = \frac{v_+ - v_0}{R_2}$$

$$\Rightarrow \frac{v_i}{R_1} + \frac{v_o}{R_2} = v_+\left(\frac{1}{R_2} + \frac{1}{R_1}\right) = v_+\left(\frac{R_1+R_2}{R_2R_1}\right)$$

$$\Rightarrow v_+ = \frac{R_2}{R_1+R_2} v_i + \frac{R_1}{R_1+R_2} v_o$$



Let's consider, initially, $v_o = v_L$. So, $v_+ = \frac{R_2}{R_1+R_2} v_i + \frac{R_1}{R_1+R_2} v_L$

To have $v_o = v_H$, $v_+ > 0$ [see (1)], $So, \frac{R_2}{R_1+R_2} v_i + \frac{R_1}{R_1+R_2} v_L > 0$ $\Rightarrow v_i > -\frac{R_1}{R_2} v_L$ ⟵ **Upper threshold** (v_{TU})

Let's consider, initially, $v_o = v_H$. So, $v_+ = \frac{R_2}{R_1+R_2} v_i + \frac{R_1}{R_1+R_2} v_H$

To have $v_o = v_L$, $v_+ < 0$ [see (2)], $So, \frac{R_2}{R_1+R_2} v_i + \frac{R_1}{R_1+R_2} v_H < 0$ $\Rightarrow v_i < -\frac{R_1}{R_2} v_H$ ⟵ **Lower threshold** (v_{TL})

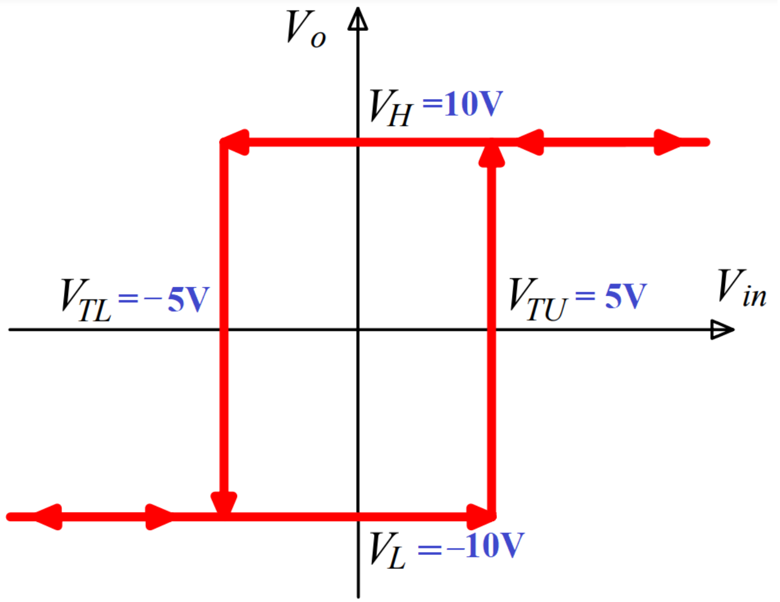
Example

If R_1=2 kΩ, R_2=2×2=4 kΩ, v_L= − 10V, v_H=10V

Calculate the threshold voltages and draw the transfer characteristics.

Upper threshold, $v_{TU} = -\frac{R_1}{R_2} v_L = -\frac{2}{4}(-10) = 5V$

Lower threshold, $v_{TL} = -\frac{R_1}{R_2} v_H = -\frac{2}{4}(+10) = -5V$

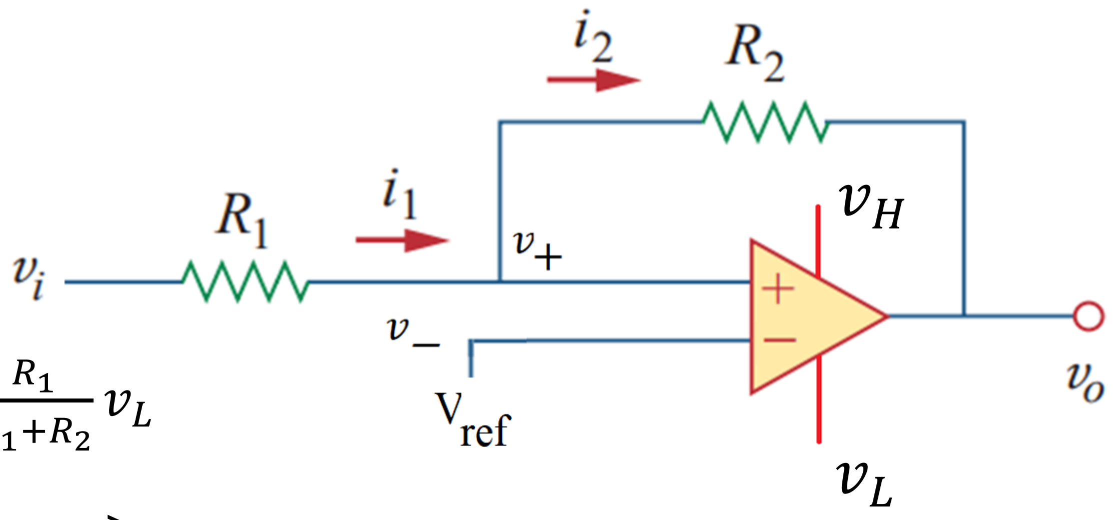


- Note that if v_o =−10V, it can't be +10V again until v_i is above $5V$ (v_{TU}).
- Similarly, if v_o =+10V, it can't be − 10V again until v_i is below$-5V$ (v_{TL})

Schmitt Trigger with Reference Voltage

(3) If $v_+ > v_-$, i.e., if $v_+ > v_{ref}, v_o = v_H$,

(4) If $v_+ < v_-$, i.e., if $v_+ < v_{ref}, v_o = v_L$



Let's consider, initially, $v_+ = v_L$. So, $v_+ = \frac{R_2}{R_1+R_2} v_i + \frac{R_1}{R_1+R_2} v_L$

To have $v_o = v_H$, $v_+ > v_{ref}$ [see (3)], So, $\frac{R_2}{R_1+R_2} v_i + \frac{R_1}{R_1+R_2} v_L > v_{ref}$

$$\Rightarrow v_i > -\frac{R_1}{R_2} v_L + v_{ref} \frac{R_1+R_2}{R_2}$$

$\longleftarrow$ **Upper threshold** (v_{TU})

Let's consider, initially, $v_+ = v_H$. So, $v_+ = \frac{R_2}{R_1+R_2} v_i + \frac{R_1}{R_1+R_2} v_H$

To have $v_o = v_L$, $v_+ < v_{ref}$ [see (4)], So, $\frac{R_2}{R_1+R_2} v_i + \frac{R_1}{R_1+R_2} v_H < v_{ref}$

$$\Rightarrow v_i < -\frac{R_1}{R_2} v_H + v_{ref} \frac{R_1+R_2}{R_2}$$

$\longleftarrow$ **Lower threshold** (v_{TL})

Example

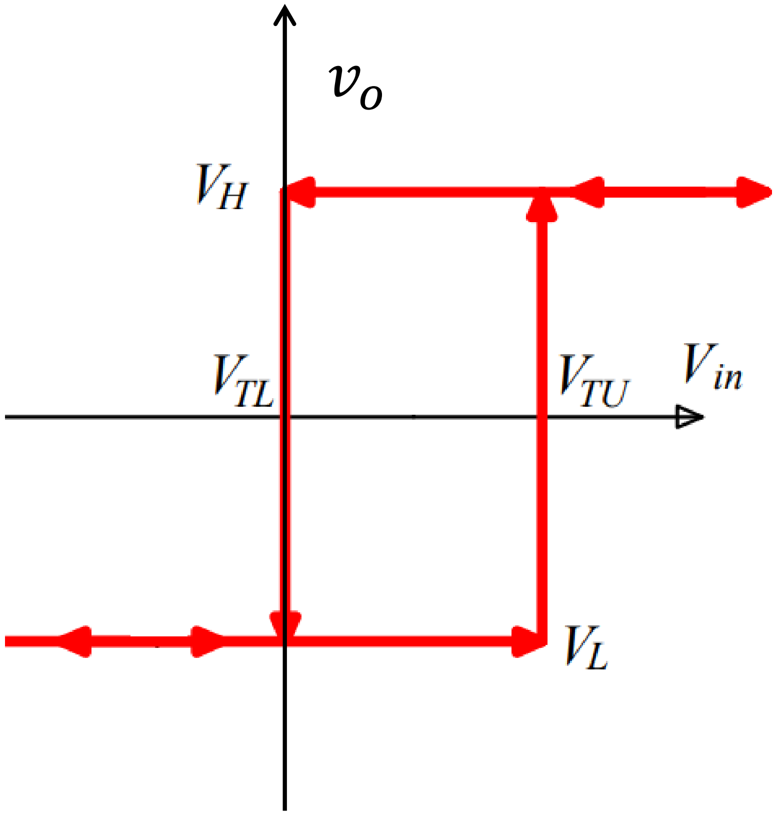
If R_1=2 kΩ, R_2=3×2=6 kΩ, v_L= − 10V, v_H=10V, v_{ref} = 2.5V

Calculate the threshold voltages and draw the transfer characteristics.

Upper threshold, $v_{TU} = -\frac{R_1}{R_2} v_L + v_{ref} \frac{R_1+R_2}{R_2}$ = 6.67 V

Lower threshold, $v_{TL} = -\frac{R_1}{R_2} v_H + v_{ref} \frac{R_1+R_2}{R_2}$ = 0 V

- Note that if v_o =−10V, it can't be +10V again until v_i is above 6.67V (v_{TU}).
- Similarly, if v_o =+10V, it can't be − 10V again until v_i is below $0V$ (v_{TL})



How can we utilize this circuit for **water level detection**?