

SOP & POS

- **Sum-of-Products (SOP) Expression:** a product term or a logical sum (OR) of several product terms.
Examples: x , $x+yz'$, $xy'+x'yz$, $AB+A'B'$
- **Product-of-Sums (POS) Expression:** a sum term or a logical product (AND) of several sum terms.
Examples: x , $x(y+z')$, $(x+y')(x'+y+z)$, $(A+B)(A'+B')$
- Every boolean expression can either be expressed as sum-of-products or product-of-sums expression.

Examples:

SOP: $x'y + xy' + xyz$

POS: $(x + y')(x' + y)(x' + z')$

MIN & MAX TERM

- Minterms are sum terms.
- For Boolean functions, the minterms of a function are the terms for which the result is 1.
- Boolean functions can be expressed as sum of-Minterms
- Maxterms are Product terms.
- For Boolean functions, the maxterms of a function are the terms for which the result is 0.
- Boolean functions can be expressed as Products-of-Maxterms.

	A	B	C	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

Min Terms : 0,1,4,5

[000,001,100,101]

$F = \sum(0,1,4,5)$

Max Terms : 2,3,6,7

[010,011,110,111]

$F = \prod(2,3,6,7)$

MIN-SOP and MAX-POS

Minterms				Maxterms	
x	y	term	notation	term	notation
0	0	$x'y'$	m0	$x+y$	M0
0	1	$x'y$	m1	$x+y'$	M1
1	0	xy'	m2	$x'+y$	M2
1	1	xy	m3	$x'+y'$	M3

Each minterm is the **complement** of the corresponding maxterm:

Example: $m_2 = xy'$ $m_2' = (xy')' = x' + (y')' = x' + y = M_2$

	A	B	C	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

Min Terms : 0,1,4,5

[000,001,100,101]

$F = \sum(0,1,4,5)$

$F = A'B'C' + A'B'C + AB'C' + AB'C$
 $(A' + B' + C')$

Max Terms : 2,3,6,7

[010,011,110,111]

$F = \prod(2,3,6,7)$

$F = (A + B' + C)(A + B' + C')(A' + B' + C)$

Conversion between MIN & MAX

$$F2 = \sum(m1, m4, m5, m6, m7)$$

x	y	z	F2	F2'
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

The complement function of F2 is:

$$F2' = \sum(m0, m2, m3) = m0 + m2 + m3$$

$$F2 = (m0 + m2 + m3)'$$

$$= m0' \cdot m2' \cdot m3'$$

$$= M0 \cdot M2 \cdot M3$$

$$= \prod(M0, M2, M3)$$

Every Boolean function can be expressed as either Sum-of-Minterms or Product-of-Maxterms.

Simplified Function to SOP & POS

How to Convert into SOP:

Check if each term contains all variable, if not then AND ($x+x'$) if x is the missing term

Simplified Function, $F=A+B'C$

$$\begin{aligned} F &= A+B'C \\ &= A(B+B')(C+C') + B'C(A+A') \\ &= (AB+AB')(C+C') + B'C(A+A') \\ &= AB(C+C') + AB'(C+C') + B'C(A+A') \\ &= ABC+ABC' + AB'C+AB'C' + AB'C+A'B'C \\ &= ABC+ABC' + AB'C+AB'C' + A'B'C \\ &= \sum(1,4,5,6,7) \end{aligned}$$

How to Convert into POS:

*1. Often distributive law ($x+yz$)= $(x+y)(x+z)$) is used
2. If then terms, like x , are missing, OR xx'
3. Each POS is missing a term so OR missing terms
Again applying distributive law*

Simplified Function, $F=A+B'C$

$$\begin{aligned} F &= A+B'C \\ &= (A+B')(A+C) \\ &= (A+B'+CC')(A+BB'+C) \\ &= \{(A+B')+C\}\{(A+B')+C'\}\{(A+C)+B\}\{(A+C)+B'\} \\ &= (A+B'+C)(A+B'+C')(A+B+C)(A+B'+C) \\ &= (A+B'+C)(A+B'+C')(A+B+C) \\ &= \square(2,3,0) \\ &= \square(0,2,3) \end{aligned}$$

