CSE260
Digital Logic Design

SOP & POS

- **Sum-of-Products (SOP) Expression**: a product term or a logical sum (OR) of several product terms.

  Examples: x+yz', xy'+x'yz, AB+A'B’

  Product-of-Sums (POS) Expression: a sum term or a logical product (AND) of several sum terms.

  Examples: x(y+z'), (x+y')(x'+y+z), (A+B)(A'+B’)

- Every boolean expression can either be expressed as sum-of-products or product-of-sums expression.

  Examples:

  SOP: $x'y + xy' + xyz$

  POS: $(x + y')(x' + y)(x' + z')$

MIN & MAX TERM

Minterms are sum terms.

For Boolean functions, the minterms of a function are the terms for which the result is 1.

Boolean functions can be expressed as sum-of-Minterms.

Maxterms are Product terms.

For Boolean functions, the maxterms of a function are the terms for which the result is 0.

Boolean functions can be expressed as Products-of-Maxterms.

MIN and MAX

	A	B	C	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

Min Terms : 0,1,4,5
[000,001,100,101]
$F=\sum(0,1,4,5)$

Max Terms : 2,3,6,7
[010,011,110,111]
$F=\prod(2,3,6,7)$

MIN-SOP and MAX-POS

Minterms				Maxterms	
x	y	term	notation	term	notation
0	0	x'y'	m0	x+y	M0
0	1	x'y	m1	x+y'	M1
1	0	xy'	m2	x'+y	M2
1	1	xy	m3	x'+y'	M3

Each minterm is the complement of the corresponding maxterm:

Example: m2 = xy'

m2' = (xy')' = x' + (y')' = x'+y = M2

MIN-SOP and MAX-POS

	A	B	C	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

Min Terms : 0,1,4,5 [000,001,100,101]

$F=\sum(0,1,4,5)$

$F=A'B'C'+A'B'C+AB'C'+AB'C$

Max Terms : 2,3,6,7 [010,011,110,111]

$F=\prod(2,3,6,7)$

$F=(A+B'+C)(A+B'+C')(A'+B'+C)\ (A'+B'+C')$

Conversion between MIN & MAX

$F2 = \Sigma(m1,m4,m5,m6,m7)$

- The complement function of F2 is:

$$F2' = \Sigma(m0,m2,m3) = m0 + m2 + m3$$

$F2 = (m0 + m2 + m3)'$

$= \mathbf{m0' . m2' . m3'}$

$= \mathbf{M0 . M2 . M3}$

$= \Pi(M0,M2,M3)$

x	y	z	F2	F2'
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Every Boolean function can be expressed as either Sum-of-Minterms or Product-of-Maxterms.

Simplified Function to SOP & POS

How to Covert into SOP:

Check if each term contains all variable, if not then AND $(x+x')$ if x is the missing term

Simplified Function, $F=A+B'C$

How To Convert into SOP

- $F=A+B'C$
- $=A(B+B')(C+C')+B'C(A+A')$
- $=(AB+AB')(C+C')+B'C(A+A')$
- $=AB(C+C')+AB'(C+C')+B'C(A+A')$
- $=ABC+ABC'+\boldsymbol{AB'C}+AB'C'+\boldsymbol{AB'C}+A'B'C$
- $=ABC+ABC'+AB'C+AB'C'+A'B'C$
  $=\sum(1,4,5,6,7)$

How to Covert into POS:

1. Often distributive law $(x+yz)=(x+y)(x+z))$ is used

2. If then terms, like x, are missing, OR xx'

3. Each POS is missing a term so OR missing terms
Again applying distributive law

Simplified Function, $F=A+B'C$

How to Convert into POS

$$A + B'C$$

$$= (A+B')(A+C)$$

$$= (A+B'+CC')(A+BB'+C)$$

$$= \{(A+B')+C\}\{(A+B')+C'\}\{(A+C)+B\}\{(A+C)+B'\}$$

$$= (A+B'+C)(A+B'+C')(A+C+B)(A+C+B')$$

$$= (\underset{0}{A}+\underset{1}{B'}+\underset{0}{C})(\underset{0}{A}+\underset{1}{B'}+\underset{1}{C'})(\underset{0}{A}+\underset{0}{B}+\underset{0}{C})$$

$$= \pi(2,3,0)$$

$$= \pi(0,2,3)$$

Thanks