Midterm Exam
Total marks: 40
Duration: 70 minutes CSE331

There are a total of four problems. You have to solve them all.

Problem 1: Finite Automata and the Regular Operations (10 points)

Let $\Sigma = \{\texttt{0}, \texttt{1}, \texttt{\#}\}$. Consider the following two languages.

$$L_1 = \{w \in \Sigma^* : w \text{ does not contain } \texttt{\#} \text{ and the number of } \texttt{0}\text{s in } w \text{ is not a multiple of } 3\}$$

$$L_2 = \{w \in \Sigma^* : \text{the substring between any two successive occurrences of } \texttt{\#}\text{s in } w \text{ is in } L_1\}$$

Now solve the following problems.

(a) Write down a string $w \in L_2$ such that the length of w is ten. (1 point)

(b) Give the state diagram for a DFA that recognizes L_1. (4 points)

(c) Give the state diagram for a DFA that recognizes L_2. (3 points)

(d) If you use the "cross product" construction shown in class to obtain a DFA for $L_1 \cap L_2$, how many states will it have? (1 point)

(e) Give an upper bound on the number of states in the smallest DFA that recognizes $L_1 \cap L_2$. (1 point)

Problem 2: Regular Expressions (10 points)

Mike and Willy recently learned how to write regular expressions. Mike wrote the regular expression $\texttt{10}^*\texttt{1}^*$ for a language L_1 on the board and Willy wrote the regular expression $\texttt{1}^*\texttt{01}^*$ for another language L_2 below that.

(a) Write down a string that is present in the language L_1 but not in the language L_2. (2 points)

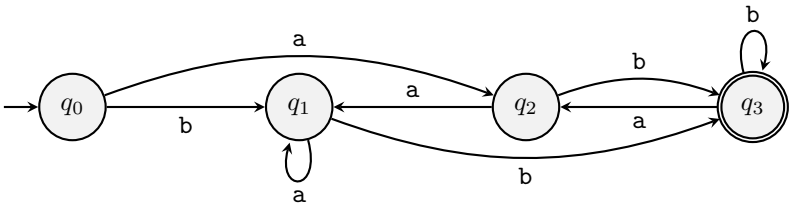
(b) Write down a string that is not present in the language L_1 but present in the language L_2. (2 points)

(c) Write down a string that is neither present in the language L_1 nor in the language L_2. (2 points)

(d) Mike and Willy asked their friend Dustin to write a regular expression for the language $L_1 \cap L_2$. Dustin came up with $\texttt{1}^*\texttt{0}^*\texttt{1}^*$. Is Dustin's regular expression correct? If you think it's not correct, then write down a correct regular expression for $L_1 \cap L_2$. (4 points)

Problem 3: Converting DFAs to Regular Expressions (10 points)

Convert the following DFA into an equivalent regular expression using the state elimination method. First eliminate q_1, then q_2 and so on. You must show work.



Problem 4: Converting Regular Expressions to NFAs (10 points)

Convert the following regular expression into an equivalent NFA.

$$(\texttt{a}^*\texttt{b}^* + (\texttt{ac} + \texttt{b}^*\texttt{c})\texttt{b})^*$$