CSE230 ASSIGNMENT 1

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SECTION: 3

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CSE 230 SPRING'22 ASSIGNMENT 01

SET & FUNCTION:

(1)(a) $A = \{3,4,5,6\}$, $B = \{5,6,7,8\}$, $C = \{3,4,5,10\}$

$(B \cap C) = \{5\}$
$A \cup (B \cap C) = \{3,4,5,6\}$

$(A \cup B) = \{3,4,5,6,7,8\}$
$(A \cup C) = \{3,4,5,6,10\}$
$(A \cup B) \cap (A \cup C) = \{3,4,5,6\}$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) = \{3,4,5,6\}$ (shown)

(b) $P \subset Q$ and $R \subset S$
for example: $(x,y) \in P \times R$
$\Rightarrow x \in P$ and $y \in R$
$\Rightarrow x \in Q$ and $y \in S$ $\Big\}$ $P \subset Q$ and $R \subset S$
$\Rightarrow (x,y) \in Q \times S$
$P \times R \subset Q \times S$ (proved)

(2) $A \times B = \{(7,a),(7,b),(8,a),(8,b)\}$
Powerset of $AB = \{(7,a)\},\{(7,b)\} \{(8,a)\},\{(8,b)\},$
$\{(7,a),(7,b)\},\{(8,a),(8,b)\},\{(7,a),(8,a)\},\{(7,a),(8,b)\}$
$\{(7,b),(8,a)\},\{(7,b),(8,b)\},\{(7,a),(7,b),(8,a)\},\{(7,a),(8,a),(8,b)\},$
$\{(7,b),(8,a),(8,b)\},\{(8,a),(8,b),(7,a)\},\{(7,a),(7,b),(8,a),(8,b)\},$
$\{\ \}$ (Ans)

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(3) ~~U~~ Total = 100

P = 20

Q = 45

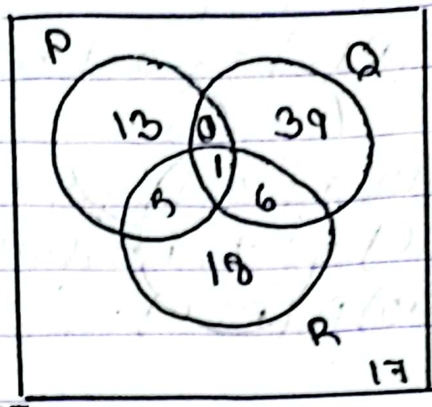
R = 30

P and Q = 1

Q and R = 6

R and P = 3

all three = 1



U = 100

(i) only Q = 39 (Ans)

(ii) do not specialise in any of the machines

= 100 - (13+1+0+3+39+6+18)

= 100 - ~~8~~ ~~20~~ 82 = 18 (Ans)

~~= 18 (Ans)~~

(-1) (i) $f(x) = 12 + \log(1-x^2)$

$12 + \log(1-x^2) \geq 0$

$\log(1-x^2) \geq -12$

$1-x^2 \geq 10^{-12}$

$-x^2 \geq 10^{-12} - 1$

$-x^2 \geq -1$

$x^2 \leq 1$

$x \leq 1, x \geq -1$ Domain: $\cancel{F}$ $(-1,1)$ (Ans)

~~Domain: $[-1,1]$ (Ans)~~ ~~Domain: $(-\infty,1) \cup (1,\infty)$ (Ans)~~

$f(x) = y$

$y = 12 + \log(1-x^2)$

$x = 12 + \log(1-y^2)$

$x - 12 = \log(1-y^2)$

$1-y^2 = 10^{x-12}$

$-y^2 = -1 + 10^{x-12}$

$y^2 = 1 - 10^{x-12}$

$y = \sqrt{1-10^{x-12}}$

$\sqrt{1-10^{x-12}} \geq \cancel{0}1$

$1 - 10^{x-12} \geq \cancel{0}1$

$10^{x-12} \leq \cancel{0}1$

$(x-12)\ln|10| \leq \ln|\cancel{0}1|$

$x - 12 \leq 0$

$x \leq 12$ Domain of $f^{-1}(x)$: $(-\infty, 12]$

Range of $f(x)$: $\cancel{[+2,}$ $(-\infty, 12]$ (Ans)

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(ii) $f(x) = \dfrac{x^2+1}{x^2+5}$

$$\begin{array}{r|l|l} x^2+5 & x^2+1 & 1 \\ & x^2+5 & \\ \hline & -4 & \end{array}$$

$1 - \dfrac{4}{x^2+5} = f(x)$

$x^2+5 \neq 0$

$x^2 \neq -5, \; x \in \mathbb{R}$

Domain : $\{(-\infty, \infty)$ (Ans)

$f(x) = y$

$y = \dfrac{x^2+1}{x^2+5}$

$x = \dfrac{y^2+1}{y^2+5}$

$x = 1 - \dfrac{4}{y^2+5}$

$x - 1 = -\dfrac{4}{y^2+5}$

$1 - x = \dfrac{4}{y^2+5}$

$y^2 = \dfrac{4}{1-x} - 5$

$y = \sqrt{\dfrac{4}{1-x} - 5}$

$= \sqrt{\dfrac{5x-1}{1-x}}$

$\cancel{\sqrt{\frac{5x-1}{1-x}}} \geq 0$

$\frac{5x-1}{\cancel{1-x}} \geq 0$

$5x-1 \geq 0$

$x \geq \frac{1}{5}$

$y \geq \frac{1}{5}$

$\cancel{\sqrt{1-x}} \geq 0$

$1-x \geq 0$

$x \leq 1$

$y < 1$

Domain of $f^{-1}(x)$: $[\frac{1}{5}, 1)$

Range of $f(x)$: ~~$[\frac{1}{5}, +\infty)$ (Ans)~~ $[\frac{1}{5}, 1)$ (Ans)

(5) (i) $f(x) = x^5 + 5$

$x_1^5 + \cancel{5} = x_2^5 + \cancel{5}$

$x_1^5 = x_2^5$

$x_1 = x_2$

$\therefore f(x)$ is injective.

~~Range: $f(x) \in R$~~

~~Codomain: R~~

~~Range = Codomain~~

~~$\therefore f(x)$ is surjective.~~

~~$f(x) = y$~~

~~$y = x^5 + 5$~~

~~$x = y^5 + 5$~~

~~$y = \sqrt[5]{x-5}$~~

~~$\sqrt[5]{x-5} \geq 0$~~

~~$x \geq 5$~~

~~Range of $f(x)$: $R - \{5\}$~~

~~$\therefore$ Range $\neq$ Codomain $\therefore f(x)$ is not surjective~~

$y = x^5 + 5$

$x = (y-5)^{\frac{1}{5}}$

$f((y-5)^{\frac{1}{5}}) = x^5 + 5 = ((y-5)^{\frac{1}{5}})^5 + 5$

$= y$

$\therefore f(x)$ is surjective.

Since $f(x)$ is both injective and surjective, it is bijective.

(ii) $f(x) = e^{4x}$

$e^{4x_1} = e^{4x_2}$

$\ln e^{4x_1} = \ln e^{4x_2}$

$4x_1 = 4x_2$

$x_1 = x_2$

$\therefore f(x)$ is injective

Codomain : $\mathbb{R}$

$y = e^{4x}$

$x = \frac{\ln y}{4}$

$y = \frac{\ln x}{4}$

$\ln x \geq 0 \ \forall \ x \geq 1$

Range of $f(x)$ = ~~$\mathbb{R}$ [1, ∞)~~

$= y \geq 1$

$f(x) = y$

$y = e^{4x}$

$x = e^{4y}$

$\ln x = 4y$

$y = \frac{1}{4} \ln x$

Range of $f(x) = \mathbb{R}$

Codomain of $f(x) = \mathbb{R}$

Range = Codomain

$\therefore f(x)$ is surjective

$y = e^{4x}$

$\ln y = 4x$

$x = \frac{\ln y}{4}$

$f\left(\frac{\ln y}{4}\right) = e^{4x} = e^{\left(\frac{\ln y}{4}\right)^4}$

$= y^4$

$\therefore f(x)$ is not surjective

Range $\neq$ Codomain

$\therefore f(x)$ is not surjective.

$f(x)$ is injective and not surjective.

$\therefore f(x)$ is not bijective.

(b)
$$f(x) = \begin{cases} x+2 & \text{if } x > 1 \\ 2 & \text{if } -1 \le x \le 1 \\ x-1 & \text{if } -3 < x < -1 \end{cases}$$

$g(x) = x^2 + 7$

(i) $(g \circ f)(-2)$

$(g \circ f)(-2) = g(x-1)$
$= (x-1)^2 + 7$

$(g \circ f)(-2) = (-2-1)^2 + 7$
$= 9 + 7$
$= 16$ (Ans)

#

(ii) $(f \circ f)(0) + (g \circ g)(1)$

$(f \circ f) = 2$

$(g \circ g) = g(x^2+7)$
$= (x^2+7)^2 + 7$
$= x^4 + 14x^2 + 49 + 7$
$= x^4 + 14x^2 + 56$

$(g \circ g)(1) = 1^4 + 14(1)^2 + 56$
$= 71$

$(f \circ f)(0) + (g \circ g)(1) = 2 + 71$
$= 73$ (Ans)

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COMBINATORICS & BINOMIAL COEFFICIENTS

(7) $(n-1)! = \cancel{a!} \rightarrow$ for all people
$\cancel{(a-1)! = b!}$ $(n-1-1)! = (n-2)! \rightarrow$ for Jim ~~one person in p~~
$\cancel{b \times 2}$ $\cancel{b! \times 2} = (n-1-1)! \times 2 \longrightarrow$ for Julie
$= (n-2)! \times 2$
$= 2(n-2)!$ (Ans)

(8) ways Jim and Julie can seat together $= 2(n-2)!$
ways Jim and Julie cannot seat together $= n - 2(n-2)!$ (Ans)

(9) ~~Boy~~ ~~the~~ different ways boys can stand $= B!$
different ways girls can stand side by side to boys
$= {}^{B+1}C_G$
different ways girls can stand $= G!$
Total $= B! \times {}^{B+1}C_G \times G!$ (Ans)

(10) no ~~case~~. 2 consecutive '0''s.
String length $= K$, Total string $= 2^K$
string length with ~~no~~ 2 consecutive '0''s $= K-2$
Total element in the string $= (K-1)$
Total bad string $= \cancel{(K-1) \times 2^K} = \cancel{(K-1) \times 2^{K-2}}$ $(K-1) \times 2^{K-2}$
Total good string $=$ Total string $-$ bad string
$= 2^K - (K-1) \times 2^{K-2}$ (Ans)

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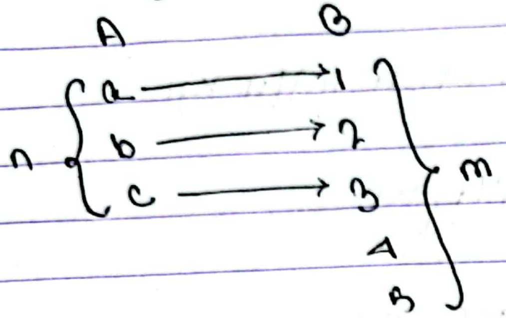
(11) $|A| = n \leq m = |B|$

if $A = \{a, b, c\}$, $B = \{1, 2, 3, 4, 5\}$



a will have 5 ways, b will have 4 ways and c will will have 3 ways. It will be $5! \times 4! \times 3!$

Therefore,

$$\underline{\underline{{}^nP_m}}$$

(12) $^nC_0 + {}^nC_2 + {}^nC_4 + \dots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots$

$^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + {}^nC_4 - {}^nC_5 = 0$

$(x+1)^n = {}^nC_0 x^n 1^0 + {}^nC_1 x^{n-1} 1^1 + {}^nC_2 x^{n-2} 1^2 + {}^nC_3 x^{n-3} 1^3 + {}^nC_4 x^{n-4} 1^4$
$+ {}^nC_5 x^{n-5} 1^5 + \dots {}^nC_n x^0 1^n$

$(-1+1)^n = {}^nC_0 (-1)^n 1^0 +$

$(1+x)^n = {}^nC_0 (1)^n x^0 + {}^nC_1 (1)^{n-1} x^1 + {}^nC_2 (1)^{n-2} x^2 + {}^nC_3 (1)^{n-3} x^3 + {}^nC_4 (1)^{n-4} x^4$
$+ {}^nC_5 (1)^{n-5} x^5 + \dots {}^nC_n (1)^0 x^n$

$(-1+1)^n = {}^nC_0 (1)^n (-1)^0 + {}^nC_1 (1)^{n-1} (-1)^1 + {}^nC_2 (1)^{n-2} (-1)^2 + {}^nC_3 (1)^{n-3} (-1)^3$
$+ {}^nC_4 (1)^{n-4} (-1)^4 + {}^nC_5 (1)^{n-5} (-1)^5 + \dots {}^nC_n (1)^0 (-1)^n$

$0 = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + {}^nC_4 - {}^nC_5$ (proved)

(13) ${}^nC_m \times {}^mC_K = {}^nC_K \times {}^{(n-K)}C_{(m-K)}$

$${}^nC_m \times {}^mC_K = \frac{n!}{(n-m)!\,m!} \times \frac{m!}{(m-K)!\,K!}$$

$$= \frac{n!}{K!} \times \frac{1}{(n-m)!\,(m-K)!}$$

$$= \frac{n!}{(n-K)!\,K!} \times \frac{(n-K)!}{(n-K-(m-K))!\,\cancel{(n-m)!}(m-K)!}$$

$$= {}^nC_K \times \frac{(n-K) \to n}{(n-K-(m-K))!\,(m-K)!}$$

${}^nC_m \times {}^mC_K = {}^nC_K \times {}^{n-K}C_{m-K}$ (proved)

(14) $\sum_{K=1}^{n} K \times {}^nC_K = n \times 2^{n-1}$ $\qquad a \le b,\ \frac{a}{b} \times {}^bC_a = {}^{(b-1)}C_{(a-1)}$

$$\Rightarrow \sum_{K=1}^{n} \frac{K}{n} \times {}^nC_K \times n$$

$$\Rightarrow n \sum_{K=1}^{n} \frac{K}{n} \times {}^nC_K$$

$$\Rightarrow n \sum_{K=1}^{n} \cancel{(n-1)}\,{}^{(n-1)}C_{(K-1)}$$

$$\Rightarrow n \sum_{K=1}^{n} \left({}^{n-1}C_{(1-1)} + {}^{n-1}C_{(2-1)} + {}^{n-1}C_{(3-1)} + \dots {}^{n-1}C_{(n-1)}\right)$$

$$\Rightarrow n \sum_{K=1}^{n} \left({}^{n-1}C_0 + {}^{n-1}C_1 + {}^{n-1}C_2 + \dots {}^{n-1}C_{n-1}\right)$$

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$\Rightarrow n\sum_{K=1}^{n}\left({}^{n-1}C_0+{}^{n-1}C_1+{}^{n-1}C_2+\dots 1\right)$ $[{}^{n}C_n=1]$

$\Rightarrow n\sum_{K=1}^{n}\left(\frac{(n-1)!}{(n-1)!\,0!}+\frac{(n-1)!}{(n-1-1)!\,1!}+\frac{(n-1)!}{(n-1-2)!\,2!}+\dots 1\right)$

$\Rightarrow n\sum_{K=1}^{n}\left(1+\frac{(n-1)!}{(n-2)!\,1!}+\frac{(n-1)!}{(n-3)!\,2!}+\dots 1\right)$

$\Rightarrow$ Since 1st value is 1 and last value is 1 (appears twice)

$\Rightarrow n\sum_{K=1}^{n}\left(1+\frac{(n-1)!}{(n-2)!\,1!}+\frac{(n-1)!}{(n-3)!\,2!}+\dots+\frac{(n-1)!}{1!\,(n-1)!}+\frac{(n-1)!}{2!\,(n-3)!}+1\right)$

$\Rightarrow n\sum_{K=1}^{n} 2^{n-1}$ $[2^k=2^{n-1}]$

$\Rightarrow \sum_{K=1}^{n} n\times 2^{n-1}$ (proved)

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