CSE230 Spring '22 Assignment 01

Set & Function:

1. **a)** If $A = \{3, 4, 5, 6\}$, $B = \{5, 6, 7, 8\}$ and $C = \{3, 4, 5, 10\}$
   Show that, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   b) Prove that, if $P \subset Q$ and $R \subset S$, then $(P \times R) \subset (Q \times S)$

2. Determine the **Power set** of $A \times B$ if $A = \{7, 8\}$ and $B = \{a, b\}$

3. A large logistic manufacturing company has 100 employees specializing in different machines. From them, 20 are proficient in machine P , 45 in machine Q , 30 in machine R , 1 in machine P and Q , 6 in machine Q and R , 5 in machine R and P , and just 1 specialist in all three machines.
   i) Determine how many of them only specialize in machine Q.
   ii) And how many of them do not specialize in any of the machines.

4. Find the Domain and Range of the following functions if $f: R \rightarrow R$
   i) $f(x) = 12 + log\,(1 - x^2)$
   ii) $f(x) = \frac{x^2+1}{x^2+5}$

5. Find if the functions are surjective, injective and bijective if $f: R \rightarrow R$
   i) $f(x) = x^5 + 5$
   ii) $f(x) = e^{4x}$

6. Find the value of the functions, given that

$$f(x) = \begin{cases} x+2 & \text{if } x > 1 \\ 2 & \text{if } -1 \leq x \leq 1 \\ x-1 & \text{if } -3 < x < -1 \end{cases}$$

$$g(x) = x^2 + 7$$

   Find the following values
   i) $(g \; o \; f)\,(-\,2)$
   ii) $(f \; o \; f)\,(0) + (g \; o \; g)\,(1)$

Combinatorics & Binomial Coefficients:

7. There are n people trying to sit around a round table for dinner. And Jim and Julie are a couple. They want to sit next to each other. In how many different ways can we accommodate them?

8. There are n people trying to sit around a round table for dinner. But Jim and Julie are enemies. They do not want to sit next to each other. In how many different ways can we accommodate them?

9. There are B boys and G girls where $B \geq G$. In how many ways can we arrange them in a row so that no two girls stand next to each other?

10. Good binary strings are strings made of 0s and 1s where there are no two consecutive 0s. For example, $1011011101,\ 011010$ etc. How many good binary strings of length k are there?
    [Hint: First you may try to find the highest possible number of 0s in such strings in terms of k . It can be done separately for odd and even k**.** Use the Floor function to generalize. Giving the final answer in summation $(\sum)$ form will suffice.**]**

11. A and B are two sets where $|A| = n \leq m = |B|$. How many injective functions can we create from A to B?

12. For an odd n, prove that,

$$n_{C_0} + n_{C_2} + n_{C_4} + \ ... = n_{C_1} + n_{C_3} + n_{C_5} + ...$$

13. For positive integers $n,\ m$ and k, where $k \leq m \leq n$

    prove that, $n_{C_m} \times m_{C_k} = n_{C_k} \times (n - k)_{C_{(m-k)}}$

14. Prove that, $\sum_{k=1}^{n} k \times n_{C_k} = n \times 2^{n-1}$ **[Hint:** For $a \leq b,\ \frac{a}{b} \times b_{C_a} = (b - 1)_{C_{(a-1)}}$ **]**