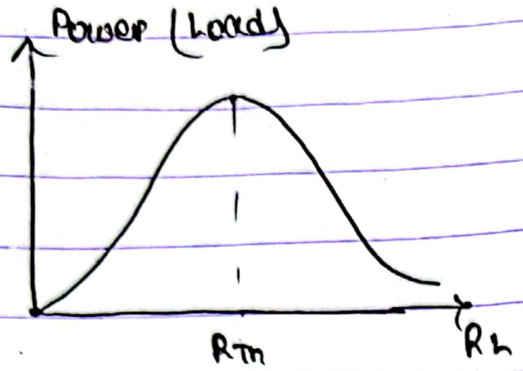
ECTURE 15

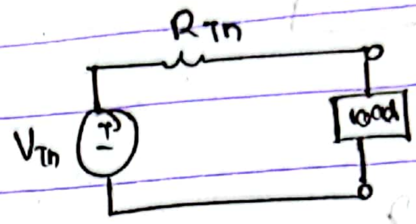
DATE: 28

MONDAY

FIRST ORDER CIRCUITS



R_N — R_{Th} → -ve



$$\frac{dP}{dR_L} = 0$$

Maxima ↙ ↘ Minima

$R_L = -R_{Th}$

* when load supplies to R_{Th} instead of voltage source, then R_{Th} comes in -ve.

LECTURE 16

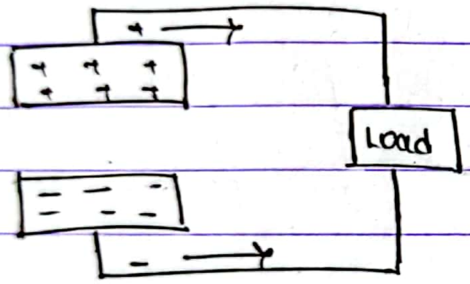
WEDNESDAY DATE: 30/11/22

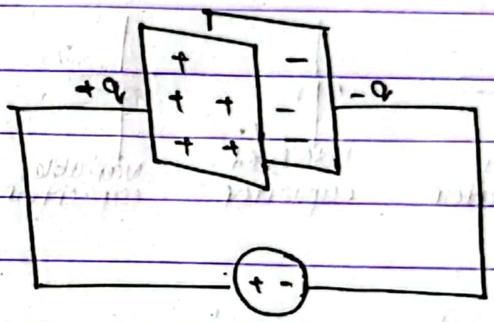
FIRST ORDER CIRCUITS

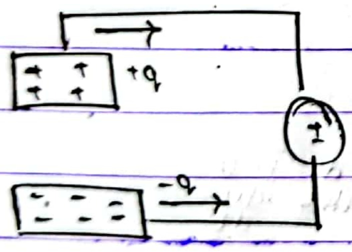
Electrical energy storage system

- mechanical
  - flywheel
  - hydro-power
- Thermal
  - salt
  - melted
- Chemical
  - H_2
  - CH_4
- Electrochemical
  - battery
- electrical
  - capacitor
- Electro-magnetical
  - inductor

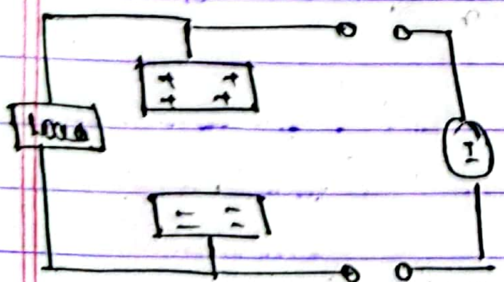
CAPACITORS







* +ve charge of lower/bottom plate is attracted to negative terminal and the bottom plate is -vely charged.
* -ve charge of top is attracted to positive terminal, and the top plate is +vely charged.
* when the plates are fully charged, the voltage source is disconnected.

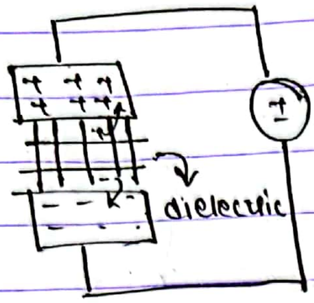


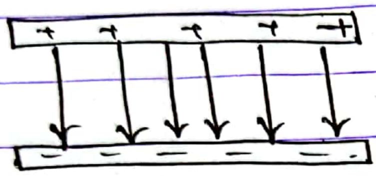
* $C = qV$ $\quad q \propto V$

(CV^{-1})

Farad (F)

* capacitance can be increased
  1. by increasing surface area of plate. Increasing surface area, increases charge.
  2. by using dielectric material.

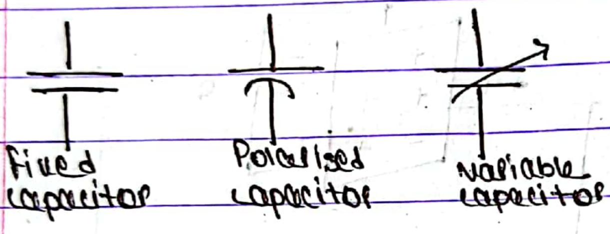




$E \propto \frac{V}{d}$

For parallel plate capacitor,

$$C = \frac{\underbrace{\varepsilon}_{\text{permitivity}} A}{d}$$



$$C = \frac{q}{V}$$

$$q = CV$$

$$\frac{dq}{dt} = C\frac{dv}{dt}$$

$i = C\frac{dv}{dt}$ → capacitor does not have fixed i-v characteristics. It depends on ~~the~~ $\frac{dv}{dt}$

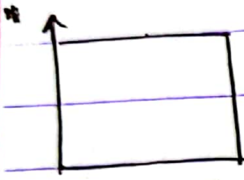
$$v(t) = \frac{1}{C} \int i(t)\,dt \quad \cancel{\Rightarrow w(t) = \frac{1}{2}Cv^2 - \frac{1}{2}Cv_0^2}$$

$$\text{power} = p = vi = Cv\frac{dv}{dt}$$

$$\text{stored energy, } w(t) = \underbrace{\frac{1}{2}Cv^2}_{\text{final voltage of capacitor}} - \underbrace{\frac{1}{2}Cv_0^2}_{\text{initial voltage of capacitor}}$$

Properties of capacitor

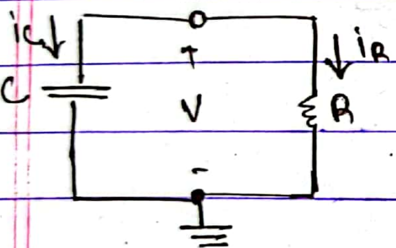
* When a ~~dc~~ ~~cure~~ capacitor is connected to a dc supply, voltage ~~ren~~ ~~flowin~~ across capacitor is constant and current is 0. Thus, an open circuit is formed.

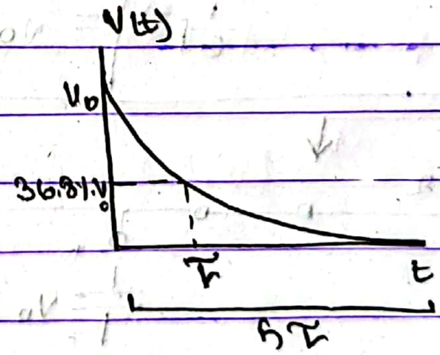
* 

$$\Delta t = t_1 - t_2 = 0$$

$$i = C\frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{\Delta V}{0} \to \infty$$

SOURCE – FREE RC CIRCUIT





$v = Ae^{-\frac{t}{RC}}$ At $t = 0$, $v(0) = A = V_0$

$v(t) = V_0 e^{-\frac{t}{RC}}$

$v(t) = V_0 e^{-\frac{t}{\tau}}$

at $t = \tau$

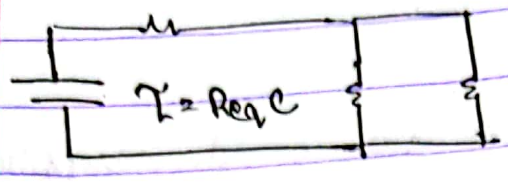
$v(t) = V_0 e^{-\frac{\tau}{\tau}}$

$= V_0 e^{-1}$

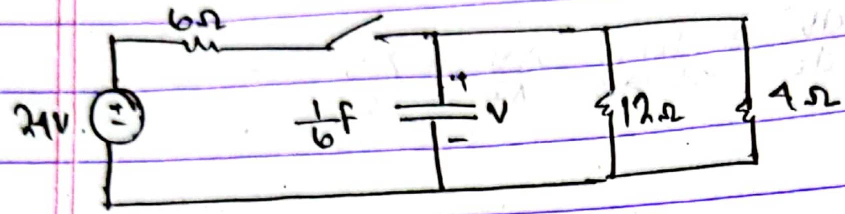
$= 0.368 V_0$

$= 36.8 V_0$

τ is the time when the discharging voltage is decreased to 36.8% of its initial voltage.



Problem 2



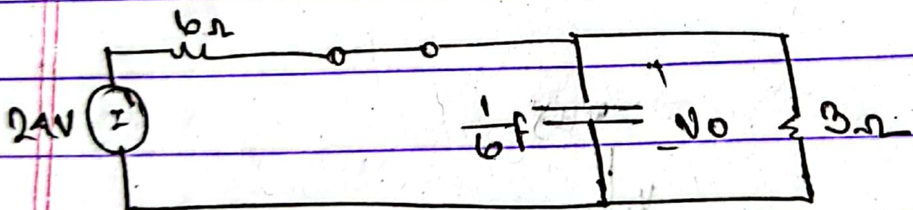
Source free

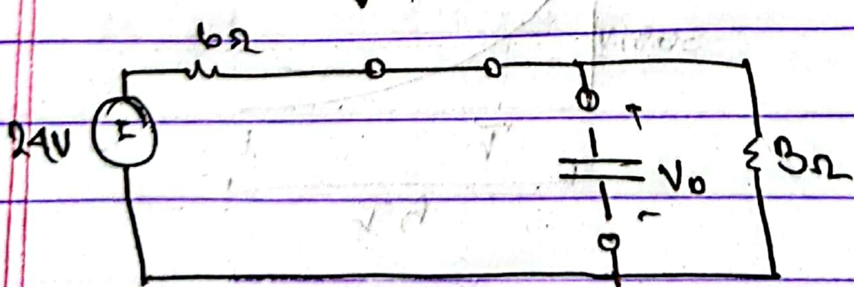
voltage across a capacitor,

$v(t) = V_0 e^{-t/\tau}$

$\tau = R_{eq} \times C \quad (t > 0)$

$t < 0 \rightarrow$ time before the switch is opened.

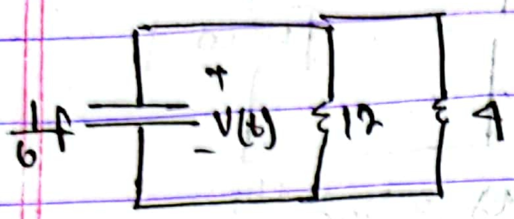




$i = C\frac{dv}{dt} = 0$ (open)

$V_0 = \frac{3}{3+6} \times 24 = 8V$

$t > 0$



$\tau = (R_{eq}) \times C = (12 \| 4) \times \frac{1}{6} = \frac{1}{2} s$

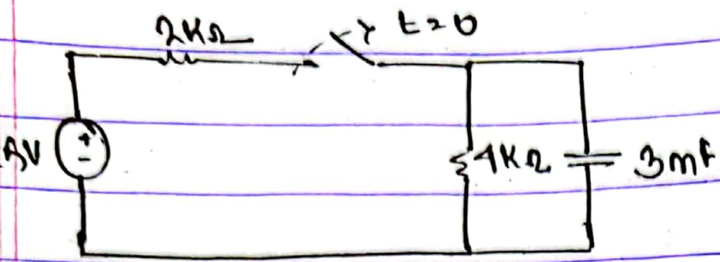
$v(t) = 8e^{-\frac{t}{1/2}} = 8e^{-2t}$ (V)

$W = \frac{1}{2}Cv_0^2 = \frac{1}{2}\left(\frac{1}{6}\right)(8)^2 = 5.333$ J

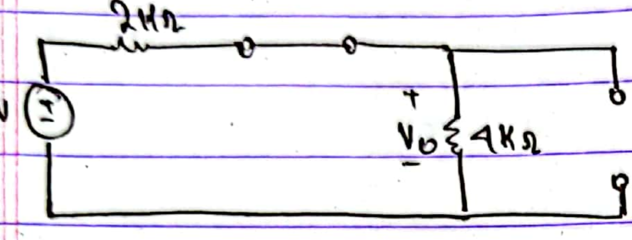
DATE: 05/12/22

MONDAY

Problem - 3

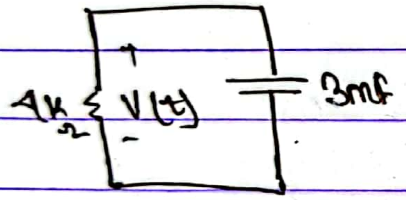


t<0



$$V_o = \frac{4K}{2K+4K} \times 15 = 10\,V$$

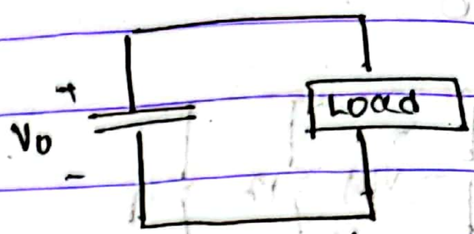
t>0



$\tau = R_{eq} C \Rightarrow (4\times10^{3}) \times (3\times10^{-3}) = 12\ s$

$V(t) = V_o e^{-t/\tau}$

$V(t) = 10 e^{-t/12}$ (V)



t>0 $\begin{cases} V(t) = V_o e^{-t/\tau} \\ \tau = R_{eq} C \end{cases}$

$W = \frac{1}{2} C V^2$

$W = \frac{1}{2} C V_o^2 e^{-2t/\tau}$

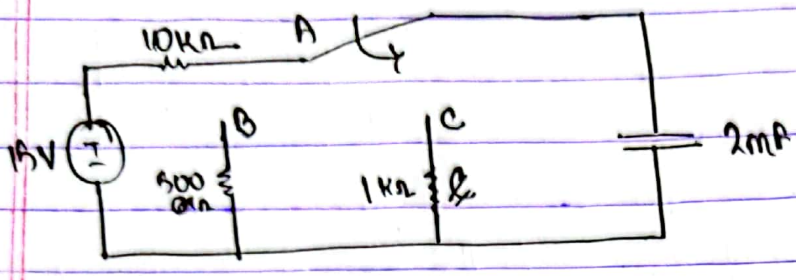
↳ stored E in capacitor at any time t.

E transferred to load over time,

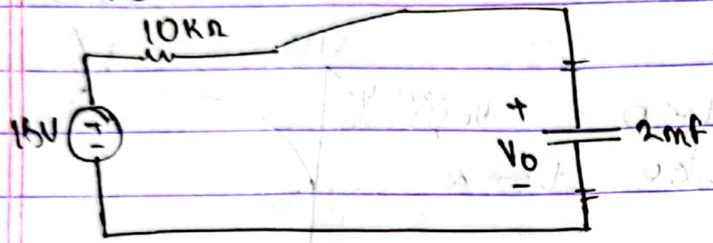
$= \frac{1}{2} C V_o^2 - \frac{1}{2} C V_o^2 e^{-2t/\tau}$

$= \frac{1}{2} C V_o^2 (1 - e^{-2t/\tau})$

Problem 3

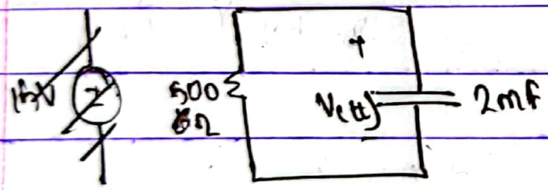


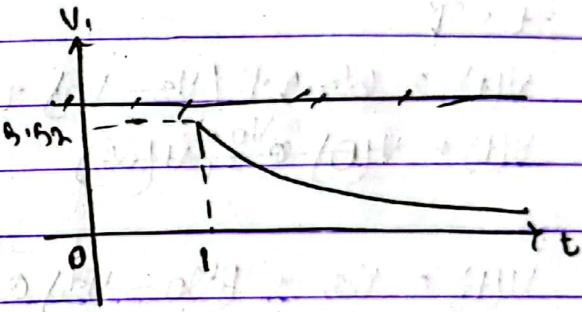
t < 0



$V_0 = 15V$

~~t > 0~~ $1 > t > 0$

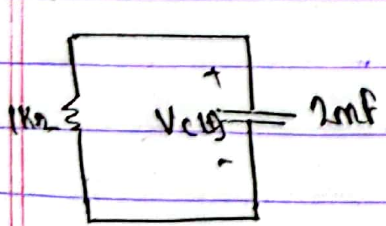




$\tau = (500 \cancel{\times 10^{3}}) \times (\cancel{10} 2 \times 10^{-3}) = \cancel{1000}\ 1\,s$

$V_0(t) = 15 e^{-t}$ (V) (for $1 > t > 0$)

$t > 1$



$V_c(t) = V_0 e^{-t/\tau}$ $\tau = (1 \times 1000)(2 \times 10^{-3}) = 2\,s$

From $1 > t > 0$,

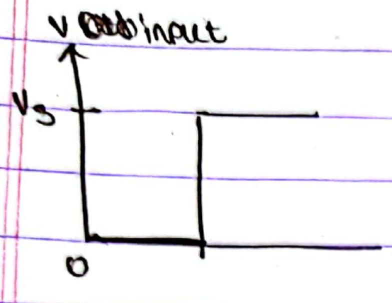
$V_0 = (15 e^{-1}) e^{-t/\tau}$

$= (15 e^{-1}) e^{-t/2}$

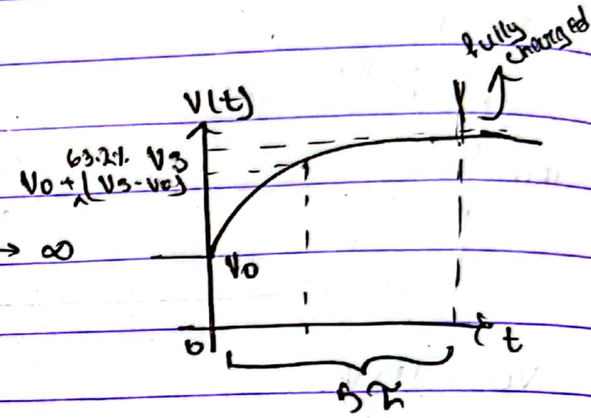
~~$= 5.52 e^{-t/2}$ $(t > 1)$~~

$= 5.52 e^{-(t-1)/2}$ $(t > 1)$

STEP RESPONSE OF A RC CIRCUIT



$$V(t) = V_s + (V_0 - V_s)e^{-t/RC}$$



$$V(t) = \begin{cases} V_0 & t < 0 \\ V_s + \underbrace{(V_0 - V_s)e^{-t/RC}}_{\to 0} & t \geq 0 \end{cases} \quad t \to \infty \quad \Rightarrow V_s$$

$t = \tau$

$V(t) = 63.2\% (V_s - V_0) + V_0$

~~$V(t) = V(0) = V_0, V(\infty)$~~

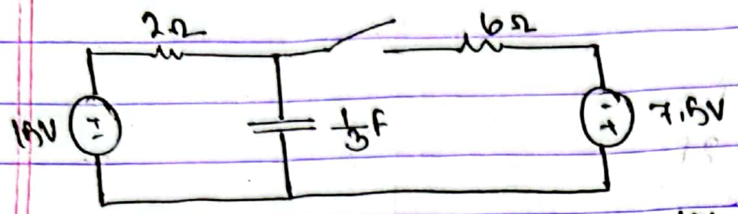
$V(t) = V_s$
$\downarrow$
5τ

$\left.\begin{matrix} 0 - 5V \\ 3 - 5V \end{matrix}\right\}$ same time (5τ)

$$V(t) = \underset{V(\infty)}{V_s} + (\underset{V(0)}{V_0} - V_s)e^{-t/RC}$$

$$V(t) = V(\infty) + (V(0) - V(\infty))e^{-t/\tau}$$

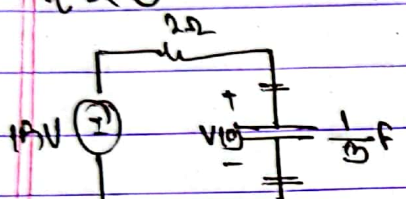
PROBLEM 11



$$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$

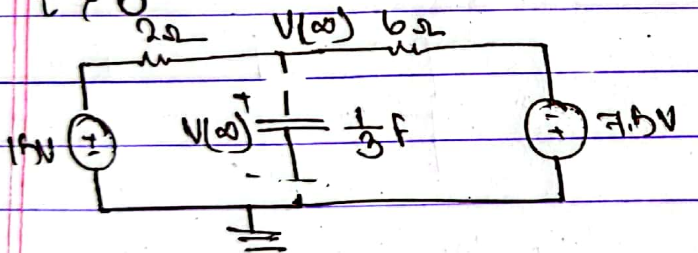
if $V(\infty) = 0$, source free $\rightarrow V(t) = V(0)e^{-t/\tau}$

$t < 0$



$V(0) = 15\,V$

$t > 0$



*If there is any independent DC source, then the capacitor will be open.

$$\frac{V(\infty) - 15}{2} + \frac{V(\infty) + 7.5}{6} = 0$$

$$\frac{2}{3}V(\infty) = \frac{25}{4} \Rightarrow V(\infty) = 9.375\,V$$

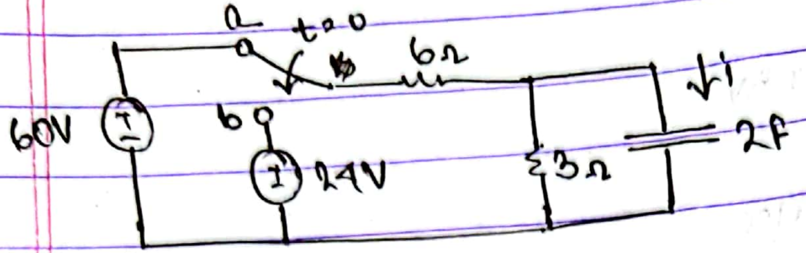
$$\tau = (2\|6) \times \frac{1}{3} = (1.5)\left(\frac{1}{3}\right) = \frac{1}{2} = 0.5\,s$$

$$V(t) = 9.375 + (15 - 9.375)e^{-t/0.5}$$

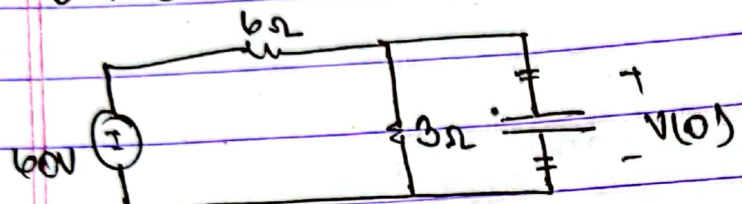
$$V(t) = 9.375 + 5.625e^{-t/0.5}$$

$$V(0.5) = 9.375 + 5.625e^{-0.5/0.5} = 11.44\,V$$

Problem 13

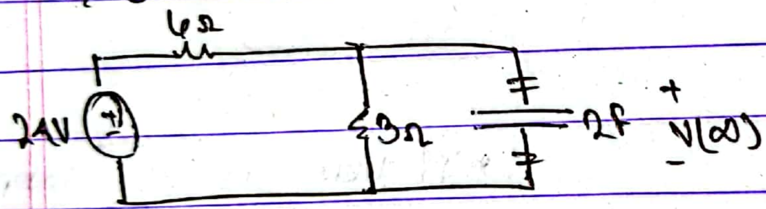


t < 0



$$V(0) = \frac{3}{3+6} \times 60 = 20\ V$$

t > 0



$$V(\infty) = \frac{3}{6+3} \times 24 = 8V$$

$$\tau = (6 \| 3) \times C = 4s$$

$$V(t) = V(\infty) + [V(0) - V(\infty)] e^{-t/\tau}$$

$$= 8 + [20 - 8] e^{-t/\tau}$$

$$V(t) = 8 + 12 e^{-0.25t}$$

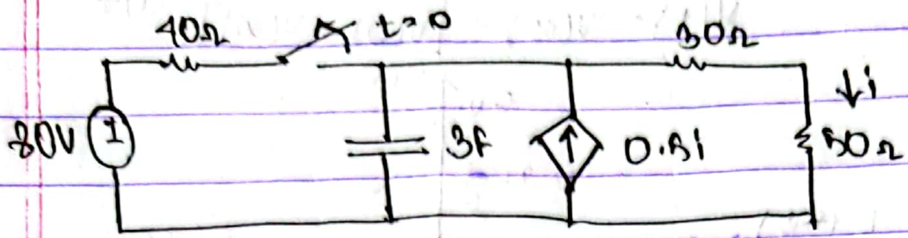
$$i(t) = C \frac{dv}{dt}$$

$$= C \frac{d(8 + 12 e^{-0.25t})}{dt}$$

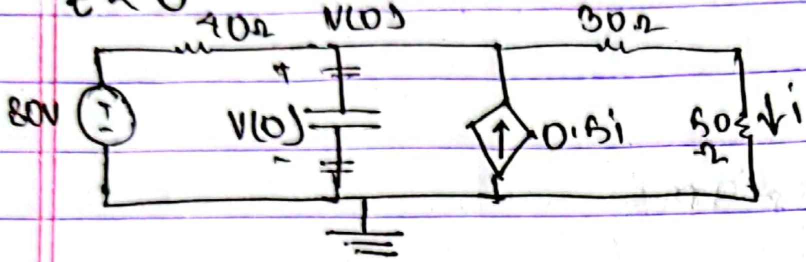
$$= 2 [12(-0.25) e^{-0.25t}]$$

$$i(t) = 6 e^{-0.25t} \quad (A)$$

Problem 14



t < 0

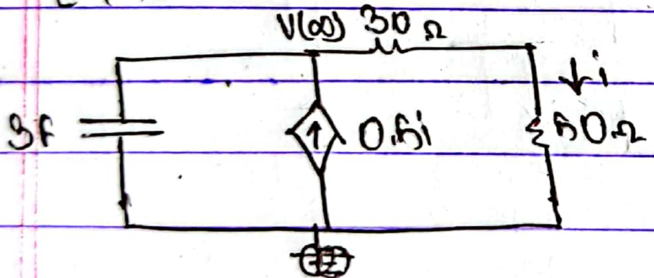


$$\frac{V(0)-80}{40} + \frac{V(0)}{80} = 0.5i \qquad i = \frac{V(0)-0}{80}$$

$$\frac{V(0)-80}{40} + \frac{V(0)}{80} = 0.5\frac{V(0)}{80}$$

$V(0) = 106.67 \quad (t<0)$

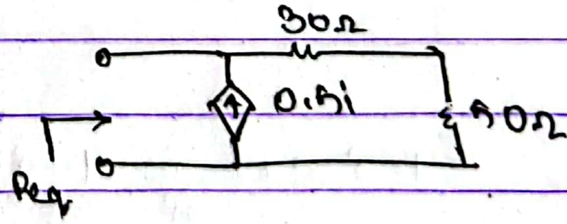
t > 0



$V(\infty) = 0\ V$

$i(\infty) = 0\ A$

τ



CTURE

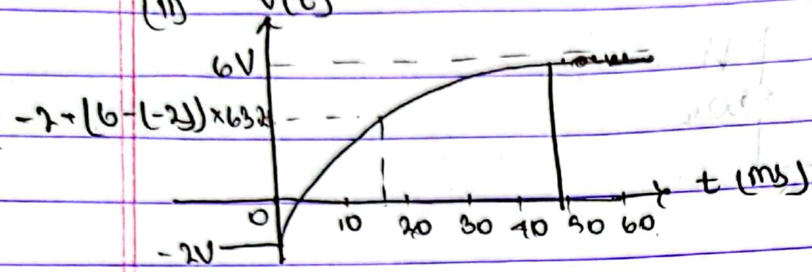
DATE: 07/12/22

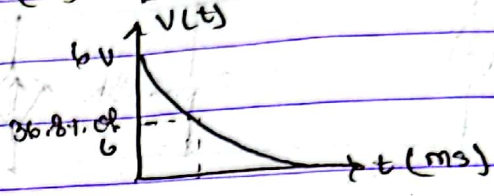
WEDNESDAY

Problem 15

$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$

(ii)





$5\tau = 45 \Rightarrow \tau = 9\ ms$

(iii) $V(\infty) = 6V$, $V(0) = -2V$, $\tau = 9\ ms$

$V(t) = 6 + (-2-6)e^{-t/9}$

$= 6 - 8e^{-t/9}$

(iv) $W = \frac{1}{2}CV_0^2$

$W = \frac{1}{2}(2.25)(-2)^2$

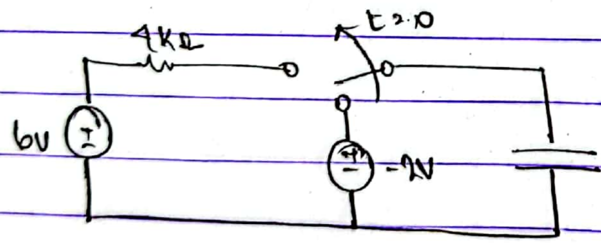
$= 4.5\ \mu J$

$\tau = R_{eq} \times C$

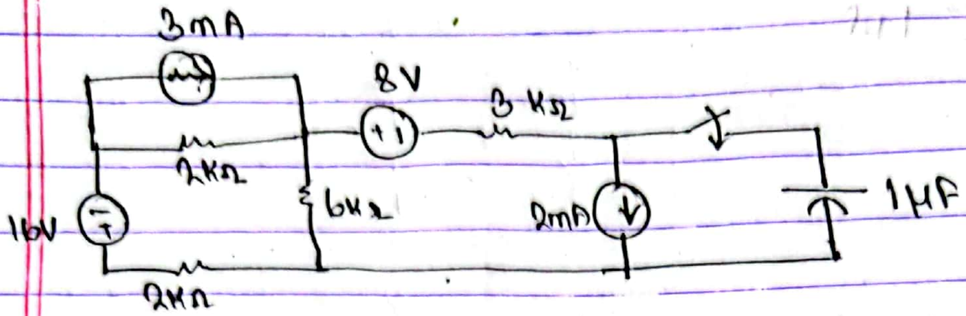
$9 = 4 \times C$

$C = 2.25\ \mu F$

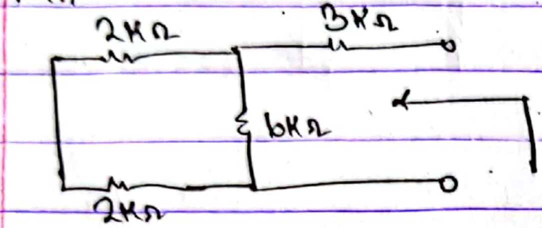
(v)



Problem 16

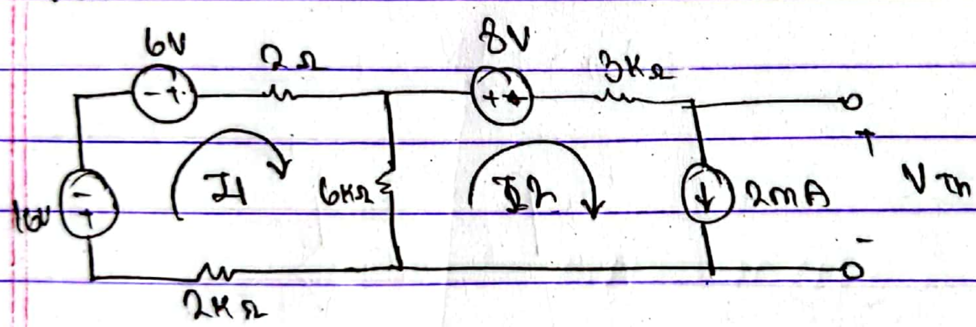


RTh →



Req = $(2+2)||6 + 3 = 5.4 K\Omega$

VTh



$I2 = 2A$

$16 - 6 + 2I1 + 6(I1 - I2) + 2I1 = 0$

$\cancel{10 I_1 - 6 I2 = -10}$

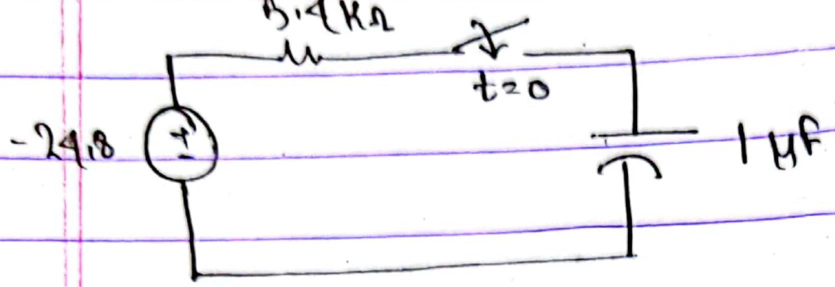
$16 - 6 + 10 I1 - I2 = 0$ — ①

$10 I1 - 6 I2 = -10 \qquad I2 = 2$

$I1 = 0.2 mA$

$-V_{Th} - 3I_2 - 8 + 6(I_1 - I_2) = 0$

$V_{Th} = -24.8\ V$



$t < 0, \; v(0) = 0\,V$

$t > 0, \; v(\infty) = -24.8$

$\tau = R_{eq} \times C = (5.4)(1) = 5.4\,ms$

$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$

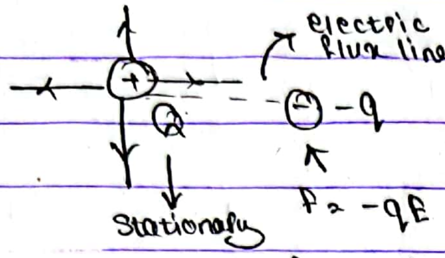
$= -24.8 \; [\cancel{v(0)}^{0} - (-24.8)]e^{-t/5.4}$

$v(t) = -24.8 + 24.8e^{-t/5.4}$

$i(t) = C\frac{d(-24.8 + 24.8e^{-t/5.4})}{dt}$

$i(t) = (1)(\cancel{-24.8} - 4.96e^{-t/5.4})$

INDUCTORS



$$F = \frac{1}{4\pi\epsilon}\cdot\frac{-Qq}{r^2}$$

$$E = \frac{1}{4\pi\epsilon}\cdot\frac{Q}{r^2}$$

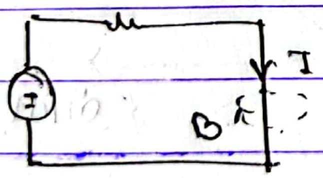
$\vec{E}$ & $\vec{H}$ are same $\vec{D}$ and $\vec{B}$ are same

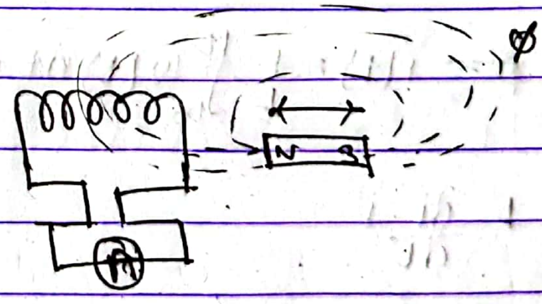
flux
$\vec{D} = \epsilon\vec{E}$ ← electric field density

$\vec{B} = \mu_0\vec{H}$ ← magnetic field density

$B = \frac{\phi}{A}$ $(Wb/m^2 = T = 10^4 \text{ Gauss})$

Faraday's law





* An electric field can produce a magnetic field. But, a magnetic field can produce an electric field with a ~~moving ma~~ changing magnetic field.
* To create changing magnetic field, we have to move either the magnet / ~~coil~~ rod perpendicularly.

Inductance

$\phi \propto I$

$\phi = LI$ → inductance (L)

$N\phi = LI$

$L = \frac{N\phi}{I}$ * L does not depend on ϕ and I as increasing I, increases ϕ

* L depends on length, surface area, material

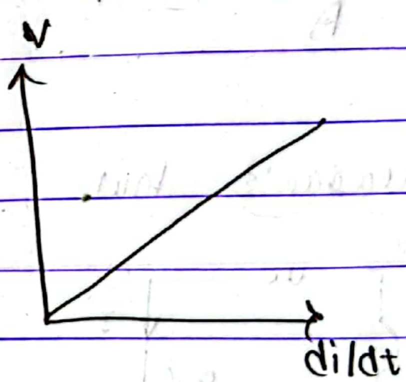
Lenz's law

$$\frac{d\phi}{dt} = \frac{L}{N}\frac{dI}{dt}$$

$$e = L\frac{dI}{dt}$$

—x—x—

$$V = L\frac{di}{dt}$$



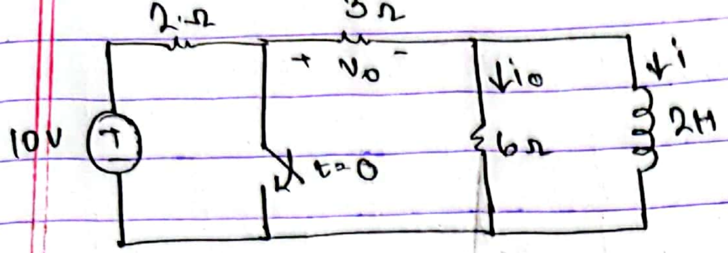
$\cancel{i(t) = \frac{1}{L}}$ $\Rightarrow i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + v(t_0)$

$$p = V_L i_L = L\frac{di}{dt}i$$

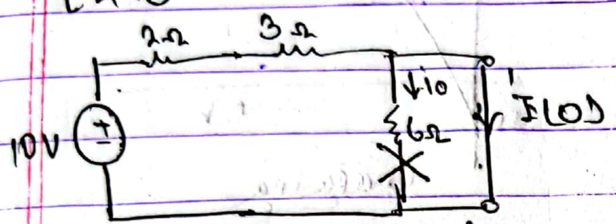
$$w(t) = \frac{1}{2}Li^2$$

current cannot be found out directly ← C	voltage ~~current~~ cannot be found directly ← L
$I_C = C\frac{dV}{dt}$	$V_L = L\frac{di_L}{dt}$
$w = \frac{1}{2}CV^2$	$w = \frac{1}{2}LI^2$
$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$	$I(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$
$\tau = R_{eq}C$	$\tau = \frac{L}{R_{eq}}$
C open at DC	L short at DC

Problem 6

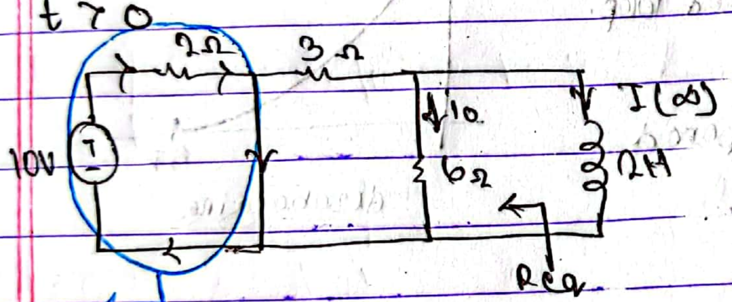


t < 0



$I(0) = \frac{10}{2+3} = 2A$

t > 0



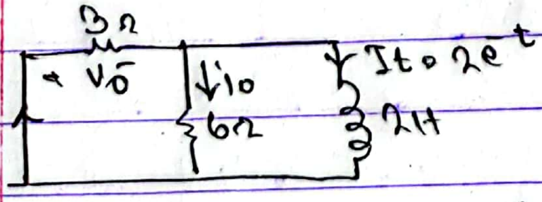
is shorted
so no contribution
in the circuit anymore
to find $I(\infty)$

$I(\infty) = 0$

$R_{eq} = 6 \| 3 = 2\,\Omega$

$\tau = \frac{L}{R_{eq}} = \frac{2}{2} = 1\ ms$

$I_t = 2e^{-t}\ (A)$



$I_o = \frac{3 \| 6}{6} \times 2e^{-t}$

$= \frac{2}{6}(2e^{-t}) = \frac{2}{3}e^{-t}$

$V_o = (I(t) - I_o) \times 3$

$= (2e^{-t} - \frac{2}{3}e^{-t}) \times 3$

$=$

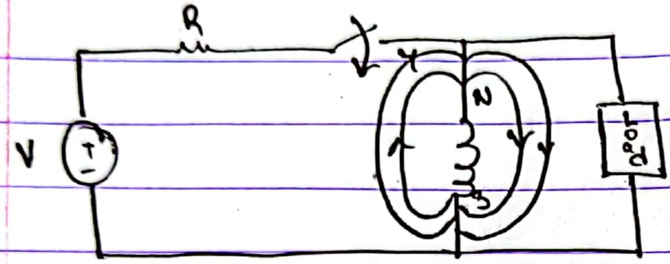
DATE: 12/12/22

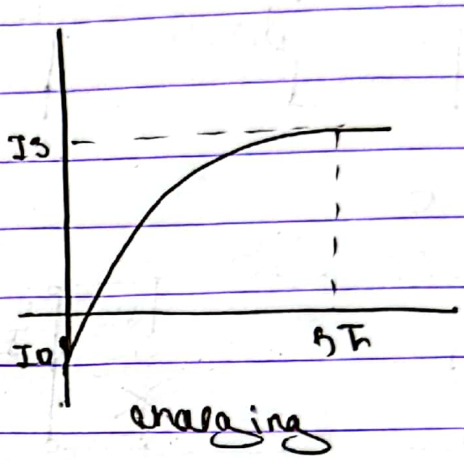
MONDAY

RL CIRCUITS

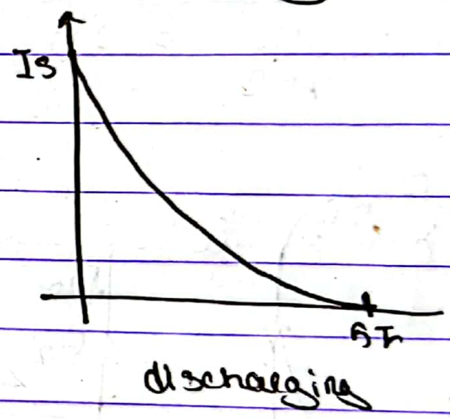
$I(t) = I(\infty) + [I(0) - I(\infty)]e^{-t/\tau}$

$\tau = \frac{L}{R_{eq}}$





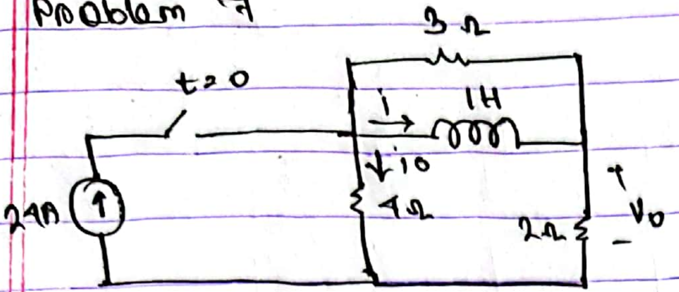
* Inductor current can't be turned off abruptly.
* Current in inductor needs a closed loop.
* when an inductor is open circuit, a spark is created at the opened switch to make the current 0.



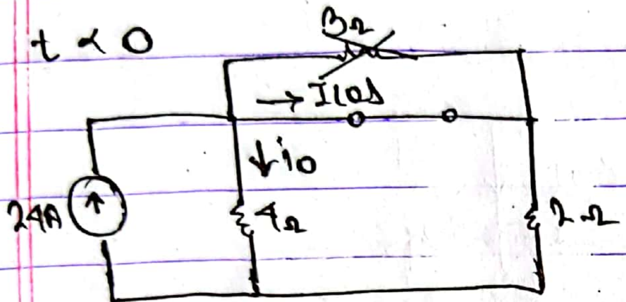
$W = \frac{1}{2} C V_0^2 (e^{-2t/\tau}) \rightarrow$ capacitor

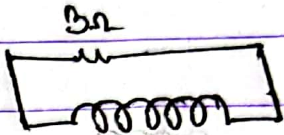
$W = \frac{1}{2} L I_0^2 (e^{-2t/\tau}) \rightarrow$ inductor

Problem 7



t < 0



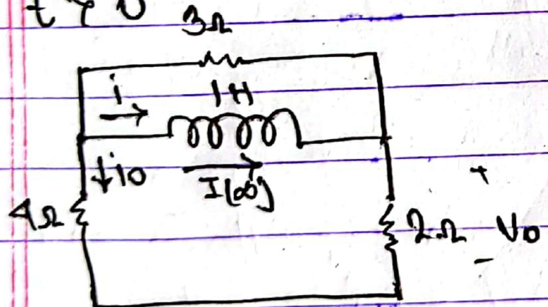


$I(0) = \frac{(2\|4)}{2} * 24 = 16A$

$V_o = 2(16) = 32V$

$i_o = 24 - 16 = 8A$

t > 0



$I(\infty) = 0A$

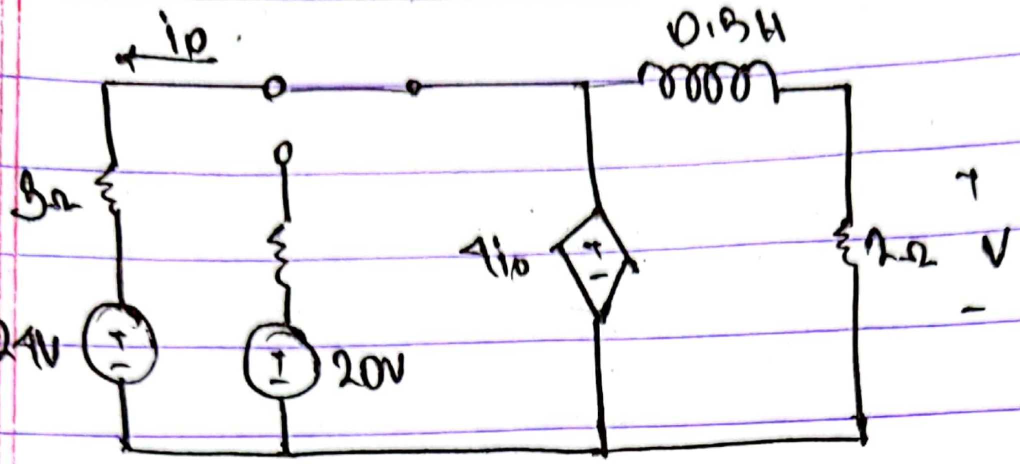
$\tau = \frac{L}{R_{eq}} = \frac{1}{(4+2)\|3} = 0.5\ (S)$

$I(t) = 16e^{-2t}$

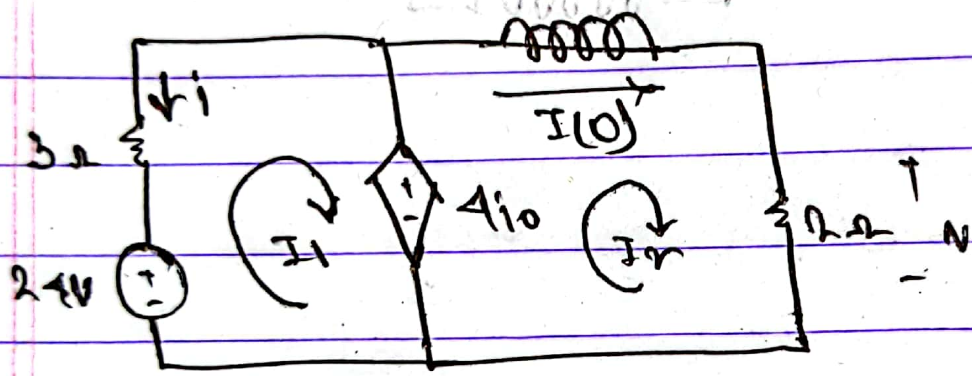
$i_o = -\frac{3\|6}{6} \times 16e^{-2t} = -\frac{16}{3}e^{-2t}\ (A)$

$V_o = -i_o \times 2 = \frac{+16}{3}e^{-2t} \times 2 = \frac{32}{3}e^{-2t}\ (V)$

Problem 19



$t < 0$



$-24 + 3I_1 + 4i_0 = 0 \qquad i_0 = -I_1$

$-24 + 3I_1 - 4I_1 = 0$

$I_1 = -24A \qquad i_0 = 24A$

$-4i_0 + \cancel{I(0)} + 2I_2 = 0$

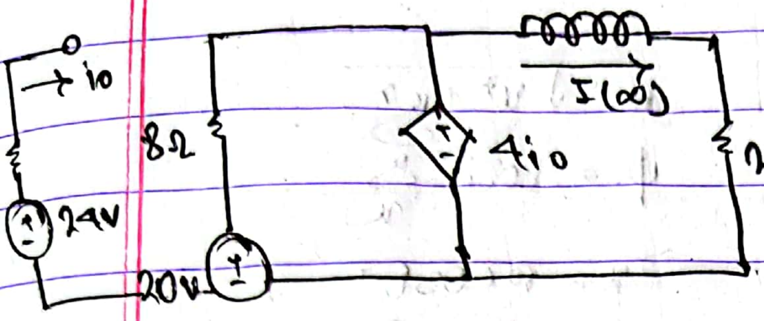
$-4I_1 + 2I_2 = 0$

$I_2 = 48\,A$

$I(0) = I_2 = 48\,A$

$V_0 = 2 \times I(0) = 2 \times 48 = 96\,V.$

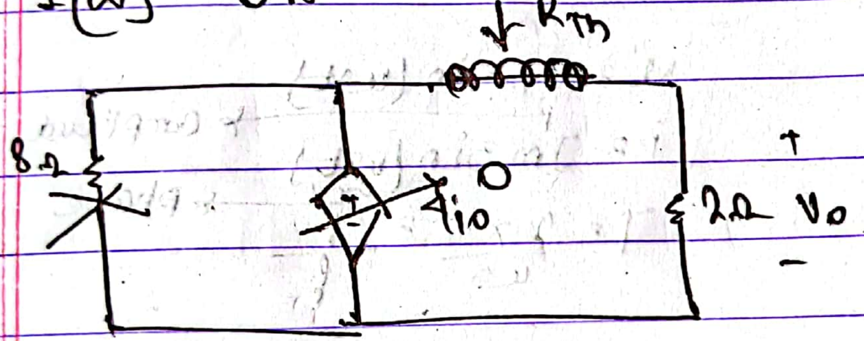
t > 0



io = 0

4io = 0 (Dependent voltage source is shorted out)

I(∞) = 0A



$R_{Th} = 2\Omega$

$\tau = \frac{L}{R_{Th}} = \frac{0.5}{2} = \frac{1}{4}$

$I(t) = 48e^{-\frac{t}{\frac{1}{4}}} = 48e^{-4t}$

$V_0 = 2(48e^{-4t}) = 96e^{-4t}$ (V)

AC FUNDAMENTALS AND AC CIRCUITS

Complex Number

$z = x + iy$ → Rectangular form

$z = r\angle\phi$ → Polar form

$z = re^{i\theta}$ → Exponential form

$r = \sqrt{x^2 + y^2}$

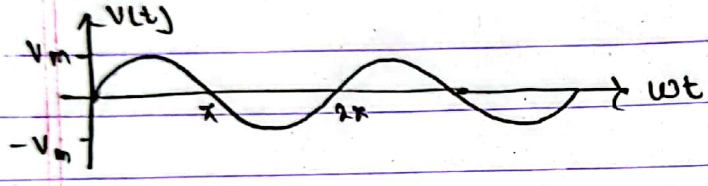
$\phi = \tan^{-1}\frac{y}{x}$

$x = r\cos\theta$

$y = r\sin\theta$

$x + iy = r\underbrace{(\cos\theta + i\sin\theta)}_{e^{i\theta}}$

Sinusoid



$v = V_m \sin(\omega t)$ → amplitude

$i = I_m \sin(\underbrace{\omega t}_{})$ → phase

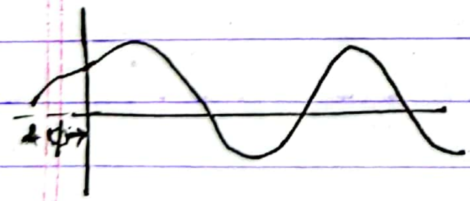
$T = \frac{2\pi}{\omega} = \frac{1}{f}$

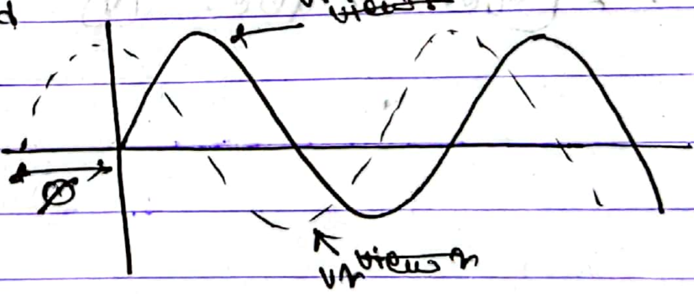
* A circular motion provides sinusoidal voltage.

* 50 Hz = 50 cycles/sec

Leading & lagging Sinusoid

$V(t) = V_m \sin(\omega t + \phi)$

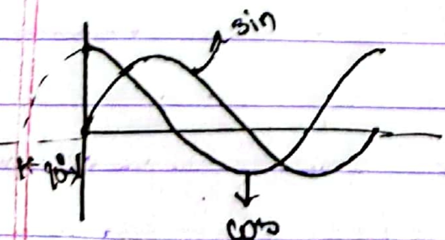


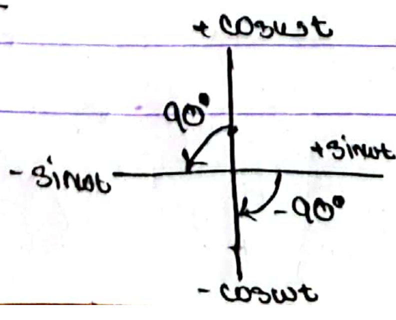


* view 2 leads view 1 by ϕ and lags by $(360 - \phi)$

* ~~view 1 leads~~

* V1 lags V2 by ϕ





$\sin(\omega t \pm 180°) = -\sin\omega t$

$\cos(\omega t \pm 180°) = -\cos\omega t$

$\sin(\omega t \pm 90°) = \pm\cos\omega t$

$\cos(\omega t \pm 90°) = \mp\sin\omega t$

~~Is(iii)~~ Problem 3

$I_m \cos(\omega t + \phi)$

(iii) $I_3 = -20\cos(314t - 30)$

amp = 20 (A)

$\omega = 314$ rad/s

$\omega = 2\pi f$

$f = \frac{314}{2\pi}$

$T = \frac{1}{f} = \frac{1}{\frac{314}{2\pi}} =$

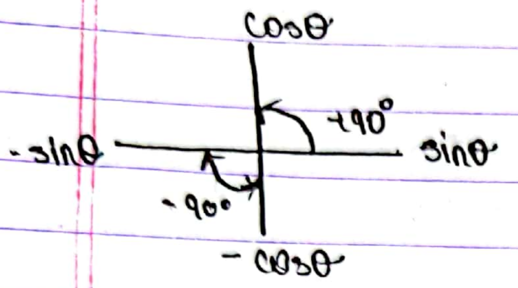
$20\cos(314t - 30^\circ + 180^\circ)$

$= 20\cos(314t + 180^\circ)$

$\hookrightarrow \phi$

LECTURE

Problem 4



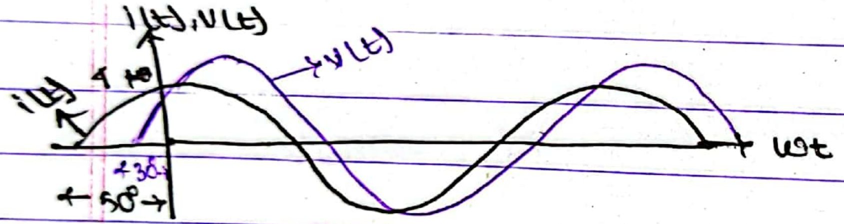
(i) $v(t) = 10\cos(4t - 60°)$

$= 10\sin(4t - 60° + 90°)$

$= 10\sin(4t + 30°)$

$i(t) = 4\sin(4t + 50°)$

* from cos to sin, we are going 90° backward. To compensate for the backward 90°, we add 90°
[~~cos~~ $\sin\theta + 90° = \cos\theta$]



i leads v by 20°

OR

$i(t) = 4\sin(4t + 50°)$

$= 4\cos(4t + 50° - 90°)$

$= 4\cos(4t - 40°)$

[~~sin θ~~ $\cos\theta - 90° = \sin\theta$]

(iv) $i_1(t) = -4\sin(377t + 25°)$ $\quad$ $i_2(t) = 5\cos(377t - 65°)$

$= 4\cos(377t + 25° + 90°)$

PHASOR

$z = x + jy = r\angle\theta = re^{j\theta}$

$\quad\quad (\cos\theta + j\sin\theta)$

$v(t) = V_m\cos(\omega t + \phi)$

$= Re\{V_m e^{j(\omega t + \phi)}\}$

$= Re[V_m\{\cos(\omega t + \phi) + j\sin(\omega t + \phi)\}]$

$= Re\{V_m e^{j\omega t} e^{j\phi}\}$

$= Re\{\underbrace{V_m e^{j\phi}}_{\text{phasor}} e^{j\omega t}\}$

$= Re\{V e^{j\omega t}\}$

$V_m\cos(\omega t + \phi)$

phasor, $V = V_m e^{j\theta} = V_m\angle\theta$

Complex mode

$\boxed{\text{MODE}}$ $\boxed{2}$

$\boxed{1}$ $\boxed{+}$ $\boxed{2}$ $\boxed{\text{ENG}}$ $\boxed{=}$

$\boxed{\text{OPTN}}$ $\boxed{\square}$ $\boxed{\blacktriangledown}$ $\boxed{1}$ $\boxed{=}$

$\boxed{\text{AC}}$ $\boxed{\text{Ans}}$ $\boxed{=}$

Problem 5

(i) $-14\sin(5t-22^\circ)$
$= 14\sin(5t-22^\circ+180^\circ)$
$= 14\sin(5t+158^\circ) \equiv 14\angle 158^\circ$

(ii) $-20\cos(314t-30^\circ)$
$= 20\cos(314t-30^\circ+180^\circ)$
$= 20\cos(314t+150^\circ) \equiv 20\angle 150^\circ$

PHASOR RELATIONSHIPS FOR CIRCUIT ELEMENTS (R)

Resistor,
$i(t) = I_m\cos(\omega t+\phi)$
$v(t) = \cancel{RI_m}\ Ri = RI_m\cos(\omega t+\phi)$

$V = RI_m\angle\phi \rightarrow \underset{I_m\angle\phi}{V} = R\underset{\text{phasor}}{I}$

Inductor
$i(t) = I_m\cos(\omega t+\phi)$
$v(t) = L\dfrac{di}{dt} = -LI_m\sin(\omega t+\phi)$ — $\sin \rightarrow -\cos$
$= -L\omega I_m\cos(\omega t+\phi-90^\circ)$
$= L\omega I_m\cos(\omega t+\phi-90^\circ+180^\circ)$ — $-\cos \rightarrow \cos$
$= L\omega I_m\cos(\omega t+\phi+90^\circ)$

Phasor form, $V = \omega L I_m\angle\phi+90^\circ$
$V = \omega L I_m\angle\phi\, e^{j90^\circ}$
$V = j\omega L I_m\angle\phi$ $\qquad V = j\omega L I$

R

$V = RI$ — complex number

L

$V = \underbrace{j\omega L}_{\text{resistance}} I$

C

$V = \underbrace{\frac{1}{j\omega C}}_{\text{resistance}} I$

$V = V_m \angle \theta$

$I = I_m \angle \theta$

DC, $\omega = 0$

L, $R = 0$ | C, $R = \infty$

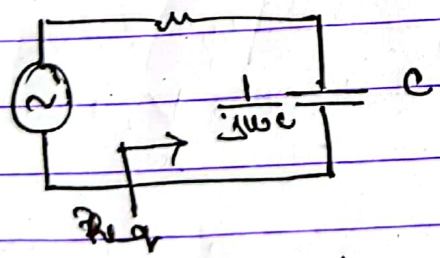
Impedance

$\frac{V}{I} = R$ | $\frac{V}{I} = j\omega L$ | $\frac{V}{I} = \frac{1}{j\omega C}$

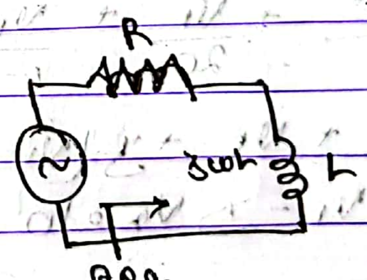
Impedance

* Impedance depends on frequency of

$Z_L = j\omega L$ | $Z_C = \frac{1}{j\omega C}$

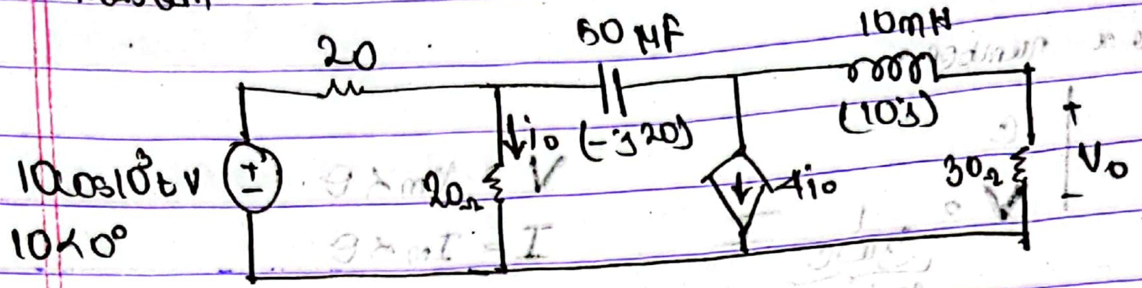


$Z_{eq} = R + \frac{1}{j\omega C}$



$Z_{eq} = R + j\omega L$

Problem 7



Impedance of 50 μF capacitor

$$Z_C = \frac{1}{j\omega C}$$

$$= \frac{1}{j \times 1000 \times 50 \times 10^{-6}} = -20j$$

$$Z_L = j\omega L$$

$$= j \times 1000 \times 10 \times 10^{-3} = 10j$$

KCL at node 1,

$$\frac{V_1 - 10\angle 0^\circ}{20} + \frac{V_1}{20} + \frac{V_1 - V_2}{(-20j)} = 0$$

$$\Rightarrow V_1 - 10 + V_1 + j(V_1 - V_2) = 0$$

$$\Rightarrow (2 + j)V_1 - jV_2 = 10 \quad \text{———} \quad (1)$$

KCL at node 2,

$$4 i_o + \frac{V_2 - V_1}{-j20} + \frac{V_2 - 0}{30 + j10} = 0$$

$$4 \times \frac{V_1}{20} + \frac{V_2 - V_1}{-j20} + \frac{V_2}{30 + j10} = 0$$

$$\Rightarrow 0.2V_1 + j0.05(V_2 - V_1) + (0.03 - j0.01)V_2 = 0$$

$$(0.2 - j0.05)V_1 + (0.03 + j0.04)V_2 = 0 \quad \text{———} \quad (\text{ii})$$

Cramer's Rule

$$\Delta = \begin{bmatrix} 2+1j & -1j \\ \cancel{2+1j}\ 0.2-j0.08 & 0.03+j0.04 \end{bmatrix} \Rightarrow \Delta = 0.07 + j0.31$$

$$\Delta 1 = \begin{bmatrix} 10 & \cancel{jj}-1j \\ 0 & 0.03+j0.04 \end{bmatrix} \Rightarrow \Delta 1 = 0.3 + j0.4$$

$$\Delta 2 = \begin{bmatrix} 2+1j & 10 \\ 0.2-j0.08 & 0 \end{bmatrix} \Rightarrow \Delta 2 = -2 + j0.8$$

$$V_1 = \frac{\Delta}{\Delta 1} =$$

$$V_2 = \frac{\Delta}{\Delta 2} =$$

Problem 11

KCL at nodes 1 & 2,

$$\frac{V_1}{2} + \frac{V_1 - V_3}{12 \angle 0 0 j2} + \frac{V_2}{-j4} + \frac{V_2 - V_3}{4} = 0 \quad \text{—— (i)}$$

KCL at node 3,

$$0.2V_0 = \frac{V_3 - V_2}{4} + \frac{V_3 - V_1}{j2}$$

$$0.2V_1 = \frac{V_3 - V_2}{4} + \frac{V_3 - V_1}{j2} \quad [V_0 = V_1] \quad \text{—— (ii)}$$

$$V_1 - V_2 = 12\angle 0^\circ \quad \text{—— (iii)}$$

$$\Delta = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} \quad \Delta_1 = [\quad], \Delta_2 = [\quad], \Delta_3 = [\quad]$$

$$V_1 = \frac{\Delta}{\Delta_1} = \qquad V_2 = \frac{\Delta}{\Delta_2} \qquad V_3 = \frac{\Delta}{\Delta_3}$$

~~V_0~~

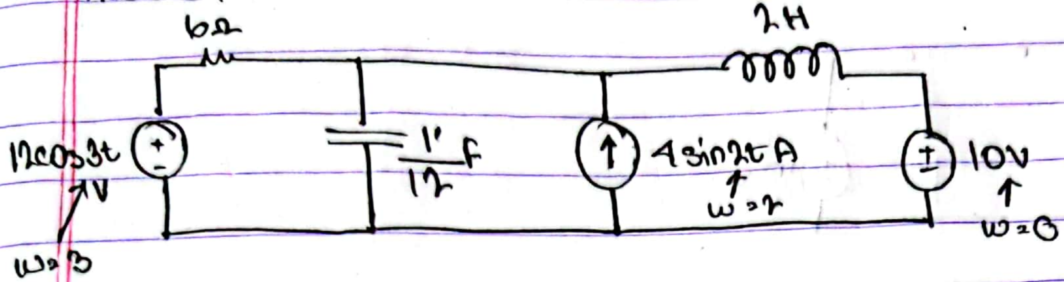
$$V_0 = 7.682 \angle 50.19^\circ$$

$$= 7.682 \cos(\omega t + 50.19^\circ)$$

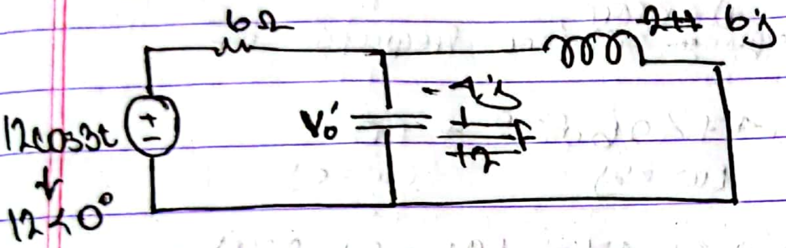
ECTURE

MONDAY   DATE:

Problem 19



Only the 12cos3t (V) source is active,



$\frac{1}{12} f = \frac{1}{j \times 3 \times \frac{1}{12}} = -j4$

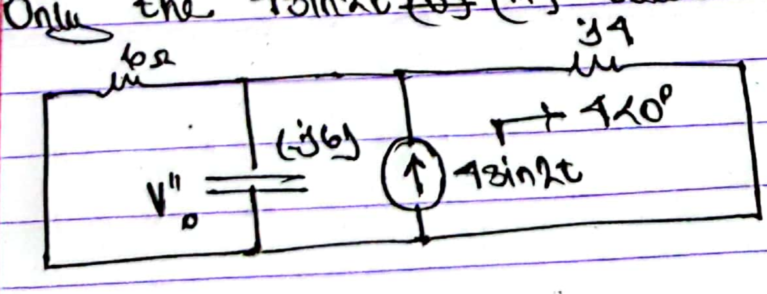
$2H = j \times 3 \times 2 = j6$

$V_o' = \frac{(-j4 \| j6)}{(-j4 \| j6) + 6} \times 12\angle 0^\circ$

$V_o' = \left(\frac{4}{5} - \frac{2}{5} i\right) \times 12\angle 0^\circ$

$= 10.73\angle -26.57^\circ$

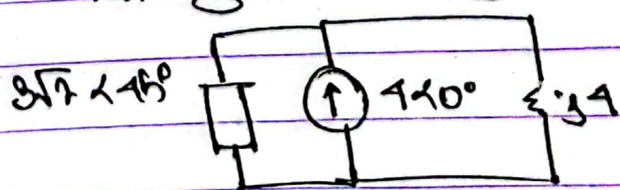
Only the 4sin2t ~~(V)~~ (A) source is active



$\omega = 2$ rad/s

$\frac{1}{12} f = \frac{1}{j \times 2 \times 12} = -j6$

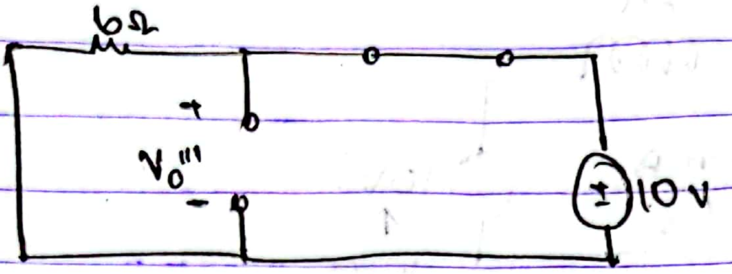
$2H = j \times 2 \times 2 = 4j$



$V_o'' = \underbrace{3\sqrt{2}\angle 45^\circ}_{Z} \times \frac{(3\sqrt{2}\angle 45^\circ \| j4)}{\underbrace{3\sqrt{2}\angle -45^\circ}_{\text{current}}} \times 4\angle 0$

$V_o'' = 21.47\angle 26.57^\circ$

Only 10V source is active



$V_0''' = 10 \text{ (V)}$

According to superposition theorem,
* ~~frequency different frequency wi~~

$V_0 = V_0' + V_0'' + V_0'''$

$= 10.73\angle -26.57° + 21.47\angle 26.57° + 10$
$(\omega = 3) \quad (\omega = 2) \quad (\omega = 0)$

$V_0 = 10.73\cos(3t - 26.57°) + 21.47\sin(2t + 26.57°) + 10$

*

INSTANTANEOUS POWER

$P = VI$

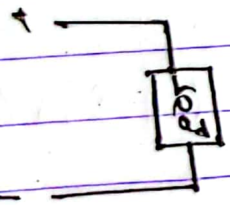
$P(t) = V(t)I(t)$

$\quad = V_m I_m \cos(\omega t)$

$$P(t) = \underbrace{\frac{1}{2} V_m I_m \cos(\theta_V - \theta_I)}_{\text{Time independent}} + \underbrace{\frac{1}{2} V_m I_m \cos(2\omega t + \theta_V + \theta_I)}_{\text{Time dependent and periodic}}$$

$$\frac{\sum P(t)}{\text{time}} = \frac{\int_0^T P(t)\, dt}{T} \rightarrow \frac{(Ws)}{(s)} = W$$

$P = \frac{1}{2} V_m I_m (\theta_V - \theta_I) \rightarrow$ (average power)



$Z = \frac{V}{I}$ (phasor)

$Z = R + jX$

$\leftarrow \theta_V > \theta_I \rightarrow$ Inductive Load ~~$\theta_V - \theta_I = 90^\circ$~~ for

$\leftarrow \theta_I > \theta_V \rightarrow$ Capacitive Load ~~$(Z = -j\square)$~~ ~~$\theta_I - \theta_V = 90^\circ$~~

$\leftarrow \theta_V = \theta_I \rightarrow$ Resistive Load

* When load has only capacitance, it is a pure ~~capacitator~~. $\rightarrow$

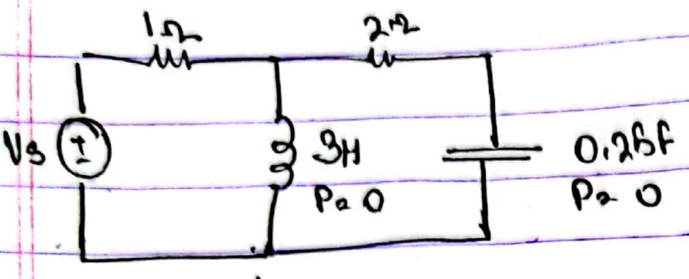
$\theta_I - \theta_V = 90^\circ$, $Z = -j\square$, Power $= 0$

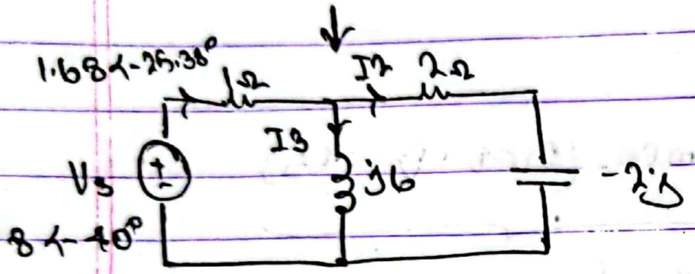
* When load has only inductor, it is a pure inductor. $\rightarrow$

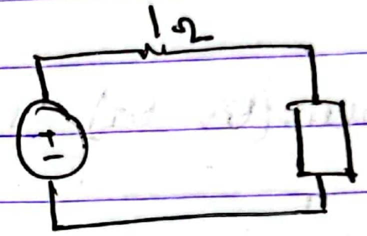
$\theta_I - \theta_V = 90^\circ$, Power $= 0$

Problem 22

$V_s = 8\cos(2t - 40°)$







$(2 - j2) \parallel j6$

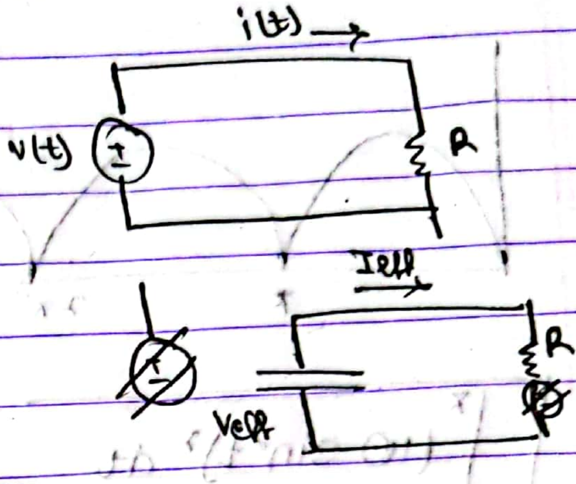
$= 3.6 - j1.2$

$$I = \frac{8\angle -40°}{1 + 3.6 - j1.2} = 1.68\angle -25.38°$$

$$I_2 = \frac{3.6 - j1.2}{(2 - 2j)} \times 1.68\angle -25.38° = 2.25\angle 1.19°$$

$$I_3 = \frac{3.6 - j1.2}{j6} \times 1.68\angle -25.38 = 1.06\angle -133.66°$$

EFFECTIVE OR RMS VALUE



$$P = \frac{1}{T}\int_0^T i^2 R\, dt = R\frac{1}{T}\int_0^T i^2\, dt$$

$$P = I_{eff}^2 R$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt}$$

RMS ~~of~~ value of SINUSOIDS

$$P = I_{rms}^2 R$$

$$V(t) = V_m \cos(\omega t + \phi)$$

$$\Downarrow$$

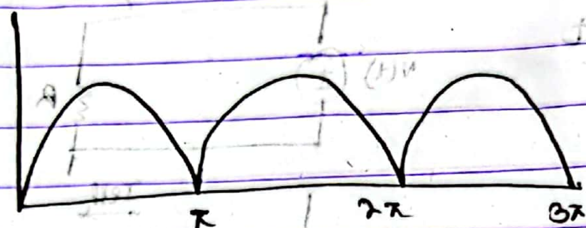
$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

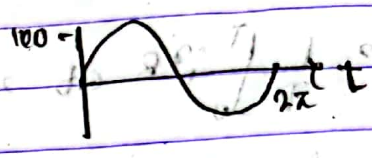
$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_I)$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_I)$$

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_I)$$

Problem 23





$T = 2\pi = \frac{1}{f}$

$v(t) = 100 \sin(2\pi f + \phi)$

$= 100 \sin(2\pi \frac{1}{2\pi} + \phi)$

$= 100 \sin t$

$$rms = \sqrt{\frac{\int_0^{\pi} (10 \sin^2 t)^2 \, dt}{\pi}} = 0.0134$$