CSC 422

State space search

Learning content

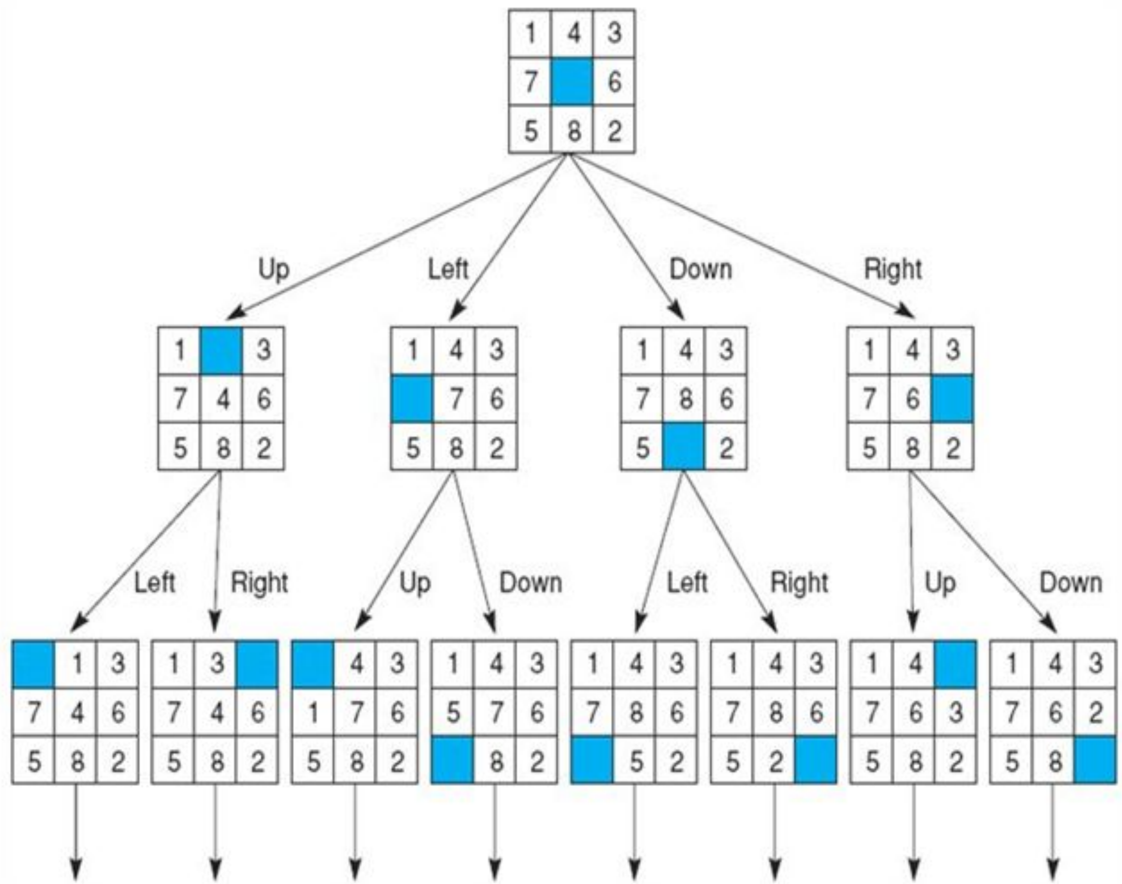
- What is **search** (a.k.a. **state-space search**)?
- What are these concepts in search?
  - Initial state
  - Actions / transition model
  - State space graph
  - Step cost / path cost
  - Goal test (cf. goal)
  - Solution / optimal solution
- What is the difference between **expanding** a state and **generating** a state?
- What is the **frontier** (a.k.a. **open list**)?

Representing actions

- The number of actions / operators depends on the **representation** used in describing a state.
  - In the 8-puzzle, we could specify 4 possible moves for each of the 8 tiles, resulting in a total of **4*8=32 operators**.
  - On the other hand, we could specify four moves for the “blank” square and we would only need **4 operators**.
- Representational shift can greatly simplify a problem!

State Space Search - Example

- generated by "move blank" operations
- ↑ -- up
- ← -- left
- ↓ -- down
- → -- **Right**



Representing states

- What knowledge needs to be represented in a state description to adequately describe the current state or situation of the world?
- The **size of a problem** is usually described in terms of the **number of states** that are possible.
  - **Tic-Tac-Toe has about 3^9 states.**
  - **Checkers has about 10^{40} states.**
  - **Rubik's Cube has about 10^{19} states.**
  - **Chess has about 10^{120} states in a typical game.**

Formalizing Search in a State Space

- A state space is a directed **graph**, (V, E) where V is a set of **nodes** and E is a set of **arcs**, where each arc is directed from a node to another node
- **node:** a **state**
  - state description
  - plus optionally other information related to the parent of the node, operation used to generate the node from that parent, and other *bookkeeping data*

Remember, state space is not a solution space

- **arc:** an instance of an (applicable) action/operation.
  - the source and destination nodes are called as **parent** **(immediate predecessor)** and **child** (**immediate successor**) nodes with respect to each other
  - ancestors (predecessors) and descendents (successors)
  - each arc has a fixed, non-negative **cost** associated with it, corresponding to the cost of the action

Formalizing Search in a State Space

- **State-space search** is the process of searching through a state space for a solution
- This is done by **making explicit** a sufficient portion of an **implicit** state-space graph to include a goal node.
  - Initially V={S}, where S is the start node; when S is expanded, its successors are generated and those nodes are added to V and the associated arcs are added to E.
  - This process continues until a goal node is generated (included in V) and identified (by goal test)
- During search, a node can be in **one of the three categories**:
  - Not generated yet (has not been made explicit yet)
  - **OPEN**: generated but not expanded
  - **CLOSED**: expanded
- Search strategies differ mainly on how to select an OPEN node for expansion at each step of search

State Space Search Algorithm

STATE SPACE SEARCH

A *state space* is represented by a four-tuple [**N,A,S,GD**], where:

N is the set of nodes or states of the graph. These correspond to the states in a problem-solving process.

A is the set of arcs (or links) between nodes. These correspond to the steps in a problem-solving process.

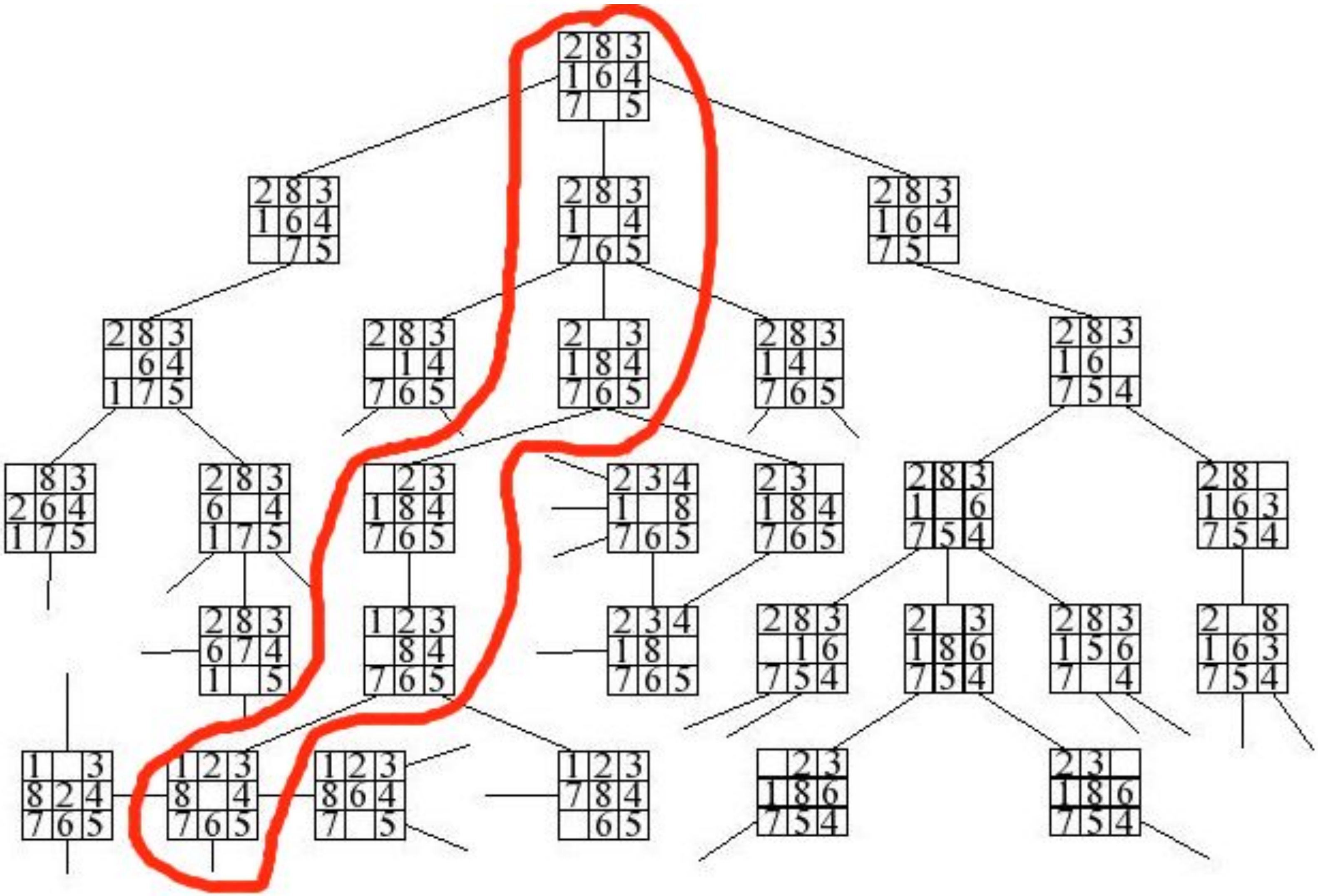
S, a nonempty subset of **N**, contains the start state(s) of the problem.

GD, a nonempty subset of **N**, contains the goal state(s) of the problem. The states in **GD** are described using either:

1. A measurable property of the states encountered in the search.
2. A property of the path developed in the search, for example, the transition costs for the arcs of the path.

A *solution path* is a path through this graph from a node in **S** to a node in **GD**.

Example of State Space Search Algorithm



Key procedures to be defined

- EXPAND -Generate all successor nodes (leaf nodes)
- GOAL-TEST - Test if state satisfies all goal conditions
- QUEUEING-FUNCTION
  - Used to maintain a ranked list of nodes that are candidates for expansion
- Typical node data structure includes:
  - State at this node
  - Parent node (root)
  - Depth of this node (number of operator applications since initial state)
  - Cost of the path (sum of each operator application so far)

Some issues

- Search process constructs a search tree, where
  - **root** is the initial state and
  - **leaf nodes** are nodes
    - not yet expanded (i.e., they are in the list “nodes”) or
    - having no successors (i.e., they’re “dead ends” because no operators were applicable and yet they are not goals)
- Search tree may be infinite because of loops even if state space is small
- Return a path or a node depending on problem.
  - 8-puzzle returns a path
- Changing definition of the QUEUEING-FUNCTION leads to different search strategies

Evaluating search strategies

- A **solution** is a sequence of operators that is associated with a path in a state space from a start node to a goal node.
- The **cost of a solution** is the sum of the arc costs on the solution path.
  - If all arcs have the same (unit) cost, then the solution cost is just the length of the solution (number of steps / state transitions)

Evaluating search strategies

- **Completeness**
  - Guarantees finding a solution whenever one exists
- **Time complexity**
  - How long (worst or average case) does it take to find a solution? Usually measured in terms of the number of nodes expanded
- **Space complexity**
  - How much space is used by the algorithm? Usually measured in terms of the maximum size of the "nodes" list during the search
- **Optimality/Admissibility**
  - If a solution is found, is it guaranteed to be an optimal one? That is, is it the one with minimum cost?

Water Jug Problem

Given a full 5-gallon jug and an empty 2-gallon jug, the goal is to fill the 2-gallon jug with exactly one gallon of water.

- State = (x,y), where x is the number of gallons of water in the 5-gallon jug and y is # of gallons in the 2-gallon jug
- Initial State = (5,0)
- Goal State = (*,1), where * means any amount

Operator table

Name	Cond.	Transition	Effect
Empty5	–	$(x,y)\rightarrow(0,y)$	Empty 5-gal. jug
Empty2	–	$(x,y)\rightarrow(x,0)$	Empty 2-gal. jug
2to5	$x \leq 3$	$(x,2)\rightarrow(x+2,0)$	Pour 2-gal. into 5-gal.
5to2	$x \geq 2$	$(x,0)\rightarrow(x-2,2)$	Pour 5-gal. into 2-gal.
5to2part	$y < 2$	$(1,y)\rightarrow(0,y+1)$	Pour partial 5-gal. into 2-gal.

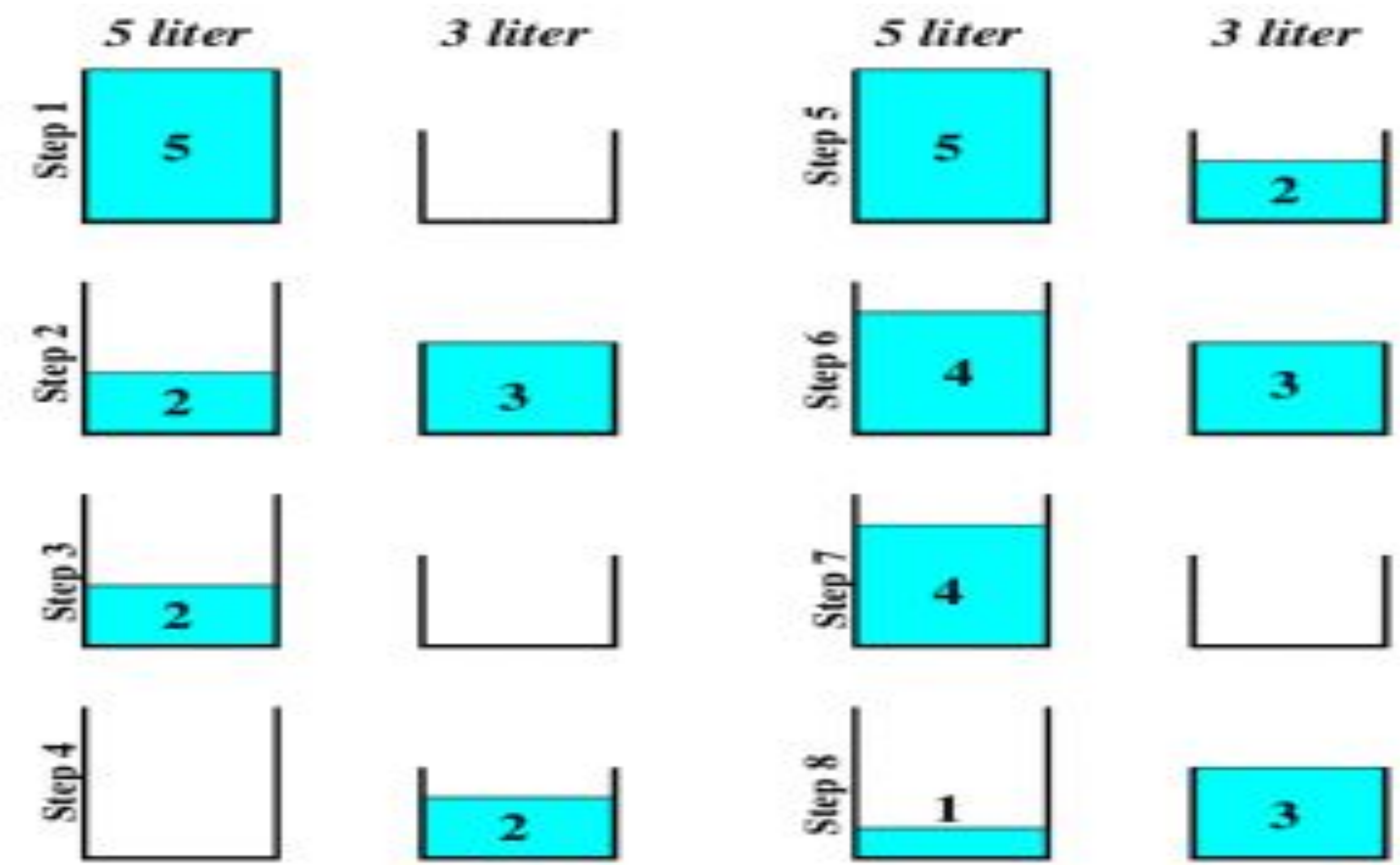
Water Jug Problem (cont.)

- To solve this we have to make some assumptions not mentioned in the problem. They are
- 1. We can fill a jug from the pump.
- 2. we can pour water out of a jug to the ground.
- 3. We can pour water from one jug to another.
- 4. There is no measuring device available.

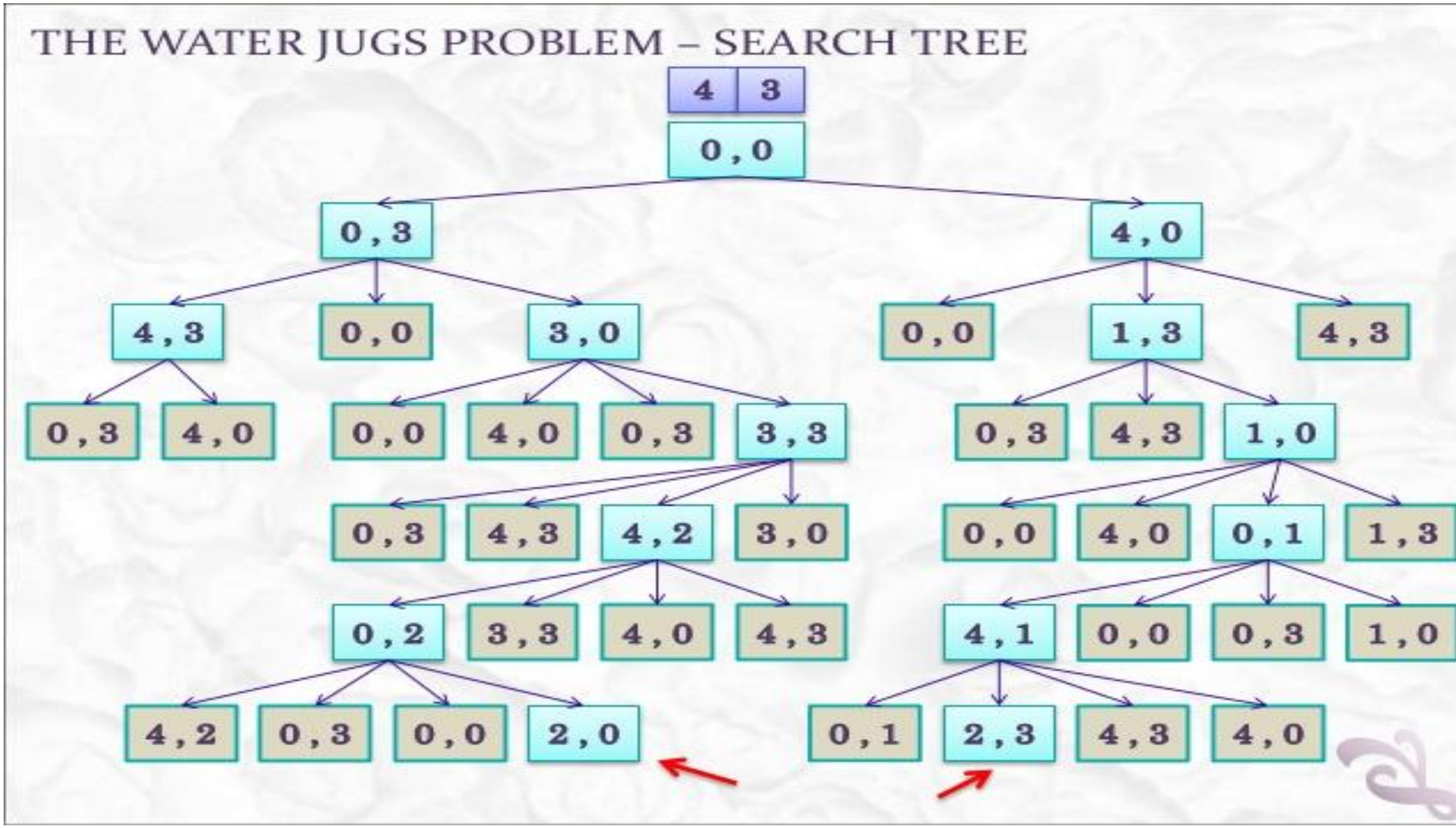
This is one of the solution

0-3
3-0
3-3
4-2
0-2
2-0

Water Jug Problem (cont.)



Water Jug Problem - Example



State Rules/Condition

1.	(X, Y) if $X < 4 \rightarrow (4, Y)$	Fill the 4-gallon jug
2.	(X, Y) if $Y < 3 \rightarrow (X, 3)$	Fill the 3-gallon jug
3.	(X, Y) if $X = d$ & $d > 0 \rightarrow (X\text{-}d, Y)$	Pour some water out of the 4-gallon jug
4.	(X, Y) if $Y = d$ & $d > 0 \rightarrow (X, Y-d)$	Pour some water out of 3-gallon jug
5.	(X, Y) if $X > 0 \rightarrow (0, Y)$	Empty the 4-gallon jug on the ground
6.	(X, Y) if $Y > 0 \rightarrow (X, 0)$	Empty the 3-gallon jug on the ground
7.	(X, Y) if $X + Y \leq 4$ and $Y > 0 \rightarrow 4, (Y - (4 - X))$	Pour water from the 3-gallon jug into the 4-gallon jug until the gallon jug is full.
8.	(X, Y) if $X + Y \geq 3$ and $X > 0 \rightarrow (X - (3 - Y), 3))$	Pour water from the 4-gallon jug into the 3-gallon jug until the 3-gallon jug is full.
9.	(X, Y) if $X + Y \leq 4$ and $Y > 0 \rightarrow (X + Y, 0)$	Pour all the water from the 3-gallon jug into the 4-gallon jug
10.	(X, Y) if $X + Y \leq 3$ and $X > 0 \rightarrow (0, X + Y)$	Pour all the water from the 4-gallon jug into the 3-gallon jug
11.	$(0, 2) \rightarrow (2, 0)$	Pour the 2-gallons water from 3-gallon jug into the 4;gallon jug
12.	$(2, Y) \rightarrow (0, Y)$	Empty the 2-gallons in the 4-gallon jug on the ground.

Fig. 2.3. *Production rules (operators) for the water jug problem.*

Water Jug Problem - Example

Water in 4-gallon jug (X)	Water in 3-gallon jug (Y)	Rule applied
0	0	
0	3	2
3	0	9
3	3	2
4	2	7
0	2	5 or 12
2	0	9 or 11

Fig. 2.4 (*a*). *A solution to water jug problem.*

X	*Y*	Rule applied (Control strategy)
0	0	
4	0	I-
1	3	8
1	0	6
0	1	10
4	1	1
2	3	8

Fig. 2.4 (*b*). *2nd solution to water jug problem.*

CLASS EXERCISE

- Representing a Sudoku puzzle as a search space
  - What are the states?
  - What are the operators?
  - What are the constraints (on operator application)?
  - What is the description of the goal state?

- Let’s try it!

	3		
			1
3			
		2	