# Game Playing

# Today's class

- Game playing
  - State of the art and resources
  - Framework
- Game trees
  - Minimax
  - Alpha-beta pruning
  - Adding randomness

# Why study games?

- Clear criteria for success
- Offer an opportunity to study problems involving {hostile, adversarial, competing} agents.
- Historical reasons
- Fun
- Interesting, hard problems which require minimal “initial structure”
- Games often define very large search spaces
  - chess $35^{100}$ nodes in search tree, $10^{40}$ legal states

# State of the art

- How good are computer game players?
  - **Chess**:
    - Deep Blue beat Gary Kasparov in 1997
    - Garry Kasparav vs. Deep Junior (Feb 2003): tie!
    - Kasparov vs. X3D Fritz (November 2003): tie! http://www.thechessdrum.net/tournaments/Kasparov-X3DFritz/index.html
    - Deep Fritz beat world champion Vladimir Kramnik (2006)
  - **Checkers**: Chinook (an AI program with a *very large* endgame database) is the world champion and can provably never be beaten. Retired in 1995
  - **Go**: Computer players have finally reached tournament-level play
  - **Bridge**: "Expert-level" computer players exist (but no world champions yet!)
- Good places to learn more:
  - http://www.cs.ualberta.ca/~games/
  - http://www.cs.unimass.nl/icga

# Typical case

- 2-person game
- Players alternate moves
- **Zero-sum**: one player's loss is the other's gain
- **Perfect information**: both players have access to complete information about the state of the game. No information is hidden from either player.
- No chance (e.g., using dice) involved
- Examples: Tic-Tac-Toe, Checkers, Chess, Go, Nim, Othello
- Not: Bridge,  Solitaire, Backgammon, ...

# How to play a game

- A way to play such a game is to:
  - Consider all the legal moves you can make
  - Compute the new position resulting from each move
  - Evaluate each resulting position and determine which is best
  - Make that move
  - Wait for your opponent to move and repeat
- Key problems are:
  - Representing the “board”
  - Generating all legal next boards
  - Evaluating a position

# Evaluation function

- **Evaluation function** or **static evaluator** is used to evaluate the "goodness" of a game position.
  - Contrast with heuristic search where the evaluation function was a non-negative estimate of the cost from the start node to a goal and passing through the given node
- The zero-sum assumption allows us to use a single evaluation function to describe the goodness of a board with respect to both players.
  - **f(n) >> 0**: position n good for me and bad for you
  - **f(n) << 0**: position n bad for me and good for you
  - **f(n) near 0**: position n is a neutral position
  - **f(n) = +infinity**: win for me
  - **f(n) = -infinity**: win for you

# Evaluation function examples

- Example of an evaluation function for Tic-Tac-Toe:

  f(n) = [# of 3-lengths open for me] - [# of 3-lengths open for you]

  where a 3-length is a complete row, column, or diagonal

- Alan Turing’s function for chess
  - **f(n) = w(n)/b(n)** where w(n) = sum of the point value of white’s pieces and b(n) = sum of black’s

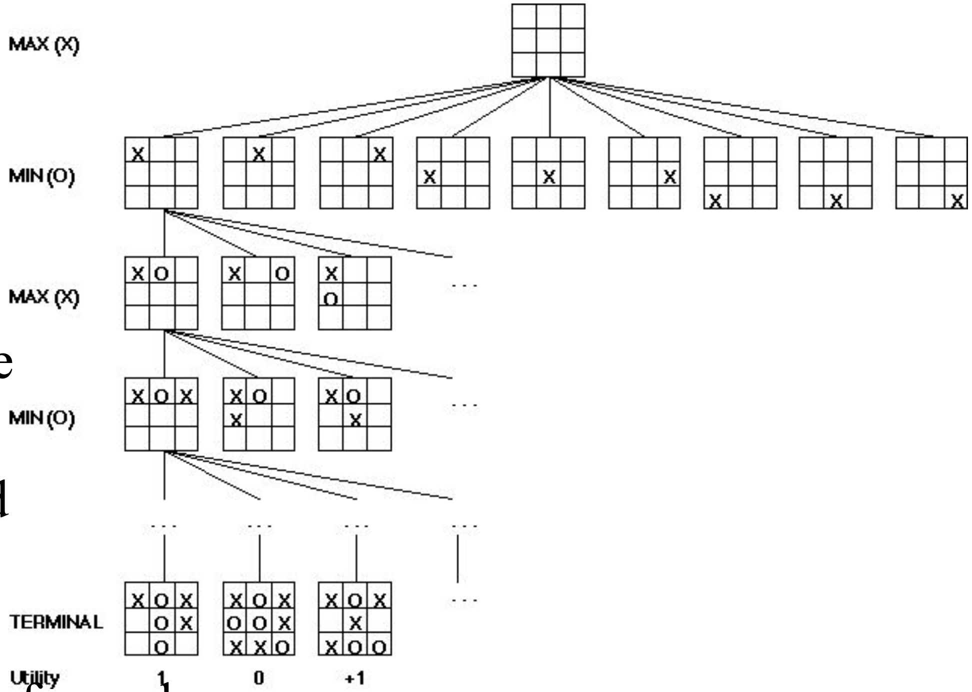
- Most evaluation functions are specified as a weighted sum of position features:

  $f(n) = w_1 * feat_1(n) + w_2 * feat_2(n) + ... + w_n * feat_k(n)$

- Example features for chess are piece count, piece placement, squares controlled, etc.

- Deep Blue had over 8000 features in its evaluation function

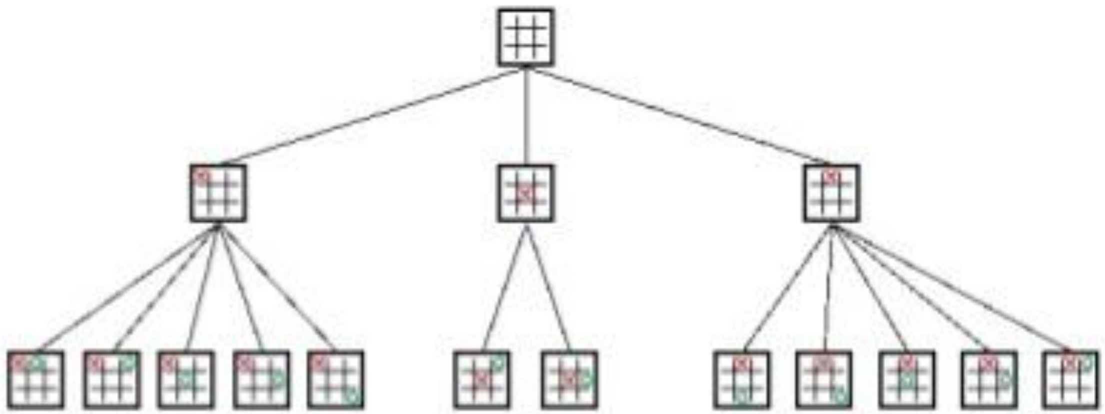
# Game trees




- Problem spaces for typical games are represented as trees
- Root node represents the current board configuration; player must decide the best single move to make next
- **Static evaluator function** rates a board position. f(board) = real number with f>0 “white” (me), f<0 for black (you)
- Arcs represent the possible legal moves for a player
- If it is **my turn** to move, then the root is labeled a "**MAX**" node; otherwise it is labeled a "**MIN**" node, indicating **my opponent's turn**.
- Each level of the tree has nodes that are all MAX or all MIN; nodes at level i are of the opposite kind from those at level i+1

# MinMax - Overview

- Search tree
  - *Squares* represent decision states (ie- after a move)
  - *Branches* are decisions (ie- the move)
  - Start at root
  - Nodes at end are leaf nodes
  - Ex: Tic-Tac-Toe (symmetrical positions removed)

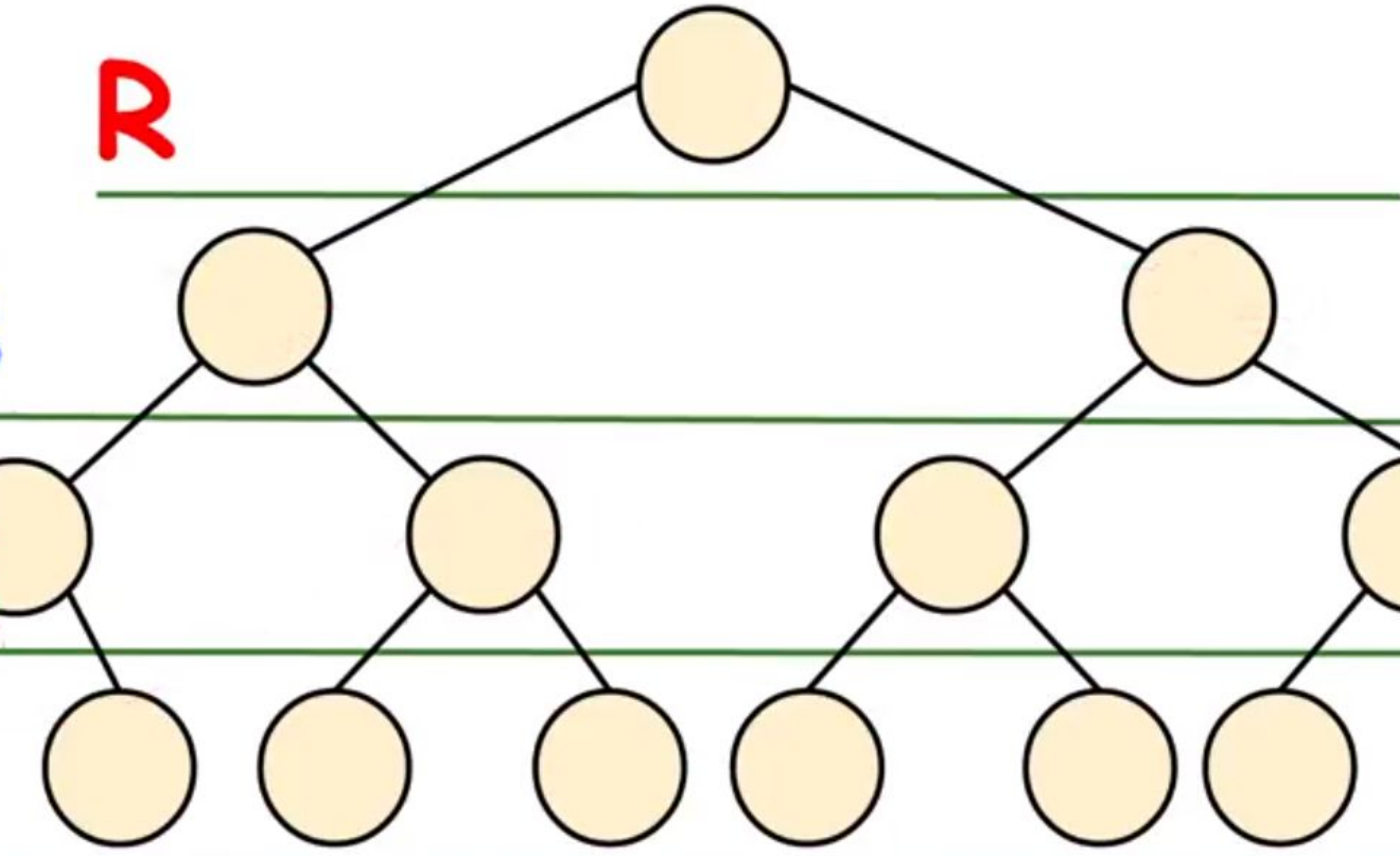


- Unlike binary trees can have any number of children
  - Depends on the game situation
- Levels usually called *plies* (a *ply* is one level)
  - Each ply is where "turn" switches to other player
- Players called *Min* and *Max* (next)

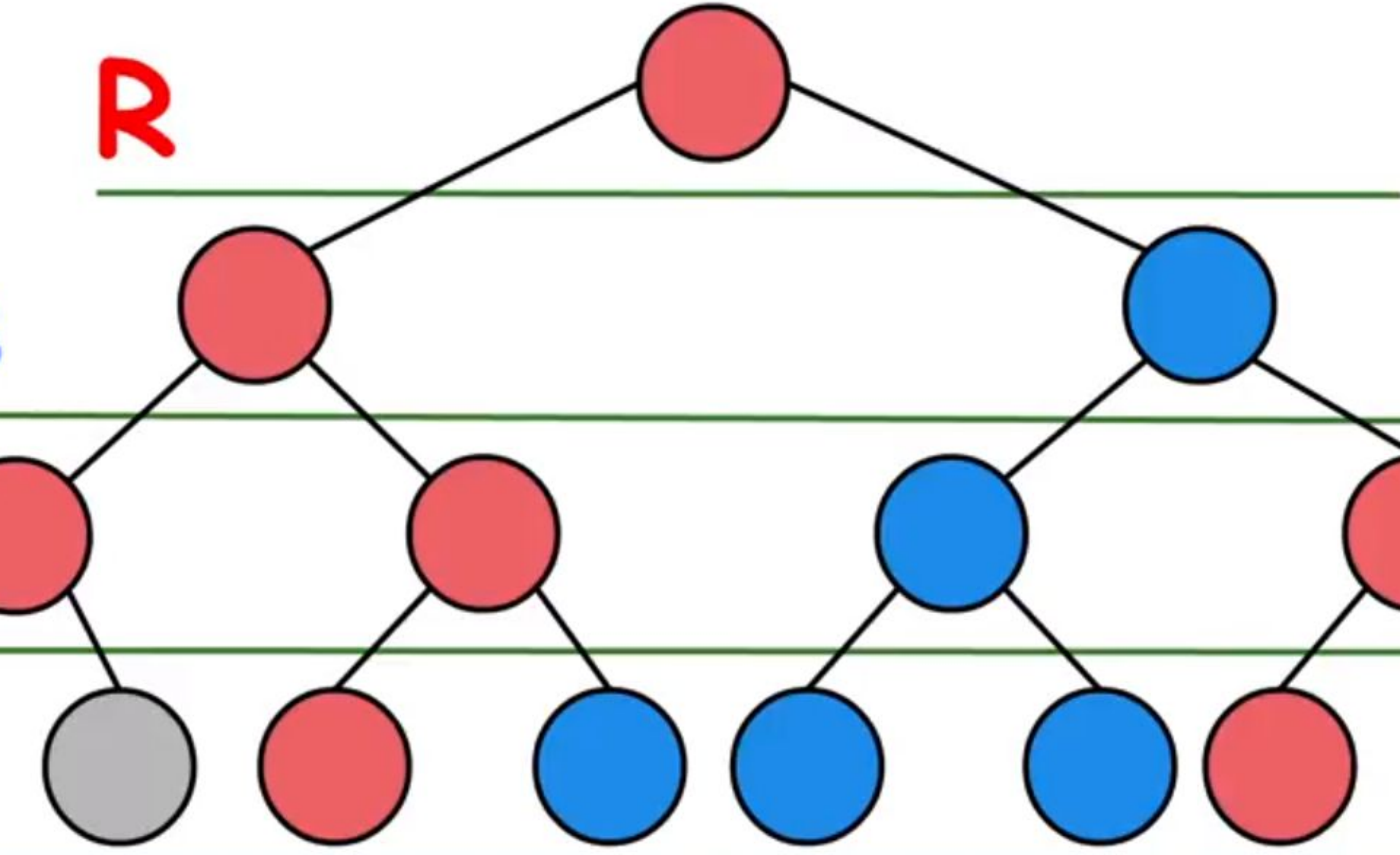
# Minimax procedure

- Create start node as a MAX node with current board configuration
- Expand nodes down to some **depth** (a.k.a. **ply**) of lookahead in the game
- Apply the evaluation function at each of the leaf nodes
- "Back up" values for each of the non-leaf nodes until a value is computed for the root node
  - At MIN nodes, the backed-up value is the **minimum** of the values associated with its children.
  - At MAX nodes, the backed-up value is the **maximum** of the values associated with its children.
- Pick the operator associated with the child node whose backed-up value determined the value at the root

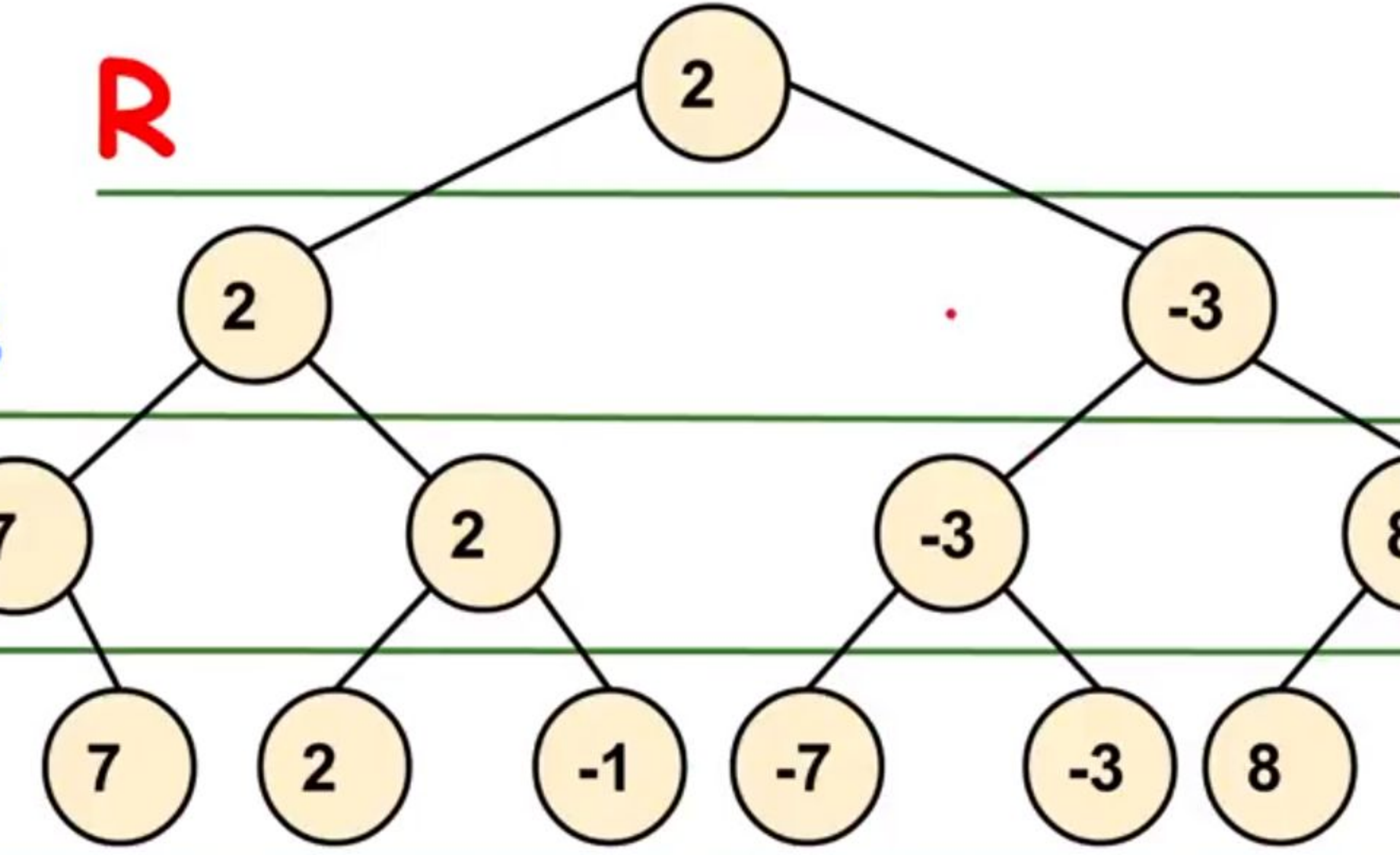
# Minimax



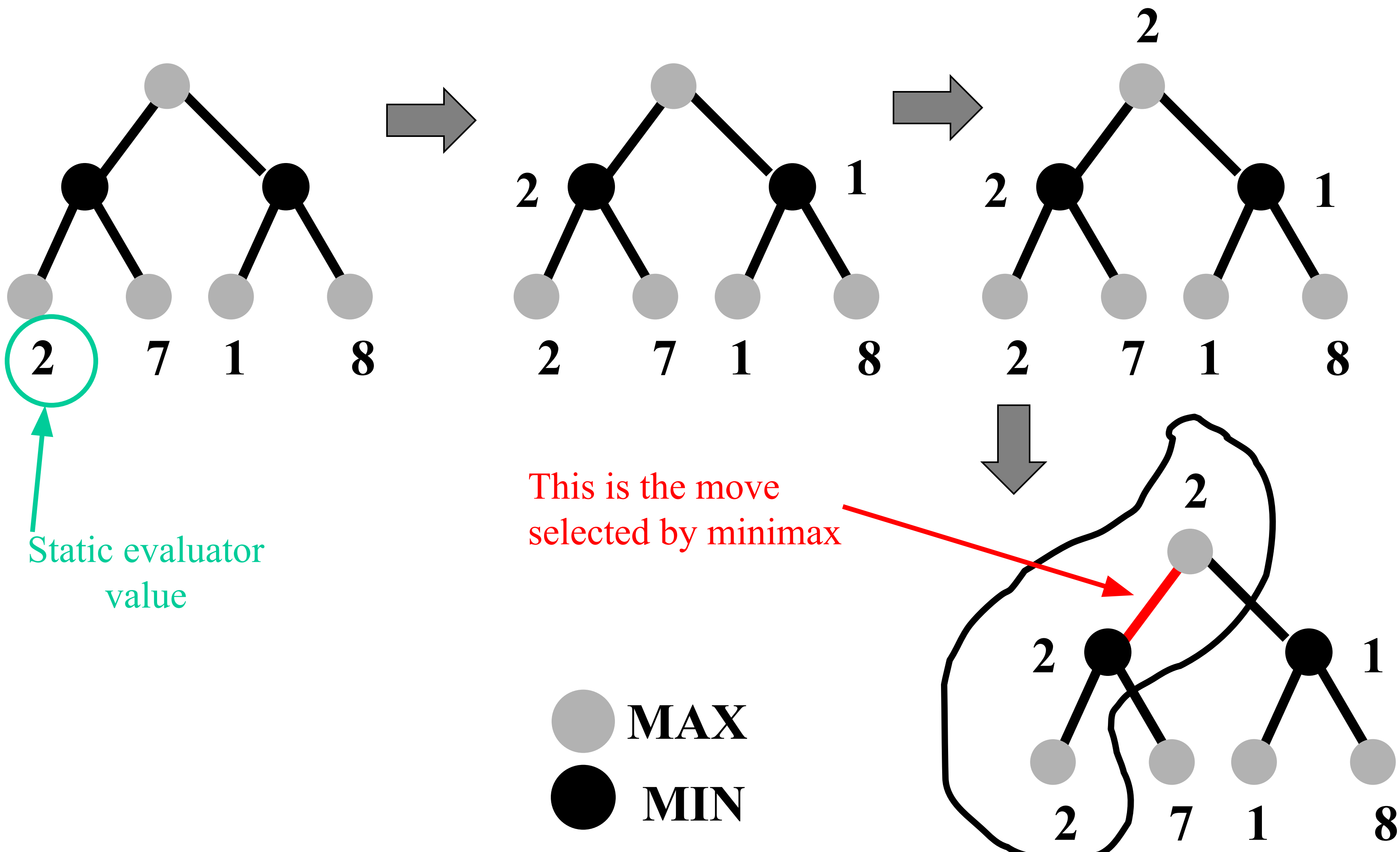
# Minimax

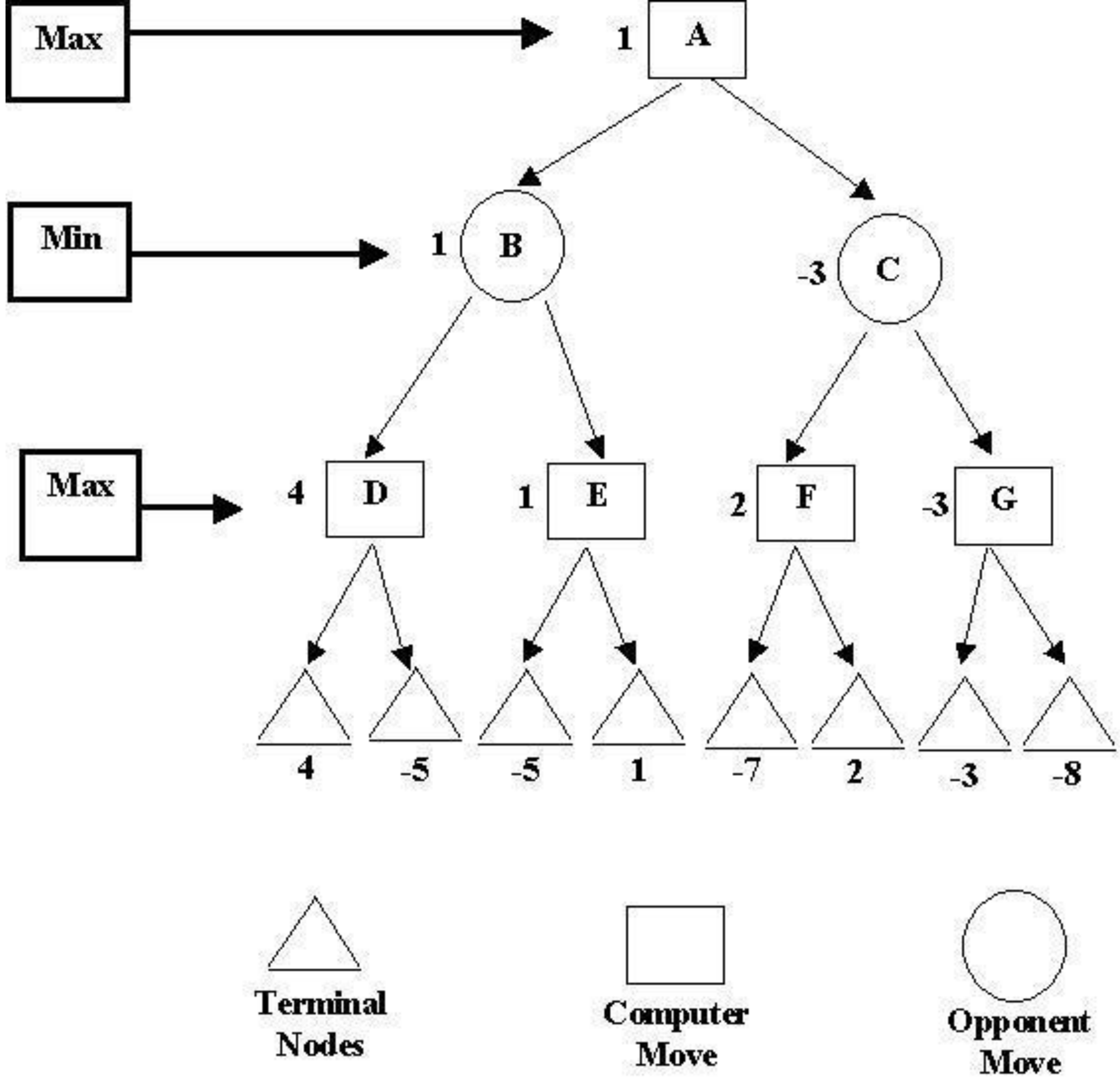


# Minimax

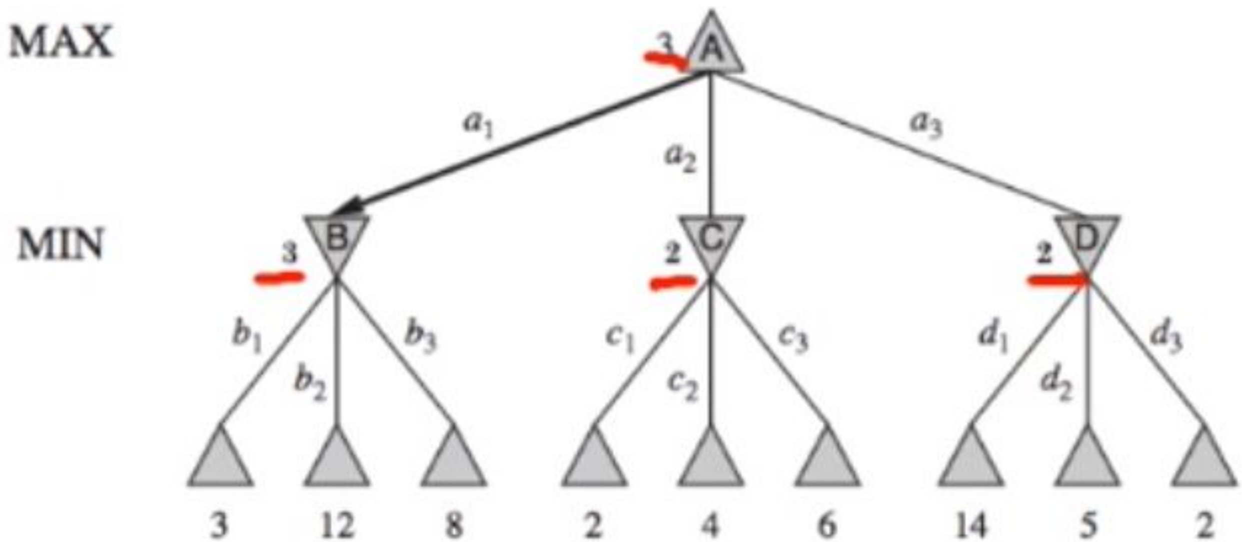


# Minimax Algorithm



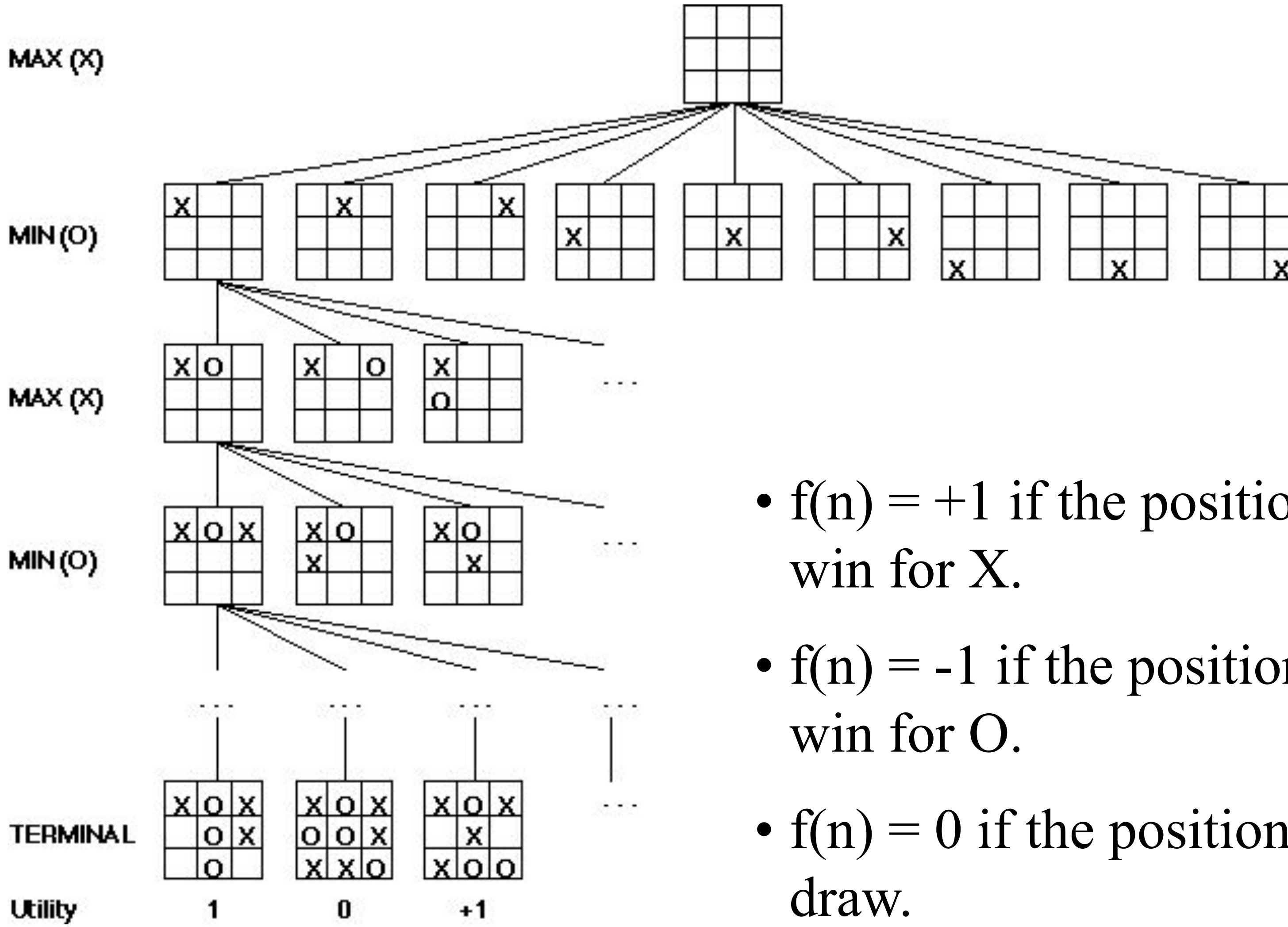

Max
1
A
Min
1
B
-3
C
Max
4
D
1
E
2
F
-3
G
4
-5
-5
1
-7
2
-3
-8
Terminal Nodes
Computer Move
Opponent Move

# Minimax
## Picking my best move against your best move



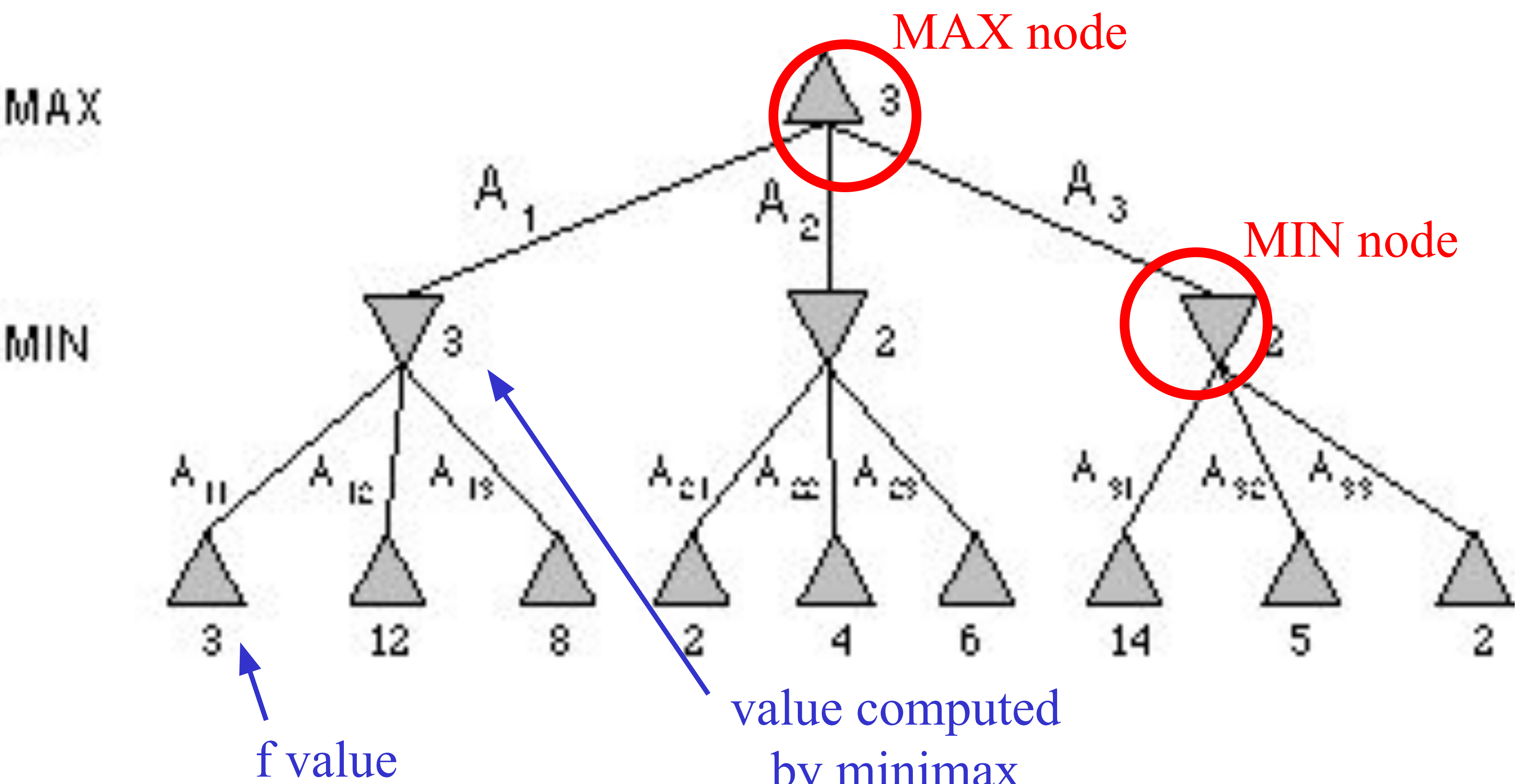

$$minimax(s) = \begin{cases} \underline{utility(s)} & \text{if } \underline{terminal(s)} \\ \max_{a \in action(s)} minimax(result(s,a)) & \text{if } \underline{player(s) = MAX} \\ \min_{a \in action(s)} minimax(result(s,a)) & \text{if } player(s) = MIN \end{cases}$$

# Partial Game Tree for Tic-Tac-Toe




- f(n) = +1 if the position is a win for X.
- f(n) = -1 if the position is a win for O.
- f(n) = 0 if the position is a draw.

# Minimax Tree

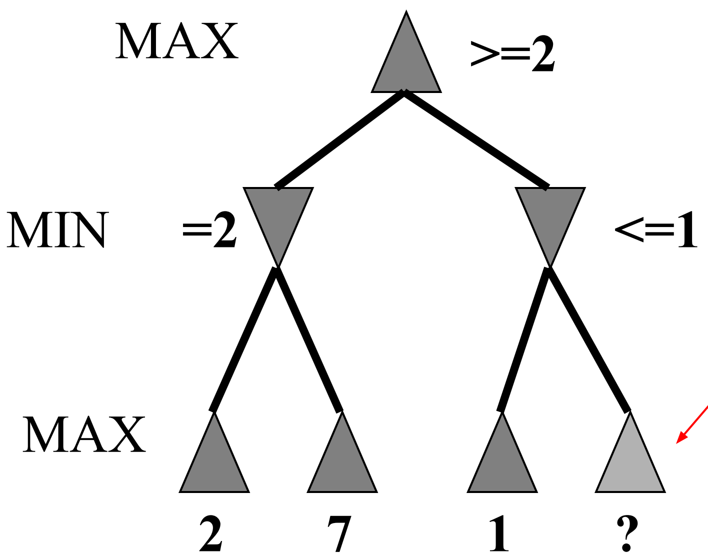


# Minimax
## Discussion

- Complete depth first exploration
- Depth $m$ with $b$ legal moves. $O(b^m)$
- Space complexity (memory) $O(bm)$
- Chess: $m \approx 35$; on average:$50 \leq b \leq 100$
- Impractical for most games, but basis of other algs.

# Alpha-beta pruning

- We can improve on the performance of the minimax algorithm through **alpha-beta pruning**
- Basic idea: *"If you have an idea that is surely bad, don't take the time to see how truly awful it is."* -- Pat Winston

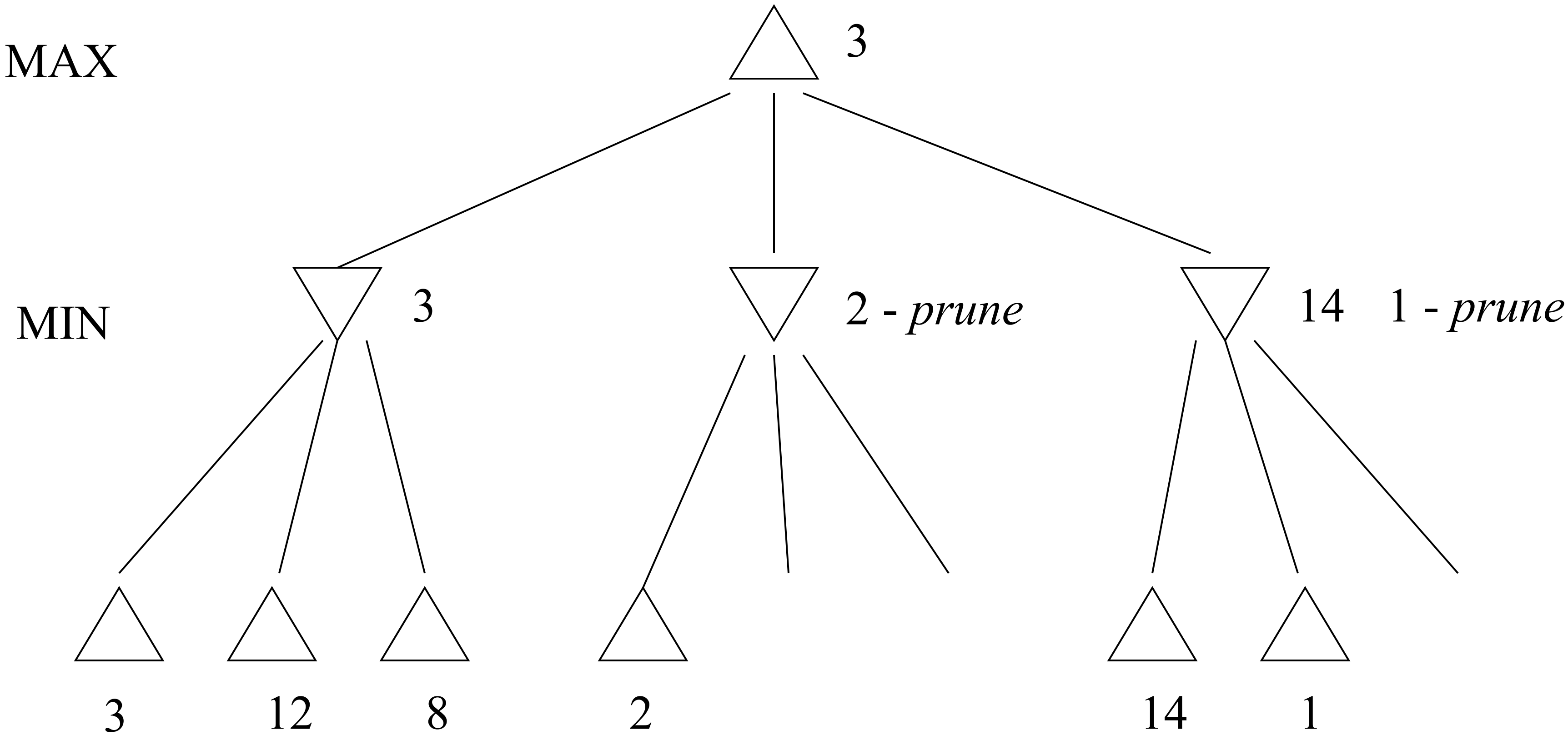



- We don't need to compute the value at this node.
- No matter what it is, it can't affect the value of the root node.

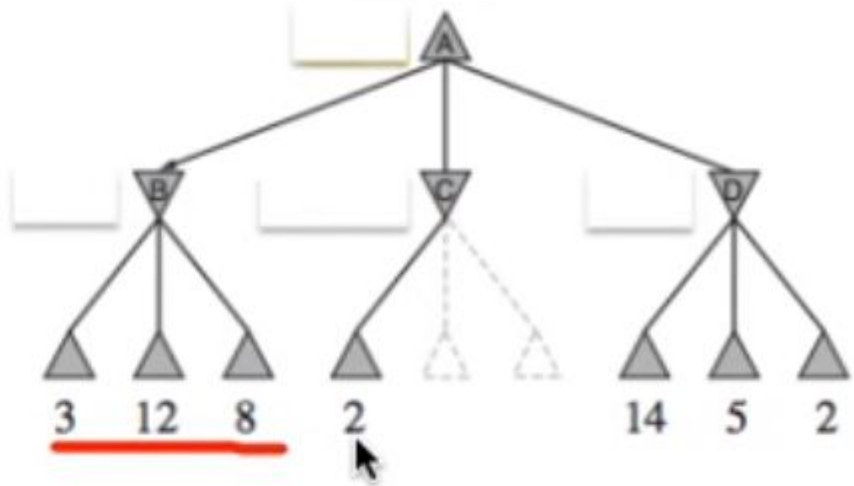
# Alpha-beta pruning

- Traverse the search tree in depth-first order
- At each **MAX** node n, **alpha(n)** = maximum value found so far
- At each **MIN** node n, **beta(n)** = minimum value found so far
  - Note: The alpha values start at -infinity and only increase, while beta values start at +infinity and only decrease.
- **Beta cutoff**: Given a MAX node n, cut off the search below n (i.e., don’t generate or examine any more of n’s children) if alpha(n) >= beta(i) for some MIN node ancestor i of n.
- **Alpha cutoff:** stop searching below MIN node n if beta(n) <= alpha(i) for some MAX node ancestor i of n.

# Alpha-beta example



# Alha-Beta prunning
## Intuition




Do we need to expand all nodes?

$$
\begin{aligned}
minimax(root) &= max(min(3, 12, 8), min(2, x, y), min(14, 5, 2)) \\
&= max(3, min(2, x, y), 2) \\
&= max(3, z, 2) \\
&= 3
\end{aligned}
$$

Do we need $z$?

# Alpha-beta algorithm

```
function MAX-VALUE (state, α, β)
    ;; α = best MAX so far; β = best MIN
if TERMINAL-TEST (state) then return UTILITY(state)
v := -∞
for each s in SUCCESSORS (state) do
    v := MAX (v, MIN-VALUE (s, α, β))
    if v >= β then return v
    α := MAX (α, v)
end
return v

function MIN-VALUE (state, α, β)
if TERMINAL-TEST (state) then return UTILITY(state)
v := ∞
for each s in SUCCESSORS (state) do
    v := MIN (v, MAX-VALUE (s, α, β))
    if v <= α then return v
    β := MIN (β, v)
end
return v
```

# Alpha–Beta prunning

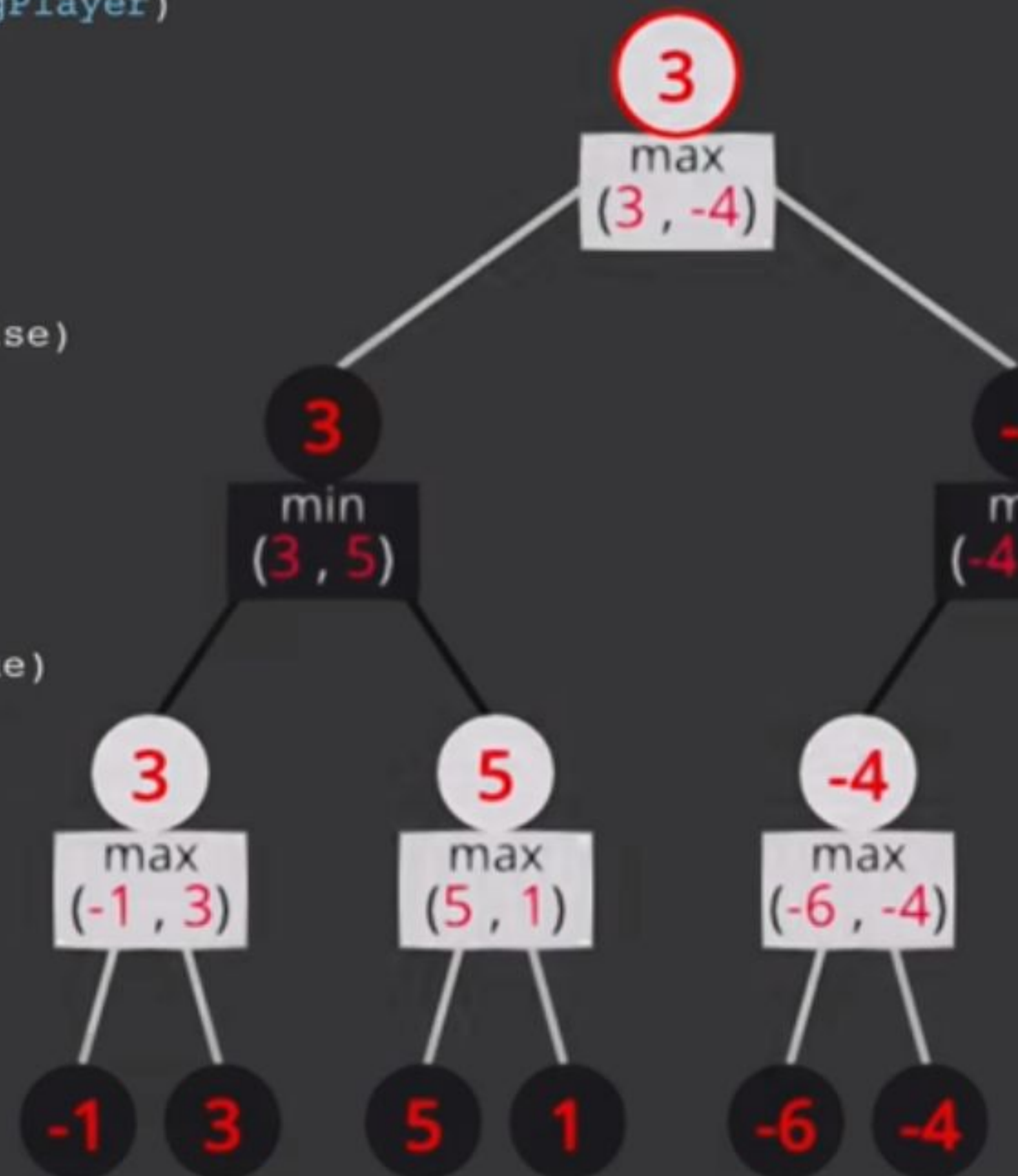
Two values:

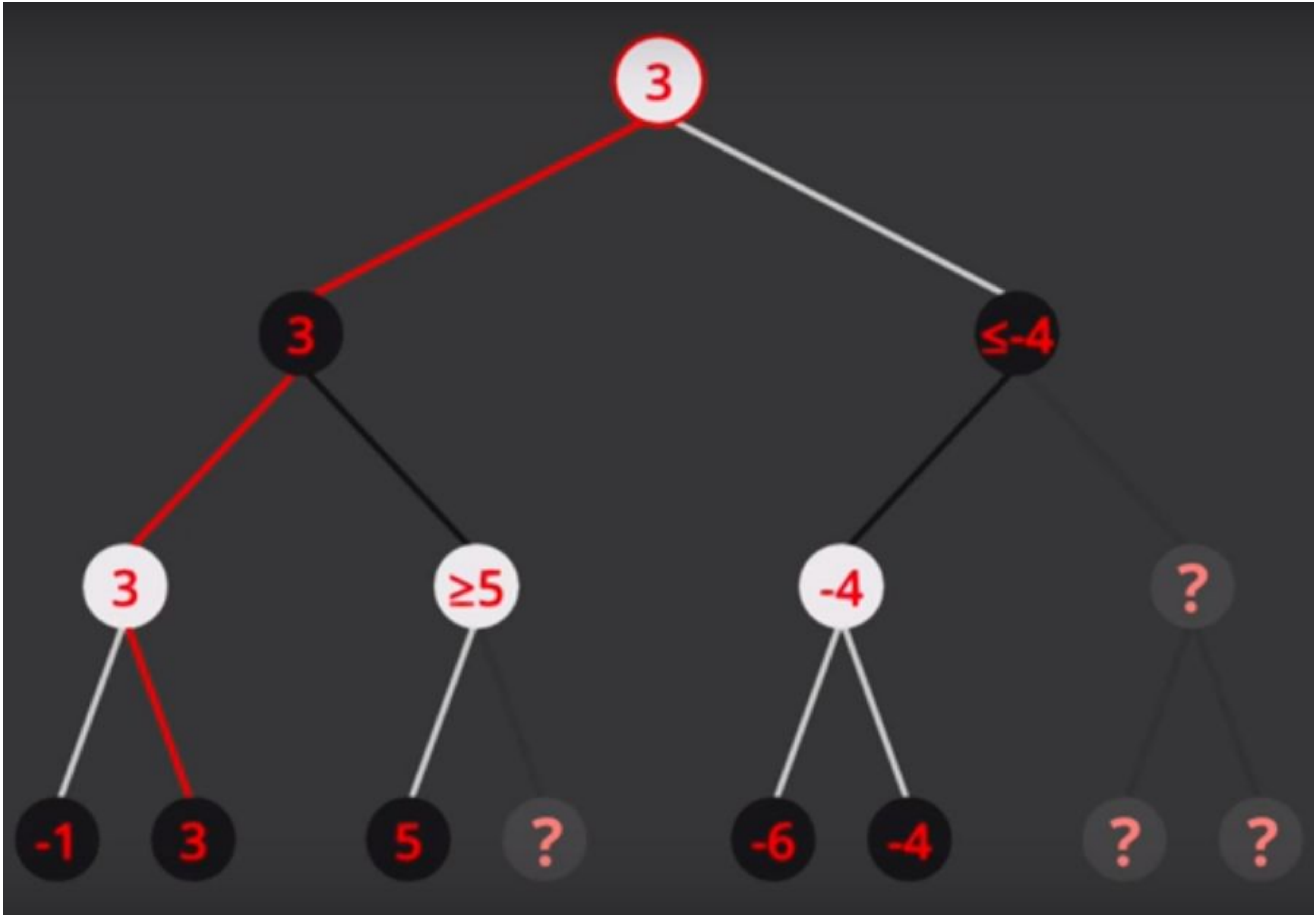
- $\alpha =$ value of best choice so far for MAX (highest-value)
- $\beta =$ value of best choice so far for MIN (lowest-value)
- Each node keeps track of its $[\alpha, \beta]$ values

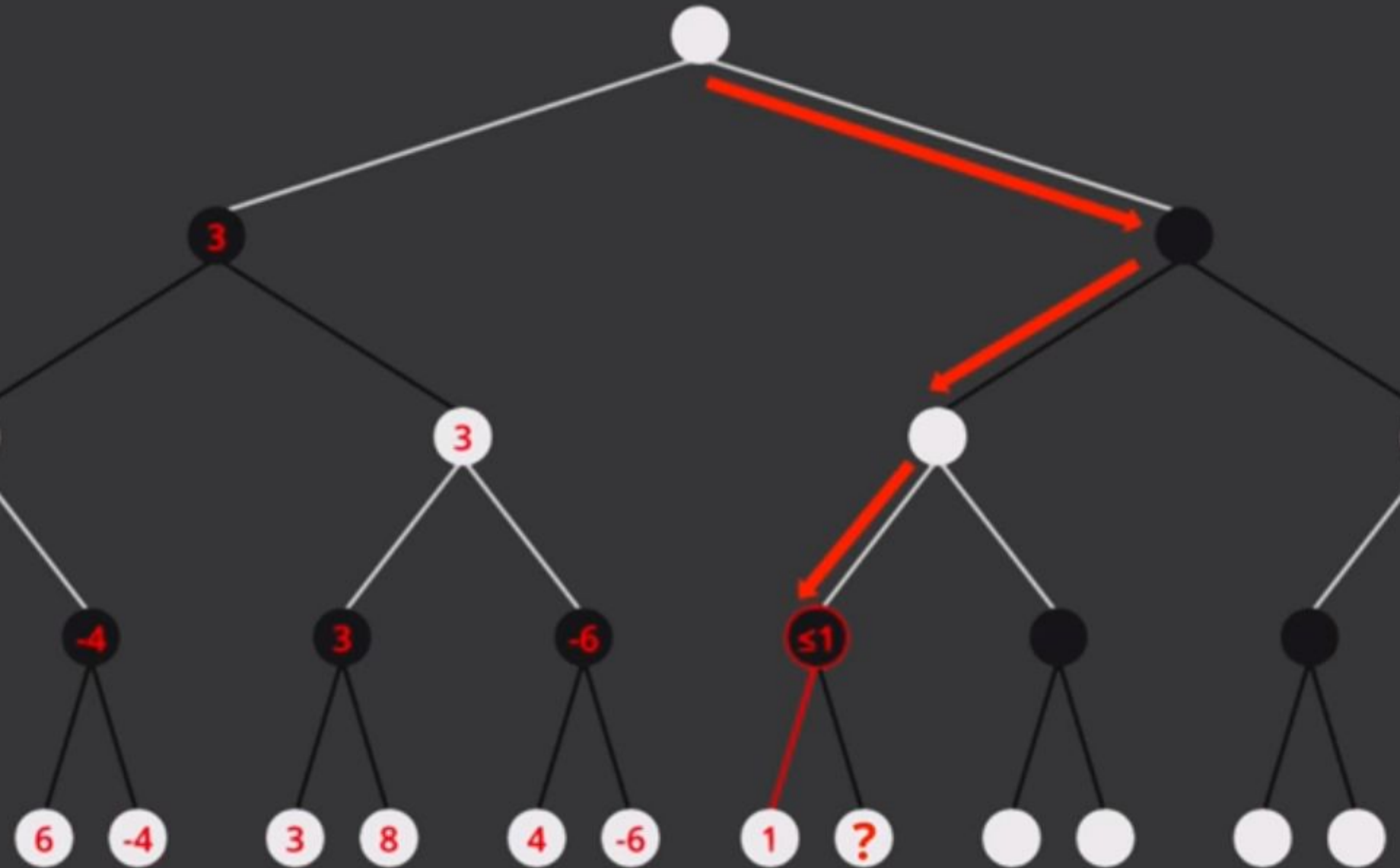
# Alpha-Beta Prunning
## Properties

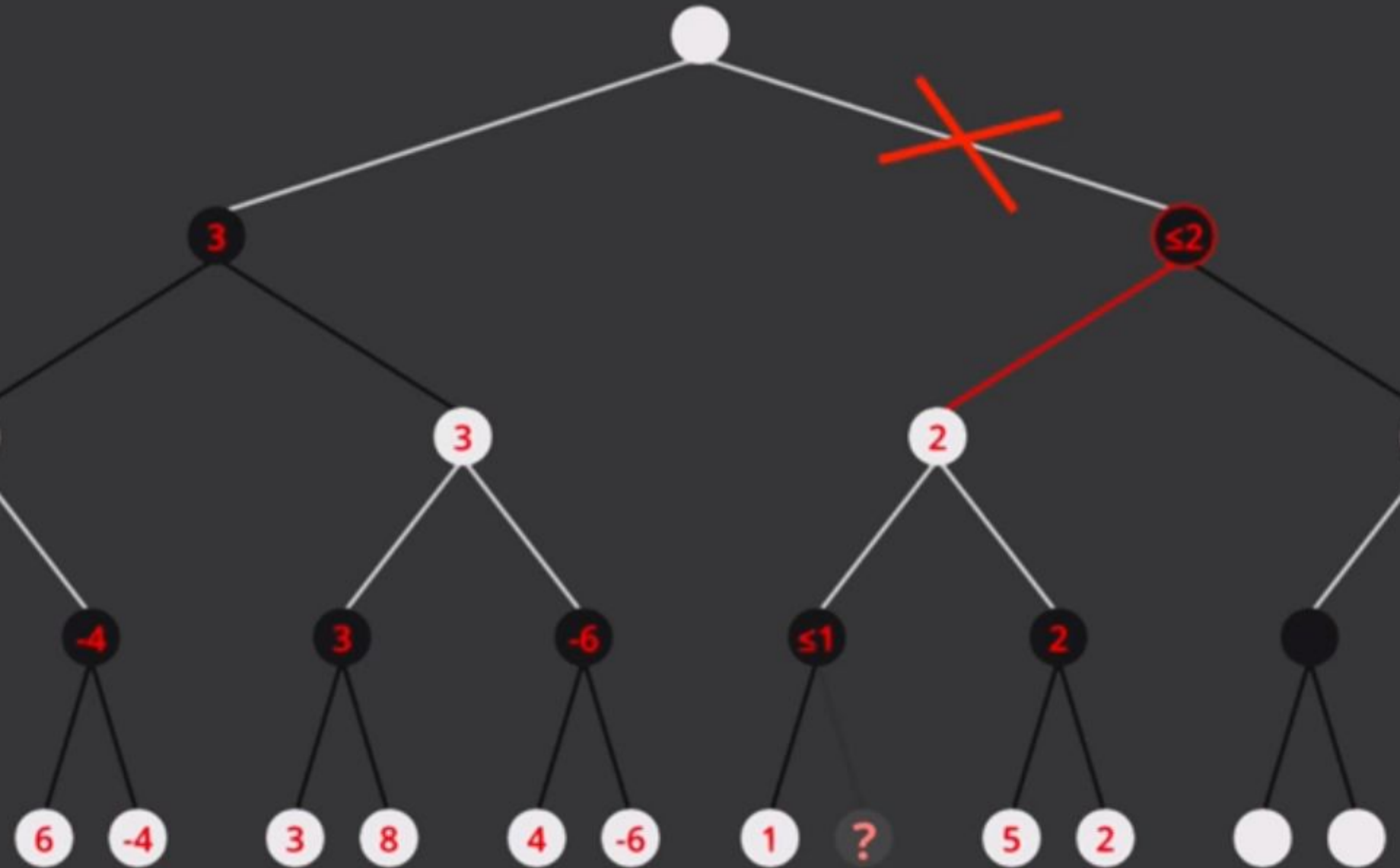
- Prunning does not affect final outcome
- Sorting moves by result improves $\alpha - \beta$ performance
- Perfect ordering: $O(b^{\frac{m}{2}})$
- An exercise on **metareasoning**

```
max(position, depth, maximizingPlayer)
= 0 or game over in position
static evaluation of position

ingPlayer
 = -infinity
 child of position
= minimax(child, depth - 1, false)
al = max(maxEval, eval)
naxEval


 = +infinity
 child of position
= minimax(child, depth - 1, true)
al = min(minEval, eval)
ninEval


ll
ntPosition, 3, true)
```



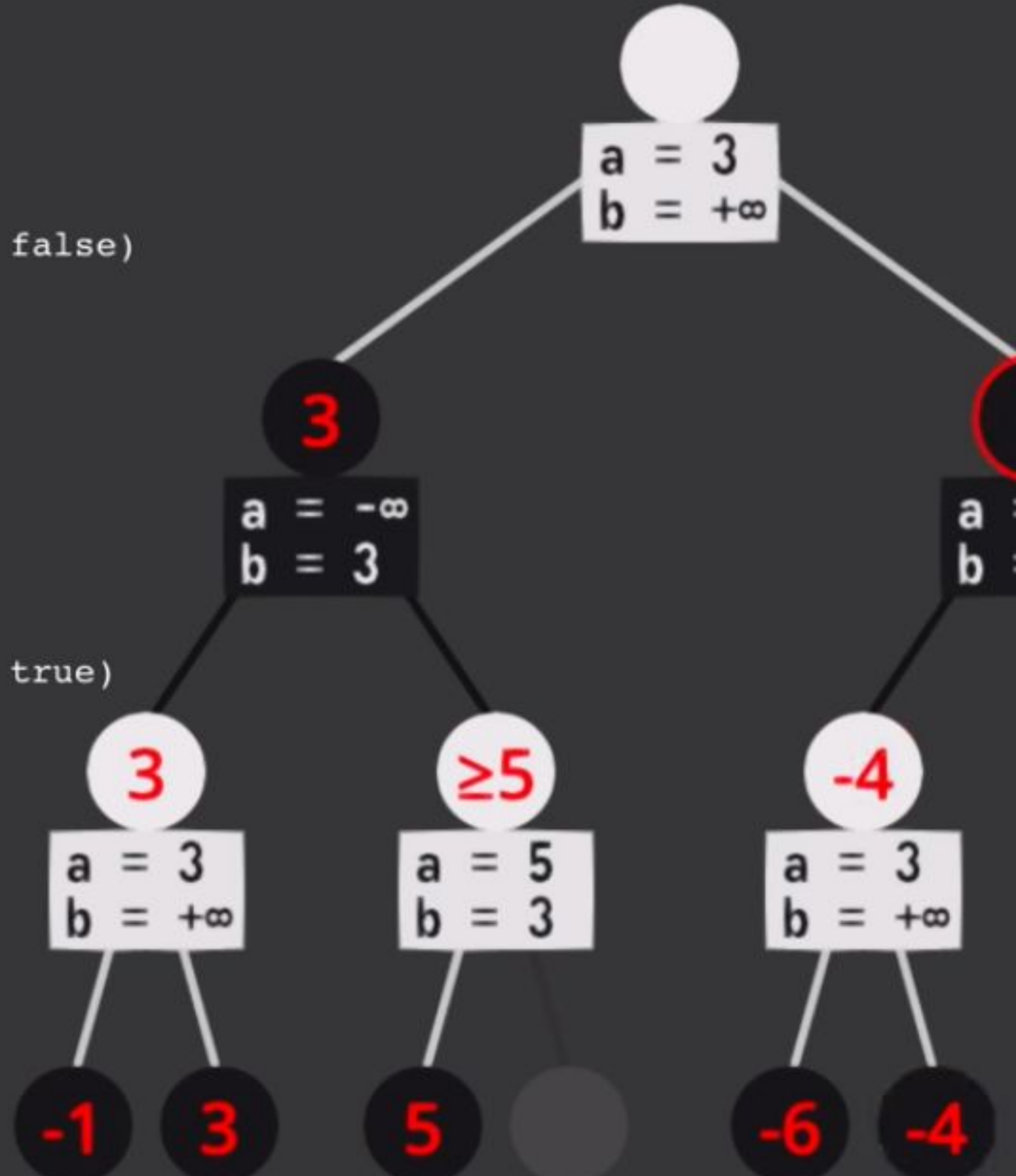

3
3
≤-4
3
≥5
-4
?
-1
3
5
?
-6
-4
?
?


3
3
-4
3
-6
≤1
6
-4
3
8
4
-6
1
?


3
≤2
3
2
-4
3
-6
≤1
2
6
-4
3
8
4
-6
1
?
5
2

```
x(position, depth, alpha, beta, maximizingPlayer)
 or game over in position
tic evaluation of position

Player
-infinity
hild of position
inimax(child, depth - 1, alpha, beta, false)
= max(maxEval, eval)
max(alpha, eval)
<= alpha

Eval


+infinity
hild of position
inimax(child, depth - 1, alpha, beta, true)
= min(minEval, eval)
in(beta, eval)
<= alpha

Eval
```

```
ll
ntPosition, 3, -∞, +∞, true)
```



# Effectiveness of alpha-beta

- Alpha-beta is guaranteed to compute the same value for the root node as computed by minimax, with less or equal computation
- **Worst case:** no pruning, examining $b^d$ leaf nodes, where each node has b children and a d-ply search is performed
- **Best case:** examine only $(2b)^{d/2}$ leaf nodes.
  - Result is you can search twice as deep as minimax!
- **Best case** is when each player's best move is the first alternative generated
- In Deep Blue, they found empirically that alpha-beta pruning meant that the average branching factor at each node was about 6 instead of about 35!

# Games of chance

- Backgammon is a two-player game with **uncertainty**.
- Players roll dice to determine what moves to make.
- White has just rolled *5 and 6* and has four legal moves:
    - 5-10, 5-11
    - 5-11, 19-24
    - 5-10, 10-16
    - 5-11, 11-16
- Such games are good for exploring decision making in adversarial problems involving skill and luck.