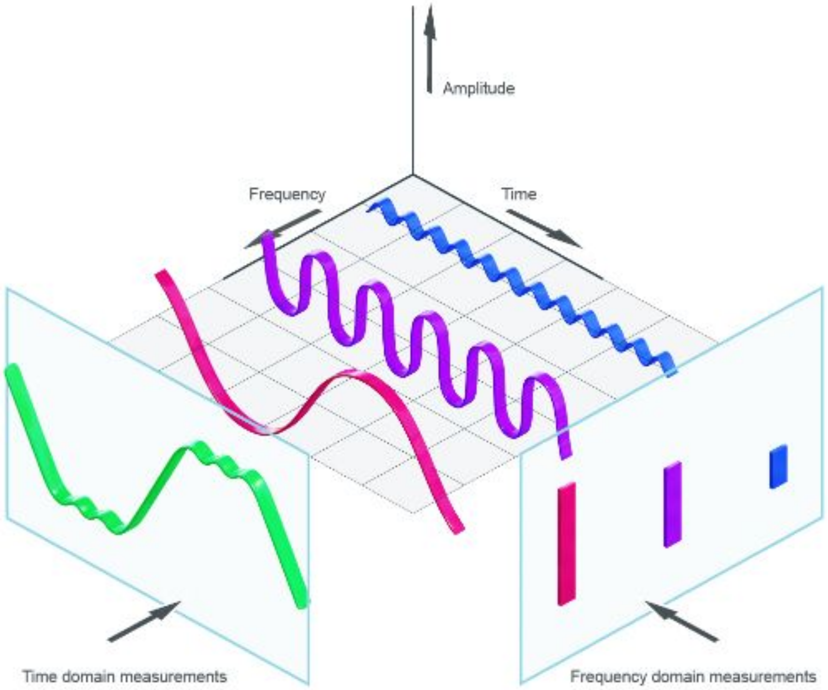
Introduction to Robotics

CSE 461

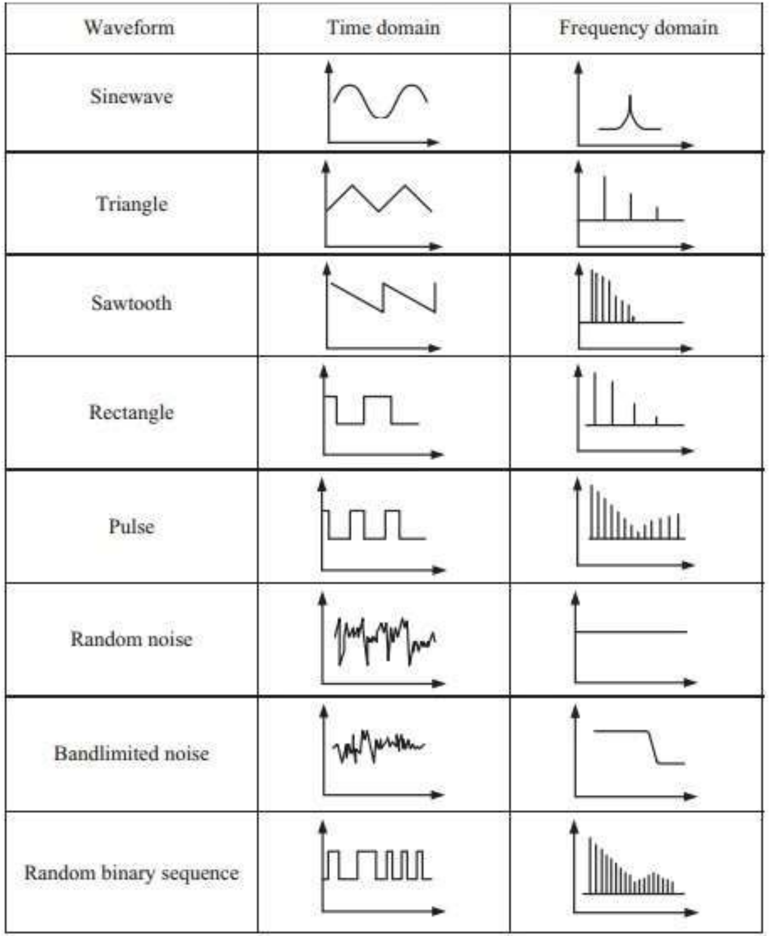
Lecture 11: Introduction to Control System Theory (Part 2)

Niloy Irtisam
Lecturer, Dept. of Computer Science and Engineering
Brac University

Time Domain to Frequency Domain



Signals

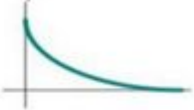


Waveform
Time domain
Frequency domain
Sinewave
Triangle
Sawtooth
Rectangle
Pulse
Random noise
Bandlimited noise
Random binary sequence

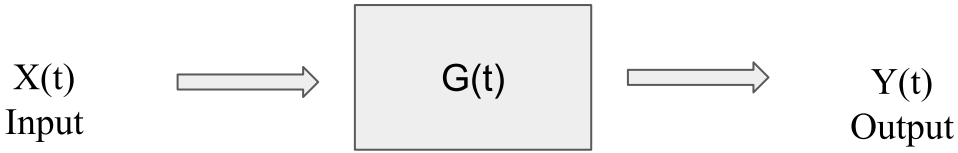
$$x(t) \Longleftrightarrow x(s)$$

$$x(t) \xleftrightarrow[\text{Laplace Transform}]{} x(s)$$

Common Laplace Transform

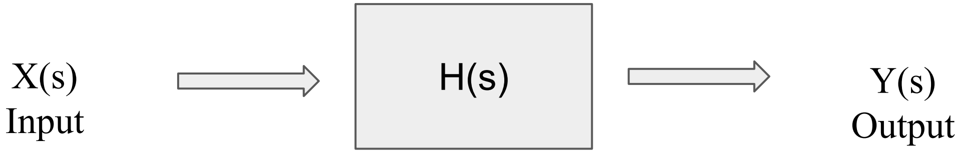
Name	$f(t)$		$F(s)$
Impulse δ	$f(t)=\begin{cases}1 & t=0\\ 0 & t>0\end{cases}$		1
Step	$f(t)=1$		$\frac{1}{s}$
Ramp	$f(t)=t$		$\frac{1}{s^2}$
Exponential	$f(t)=e^{-at}$		$\frac{1}{s+a}$
Sine	$f(t)=\sin(\omega t)$		$\frac{\omega}{s^2+\omega^2}$
Damped Sine	$f(t)=e^{-at}\sin(\omega t)$		$\frac{\omega}{(s+a)^2+\omega^2}$

Block Diagram



$$Y(t) = X(t)*G(t)$$
$$Y(t)/X(t) = G(t)$$
$$\text{Gain} = Y(t)/X(t) = \text{Output}/\text{Input}$$

Transfer Function



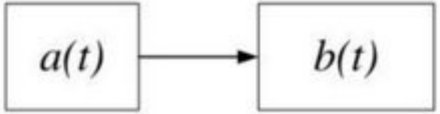
- Definition: Transfer function is defined as the ratio of LT of output to the L.T of input. When all the initial condition assume to be zero.

$$H(s) = Y(s) / X(s)$$

- Relates the output of a linear system to its input.
- Describes how a linear system responds to an impulse, called impulse response

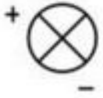
Block Diagram

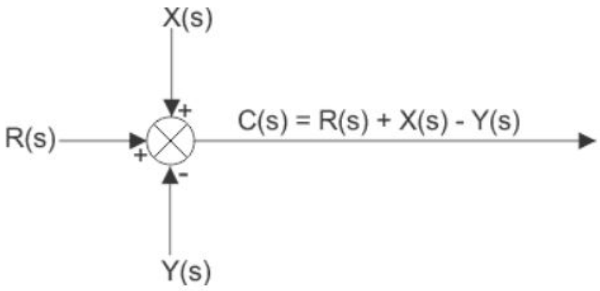
- Rules
  - Cascaded elements: convolution



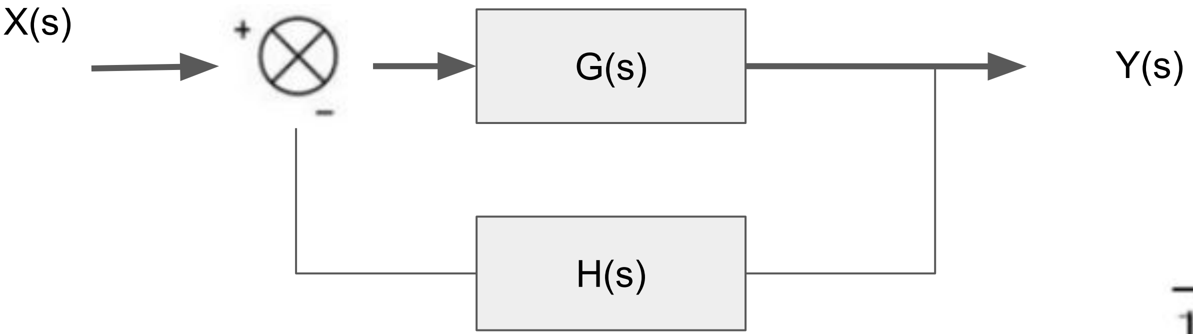
$$= a(t) * b(t)$$

  - Summation and deference elements



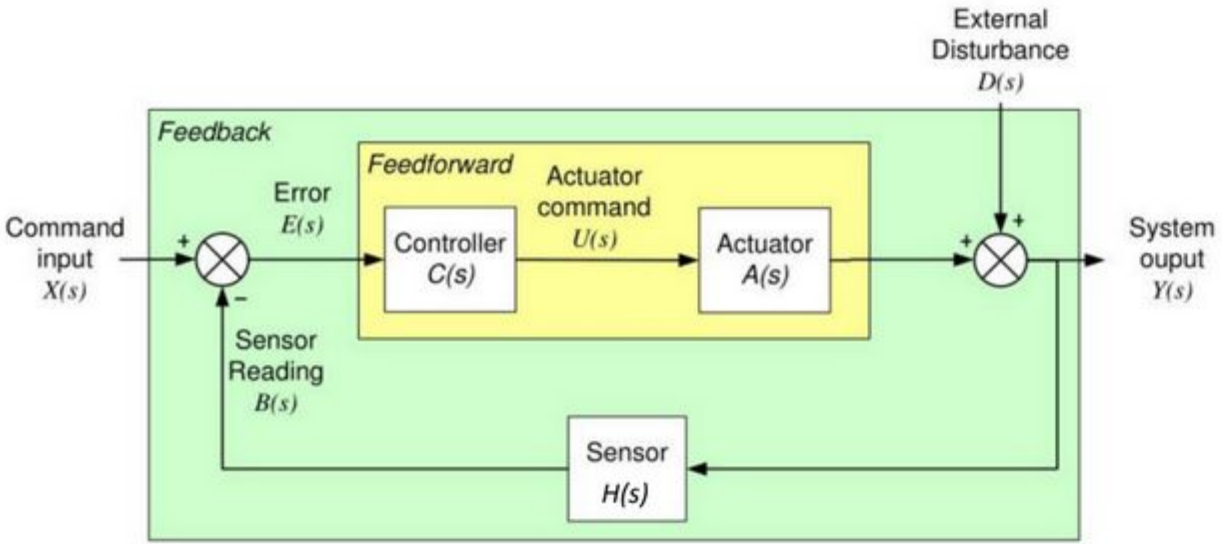


- Feedback Connection



$$\frac{G(s)}{1 \mp G(s)H(s)} * X(s)$$

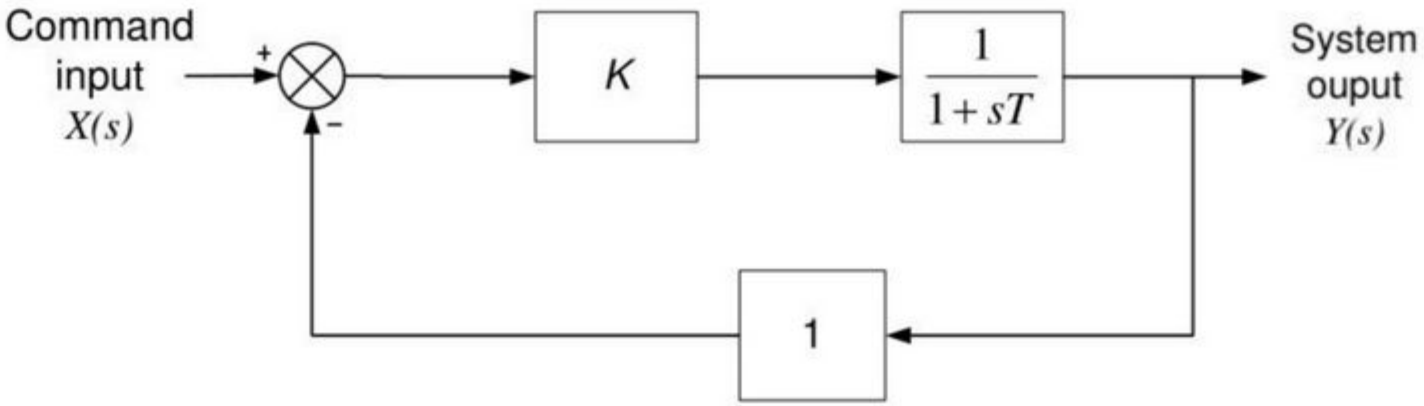
Key Transfer Function



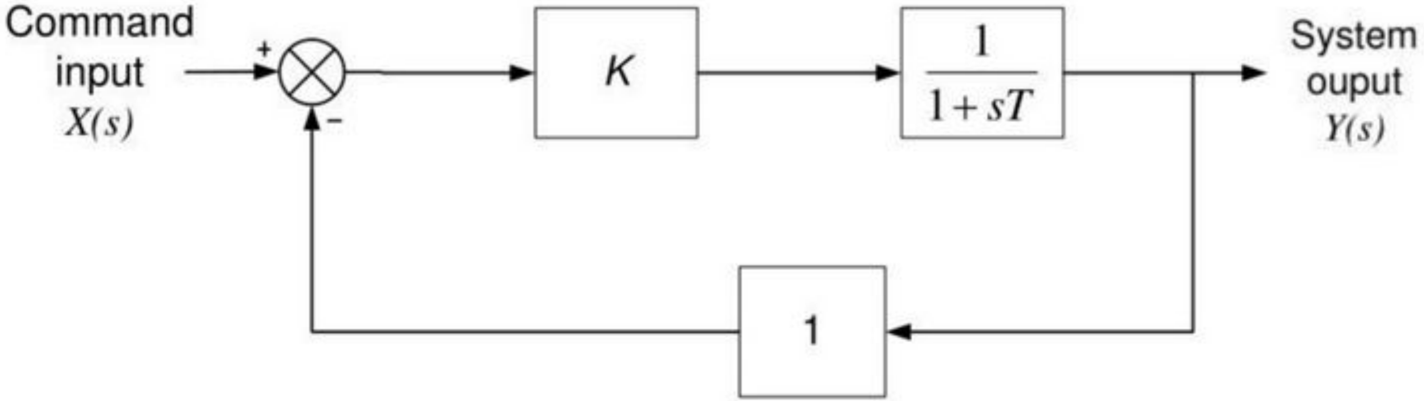
$$\text{Feedforward}: \frac{Y(s)}{E(s)} = C(s)A(s)$$

$$\text{Feedback}: \frac{Y(s)}{X(s)} = \frac{C(s)A(s)}{1+C(s)A(s)H(s)}$$

Transfer Function



Transfer Function

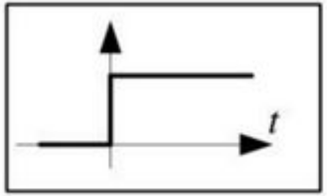


$$\frac{Y(s)}{X(s)} = \frac{K}{1+K+sT}$$

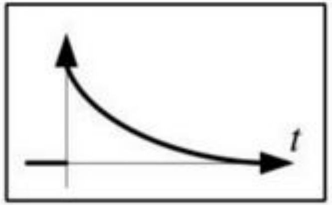
Steady State Vs Transient

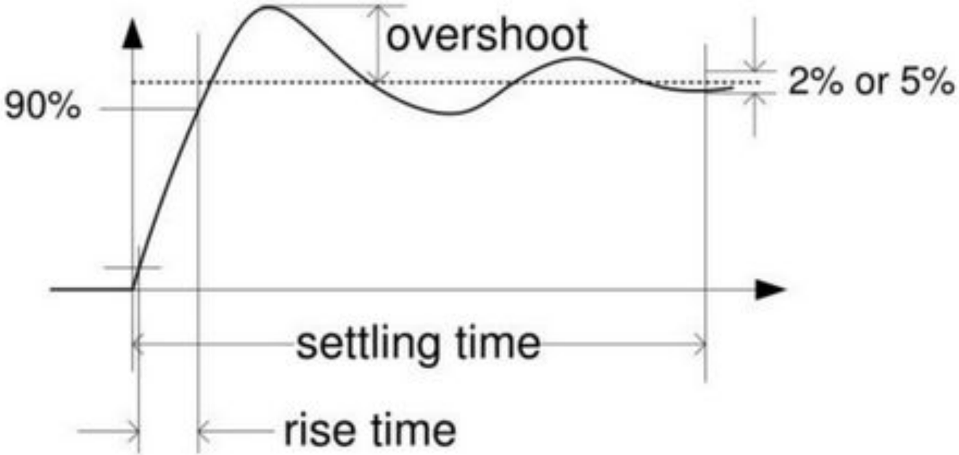
- Step Response illustrates how a system response can be decomposed into two components

- Steady-state part:



- Transient





overshoot -- % of final value exceeded at first *oscillation*

rise time -- time to span from 10% to 90% of the final value

settling time -- time to reach within 2% or 5% of the final value

PID Controller

$$\text{Proportional control}: \quad u(t) = K_p e(t) \qquad \frac{U(s)}{E(s)} = K_p$$

$$\text{Integral control}: \quad u(t) = K_i \int_0^t e(t)dt \qquad \frac{U(s)}{E(s)} = \frac{K_i}{s}$$

$$\text{Differential control}: \quad u(t) = K_d \frac{d}{dt} e(t) \qquad \frac{U(s)}{E(s)} = K_d s$$

- It produces an output, which is the combination of the outputs of proportional , integral & derivative controllers

$$u(t) \propto e(\mathrm{t}) + \int e(t) + \frac{\mathrm{d}}{\mathrm{d}t} e(t)$$

$$\gg u(t) = K_P e(\mathrm{t}) + K_I \int e(t) + K_D \frac{\mathrm{d}}{\mathrm{d}t} e(t)$$

Laplace transform in both side

$$U(S) = K_P E(\mathrm{S}) + \frac{K_I}{S} E(\mathrm{S}) + K_D S E(\mathrm{S})$$

$$U(S) = E(\mathrm{S}) \left(K_P + \frac{K_I}{S} + K_D S \right)$$

$$\frac{U(S)}{E(\mathrm{S})} = \left(K_P + \frac{K_I}{S} + K_D S \right) = \frac{K_P S + K_I + K_D S^2}{S}$$

Effect of Controller Functions

- Proportional Action

Simplest Controller Function, The P term helps to reduce the steady-state error and improve the system's responsiveness. However, too high of a proportional gain can lead to instability and oscillations in the system's response.

- Integral Action

Eliminates steady-state error, The I term helps to improve the system's stability and eliminate any bias in the system. However, too high of an integral gain can lead to overshoot and instability in the system's response.

- Derivative Action (“rate control”)

Effective in transient periods, The D term helps to improve the system's stability and reduce the effects of disturbances in the system. However, too high of a derivative gain can lead to noise amplification and instability in the system's response.

How to get the PID parameter values ?

- If we know the transfer function, analytical methods can be used (e.g., root-locus method) to meet the transient and steady-state specs.
- When the system dynamics are not precisely known, we must resort to experimental approaches.

Ziegler-Nichols Rules for Tuning PID Controller

Using only Proportional control, turn up the gain until the system oscillates without dying down, i.e., is marginally stable. Assume that K and P are the resulting gain and oscillation period, respectively.

Then Use

for P control	for PI control	for PID control
$K_p = 0.5\,K$	$K_p = 0.45\,K$	$K_p = 0.6\,K$
	$K_i = 1.2 / P$	$K_i = 2.0 / P$
		$K_d = P / 8.0$

Ziegler-Nichols Tuning for second or higher order systems

- PID control---most widely used control strategy today
- Over 90% of control loops employ PID control, often the derivative gain set to zero (PI control)
- The three terms are intuitive---a non - specialist can grasp the essentials of the PID controller’s action. It does not require the operator to be familiar with advanced math to use PID controllers
- Engineers prefer PID controls over untested solutions

Next Class

Machine Learning