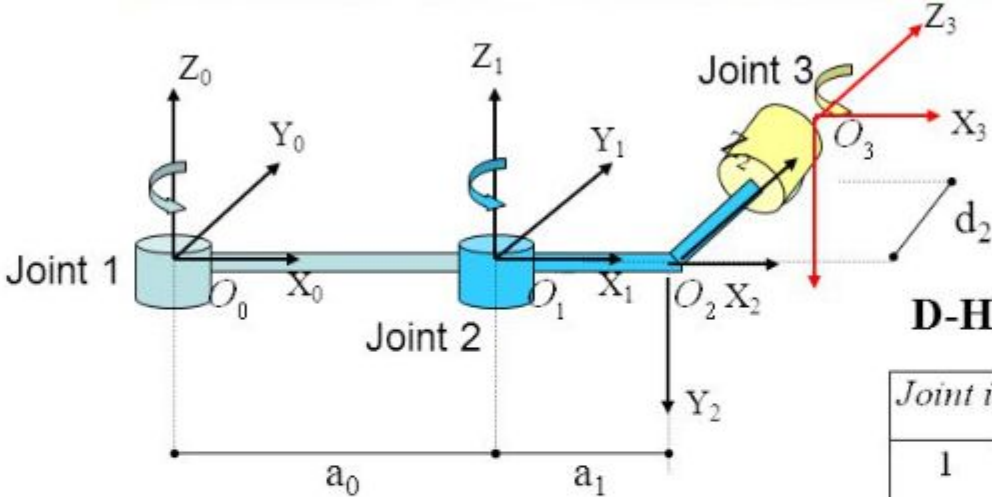
D-H Link Parameter Example



D-H Link Parameter Table

Joint i	α_i	a_i	d_i	θ_i
1	0	a_0	0	θ_1
2	-90	a_1	0	θ_2
3	0	0	d_2	θ_3

α_i : rotation angle from Z_{i-1} to Z_i about X_i

a_i : distance from intersection of Z_{i-1} & X_i to origin of i coordinate along X_i

d_i : distance from origin of (i-1) coordinate to intersection of Z_{i-1} & X_i along Z_{i-1}

θ_i : rotation angle from X_{i-1} to X_i about Z_{i-1}

Transformation Matrices

Joint i	α_i	a_i	d_i	θ_i
1	0	a_0	0	θ_1
2	-90	a_1	0	θ_2
3	0	0	d_2	θ_3

$$T_{i-1}^{i} = \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = (T_0^1)(T_1^2)(T_2^3)$$

$$T_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_0\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & a_0\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 & a_1\cos\theta_2 \\ \sin\theta_2 & 0 & \cos\theta_2 & a_1\sin\theta_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Direct Kinematics of 3 DOF Manipulator

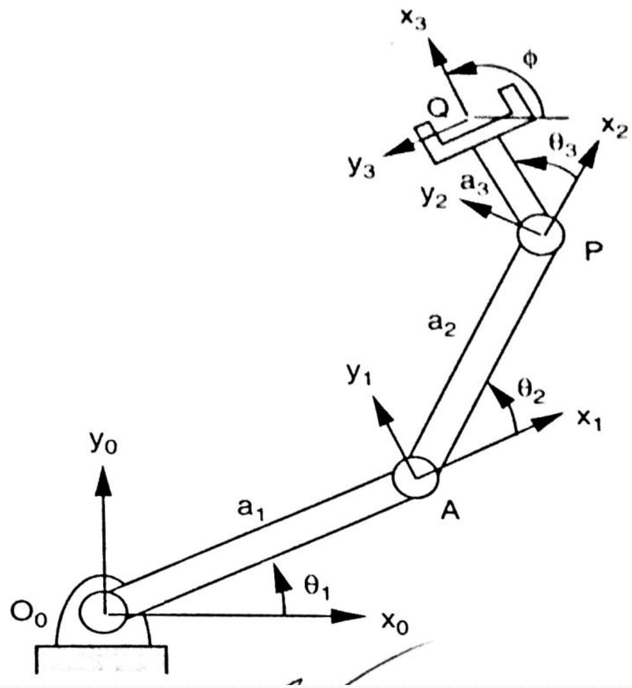


TABLE 2.1. D-H Parameters of a 3-DOF Manipulator

Joint i	α_i	a_i	d_i	θ_i
1	0	a_1	0	θ_1
2	0	a_2	0	θ_2
3	0	a_3	0	θ_3

$$^{0}A_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_1c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & a_1s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$^{2}A_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_3c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & a_3s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Direct Kinematics of 3 DOF Manipulator

Overall transformation matrix?

$$ {}^0A_3 = \begin{bmatrix} c\theta_{123} & -s\theta_{123} & 0 & a_1 c\theta_1 + a_2 c\theta_{12} + a_3 c\theta_{123} \\ s\theta_{123} & c\theta_{123} & 0 & a_1 s\theta_1 + a_2 s\theta_{12} + a_3 s\theta_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

Direct Kinematics of 3 DOF Manipulator

Initial position vector :

$$^{3}\mathbf{q} = [0, 0, 0, 1]^{T}$$

Final position vector?

$$\begin{bmatrix} q_x \\ q_y \\ q_z \\ 1 \end{bmatrix} = {}^{0}A_3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 c\theta_1 + a_2 c\theta_{12} + a_3 c\theta_{123} \\ a_1 s\theta_1 + a_2 s\theta_{12} + a_3 s\theta_{123} \\ 0 \\ 1 \end{bmatrix}$$

Direct Kinematics of 3 DOF Manipulator

Initial position vector :

$$^{3}\mathbf{g} = [g_u, g_v, 0, 1]^T$$

Final position vector?

$$\begin{bmatrix} g_x \\ g_y \\ g_z \\ 1 \end{bmatrix} = {}^{0}A_3 \begin{bmatrix} g_u \\ g_v \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} g_u c\theta_{123} - g_v s\theta_{123} + a_1 c\theta_1 + a_2 c\theta_{12} + a_3 c\theta_{123} \\ g_u s\theta_{123} + g_v c\theta_{123} + a_1 s\theta_1 + a_2 s\theta_{12} + a_3 s\theta_{123} \\ 0 \\ 1 \end{bmatrix}$$

Direct Kinematics of SCARA Arm (4 DOF Manipulator)

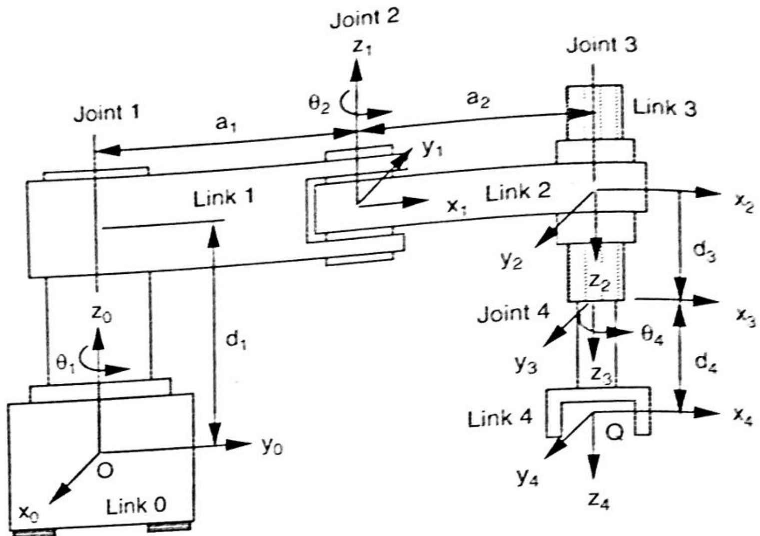


FIGURE 2.4. Schematic diagram of a SCARA arm.

TABLE 2.2. D-H Parameters of the SCARA Arm

Joint i	α_i	a_i	d_i	θ_i
1	0	a_1	d_1	θ_1
2	π	a_2	0	θ_2
3	0	0	d_3	0
4	0	0	d_4	θ_4

$$^0A_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & a_1c\theta_1 \\ s\theta_1 & c\theta_1 & 0 & a_1s\theta_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$^1A_2 = \begin{bmatrix} c\theta_2 & s\theta_2 & 0 & a_2c\theta_2 \\ s\theta_2 & -c\theta_2 & 0 & a_2s\theta_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^2A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^3A_4 = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & 0 \\ s\theta_4 & c\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Direct Kinematics of SCORBOT Robot (5 DOF Manipulator)

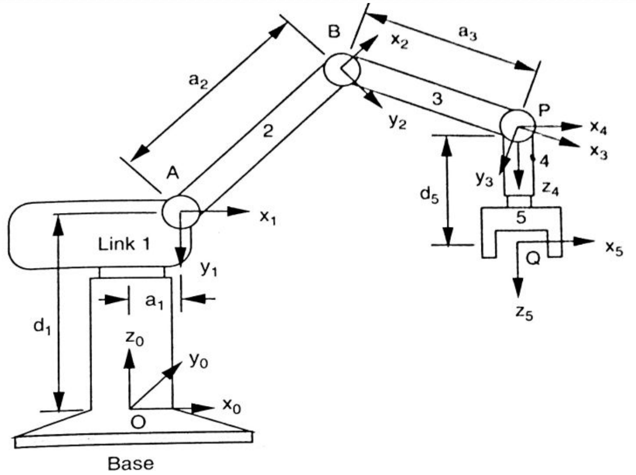


TABLE 2.3. D-H Parameters of a 5-DOF Manipulator

Joint i	α_i	a_i	d_i	θ_i
1	$-\pi/2$	a_1	d_1	θ_1
2	0	a_2	0	θ_2
3	0	a_3	0	θ_3
4	$-\pi/2$	0	0	θ_4
5	0	0	d_5	θ_5

$$^0A_1 = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & a_1 c\theta_1 \\ s\theta_1 & 0 & c\theta_1 & a_1 s\theta_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

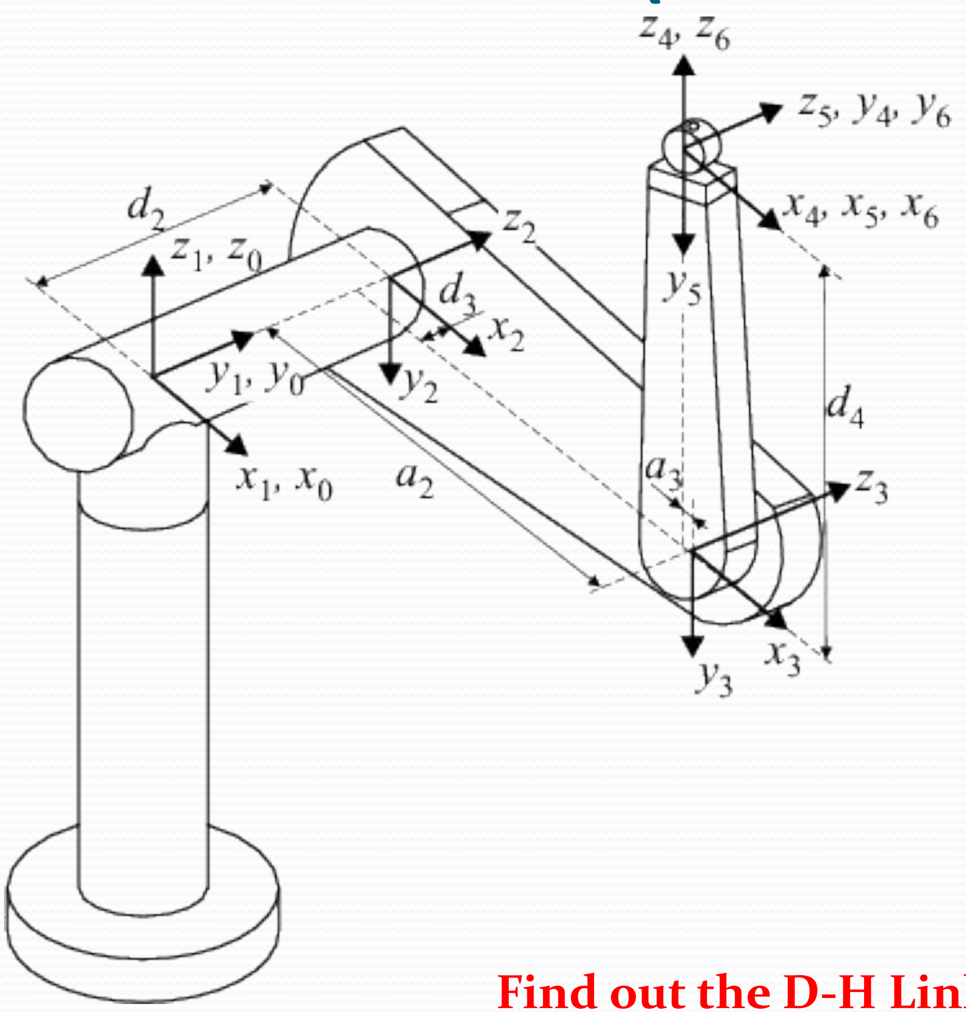
$$^1A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2A_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & a_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^3A_4 = \begin{bmatrix} c\theta_4 & 0 & -s\theta_4 & 0 \\ s\theta_4 & 0 & c\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^4A_5 = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

PUMA 560 (6 DOF Manipulator)

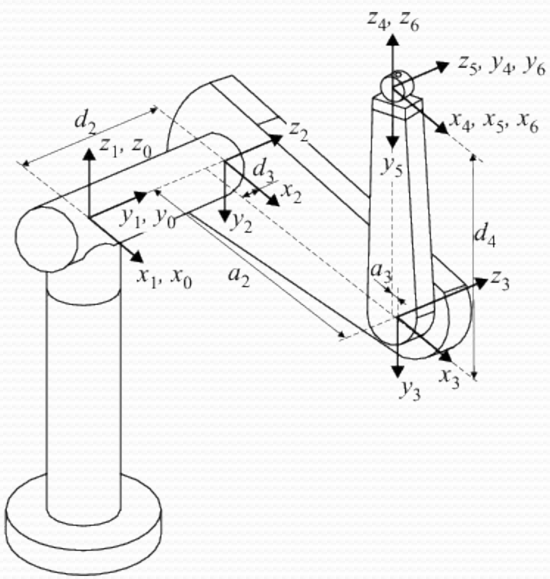


PUMA 560 robot arm link coordinate parameters

Joint i	θ_i	α_i	$a_i(mm)$	$d_i(mm)$
1	θ_1	-90	0	0
2	θ_2	0	431.8	149.09
3	θ_3	90	-20.32	0
4	θ_4	-90	0	433.07
5	θ_5	90	0	0
6	θ_6	0	0	56.25

Find out the D-H Link Parameters for each joint

PUMA 560 (6 DOF Manipulator)



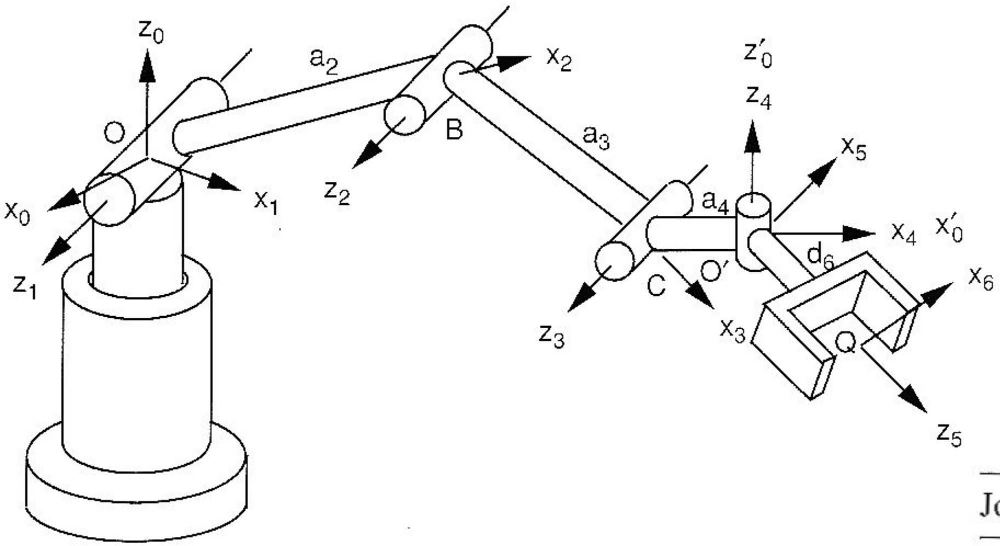
$$ {}^0T_1 = \begin{pmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^1T_2 = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^2T_3 = \begin{pmatrix} \cos\theta_3 & 0 & \sin\theta_3 & a_3\cos\theta_3 \\ \sin\theta_3 & 0 & -\cos\theta_3 & a_3\sin\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^3T_4 = \begin{pmatrix} \cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^4T_5 = \begin{pmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad {}^5T_6 = \begin{pmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Derive Overall Transformation matrix

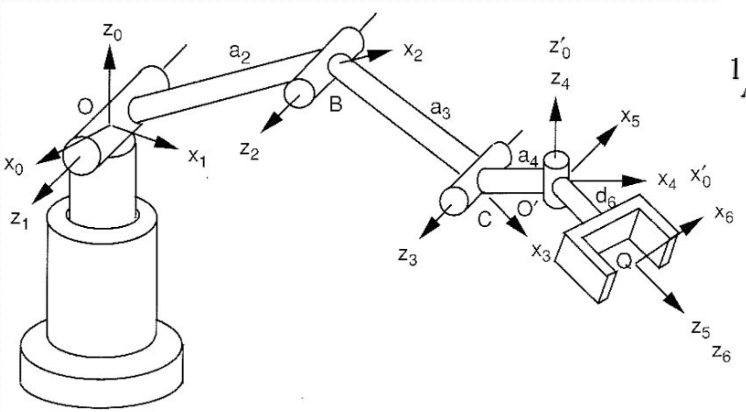
ELBOW Manipulator (6 DOF Manipulator)



Joint i	α_i	a_i	d_i	θ_i
1	$\pi/2$	0	0	variable
2	0	a_2	0	variable
3	0	a_3	0	variable
4	$-\pi/2$	a_4	0	variable
5	$\pi/2$	0	0	variable
6	0	0	d_6	variable

Find out the D-H Link Parameters for each joint

ELBOW Manipulator



$$^{1}A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}A_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_3c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & a_3s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}A_4 = \begin{bmatrix} c\theta_4 & 0 & -s\theta_4 & a_4c\theta_4 \\ s\theta_4 & 0 & c\theta_4 & a_4s\theta_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

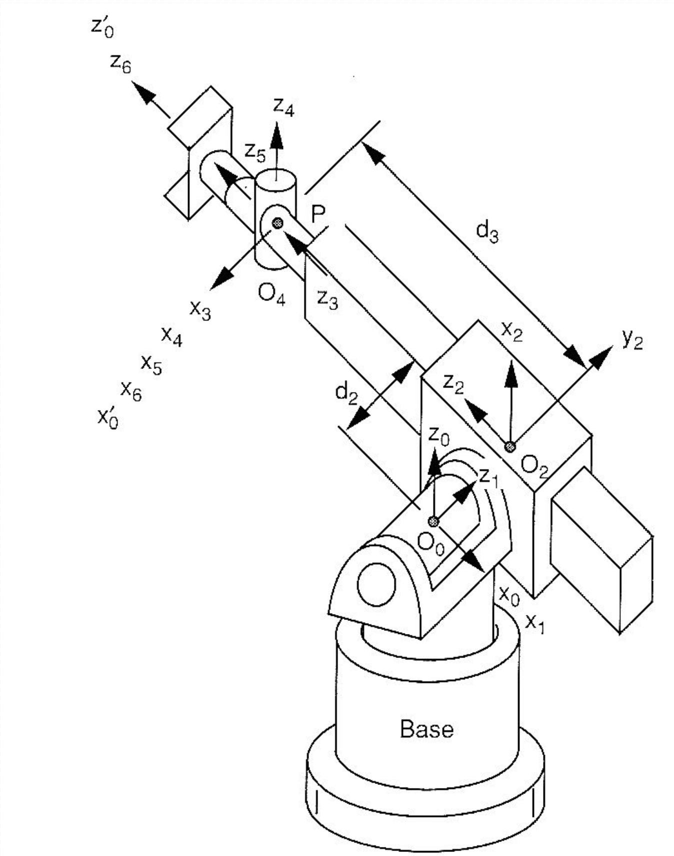
$$^{4}A_5 = \begin{bmatrix} c\theta_5 & 0 & s\theta_5 & 0 \\ s\theta_5 & 0 & -c\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}A_1 = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{5}A_6 = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Derive Overall Transformation matrix

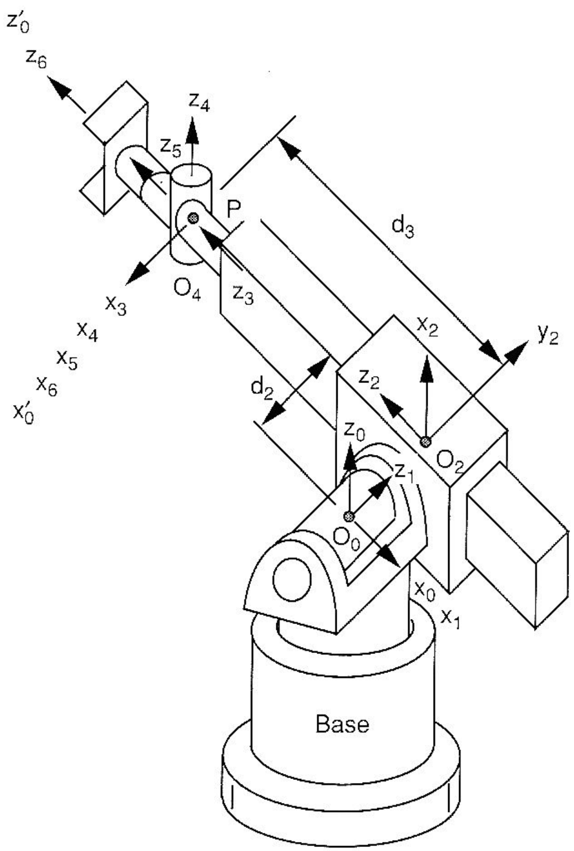
Stanford Manipulator (6 DOF Manipulator)



Joint i	α_i	a_i	d_i	θ_i
1	$-90°$	0	0	θ_1 (variable)
2	$90°$	0	d_2 (constant)	θ_2 (variable)
3	$0°$	0	d_3 (variable)	$-90°$ (constant)
4	$-90°$	0	0	θ_4 (variable)
5	$90°$	0	0	θ_5 (variable)
6	$0°$	0	0	θ_6 (variable)

Find out the D-H Link Parameters for each joint

Stanford Manipulator



$$^{0}A_1 = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & 0 \\ s\theta_1 & 0 & c\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}A_2 = \begin{bmatrix} c\theta_2 & 0 & s\theta_2 & 0 \\ s\theta_2 & 0 & -c\theta_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

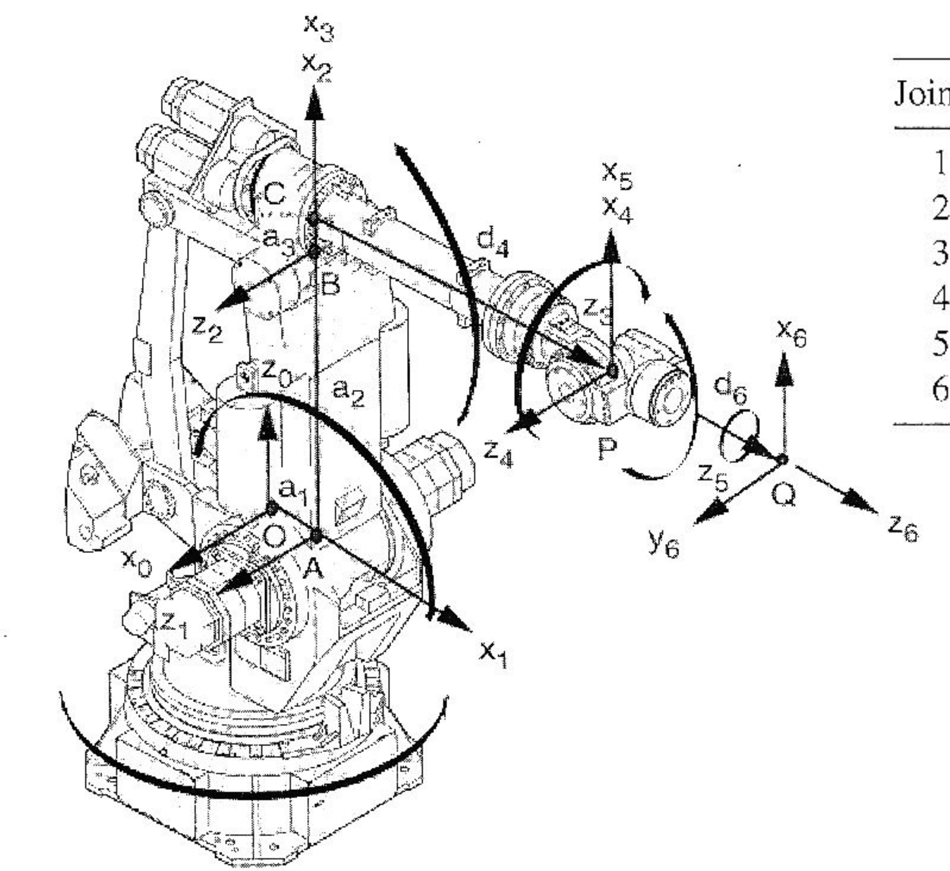
$$^{3}A_4 = \begin{bmatrix} c\theta_4 & 0 & -s\theta_4 & 0 \\ s\theta_4 & 0 & c\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{4}A_5 = \begin{bmatrix} c\theta_5 & 0 & s\theta_5 & 0 \\ s\theta_5 & 0 & -c\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{5}A_6 = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Derive Overall Transformation matrix

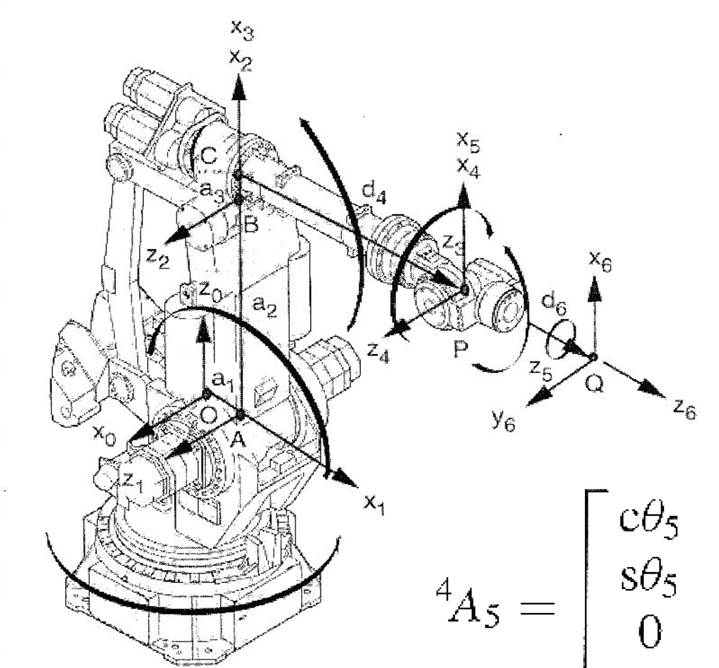
FANUC S-900W (6 DOF Manipulator)



Joint i	α_i	a_i	d_i	θ_i
1	$\pi/2$	a_1	0	θ_1
2	0	a_2	0	θ_2
3	$\pi/2$	a_3	0	θ_3
4	$-\pi/2$	0	d_4	θ_4
5	$\pi/2$	0	0	θ_5
6	0	0	d_6	θ_6

Find out the D-H Link Parameters for each joint

FANUC S-900W



$$^0A_1 = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & a_1 c\theta_1 \\ s\theta_1 & 0 & -c\theta_1 & a_1 s\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$^1A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$^2A_3 = \begin{bmatrix} c\theta_3 & 0 & s\theta_3 & a_3 c\theta_3 \\ s\theta_3 & 0 & -c\theta_3 & a_3 s\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$^3A_4 = \begin{bmatrix} c\theta_4 & 0 & -s\theta_4 & 0 \\ s\theta_4 & 0 & c\theta_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$^4A_5 = \begin{bmatrix} c\theta_5 & 0 & s\theta_5 & 0 \\ s\theta_5 & 0 & -c\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^5A_6 = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$