BRAC UNIVERSITY, CSE 461

Robot Kinematics (Lecture Note)

Instructor: Mr. RIAD AHMED

BRAC UNIVERSITY

Inspiring Excellence

Riad Ahmed

Summer 2023

Contents

1 Introduction to Robot kinematics

1.1 Introduction

A mechanical manipulator is made up of several links connected by joints. One link is typically fixed to the ground and it is called Base link while another is designed as the output link which is called end-effector. Some of the joints in the manipulator are actuated they are called active links and others are passive links because of not being actuated. Robot manipulators are widely used in industrial automation, manufacturing, assembly, and other fields.

1.2 Components of Robot Manipulator

Links: The robot manipulator consists of rigid links or segments that form the structure of the robot arm. Links are typically made of lightweight materials such as aluminum or composite materials.

Joints: Joints are the connections between the links, allowing relative motion between them. Common types of joints include:

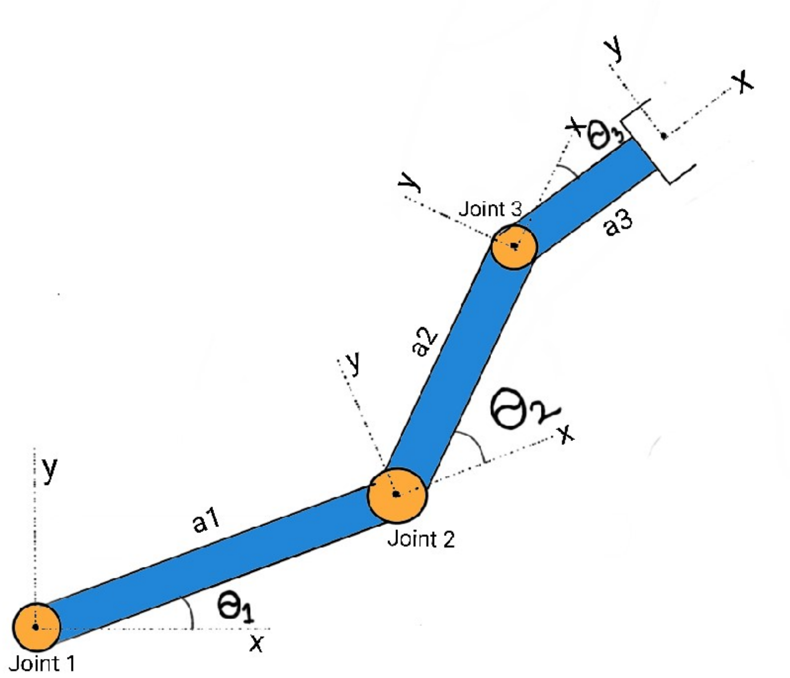
- Revolute Joint: Enables rotation about a single axis, similar to a human elbow joint.
- Prismatic Joint: Allows linear motion along a single axis, like a sliding drawer.

End Effector: The end effector is the tool or device attached to the last link of the manipulator. It can be a gripper for picking and placing objects, a welding tool, a cutting tool, or any other tool specific to the application.

Actuators: Actuators are the components responsible for driving the joints of the manipulator. Electric motors, hydraulic cylinders, or pneumatic cylinders are commonly used as actuators.

1.3 Degrees of Freedom

Typically the number of actuated joints is equal to the number of degrees of freedom so that the manipulator can be controlled at will. In other word, the number of free movements of the links of a manipulator body in total is called degrees of freedom.



In the given figure, we can see there are three active joints that means all three joints are actuated. So the degrees of freedom for this specific robot is 3.

1.4 Kinematics

Robot kinematics refers to the study of the motion of robot manipulators. It focuses on understanding and describing the geometric relationships between the different components of a robot, such as the links and joints, and how these relationships determine the robot's overall motion.

There are two primary aspects of robot kinematics:

- **Forward Kinematics:** Forward kinematics deals with determining the position (x, y, z) and orientation of the robot's end effector based on the joint angles $(\theta_1, \theta_2 .....\theta_n)$ for revolute joints or joint displacements (d) for prismatic joints. This analysis is essential for controlling and planning the robot's movements, as it allows us to know the exact location of the end effector given the joint configurations.
- **Inverse Kinematics:** Inverse kinematics, on the other hand, involves finding the $(\theta_1, \theta_2 ..\theta_n)$ for revolute joints or joint displacements (d) for prismatic joints required to achieve a desired position (x, y, z) and orientation of the end effector.

1.5 Denavit-Hartenberg (D-H) link parameter

The Denavit-Hartenberg (D-H) parameters are a widely used convention in robotics to describe the kinematic properties and geometry of robot manipulators. They provide a systematic way to represent the relationships between consecutive links and joints in a robot manipulator. The D-H parameters consist of four parameters associated with each link-joint connection:

- Link Offset (d): It represents the distance between the z-axes of consecutive coordinate frames along the common normal. In simpler terms, it is the distance between the origins of the two consecutive frames along the z-axis. The length is measured along the z-axis of the current frame.
- Joint Angle (θ): It defines the angle between the x-axes of consecutive coordinate frames about the common normal. It represents the rotation about the z-axis to align the x-axis of the current frame with the x-axis of the next frame.
- Link Length (a): It represents the distance between the origins of the two consecutive frames along the common x-axis. In other words, it is the distance between the origins of the two frames along the x-axis of the current frame.
- Link Twist (α): It defines the angle between the z-axes of consecutive coordinate frames about the common x-axis. It represents the rotation about the x-axis to align the z-axis of the current frame with the z-axis of the next frame.

1.6 Calculation Strategy of Denavit-Hartenberg (D-H) link parameter

The numbers of joints are needed to be determined firstly. To calculate the joint numbers, we can check how many individual coordinate axis are given on the manipulator structure.

Number of individual spatial coordinate axis = Number of Joints except the end effector's axis frame.

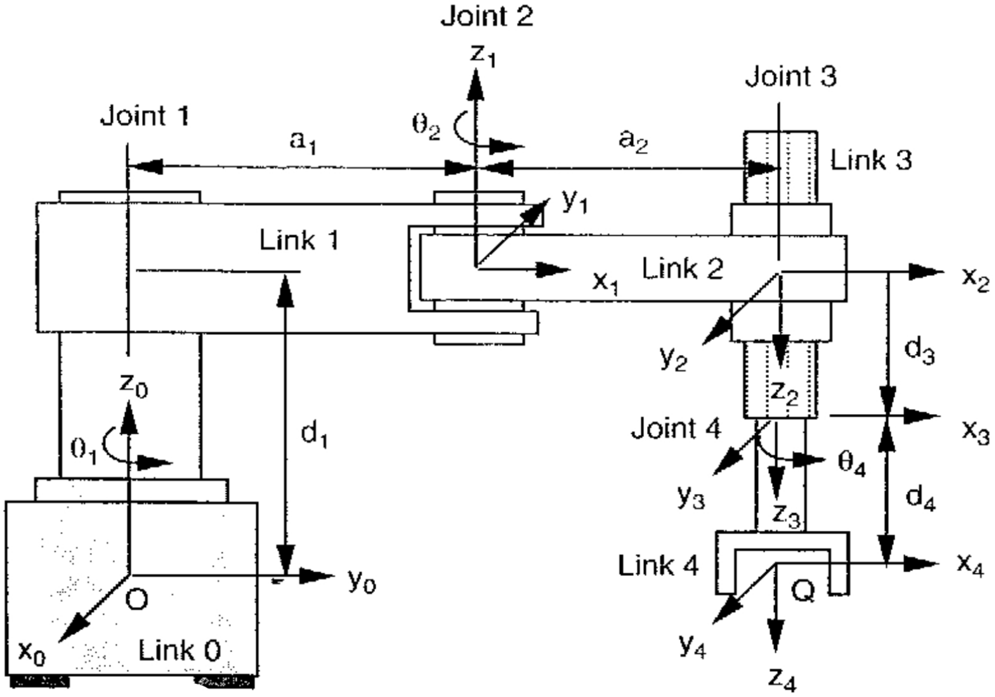
Suppose we are calculating for Joint 1.

- Link Twist (α): Angle between Joint 1's and Joint 2's z-axes. Rotate joint 1's z-axis to joint 2's z-axis to find the direction of the angle; this direction is known as the direction of rotation. The α will be negative if the direction is clockwise and positive if the direction is counterclockwise.
- Link Length (a): The distance between origin/center of Joint 1 to origin/center of Joint 2 along x-axis of Joint 2.
- Link Offset (d): The distance between origin/center of Joint 1 to origin/center of Joint 2 along z-axis of Joint 1.
- Joint Angle (θ):This defines the angle between the x-axis of Joint 1 and Joint 2 generally. However, this angle may typically be determined more simply. Check to see if Joint 1 is

Prismatic or Revolute. If the joint is Revolute then the given θ will be the Joint angle.

SCARA Robot

A SCARA Robot is constrcuted with 4 Joints. First 2 and 4th are revolute joints, and the third is a prismatic joint. Fig shows a schemetic diagram of a SCARA Robot. And the corresponding D-H parameters are given in 1.1 .



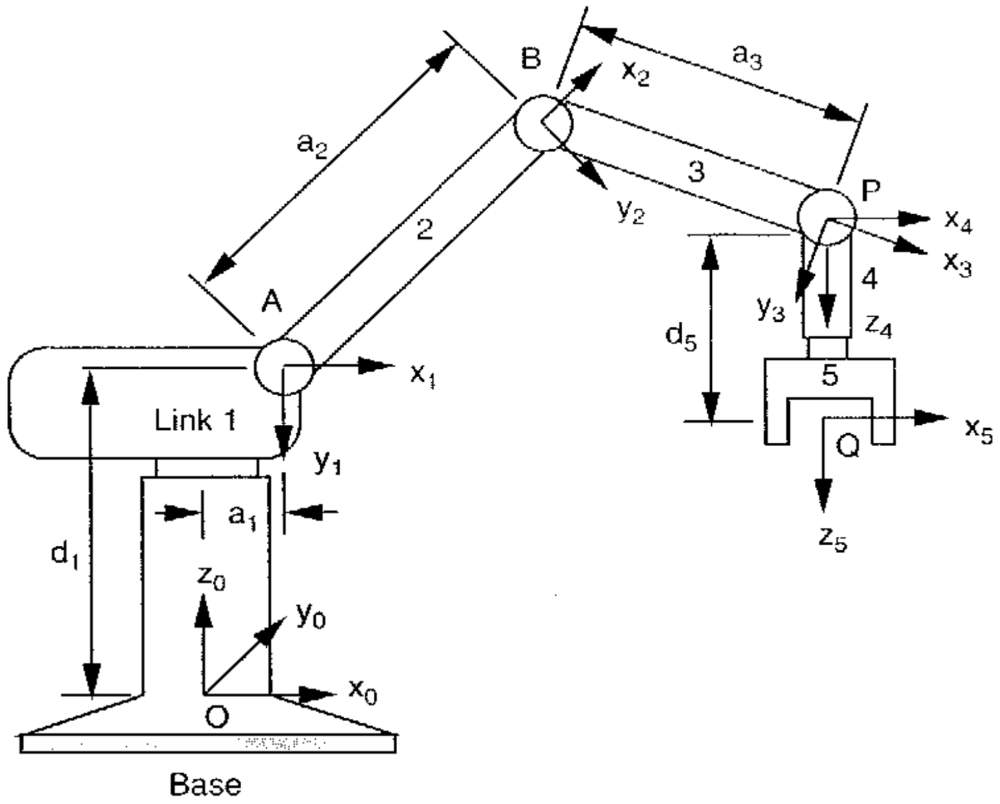
[redacted] Schematic diagram of a SCARA arm.

Joint	α	a	d	θ
1	0	a_1	d_1	θ_1
2	π	a_2	0	θ_2
3	0	0	d_3	0
4	0	0	d_4	θ_4

Table 1.1: **D-H Parameters of the SCARA Arm**

SCORBOT Robot

This robot has 5 joints. All of the joints are revolute joints.



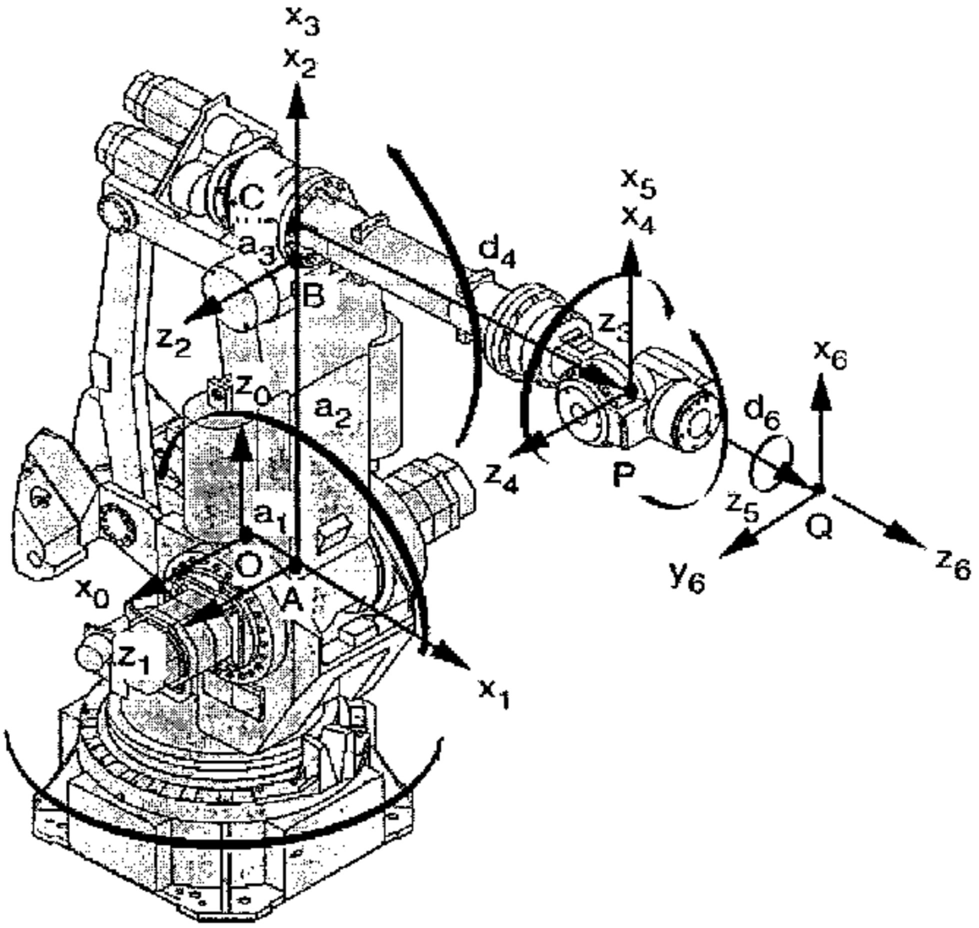
Schematic diagram of the Scorbot robot.

Joint	α	a	d	θ
1	$-\pi/2$	a_1	d_1	θ_1
2	0	a_2	0	θ_2
3	0	a_3	0	θ_3
4	$-\pi/2$	0	0	θ_4
5	0	0	d_5	θ_5

Table 1.2: **D-H Parameters of the SCORBOT Robot**

FANUC Robot

This robot has 6 joints. All of the joints are revolute joints.



Fanuc S-900W robot. (Courtesy of Fanuc Robotics North Amer

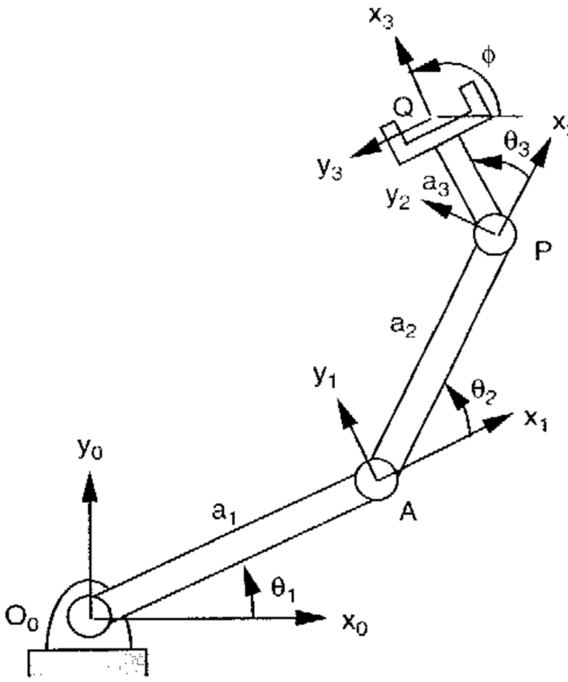
Joint	α	a	d	θ
1	$\pi/2$	a_1	0	θ_1
2	0	a_2	0	θ_2
3	$\pi/2$	a_3	0	θ_3
4	$-\pi/2$	0	d_4	θ_4
5	$\pi/2$	0	0	θ_5
6	0	0	d_6	θ_6

Table 1.3: **D-H Parameters of the FANUC Robot**

1.7 D-H (Denavit-Hartenberg) homogeneous transformation matrix

1.8 Forward Kinematics

We will see a forward kinematics problem of a 3 DOF Planar Manipulator first.



Angles of three joints are 30 degree, 45 degree and 10 degree. Length of arms are 20 cm, 32 cm and 10 cm.

Solution:

Given That, $a_1 = 20cm$, $a_2 = 32cm$, $a_3 = 10cm$ and

$\theta_1 = 30°$, $\theta_2 = 45°$, $\theta_3 = 10°$

By using there values the D-H parameters table is generated that is given below.

Joint	α	a	d	θ
1	0	20	0	30°
2	0	32	0	45°
3	0	10	0	10°

Table 1.4: **D-H Parameters of the Planar Robot**

We know the D-H homogeneous matrix is

$$T_i = \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We will substitute the D-H parameters of Joint 1 to the T_i to get T_1

$$T_1 = \begin{bmatrix} cos30^\circ & -cos0^\circ sin30^\circ & sin0^\circ sin30^\circ & 20cos30^\circ \\ sin30^\circ & cos0^\circ cos30^\circ & -sin0^\circ cos30^\circ & 20sin30^\circ \\ 0 & sin0^\circ & cos0^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} 0.87 & -0.50 & 0 & 17.32 \\ 0.5 & 0.87 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now substitute the D-H parameters of Joint 2 to the T_i to get T_2 similar way.

$$T_2 = \begin{bmatrix} 0.71 & -0.71 & 0 & 22.72 \\ 0.71 & 0.71 & 0 & 22.72 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By substituting D-H parameters of Joint 3 to the T_i, T_3 can be calculated.

$$T_3 = \begin{bmatrix} 0.98 & -0.17 & 0 & 9.85 \\ 0.17 & 0.98 & 0 & 1.74 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We know Overall Transformation Matrix $T = T_1.T_2.T_3$

$$T_1.T_2 = \begin{bmatrix} 0.87 & -0.50 & 0 & 17.32 \\ 0.5 & 0.87 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 0.71 & -0.71 & 0 & 22.72 \\ 0.71 & 0.71 & 0 & 22.72 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.26 & -0.97 & 0 & 25.73 \\ 0.97 & 0.26 & 0 & 41.13 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally $T = (T_1.T_2).T_3 = \begin{bmatrix} 0.26 & -0.97 & 0 & 25.73 \\ 0.97 & 0.26 & 0 & 41.13 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 0.98 & -0.17 & 0 & 9.85 \\ 0.17 & 0.98 & 0 & 1.74 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0.09 & -0.99 & 0 & 26.6 \\ 0.99 & 0.09 & 0 & 51.14 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So from overall Transformation Matrix, $x = 26.6, y = 51.14$ and $z = 0$

Instructor: Riad Ahmed