

MAT 120 ASSIGNMENT 3

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SECTION: 13

MAT120 ASSIGNMENT 3

$$\begin{aligned} (1) \quad z &= \sqrt{u^2 + v^2} \\ z &= \sqrt{r^2} \\ z &= r \end{aligned}$$

$$x^2 + y^2 = r^2$$

$$\begin{aligned} z &= 2 - u^2 - v^2 \\ z &= 2 - (u^2 + v^2) \\ z &= 2 - r^2 \end{aligned}$$

$$r^2 + r - 2 = 0$$

$$(r+2)(r-1) = 0$$

$$r = 0, r = 1, r \neq -2$$

$$\text{limit for } r, r = 0, r = 1$$

$$\text{limit for } z, z = r, z = 2 - r^2$$

$$\text{limit for } \theta, \theta = 0, \theta = 2\pi$$

$$\int_0^{2\pi} \int_0^1 \int_r^{2-r^2} r \, dz \, dr \, d\theta$$

$$\Rightarrow \int_0^{2\pi} \int_0^1 r z \Big|_r^{2-r^2} dr \, d\theta$$

$$\Rightarrow \int_0^{2\pi} \int_0^1 r(2-r^2) - r(r) dr \, d\theta$$

$$\Rightarrow \int_0^{2\pi} \int_0^1 2r - r^3 - r^2 dr \, d\theta$$

$$\Rightarrow \int_0^{2\pi} \left[r^2 - \frac{r^4}{4} - \frac{r^3}{3} \right]_0^1 d\theta$$

$$\Rightarrow \int_0^{2\pi} \left(1 - \frac{1}{4} - \frac{1}{3} \right) d\theta$$

$$\Rightarrow \int_0^{2\pi} \frac{5\pi}{12} d\theta$$

$$\Rightarrow \left[\frac{5\pi}{12} \theta \right]_0^{2\pi} \Rightarrow \frac{5\pi(2\pi)}{12} \Rightarrow \frac{5\pi}{6} \text{ (Ans)}$$

$$(2) \quad u = x - 2y \quad v = 2x + y$$

$$\begin{array}{r} u = x - 2y \\ + \quad v = 2x + y \quad \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} u + v = 3x - y \\ u + 2v = 5x \end{array}$$

$$\therefore x = \frac{1}{5}(u + 2v)$$

$$x = \frac{1}{5}u + \frac{2}{5}v$$

$$\begin{array}{r} u = x - 2y \quad \times 2 \\ - \quad v = 2x + y \\ \hline \end{array}$$

$$u - v = -3y$$

$$y = \frac{1}{3}(v - u)$$

$$y = \frac{1}{3}v - \frac{1}{3}u$$

$$\frac{\partial x}{\partial u} = \frac{1}{5}$$

$$\frac{\partial x}{\partial v} = \frac{2}{5}$$

$$\frac{\partial y}{\partial u} = -\frac{1}{3}$$

$$\frac{\partial y}{\partial v} = \frac{1}{3}$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{1}{3} & \frac{1}{3} \end{vmatrix} = \left(\frac{1}{5} \times \frac{1}{3} \right) - \left(\frac{2}{5} \times -\frac{1}{3} \right)$$

$$= \frac{1}{15} + \frac{2}{15} = \frac{1}{5}$$

$$\iint_R \frac{x - 2y}{2x + y} dA = \int_1^4 \int_1^3 \frac{u}{v} |J| du dv$$

$$u = 1, u = 4$$

$$v = 1, v = 3$$

$$= \int_{v=1}^3 \int_{u=1}^4 \frac{u}{v} \left(\frac{1}{5} \right) du dv$$

$$= \frac{1}{5} \int_{v=1}^3 \frac{1}{v} \int_{u=1}^4 u du dv$$

$$= \frac{1}{5} \int_{v=1}^3 \frac{1}{v} \cdot \frac{u^2}{2} \Big|_1^4 dv$$

$$= \frac{1}{5} \int_{v=1}^3 \frac{1}{v} \cdot \left[\frac{4^2}{2} - \frac{1^2}{2} \right] dv$$

$$= \frac{1}{5} \int_{v=1}^3 \frac{1}{v} \cdot \frac{15}{2} dv$$

$$= \frac{15}{10} \int_{v=1}^3 \frac{1}{v} dv$$

$$= \frac{15}{10} \ln|v| \Big|_1^3$$

$$= \frac{15}{10} (\ln|3| - \ln|1|)$$

$$= \frac{15}{10} (\ln 3 - 0)$$

$$= \frac{3}{2} \ln 3 \text{ (Ans)}$$

$$(3) \frac{dy}{dt} + 2y = 1$$

$$\Rightarrow \frac{dy}{dt} = 1 - 2y$$

$$\Rightarrow \frac{dy}{1-2y} = dt$$

$$\Rightarrow \int \frac{1}{1-2y} dy = \int dt$$

$$\Rightarrow -\frac{1}{2} \ln|1-2y| = t + C$$

$$y(0) = \frac{5}{2}$$

$$-\frac{1}{2} \ln|1-2(\frac{5}{2})| = 0 + C$$

$$\Rightarrow -\frac{1}{2} \ln|-4| = C$$

$$-\frac{1}{2} \ln |1-2y| = t - \frac{1}{2} \ln |-4|$$

$$-\frac{1}{2} \ln |1-2y| + \frac{1}{2} \ln |-4| = t$$

$$\frac{1}{2} \ln \left| \frac{-4}{1-2y} \right| = t$$

$$\frac{1}{2} \ln \left| \frac{4}{2y-1} \right| = t$$

$$\ln \left| \frac{4}{2y-1} \right| = 2t$$

$$e^{\ln \left| \frac{4}{2y-1} \right|} = e^{2t}$$

$$\frac{4}{2y-1} = e^{2t}$$

$$4 = 2e^{2t} - e^{2t}$$

$$2e^{2t} - 4 = e^{2t}$$

$$4 + e^{2t} = 2ue^{2t}$$

$$2u^2 \frac{1}{e^{2t}} + \frac{e^{2t}}{e^{2t}}$$

$$u^2 \frac{2}{e^{2t}} + \frac{1}{2}$$

$$u^2 2e^{-2t} + \frac{1}{2} \quad \underline{\underline{\text{(Ans)}}}$$

$$(1) (x+1) \frac{du}{dx} + u = \ln x$$

$$\frac{(x+1)}{(x+1)} \frac{du}{dx} + \frac{u}{(x+1)} = \frac{\ln x}{(x+1)}$$

$$\frac{du}{dx} + \frac{1}{(x+1)} u = \frac{\ln x}{(x+1)}$$

$$P(x) = \frac{1}{(x+1)}$$

$$\begin{aligned} \int P(x) dx &= \int \frac{1}{x+1} dx \\ &= \ln(x+1) \\ &= (x+1) \end{aligned}$$

$$Q(x) = \frac{\ln x}{(x+1)}$$

$$\begin{aligned} u &= e^{\int P(x) dx} = \int e^{\int P(x) dx} Q(x) dx \\ u(x+1) &= \int (x+1) \cdot \frac{\ln x}{(x+1)} dx \end{aligned}$$

$$\begin{aligned} u(x+1) &= \int \ln x dx \\ &= x \ln x - \int \frac{1}{x} dx \\ &= x \ln(x) - \int \frac{1}{x} dx \end{aligned}$$

$$\begin{aligned} &= x \ln x - \int dx \\ &= x \ln x - x + C \end{aligned}$$

$$u = \frac{1}{(x+1)} (x \ln x - x + C)$$

$$\begin{aligned} u &= \ln x \\ du &= \frac{1}{x} dx \end{aligned}$$

$$\neq dx = x du$$

$$e^u = x$$

$$dv = \int du$$

$$\cancel{dx = x du}$$

$$v = x$$

$$y(1) = 10$$

$$x=1,$$

$$y(1) = \frac{1}{(1+1)} [(1) \ln(1) - 1 + C]$$

$$10 = \frac{1}{2} [0 - 1 + C]$$

$$10 = -\frac{1}{2} + C$$

$$C = 21$$

$$y = \frac{1}{(u+1)} (u \ln u - u + 21) \quad \underline{\underline{(Ans)}}$$

$$(b) \int_1^3 \int_x^{x^2} \int_0^{\ln z} x e^y \, dy \, dz \, dx$$

$$\Rightarrow \int_1^3 \int_x^{x^2} x e^y \Big|_0^{\ln z} \, dz \, dx$$

$$\Rightarrow \int_1^3 \int_x^{x^2} (x e^{\ln z} - x e^0) \, dz \, dx$$

$$= \int_1^3 \int_x^{x^2} (xz - x) \, dz \, dx$$

$$= \int_1^3 \int_x^{x^2} \frac{xz^2}{2} - xz \Big|_x^{x^2} \, dx$$

$$= \int_1^3 \left[\frac{x(x^2)^2}{2} - x(x^2) \right] - \left[\frac{x(x)^2}{2} - x(x) \right] \, dx$$

$$= \int_1^3 \left(\frac{x^5}{2} - x^3 - \frac{x^3}{2} + x^2 \right) \, dx$$

$$= \int_1^3 \left(\frac{x^5}{2} - \frac{3x^3}{2} + x^2 \right) \, dx$$

$$= \left[\frac{x^6}{12} - \frac{3x^4}{8} + \frac{x^3}{3} \right]_1^3$$

$$= \left[\frac{(3)^6}{12} - \frac{3(3)^4}{8} + \frac{(3)^3}{3} \right] - \left[\frac{(1)^6}{12} - \frac{3(1)^4}{8} + \frac{(1)^3}{3} \right]$$

$$= \frac{315}{8} - \frac{1}{24}$$

$$= \frac{118}{3} \text{ (Ans)}$$