MAT120 ASSIGNMENT 2

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SECTION: 13

MAT120 ASSIGNMENT 2

(1) $y = \sqrt{4-x^2}$, $-1 \leq x \leq 1$

$$\frac{dy}{dx} = -2x \cdot \frac{1}{2} \cdot (4-x^2)^{-\frac{1}{2}}$$

$$= \frac{-x}{(4-x^2)^{\frac{1}{2}}}$$

$$\left(\frac{dy}{dx}\right)^2 = \left(\frac{-x}{\sqrt{4-x^2}}\right)^2$$

$$= \frac{x^2}{4-x^2}$$

$$\cancel{S = \int_{-1}^{1} 2\pi f(x)\sqrt{1+}}$$

$$S = \int_{-1}^{1} 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_{-1}^{1} 2\pi(\sqrt{4-x^2})\sqrt{1+\left(\frac{x^2}{4-x^2}\right)}\, dx$$

$$S = 2\pi \int_{-1}^{1} \sqrt{4-x^2}\sqrt{\frac{4-x^2+x^2}{4-x^2}}\, dx$$

$$S = 2\pi \int_{-1}^{1} \sqrt{4-x^2}\sqrt{\frac{4}{4-x^2}}\, dx$$

$$S = 2\pi \int_{-1}^{1} \sqrt{4-x^2} \cdot \frac{\sqrt{4}}{\sqrt{4-x^2}}\, dx$$

$$S = 2\pi \int_{-1}^{1} \cancel{\sqrt{4}\, dx}\ 2\, dx$$

$$S = 2\pi [2x]_{-1}^{1} \Rightarrow S = 2\pi[2(1) - 2(-1)]$$

$$= 2(4\pi)$$

$$= 8\pi = 25.1 \quad \text{(Ans)}$$

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(2) $x = \frac{1}{8}y^4 + \frac{1}{4}y^{-2}$, $1 \le y \le 4$

$$\frac{dx}{dy} = \frac{1}{2}y^3 - \frac{1}{2}y^{-3} = \frac{1}{2}y^3 - \frac{1}{2y^3}$$

$$\left(\frac{dx}{dy}\right)^2 = \left(\frac{1}{2}y^3 - \frac{1}{2y^3}\right)^2$$

$$= \left(\frac{y^6 - 1}{2y^3}\right)^2$$

$$= \frac{(y^6-1)^2}{(2y^3)^2} = \frac{y^{12} - 2y^6 + 1}{4y^6}$$

$$L = \int_b^a \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$L = \int_1^4 \sqrt{1 + \frac{y^{12} - 2y^6 + 1}{4y^6}}\, dy$$

$$= \int_1^4 \sqrt{\frac{4y^6 + y^{12} - 2y^6 + 1}{4y^6}}\, dy$$

$$= \frac{1}{2}\int_1^4 \frac{\sqrt{y^{12} + 2y^6 + 1}}{y^3}\, dy$$

$$= \frac{1}{2}\int_1^4 \frac{\sqrt{(y^6 + 1)^2}}{y^3}\, dy$$

$$= \frac{1}{2}\int_1^4 \frac{y^6 + 1}{y^3}\, dy$$

$$= \frac{1}{2}\int_1^4 (y^3 + y^{-3})\, dy$$

$$L = \frac{1}{2}\left[\frac{y^4}{4} - \frac{1}{2y^2}\right]_1^4$$

$$L = \frac{1}{2}\left[\left\{\frac{(4)^4}{4} - \frac{1}{2(4)^2}\right\} - \left\{\frac{(1)^4}{4} - \frac{1}{2(1)^2}\right\}\right]$$

$$= \frac{1}{2}\left(\frac{513}{8}\right) = 32.1 \text{ (Ans)}$$

(3) $\int_{-4}^{0}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} 3x \, dy\, dx$

$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_{0}^{4} 3r^2\cos\theta \, dr\, d\theta$

$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left[\frac{3r^3\cos\theta}{3}\right]_0^4 d\theta$

$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left\{\frac{3(4)^3\cos\theta}{3} - 0\right\} d\theta$

$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 64\cos\theta \, d\theta$

$= \left[64\sin\theta\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$

$= 64\sin\left(\frac{3\pi}{2}\right) - 64\sin\left(\frac{\pi}{2}\right)$

$= -64 - 64$

$= -128$ (Ans)

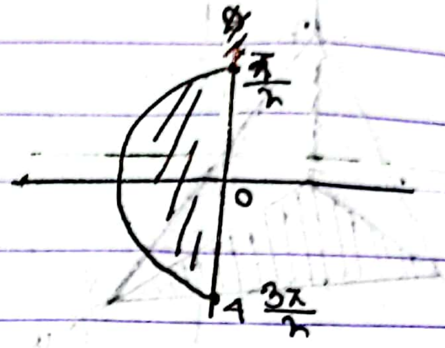
$x^2 + y^2 = r^2$

$y = \sqrt{r^2 - x^2}$

$y = \sqrt{4^2 - x^2}$

$r = 4$

$r \in [0, 4]$



$x = r\cos\theta$

$y = r\sin\theta$

$3x = 3(x) = 3(r\cos\theta)$

$= (3r)\cos\theta$

$= 3r^2\cos\theta$

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(4) $2x + y + z = 2$

$x = 0, y = 0, z = 2$

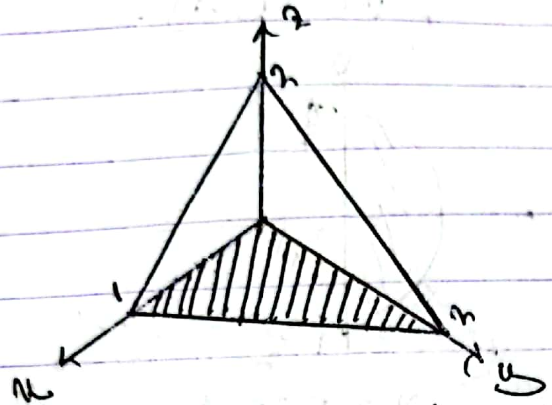
$x = 0, z = 0, y = 2$

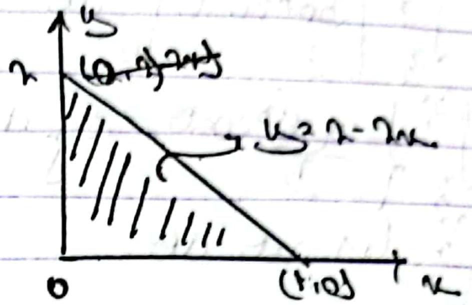
$y = 0, z = 0, x = 1$

$z = 0, \; y + 2x + 0 = 2$

$y = 2 - 2x$

$z = 2 - 2x - y$





$$V = \int_1^2 \int_0^{2-2x} z \, dy \, dx = \int_0^1 \int_0^{2-2x} (2 - 2x - y) \, dy \, dx$$

$$~~= \int_1^2 \int_0^{2-2x} (2-2x) \, dy \, dx~~ = \int_0^1 \left[2y - 2xy - \frac{y^2}{2} \right]_0^{2-2x} dx$$

$$= \int_0^1 2(2-2x) - 2x(2-2x) - \frac{(2-2x)^2}{2} \, dx$$

$$= \int_0^1 4 - 4x - 4x + 4x^2 - \frac{1}{2}(2-2x)^2 \, dx$$

$$= \int_0^1 4 - 8x + 4x^2 - \frac{1}{2}(4 - 8x + 4x^2) \, dx$$

$$= \int_0^1 (4 - 8x + 4x^2 - 2 + 4x - 2x^2) dx$$

$$= \int_0^1 (2x^2 - 4x + 2) \, dx$$

$$= \left[\frac{2x^3}{3} - 2x^2 + 2x \right]_0^1 = \left\{ \frac{2(1)^3}{3} - 2(1)^2 + 2(1) \right\} - 0$$

$$~~= \left\{ \frac{2(2)^3}{3} - 2(2)^2 + 2(2) \right\} - \left\{ \frac{2(1)^3}{3} - 2(1)^2 + 2(1) \right\}~~$$

$$= \cancel{\frac{4}{3}} \; \frac{2}{3}$$

$$= \frac{2}{3} \text{ (Ans)}$$

$= 0.667$ (Ans)

(5) $\int_{-2}^{0}\int_{-1}^{2} (x^2+y^2)\,dx\,dy$

$\Rightarrow \int_{-2}^{0} \left[\frac{x^3}{3} + y^2 x\right]_{-1}^{2} dy$

$\Rightarrow \int_{-2}^{0} \left\{\frac{(2)^3}{3} + y^2(2)\right\} - \left\{\frac{(-1)^3}{3} + y^2(-1)\right\} dy$

$\Rightarrow \int_{-2}^{0} (3+3y^2)\,dy$

$\Rightarrow \left[3y + y^3\right]_{-2}^{0}$

$\cancel{\Rightarrow \left[3(-2)+(-2)^3\right] - 0}$ $0 - \left[3(-2)+(-2)^3\right] = 0 - -14$

$\Rightarrow 14$ (Ans)