MAT120 ASSIGNMANT 1

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SECTION: 13

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MAT120 Assignment 01

(1) $y = 9-x^2$ $[0,3]$

$a = 0, b = 3$

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

$$x^*_k = a + k\Delta x$$

$$= \cancel{0} + k\left(\frac{3}{n}\right) = \frac{3k}{n}$$

$$f(x^*_k)\Delta x = (9 - x^{*2}_k)\frac{3}{n}$$

$$= \left(9 - \left(\frac{3k}{n}\right)^2\right)\frac{3}{n}$$

$$= \left[9 - \left(\frac{9k^2}{n^2}\right)\right]\frac{3}{n}$$

$$= \frac{27}{n} - \frac{27k^2}{n^3}$$

$$\cancel{A = \lim_{n\to\infty}\sum_{k=1}^{n} f(x^*_k) = \lim_{n\to\infty}\sum_{k=1}^{n}\left[\frac{1}{n}(27)\frac{1}{n} - 27\left(\frac{k^2}{n^3}\right)\right]}$$

$$\cancel{= (1)(27) - 27\left(\frac{3^2}{3^3}\right)}$$

$$\cancel{= 27 - 27\left(\frac{1}{3}\right)}$$

$$\cancel{= 18\ (\text{Ans})}$$

$$\text{Area} = \lim_{n\to\infty}\sum_{k=1}^{n} f(x^*_k) = \lim_{n\to 0}\sum_{k=1}^{n}\left[\frac{1}{n}(27) - \left(\frac{k^2}{n^3}\right)(27)\right]$$

$$= (1)(27) - 27\left(\frac{1}{3}\right) \qquad \left[\frac{1}{n^3}\sum_{k=1}^{n} k^2 = \frac{1}{3}\right]$$

$$= 27 - 9 \ \cancel{= 18\ (\text{Ans})}$$

$$= 18\ (\text{Ans})$$

(2) $\int_0^{+\infty} e^{-2x}\,dx$

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(2) $\int_0^{+\infty} e^{-2x}\,dx$

$= \lim_{a \to +\infty} \int_0^{+a} (e^{-2x})\,dx$

$= \lim_{a \to +\infty} \left[-\frac{1}{2} e^{-2x}\right]_0^a$

$= \lim_{a \to +\infty} \left[-\frac{1}{2} e^{-2a} + \frac{1}{2} e^{-2(0)}\right]$

~~$= \lim_{a \to +\infty} \left[-\frac{1}{2} e^{-2a} + \frac{1}{2}\right]$~~

$= \lim_{a \to +\infty} \left(-\frac{1}{2} e^{-2a}\right) + \lim_{a \to +\infty} \left(\frac{1}{2} e^{-2(0)}\right)$

$= -\frac{1}{2} e^{-2\infty} + \frac{1}{2}$

$= \frac{-1}{2e^{2\infty}} + \frac{1}{2} = \frac{-1}{2e^{2\infty}}$

$= \frac{-\cancel{1}}{\cancel{2}(\infty)} + \frac{\cancel{1}}{2}$

$= \frac{-1}{\infty} + \frac{1}{2}$

$= 0 + \frac{1}{2} = \frac{1}{2}$

Since ~~integr~~ there is a real value ~~infinity~~, while calculating for the integration of $-\frac{1}{2} e^{-2x}$ ~~at $a = +\infty$~~, the integral ~~diverges~~ converges.

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(3) $\int \frac{2x^2-9x-9}{x^3-9x}\, dx$

using partial fraction,

~~$2x^2-6x-3x-9$~~ $\frac{2x^2-9x-9}{x(x^2-9)}$

$\Rightarrow \frac{2x^2-9x-9}{(x)(x-3)(x+3)} = \frac{A}{x} + \frac{B}{(x-3)} + \frac{C}{(x+3)}$

$= \frac{A(x-3)(x+3) + B(x)(x+3) + C(x)(x-3)}{x(x-3)(x+3)}$

$2x^2-9x-9 = A(x-3)(x+3) + B(x)(x+3) + C(x)(x-3)$

$= A(x^2-9) + B(x^2+3x) + C(x^2-3x)$

$= Ax^2 - 9A + Bx^2 + 3Bx + Cx^2 - 3Cx$

$= (A+B+C)x^2 + (\cancel{-9A} + 3B - 3C)x - 9A$

$A+B+C = 2$	$3B-3C = -9$	$-9A = -9$
$1+B+C = 2$	$B-C = -3$ — (ii)	$A = 1$
$B+C = 1$ — (i)		

Using (i) + (ii):

$B + \cancel{C} = 1$

$B - \cancel{C} = -3$

$2B = -2$

$B = -1$

$B+C = 1$

$(-1)+C = 1$

$C = 2$

$\frac{1}{x} - \frac{1}{(x-3)} + \frac{2}{(x+3)}$

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$$\int \frac{1}{x} - \frac{1}{(x-3)} + \frac{2}{(x+3)}\, dx$$

$$\Rightarrow \int \frac{1}{x}\,dx - \int \frac{1}{(x-3)}\,dx + 2\int \frac{1}{(x+3)}\,dx$$

$$\Rightarrow \ln|x| - \ln|x-3| + 2\ln|x+3| + c$$

$$\cancel{\Rightarrow \ln\left|\frac{x(x+3)^2}{(x-3)}\right|} \quad \ln|x| + \ln|x+3|^2 - \ln|x-3|$$

$$\Rightarrow \ln\left|\frac{x(x+3)^2}{(x-3)}\right| + c \quad \text{(Ans)}$$

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(4) $\int \frac{du}{\sqrt{u^2-9}}$

$\Rightarrow \int \frac{du}{\sqrt{9(\frac{x^2}{9}-1)}}$

$\Rightarrow \frac{1}{3}\int \frac{1}{\sqrt{(\frac{x}{3})^2-1}}\, dx$

$\Rightarrow \int \frac{1}{\sqrt{sec^2\theta-1}} \cdot sec\theta \tan\theta \, d\theta$

$\Rightarrow \int \frac{1}{\tan\theta} \cdot sec\theta \tan\theta \, d\theta$

$\Rightarrow \int sec\theta \, d\theta$

$\Rightarrow \ln|sec\theta + \tan\theta| + c$

$\Rightarrow \ln\left| \frac{u}{3} + \sqrt{(\frac{u}{3})^2-1} \right| + c$

$\Rightarrow \ln\left| \frac{u}{3} + \sqrt{\frac{u^2-9}{9}} \right| + c$

$\Rightarrow \ln\left| \frac{u}{3} + \frac{\sqrt{u^2-9}}{3} \right| + c$

$\Rightarrow \frac{1}{3} \ln\left| x + \sqrt{u^2-9} \right| + c$ (Ans)

$1+\tan^2\theta = sec^2\theta$

$\frac{x}{3} = \cancel{\tan\theta}\, sec\theta$

$\frac{1}{3}\, dx = sec\theta \tan\theta \, d\theta$

$\tan\theta = \sqrt{sec^2\theta-1}$

(5) $\int_0^\infty e^{-y^2} y^5 \, dy$

$u = y^2$

$du = 2y \, dy$

$dy = \frac{du}{2y} = \frac{1}{2\sqrt{u}} du$

$y = \sqrt{u}$

$\cancel{\int_0^\infty u e^{-p}}$

$\int_0^\infty e^{-u} (\sqrt{u})^5 \cdot \frac{1}{2\sqrt{u}} du$

$= \int_0^\infty e^{-u} \cdot u^{\frac{5}{2}} \cdot \frac{1}{2u^{\frac{1}{2}}} du$

$= \frac{1}{2} \int_0^\infty e^{-u} \cdot u^{\frac{5}{2}-\frac{1}{2}} du$

$= \frac{1}{2} \int_0^\infty e^{-u} \cdot u^2 du$

~~$\int_0^\infty e^{-x+} (x+1)(x)^{n-1} dt \rightarrow$ Gamma function~~

$\int_0^\infty e^{-x} (x)^{n-1} dx \rightarrow$ ~~Gamma~~ $\rightarrow$ Gamma function

$\int_0^\infty e^{-u} (x)^{n-1} dx = \frac{1}{2} \int_0^\infty e^{-u} \cdot u^2 du$

$n-1 = 2$ | $x = u$

$n = 3$

$\frac{1}{2} \Gamma(3) = \frac{1}{2} (3-1)!$

$= \frac{1}{2} \times 2!$

$= \frac{1}{2} \times 2 \times 1$

$= 1$ (Ans)