MAT120 Assignment 4

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SECTION: 13

MAT120 ASSIGNMENT ~~#~~ 1

(1) $(x^3+y^3)dx + 3xy^2dy = 0$

$Mdx + Ndy = 0$

$M = x^3+y^3$ $\qquad$ $N = 3xy^2$

$\frac{\partial M}{\partial y} = 3y^2$ $\qquad$ $\frac{\partial N}{\partial x} = 3y^2$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, the equation is an exact differential equation.

$\frac{\partial f}{\partial x} = M = x^3+y^3$

$\int \partial f = \int (x^3+y^3)\partial x$

$f = \frac{x^4}{4} + xy^3 + \phi(y)$ —— ①

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[\frac{x^4}{4} + xy^3 + \phi(y)\right]$

$= 0 + 3xy^2 + \phi'(y)$

$= 3xy^2 + \phi'(y)$

$N = \frac{\partial f}{\partial y}$

$\cancel{3xy^2} = \cancel{3xy^2} + \phi'(y)$

$\phi'(y) = \cancel{-x}\ 0$

$\int \phi'(y)dy = \int \cancel{\ldots}\, 0\, dy$

$\phi(y) = \cancel{\ldots} + C_0$

$f(x,y) = \frac{x^4}{4} + xy^3 + \cancel{\ldots} + C_0 = c$

$\therefore f(x,y) = c$ (Ans)

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(2) $(4y+2t-5)dt + (6y+4t-1)dy = 0$

$M dt + N dy = 0$

$M = 4y+2t-5$ $\qquad$ $N = 6y+4t-1$

$\frac{\partial M}{\partial y} = 4$ $\qquad$ $\frac{\partial N}{\partial t} = 4$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$, the equation is an exact ~~direct~~ differential equation.

$\frac{\partial f}{\partial t} = M = 4y+2t-5$

$\int \partial f = \int (4y+2t-5)\,dt$

$f = 4yt + t^2 - 5t + \phi(y)$ — (1)

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[4yt + t^2 - 5t + \phi(y)\right]$

$\frac{\partial f}{\partial y} = 4t + \phi'(y)$

$N = \frac{\partial f}{\partial y}$

$6y + \cancel{4t} - 1 = \cancel{4t} + \phi'(y)$

$\phi'(y) = 6y - 1$

$\int \phi'(y) = \int (6y-1)\,dy$

$\phi(y) = 3y^2 - y + C_0$

$f = 4yt + t^2 - 5t + 3y^2 - y + C_0$

$f(x,y) = C$

$4yt + t^2 - 5t + 3y^2 - y + C_0 = C$

$4yt + t^2 - 5t + 3y^2 - y = C - C_0 = C$

$4yt + t^2 - 6t + 3y^2 - y = c$

$y(-1) = 2$

$t = -1, y = 2$

$4(2)(-1) + (-1)^2 - 6(-1) + 3(2)^2 - 2 = c$

$c = \cancel{17}\ 8$

$f(x,y) = 4yt + t^2 - 6t + 3y^2 - y = 8$ (Ans)

$f(x,y) = 4yt + t^2 - 6t + 3y^2 - y = 8$ (Ans)

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(3) $\frac{d^2y}{dx^2} + y = 0$

$y = e^{rx}$, $y'' = r^2 e^{rx}$

$\Rightarrow r^2 e^{rx} + e^{rx} = 0$

$\Rightarrow \frac{r^2 e^{rx}}{e^{rx}} + \frac{e^{rx}}{e^{rx}} = \frac{0}{e^{rx}}$

$\Rightarrow r^2 + 1 = 0$

$\Rightarrow r^2 = \cancel{\sqrt{+}} -1$ $\qquad i = \sqrt{-1}$

$\Rightarrow r = \sqrt{-1}$

$\Rightarrow r = i$

$y = C_1 e^{ix}$ $\qquad e^{ix} = \cos x + \sin x$

$y = C_1 \cos x + C_2 \sin x$

$y(\frac{\pi}{3}) = 0$ $[x = \frac{\pi}{3}, y = 0]$

$0 = C_1 \cos(\frac{\pi}{3}) + C_2 \sin(\frac{\pi}{3})$

$0 = \frac{1}{2} C_1 + \frac{\sqrt{3}}{2} C_2$ ——— (i)

$y'(\frac{\pi}{3}) = 2$ $[x = \frac{\pi}{3}, y' = 2]$

$y' = -C_1 \sin x + C_2 \cos x$

$2 = -C_1 \sin(\frac{\pi}{3}) + C_2 \cos(\frac{\pi}{3})$

$2 = -\frac{\sqrt{3}}{2} C_1 + \frac{1}{2} C_2$ ——— (ii)

$\cancel{\frac{1}{2} C_1} + \frac{\sqrt{3}}{2} C_2 = 0 \times \sqrt{3}$

$\cancel{-\frac{\sqrt{3}}{2} C_1} + \frac{1}{2} C_2 = 2$

$2 C_2 = 2$

$C_2 = 1$

$0 = \frac{1}{2} C_1 + \frac{\sqrt{3}}{2}(1)$

$\frac{1}{2} C_1 = -\frac{\sqrt{3}}{2} \Rightarrow C_1 = -\sqrt{3}$ $\qquad y = -\sqrt{3} \cos x + \sin x$ (Ans)

(4) $y'' - 4y' - 12y = 2t^3 - t + 3$

for complimentary function:-

$y'' - 4y' - 12y = 0$

$y = e^{rt}$

$y' = re^{rt}$

$y'' = r^2 e^{rt}$

$r^2 e^{rt} - 4re^{rt} - 12e^{rt} = 0 \quad \ldots (e^{rt})$

$r^2 - 4r - 12 = 0$

$(r-6)(r+2) = 0$

$r = 6, r = -2$

$y_c = C_1 e^{6t} + C_2 e^{-2t}$

for particular solution

~~$y_p = Ax^3 + Bx^2 + Cx + D$~~

$y_p = At^3 + Bt^2 + Ct + D \qquad \because g(t) = 2t^3 - t + 3$

$y_p' = 3At^2 + 2Bt + C$

$y_p'' = 6At + 2B$

$y'' - 4y' - 12y = 2t^3 - t + 3$

$(6At + 2B) - 4(3At^2 + 2Bt + C) - 12(At^3 + Bt^2 + Ct + D)$
$= 2t^3 - t + 3$

$\Rightarrow 6At + 2B - 12At^2 - 8Bt - 4C - 12At^3 - 12Bt^2 - 12Ct - 12D$
$= 2t^3 - t + 3$

$\Rightarrow -12At^3 + (-12At^2 - 12Bt^2) + (6At - 12Ct - 8Bt) + (2B - 4C - 12D)$
$= 2t^3 - t + 3$

~~Equa~~

Equating coefficients of like terms

$-12A = 2$
$A = -\frac{1}{6}$

$-12A - 12B = 0$
$-12\left(-\frac{1}{6}\right) - 12B = 0$
$2 - 12B = 0$
$B = \frac{1}{6}$

$6A - 12C - 8B = -1$
$6\left(-\frac{1}{6}\right) - 12C - 8\left(\frac{1}{6}\right) = -1$
$\cancel{-1} - 12C - \frac{4}{3} = \cancel{-1}$
$-12C = \frac{4}{3}$
$C = -\frac{1}{9}$

$2B - 4C - 12D = 3$
$2\left(\frac{1}{6}\right) - 4\left(-\frac{1}{9}\right) - 12D = 3$
$\frac{1}{3} + \frac{4}{9} - 12D = 3$
$\frac{7}{9} - 12D = 3$
$-12D = \frac{20}{9}$
$D = -\frac{5}{27}$

$$y_p = -\frac{1}{6}t^3 + \frac{1}{6}t^2 - \frac{1}{9}t + \frac{7}{27}$$

$$y = y_c + y_p = c_1 e^{6t} + c_2 e^{-2t} - \frac{1}{6}t^3 + \frac{1}{6}t^2 - \frac{1}{9}t - \frac{5}{27} \text{ (Ans)}$$

(5) $y^5 + 5y^4 - 2y''' - 10y'' + y' + 5y = 0$

$y = e^{rx}$

$y' = re^{rx}$

$y'' = r^2 e^{rx}$

$y''' = r^3 e^{rx}$

$y^4 = r^4 e^{rx}$

$y^5 = r^5 e^{rx}$

$r^5 e^{rx} + 5r^4 e^{rx} - 2r^3 e^{rx} - 10r^2 e^{rx} + re^{rx} + 5e^{rx} = 0 \quad \dots (\div e^{rx})$

$r^5 + 5r^4 - 2r^3 - 10r^2 + r + 5 = 0$

~~$f(x)$~~

$f(r) = r^5 + 5r^4 - 2r^3 - 10r^2 + r + 5$

$(r+1)$ and $(r-1)$ are factors of $f(x)$

$(r+1)(r-1) = r^2 - 1$

$$
\begin{array}{l}
r^2 - 1 \,\big)\, \cancel{r^5} + 5r^4 - 2r^3 - 10r^2 + r + 5 \,\big(\, r^3 + 5r^2 - r - 5 \\
\quad\quad\; \cancel{r^5} - r^3 \\
\quad\quad\; - \;\; + \\
\hline
\quad\quad\quad \cancel{5r^4} - r^3 \\
\quad\quad\quad \cancel{5r^4} - 5r^2 \\
\quad\quad\quad - \;\;\; + \\
\hline
\quad\quad\quad\quad -\cancel{r^3} - 5r^2 \\
\quad\quad\quad\quad -\cancel{r^3} + r \\
\quad\quad\quad\quad + \;\;\; - \\
\hline
\quad\quad\quad\quad\quad -\cancel{5r^2} + 0 + \cancel{5} \\
\quad\quad\quad\quad\quad -\cancel{5r^2} \;\; + \cancel{5} \\
\quad\quad\quad\quad\quad + \;\;\;\;\;\; - \\
\hline
\end{array}
$$

$(r^2 - 1)(r^3 + 5r^2 - r - 5) = Q(r) \qquad \underline{Q(r)} = r^3 + 5r^2 - r - 5$

$(r-1)$ and $(r+1)$ are factors of $Q(r)$

$$r^2-1\ \big)\ r^3+5r^2-r-5\ \big(\ r+5$$

$$\underline{\cancel{r^3} - \cancel{r}}$$

$$5r^2-5$$

$$\underline{\cancel{5r^2}-\cancel{5}}$$

$$(r^2-1)(r^2-1)(r+5)=0$$

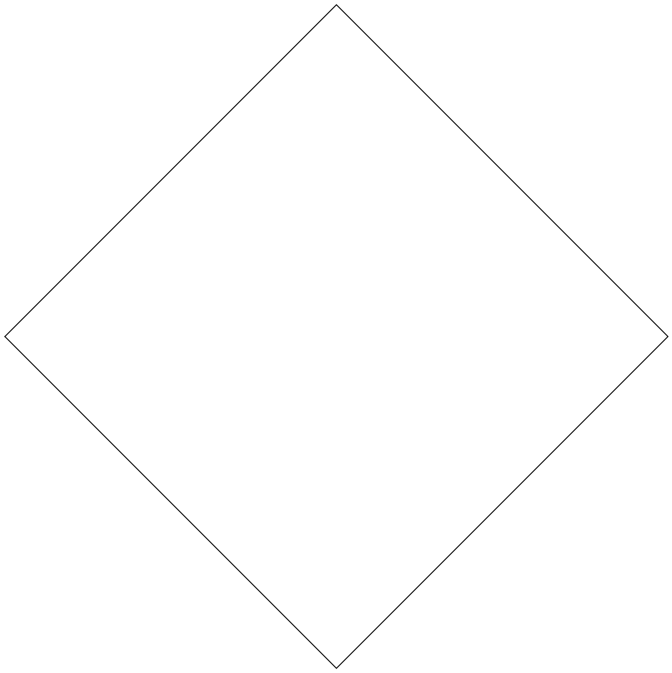
$r^2=1$	$r^2=1$	$r+5=0$
$r=1,\ r=-1$	$r=1,\ r=-1$	$r=-5$

$$y(x,y)=c_1e^{x}+c_2xe^{x}+c_3e^{-x}+c_4xe^{-x}+c_5e^{-5x}\ \text{(Ans)}$$

Answer for 6(a)

In[1]:=
```
Graphics[Line[{{0, 0}, {1, 1}, {2, 0}, {1, -1}, {0, 0}}]]
```

Out[1]=



In[2]:=
```
Solve[{u == x - y, v == x + y}, {x, y}]
```

Out[2]= $\left\{\left\{x \to \frac{u+v}{2},\ y \to \frac{1}{2}(-u+v)\right\}\right\}$

$x = \frac{u+v}{2};$

$y = \frac{1}{2}(-u+v);$

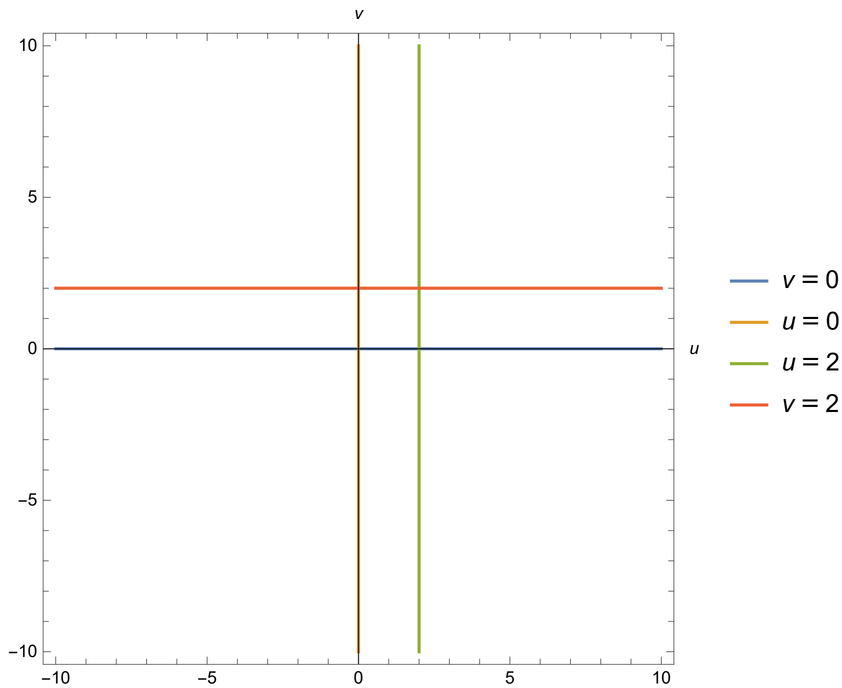
$(0, 0),\ (1, 1),\ (2, 0),\ (1, -1)$

In[6]:= $\text{Simplify}\left[\text{Solve}\left[\frac{y - y1}{x - x1} == \frac{y2 - y1}{x2 - x1}, v\right] /. \{x1 \to 0, y1 \to 0, x2 \to 1, y2 \to -1\}\right]$

Out[6]= $\{\{v \to 0\}\}$

```
In[7]:= plot = ContourPlot[{ v == 0, u == 0 , u == 2, v == 2}, {u, -10, 10},
          {v, -10, 10}, Axes → True , AxesLabel → Automatic , PlotLegends → "Expressions"]
```

Out[7]=



Answer for 6(b)

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In[8]:= jac = Det[D[{x, y}, {{u, v}}]]
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Out[8]= $\frac{1}{2}$

Answer for 6(c)

In[9]:= $\int_0^2 \int_0^2 x\, y\, (\text{jac})\, dv\, du$

Out[9]= 0