MAT120 ASSIGNMENT 3

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SECTION: 13

MAT120 ASSIGNMENT 3

(1) $z = \sqrt{x^2+y^2}$ $\qquad\qquad$ $x^2+y^2 = r^2$

$z = \sqrt{r^2}$

$z = r$

$z = 2-x^2-y^2$

$z = 2-(x^2+y^2)$

$z = 2-r^2$

$r^2+r-2 = 0$

$(r-1)(r+2) = 0$

$r \geq 0, r = 1, r \neq -2$

limit for r, $r = 0$, $r = 1$

limit for z, $z = r$, $z = 2-r^2$

limit for θ, $\theta = 0$, $\theta = 2\pi$

$$\int_0^{2\pi}\int_0^1\int_r^{2-r^2} r \, dz\,dr\,d\theta$$

$$\Rightarrow \int_0^{2\pi}\int_0^1 rz\Big|_r^{2-r^2} dr\,d\theta$$

$$\Rightarrow \int_0^{2\pi}\int_0^1 r(2-r^2) - r(r) \, dr\,d\theta$$

$$\Rightarrow \int_0^{2\pi}\int_0^1 2r - r^3 - r^2 \, dr\,d\theta$$

$$\Rightarrow \int_0^{2\pi} r^2 - \frac{r^4}{4} - \frac{r^3}{3}\Big|_0^1 d\theta$$

$$\Rightarrow \int_0^{2\pi} (1)^2 - \frac{(1)^4}{4} - \frac{(1)^3}{3} \, d\theta$$

$$\Rightarrow \int_0^{2\pi} \frac{5\pi}{12} \, d\theta$$

$$\Rightarrow \left[\frac{5\theta}{12}\right]_0^{2\pi} \Rightarrow \frac{5\pi(2\pi)}{12} \Rightarrow \frac{5\pi}{6} \text{ (Ans)}$$

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(2) $u = x - 2y \qquad v = 2x + y$

$$\begin{aligned} u &= x - 2y \\ + \quad v &= 2x + y \quad \times 2 \\ \hline \cancel{u + v = 3x - y} \\ u + 2v &= 5x \end{aligned}$$

$\therefore \; x = \frac{1}{5}(u + 2v)$

$x = \frac{1}{5}u + \frac{2}{5}v$

$$\begin{aligned} u &= x - 2y \quad \times 2 \\ - \quad v &= 2x + y \\ \hline u - v &= -3y \end{aligned}$$

$y = \frac{1}{3}(v - u)$

$y = -\frac{1}{3}u + \frac{1}{3}v$

$\frac{\partial x}{\partial u} = \frac{1}{5} \qquad \frac{\partial x}{\partial v} = \frac{2}{5}$

$\frac{\partial y}{\partial u} = -\frac{1}{3} \qquad \frac{\partial y}{\partial v} = \frac{1}{3}$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{1}{3} & \frac{1}{3} \end{vmatrix} = \left(\frac{1}{5} \times \frac{1}{3}\right) - \left(\frac{2}{5} \times -\frac{1}{3}\right)$$

$$= \frac{1}{15} + \frac{2}{15} = \frac{1}{5}$$

$$\iint_R \frac{x - 2y}{2x + y}\, dA = \int \int_1 \frac{u}{v} |J|\, du\, dv$$

$u = 1, u = 4$

$v = 1, v = 3$

$$= \int_{v=1}^{3} \int_{u=1}^{4} \frac{u}{v}\left(\frac{1}{5}\right) du\, dv$$

$$= \frac{1}{5} \int_{v=1}^{3} \frac{1}{v} \int_{u=1}^{4} u\, du\, dv$$

$$= \frac{1}{5} \int_{v=1}^{3} \frac{1}{v} \cdot \frac{u^2}{2}\Big|_1^4\, dv$$

$$= \frac{1}{5} \int_{v=1}^{3} \frac{1}{v} \cdot \left[\frac{4^2}{2} - \frac{1^2}{2}\right] dv$$

$$= \frac{1}{5} \int_{v=1}^{3} \frac{1}{v} \cdot \frac{15}{2}\, dv$$

$$= \frac{15}{10}\int_{v=1}^{3} \frac{1}{v}\, dv$$

$$= \frac{15}{10} \ln|v| \Big|_1^3$$

$$= \frac{15}{10}\left(\ln|3| - \ln|1|\right)$$

$$= \frac{15}{10}(\ln 3 - 0)$$

$$= \frac{3}{2}\ln 3 \text{ (Ans)}$$

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(3) $\frac{dy}{dt} + 2y = 1$

$\Rightarrow \frac{dy}{dt} = 1 - 2y$

$\Rightarrow \frac{dy}{1-2y} = dt$

$\Rightarrow \int \frac{1}{1-2y}\, dy = \int dt$

$\Rightarrow -\frac{1}{2}\ln|1-2y| = t + c$

$y(0) = \frac{5}{2}$

$-\frac{1}{2}\ln\left|1-2\left(\frac{5}{2}\right)\right| = 0 + c$

$\Rightarrow -\frac{1}{2}\ln|-4| = c$

$$-\frac{1}{2}\ln|1-2y| = t - \frac{1}{2}\ln|-4|$$

$$-\frac{1}{2}\ln|1-2y| + \frac{1}{2}\ln|-4| = t$$

$$\frac{1}{2}\ln\left|\frac{-4}{1-2y}\right| = t$$

$$\frac{1}{2}\ln\left|\frac{4}{2y-1}\right| = t$$

$$\ln\left|\frac{4}{2y-1}\right| = 2t$$

$$e^{\ln\left|\frac{4}{2y-1}\right|} = e^{2t}$$

$$\frac{4}{2y-1} = e^{2t}$$

$$4 = 2e^{2t}y - e^{2t}$$

$$\cancel{2e^{2t}y = 4 \quad e^{2t}}$$

$$4 + e^{2t} = 2y e^{2t}$$

$$2y = \frac{4}{e^{2t}} + \frac{e^{2t}}{e^{2t}}$$

$$y = \frac{2}{e^{2t}} + \frac{1}{2}$$

$$y = 2e^{-2t} + \frac{1}{2} \quad \text{(Ans)}$$

(4) $(x+1)\frac{dy}{dx} + y = \ln x$

$$\frac{(x+1)}{(x+1)}\frac{dy}{dx} + \frac{y}{(x+1)} = \frac{\ln x}{(x+1)}$$

$$\frac{dy}{dx} + \frac{1}{(x+1)}y = \frac{\ln x}{(x+1)}$$

$$P(x) = \frac{1}{(x+1)}$$

$$e^{\int P(x)dx} = e^{\int \frac{1}{(x+1)}dx}$$
$$= e^{\ln(x+1)}$$
$$= (x+1)$$

$$Q(x) = \frac{\ln x}{(x+1)}$$

$$\cancel{y\,P(x) = \int}$$
$$y \cdot e^{\int P(x)dx} = \int e^{\int P(x)dx} Q(x)\,dx$$
$$\Rightarrow y(x+1) = \int (x+1)\cdot\frac{\ln x}{(x+1)}\,dx$$

$$y(x+1) = \int \ln x\,dx$$
$$= uv - \int v'\,du\,dx$$
$$= \ln x(x) - \int x\cdot\frac{1}{x}\,dx$$
$$= x\ln x - \int dx$$
$$= x\ln x - x + c$$
$$y = \frac{1}{(x+1)}\left(x\ln x - x + c\right)$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\Rightarrow dx = x\,du$

$\cancel{e^{u} = x}$

$dv = \int 1\,dx$

$\cancel{dv = x\,dx}$

$v = x$

~~at~~ $y(1) = 10$
$x = 1,$

$$y(1) = \frac{1}{(1+1)}\left[(1)\ln(1) - 1 + c\right]$$

$$10 = \frac{1}{2}\left[0 - 1 + c\right]$$

$$10 = -\frac{1}{2} + c$$

$$c = 21$$

$$y = \frac{1}{(x+1)}(x\ln x - x + 21) \quad \underline{\underline{(\text{Ans})}}$$

(5) $\int_1^3 \int_x^{x^2} \int_0^{\ln z} x e^y \, dy \, dz \, dx$

$\Rightarrow \int_1^3 \int_x^{x^2} x e^y \Big|_0^{\ln z} dz \, dx$

$\Rightarrow \int_1^3 \int_x^{x^2} (x e^{\ln z} - x e^0) \, dz \, dx$

$= \int_1^3 \int_x^{x^2} (xz - x) \, dz \, dx$

$= \int_1^3 \int_x^{x^2} \frac{xz^2}{2} - xz \Big|_x^{x^2} dx$

$= \int_1^3 \left[\left(\frac{x(x^2)^2}{2} - x(x^2) \right) - \left(\frac{x(x)^2}{2} - x(x) \right) \right] dx$

$= \int_1^3 \left(\frac{x^5}{2} - x^3 - \frac{x^3}{2} + x^2 \right) dx$

$= \int_1^3 \left(\frac{x^5}{2} - \frac{3x^3}{2} + x^2 \right) dx$

$= \left[\frac{x^6}{12} - \frac{3x^4}{8} + \frac{x^3}{3} \right]_1^3$

$= \left[\frac{(3)^6}{12} - \frac{3(3)^4}{8} + \frac{(3)^3}{3} \right] - \left[\frac{(1)^6}{12} - \frac{3(1)^4}{8} + \frac{(1)^3}{3} \right]$

$= \frac{315}{8} - \frac{1}{24}$

$= \frac{118}{3}$ (Ans)

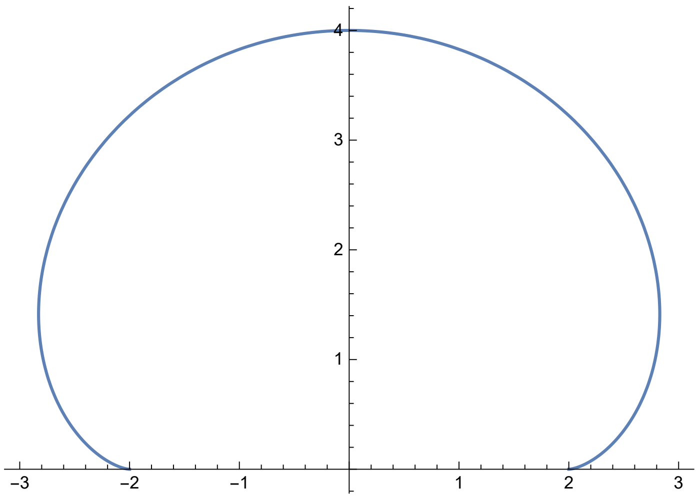
Answer for 6(a)

```
x = 3 Cos[t ] - Cos[3 t];

y = 3 Sin[t ] - Sin[3 t];
```

In[1]:=
```
ParametricPlot[{3 Cos[t ] - Cos[3 t] , 3 Sin[t ] - Sin[3 t]}, {t, 0, π}, PlotRange → Full]
```

Out[1]=



Answer for 6(b)

In[2]:=
```
D[3 Cos[t ] - Cos[3 t], t]
```

Out[2]= $-3\,\mathrm{Sin}[t] + 3\,\mathrm{Sin}[3t]$

In[3]:=
```
D[3 Sin[t ] - Sin[3 t], t]
```

Out[3]= $3\,\mathrm{Cos}[t] - 3\,\mathrm{Cos}[3t]$

In[4]:= $L = \int_0^{\pi} \sqrt{\left(-3\,\mathrm{Sin}[t] + 3\,\mathrm{Sin}[3t]\right)^2 + \left(3\,\mathrm{Cos}[t] - 3\,\mathrm{Cos}[3t]\right)^2}\, dt$

Out[4]= 12

Answer for 6(c)

```
x = Sin[ϕ] Cos[θ] ;

y = Sin[ϕ] Sin[θ] ;

z = Cos[ϕ] ;
```

In[5]:=
```
ParametricPlot3D[{Sin[ϕ] Cos[θ], Sin[ϕ] Sin[θ], Cos[ϕ]}, {θ, 0, 2 π}, {ϕ, 0, π},
 ColorFunction → "Rainbow", Background → LightBlue, Axes → False, Boxed → False]
```

Out[5]=