

MAT216 Midterm Exam Spring 2022

May 2022

Instructions

- Cheating is strictly prohibited. Punishment for your first strike will be a change of seats. Punishment for the second strike will be a deduction of 20 marks.
- The total exam is in 50 marks.
- To get 50 marks, you have to answer all the questions.
- You have 90 minutes to attempt all questions.

Problem 1

15 Marks

You have been given a system of linear equations:

$$\begin{aligned}x_1 + x_2 - x_3 &= 0 \\6x_1 + 4x_2 + 3x_3 &= 21 \\-x_1 + x_2 + 5x_3 &= 12\end{aligned}$$

a.

Find the associated augmented matrix. [2 Marks]

b.

What is the rank of the left-hand side matrix? [1 Mark]

c.

Decompose the left-hand side matrix into LU form. Find both the L and the U matrices. [4+4=8 Marks]

d.

Find the solution to this system using the LU decomposition and back-substitution.
[4 Marks]

Problem 2

10 Marks

You have been given four vectors in R^4 :

$$v_1 = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 4 \\ 8 \\ 3 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 3 \end{bmatrix}, v_4 = \begin{bmatrix} 9 \\ 0 \\ 2 \\ 3 \end{bmatrix}$$

a.

Are these four vectors linearly independent? You have to do the necessary calculations before coming to a decision. [5 Marks]

b.

Find all c_1, c_2 and c_3 such that $c_1 * v_1 + c_2 * v_2 + c_3 * v_3 = v_4$ [5 Marks]

Problem 3

10 Marks

Find if the following statements are true (you have to show the appropriate reasoning/calculations).

a.

The line $x + 2y = 0$ is a subspace of R^2 . [2/5 Marks]

b.

The line $x + 2y = 1$ is a subspace of R^2 . [2.5 Marks]

c.

$\left\{ \begin{bmatrix} x \\ x \\ 0 \end{bmatrix} : x \in R \right\}$ is a subspace of R^3 . [2.5 Marks]

d.

$\left\{ \begin{bmatrix} 2x \\ 2x+3 \\ 1 \end{bmatrix} : x \in R \right\}$ is a subspace of R^3 . [2.5 Marks]

Problem 4

15 Marks

The coefficients of a vector v in R^3 is $(2, 4, 1)$, but the basis being used is not the standard basis, but instead the basis set, $B = \{v_1, v_2, v_3\}$ where

$$v_1 = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -8 \\ 5 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix}$$

a.

What are the coefficients of the vector using the standard basis set, i.e. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$?
[4 Marks]

b.

If we want to change the basis to $B_n = \left\{ \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -8 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix} \right\}$, what will be our matrix of "change of basis" from B to B_n ? [5 Marks]

c.

What will be the coordinates of v using B_n as the basis? [5 Marks]

d.

What are the two most satisfying parts of this course? What are the two most dissatisfying parts? [2 Marks]

