

# MAT216 Final Exam Spring 2022

Section 9

12th May, 2022

## Instructions

- **Total Marks : 60**
- **Exam Duration : 1 hour and 30 minutes**
- You need to answer all the 5 questions.
- Each of the questions carries 12 marks.

## Problem 1

**12 Marks**

Find bases for the four fundamental subspaces of the matrix:

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

## Problem 2

**12 Marks**

Find the projection of the vector  $\mathbf{v}$  onto the subspace  $\mathbf{S}$ .

$$\mathbf{S} = \text{span}\left\{ \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

### Problem 3

**12 Marks**

Use the Gram-Schmidt orthonormalization process to transform the given basis for a subspace of  $\mathbf{R}^4$  into an orthonormal basis for the subspace.

$$\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right\}$$

### Problem 4

**12 Marks**

Find (if possible) a nonsingular matrix  $P$  such that  $P^{-1}AP$  is diagonal. Verify that  $P^{-1}AP$  is a diagonal matrix with the eigenvalues on the diagonal.

$$\mathbf{A} = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

### Problem 5

**12 Marks**

$$\mathbf{A} = \begin{bmatrix} -5 & 0 & 0 \\ 3 & 7 & 0 \\ 4 & -2 & 3 \end{bmatrix}$$

For the given matrix  $A$ , find

(a) the characteristic equation

[2 marks]

(b) the eigenvalues and the corresponding eigenvectors of the matrix.

[8 marks]

(c) the eigen spaces corresponding to each eigenvalues.

[2 marks]