

MAT216 Midterm Exam Spring 2022

Section 8

16th March, 2022

Instructions

- **Total Marks : 50**
- **Exam Duration : 1 hour and 30 minutes**
- You need to answer 4 questions.
- **Problem 1 is mandatory.** Answer any **three** out of the remaining 4 questions.
- Each of the questions carries 12.5 marks.
- Plagiarism is strictly prohibited. Punishment for your first strike will be a change of seats and for the second strike will be a deduction of 20 marks. I hope the punishment for the third strike need not be decided.

Problem 1 (Mandatory)

12.5 Marks

Consider the following system of linear equations:

$$2x + 3y + 3z = 3$$

$$6x + 6y + 12z = 13$$

$$12x + 9y - z = 2$$

- (a) Find the associated augmented matrix.

[1 marks]

- (b) Decompose the left-hand side matrix into LU form. Find both the L and the U matrices.

[4+4=8 Marks]

- (c) Find the solution to this system using the LU decomposition and back-substitution.

[3.5 Marks]

Problem 2

12.5 Marks

Solve the given system of linear equations using either Gaussian elimination with back-substitution or Gauss-Jordan elimination.

$$x + y - 5z = 0$$

$$x - 2z = 21$$

$$2x - y - z = 12$$

Problem 3

12.5 Marks

Let $M_{3,3}(R)$ be the set of all 3×3 matrices over \mathbf{R} and $D_{3,3}(R)$ be the set of all 3×3 diagonal matrices over \mathbf{R} .

- (a) Show that $M_{3,3}(R)$ is a real vector space under the standard matrix addition and scalar multiplication.

[7 marks]

- (b) Determine whether the set $D_{3,3}(R)$, together with the mentioned operations, is a subspace of $M_{3,3}(R)$.

[5.5 marks]

Problem 4

12.5 Marks

Find the coordinate matrices of $v = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ relative to the following bases for R^3 .

(a) $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

[2 marks]

(b) $B = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \right\}$

[4 marks]

$$(c) F = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

[6.5 marks]

Problem 5

12.5 Marks

$$\mathbf{S} = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 2 \end{bmatrix} \right\}$$

Determine whether the set of vectors \mathbf{S} in $M_{4,1}$ is linearly independent or linearly dependent.

