BONUS ASSIGNMENT

(1) $u = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ 0 \end{bmatrix}$ $\quad v = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ 0 \end{bmatrix}$

$$u + v = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ 0 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ 0 \end{bmatrix} = \begin{bmatrix} y_1 + z_1 \\ y_2 + z_2 \\ y_3 + z_3 \\ 0 \end{bmatrix} \in W$$

$$cu = c \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ 0 \end{bmatrix} = \begin{bmatrix} cy_1 \\ cy_2 \\ cy_3 \\ 0 \end{bmatrix} \in W$$

$\therefore$ W is a subspace of V.

(2) $u = \begin{bmatrix} x_1 \\ y_1 \\ 2x_1 - 3y_1 \end{bmatrix}$ $\quad v = \begin{bmatrix} x_2 \\ y_2 \\ 2x_2 - 3y_2 \end{bmatrix}$

$$u + v = \begin{bmatrix} x_1 \\ y_1 \\ 2x_1 - 3y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ 2x_2 - 3y_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \\ (2x_1 + 2x_2) - (3y_1 + 3y_2) \end{bmatrix} \in W$$

$$cu = c \begin{bmatrix} x_1 \\ y_1 \\ 2x_1 - 3y_1 \end{bmatrix} = \begin{bmatrix} cx_1 \\ cy_1 \\ 2cx_1 - 3cy_1 \end{bmatrix} \in W$$

$\therefore$ W is a subspace of V

(3) Let $A = \begin{bmatrix} 0 & a_1 \\ b_1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & a_2 \\ b_2 & 0 \end{bmatrix}$

$$A + B = \begin{bmatrix} 0 & a_1 \\ b_1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & a_2 \\ b_2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & a_1 + a_2 \\ b_1 + b_2 & 0 \end{bmatrix} \in W$$

$$cA = c \begin{bmatrix} 0 & a_1 \\ b_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & ca_1 \\ cb_1 & 0 \end{bmatrix} \in W$$

$\therefore W$ is a subspace of V.

(4) $A = \begin{bmatrix} a_1 & b_1 \\ a_1 + b_1 & 0 \\ 0 & c_1 \end{bmatrix}$ $\quad B = \begin{bmatrix} a_2 & b_2 \\ a_2 + b_2 & 0 \\ 0 & c_2 \end{bmatrix}$

$$A + B = \begin{bmatrix} a_1 & b_1 \\ a_1 + b_1 & 0 \\ 0 & c_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ a_2 + b_2 & 0 \\ 0 & c_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ (a_1 + a_2) + (b_1 + b_2) & 0 \\ 0 & c_1 + c_2 \end{bmatrix} \in W$$

$$cA = c \begin{bmatrix} a_1 & b_1 \\ a_1 + b_1 & 0 \\ 0 & c_1 \end{bmatrix} = \begin{bmatrix} ca_1 & cb_1 \\ ca_1 + cb_1 & 0 \\ 0 & cc_1 \end{bmatrix} \in W$$

$\therefore W$ is a subspace of V

- $W = \begin{bmatrix} x \\ y \\ -1 \end{bmatrix}$

$u = \begin{bmatrix} x_1 \\ y_1 \\ -1 \end{bmatrix}, v = \begin{bmatrix} x_2 \\ y_2 \\ -1 \end{bmatrix}$

$u + v = \begin{bmatrix} x_1 \\ y_1 \\ -1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ -1 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \\ -2 \end{bmatrix} \notin W$

$\therefore$ W is not a subspace of V

- $W = \begin{bmatrix} x \\ 1 \end{bmatrix}$

$u = \begin{bmatrix} x_1 \\ 1 \end{bmatrix}, v = \begin{bmatrix} x_2 \\ 1 \end{bmatrix}$

$u + v = \begin{bmatrix} x_1 \\ 1 \end{bmatrix} + \begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ 2 \end{bmatrix} \notin W$

$\therefore$ W is not a subspace of V

- $W = \begin{bmatrix} x \\ y \end{bmatrix}$ ~~$W \in$ rational~~ $W \in$ rational number

$u = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, v = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$u + v = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix} \notin$ ~~rational number~~ W

e.g. $\begin{bmatrix} \frac{1}{2} + \frac{1}{2} \\ \frac{25}{2} + \frac{3}{2} \end{bmatrix} \notin$ ~~rational number~~ W

$\therefore W$ is not a subspace of V

- $W = \begin{bmatrix} x \\ y \end{bmatrix}$ $W \in$ integer

$u = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ $v = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$u + v = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix} \notin W$

e.g. $\begin{bmatrix} 0.5 + 1.4 \\ 2 + 3.5 \end{bmatrix} \notin W$

$\therefore W$ is not a subspace of V