PHP2510

EXAM #1 EXTRA CREDIT WEEKS 1-6 ANSWER KEY

You are studying whether a new sleep medication is effective. You recruit patients to participate in an experiment: they each take the medication at 10pm then go to bed in a monitored setting. For each patient, you collect whether the patient entered REM sleep before 2am (a yes/no variable, coded 1/0). You want to estimate, at the population level, the proportion of people that will indeed enter REM sleep within 4 hours when taking the medication. But, of course, you only have your sample (of n patients in the experiment), so you need to perform statistical inference.

1. [+1] Create notation for this problem. Make sure you clearly define:
   a. Your outcome variable and its theoretical distribution
   b. The population parameter of interest (that you want to estimate)
   c. The data you actually collected

Outcome is X ~ Bernoulli(p) - 1s and 0s from the population
The population parameter of interest is p, the underlying probability of REM sleep within 4 hours
X_i ~ iid i=1, …, n draws from X was collected as data
Stating the data was Bin(n,p), but not specifying n=1, was given partial credit. Students who assumed we collected the total number of successes (rather than 1s/0s) had significantly more trouble on later parts (they added extra n's in many places).

2. [+1] Why would the sample mean be a reasonable statistic to use in this setting? Mathematically prove your justification, using notation from (1). *Hint: use the linearity of expectations.*

It is an unbiased estimator of p.
*E[sum(X_i)/n] = 1/n*E[sum(X_i)] = 1/n*(sum(E[X_i])) = 1/n*sum(p) = 1/n * np = p*

- *Full credit is given to "it's consistent" if it was mathematically proven.*
- *1/2 credit is given to "it's unbiased" and/or "it's consistent" if no mathematical proof is given*

- *1/2 credit is given to “the central limit theorem applies, if n is large enough” [which is true, but without a mathematical proof]*

3. [+1] Using the central limit theorem, what is the approximate distribution of your sample mean? *Hint: use the fact that a Bern(p) has variance p(1-p) – which you proved in homework #1! – or the fact that a Bin(n,p) has variance np(1-p)*

CLT says that the sample mean is approximately N(population mean, population variance / n) = N(p, p(1-p)/n).*
Note: the Bin(n,p) variance was given in case students wanted to directly calculate the variance of the sample mean (sum of X_i / n), where the numerator has the variance of np(1-p), so the overall variance is p(1-p)/n

4. [+1] Explain, in words or with a picture, how you'd decide if a certain medication effectiveness in the population (e.g. 70%) was likely or not, given your realized data. Please be concise (2-3 sentences is plenty).

If p = 0.7, that means the sample mean, from (3), is approximately N(0.7, 0.21/n). I can calculate how likely/extreme my realized sample mean is using that distribution (via area under the curve of one or two tails of the distribution).
A picture showing a normal distribution, centered at 0.7 with sd = sqrt(0.21/n), and calculating an area under the curve (of one or two tails), based on the realized value of the sample mean, is also a great answer.
Note that many students got this switched (they drew a picture centered at the sample mean, and checked if 0.7 was extreme); this was given partial credit

5. [+1] What is an “iid” sample? Explain the concept in your own non-technical words (do not just state what the letters stand for). What could go wrong with this experiment that would violate an “iid” assumption? Please be concise (2-3 sentences is plenty).

An “iid” sample is a collection of observations that are all from a single population, each drawn independently from each other. That means that observing one subject has no influence on whether I observe a different subject, and all subjects’ outcomes come from the same underlying distribution of outcomes. If this experiment had patients co-located, the “independence” assumption would be very unlikely because someone could interrupt someone else’s sleep.