Optimizing Profit for The WARP Shoe Company

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1.0 Abstract

Using the provided information given in the problem statement, we were able to formulate a relaxed integer program resulting in a production plan with the most profit for WARP Shoes Company. The decision variable chosen is x_i, representing the number of pairs of shoes of type i produced. There are 5 different constraints including raw materials budget, warehouse capacity, maximum machine time, demand for each shoe, and available raw materials, as well as an added non-negativity constraint. Using the programming language AMPL, we created various sets, parameters, variables, an objective function, and constraints to create our program. Using the Gurobi solver on AMPL, our program yielded a maximum profit of $13,458,506.80.

2.0 Introduction

At the beginning of 2006, the WARP Shoe Company – one of the oldest shoe companies in Canada – market analysts detected a double in demand in February due to the bankruptcy of their competitor. The WARP management has been consulting with the University of Toronto Industrial Engineering students to seek help in producing an optimal production plan. With this in mind, it is our goal to create a production plan to maximize the WARP Shoes Company's profit while still complying to their stated constraints and assumptions.

3.0 Methodology

3.1 Assumptions

This section identifies the assumptions provided in the problem statement and additional assumptions made to formulate a model.

- Each shoe must use one sole, lace, leather, label, and attachment.
- To determine the demand for February 2006, average all the data from February from 1997 to 2003.
- Shoes will be arbitrarily shipped to warehouses until they are fully occupied.
- The number of shoes produced will be an element of integers.
- Shoes will be produced until the demand is satisfied.
- The closing inventory of January 2006 was zero for all types of shoes.
- All sales happen at the end of the month.
- Transportation and machine set-up costs are ignored.
- During February 2006, machines must only operate up to 12 hours a day for 28 days a month (this translates to 20,160 minutes/month)

- The budget for raw materials is $10,000,000.

3.2 Integer Programming Formulation

Below is the formulation and explanation of all the components of the integer program for the WARP Shoes optimization problem.

Decision Variable:

x_i: $number\ of\ pairs\ of\ shoes\ of\ type\ i\ produced$

$i = 1, 2, 3, ..., 557$

Parameters:

p_i: $price\ of\ shoe\ type\ i$

$i = 1, 2, 3, ..., 557$

d_i: $demand\ of\ shoe\ i\ in\ total$

$i = 1, 2, 3, ..., 557$

m_k: $operation\ cost\ of\ machine\ k$

$k = 1, 2, 3, ..., 72$

c_n: $cost\ of\ raw\ material\ n$

$n = 1, 2, 3, ..., 165$

$r_{i,n}$: $quantity\ of\ raw\ material\ n\ required\ for\ shoe\ type\ i$

$i = 1, 2, 3, ..., 557$

$n = 1, 2, 3, ..., 165$

a_n: $quantity\ of\ raw\ material\ n\ available\ to\ purchase\ within\ a\ month$

$n = 1, 2, 3, ..., 165$

w_l: $warehouse\ capacity\ in\ warehouse\ l$

$l = 1, 2, 3, ..., 8$

$v_{i,k}$: *average duration on machine k to make shoe type i*

$i = 1, 2, 3, \ldots, 557$

$k = 1, 2, 3, \ldots, 72$

y_i: *binary variable for demand*

$i = 1, 2, 3, \ldots, 557$

Objective Function:

Maximize profit:

$$z = \sum_{i=1}^{557} p_i \cdot x_i - \sum_{i=1}^{557} 10(2 \cdot d_i - x_i)y_i - \frac{25}{60}\sum_{i=1}^{557}\sum_{i=1}^{557}\frac{1}{60} \cdot v_{i,k} \cdot x_i$$
$$- \sum_{i=1}^{557}\sum_{n=1}^{165} x_i \cdot c_n \cdot r_{i,n} - \sum_{k=1}^{72} m_k \cdot \sum_{i=1}^{557}\frac{1}{60} \cdot v_{i,k}$$

Constraints:

1. The cost of the total raw materials must be less than or equal to 10 million dollars.

RawMaterialsBudget: $\sum_{i=1}^{557} \sum_{n=1}^{165} x_i \cdot c_n \cdot r_{i,n} \leq 10\,000\,000$

2. The maximum amount of shoes that can be stored across all warehouses.

WarehouseCapacity: $\sum_{i=0}^{557} x_i \leq \sum_{l=1}^{8} w_l$

3. The maximum amount of time each machine must only work up to (60 minutes/hour)(12 hours/day)(28 days/month) = 20160 minutes/month.

MachineTime: $\frac{1}{60}\sum_{i=1}^{557} v_{i,k} \cdot x_i = 20160, \quad \forall k = 1, \ldots, 72$

4. The maximum of raw materials that are available to be purchased within a month.

$$\text{AvailRawMaterials: } \sum_{i=1}^{557} x_i \cdot r_{i,n} \le a_n, \quad \forall n = 1, \dots, 165$$

5. Binary constraint for demand: the demand must be greater or equal to the number of manufactured shoes.

$$y_i = \begin{cases} 0 & \text{if } (dj - xi) < 0 \\ 1 & \text{if } (dj - xi) \ge 0 \end{cases}$$

$i = 1, 2, 3, \dots, 557$

6. Non-negativity constraints: the number of pairs of type i shoes cannot be negative for the decision variable.

$x_i \ge 0$

Please refer to *warp.mod* to view how we implemented these sets, variables, parameters, the objective function, and constraints within AMPL.

4.0 Results

To solve the integer program introduced in section 3.2, the Gurobi solver was used in the AMPL program. Additionally, five files were combined to generate a solution: a mod file, dat file, run file, and an Access database. In the mod file, sets parameters, the decision variable, objective function, and constraints are declared. By implementing the ODBC handler in the dat file and reading through the respective Access tables, each parameter was instantiated with values. Finally, the run file was essential in incorporating the Gurobi solver and printing relevant values and information.

The problem was initially modeled as an integer program; however, the computation time was significantly greater. Therefore, the integer program was relaxed to a linear program and an objective function value of 13,458,506.80 was calculated. In other words, the maximum profit is $13,458,506.80. The number of shoes created of each type can be found in the *warp.out* file.

Question 1: How should you estimate the demand for the month of February?

The provided database file contained demand data from the years of 1997 to 2003 for each month and each shoe type. Therefore, some calculations had to be performed to estimate the demand for February 2006. By leveraging Excel formulas, the average of the demand for each shoe was taken during February for all years. Further details about this process can be found in Appendix 6.1. These obtained averages were then inserted into the *Product_Master* file in the Access database and were extracted in the AMPL dat file. Moreover, the store numbers associated with the given demand values were not considered since transportation costs are ignored. Therefore, the LP formulation ensures that the demand for the shoe type is met.

Question 2: How many variables and constraints do you have?

There are 10 types of variables in total with 8 parameters, 1 binary variable, and 1 decision variable. Additionally, there are 6 constraints. These are all presented below.

Parameters:
p[i] : 557 parameters
d[i]: 557 parameters
m[k]: 72 parameters
c[n]: 165 parameters
r[i, n]: $557 \cdot 165$ = 91,905 parameters
a[n]: 165 parameters
w[l]: 8 parameters
v[i, k]: $557 \cdot 72$ = 40,104 parameters

Variables:
Binary Variable:
y[i]: 557 variables

Decision Variable:
x[i]: 557 variables

Constraints:
RawMaterialsBudget: 1 constraint
WarehouseCapacity: 1 constraint
MachineTime: 72 (k = 72) constraints
Demand: 557 (i = 557) constraints
AvailRawMaterials: 165 (n = 165) constraints
Non-negativity constraint: 1 constraint

Question 3: If you had to relax your integer program to an LP, how many constraints were violated after rounding the LP solution to the closest integer solution?

After relaxing the integer program to a linear program, one type of constraint was violated: available raw materials. Within this type of constraint, there are 165 individual constraints and the violated ones can be found in the *warp.out* file.

Question 4: Which constraints are binding, and what is the real-world interpretation of those binding constraints?

Several available raw materials constraints are binding; these are printed in the *warp.out* file. A binding constraint occurs when the left-hand side equals the right-hand side and this demonstrates the constraint is satisfied through the decision variable's optimal value. For certain materials, the available raw materials constraints are binding, which indicates all the available material is being used. Thus, a decrease to the right-hand side of the constraints will "worsen" the objective function value and an increase will "improve" it.

Question 5: Assume that some additional warehouse space is available at the price of $10/box of shoes. Is it economical to buy it? What is the optimal amount of space to buy in this situation?

The warehouse constraint ("WarehouseCapacity") is not binding; therefore, there is still additional space available. With that said, increasing the warehouse capacity will only create more additional space and will not result in any significant changes to the objective function value. It is not economical to buy it.

Question 6: Imagine that machines were available for only 8 hours per day. How would your solution change? Which constraints are binding now? Does the new solution seem realistic to you?

The objective function value would undergo minimal changes because all the machine time constraints are nonbinding. This means that for every machine, there is still extra time available to use. After implementing the change of 8 hours per day, the same constraints are binding ("AvailRawMaterials") and the objective function value changes from $13,458,506.80 to $13,458,626.86, undergoing an increase of $120.06. This is unrealistic because a decrease in the hours (i.e. less time to make shoes) should decrease the objective function value. This discrepancy may be attributed to the IP relaxation.

Question 7: If in addition there were a $7,000,000 budget available to buy raw materials, what would you do? Change your formulation and solve again.

A 7,000,000 increase in the "RawMaterialsBudget " would not result in a notable change since there are already extra funds in the original budget due to the nonbinding nature of this constraint. After solving again, a value of $13,458,601.49 is obtained, increasing by $94.69. Evidently, this is a very insignificant change.

5.0 Conclusion

In conclusion, the most profitable production plan for the Warp Shoe Company, accounting for the doubled demand in February 2006, resulted in a profit of $13,458,506.80. Within our relaxed integer program, the defined sets, variables, parameters, objective function, constraints, and the Gurobi solver on AMPL help generate this maximum profit. Through the *warp.run* file and *warp.out* file, along with the final maximized profit, the decision variable – optimal number of shoes to be produced – can also be identified.

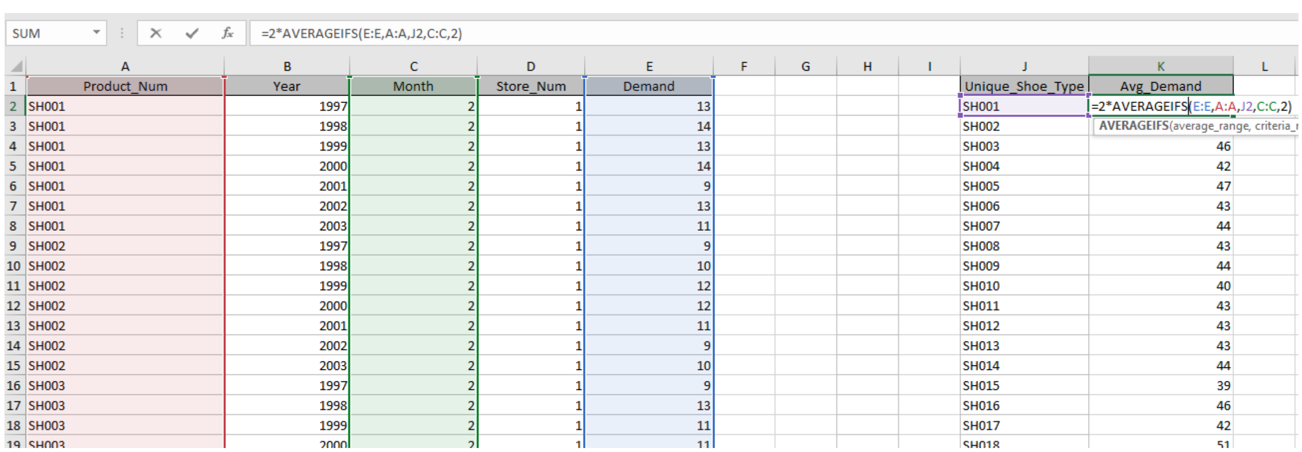
6.0 Appendices

Appendix 6.1 Demand Calculations

To calculate the average demand for shoes during February 1997-2003, an Excel sheet, which is attached with the submission, was used to pull the data from the *Product_Demand* table in the mdb file. Two new columns were created and called “Unique_Shoe_Type” and “Avg_Demand” which represent the product number of each shoe and the calculated average demand. This average demand was calculated by using this equation:

=2 * AVERAGEIFS(E:E, A:A,J2,C:C,2)

Please refer to Figure 1 below to view how this equation is used in Excel.



SUM =2*AVERAGEIFS(E:E,A:A,J2,C:C,2)

	A	B	C	D	E	J	K
1	Product_Num	Year	Month	Store_Num	Demand	Unique_Shoe_Type	Avg_Demand
2	SH001	1997	2	1	13	SH001	=2*AVERAGEIFS(E:E,A:A,J2,C:C,2)
3	SH001	1998	2	1	14	SH002	AVERAGEIFS(average_range, criteria_r
4	SH001	1999	2	1	13	SH003	46
5	SH001	2000	2	1	14	SH004	42
6	SH001	2001	2	1	9	SH005	47
7	SH001	2002	2	1	13	SH006	43
8	SH001	2003	2	1	11	SH007	44
9	SH002	1997	2	1	9	SH008	43
10	SH002	1998	2	1	10	SH009	44
11	SH002	1999	2	1	12	SH010	40
12	SH002	2000	2	1	12	SH011	43
13	SH002	2001	2	1	11	SH012	43
14	SH002	2002	2	1	9	SH013	43
15	SH002	2003	2	1	10	SH014	44
16	SH003	1997	2	1	9	SH015	39
17	SH003	1998	2	1	13	SH016	46
18	SH003	1999	2	1	11	SH017	42
19	SH003	2000	2	1	11	SH018	51

Figure 1. Average Demand Calculations in Excel