Counting Problems:

1. UNUSUAL

   * 5 letters that don't repeat
   * # of strings that can be formed } 5!

   $5 \times 4 \times 3 \times 2 \times 1 = 120$

* There is one unique subset that is possible
* 120 different strings can be made.

2. num of ways to form a 5-card hand with 2 pairs } $\frac{13!}{2! \times 11!} \cdot \frac{4!}{2! \times 2} \cdot \frac{4!}{2! \times 2} \cdot 44 =$

   $= 123,552$

3. # of songs : 16

   # of couples : 7

formula: $\frac{n!}{r! \times (n-r)!}$

* one is having fight; at most 1 song to be played

Total ways of distributing } ${}_{16}C_1 \times {}_{15}C_6$

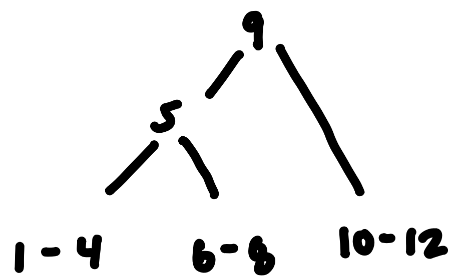
$\frac{16!}{1!\,(16-1)!} \cdot \frac{15!}{6!\,(15-6)!} = 16(5005) = 80080$

There are 80,080 ways that the songs can be distributed amongst the couples.

4. BST with 12 nodes

node value: between 1 & 12

root: 9, left-child: 5



$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\frac{2(4)}{4+1} \qquad \frac{2(3)}{3+1} \qquad \frac{2(3)}{3+1}$$

14   5   5

$14 \times 5 \times 5 = 350$

Total possibilities = 350

5. 10 friends per slot : $\frac{10!}{(10-1)!\,1!} = 10$

4 nurses : $\frac{4!}{(4-1)!\,1!} = \frac{4!}{3!} = 4$

$= \frac{10!}{(10-4)!\,4!} = 210$ mins

$= \frac{10!}{(10-4)!} = 5040$

There are 5040 different combinations.