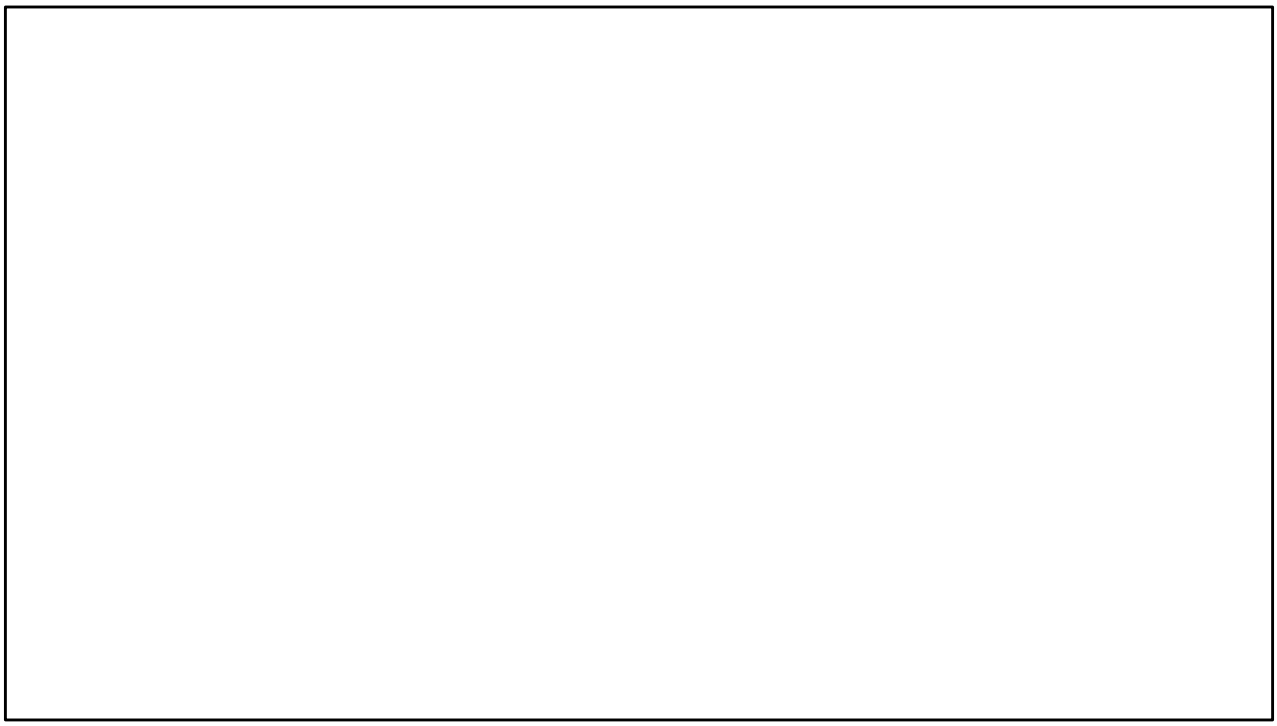


Today's class

- Randomized quick sort
- Pointers
- Dynamic arrays

Randomized Quick sort

```
void quicksort(int arr[], int lo, int hi)
{
  if (lo >= hi) {
    return;
  }
  int p = partition(arr, lo, hi);
  quicksort(arr, lo, p - 1);
  quicksort(arr, p + 1, hi);
}
```

```
int partition(int arr[], int lo, int hi) {
  int idx = get_random_idx(lo, hi);
  exchange(arr, lo, idx);
  int pivot = arr[lo];
  int left = lo + 1;
  int right = hi;

  while (left <= right) {
    while (left <= right && arr[left] < pivot) {
      left += 1;
    }
    while (right >= left && arr[right] > pivot) {
      right -= 1;
    }
    if (left <= right) {
      exchange(arr, left, right);
      left++; right--;
    }
  }

  exchange(arr, lo, right);
  return right;
}
```

In the randomize quick sort algorithm, we pick the pivot element randomly. If the pivot is truly random, then the probability of getting the final position of the pivot as 0, 1, 2, …, N-1 would be 1/N. However, generating a truly random number is hard, and that is a separate discussion outside the scope of this course. This algorithm is suitable for average time complexity.

Time complexity

```
void quicksort(int arr[], int lo, int hi)
{
  if (lo >= hi) {
    return;
  }
  int p = partition(arr, lo, hi);
  quicksort(arr, lo, p - 1);
  quicksort(arr, p + 1, hi);
}
```

Average case:

In the randomized algorithm, the probabilities of getting the final position of the pivot as 0, 1, 2, …, n-1 are equal. So, the average time complexity is the arithmetic mean of the number of operations in all N possible partitions.

$$T(n) = \frac{1}{n}\sum_{i=0}^{n-1}(T(i) + T(n-i-1)) + c_1 n + c_2$$

```
int partition(int arr[], int lo, int hi) {
  int idx = get_random_idx(lo, hi);
  exchange(arr, lo, idx);
  int pivot = arr[lo];
  int left = lo + 1;
  int right = hi;

  while (left <= right) {
    while (left <= right && arr[left] < pivot) {
      left += 1;
    }
    while (right >= left && arr[right] > pivot) {
      right -= 1;
    }
    if (left <= right) {
      exchange(arr, left, right);
      left++; right--;
    }
  }

  exchange(arr, lo, right);
  return right;
}
```

If the partitioning algorithm returns an index i, the first recursive call performs T(i) operations and the second recursive call performs T(n-i-1) operations. If we take the average of all possible outcomes for i, we get the recurrence relation shown on this slide.

Time complexity (average case)

$T(1) = c$

$$T(n) = \frac{1}{n}\left(\sum_{i=0}^{n-1} (T(i) + T(n-1-i))\right) + c_1 n + c_2$$

$$nT(n) = \left(\sum_{i=0}^{n-1} (T(i) + T(n-1-i))\right) + c_1 n^2 + c_2 n$$

$$nT(n) = \left(\sum_{i=0}^{n-1} T(i) + \sum_{i=n-1}^{0} T(i)\right) + c_1 n^2 + c_2 n$$

$$nT(n) = 2\sum_{i=0}^{n-1} T(i) + c_1 n^2 + c_2 n$$

$$(n-1)T(n-1) = 2\sum_{i=0}^{n-2} T(i) + c_1 (n-1)^2 + c_2 (n-1)$$

$$nT(n) - (n-1)T(n-1) = 2T(n-1) + c_1(2n-1) + c_2$$

$$nT(n) = 2T(n-1) + (n-1)T(n-1) + 2c_1 n + c_3$$

To solve the recurrence relations in which the entire history is present, we can compute the value of T(n-1) and subtract it from T(n). This will ensure that most of the history will be canceled out and the corresponding equation is in the form that can be solved using the expansion method.

Time complexity (average case)

$$nT(n) = (n+1)T(n-1) + 2c_1 n + c_3$$

$$T(n) = \frac{n+1}{n} T(n-1) + 2c_1 + \frac{c_3}{n}$$

$$T(n) \le \frac{n+1}{n} T(n-1) + 2c_1$$

TYPO: Use c_4 instead of $2c_1$ from here onwards

$$\le \frac{n+1}{n}\left(\frac{n}{n-1} T(n-2) + 2c_1\right) + 2c_1$$

$$\le \frac{n+1}{n-1} T(n-2) + 2c_1\left(1 + \frac{n+1}{n}\right)$$

$$\le \frac{n+1}{n-1}\left(\frac{n-1}{n-2} T(n-3) + 2c_1\right) + 2c_1\left(1 + \frac{n+1}{n}\right)$$

$$\le \frac{n+1}{n-2} T(n-3) + 2c_1\left(1 + \frac{n+1}{n} + \frac{n+1}{n-1}\right)$$

$$\le \frac{n+1}{n-k+1} T(n-k) + 2c_1\left(1 + \frac{n+1}{n} + \frac{n+1}{n-1} + \cdots + \frac{n+1}{n-k+2}\right)$$

Here, we are assuming that c_3/n will always be less than some other constant. Therefore, we can use another constant c_4 instead of $2c_1 + c_3 / n$.

Time complexity (average case)

$$T(n) \le \frac{n+1}{n-K+1} T(n-K) + 2c_1 \left(1 + \frac{n+1}{n} + \frac{n+1}{n-1} + \cdots + \frac{n+1}{n-K+2}\right)$$

Substitute : $n - K = 1$

$$T(n) \le \frac{n+1}{2} T(1) + 2c_1 \left(1 + \frac{n+1}{n} + \frac{n+1}{n-1} + \cdots + \frac{n+1}{3}\right)$$

$$\le \frac{n+1}{2} \times c + 2c_1 (n+1) \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n+1} - \frac{3}{2}\right)$$

$$\le \frac{n+1}{2} \times c + 2c_1 (n+1) \times \left(c_2 \log(n+1) - \frac{3}{2}\right)$$

$$O(n \log n)$$

Harmonic series sum

- https://en.wikipedia.org/wiki/Harmonic_series_(mathematics)
  - Comparison test
  - Integral test

Harmonic series sum

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

$$\leq 1 + \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) + \dots +$$

Let's say there are k phases

$$\leq k$$

$$1 + 2 + 2^2 + \dots + 2^{k-1} = n$$

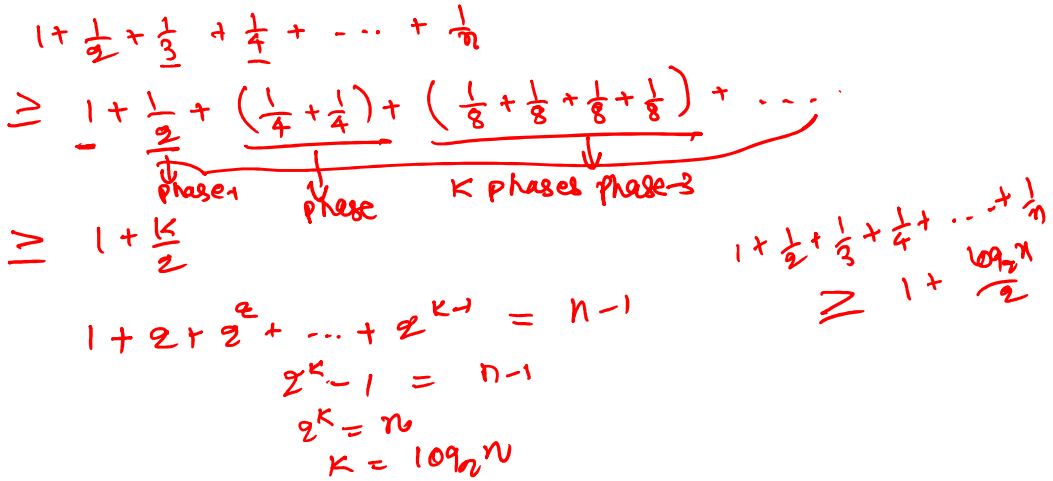
$$2^k - 1 = n$$

$$k = \log_2(n+1)$$

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \leq \log_2(n+1)$$

To find an upper bound, we can replace 1/3 with 1/2; 1/5, 1/6, 1/7 with 1/4; 1/9,1/10, …, 1/15 with 1/8, and so on until we have replaced all terms. Let's say the group of all the terms with common denominators is a phase, and there are k phases. Therefore, the sum of the harmonic series would be k. Now, let's add the number of terms in each phase to get the total number of terms. The number of terms in phase-1 is 1, the number of terms in phase-2 is 2, the number of elements in phase 3 is 2^2, and so on. Therefore, the number of elements in the kth phases would be 2^(k-1). The sum of the number of terms in all phases is the sum of the geometric series: 1, 2, 2^2, 2^3, …, 2^(k-1), which is equal to the total number of terms n. Solving these equations gives us the upper bound as $\log_2(n+1)$.

Harmonic series sum



To find a lower bound, we can group them differently. We can replace 1/3 with 1/4; 1/5, 1/6, 1/7 with 1/8; 1/9,1/10, …, 1/15 with 1/16, and so on until we have replaced all terms. Let's say the group of all the terms with common denominators except the first term is a phase, and there are k phases. The sum of the harmonic series would be 1 + k/2. Now, let's add the number of terms in each phase to get the total number of terms. The number of terms in phase-1 is 1, the number of terms in phase-2 is 2, the number of elements in phase 3 is 4, and so on. Therefore, the number of elements in the kth phases is 2^(k-1). The sum of the number of terms in all phases is the sum of the geometric series: 1, 2, 2^2, 2^3, …, 2^(k-1), which is equal to n-1. Solving these equations gives us the lower bound as 1 + ($\log_2$(n)/2).

Time complexity (average case)

- Time complexity, when the portioning algorithm returns i

$$T(n) = T(i) + T(n-1-i) + c_1 n + c_2$$

If each index has the same probability of being selected as a target position for the pivot, then the average complexity is

$$\mathrm{T(n)} = \frac{1}{n}\left(\sum_{i=0}^{n-1}(T(i) + T(n-1-i))\right) + \mathrm{c_1 n} + \mathrm{c_2}$$

Time complexity (average case)

$$\mathrm{T(n)} = \frac{1}{n}\left(\sum_{i=0}^{n-1}(T(i) + T(n-1-i))\right) + (\mathrm{c_1 n} + \mathrm{c_2})$$

$$\mathrm{T(n)} = \frac{2}{n}\left(\sum_{i=0}^{n-1}(T(i))\right) + (\mathrm{c_1 n} + \mathrm{c_2})$$

(Full history recurrence relation)

$$\mathrm{nT(n)} = 2\left(\sum_{i=0}^{n-1}(T(i)\right) + \mathrm{c_1 n^2} + \mathrm{c_2 n}$$

$$\mathrm{(n-1)T(n-1)} = 2\left(\sum_{i=0}^{n-2}(T(i)\right) + \mathrm{c_1(n-1)^2} + \mathrm{c_2(n-1)}$$

$$\mathrm{nT(n)} - \mathrm{(n-1)}T(n-1) = 2T(n-1) + c_1(2n-1) + c_2$$

$$T(n) = \frac{n+1}{n}T(n-1) + 2c_1 + \frac{c_2 - c_1}{n} \leq \frac{n+1}{n}T(n-1) + c$$

Here, we are assuming that "$(c_2 - c_1)/n$" will always be less than some other constant c_3, and $2c_1 + c_3 = c$.

Time complexity(average case)

$$T(n) \leq \frac{n+1}{n} T(n-1) + c$$

$$= \frac{n+1}{n}\left(\frac{n}{n-1} T(n-2) + c\right) + c = \frac{n+1}{n-1} T(n-2) + c\left(1 + \frac{n+1}{n}\right)$$

$$= \frac{n+1}{n-1}\left(\frac{n-1}{n-2} T(n-3) + c\right) + c\left(1 + \frac{n+1}{n}\right) = \frac{n+1}{n-2} T(n-3) + c\left(1 + \frac{n+1}{n} + \frac{n+1}{n-1}\right)$$

$$= \ \ldots$$

$$= \frac{n+1}{n-k+1} T(n-k) + c\left(1 + \frac{n+1}{n} + \frac{n+1}{n-1} + \ \ldots + \frac{n+1}{n-k+2}\right)$$

Time complexity (average case)

Substituting k = n – 1

$$T(n) \leq \frac{n+1}{2} T(1) + c\left(1 + \frac{n+1}{n} + \frac{n+1}{n-1} + \ \ldots + \frac{n+1}{3}\right)$$

$$= \frac{n+1}{2} c_3 + c(n+1)\left(1 + \frac{1}{2} + \frac{1}{3} + .. + \frac{1}{n+1} - \frac{3}{2}\right)$$

Because, $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ *is* $O(\log n)$

$$= \frac{n+1}{2} c_3 + c(n+1)\left(c_4(\log(n+1)) - \frac{3}{2}\right) = O(n \log n)$$

Harmonic series sum

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

$$\leq 1 + \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) + \cdots$$

$$\leq 1 + 1 + 1 + \ \ldots upto\ k\ terms\ =\ \mathrm{k}$$

$$1 + 2 + 2^2 + \cdots + 2^{k-1} = n$$
$$2^k - 1 = n$$
$$k = \log_2(n+1)$$
$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \leq \log_2(n+1)$$

Harmonic series sum

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

$$\geq 1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right) + \cdots$$

$$\geq 1 + \frac{k}{2}$$

$1 + 2 + 2^2 + \cdots + 2^{k-1} = n - 1$
$2^k - 1 = n - 1$
$k = \log_2 n$
$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \geq 1 + \frac{\log_2 n}{2}$

Quick sort vs merge sort

Quick sort

- The worst-case time complexity is O(n^2)
- Require very little extra space (an in-place algorithm)
- Empirically, quick sort performs better than merge sort

Merge sort

- The worst-case time complexity is O($n \log n$)
- Requires extra space during the merge operation (not in-place)
- Empirically, merge sort performs poorly than quick sort

Quick sort

- C++ standard library implements a hybrid sorting algorithm, Intro-sort, to mitigate the worst-case behavior of the quick sort
  - https://en.wikipedia.org/wiki/Introsort
  - Quick sort + Heap sort (worst case $n \log n$) + Insertion sort ($O(n^2)$)

Homework

- Modify the randomize quick sort algorithm to find the median of n numbers
  - No need to sort the entire array
- There is also an algorithm with O(n) worst-case complexity for finding median of n numbers
  - You will get a chance to implement that algorithm in the bonus assignment

Asymptotic analysis

Shortcomings of asymptotic analysis

- Ignores the highest-order constant factors
  - An algorithm that takes 100n operations is considered the same as an algorithm that takes 2n operations, but the later is 50x faster
    - As asymptotic analysis is done at a very high level, computing the correct constant factor is challenging because of its dependency on the underlying hardware and compiler
- Other constant factors are also ignored
  - $2n^2$ is better than 1000n + 10000 for small inputs, but asymptotic analysis ignores it
- Only consider the worst-case performance
  - Even though the average running time is significantly lower than the worst case, the algorithm may be still be considered bad because the average-time analysis is extremely complex for many problems

Pointers

Assignment

- We can only do an assignment if the type of the variable in LHS and the type of the value in the RHS are the same
  - Otherwise, it is an error
- When we do an assignment: a = b, where the type of a and b is Ty, sizeof(Ty) bytes are copied from b to a

Assignment

```
int a;

a = 10;
// Is this a valid assignment?  Yes
// How many bytes are copied during the assignment?  4
```

a = 10 is a valid assignment because both sides are of type int. Four bytes will be copied because the size of int is 4.

Assignment

```
struct record {
   int a;
   int b;
   int c[20];
};

struct record var1;
struct record var2;
var1.a = 10;
var1.b = 20;
var2 = var1;
// Is this a legal assignment?
// How many bytes will be copied?

printf("%d %d %d %d\n", var2.a, var2.b, var2.c[0], var2.c[1]);
// what will be the output of the printf?
```

"var2 = var1" is a legal assignment because both are of type struct record. Eighty-eight bytes will be copied because the size of struct record is 88.

Assignment

```
struct record {
   int a;
   int b;
   int c[20];
};

struct record var1;
int var2;
var1.a = 10;
var1.b = 20;
var2 = var1;
// Is this a legal assignment?
// How many bytes will be copied?

printf("%d %d %d %d\n", var2.a, var2.b, var2.c[0], var2.c[1]);
// what will be the output of the printf?
```

No

var2 = var1 is not a legal assignment because their types are different.

Assignment

```
struct record {
   int a;
   int b;
   int c[20];
};

struct record var1;
struct record var2;
var1.a = 10;
var1.b = 20;
var2.a = var1;
// Is this a legal assignment?
// How many bytes will be copied?

printf("%d %d %d %d\n", var2.a, var2.b, var2.c[0], var2.c[1]);
// what will be the output of the printf?
```

NO

var2.a = var1 is not a legal assignment because their types are different.

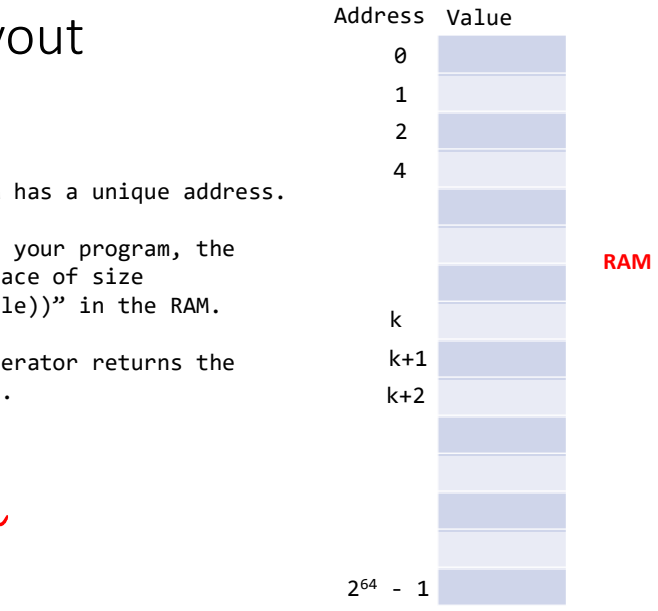
Memory layout

Every byte of the RAM has a unique address.

For every variable in your program, the compiler allocates space of size "sizeof(typeof(variable))" in the RAM.

The address of (&) operator returns the address of a variable.

int a;



Address of (&) operator

```
struct record {
   int a;
   int b;
   int c[20];
};

struct record var1;
var1.a = 10;
var1.b = 20;

printf("%p\n", &var1);
// What will be the output of the printf?
// How can we store the address of var1 in a variable?
```

The & operator returns the address of a variable in the RAM.

Pointers

```
int a;
```

The **type** of **&a** is **int***. **int*** is called **pointer** to an integer.

If a variable “a” is of type “int”, the type of “&a” is “int*”.

Pointers

- A pointer variable stores the address of a variable (i.e., a RAM address)

```
int a = 10;
int *x = &a;    // the type of &a is int*
int **y = &x;   // the type of &x is int**
int ***z = &y; // the type of &z is int***

struct record r;
What is the type of variable that can store the address of variable r?
```

struct record *v = &r;

Because "a" is of type "int", the type of "&a" is "int*". Therefore, x = &a is a valid assignment because both sides are of the same type. Similarly, the type of &x is int** because the type of x is int*. In general, the type of the address of a variable of type Ty followed by k stars is Ty followed by (k+1) stars. If we want to store the address of a variable of type "struct record", we can store it in a variable of type struct record*.

Pointers

```
struct record {
  int a;
  int b;
  struct record *next;
};

struct record var1;
var1.a = 10;
var1.b = 20;
var1.next = 30;
```

Is this a legal program? No

This is not a legal program because at the line "var1.next = 30", the type of var1.next is "struct record*", and the type of 30 is int. Therefore, it is not a valid assignment.

Pointers

```
struct record {
   int a;
   int b;
   struct record *next;
};

struct record var1;
var1.a = 10;
var1.b = 20;
var1.next = &var1;
```

Is this a legal program? Yes

This is a legal program because the statement “var1.next = &var1” is valid. The type of var1.next is "struct record*", and the type of &var1 is also "struct record*".

Pointers

```
struct record {
   int a;
   int b;
   struct record *next;
};

struct record var1;
struct record *var2;
var2 = &var1;
var1.a = 10;
var1.b = 20;
var1.next = &var1;
```

Is this a legal program? yes

This is a legal program because the assignment var2 = &var1 is valid. The type of var2 is "struct record*", and the type of &var1 is also "struct record*".

Copying address

- We can copy the RAM address stored in the pointer variable ptr in a variable var using:

var = ptr;

```
int a = 10;
int *x = &a;
int y;
int *z;

y = x;
// Is this a legal assignment?
// How many bytes will be copied?
// What will be the value of y?

z = x;
// Is this a legal assignment?
// How many bytes will be copied?
// What will be the value of z?
```

No

Yes 8 &a

We can copy the RAM address stored in a pointer variable into another pointer variable of the same type by a simple assignment operation. The assignment y = x is not legal because the type of x is int* and y is int. The statement z = x is legal because the type of x and z both are int*. This statement copies the 8-byte RAM address stored in x to z. Notice that the size of the type int* is eight.

Loading from RAM

- We can load the value stored in the RAM at an address stored in the pointer variable ptr into a variable var using:

```
var = *(ptr)
```

```
int a = 10;
int *x = &a;
int **y = &x;
int ***z = &y;

The type of *(x) is int
The type of *(y) is int *
The type of *(z) is int **

What is the type of **(z)?
What is the type of ***(z)?
What is the type of **(y)?
What is the type of **(x)?
What is the type of *(a)?
```

If the type of ptr is int*, the type of *(ptr) is int. If the type of ptr is int**, the type of *(ptr) will be int*. In general, if the type of a variable ptr is Ty followed by i stars, then the type of *(ptr) would be Ty followed by (i-1) stars, the type of **(ptr) would be Ty followed by (i-2) stars and so on. Dereferencing a variable v using *v, where the type of v is not a pointer type, is an error.

Loading from RAM

- We can load the value stored in the RAM at an address stored in the pointer variable ptr into a variable var using:

```
var = *(ptr)
```

```
struct record {
  int a;
  int b;
  struct record *next;
};

struct record var1;
struct record *var2;
struct record var3;

var1.a = 10;
var1.b = 20;
var1.next = &var1;
var2 = &var1;
// What is the type of &var1?
// Is this a legal assignment?
// How many bytes will be copied?

var3 = *(var2);
// What is the type of *(var2)?
// Is this a legal assignment?
// How many bytes will be copied?

printf("%d %d %p\n", var3.a, var3.b, var3.next);
// What will be the output of the printf?
```

var2 = &var1 is a legal assignment because both sides are of type struct record*. var3 = *(var2) is also a legal assignment because both are of type struct record. During var3 = *(var2), 16 bytes will be copied because the type of both sides is "struct record". When we do a pointer dereference in this case, it will load the 16-byte value into var3 from the RAM starting from the address stored in var2. Because var2 contains the address of var1 at this point, the loading from RAM will eventually copy var1 to var3.

Storing in RAM

- You can store the value of a variable var in RAM at an address stored in the pointer variable ptr using:

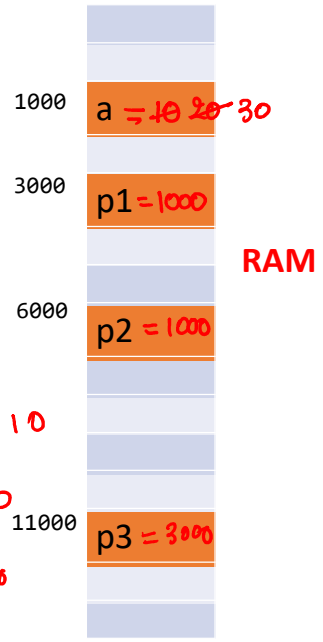
```
*(ptr) = var
```

```
struct record {
  int a;
  int b;
  struct record *next;
};

struct record var1;
struct record *var2;
struct record var3;

var1.a = 10;
var1.b = 20;
var1.next = &var1;
var2 = &var3;
// What is the type of &var3?
// Is this a legal assignment?
// How many bytes will be copied?

*(var2) = var1;
// What is the type of *(var2)?
// Is this a legal assignment?
// How many bytes will be copied?

printf("%d %d %p\n", var3.a, var3.b, var3.next);
// What will be the output of the printf?
```

var2 = &var3 is a legal assignment because both sides are of type "struct record*". *(var2) = var is also a legal assignment because both are of type "struct record". During "*(var2) = var1", 16 bytes will be copied because the type of both sides is "struct record". At this statement, a 16-byte value of var1 will be stored in RAM starting from the address stored in var2. Because var2 contains the address of var3 at this point, storing in RAM will eventually copy var1 to var3.

Pointers

```
int a;
int *p1;
int *p2;
int **p3;

a =10;
printf("%p %d\n", &a, a);
p1 = &a;
printf("%p %p %d\n", &p1, p1, *(p1));
p2 = p1;
printf("%p %p %d\n", &p2, p2, *(p2));
p3 = &p1;
printf("%p %p %p %d\n", &p3, p3, *(p3), **(p3));
*(p1) = 20;
printf("%p %d %p %d\n", p1, *(p1), p2, *(p2));
*(p2) = 30;
printf("%p %d %p %d\n", p1, *(p1), p2, *(p2));
```



We are assuming that the starting address of a is 1000, p1 is 3000, p2 is 6000, and p3 is 11000. These addresses are assigned at runtime. When you execute this program, you may get different addresses on different machines.

Pointer arithmetic

- The type of the expression (ptr + i), where i is an integer and ptr is a pointer variable, is the same as the type of ptr

```
struct record *p;
struct record **q;
int i;
```

What is the type of "p + i"? struct record *
What is the type of "p - i"? struct record *
What is the type of "q + i"? struct record **
What is the type of "q - i"? struct record **

Pointer arithmetic

- The pointer arithmetic rules are different from integer arithmetic
  - (p + i) is the RAM address of type typeof(p) after skipping i values of type typeof(*(p)) starting from p
  - (p - i) is the RAM address of type typeof(p) after skipping i values of type typeof(*(p)) starting from p in the reverse direction

Pointer arithmetic

- The rules for pointer arithmetic are different from integer arithmetic

```
int *p;
int x;
int *q;

p = &x;
// Let's assume, &x == 1000

q = p + 1;
What will be the value of q?  1004
```

Because the type of p is int*, p +1 is the address after skipping one "int" value starting from p. If the value of p is 1000, p + 1 will be 1004. The type of p+1 is the same as p, i.e., int*. q = p + 1 is a valid assignment because the type of both q and p+1 is int*.

Pointer arithmetic

- The rules for pointer arithmetic are different from integer arithmetic

```
struct record {
   int a[20];
   int b;
};

struct record *p;
struct record x;
struct record *q;

p = &x;
// Let's assume, &x == 1000

q = p + 5;
What will be the value of q?
```

1000 + (84×5)

Because the type of p is "struct record *", the address p + 5 will be computed as the address after skipping five "struct record" values. The size of "struct record" is 84; therefore, the actual RAM address corresponding to p+5 is 1000 + (84*5).

Arrays and Pointers

Array

- The array elements are stored in contiguous memory locations

a[0]	a[1]	a[2]	a[3]	a[4]	a[5]	a[6]	a[7]	a[8]	a[9]

```
int a[10];

The base (starting) address of the array “a” is &a[0].

If &a[0] == X, where (0 ≤ X ≤ 2^64 − 1).

In this case,
&a[1] == X + 4
&a[2] == X + 8
&a[3] == X + 12
…
&a[i]  == X + ?
```

Array

- The array elements are stored in contiguous memory locations

a[0]	a[1]	a[2]	a[3]	a[4]	a[5]	a[6]	a[7]	a[8]	a[9]

```
struct record {
  int a;
  struct record *next;
};

struct record a[10];
If &a[0] == X, where (0 ≤ X ≤ 2^64 − 1).

In this case,
&a[1] == X + 12
&a[2] == X + 24
&a[3] == X + 36
…
&a[i]  == X + (i*12)
```