

HOMEWORK-9

Total Points: 62

1. [12 Points] Insert 1, 2, 3, 4, 5, 6, 7, 9, 8, 10, 11, 13, 12 one by one in a max-heap. Show the max-heap after every insertion.
2. [12 Points] Build a max-heap from the given numbers 1, 2, 3, 4, 5, 6, 7, 9, 8, 10, 11, 13, 12 using an $O(n)$ algorithm. All the numbers are already given to you in an array starting at index 1. Show all intermediate steps.
3. [20 Points] We want to store (key, value) pairs of type "struct pair" in a data structure called priority-queue. The priority-queue supports three operations: insert, find, and delete. The insert operation inserts a (key, value) pair in $O(\log n)$ operations. The find operation returns a (key, value) pair corresponding to the largest key. If multiple pairs contain the largest key, find returns the one that was inserted first. The time-complexity of the find operation is $O(1)$. The delete operation deletes a pair that contains the largest key. If multiple pairs contain the largest key, the one that was inserted first is deleted. The time complexity of the delete operation is $O(\log n)$. Give a pseudocode for the insert, find, and delete operations. What is the type of node in the priority-queue? The type "struct pair" is as follows.

```
struct pair {  
    int key, value;  
};
```

4. [6 Points] Create a max-heap containing 15 numbers 1, 2, 3, ..., 15 in such a way that the post-order traversal of the max-heap gives us a sorted sequence in increasing order.
5. [4 Points] Give an algorithm to find the minimum element from a max-heap of integers. What is the time complexity of your algorithm?
6. [4 Points] What are the minimum and maximum number of connected components in a graph with n vertices? Justify your answer.
7. [4 Points] What are the minimum and maximum number of edges in a simple undirected graph with n vertices? Justify your answer.