

④ det, \exists a fn. 'f' $\mathbb{R} \rightarrow \mathbb{R}$ which is diff. on (a, b) s.t

$$f'(x) = h(x)$$

$$h(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

$$0 = f'(-1) < 1/2 < f'(0) = 1$$

By IMV of diff. fn. there must exist $c \in \mathbb{R}$ s.t $f'(c) = 1/2$
But \exists no point $c \in \mathbb{R}$ s.t $f'(c) = h(c) = 1/2$.

(FALSE)

⑤ $\lim_{x \rightarrow \infty} \frac{x - \sin x}{2x + \sin x}$

$$f(x) = x - \sin x; \quad g(x) = 2x + \sin x$$

$$f'(x) = 1 - \cos x$$

$$g'(x) = 2 - \cos x$$

But, $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} g(x)$ doesn't exist.

(FALSE)

$$\textcircled{c} \quad W = (u-s)^2 + (u+s)^2 \quad (\text{Time})$$

$$= 2u^2 + 2s^2$$

$$W_u = 4u$$

$$\textcircled{d} \quad f(x) = |x|^3$$

$$f'(x) = 3x|x|$$

$$f''(x) = \begin{cases} 0 & x=0 \\ \frac{6x^2}{|x|} & x \neq 0 \end{cases} \quad (\text{False})$$

$$\text{So, } f''(0^+) = f''(0) = f''(0^-) = 0$$

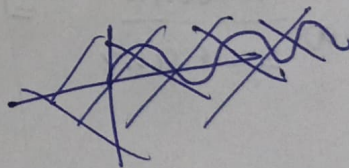
$$\text{as } f''(0^+) / f''(0^-):$$

$$f''(0^+) = \frac{6x^2}{x} = 6x = 0$$

$$f''(0^-) = \frac{6x^2}{-x} = -6x = 0$$

$$\textcircled{e} \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad (\text{False})$$

$$f'(x) = \lim_{h \rightarrow 0^-} \frac{|\sin(x+h)| - |\sin x|}{h} \quad k\pi$$



$$f(x) = |\sin x|$$

$$\text{for } x = k\pi \quad (k=0, 1, \dots)$$

$$\lim_{h \rightarrow 0^+} \frac{f(k\pi+h) - f(k\pi)}{h} = \frac{|\sin(k\pi+h)| - |\sin k\pi|}{h} =$$

$$\frac{|\sin k\pi \cos h + \cos k\pi \sin h| - 0}{h} = \frac{|\sin h|}{h} = 1$$

$$\lim_{h \rightarrow 0^-} \frac{f(k\pi + h) - f(k\pi)}{h} = \frac{| \sin h |}{h} = -1$$

as $h \rightarrow 0^-$ hence $= -1$

so, $LHL \neq RHL$;

$|\sin x|$ not differentiable at all points.

(f)

(True)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^6+y^2}$$

let $x = r \cos \theta$, $y = r \sin \theta$

then
$$\frac{(r \cos \theta)(r^2 \sin^2 \theta)}{r^4 r^6 \cos^6 \theta + r^2 \sin^2 \theta} = \frac{r \cos \theta \sin^2 \theta}{r^4 \cos^6 \theta + \sin^2 \theta}$$

$$\lim_{r \rightarrow 0} \frac{r \cos \theta \sin^2 \theta}{r^4 \cos^6 \theta + \sin^2 \theta} = 0$$

g

Let f be an onto function $f: [a, b] \rightarrow \mathbb{R}$.

Then for every $(n > 0)$ there is $x_n \in [a, b]$ s.t. $f(x_n) = n$

x_n is bounded in $[a, b]$

hence x_n will have convergent subsequence, let's say x_{n_k}

$$\text{Let } \lim_{k \rightarrow \infty} x_{n_k} = x \in [a, b]$$

Then;

$$\lim_{k \rightarrow \infty} f(x_{n_k}) = f(x) \quad \{\text{by continuity}\}$$

while;

$$\lim_{k \rightarrow \infty} (f(x_{n_k})) = \lim_{k \rightarrow \infty} (n_k) = \infty$$

(True)

So, By contradiction; $f(x) = \infty$.

h) $\lim_{x \rightarrow c} g(x) \cdot \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x)f(x)$

(False)

i

$$f(x) = 0$$

$$g(x) = |x| \quad ; \quad g(f(x)) = |f(x)| = 0$$

$$g(f(x)) = \text{differentiable at } x.$$

$$f(x) = \text{differentiable at } x.$$

$$g(x) = \text{not differentiable at } x=0.$$

(False)

j

$$\text{Let } |f(x)| \leq M \text{ and } |g(x)| \leq N$$

($\forall x, y$ in A and some constants $M, N > 0$)

Then,

$$|f(x)g(x) - f(y)g(y)| = |f(x)(g(x) - g(y)) + f(x)g(y) - f(y)g(y)|$$

$$= f(x)(g(x) - g(y)) + g(y)(f(x) - f(y))$$

$$\leq |f(x)| |g(x) - g(y)| + |g(y)| |f(x) - f(y)|$$

$$\leq M |g(x) - g(y)| + N |f(x) - f(y)|$$

$$\leq M_d |x - y| + N_c |x - y|$$

$$= C |x - y|$$

(True)

hence, $f \cdot g$ is Lipschitz fn. bounded on A .