

Assignment 7

October 31, 2023

1. Prove using $(\varepsilon - \delta)$ definition that $f : [0, \pi] \rightarrow \mathbb{R}$ defined by $f(x) = \sin x$ or $(\cos x)$ is continuous function.
2. Define what the Lipschitz function is. Is the Lipschitz function always continuous?
3. Let $f : [0, 1] \rightarrow \mathbb{R}$. Suppose that $f(x)$ is rational for irrational x and that $f(x)$ is irrational for rational x . Show that f cannot be continuous.
4. Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is differentiable function at $x = c$ and that $f(c) = 0$. Show that $g(x) := |f(x)|$ is differentiable at c iff $f'(c) = 0$.
5. Determine where each of the following functions from \mathbb{R} to \mathbb{R} is differentiable and find the derivative a) $f(x) = |x| + |x + 1|$, b) $g(x) = 2x + |x|$, c) $h(x) = x|x|$ and $k(x) = |\sin x|$.
6. Let $f : [0, 1] \rightarrow \mathbb{R}$ be differentiable and $f(0) = 0$ and $f(1) = 1$. Show that the equation $f'(x) = 2x$ has a solution on $(0, 1)$.