Indraprastha Institute of Information Technology Delhi

IIITD

Mid Semester Examination

Course Title : Real Analysis -1 | Time Duration : 90 min
Date : October 31, 2022 | Total Marks : 30
Course Code : MTH-240

Q.1a) Determine the convergence of $\sum\limits_{n=1}^{\infty} \dfrac{n!n!}{(2n)!}$ 2-marks

Q.1b) Determine the convergence of $\sum_{n=1}^{\infty} \dfrac{2\sqrt{n}+3}{n^3-n+1}$. 2-marks

Q.1c) Give an example where $\sum_{n=1}^{\infty} a_n^2$ converges but not $\sum_{n=1}^{\infty} a_n$. 1-mark

Q.2a) If f, g are continuous at c, then $max(f, g)$ and $min(f, g)$ are continuous at c. 1.5-marks

Q.2b) Using the above prove the following: Let $I := [0, \frac{\pi}{2}]$ and let $f : I \to \mathbb{R}$ be defined by $f(x) := sup\{x^2,\ \cos x\}$ for $x \in I$. Show there exists an absolute minimum (or infimum) point for f on I. 1.5-marks

Q.2c) Does there exist a non-constant continuous function $f : [0, 1] \to \mathbb{R}$ which takes only rational values? 2-marks

Q.3a) Prove that the function defined by $f(x) = x|x|$ is differentiable at $x = 0$. 1-mark

Q.3b) Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Suppose $a < b$ be any two real numbers such that $f(a) = f'(a) = f(b) = 0$. Prove that there is a point $x_1 \in (a, b)$ such that $f''(x_1) = 0$ 4-marks

Q.4a) When can you say a function $f : D \to \mathbb{R}$ is uniformly continuous.1-marks

Q.4b) Using the definition show that $f : [1, 2] \to \mathbb{R}$ defined by $f(x) = x^{3/2}$ is uniformly continuous. 2.5-marks

Q.4c) Is $f : (0, \infty) \to \mathbb{R}$ defined by $f(x) = \frac{1}{x^3}$ uniformly continuous? 1.5-marks

Q.5a) If a function has bounded derivative then it is uniformly continuous.2.5-marks

Q.5b) Show that $f : (0, 1) \to \mathbb{R}$ defined by $f(x) = \sqrt{1 + x^2}$ is uniformly continuous. 2.5-marks

Q.6a) Find the sum of $\sum\limits_{n=1}^{\infty} \dfrac{n}{(2n-1)^2(2n+1)^2}$ 1.5-marks

Q.6b) Assume $f : [a, b] \to \mathbb{R}$ is a continuous function. It is twice differentiable function on (a, b) and has at least three distinct zeros, then the equation $f(x) + f''(x) = 2f'(x)$ has at least one root in (a, b). 3.5-marks