Q1) First Fundamental Theorem of Calculus.

a) Let f be integrable on $[a,b]$. If there exists a differentiable f^n F on $[a,b]$ s.t $F'(x) = f(x)$ $\forall x \in (a,b)$, we have 0.5 marks

$$\int_a^b F'(x)\,dx = \int_a^b f(x)\,dx = F(b) - F(a)$$ 1 marks

or If g is a continuous f^n on $[a,b]$ & differentiable on (a,b) & if g' is integrable on $[a,b]$, then 0.5 marks

$$\int_a^b g'(x)\,dx = g(b) - g(a).$$ 1 marks

$32 - \frac{64}{3}$

$\frac{96-64}{3}$ $\frac{42}{3}$

1)b) Consider $f(x) = 4x - x^2$ $x \in [0,4]$

$F: [0,4] \to \mathbb{R}$

$\forall$ ~~Hence~~ $F(x) = 2x^2 - \frac{x^3}{3} + c$ c is a const.

(then F is continuous on $[0,4]$ & diff on $(0,4)$) & $F'(x) = f(x)$ $\forall x \in (0,4)$ 0.5 marks 0

$F(4) = \frac{32}{3} + c$ & $F(0) = c$

Hence $$\int_0^4 (4x - x^2)\,dx = \int_0^4 f(x)\,dx = F(4) - F(0)$$

$$= 32/3.$$ 1 marks

by 1st fundamental theorem of calculus

2.(b)

$$f(x,y) = \begin{cases} \dfrac{3x^2y}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Let $\varepsilon > 0$ be any given real no.

Now consider,

$$\left[|f(x,y) - f(0,0)| = \left| \frac{3x^2y}{x^2+y^2} - 0 \right| = \frac{|3x^2y|}{x^2+y^2} \right.$$

$$= \frac{3x^2|y|}{x^2+y^2}$$

$$= \frac{3}{2} \frac{(2|x||y|)\cdot|x|}{x^2+y^2}$$

$$\le \frac{3}{2} \frac{x^2+y^2}{x^2+y^2} \times \sqrt{x^2+y^2} = \left. \frac{3}{2}\sqrt{x^2+y^2} \right]$$ 1 mark

[We know, $2|x||y| \le |x|^2 + |y|^2 = x^2+y^2$ $\quad ((|x|-|y|)^2 \ge 0)$

$\&$ $|x| \le \sqrt{x^2+y^2}$] 0.5 mark

[If we choose $\delta_\varepsilon = \frac{2}{3}\varepsilon$, ~~than~~

then $\forall (x,y)$ with

$$0 < \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2+y^2} < \delta_\varepsilon = \frac{2}{3}\varepsilon$$

will imply,

$$|f(x,y) - f(0,0)| \le \frac{3}{2}\sqrt{x^2+y^2} < \frac{3}{2} \times \frac{2}{3}\varepsilon = \varepsilon$$

$$\Rightarrow \quad |f(x,y) - f(0,0)| < \varepsilon$$

Hence, $\lim_{(x,y)} f(x,y) = f(0,0)$

So, $f(x,y)$ is continuous at $(0,0)$] 1 mark will imply

$|f(x,y)-f(0,0)| < \frac{3}{2}\sqrt{x^2+y^2} < \varepsilon.$

Q3)a) Let $\hat{u} = u_1\hat{i} + u_2\hat{j}$ be any unit vector. Then the directional derivative of $f(x,y)$ at (x_0, y_0) in the direction $\hat{u}$ is given by

use $\hat{p} = p_1\hat{i} + p_2\hat{j}$ unit.

$$D_{\hat{u}}(x_0,y_0) = \lim_{s\to 0} \frac{f(x_0 + su_1, y_0 + su_2) - f(x_0,y_0)}{s}$$

Q3)b) $f(x,y) = x^2y^3 - 4y$

① $f_x(x,y) = 2xy^3$ & $f_y(x,y) = 3x^2y^2 - 4$

so $\nabla f(x,y) = (2xy^3)\hat{i} + (3x^2y^2 - 4)\hat{j}$

$\nabla f(2,-1) = -4\hat{i} + 8\hat{j}$

$\vec{v} = 2\hat{i} + 5\hat{j}$

$|v| = \sqrt{29}$

$\hat{u} = \frac{\hat{v}}{|v|} = \frac{2}{\sqrt{29}}\hat{i} + \frac{5}{\sqrt{29}}\hat{j}$. — 0.5

so we know

$D_u f(x_0, y_0) = \nabla f_0(x_0, y_0) \cdot \hat{u}$ — 0.5

Hence $D_{\hat{u}} f(2,-1) = \nabla f(2,-1) \cdot u$

$= (-4\hat{i} + 8\hat{j})\left(\frac{2}{\sqrt{29}}\hat{i} + \frac{5}{\sqrt{29}}\hat{j}\right)$

$= \frac{-8+40}{\sqrt{29}} = \frac{32}{\sqrt{29}}$ — 0.5

n on $[a, b]$ that

Ans → 4

<u>Use Result</u>: (2nd Fundamental Thm of Calculus)

Let f be integrable on $[a,b]$.
For $a \le x \le b$, let $F(x) = \int_a^x f(t)\,dt$

If f is cont. on $[a,b]$ then $F(x)$ is diff.
& $F' = f$. → 1

<u>Given</u>: $\int_\alpha^\beta f(t)\,dt = 0$ for each $a \le \alpha, \beta \le b$

Take $\alpha = a$ and varying β : $a \le \beta \le b$.

we get $F(\beta) = \int_a^\beta f(t)\,dt$ — 1

$F(\beta) = 0$

∵ β is any arb. pt. b/w a & b — 1

$F = 0$

$F' = 0$ — 1

$\Rightarrow f = 0$