Assignment- 7

November 13, 2022

1. Suppose $h : [a, b] \to \mathbb{R}$ be a function. If $h \in \mathcal{R}[a, b]$ exists then so is $h^2 \in \mathcal{R}[a, b]$.

2. Using the above prove that if $f, g \in \mathcal{R}[a, b]$, so is fg.

3. Suppose $\int_a^b f(x)dx$ exists, then prove that $\lim_{t \to a+} \int_a^t f(x)dx = \int_a^b f(t)dt$.

4. In each of the following cases, show that f is integrable using the Riemann criterion.
   (a) $f(x) = x$ on $[0, 1]$.
   (b) $f(x) = x^3$ on $[3, 7]$.
   (c) $f(x) = \frac{1}{x^2}$ on $[1, 2]$.

5. If g is a continuous non-negative function on $[a, b]$ and if $\int_a^b g(t)dt = 0$ then g is identically 0 on $[a, b]$.

6. Let f be a continuous function on $\mathbb{R}$ and define

$$G(x) = \int_0^{\sin x} f(t)dt \quad \text{for } x \in \mathbb{R}$$

   then G is differentiable and compute G'.