Quiz 2

Course Title : *Real Analysis 1*   Time Duration: 50 min

Date: October 19, 2023   Total Mark: 15

Course Code: MTH-240   Time: 9.30-10.20 am

Give proper justifications for your answer. Mention the results or Theorems which you are using. Do not use the method of differentiation.

Q.1)a) Give $\varepsilon - \delta$ definition of a continuous function.

Q.1)b) Give an example of a function f such that $\lim_{x \to c} f(x)$ exists but not equal to $f(c)$. $2 + 1 = 3$-marks

Q.2)a) Show that if $f : (a, \infty) \to \mathbb{R}$ is such that $\lim_{x \to \infty} x f(x) = L$ where $L \in \mathbb{R}$, then $\lim_{x \to \infty} f(x) = 0$.

Q.2)b) Can you give example of a continuous function such that $\lim_{x \to \infty} f(x) = L$ such that L is a finite number. $2.5 + 0.5 = 3$-marks

Q.3)a) Suppose $f : [a, b] \to \mathbb{R}$ is continuous. If $c \in (a, b)$ is such that $f(c) > 0$, and if $0 < \beta < f(c)$, then show that there exists $\delta > 0$ such that $f(x) > \beta$ for all $x \in (c - \delta, c + \delta) \subseteq [a, b]$.

Q.3)b) Locate a root of the equation $1 - \frac{x^2}{4} = \cos x$ other than 0. (You can assume $\pi^2 = 9.87$) $1.5 + 1.5 = 3$-marks

Q.4)a) Let $f : [0, \pi] \to \mathbb{R}$ be defined by $f(0) = 0$ and $f(x) = x \sin \frac{1}{x} - \frac{1}{x} \cos \frac{1}{x}$ for $x \neq 0$. Is f continuous at $x = 0$?

Q.4)b) If $\lim_{x \to 0+} f(x) = A$ and $\lim_{x \to 0-} f(x) = B$ and A may not be equal to B, then what is $\lim_{x \to 0+} f(x^3 - x)$? $1.5 + 1.5 = 3$-marks

Q.5)a) Using ε-δ definition can you prove $f : (0, \infty) \to \mathbb{R}$ be defined by $f(x) = \frac{1}{\sqrt{x}}$ is a continuous function.

Q.5)b) Can you give an example of a function which is continuous only at one point in its domain? $2 + 1 = 3$-marks