Assignment 8

November 21, 2023

1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = 1$ if $x = 0$ or $y = 0$ and $f(x,y) = 0$ otherwise. Show that $f_x(0,0) = f_y(0,0) = 0$ but f is not continuous at $(0,0)$.

2. Let $f, g : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = |x| + |y|$ and $g(x,y) = |xy|$ for $(x,y) \in \mathbb{R}^2$. Show that
(a) $f_x(0,0)$ and $f_y(0,0)$ do not exist whereas $g_x(0,0)$ and $g_y(0,0)$ exist.
(b) for $x_0 \neq 0$, $g_y(x_0,0)$ does not exist and for $y_0 \neq 0$, $g_x(0,y_0)$ does not exist.

3. Discuss the differentiability of $f(x,y) = x^2 + \sin y + y^2 e^x$ at $(0,0)$.

4. Discuss the differentiability of
$$f(x,y) = \begin{cases} \frac{2x^2y}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$
at $(0,0)$.

5. Discuss the differentiability of
$$f(x,y) = \begin{cases} \frac{x^2y}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$
at $(0,0)$.

6. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) = (x-y)^2 \sin \frac{1}{(x-y)}$ if $x \neq y$ and $f(x,x) = 0$. Show that
(a) f_x and f_y exist at all points of $\mathbb{R}^2$.
(b) f is differentiable at $(0,0)$.
(c) f_x and f_y are not continuous on the line $y = x$..

7. Let $z = f(x,y)$, $x = rcos\theta$ and $y = rsin\theta$.
(a) Show that $\frac{\partial z}{\partial r} = \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$ and $\frac{1}{r}\frac{\partial z}{\partial \theta} = -\frac{\partial f}{\partial x}\sin\theta + \frac{\partial f}{\partial y}\cos\theta$.
(b) If $f(x,y) = x^2 + 2xy$, show that $\frac{\partial z}{\partial \theta} = 2(x^2 - xy - y^2)$.

8. (a) express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ as functions of u and v both by using the Chain Rule and by expressing z directly in terms of u and v before differentiating. Then (b) evaluate $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ at the given point (u,v)
Consider $z = 4e^x lny$, $\quad x = ln(u\cos v)$, $\quad y = u\sin v$; $\quad (u,v) = (2, \pi/4)$.