Ques 1 (a) $3^{4n+2} + 5^{2n+1}$ is divisible by 14

Let $P(n)$: $3^{4n+2} + 5^{2n+1}$ is divisible by 14

$S = \{ n \in \mathbb{N} ; P(n) \text{ is true} \}$

Base case: $n = 1$ so,

$3^{4+2} + 5^{2+1} = 729 + 125 = 854 = 14 \times 61$

so, $1 \in S$ — 0.5

Inductive Hypo: for some $K \in \mathbb{N}$; $P(K)$ is true. — 0.25

Inductive step: for $(K+1) \in \mathbb{N}$

$$\left.\begin{aligned} &3^{4(K+1)+2} + 5^{2(K+1)+1} \\ &= 3^{4K+6} + 5^{2K+3} \end{aligned}\right\} \text{— 0.25}$$

and by inductive hypo. we know,

$$3^{4K+2} + 5^{2K+1} = 14m \quad (m \in \mathbb{N})$$

(1) so, $\left(3^{4K+6} + 5^{2K+3}\right) = 3^4\left(3^{4K+2} + 5^{2K+1}\right) - 3^4 5^{2K+1} + 5^{2K+3}$

$$= 3^4(14m) - 5^{2K+1}(3^4 - 5^2)$$

$$= 3^4(14m) - 5^{2K+1}(56)$$

$$= 14(3^4 m - 20 \cdot 5^{2K})$$

Hence ; $P(K+1)$ is true. so, by PMI proved !!

Ques 1. ARCHIMEDEAN PROPERTY of $\mathbb{R}$ states that -

(b) If $x, y \in \mathbb{R}$ and $x > 0$, then there is an $n_0 \in \mathbb{N}$ s.t $n_0 x > y$.

$0.25 \longrightarrow x, y \in \mathbb{R}$

$0.25 \longrightarrow x > 0$

$0.25 \longrightarrow n_0 \in \mathbb{N}$

$0.25 \longrightarrow n_0 x > y$.