ASSIGNMENT 3

(1) If $\{x_n\}$ and $\{y_n\}$ are two sequences such that $\{x_n\}$ and $\{x_n + y_n\}$ are convergent, then $\{y_n\}$ is convergent.

(2) If $\{x_n\}$ and $\{y_n\}$ are two sequences such that $\{x_n\}$ converges to $x \neq 0$ and $\{x_n y_n\}$ is convergent, then $\{y_n\}$ is convergent.

(3) Suppose that $x_n \geq 0$ for all $n \in \mathbb{N}$ and that $\lim_{n\to\infty}((-1)^n)x_n$ exists. Show that $\{x_n\}$ converges.

(4) Show that the following sequences $\{x_n\}$ (given below) are not convergent.
a) $x_n = \frac{(-1)^n n}{n+1}$,
b) $x_n = 2^n$.

(5) For any $a \in \mathbb{R}$ show that there is a monotonic sequence of rational numbers that converges to a.

(6) Let $\{x_n\}$ be a bounded sequence and let $s = Sup\{x_n;\ n \in \mathbb{N}\}$ and $s \notin \{x_n;\ n \in \mathbb{N}\}$. Show that there is a subsequence of $\{x_n\}$ that converges to s.

(7) For a monotonic sequence $\{x_n\}$ (increasing or decreasing) show that $\{x_n\}$ converges iff $\{|x_n|\}$ converges.

(8) Let $a_n = (1 + \frac{1}{n})^n$ and $b_n = (1 + \frac{1}{n})^{n+1}$ for $n \in \mathbb{N}$. Then
a) the sequence $\{a_n\}$ strictly increasing.
b) the sequence $\{b_n\}$ is strictly decreasing.
Show that $\{a_n\}$ and $\{b_n\}$ both have the same limit and the limit is denoted by e. (called Euler's number).

(9) Given an example of an unbounded sequence that has a convergent subsequence.