Assignment 6

November 1, 2022

1. Let $f : [a, b] \to \mathbb{R}$ be a continuous function. Then f is uniformly continuous.

2. Let $f : A \to \mathbb{R}$ be a uniformly continuous function and $\{x_n\}$ is a Cauchy sequence in A, then $\{f(x_n)\}$ is also a Cauchy sequence.

3. Uniformly continuous maps bounded sets to bounded sets.

4. Let f be continuous on the interval $I := [a, b]$ and let c be an interior point of I. Assume that f is differentiable on (a, c) and (c, b). Then:
   a) If there is a neighborhood $(c - \delta, c + \delta)$ such that $f'(x) \geq 0$ for $c - \delta < x < c$ and $f'(x) \leq 0$ for $c < x < c + \delta$, then f has a local maximum at c.
   b) If there is a neighborhood $(c - \delta, c + \delta)$ such that $f'(x) \geq 0$ for $c < x < c + \delta$ and $f'(x) \leq 0$ for $c - \delta < x < c$, then f has a local minimum at c. Show that the converse of the above is not true. That means f may have local maximum and minimum at $x = c$ but that its derivative has both positive and negative values in $(c - \delta, c)$ as well as $(c, c + \delta)$ for every $\delta > 0$.

5. Let $f : [a, b] \to \mathbb{R}$ be a continuous function and differentiable on (a, b). If the derivative $f'(x) \neq 0$ for all $x \in (a, b)$, then either $f'(x) > 0$ for all $x \in (a, b)$ or $f'(x) < 0$ for all $x \in (a, b)$.

6. Suppose f is differentiable on $(0, \infty)$ and $\lim_{x\to\infty} f'(x) = 0$ Prove that $\lim_{x\to\infty}[f(x + 1) - f(x)] = 0$.