Assignment 2

September 12, 2022

Let A be a non-empty set of real numbers. Then the set A is countable iff there exists a one-to-one function $f : A \to N$ (not necessarily onto!).

1. The set of integers $\mathbb{Z}$ is a countable set.
2. The set of rational numbers $\mathbb{Q}$ is countable.
3. The set of real numbers is uncountable.
4. Prove that $\lim_{n\to\infty} \frac{b}{n^2} = 0$ for any rel number b.
5. Prove that $\lim_{n\to\infty}(\sqrt{n+1} - \sqrt{n}) = 0$.
6. If $\lim_{n\to\infty} x_n = x > 0$, show that there exists a natural number k such that if $n \geq k$, then $\frac{x}{2} < x_n < 2x$.
7. Let $\{x_n\}$ be a sequence of real numbers and let $x \in \mathbb{R}$. If $\{a_n\}$ is a sequence of positive real numbers with $\lim_{n\to\infty} a_n = 0$ and if for some constant $C > 0$ and some $m \in \mathbb{N}$ we have

$$|x_n - x| < Ca_n \quad \forall\, n \geq m$$

then it follows that $\lim_{n\to\infty} x_n = x$.

8. If $c > 0$, then $\lim_{n\to\infty} c^{\frac{1}{n}} = 1$.

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