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Case 1: If $a \in \mathbb{Q}$, then $x_n = a + \frac{\sqrt{2}}{n}$ is a monotonically decreasing sequence of irrational numbers which converges to a.

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} a + \frac{\sqrt{2}}{n} = a + \lim_{n\to\infty} \frac{\sqrt{2}}{n} = a + 0 = a$$

Case 2: If $a \notin \mathbb{Q}$

By density property of irrational numbers in $\mathbb{R}$ we know that $\exists$ an irrational no. b_1 such that $a < b_1 < a+1$

Case 1 - If $a+1/2 < b_1$, then again by density property of irrational numbers in $\mathbb{R}$ $\exists$ an irrational no. b_2' s.t.

$$a < b_2' < a+1/2 < b_1$$

Case 2 - If $a+1/2 > b_1$, again by density property $\exists$ an irrational no. b_2'' s.t.

$$a < b_2'' < b_1 < a+1/2$$

Take $b_2 = \begin{cases} b_2' & \text{if } a+1/2 < b_1 \\ b_2'' & \text{if } a+1/2 > b_1 \end{cases}$

From this we can see that in any case $b_2 < b_1$ and $b_2 < a+1/2$. Similarly we can choose no's $b_3, b_4, b_5, b_6, \ldots$ and so on such that $b_{n+1} < b$ and $b_{n+1} < a + \frac{1}{n+1}$ $\forall\, n \geq 1$

So we get $\{b_n\}$ as a monotonically decreasing sequence and $a < b_n < a + 1/n$

Let $x_n = a$, $y_n = b_n$ and $z_n = a + 1/n$

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} a = a$$

$$\lim_{n\to\infty} z_n = \lim_{n\to\infty} a + \frac{1}{n} = a + \lim_{n\to\infty} \frac{1}{n} = a$$

By sandwich theorem $\lim_{n\to\infty} y_n = \lim_{n\to\infty} b_n$ is also equal to a.

OR

<u>case 1:</u> $a \in \mathbb{Q}$

then we take $a + \frac{\sqrt{2}}{n} = x_n$

It is a sequence of irrational no.s

$$\lim_{n\to\infty} a + \frac{\sqrt{2}}{n} = a + \lim_{n\to\infty} \frac{\sqrt{2}}{n} = a + 0 = a$$

<u>case 2:</u> $a \notin \mathbb{Q}$

then we take $x_n = a + \frac{1}{n}$, this is a sequence of irrational numbers

$$\lim_{n\to\infty} a + \frac{1}{n} = a + \lim_{n\to\infty} \frac{1}{n} = a + 0 = a$$