INDRAPRASTHA INSTITUTE *of* INFORMATION TECHNOLOGY DELHI

Mid Semester

Course Title : *Real Analysis 1*	Time Duration: 90 min
Date: October 9, 2023	Total Mark: 30
Course Code: MTH-240	Time: 3-5 pm

Q.1)a) What is an infinite series? When we say the infinite series converges? $0.5 + 1.5 = 2$-marks

Q.1)b) Check whether the series $\displaystyle\sum_{n=3}^{\infty} 9^{-n+2}4^{n+1}$ converges. If yes, what is the sum? $1.5 + 1.5 = 3$-marks

Q.2)a) What is the Cauchy condensation test? Using this test can you check whether the series $\sum_{n=1}^{\infty} \frac{1}{n(\log n)^c}$ converges where $c > 1$? $1.5 + 1.5 = 3$-marks

Q.2)a) What is the Ratio test? Can you apply ratio test to the series $\displaystyle\sum_{n=0}^{\infty} \frac{n+2}{2n+7}$ to test its convergence or divergence. $1.5 + 0.5 = 2$-marks

Q.3)a) If $\sum_{n=1}^{\infty} a_n$ converges with $a_n > 0$ then is always $\sum_{n=1}^{\infty} \sqrt{a_n}$ convergent? Either prove it or give a counterexample. 0.5-marks

Q.3)b) Check the convergence of the series $\sum_{n=1}^{\infty} \frac{(2n)!}{n!3^{2n}}$. 1.5-marks.

Q.3)c) Let $a_n \geq 0$. Then show that both the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$ converge or diverge together. 3-marks

Q.4)a) What is the completeness axiom of $\mathbb{R}$? 1-mark

Q.4)b) What is the infimum of a set? Let A be a nonempty bounded subset of strictly positive real numbers. Let $\frac{1}{A} = \{\frac{1}{x},\ x \in A\}$. Let $Inf A > 0$. What is the $Sup \frac{1}{A}$ (Give Justification)? $1.5 + 2.5 = 4$-marks

Q.5)a) What is the rational zeros theorem? 1.5-marks.

Q.5)b). Let $\{a_n\}$ and $\{b_n\}$ be two Cauchy sequences. Show that the sequence defined by

$$\{a_1, b_1, a_2, b_2, \cdots, a_n, b_n, \cdots\}.$$

is Cauchy iff $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$. 3.5-marks.

Q.6)a) Let $\{x_n\}$ be a sequence defined by $x_1 = 1$ and $x_{n+1} = x_n \left(1 + \frac{\sin n}{2^n}\right)$ for $n \geq 1$. Discuss the convergence of the sequence $\{x_n\}$. 3-marks

Q.6)b) If $\{x_n\}$ is an unbounded sequence. Then prove that there exists a convergent subsequence $\{x_{n_k}\}$ such that $\lim_{k\to\infty} \frac{1}{x_{n_k}} = 0$. 2-marks