Ans. 1)

a) Let $\{a_n\}$ be a sequence of real numbers. An expression of the form

$$a_1 + a_2 + \cdots + a_n + \cdots$$

is called an infinite series. $\boxed{-0.5 \text{ Marks}}$

The sequence $\{S_n\}$ defined by $S_n = \sum_{k=1}^{n} a_k$ is called the sequence of partial sums of series. If the sequence $\{S_n\}$ of partial sums converges to a limit L, we say that the series $\sum_{n=1}^{\infty} a_n$ converges and its sum is L. $\boxed{-1.5 \text{ Marks}}$

b) $$\sum_{n=3}^{\infty} 9^{-n+2} 4^{n+1} = 4 \cdot 9^2 \sum_{n=3}^{\infty} \left(\frac{4}{9}\right)^n = 324 \sum_{n=3}^{\infty} \left(\frac{4}{9}\right)^n$$

$r = \frac{4}{9}$, GP with $r < 1$, hence the series converges

$\boxed{\text{OR}}$

Ratio Test –

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(4/9)^{n+1}}{(4/9)^n}\right| = \lim_{n\to\infty} \frac{4}{9} = \frac{4}{9} < 1$$

$\boxed{\text{OR}}$

Root Test –

$$\lim_{n\to\infty} |a_n|^{1/n} = \lim_{n\to\infty} \left(\left(\frac{4}{9}\right)^n\right)^{1/n} = \lim_{n\to\infty} \left(\frac{4}{9}\right) = \frac{4}{9} < 1$$

$\boxed{-1.5 \text{ Marks}}$

Sum of the series →

$$324 \sum_{n=3}^{\infty} \left(\frac{4}{9}\right)^n = 324 \left(\sum_{n=1}^{\infty} \left(\frac{4}{9}\right)^n - \sum_{n=0}^{2} \left(\frac{4}{9}\right)^n \right)$$

$$= 324\left(\frac{1}{1-\left(\frac{4}{9}\right)} - \left(1+\frac{4}{9}+\left(\frac{4}{9}\right)^2\right)\right)$$

$$= 324\left(\frac{9}{5} - \frac{81+36+16}{81}\right)$$

$$= 324\left(\frac{729-665}{405}\right) = \frac{\cancel{324}^{\,4} \times 64}{\cancel{405}_{\,5}} = \frac{4^4}{5} = \frac{256}{5}$$

$\boxed{-1.5 \text{ Marks}}$

Ans. 20)

a) Cauchy condensation Test — Suppose $\{a_n\}$ is decreasing sequence of positive terms. Then the series $\sum_{n=1}^{\infty} a_n$ converges iff the series $\sum_{k=1}^{\infty} 2^k a_{2^k}$ converges.

0.25	–	For writing decreasing sequence
0.25	–	For mentioning terms are positive
0.25	–	For mentioning iff (if & only if) condition
0.75	–	Mentioning both series correctly

$$\sum_{n=1}^{\infty} \frac{1}{n(\log n)^c}$$

- All terms of the sequence are positive
- As $n(\log n)^c < n+1(\log(n+1))^c \Rightarrow \frac{1}{n(\log n)^c} > \frac{1}{n+1(\log(n+1))^c}$

Hence the sequence is decreasing.

→ We can apply Cauchy condensation Test

- No marks have been deducted for these steps but these are important

$$\sum_{k=1}^{\infty} 2^k \frac{1}{2^k (\log 2^k)^c} = \sum_{k=1}^{\infty} \cancel{2^k} \frac{1}{\cancel{2^k} (k \log 2)^c}$$

$$= \frac{1}{(\log 2)^c} \sum_{k=1}^{\infty} \frac{1}{k^c}$$ $\boxed{-\ 1 \text{ Mark}}$

As $c > 1$ this series converges by the p-series test.

As $\sum_{k=1}^{\infty} 2^k a_{2^k}$ converges, $\sum_{n=1}^{\infty} a_n$ converges (by cauchy condensation test)

Hence $\sum_{n=1}^{\infty} \frac{1}{n(\log n)^c}$ converges. $\boxed{-\ 0.5 \text{ Mark}}$

0.25 have been deducted if p-series test is not mentioned.