Assignment 5

October 11, 2022

1. Suppose $f : [a, b] \to \mathbb{R}$ is continuous. If $c \in (a, b)$ is such that $f(c) > 0$, and if $0 < \beta < f(c)$, then show that there exists $\delta > 0$ such that $f(x) > \beta$ for all $x \in (c - \delta, c + \delta) \subseteq [a, b]$.
2. Suppose f is a function from $f : \mathbb{R} \to \mathbb{R}$ is a continuous function such that $\lim_{|x|\to\infty} f(x) = 0$. Prove that f is bounded on $\mathbb{R}$ and attains either an absolute maximum or an absolute minimum!
3. There does not exist a continuous function f from $[0, 1]$ onto $\mathbb{R}$ – Why?
4. Find a continuous function f from $(0, 1)$ onto $\mathbb{R}$.
5. Let $f : [a, b] \to \mathbb{R}$ be a continuous function such that for each $x \in [a, b]$ there exists $y \in [a, b]$ such that $|f(y)| \leq \frac{1}{2}|f(x)|$. Prove there exists a point c in $[a, b]$ such that $f(c) = 0$.
6. $f : A \to \mathbb{R}$ is uniformly continuous on $\mathbb{R}$, and $|f(x)| \geq k > 0$ for all $x \in A$, show that $\frac{1}{f(x)}$ is uniformly continuous on A.
7. $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) := \frac{1}{1+x^2}$ is uniformly continuous on $\mathbb{R}$.