

$$f(x) - f(c) \le 3|x-c| < \varepsilon$$

80, convergending to $\varepsilon > 0$, we have found $\delta_0(\varepsilon) = \min_{\varepsilon \in S_0} \{2\delta_{y_2}, 2\varepsilon, 2\varepsilon\} < \varepsilon$

inflies $|\sin x - \sin c| < \varepsilon$.

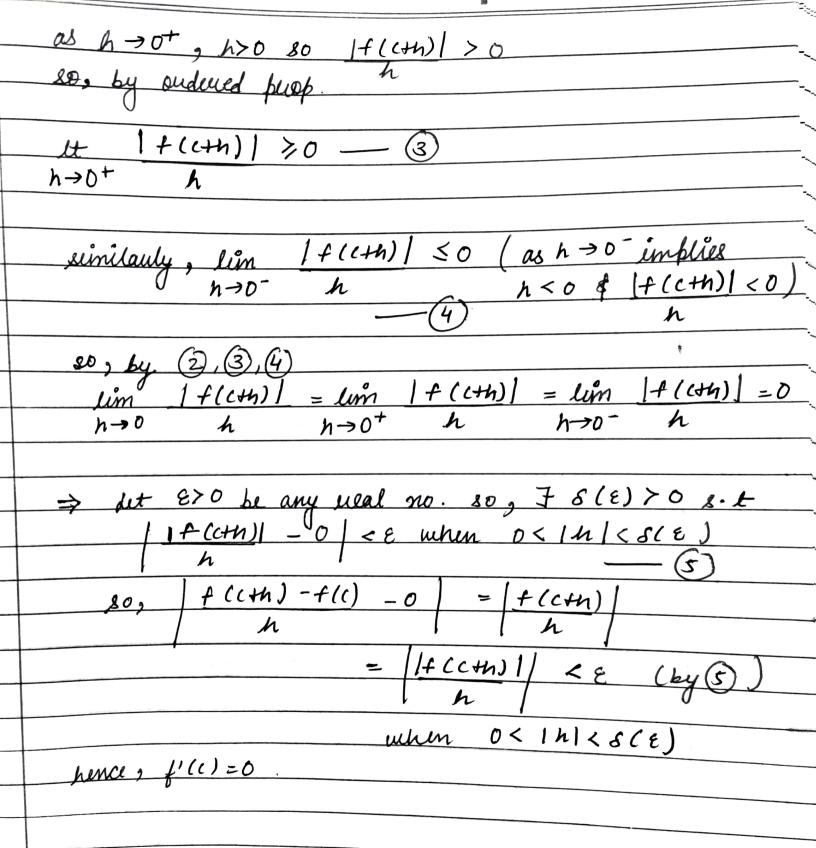
raye

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cues 2.
           A function it such that |f(x)-f(y)| \leq C|x-y| for
            all x and y, where 'C' is a constant independent of x and y, is called a LIPSCHITZ FUNCTION.
            Yes, every sipschitz for is continous
            (By \varepsilon-S defin of cont. fn. at c)

choose S = \varepsilon/c
                  then, |f(x)-f(y)| < \varepsilon whenever, |x-y| < \delta
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ours. Suppose f: [0,1] -R is continous fn. consider g:[0,1] -> R is continous for Let g(x) = f(x) - x. When $x \in Q$; $f(x) \notin Q$; so, $g(x) \notin Q$ If $x \notin Q$ then $f(x) \notin Q$; but $g(x) \notin Q$ So, q(x) \$ Q + x ∈ [0,1] Suppose $x_1, x_2 \in [0,1]$ s.t $g(x_1) \neq g(x_2)$. Then $g(x_1)$ and $g(x_2)$ are 2 invational numbers and between any 2 invational no. f a extrional no., so f req s.t $g(x_1) < u < g(x_2)$. But I no x. 6 [b,1] sot g(x) = u. So g fails to satisfy IVP. Hence, g can't be continous and so is f.

Let us assume f'(c) = 0Dues-4 Let $\varepsilon>0$, since $\lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$ exists ε equal to f'(c) = 080, cour to E>0 7 S(E)>0 8.+ 0< 1 h-0/<S(E) implies $\left| \frac{f(c+h)-f(c)}{f(h)} - f'(c) \right| < \varepsilon$ that is, $\left| \frac{f(c+h)}{f(h)} \right| < \varepsilon$ (as f(c) = 0) Now, let 0< 1h-01< s(E) then, $\left| \frac{g(c+h) - g(c)}{h} - o \right| = \left| \frac{1f(c+h)}{h} \right| = \left| \frac{f(c+h)}{h} \right| < \varepsilon$ $\frac{(by0)}{2 \cdot (by0)}$ hence, cowe. to $\varepsilon > 0$, we have found, $s(\varepsilon) > 0$ s.t. $|g(c+n) - g(c) - 0| < \varepsilon$ when $o<|h|< s(\varepsilon)$ so, lt g(cth) - g(c) exists and equal to 0. $h \rightarrow 0$ h so, g'(c) = 0conversely, let g(x) = |f(x)| is differentiable at x = c. that means $\lim_{h\to 0} |f(c+h)| - |f(c)|$ exists i.e. $\frac{d}{h \to 0^+} \frac{|f(c+h)|}{h} = \frac{d}{h} \frac{|f(c+h)|}{h} - \frac{2}{h}$

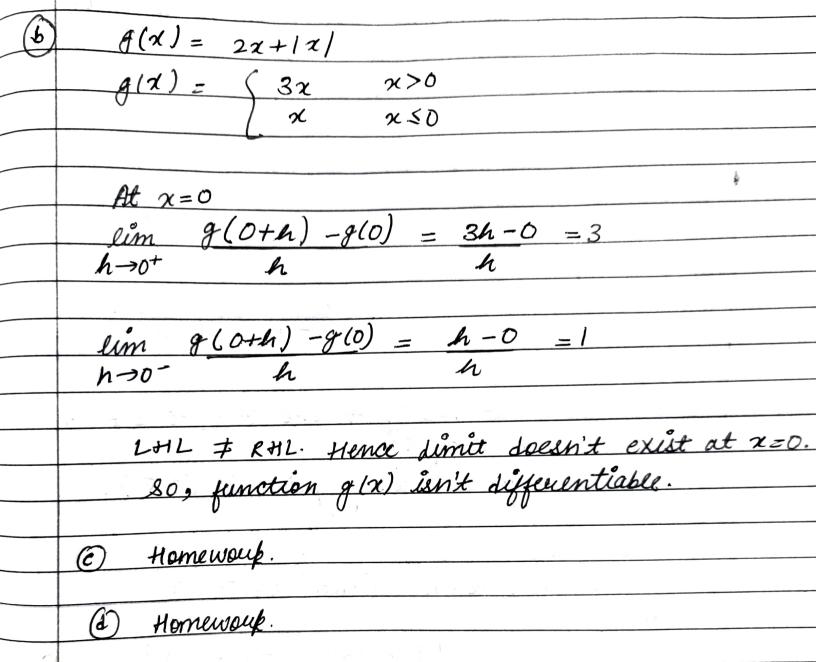


lim
$$f(0+h) - f(0) = lt(2h+1) - 1 = 2$$

 $h \to 0+$
 $h \to 0+$

h > D -

LHL # RHL. dinit doesn't exist Therefore, An. f(x) isn't differentiable



ouls 6. Guinen: +: [0,1] → R +: differentiable fn. f(0) = 0 + f(1) = 1 $f'(\chi) = 2\chi$ $\det g(x) = f(x) - x^2$ (as $f(x) = differentiable for and <math>x^2 = continues for on$ IR, nence g(x) is also a continue for.) (similarly g(x) is also differentiable) [algebra on diff. fns.] so, g(0) = 0, g(1) = 0 g(c) = g(1) - g(0) = 080, g'(c) = f'(c) -2c=0 f'(c) = 2cHence, this shows that I a point c in (0,1) s.t +1(c)= 2c.