Assignment 8

December 5, 2022

1. Discuss the differentiability of $f(x, y) = x^2 + \sin y + y^2 e^x$ at $(0, 0)$.

2. Let $f(x, y) = |xy|$ for all $(x, y) \in \mathbb{R}^2$, then
(a) f is differentiable at $(0, 0)$.
(b) $f_x(0, y_0)$ does not exist if $y_0 \neq 0$.

3. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = (x-y)^2 \sin \frac{1}{(x-y)}$ if $x \neq y$ and $f(x, x) = 0$. Show that
(a) f_x and f_y exist at all points of $\mathbb{R}^2$.
(b) f is differentiable at $(0, 0)$.
(c) f_x and f_y are not continuous on the line $y = x$..

4. Let $z = f(x, y)$, $x = rcos\theta$ and $y = rsin\theta$.
(a) Show that $\frac{\partial z}{\partial r} = \frac{\partial f}{\partial x} \cos\theta + \frac{\partial f}{\partial y} \sin\theta$ and $\frac{1}{r}\frac{\partial z}{\partial \theta} = -\frac{\partial f}{\partial x} \sin\theta + \frac{\partial f}{\partial y} \cos\theta$.
(b) If $f(x, y) = x^2 + 2xy$, show that $\frac{\partial z}{\partial \theta} = 2(x^2 - xy - y^2)$.

5. An ice block of rectangular shap is melting. Suppose that at a given instant, the block has a height of 5 ft, a length of 10 ft and a width of 12 ft. If each of the dimension is decreasing at a rate of 5 ft per hour, at what rate is the volume of the block decreasing at the given instant ?.

6. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be differentiable. Suppose $f(1, 2) = f(3, 4) = 0$. Show that there exists a point (x_1, y_1) lying in the line segment joining $(1, 1)$ and $(3, 4)$ such that $f_x(x_1, y_1) = -f_y(x_1, y_1)$.

7. What is the t-derivative of $z = f(x(t), y(t))$ at $t = 1$ if $x(1) = 2$, $y(1) = 3$, $x'(1) = -4$, $y'(1) = 5$, $f_x(2, 3) = -6$ and $f_y(2, 3) = 7$?

8. Find all local maxima and minima of $f(x, y) = (x - 2)^4 + (x - 2y)^2$