Indraprastha Institute of Information Technology Delhi

IIITD

Quiz-2

Course Title : Real Analysis -1 Time Duration : 45 min
Date of Examination : November 18, 2022 Total Mark : 15
Course Code : MTH-240

Q.1) Let $f : (0, 1) \to \mathbb{R}$ be a function defined by $f(x) = x^{\frac{1}{3}}(1 - x)^{\frac{2}{3}}$. Determine all the local extremum points. 3-marks

Q.2) Show that $x + \frac{x^2}{2} - \frac{x^3}{6} < (x + 1)\log(1 + x) < x + x^2$ for $x > 0$? 4-marks

Q.3) Let $f : [a, b] \to \mathbb{R}$ be a bounded continuous function. Suppose that there is a partition P of $[a, b]$ such that $L(P, f) = U(P, f)$. Show that f is a constant function. 4-marks

Q.4) Let $f : [0, 1] \to \mathbb{R}$ be a bounded function defined by

$$f(x) = \begin{cases} 0 \text{ if } 0 \leq x < \frac{1}{2} \\ 10 \text{ if } x = \frac{1}{2} \\ 1 \text{ if } \frac{1}{2} < x \leq 1 \end{cases}$$

Using Riemann integrability criterion show that f is integrable. 4-marks
The above is an example of a non continuous function which is Riemann integrable.