Indraprastha Institute of Information Technology Delhi

IIITD

Quiz-3

Course Title : Real Analysis -1 Time Duration : 45 min
Date of Examination : November 30, 2022 Total Mark : 15
Course Code : MTH-240

Q.1) a) State First Fundamental Theorem of Calculus.
b) Using the first fundamental theorem evaluate $\int_0^4 (4x - x^2)\,dx$ 1.5+1.5=3-marks

Q.2) a) Give definition of a continuous function $z = f(x, y)$ at (x_0, y_0). 1.5-marks

Q.2) b) Using $\varepsilon - \delta$ definition prove that the following function is continuous at $(x, y) = (0, 0)$?
2.5-marks

$$f(x,y) = \begin{cases} \frac{3x^2y}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Q.3)a) Give the definition of directional derivative of $z = f(x, y)$ at (x_0, y_0) in the direction of $\bar{u} = u_1\bar{i} + u_2\bar{j}$.
1.5-marks
b) Let $f(x, y) = x^2y^3 - 4y$, find the directional derivative of f at the point $(2, -1)$ in the direction of the vector $\bar{v} = 2\bar{i} + 5\bar{j}$.
2.5-marks

Q.4) Show that if f is continuous function on $[a, b]$ and for each α, β we have $a \leq \alpha \leq \beta \leq b$,

$$\int_\alpha^\beta f(x)dx = 0$$

then f is identically zero.
4-marks