Assignment 4

September 19, 2023

1. Test the convergence or divergence of the series

   (a) $\sum_{n=1}^{\infty} \frac{(3n)!+4^{n+1}}{(3n+1)!}$.

   (b) $\sum_{n=1}^{\infty} \frac{n^2}{2n^2+1}$.

   (c) $\sum_{n=1}^{\infty} \frac{5}{2^{\frac{1}{n}}+1}$

   (d) $\sum_{n=1}^{\infty} \frac{2}{n^2+2n}$.

   (e) $\sum_{n=1}^{\infty} \frac{1}{2n^2+3n-5}$.

2. Suppose $\{a_n\}$ and $\{b_n\}$ are sequences of non negative real numbers, such that $\sum_{n=1}^{\infty} a_n^2$ and $\sum_{n=1}^{\infty} b_n$ both converge, then prove that $\sum_{n=1}^{\infty} a_n b_n$ converges.

3. Can you give an example of a convergent series $\sum_{n=1}^{\infty} x_n$ and a divergent series $\sum_{n=1}^{\infty} y_n$. such that $\sum_{n=1}^{\infty}(x_n + y_n)$ is convergent? Explain.

4. Prove that if $\sum_{n=1}^{\infty} a_n$ is a convergent series of non negative numbers and $p > 1$, then $\sum_{n=1}^{\infty} a_n^p$ converges.

5. If $\sum_{n=1}^{\infty} a_n$ converges with $a_n > 0$ then is always $\sum_{n=1}^{\infty} \sqrt{a_n}$ convergent? Either prove it or give a counterexample.

6. If $\sum_{n=1}^{\infty} a_n$ converges with $a_n > 0$ then is always $\sum_{n=1}^{\infty} \sqrt{a_n a_{n+1}}$ convergent? Either prove it or give a counterexample.

7. If $\sum_{n=1}^{\infty} a_n$ converges with $a_n > 0$ then $\sum_{n=1}^{\infty} b_n$ where $b_n = \frac{a_1+a_2+\cdots+a_n}{n}$ always divergent?