Ques. 3 (a) No; — 0.25

eg: $a_n = 1/n^2$ which is convergent series but — 0.25

$\sqrt{a_n} = 1/n$ isn't.

Ques. 3 (b)

$$\sum_{n=1}^{\infty} \frac{(2n)!}{n!\, 3^{2n}}$$

If series a_n converges $\Rightarrow \lim_{n\to\infty} a_n = 0$. (Lemma 2-4) — 0.25

$$a_n = \frac{(2n)!}{n!\, 3^{2n}} = \frac{(2n)(2n-1) \cdots (n+1)}{3^{2n}} \quad — 0.25$$

If $n \geq 9$; $n+1 > 9$

$n+2 > 9$

$\vdots$

$2n > 9$

so; $(n+1)(n+2) \cdots (2n-1)(2n) > 9^n$ — 1

as $a_n = \dfrac{(n+1)(n+2) \cdots 2n}{9^n}$ — 0.5

for $n \geq 9$;

By 1

$a_n \geq 1$; so, as $\lim_{n\to\infty} a_n \geq 1$

0.5 —

But, for convergence $\lim_{n\to\infty} a_n = 0$; hence this series isn't convergent.

Ques. 2 (b)

Ratio Test :

A series $\sum_{n=1}^{\infty} a_n$ of nonzero terms.

Let $\alpha = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$

(i) converges absolutely if $\alpha < 1$ — 0.5

(ii) diverges if $\alpha > 1$ — 0.5

(iii) $\alpha = 1$; test gives no information — 0.5

series : $\sum_{n=0}^{\infty} \frac{n+2}{2n+7}$

$a_n = \frac{n+2}{2n+7}$; $a_{n+1} = \frac{(n+1)+2}{2(n+1)+7} = \frac{n+3}{2n+9}$

$\left| \frac{a_{n+1}}{a_n} \right| = \left(\frac{n+3}{2n+9} \right) \left(\frac{2n+7}{n+2} \right) = \frac{2n^2 + 13n + 21}{2n^2 + 13n + 18}$

0.25 (for reason)

so; $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$.

so by (iii) $\alpha = 1$ gives no info. hence we can't predict if series is divergent / convergent.

Hence, ans = NO. — 0.25