Theory of Computation '23 Quiz 1
Time : 40 minutes

Name : **Roll :** **Marks :**

Problem 1. State True/False for the following with a valid formal reason (zero credit for just writing True/False)

a) (2 points) If L is a regular language, then every subset of L is regular

Solution. False. Consider the trivial regular language $L = \{w \in \{a, b\}^\star\}$. There are many subsets of this language which are not regular, for example $\{a^k b^k | k \geq 0\}$.

(+2 for correct answer and proper example. If answer is correct but example is wrong, +1. If only answer correct, but no example attempted, then 0).

b) (3 points) If L_1, L_2 are regular languages, then $L_3 = \{w | w \in L_1 \text{ or } w \in L_2 \text{ but } w \notin \text{ both } \}$ is regular **Solution.** True. L_3 is essentially

$$(L_1 \setminus L_2) \cup (L_2 \setminus L_1)$$

Now from pset2, we know each of the individual terms are regular and finally they are connected by a *cup* and hence the result is regular as well. (+3 for correct answer and proper reasoning. If answer is correct but reasoning is somewhat fine +2. If only answer correct, but reasoning is completely wrong or not present, 0).

Problem 2. (5 points) Suppose an NFA A with n states accepts a language $L(A)$ which is a non-empty language. Then there exists a string in $L(A)$ which is of length *at most* n.

Solution. Suppose there is no string of length at most n in $L(A)$ - consider any such string $|w|$ and the let us look at the run of this string on the NFA starting at the start state. By pigeon hole principle, since the number of states in n, this run will have at least one repeated state - say q. Now delete the portion of w between the two consecutive visits to q. It is straightforward to observe that the new string has strictly smaller length and is also in the language $L(A)$. We can carry out this exercise until the length of the string becomes strictly less than n and hence arrive at a contradiction to our initial assumption.

Remark. You can also use the Pumping Lemma and the idea of 'pumping down' to achieve the same effect.

(+5 for the correct proof. The evaluation for this is going to be subjective.)

Problem 3. (10 points) Design a DFA for the following language with at most 5 states

$$L = \{w \in \{a, b\}^\star |\text{ does not contain two consecutive occurences of } ab\}$$

Example : *abaab* is in L, but *abaababab* or *bababbb* is not.

(+10 for the correct answer. -0.5 for each missing transition. -1 for each missed accept state.)