Theory of Computation '23
Problem Set 5

Problem 1. A Turing machine with stay put instead of left is similar to an ordinary Turing machine, but the transition function has the form

$$\delta : Q \times T \rightarrow Q \times T \times \{R, S\}$$

At each point the machine can move its head right or let it stay in the same position. Show that this Turing machine variant is not equivalent to the usual version. (Hint: Show that these machines only recognize regular languages).

Problem 2. Show that every infinite Turing-recognizable language has an infinite decidable subset. (Hint: Use the a result from another problem that you did in pset4).

Problem 3. Let C be a language. Prove that C is Turing-recognizable iff a decidable language D exists such that

$$C = \{x \mid \exists y(\langle x, y\rangle \in D)\}$$

Problem 4. Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.

Problem 5. A *one-counter automaton* is an automaton with a finite set of states Q, a two-way *read-only* input head (so essentially it does not have a work/write tape) and a separate *counter* that can hold any non-negative integer. The input $x \in \Sigma^\star$ is enclosed in endmarkers $\vdash, \dashv$ and the input head may not go outside the endmarkers. The machine starts in its start state s with its counter empty and its input head pointing towards the left endmarker $\vdash$. At each step, it can test it's counter for 0. Based on this information, its current state and the symbol its head is reading, it can either add 1 or subtract 1 to its counter, move the head left or right and enter a new state. It accepts by entering an explicit *accept* state.

a. Give a formal definition of the above automaton including a definition of acceptance. Your definition should read as follows "A one-counter automaton is a septuple

$$M = (Q, \Sigma, \vdash, \dashv, s, t, \delta$$

where $\cdots$ "

b. Prove that the membership problem of one-counter automaton : Given a one-counter automaton M and string x, does M accept x - is decidable.

c. Prove that the emptiness problem of one-counter automaton : Given a one-counter automaton M, is $L(M) = \varphi$ decidable ?