Theory of Computation '23
Problem Set 6

Problem 0. A *one-counter automaton* is an automaton with a finite set of states Q, a two-way *read-only* input head (so essentially it does not have a work/write tape) and a separate *counter* that can hold any non-negative integer. The input $x \in \Sigma^\star$ is enclosed in endmarkers $\vdash, \dashv$ and the input head may not go outside the endmarkers. The machine starts in its start state s with its counter empty and its input head pointing towards the left endmarker $\vdash$. At each step, it can test it's counter for 0. Based on this information, its current state and the symbol its head is reading, it can either add 1 or subtract 1 to its counter, move the head left or right and enter a new state. It accepts by entering an explicit *accept* state.

a. Give a formal definition of the above automaton including a definition of acceptance. Your definition should read as follows "A one-counter automaton is a septuple

$$M = (Q, \Sigma, \vdash, \dashv, s, t, \delta$$

where $\cdots$ "

b. Prove that the membership problem of one-counter automaton : Given a one-counter automaton M and string x, does M accept x - is decidable.

c. Prove that the emptiness problem of one-counter automaton : Given a one-counter automaton M, is $L(M) = \phi$ is undecidable.

Remark: This one is from the previous problem set but there was a mistake. The last question needs you to prove undecidability and not decidability.

Problem 1. Tell whether or not the following sets are Turing recognizable. Give proper proof.

$$\{(M, N) \mid M \text{ takes fewer steps than N on input } \varepsilon\}$$

$$\{M \mid M \text{ takes fewer than } 481^{481} \text{ steps on some input}\}$$

$$\{M \mid M \text{ takes fewer than } 481^{481} \text{ steps on at least } 481^{481} \text{ different inputs}\}$$

$$\{M \mid M \text{ takes fewer than } 481^{481} \text{ steps on all inputs}\}$$

Problem 2. Let

$$T = \{\langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$$

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where w^R is the reverse of w. Show that T is undecidable.

Problem 3. Give an example of an undecidable language L where $L \leq_m \overline{L}$. Prove your answer.