

Theory of Computation '23

Problem Set 7

Problem 1. Prove that if a language is decidable by a multi-tape Turing machine in time $\mathcal{O}(t(n))$, then it is decidable by a single-tape Turing machine in time $\mathcal{O}(t^2(n))$

Problem 2.

1. Show that EQ_{DFA} is in P .
2. A language A is said to be *star-closed* if $A = A^*$. Show that deciding if a DFA recognizes a star-closed in P

Problem 3. Let G represent an undirected graph. Also let

$$SPATH = \{ \langle G, a, b, k \rangle \mid G \text{ contains a simple path of length at most } k \text{ from } a \text{ to } b \}$$

, and

$$LPATH = \{ \langle G, a, b, k \rangle \mid G \text{ contains a simple path of length at least } k \text{ from } a \text{ to } b \}$$

- a. Show that $SPATH \in P$.
- b. Show that $LPATH$ is NP -complete.

Problem 4. Problem 7.26 from Sipser (~~≠~~-assignment)

Problem 5. Problem 7.27 from Sipser (MAX-CUT)

Problem 6. Problem 7.29 from Sipser (Coloring)