Theory of Computation '23 Quiz 3
Time : 35 minutes

Name : **Roll :** **Marks :**

Problem 1. (3×3) State TRUE or FALSE with a very brief explanation that fits in the space provided. Just the correct answer without explanation fetches zero credit.

1. If L_1, L_2 are both decidable languages then $L_1 \oplus L_2$ is also decidable.

   Solution. TRUE. $L_1 \oplus L_2 = (L_1 \cap \overline{L_2}) \cup (L_2 \cap \overline{L_1}$. Since decidable languages are closed under complementation, intersection and union, the result follows.

2. Recall the reverse of a language $L^R = \{w \mid w^R \in L\}$. If L is decidable then so is L^R.

   Solution. TRUE. Given a Turing machine M that decides L, We will design a Turing machine that decides L^R as follows. M^R first reverses the string written on its input alphabet. After this, M^R just behaves in exactly the same way as M does on the reversed string.

3. FALSE. All unary languages $L \subseteq \{1\}^*$ are Turing recognizable.

   Solution. FALSE. There are many ways to prove this. Here in one. Every Turing recognizable has a Turing machine that recognizes it. Now we saw in the lecture that the set of all Turing machines is countable and hence the set of all T-recognizable languages is also countable. However, also we saw in lecture that subsets of $\{1\}^*$ is an uncountable set. Hence, all unary languages cannot be recognizable.

Problem 2. (6 points) Let

$$\textsc{Non-Empty} = \{\langle M \rangle | M \text{ accepts some string }\}$$

Prove that $\textsc{Non-Empty}$ is Turing recognizable.

Solution. We simply proceed exactly as in the construction of an enumerator from a Turing machine that recognizes a language: simulate M on all strings of length at most i for i steps, and keep increasing i. We accept if the computation of M accepts some string. If L(M) is non-empty, we are certain that for some i our machine will halt and accept.