**Theory of Computation '23 Quiz 1**
**Time : 45 minutes**

**Name :** **Roll :** **Marks :**

**Problem 1.** State True/False for the following with a valid formal reason (zero credit for just writing True/False)

a) (2 points) If $L$ is a regular language, then every subset of $L$ is regular

**Solution.** FALSE. Consider the trivial regular language $L = \{a, b\}^\star$. There are many subsets of this language which are not regular, for example, $\{a^k b^k \mid k \geq 0\}$.

(+2 for correct answer and proper example. +1 if answer is correct but example is wrong. 0 If only answer correct but no example attempted).

b) (3 points) If $L_1, L_2$ are regular languages, then $L_3 = \{w \mid w \in L_1 \text{ or } w \in L_2 \text{ but } w \notin \text{ both }\}$ is regular

**Solution.** TRUE. $L_3$ is the *exclusive-or* of $L_1$ and $L_2$, i.e. $L_3$ is

$$(L_1 \setminus L_2) \cup (L_2 \setminus L_1) \text{ or } (L_1 \cap L_2^c) \cup (L_2 \cap L_1^c)$$

[From pset2]: Complement, Intersection, and Union are regular operations. Hence, the individual terms are regular, making the result regular as well.

(+3 for correct answer and proper reasoning. If answer is correct but reasoning is somewhat fine +1.5. Also +1.5 if directly claiming the closure of set differences. If only answer correct, but reasoning is completely wrong or not present, 0).

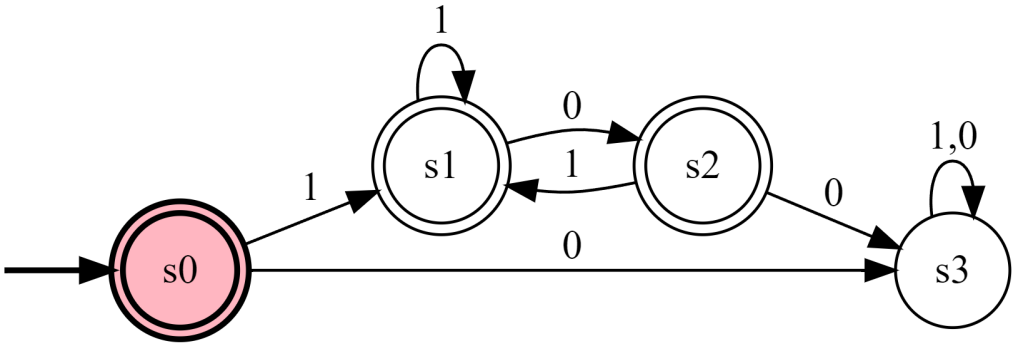
**Problem 2.** (10 points)

Design deterministic finite automata that recognize the following languages over the alphabet $\{0, 1\}$. Only the diagram or notational description is enough.

1. (5 points) $\{x \mid$ each 0 in $x$ is immediately preceded by a 1$\}$

   (0 if the DFA has more than 4 states, +5 for correct DFA, -0.5 for each missing transition, -1 if incorrect state has been marked as accept state)

   We want a DFA such that anytime a 0 occurs in a string, it must occur as a substring 10 (1 immediately precedes 0). Otherwise, we must reject the string. Note that this is different from a DFA that accepts **any** string with 10 as a substring.



2. (5 points) $\{x \mid x$ has both 01 and 10 as substrings$\}$

   (+5 for correct DFA, -0.5 for each missing transition, -1 if incorrect state has been marked as accept state. Only a (correct) DFA with at most 6 states will get full credit, at most 10 states will get half credit, and zero for anything more than that.)

   Try to think about the given language as follows. Let's say a string starts with a 0. To get accepted, the string must be in $0^+1^+0^+\Sigma^*$. Similarly, if it starts with a 1, it must be of the form $1^+0^+1^+\Sigma^*$ to get accepted. Here, $L^+ = L^* \setminus \epsilon$.