Theory of Computation '23
Problem Set 2

Notations. Let $\Sigma = \{a, b\}$. For $w \in \Sigma^\star$, let $|w|$ denote the length of w. Let $\#_a(w)$ denote the number of as in w and let $\#_b(w)$ denote the number of bs in w.

Problem 1. Let L_1, L_2 be two regular languages. Show that $L1 \setminus L2 = \{w | w \in L_1 \text{ and } w \notin L_2\}$ is also regular.

Problem 2. Given a DFA $A = (Q, \Sigma, q_0, F, \delta)$, come up with another DFA $A' = (Q', \Sigma, q_0', F', \delta')$ such that $L(A) = L(A')$, i.e. they accept the same language and for all $a \in \Sigma$ and for all $q \in Q', |\delta'(q, a)| = 1$.

Problem 3. Give an NFA for the following regular expressions. (You may simplify the expression as much as possible using one of the algebraic laws listed in Lecture 9 of the Kozen book - equations 9.1-9.13)

1. $(aa^\star + bb^\star)^\star$
2. $(ab + ba) \cdot (ab + ba) \cdot (ab + ba)$

Problem 4. Write the regular expressions corresponding to the following languages.

1. $L = \{\#_a(w) = 1(\mod 2)\}$.
2. $L = \{\text{every other letter in } w \text{ is } a\}$
3. $L = \{w \text{ contains an odd number of } a\text{'s and an even number of } b\text{'s}\}$

Problem 5 Construct a DFA that accepts strings which are the binary representations of numbers which are 0 (mod 3). Give a regular expression corresponding to this DFA using the recursive (inductive) construction done in class (given in Kozen book).