ANALYTIXLABS

Introduction to Business Analytics

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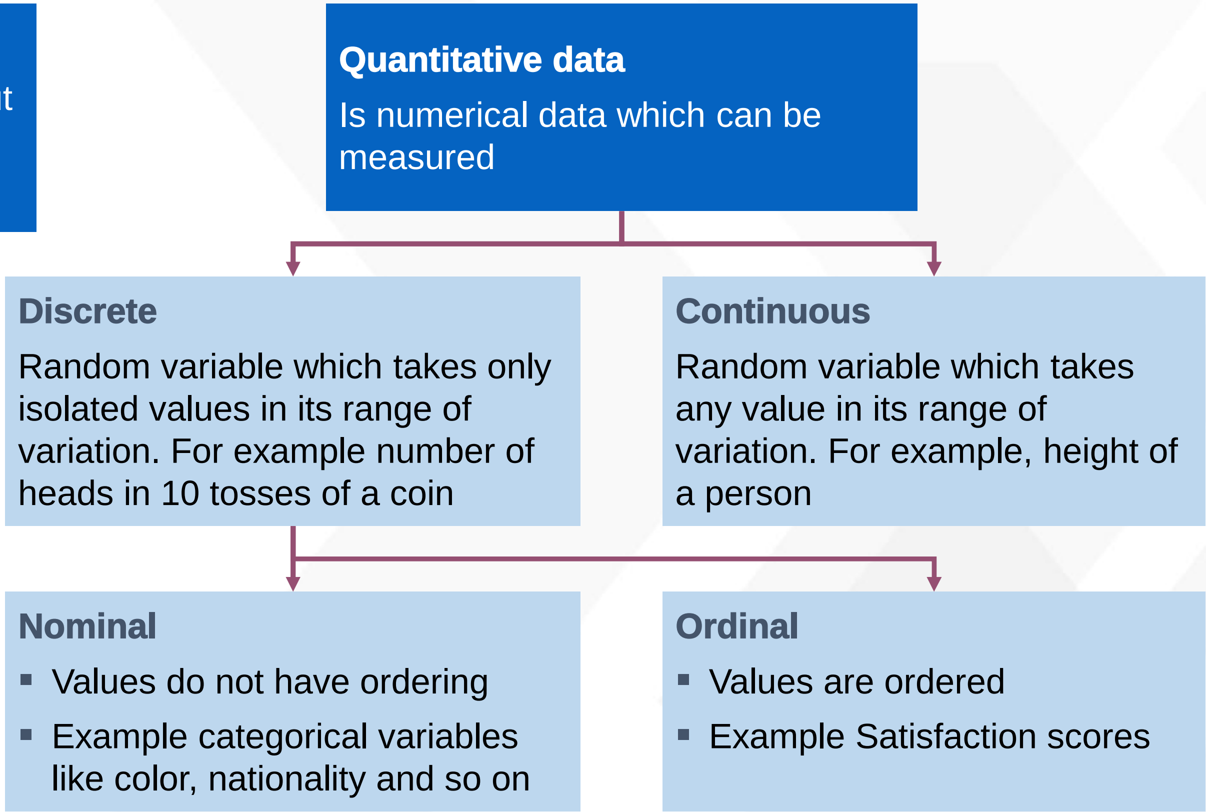
Learn to Evolve

Types of Data

Categorical data

Is non-numeric, can be observed but not measured

E.g. Favorite color, Place of Birth



Quantitative data

Is numerical data which can be measured

Discrete

Random variable which takes only isolated values in its range of variation. For example number of heads in 10 tosses of a coin

Continuous

Random variable which takes any value in its range of variation. For example, height of a person

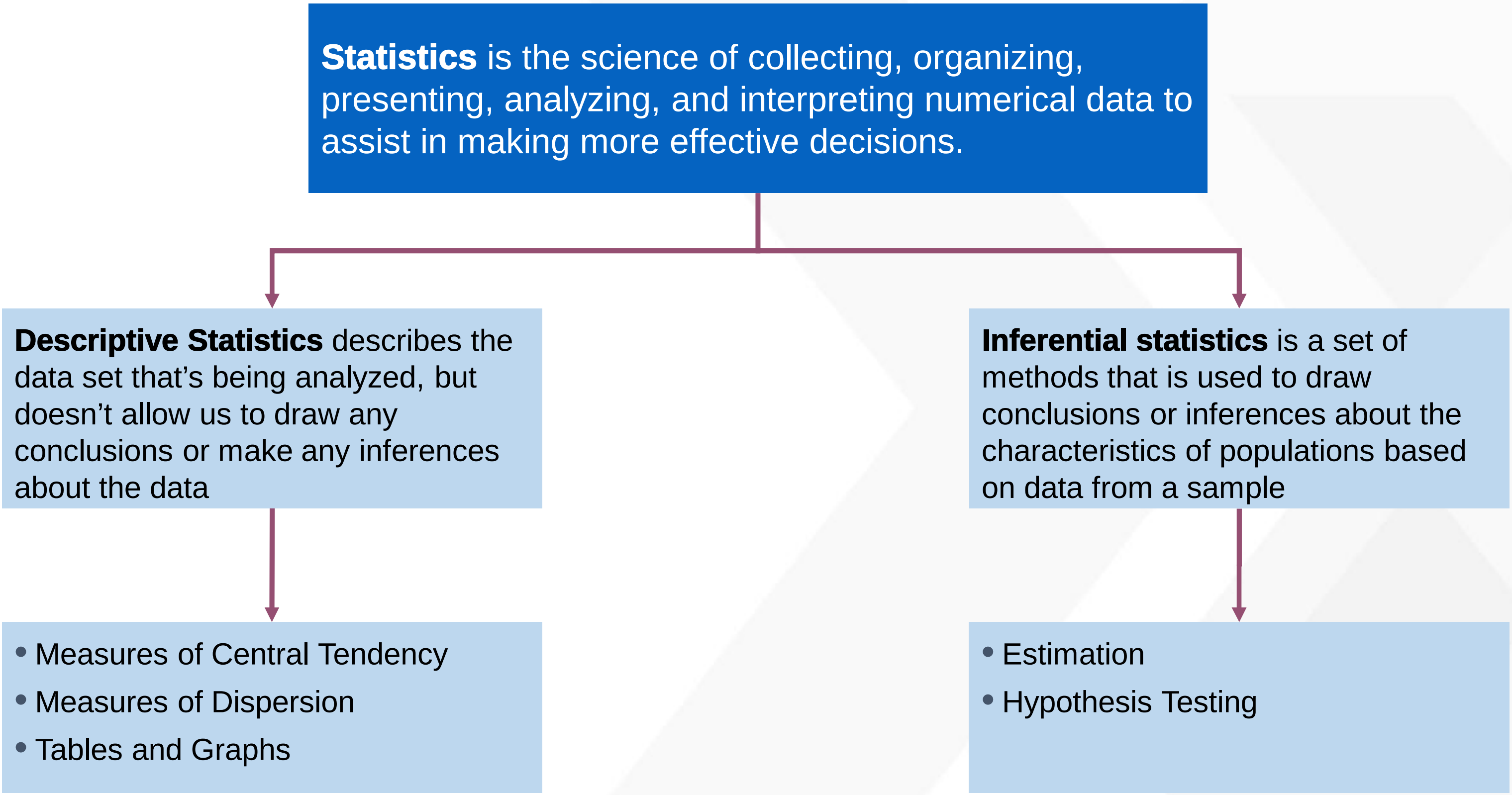
Nominal

- Values do not have ordering
- Example categorical variables like color, nationality and so on

Ordinal

- Values are ordered
- Example Satisfaction scores

Types of Statistics



Statistics is the science of collecting, organizing, presenting, analyzing, and interpreting numerical data to assist in making more effective decisions.

Descriptive Statistics describes the data set that’s being analyzed, but doesn’t allow us to draw any conclusions or make any inferences about the data

- Measures of Central Tendency
- Measures of Dispersion
- Tables and Graphs

Inferential statistics is a set of methods that is used to draw conclusions or inferences about the characteristics of populations based on data from a sample

- Estimation
- Hypothesis Testing

Measures of Central Tendency

MEAN

It is just the average of the data, computed as the sum of the data points divided by the number of points

⊕ It is the easiest metric to understand and communicate

⊖ Mean is prone to presence of outliers

Example: What is a typical student in the class doing?

MEDIAN

It is the value in the middle of the data set, when the data points are arranged from smallest to largest.

Tricky circumstances:

If there is an even number of data points, you will need to take the average of the two middle values.

⊕ Median is a more “robust” to presence of outliers

⊖ It is more complicated to communicate

Example: To compare performance of any single student against group

MODE

It is the most common value in the data set.

Tricky circumstances:

If no value occurs more than once, then there is no mode

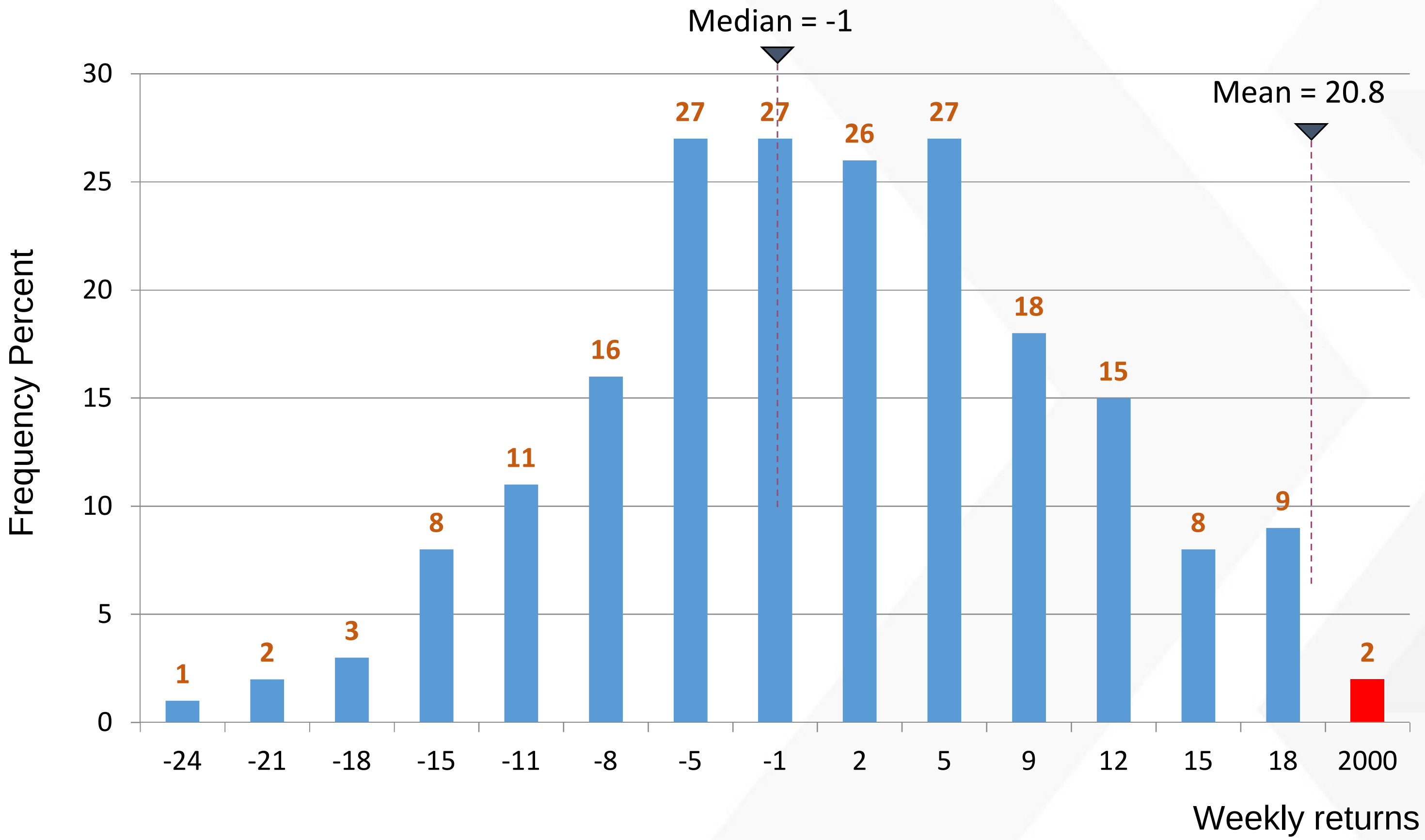
If two, or more, values occur as frequently as each other and more frequently than any other, then there are two, or more, modes.

⊕ Not very practical since it is affected by skewness

⊖ Most real life distributions are multimodal

Example: A parent wanting to know whether their child is better or worse than typical child at his grade level

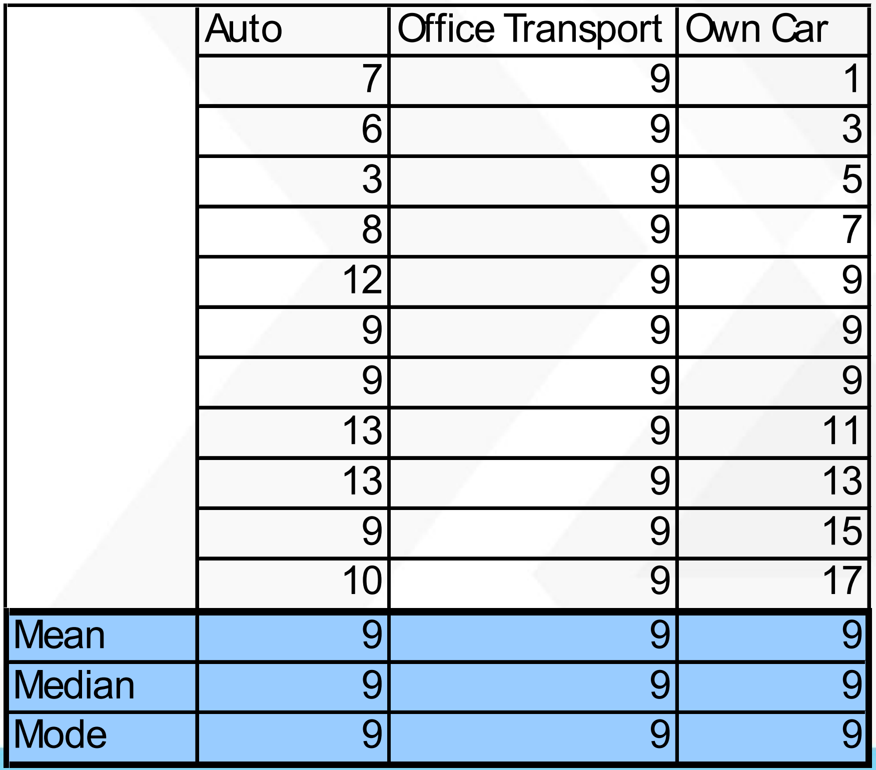
An example – Histogram of weekly returns of XYZ share prices



And Mode?

But are these sufficient?

- There is the man who drowned crossing a stream with an average depth of six inches.
- Say you were standing with one foot in the oven and one foot in an ice bucket. According to the averages, you should be perfectly comfortable.
- Time taken by different modes of transport



	Auto	Office Transport	Own Car
	7	9	1
	6	9	3
	3	9	5
	8	9	7
	12	9	9
	9	9	9
	9	9	9
	13	9	11
	13	9	13
	9	9	15
	10	9	17
Mean	9	9	9
Median	9	9	9
Mode	9	9	9

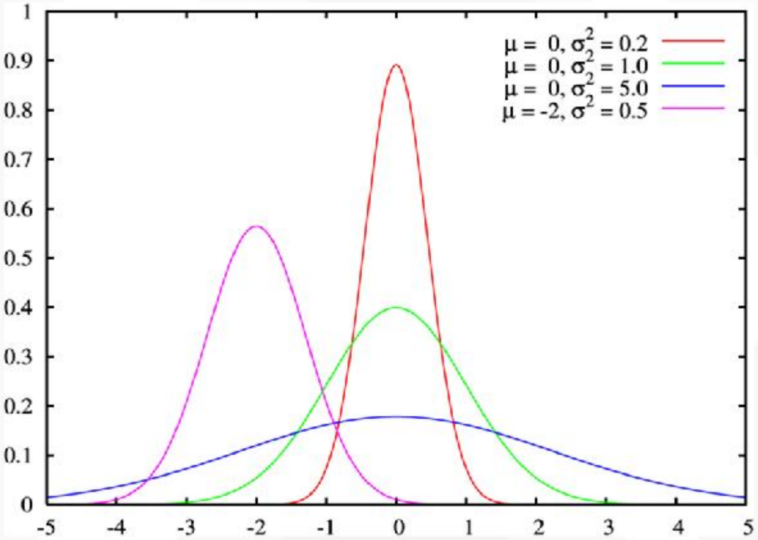
Measures of Variation/Dispersion

Dispersion refers to the spread or variability in the data. It determines how spread out are the scores around the mean.

Why is Dispersion important?

- It gives additional information that enables to judge the reliability of the measure of central tendency
- If data are widely spread the central location is less representative of data as a whole than it would be for data more closely centered around Mean
- Since problems are peculiar to widely dispersed data, dispersion enables to identify and tackle problems accordingly
- This enables to compare dispersions of various samples
- For eg. If a wide spread of values are away from center, this may be
- undesirable or presents a risk, one may avoid choosing that distribution

Distributions with different dispersions



Common measures of dispersion

Standard Deviation is a measure of how spread out numbers are

Variance is defined as the average of the squared differences from the Mean

$$\sigma^2 = \frac{\Sigma (X - \mu)^2}{N} = \frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \ldots}{N}$$

Where **X is the value of an observation in the population**

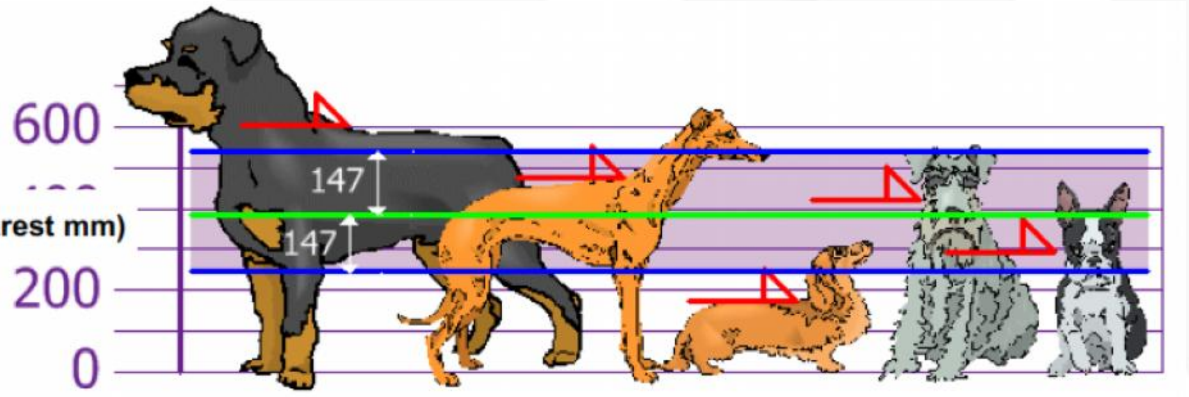
μ is the arithmetic mean of the population

N is the number of observations in the population

Example: You have just measured the heights of your dogs (in millimeters). The heights are: 600mm, 470mm, 170mm, 430mm and 300mm.
Mean = 394mm

$$\text{Variance: } \sigma^2 = \frac{206^2 + 76^2 + (-224)^2 + 36^2 + (-94)^2}{5} = 21,704$$

Standard Deviation: σ = √21,704 = 147.32... = 147 (to the nearest mm)



Using the Standard Deviation we have a "standard" way of knowing what is normal,
and what is extra large or extra small.

Now have a look….

	Auto	Office Transport	Own Car
	7	9	1
	6	9	3
	3	9	5
	8	9	7
	12	9	9
	9	9	9
	9	9	9
	13	9	11
	13	9	13
	9	9	15
	10	9	17
Mean	9	9	9
Median	9	9	9
Mode	9	9	9

Std Dev	**3.0**	**0.0**	**4.9**
Variance	**9.2**	**0.0**	**24.0**

Coefficient of Variation (CV)

It is a normalized measure of dispersion of a probability distribution. It is calculated as the ratio of the standard deviation to the mean.

- Measure of relative dispersion
- Always a %
- Shows variation relative to mean
- Used to compare 2 or more groups

Which Cricketer do you like? Who is more consistent?

Dravid	150	150	130	125	145	110	100	152	120	50	128
Sehwag	230	240	150	50	173	23	20	300	45	1	128

	Dravid	Sehwag
Mean	123.636	123.636
Median	128	128

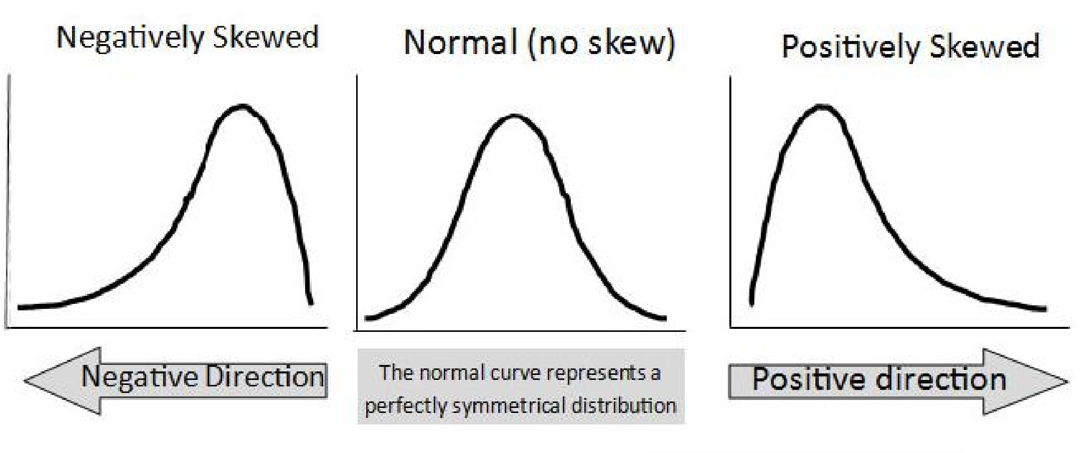
CV	24%	84%

Descriptive Statistics

Central Tendency: is the middle point of distribution. Measures of Central Tendency include Mean, Median and Mode

Dispersion: is the spread of the data in a distribution, or the extent to which the observations are scattered.

Skewness: When the data is asymmetrical ie the values are not distributed equally on both sides. In this case, values are either concentrated on low end or on high end of scale on horizontal axis.



If the trail is to the right or positive end of the scale, the distribution is said to be “positively skewed”.
If the distribution trails off to the left or negative side of the scale, it is said to be “negatively skewed”.

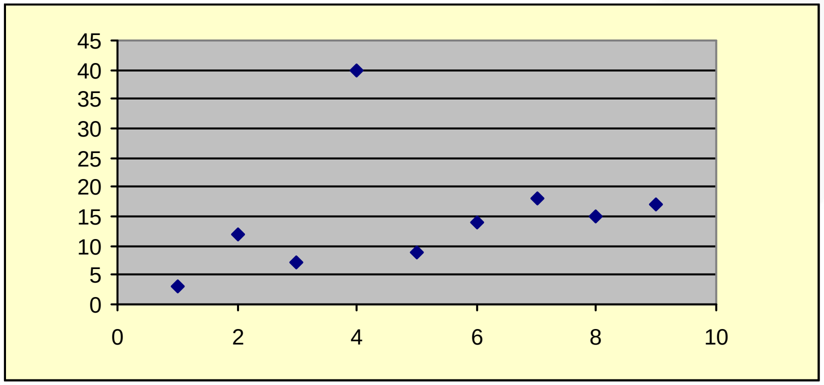
Outliers

An outlier is an observation that is numerically distant from the rest of the data.

An outlying observation, or outlier, is one that appears to deviate markedly from other members of the sample in which it occurs. Outliers can occur by chance in any distribution, but they are often indicative either of measurement error or that the population has a heavy-tailed distribution.

Example: Bill Gates makes $500 million a year. He's in a room with 9 teachers, 4 of whom make $40k, 3 make $45k, and 2 make $55k a year. What is the mean salary of everyone in the room? What would be the mean salary if Gates wasn't included? Mean With Gates: **$50,040,500** Mean Without Gates: **$45,000**

A **Scatterplot** is useful for "eyeballing" the presence of **outliers**.



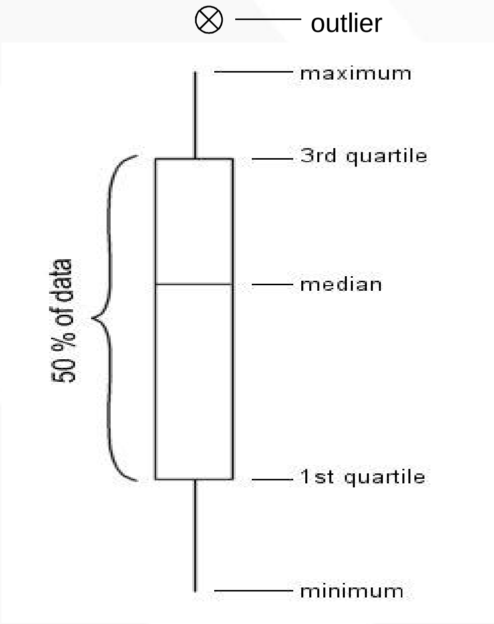
We can also use Standard Deviation to identify Outliers!

Other Measures of Dispersion

Box-plot

- Reveals the spread of the data
- Outliers defined using the

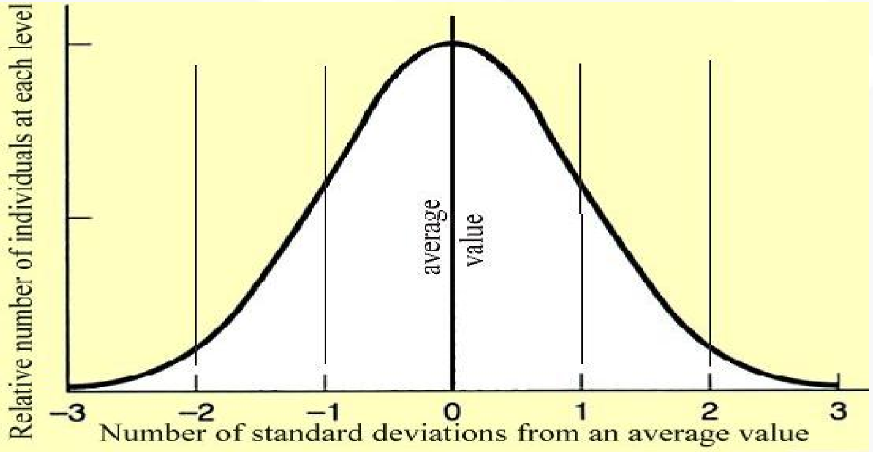
  Q1 - 1.5(Q3-Q1) and Q3 + 1.5(Q3-Q1)



Normal Distribution

Normal distribution is a pattern for the distribution of a set of data which follows a ***bell shaped curve***. This also called the *Gaussian distribution*.
The normal distribution is a theoretical ideal distribution. Real-life empirical distributions never match this model perfectly. However, many things in life do approximate the normal distribution, and are said to be “normally distributed.”

- Normal Distribution has the mean, the median, and the mode all coinciding at its peak
- The curve is concentrated in the center and decreases on either side ie most observations are close to the mean
- The bell shaped curve is symmetric and Unimodal
- It can be determined entirely by the values of mean and std dev
- Area under the curve = 1
- **The empirical *68-95-99.7 rule* states that for a normal distribution:**
  - **68.3% of the data will fall within 1 SD of mean**
  - **95.4% of the data will fall within 2 SD’s of the mean**
  - **Almost all (99.7%) of the data will fall within 3 SD’s of the mean**



Standard Normal Distribution

Standard Normal distribution is a special case of the Normal distribution which has a mean of 0 and a standard deviation of 1

Any normal distribution can be converted to a Standard normal distribution through:

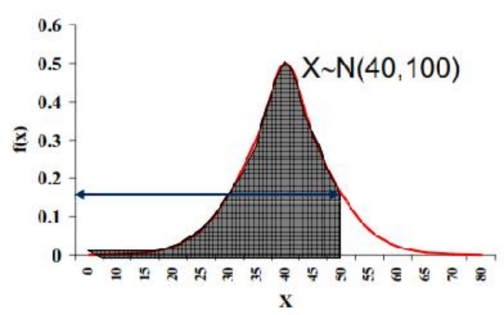
$$Z = \frac{X - \mu}{\sigma}$$

Example: If X is a continuous random variable with a mean of 40 and a standard deviation of 10, what proportion of observations are a) Less than 50 b) Less than 20 c) Between 20 and 50

a) P(x<50)?

$$Z = \frac{50-40}{10} = 1$$

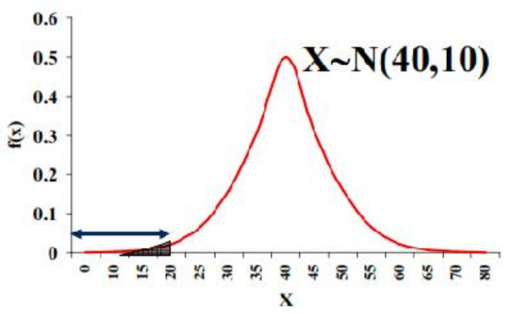
P(x<50) = P(z<1) = .8413



b) P(x<20)?

$$Z = \frac{20-40}{10} = -2$$

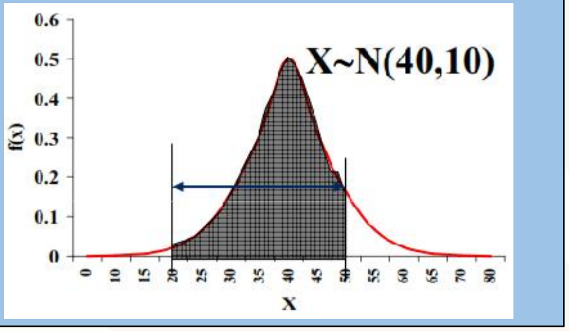
P(x<20) = P(x<-2) = .0228



c) P(20<x<50)?

$$\frac{20-40}{10} < Z < \frac{50-40}{10}$$

P(-2 < Z < 1) = .8185



Standard scores

A standard score (also called Z score) is the number of standard deviations that a given raw score is above or below the mean.

- All normal distributions can be converted into the standard normal curve by subtracting the mean and dividing by the standard deviation.
- Standardizing variables helps to get variables to the same scale, or makes the variable unit free For eg: If we want to input Income (in INR) and Number of calling minutes into the same analysis, we will have to standardize both variables to get them to the same scale

$$Z = \frac{X - «}{\pm}$$

Central Limit Theorem

It is always not possible to get the true information about the population. In this case we have to live with samples. For eg: we don't know the actual average income for India, but can estimate it based on a random sample picked from the Indian population

In this case, the average we have is not the population average μ but an estimate $\overline{X}$

If we take a similar second sample, it is extremely unlikely that the average calculated for the second sample will be the same as the average calculated for the first sample. In fact, statisticians know that repeated samples from the same population give different sample means.

They have also proven that the distribution of these sample means will always be normally distributed, regardless of the shape of the parent population. This is known as the Central Limit Theorem.

A distribution with a mean μ and variance σ^2, the sampling distribution of the mean approaches a normal distribution with a mean (μ) and a variance σ^2/N as N, the sample size increases.

The amazing and counter-intuitive thing about the central limit theorem is that *The distribution of an average tends to be Normal, even when the distribution from which the average is computed is decidedly non-Normal distribution from which the average is computed is decidedly non-Normal.*
As the sample size n increases, the variance of the sampling distribution decreases. This is logical, because the larger the sample size, the closer we are to measuring the true population sample size, the closer we are to measuring the true population parameters.

Standard Error

Since all samples drawn from a population are similar BUT NOT the same as population, we calculate a Standard Error.

Standard Error is the standard deviation of the sample means from the population mean

Also, Standard Error ultimately converges to the Standard Deviation of the population.

$$\textbf{Standard Error} = \frac{\sigma}{\sqrt{N}}$$

Populations and Samples

So far we have determined the results associated with individual observations or sample means when the true population parameters are known. In reality, the true population parameters are seldom known. We now learn how to infer **levels of confidence**, or a measure of accuracy on parameters, estimated using samples

POINT ESTIMATOR

- If we take a sample from a population, we can estimate parameters from the population, using sample statistics
- Example: Sample mean (x) is our best estimate of the population mean (μ)
- Whereas, we really don't know how close the estimate is to the true parameter
- The mean annual rainfall of Melbourne is 620mm per year

INTERVAL ESTIMATOR

- If we estimate a range or interval within which the true population parameter lies, then we are using an interval estimation method
- This is the most common method of estimation. We can also apply a level of how confident we are in the estimate
- In 80% of all years Melbourne receives between 440 and 800 mm rain

Sampling methodologies

Sampling is required because it is seldom possible to measure all the individuals in a population. Researchers hence, use samples and infer their results to the population of interest

Eg: Election polls, market research surveys, etc

For a sample to be a "good sample", it is imperative that there is a good sample size and there is no biasness in the sample.

Simple Random Sample

is one in which every member of the population is equally likely to be measured

Eg: Allocate a number to each member of the population and use a random number generator to determine which individuals will be measured

Stratified Sampling

separates the population into mutually exclusive groups and randomly samples within the groups

Eg: Randomly select a number of people within each demographic cell, while maintaining overall proportions like gender ratio, income ratio, etc

Other methodology

Cluster sampling: is used when there is a considerable variation within each group but the groups are essentially similar to each other. Here we divide the population into groups, or clusters, and then select a random sample of these clusters.

Confidence Intervals

Because we know the properties of the normal distribution so well, we can use these properties to assist us in applying confidence intervals to estimates. This is essentially the interval estimator range.

For example, we know that 68.3 % of sample means lie within one standard error of the true population mean.

Therefore, if we know the true population variance, we can infer the range within which we can be 68.3% confident that the true population mean lies

Example:

X ~ N(μ, 3.62) grams, n = 36, x = 25.5

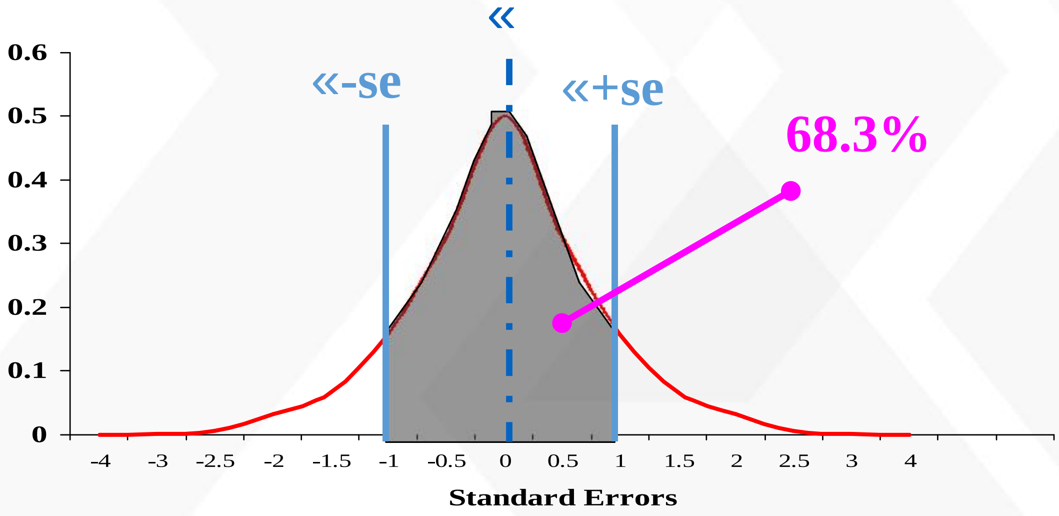
We can be 68.3% confident that μ is within the interval

x ± SE

= 25.5 ± 3.6/ √ 36

= 25.5 ± 0.6

= (24.9 to 26.1) grams



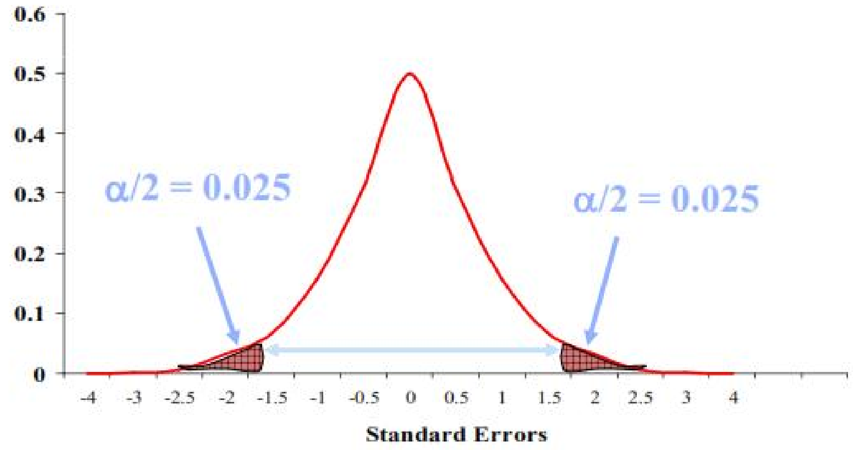
Confidence Intervals

We can extend this principle further:

- We can be *90% confident* that the true population mean lies within $x \pm 1.645(SE)$
- We can be *95% confident* that the true population mean lies within $x \pm 1.960(SE)$
- We can be *99% confident* that the true population mean lies within $x \pm 2.576(SE)$

P - value

- Furthermore, the area outside the confidence interval is cumulatively known as α (alpha)
- Confidence Interval = 1 - α
- Example: for 95% confidence interval, ∝=0.05
- α is also known as p-value.
- Hence, p-value is the probability that a randomly picked sample will have the mean **lying outside the confidence interval**.



Student’s t - distribution

- While z-distribution is for the population, t-distribution is for the sample distribution.
- Hence, the shape of ‘t’ sampling distribution is similar to that of the ‘z’ sampling distribution in that it is

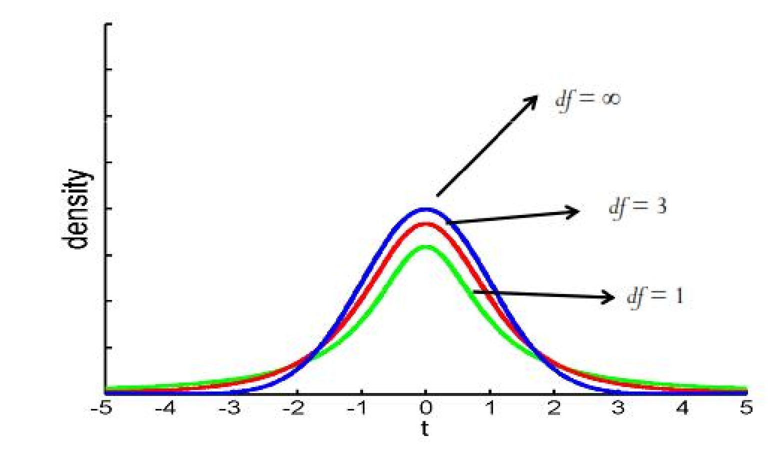
  a) Symmetrical

  b) Centered over a mean of zero

  c) Variance depends on the sample size, more specifically on the degrees of freedom (abbreviated as df)
- As the number of degrees of freedom increases , variance of the t distribution approaches more closely to that of z
- For n ≥ 30, shapes are almost similar
- For n of 30 taken as dividing point between small & large samples
- t-test for population mean is:

$$\frac{\overline{X} - \mu_0}{s/\sqrt{n}}$$

When n < 30



When to use

z-test:

- σ is known and the population is normal
- σ is known and the sample size is at least 30. (The population need not be normal)

t-test:

- Whenever σ is not known
- The population is assumed to be normal
- And n<30
- The correct distribution to use is the ‘t’ distribution with n-1 df

Hypothesis testing

- In a Test Procedure, to start with, a hypothesis is made.
- The validity of the hypothesis is tested.
- If the hypothesis is found to be true, it is accepted.
- If it is found to be untrue, it is rejected.
- The hypothesis which is being tested for possible rejection is called null hypothesis
- Null hypothesis is denoted by H_0
- The hypothesis which is accepted when null hypothesis is rejected is called Alternate Hypothesis H_a
- Ex. H_o : The drug works –it has a real effect.

  H_a : The drug doesn't work - Any effect you saw was due to chance.

Hypothesis testing

Hypothesis tests consist of the following steps:

- Null Hypothesis
- Alternative Hypothesis
- Confidence Level
- Decision Rule
- Test statistic
- Decision

Hypothesis testing

- Null hypothesis - We always assume the null hypothesis is true, or at least is the most plausible explanation before we do the test. The test can only ***disprove*** the null hypothesis.
- Alternative hypothesis - The alternative hypothesis is the hypothesis that we set out to test for. It is the hypothesis that we wish to ***prove***.
- Decision Rule - After we know the null and alternative hypotheses and the level of confidence associated with the test, we determine the points on the distribution of the test statistic where we will decide when the null hypothesis should be rejected in favor of the alternative hypothesis
- Use the terminology " **Reject H_O**" or "**Do not reject H_o**". Never say "Accept Ho"

Type I and Type II Error

Process of testing a hypothesis indicates that there is a possibility of making an error. There are two types of errors:

Type I error: The error of rejecting the null hypothesis H_0 even though H_0 was true.

Type II error: The error of accepting the null hypothesis H_0 even though H_0 was false.

Example 1

Suppose that we have been told that the price of petrol in Melbourne is normally distributed with a mean of 92 cents per litre, and a standard deviation of 3.1 cents/litre. To test whether this price is in fact true, we sample 50 service stations and obtain a mean of 93.6 cents/litre

Solution:

- Step 1: State the null and alternative hypotheses
    - Ho: $\mu = 92$
    - Ha: $\mu \neq 92$

- Step2: Determine the appropriate test statistic and it’s distribution

  Because we know the population standard deviation, we can use the z distribution

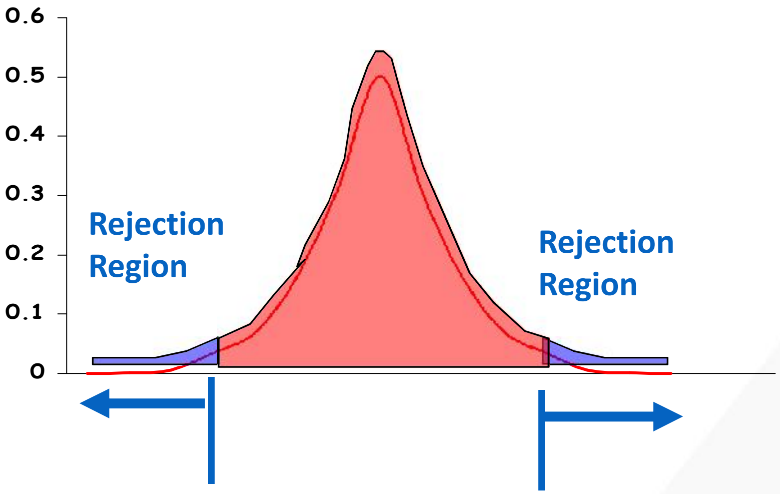
- Step3: Specify the significance level, Say $\alpha = 0.05$

Example 1 (contd.)

- Step 4: Define the decision rule.

  Using a z distribution (from tables), if $\alpha = 0.05$,

  the rejection region is > +1.96 and < -1.96



i.e., if the test statistic is

> 1.96 or < -1.96,

we will reject H_o and

accept H_a

Example 1 (contd.)

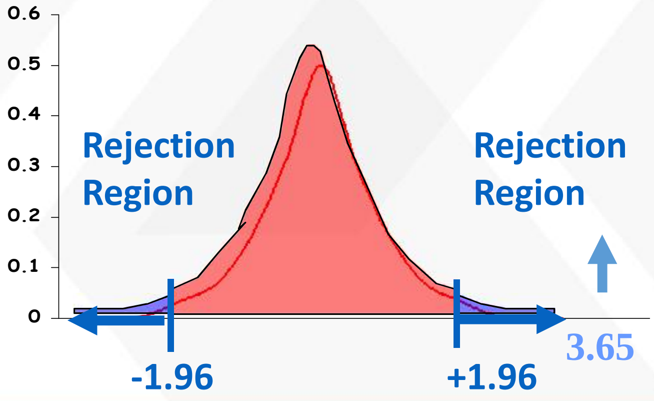
- Step 5: Calculate the test statistic

$$Z = \frac{\overline{X} - «_{\overline{x}}}{SE} = \frac{\overline{X} - «_{o}}{± / \sqrt{n}}$$

$$Z = \frac{93.6 - 92}{3.1 / \sqrt{50}} = 3.65$$

- Step 6: Make a decision and answer the question: As, 3.65 > 1.96, the test statistic is in the rejection region
  Reject H_o, accept H_a as a more plausible explanation

- Step 7: Write your conclusion in the context of the aims of the study.
  “The average price of petrol in Melbourne was significantly different to 92 cents/litre”



Example 1 (contd.) – Importance of sample sizes

- Consider the petrol prices in Melbourne example. If the sample size we had used was only 10, rather than 50, the test statistic would have been;

$$\mathbf{z} = \frac{\mathbf{93.6 - 92}}{\mathbf{3.1/\sqrt{10}}} = \mathbf{1.63}$$

In which case we would not have rejected the H_o

Example 2

A company pays production workers \$630 per week. The union claims that these workers are paid below the industry average for their work. A sample of 15 workers from other sites gives a mean wage of \$670/week with a standard deviation of \$58/week. Is the unions claim justified?

Solution:

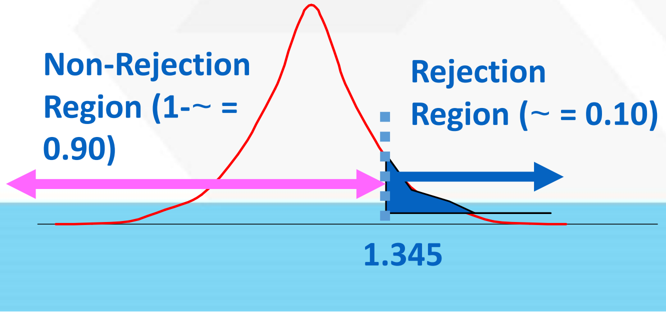
Step 1: Ho: μ =< \$630 (industry weekly average is not significantly different to \$630)

Ha: μ > \$630 (The industry weekly average is greater than \$630)

Step2: Test Statistic - As we don’t know the population variance, and the sample size is < 30, we shall use the t test.

Step3: Significance level - We will use α = 0.10 (as we want to be liberal rather than conservative)

Step 4 : Decision rule - From ‘t’ table, $t_{(0.1,\ 14df)} = 1.345$


Non-Rejection Region (1-~ = 0.90)
Rejection Region (~ = 0.10)
1.345

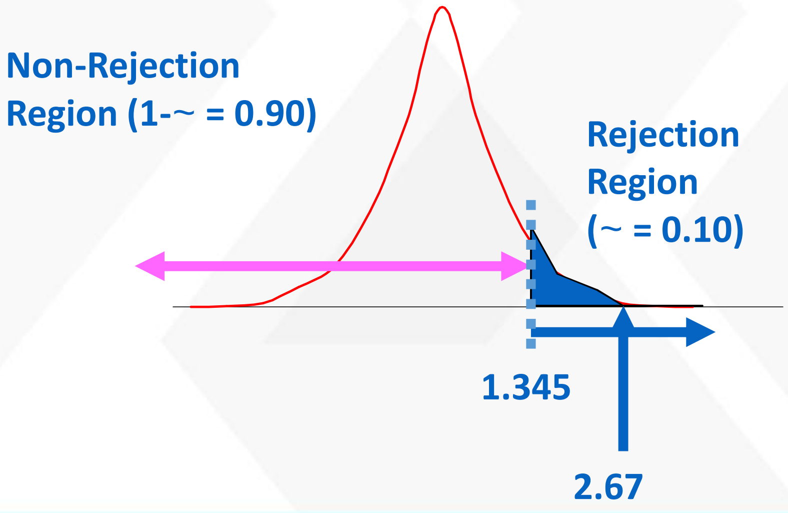
Example 2 (contd.)

Step 5: Calculate test statistic;

$$t = \frac{\overline{X} - \text{«}_{\overline{x}}}{SE} = \frac{\overline{X} - \text{«}_{\overline{x}}}{s/\sqrt{n}} \qquad t = \frac{670 - 630}{58/\sqrt{15}} = 2.67$$

Step 6: <u>Make a decision</u> - As 2.67 is > 1.345, we will reject the H_o

Step 7: <u>Conclusion</u> - “Production workers at the company earn an average of $40 per week less than the industry standard (t = 2.67, df = 14, p < 0.1)”

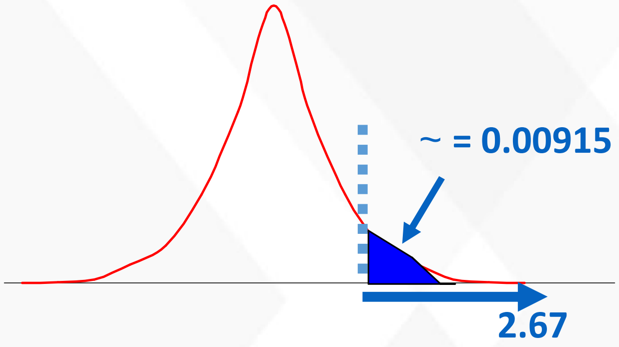


P-value

In fact, with the advent of computers it is simple to calculate the exact probability of a test statistics.

Example: $P(t \geq 2.67) = 0.00915$

i.e. The area under the curve is 0.00915



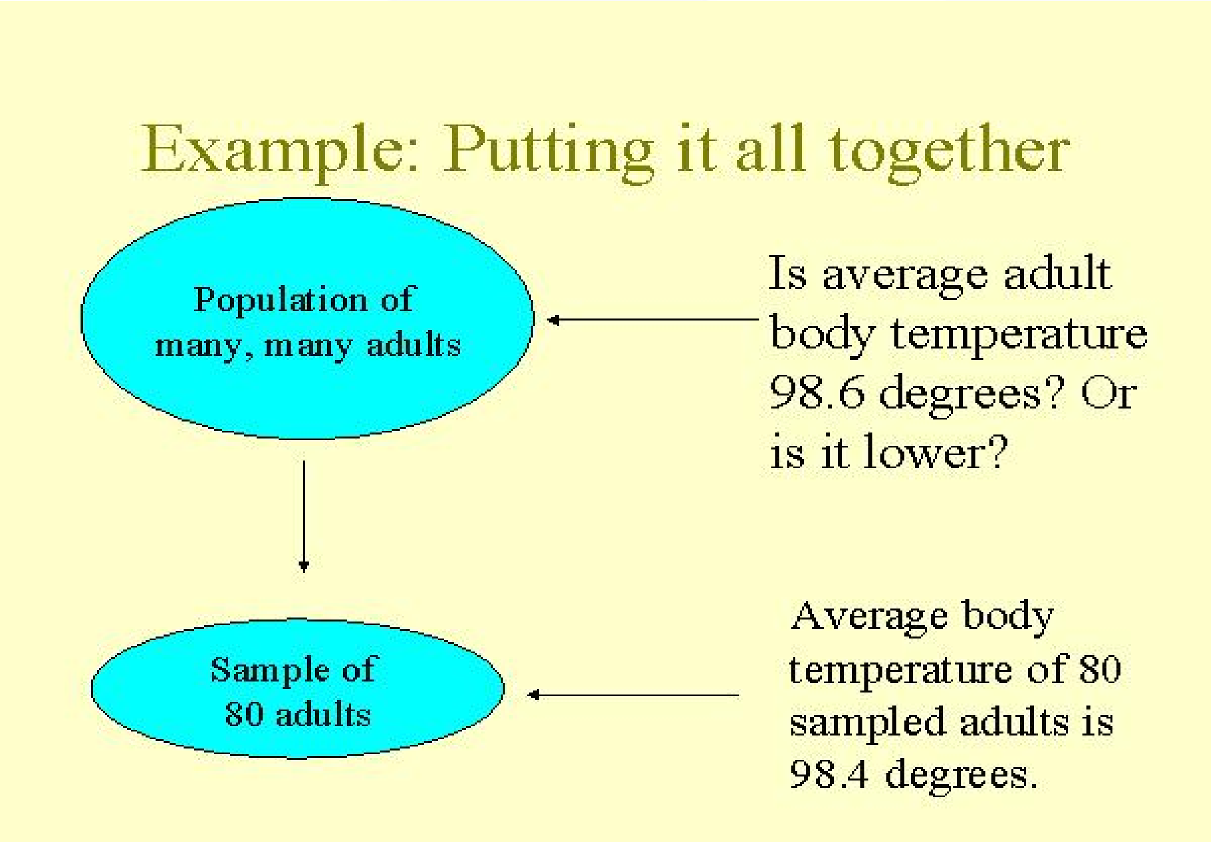
With a z test, it is possible to determine the exact probabilities using the table in the text book

Ex.: What is the exact probability of obtaining a value of $z \leq 1.5$?

From table, when $z = 1.5$, $p = 0.5 + 0.4332$,

$P(z \leq 1.5) = 0.9332$

A Practical Example



Example: Putting it all together

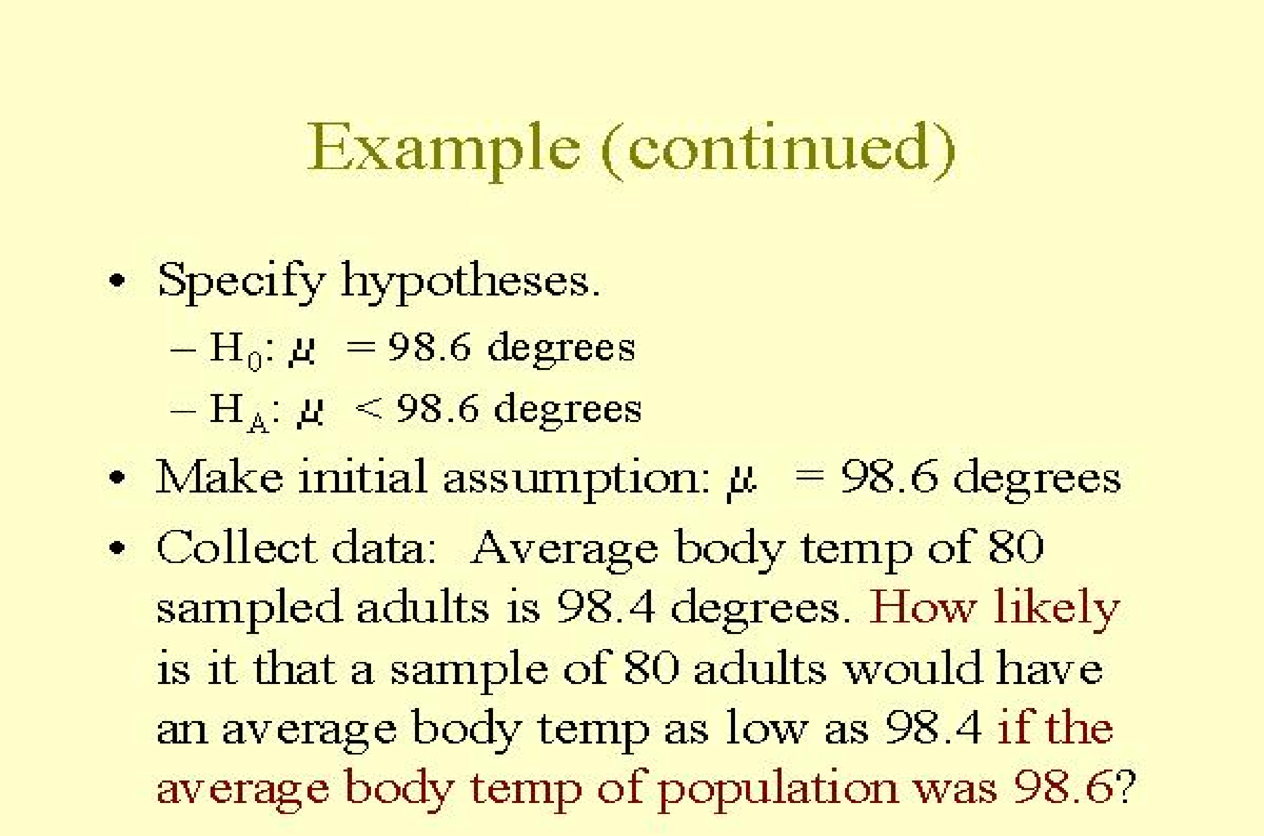
Population of many, many adults

Is average adult body temperature 98.6 degrees? Or is it lower?

Sample of 80 adults

Average body temperature of 80 sampled adults is 98.4 degrees.

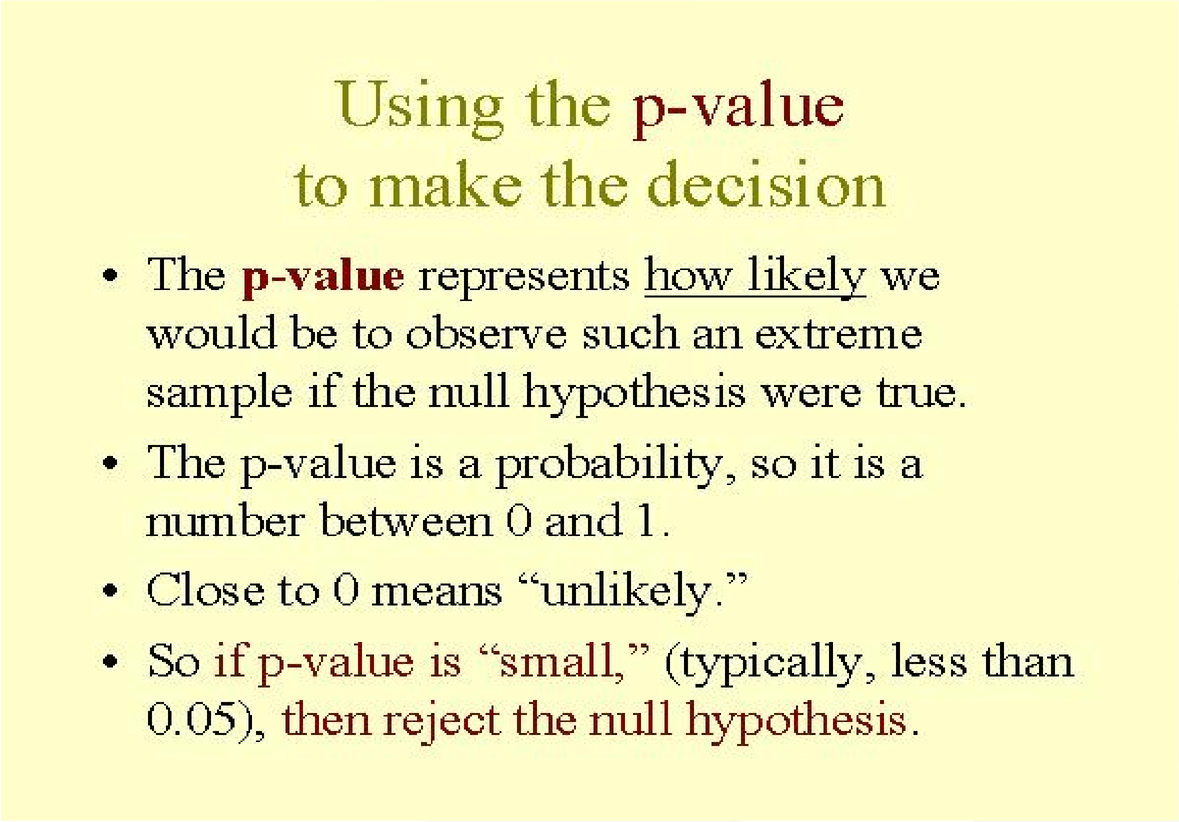
A Practical Example



Example (continued)

- Specify hypotheses.
  - $H_0: \mu = 98.6$ degrees
  - $H_A: \mu < 98.6$ degrees
- Make initial assumption: $\mu = 98.6$ degrees
- Collect data: Average body temp of 80 sampled adults is 98.4 degrees. How likely is it that a sample of 80 adults would have an average body temp as low as 98.4 if the average body temp of population was 98.6?

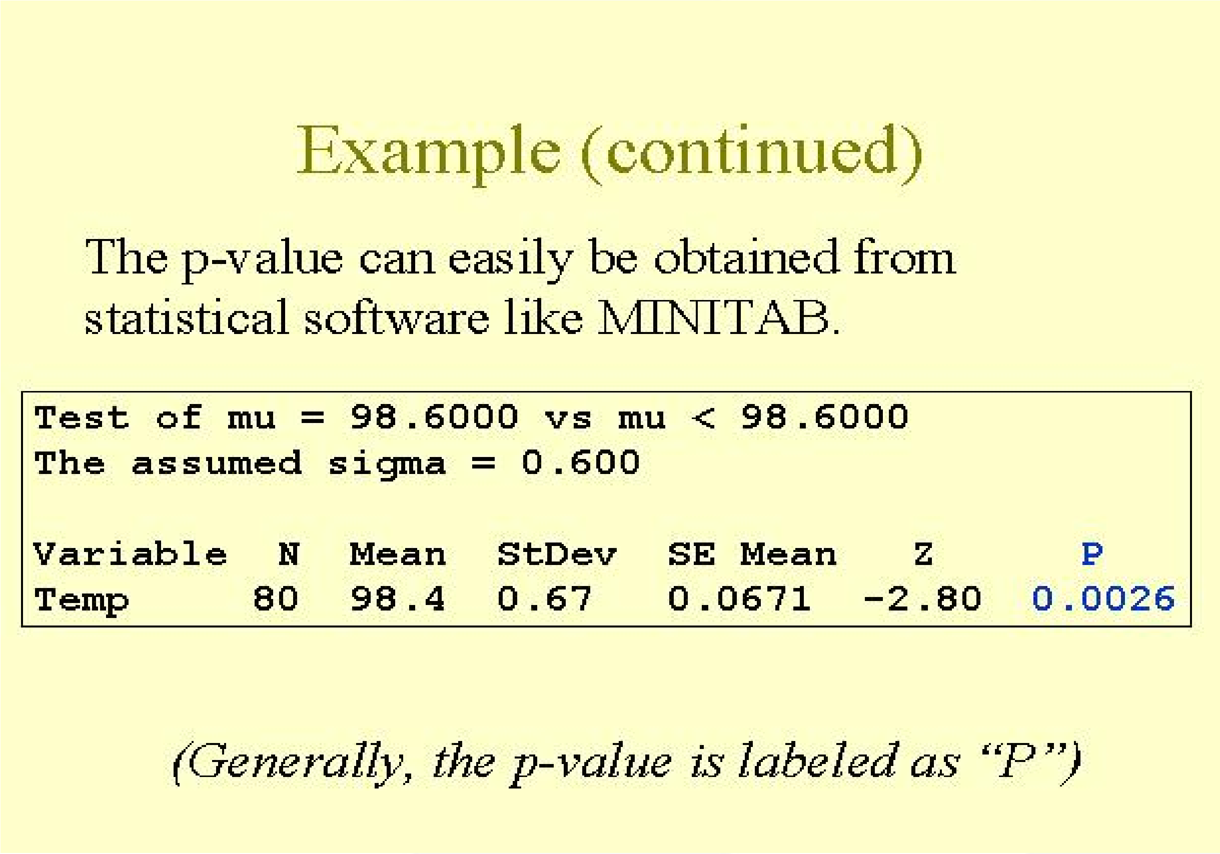
A Practical Example



Using the p-value to make the decision

- The **p-value** represents how likely we would be to observe such an extreme sample if the null hypothesis were true.
- The p-value is a probability, so it is a number between 0 and 1.
- Close to 0 means "unlikely."
- So if p-value is "small," (typically, less than 0.05), then reject the null hypothesis.

A Practical Example



Example (continued)

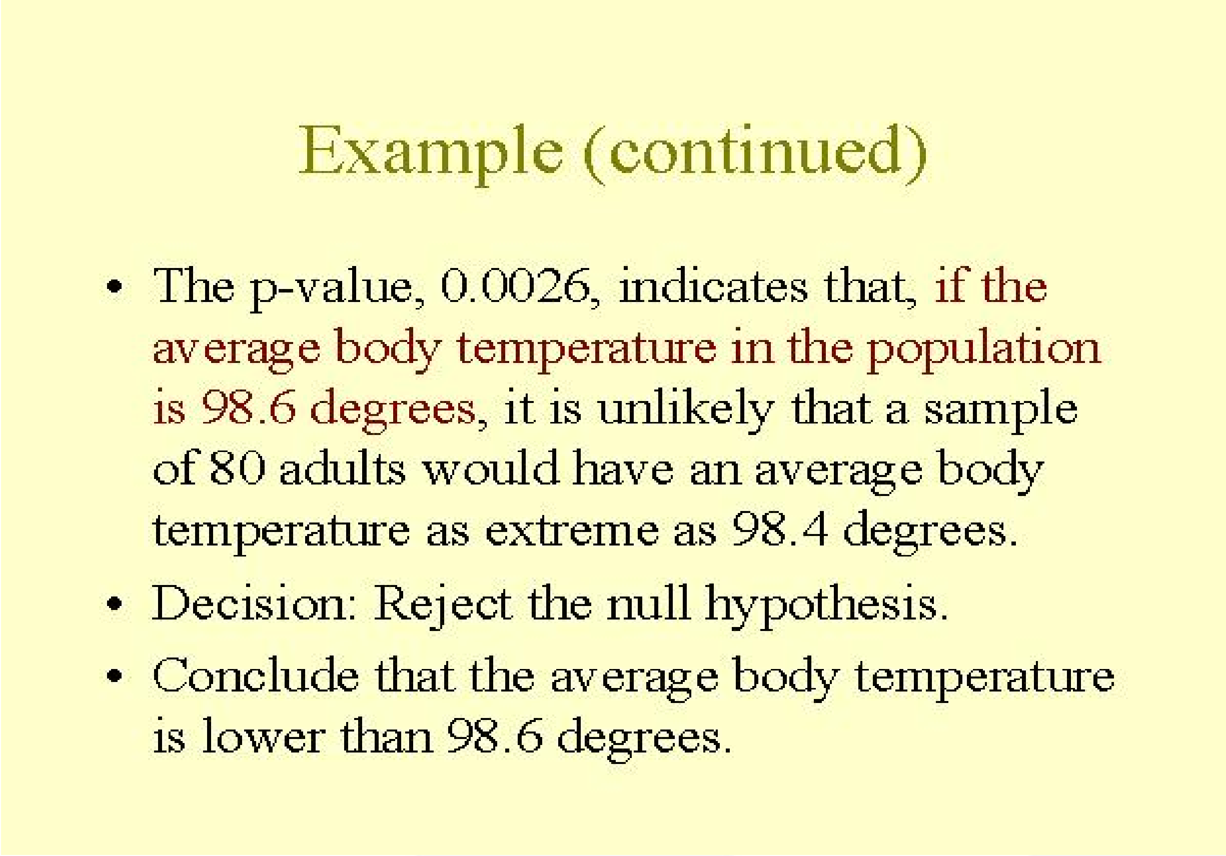
The p-value can easily be obtained from statistical software like MINITAB.

```
Test of mu = 98.6000 vs mu < 98.6000
The assumed sigma = 0.600

Variable   N   Mean   StDev   SE Mean    Z       P
Temp      80   98.4   0.67    0.0671   -2.80  0.0026
```

(Generally, the p-value is labeled as "P")

A Practical Example



Example (continued)

- The p-value, 0.0026, indicates that, if the average body temperature in the population is 98.6 degrees, it is unlikely that a sample of 80 adults would have an average body temperature as extreme as 98.4 degrees.
- Decision: Reject the null hypothesis.
- Conclude that the average body temperature is lower than 98.6 degrees.

Comparison of two populations

Hypothesis testing for two samples:

- Difference between independent samples & dependent samples
- Two sample z test for means using independent samples
- Two sample t test for means using independent Samples
- Two sample t tests for means using dependent Samples

Chi-square test

Two properties are associated if the probability of having one property affects the probability of having another. Sometimes it is not known whether two properties are associated or not. What is required is a test of association, or, what is equivalent, a test of independence.

The Chi-square (χ 2) distribution can be used as a test of independence.

Example:

A psychologist conducted a survey into the relationship between the way in which a calculator was held and the speed with which 10 arithmetical operations were performed. The calculator could be either placed on a table or held in the hand; the sums could be performed in either less than 2 minutes, between 2 and 3 minutes or more than 3 minutes.

The following results were obtained for a sample of 150 children between 12 and 13 years old.

		Mode of Computation	
		On Table	**Hand Held**
Speed Of Computation	<2	28	12
	2-3	25	35
	>3	21	29

Example (contd.)

Solution:

Step 1: Ho: Speed and mode are independent

Ha: Speed and mode are associated

Step2: In order to determine whether the two variables are associated it is necessary to calculate what the frequencies would be if there was absolutely no connection between them, or as we call them “Expected Frequencies”

	Table	**Hand**
<2	40*74/150 = 19.73	40*76/150 = 20.27
2-3	60*74/150 = 29.60	60*76/150 =30.40
>3	50*74/150 =24.67	50*76/150 = 25.33

Example (contd.)

Step 3: Now we have to use the expected and observed frequencies to calculate a test statistic.

The χ 2 test statistic is determined by $$\sum \frac{(O_i - E_i)^2}{E_i}$$

Step 4: In order to compare this with a critical value, we need to know the degrees of freedom of statistic.

v=degrees of freedom=(row number −1)· (column number −1)

$$\textit{Then}\ \text{¡}^{2}{}_{test} = 9.329 > \text{¡}^{2}{}_{critical} = 5.992$$

Step 5: Therefore, we reject H_O and accept H_1 .

The result is significant at the 0.05 or 5% level. This means that there is a 5 in 100 probability that the difference between the two conditions could have arisen by chance.

According to these results the way you use your calculator does affect the speed with which you do a calculation.

ANOVA

- Analysis of variance is a statistical technique used for comparing the means of different samples and deciding whether they are drawn from the same population or different populations.
- Main Question: Do the (means of) the quantitative variables depend on which group (given by categorical variable) the individual is in?
- The ANOVA F-statistic is a ratio of the Between Group Variation divided by the Within Group Variation:

$$F = \frac{Between}{Within} = \frac{MSTR}{MSE}$$

- A large F is evidence against H0, since it indicates that there is more difference between groups than within groups.

Covariance

- A statistical measure of the joint variability of two random variables.

- An extension to *variance* but variance – *talks about the spread of a variable* and covariance – *talks about – spread of one variable wrt to other*.

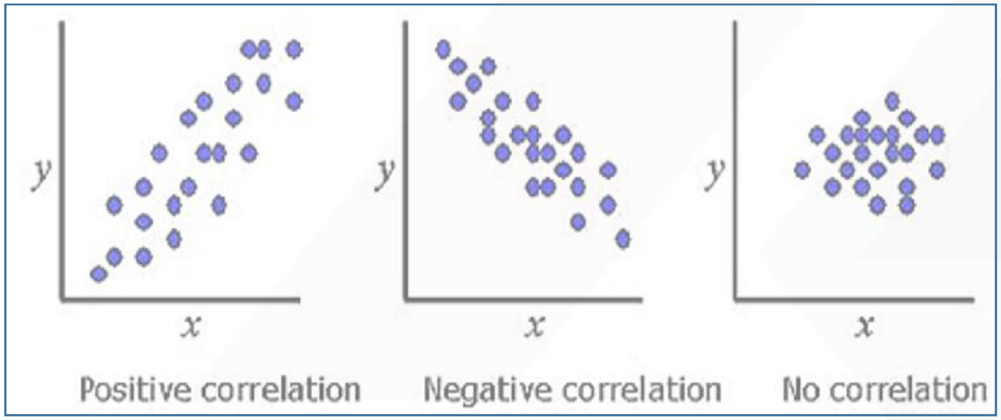
- Mostly, tries to establish a linear relationship between two variable - wherein a change in one variable reciprocated by an equivalent change in another variable

- Value ranges from –Inf to Inf and large covariance denotes a strong relationship but cannot be applicable to data with different measures and units.

- A zero value denotes no relationship.

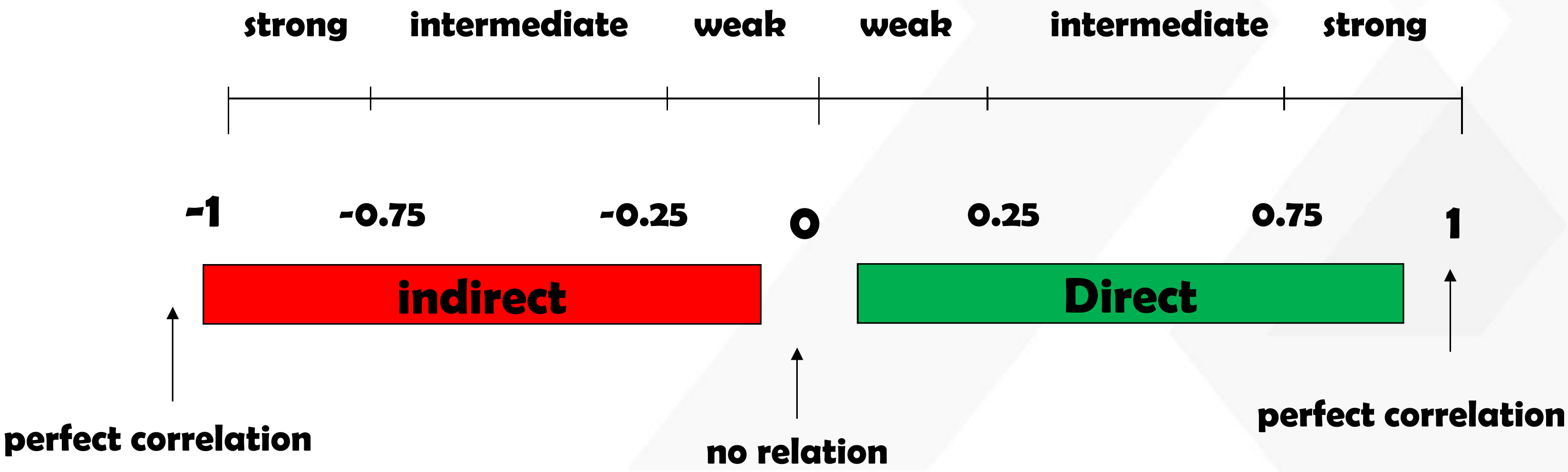
Correlation

- A statistical technique that can show whether and how strongly pairs of variables are related or whether there exists a mutual relationship or connection between two or more things.
- Positive Correlation: If the values of two variables changing with same direction.
  Ex. Demand and CostPrice, Experience or Skills and CTC
- Negative Correlation: When the values of variables change with opposite direction.
  Ex. OperatingCost and TotalProfit



Correlation Coefficient – Pearson’s Coefficient (r)

- Also called Simple correlation or product moment correlation coefficient ***it measures the nature and strength between two variables of the quantitative type.***
- The value r (-1 to 1) r denotes **strength.**
- The sign of r denotes **nature** – positive, negative or no correlation



strong intermediate weak weak intermediate strong

-1 -0.75 -0.25 0 0.25 0.75 1

indirect

Direct

perfect correlation

no relation

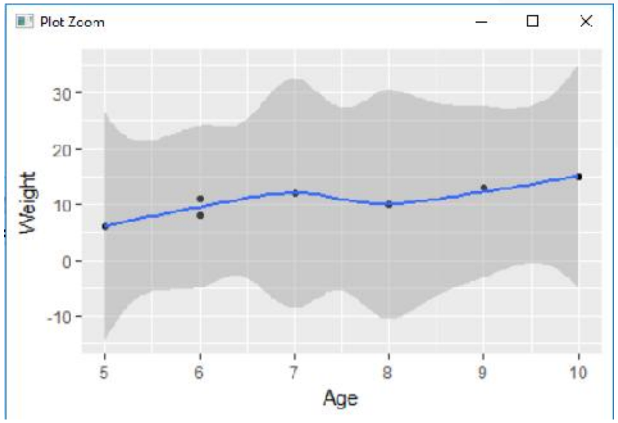
perfect correlation

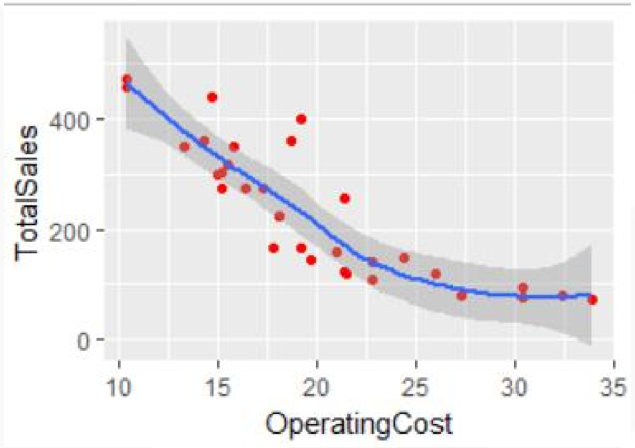
Correlation Coefficient – Spearman Rank (r_s)

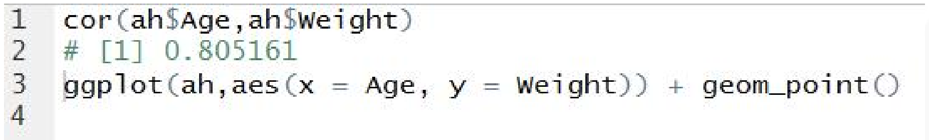
- A non-parametric measure of correlation, procedure makes use of the two sets of ranks that may be assigned to the sample values of x and Y.
- Spearman Rank correlation coefficient could be computed in the following cases:
  - Both variables - quantitative.
  - Both variables - qualitative but ordinal.
  - One variable is quantitative and the other is qualitative ordinal.
- The value of rs denotes the magnitude and nature of association giving the same interpretation as simple r.

Correlation – Examples

Age (years)	Weight (Kg)
7	12
6	8
8	12
5	10
6	11
9	13

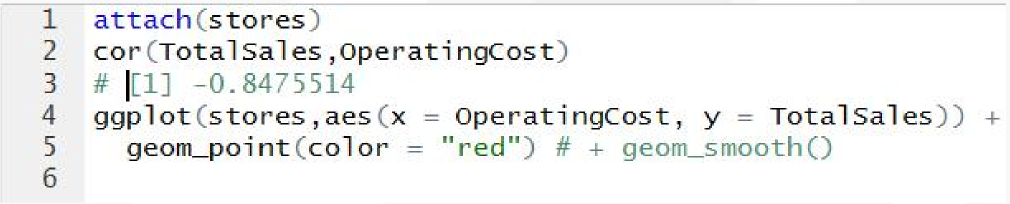






```
cor(ah$Age,ah$Weight)
# [1] 0.805161
ggplot(ah,aes(x = Age, y = Weight)) + geom_point()

```



```
attach(stores)
cor(TotalSales,OperatingCost)
# [1] -0.8475514
ggplot(stores,aes(x = OperatingCost, y = TotalSales)) +
  geom_point(color = "red") # + geom_smooth()

```

Covariance vs Correlation

BASIS FOR COMPARISON	COVARIANCE	CORRELATION
Meaning	Covariance is a measure indicating the extent to which two random variables change in tandem.	Correlation is a statistical measure that indicates how strongly two variables are related.
What is it?	Measure of correlation	Scaled version of covariance
Values	Lie between -∞ and +∞	Lie between -1 and +1
Change in scale	Affects covariance	Does not affects correlation
Unit free measure	No	Yes

Non - Parametric tests

- Deals with enumerative data
- Does not deal with specific population parameters
- Does not require assumptions about specific population distributions (in particular , the assumption of normality)
- Non-Parametric tests ignores the magnitude of information contained in observations
- Use either frequencies or ranks (categorical or ordinal)
- Non-parametric tests are called ''non-parametric'' because they do not make any assumption about a population parameter.
- In other words, when we apply a non-parametric test we do not have to make assumptions about mean of a population, its variance or background probability distribution.
- Thus, non-parametric tests are not as powerful as parametric tests they are of more general application and are available when the parametric tests fail.
- Basically, there is at least one non-parametric equivalent for each parametric general type of test
- Non-Parametric tests broadly fall into the following categories:
- Tests of differences between independent samples: The Mann-Whitney U test (t-test for independent samples), The Kruskal-Wallis H test (ANOVA), The Kolmogorov-Smirnov test (t-test for independent samples)
- Tests of differences between dependent samples: Wilcoxon Mann-Whitney Test (t-test for independent samples)

<table>
<tr><td colspan="3">Some Commonly Used Normal Statistical & NonParametric</td></tr>
<tr><th>Normal theory based test</th><th>Corresponding nonparametric test</th><th>Purpose of test</th></tr>
<tr><td>t test for independent samples</td><td>Mann-Whitney U test; Kolmogorov-Smirnov 2 sample test</td><td>Compares two independent samples</td></tr>
<tr><td>Paired t test</td><td>Wilcoxon matched pairs signed-rank test</td><td>Examines a set of differences</td></tr>
<tr><td>One way analysis of variance (F test)</td><td>Kruskal-Wallis analysis of variance by ranks</td><td>Compares three or more groups</td></tr>
<tr><td>Two way analysis of variance</td><td>Friedman Two way analysis of variance</td><td>Compares groups classified by two different factors</td></tr>
</table>

ADDITIONAL REFERENCE SLIDES

OBJECTIVE

- ✓ Identify situations that contain dependent or independent samples.
- ✓ Calculate the test statistic to test hypotheses about dependent data pairs as well as independent data pairs.
- ✓ Understand the shortcomings of comparing multiple means as pairs of hypotheses.
- ✓ Understand the steps of the ANOVA method and the method’s advantages.
- ✓ Understand the differences in situations that allow for ANOVA or MANOVA methods.
- ✓ Know the procedure of two sample t-test, ANOVA, and MANOVA and their application through technological tools.

Part I: Two Sample t-test

Part I: Two Sample t-test
Introduction
Independent Samples t-test
Dependent Samples t-test
Applications
Code in R

Introduction

- It is used when we are to perform a statistical analysis involving two samples.
- These two samples could be independent or dependent.
  - When we have independent samples we assume that the scores of one sample do not affect the other.
  - Two samples of data are dependent when each score in one sample is paired with a specific score in the other sample. In short, these types of samples are related to each other.
- In such a case, we would analyze both samples and the hypothesis would address the difference between two sample means.

Part I: Two Sample t-test

Introduction

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Dependent Samples t-test

Applications

Code in R

Independent Samples t-test

- The two sample t-test is used for comparing the means of a quantitative variable (Y) in two populations.

- Our goal is comparing μ_1 and μ_2 (which in practice is done by making inference on the difference $\mu_1 - \mu_2$).
  - The null hypotheses is Ho: $\mu_1 - \mu_2 = 0$
  - and the alternative hypothesis is one of the following (depending on the context of the problem):
    - Ha: $\mu_1 - \mu_2 < 0$
    - Ha: $\mu_1 - \mu_2 > 0$
    - Ha: $\mu_1 - \mu_2 \neq 0$

Independent Samples t-test (2)

- The two-sample t-test can be safely used when the samples are independent and at least one of the following two conditions hold:
  - The variable Y is known to have a normal distribution in both populations
  - The two sample sizes are large.

- When the sample sizes are not large (and we therefore need to check the normality of Y in both population), we look at the histograms of the two samples and make sure that there are no signs of non-normality such as extreme skewedness and/or outliers.

Independent Samples t-test (3)

- The test statistic is as follows and has a t distribution when the null hypothesis is true:

$$t = \frac{(\overline{y_1} - \overline{y_2}) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

- P-values are obtained from the output, and conclusions are drawn as usual, comparing the p-value to the significance level alpha.
- If H_o is rejected, a 95% confidence interval for μ_1 - μ_2 can be very insightful and can also be used for the two-sided test.

Part I: Two Sample t-test

Introduction

Independent Samples t-test

Dependent Samples t-test

Applications

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Dependent Samples (Paired) t-test

- The paired t-test is used to compare two population means when the two samples (drawn from the two populations) are dependent in the sense that every observation in one sample can be linked to an observation in the other sample.

- The most common case in which the matched pairs design is used is when the same subjects are measured twice, usually before and then after some kind of treatment and/or intervention.

- The idea behind the paired t-test is to reduce the data from two samples to just one sample of the differences, and use these observed differences as data for inference about a single mean—the mean of the differences, µd.

Dependent Samples (Paired) t-test (2)

- The paired t-test is therefore simply a one-sample t-test for the mean of the differences µd, where the null value is 0.
  The null hypothesis is therefore:
  H_o: $\mu_d = 0$
  and the alternative hypothesis is one of the following (depending on the context of the problem):
  H_a: $\mu_d < 0$
  H_a: $\mu_d > 0$
  H_a: $\mu_d \neq 0$

Dependent Samples (Paired) t-test (3)

- The paired t-test can be safely used when one of the following two conditions hold:
    - The differences have a normal distribution.
    - The sample size of differences is large.

- When the sample size of difference is not large (and we therefore need to check the normality of the differences), we look at the histograms of the differences and make sure that there are no signs of non-normality like extreme skewedness and/or outliers.

Dependent Samples (Paired) t-test (4)

- The test statistic is as follows and has a t distribution when the null hypothesis is true:

$$t = \frac{\overline{x_d} - 0}{\frac{s_d}{\sqrt{n}}}$$

- P-values are obtained from the output, and conclusions are drawn as usual, comparing the p-value to the significance level alpha.
- If H_o is rejected, a 95% confidence interval for μ_d can be very insightful and can also be used for the two-sided test.

Part I: Two Sample t-test

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Code in R

Applications

- With the help of two sample t-test, we can also perform hypothesis tests with two samples. We can test two independent samples which are samples that do not affect one another or dependent samples which assume that the samples are related to each other.

- We can also test the likelihood that two dependent samples are related.

Part I: Two Sample t-test

Introduction

Independent Samples t-test

Dependent Samples t-test

Applications

Code in R

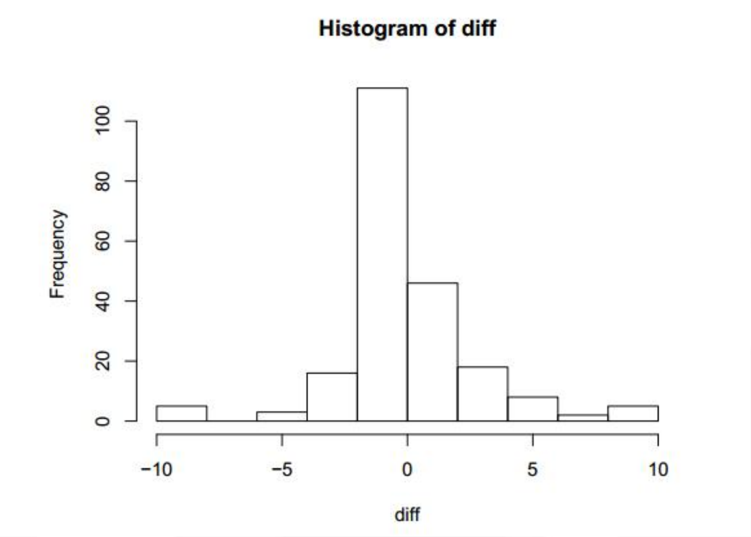
Code in R: Paired two sample t-test

- Data set of 214 undergraduate students that we surveyed both at the beginning and at the end of their semester. So among the many questions that we asked these students was this exclusive
  variable. The question that they were asked is how likely they think on a scale of 1 to 10 they would be an exclusive relationship at the end of the year.
- 1 means that there's no way they would be in an exclusive relationship. 10 means that they're definitely going to be an exclusive relationship at the end of that year.
- Now, if we wanted to compare the mean response for the exclusive question at the beginning to the mean response at the end of the semester, we're looking at two measurements within the same student. So to compare those means, we're going to be doing a **paired two sample t-test.**

Code in R: Paired two sample t-test (2)

- The assumption of a paired t-test is that the distribution of the differences in measurements is normal.

```
diff <- post$exclusive - post$post_exclusive
hist(diff)
```



- It’s not perfectly normal, but there’s no huge outliers and there’s no skewness. So we are good to go on the normality assumption of our paired sample t-test.

Code in R: Paired two sample t-test (3)

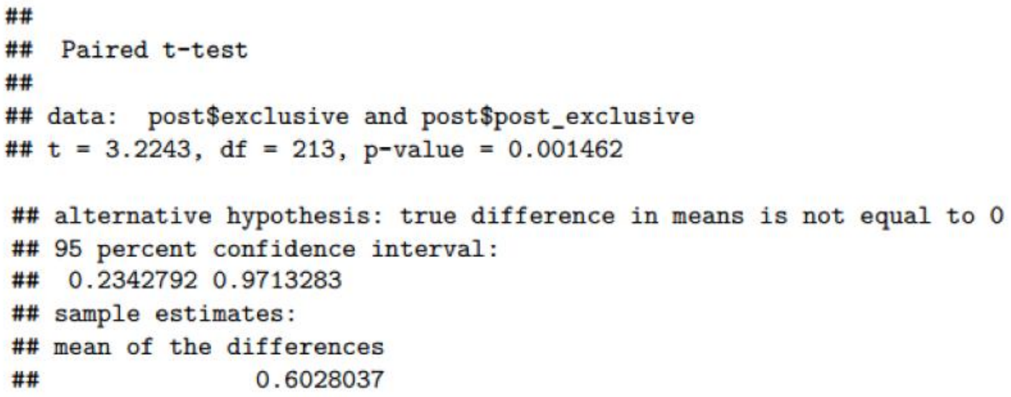
- Using the **t.test()** function.
- Pass it our two variables that we want to compare separated by comma.

```
t.test(post$exclusive, post$post_exclusive, paired = TRUE)
```

- So, this'll tell R that these two variables that we're giving it within the t.test function are actually from the same subjects.

Code in R: Paired two sample t-test (4)

- Output:



```
##
##  Paired t-test
##
## data:  post$exclusive and post$post_exclusive
## t = 3.2243, df = 213, p-value = 0.001462

## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.2342792 0.9713283
## sample estimates:
## mean of the differences
##               0.6028037
```

- The p-value is less than alpha, less than 0.05. So we would reject the null hypothesis that the mean response to the first exclusive variable is equal to the mean of the second.

Code in R: Independent Samples t-test

- Using the same data set, we’re going to look at the gender variable, where we have male or female students, and then how long they said they slept on Tuesday of that week.

- This variable is giving us an idea of how much sleep these students are getting on a typical weekday.

- So if we want to compare if the mean number of hours of sleep on Tuesday is the same for female students as it is for male students, we’re going to use the same **t.test()** function.

Code in R: Independent Samples t-test (2)

- Creating dedicated variables:

```
fsleep <- post$sleep_Tues[post$gender == "Female"]
msleep <- post$sleep_Tues[post$gender == "Male"]
```

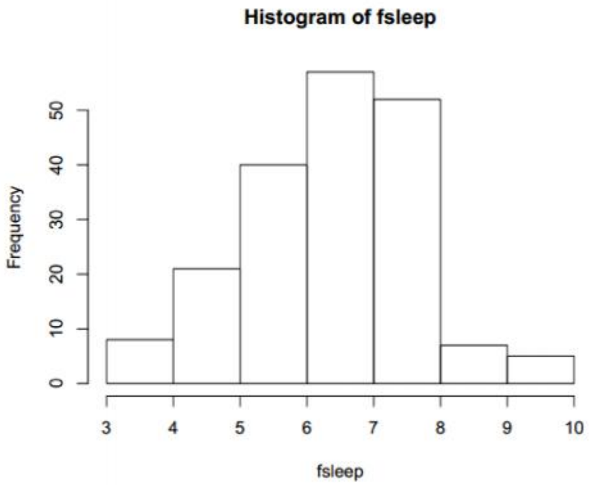
- This creates two vectors that contain the value of “sleep_Tuesday” for both the female students and then the male students.

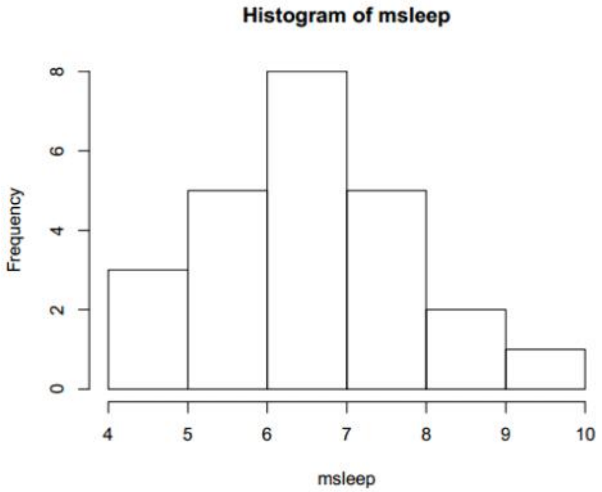
- So now I have two vectors of my two groups that I’m comparing with t-test.

Code in R: Independent Samples t-test (3)

- Checking assumptions: Are both the groups normally distributed?

```
hist(fsleep)
hist(msleep)
```

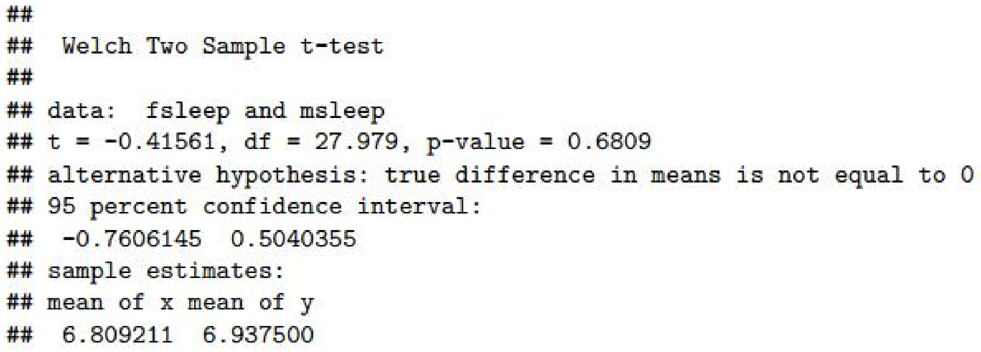




- Both the distributions are roughly symmetric. There are no outliers. These are definitely approximately normal distribution.

Code in R: Independent Samples t-test (4)

- Conducting the t-test
  `t.test(fsleep, msleep)`
- Output:



```
##
##  Welch Two Sample t-test
##
## data:  fsleep and msleep
## t = -0.41561, df = 27.979, p-value = 0.6809
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.7606145  0.5040355
## sample estimates:
## mean of x mean of y
##  6.809211  6.937500
```

- So because our p value here is greater than 0.05, we would fail to reject the null hypothesis that the means of these two groups are equal.

Part II: ANOVA, MANOVA
Introduction
ANOVA
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Code in R

Introduction

- If our objective is to test the hypothesis that multiple population means and variances of scores are equal, we use ANOVA.
- Instead, we could also conduct a series of t-tests to determine if any of the sample means differ. However, this would be tedious and the probability of making one or more Type I errors multiplies exponentially.
- So, we use **Analysis of Variance (ANOVA)**, which can be used when we want to test the means of three or more populations at once. This ANOVA is technically called “one-way” when it has just one main grouping factor. We can also have an ANOVA with more than one factor.

Part II: ANOVA, MANOVA
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ANOVA

- The ANOVA F-test is used for comparing more than two population means when the samples (drawn from each of the populations we are comparing) are independent.

- We encounter this situation when we want to examine the relationship between a quantitative response variable and a categorical explanatory variable that has more than two values.

- The hypotheses that are being tested in the ANOVA F-test are:
  - H0: μ1=μ2=...=μk
  - Ha: not all the μ's are equal

ANOVA (2)

- The idea behind the ANOVA F-test is to check whether the variation among the sample means is due to true differences among the μ's or merely due to sampling variability by looking at:

$$\left(\frac{\quad}{\quad}\right)$$

- Once we verify that we can safely proceed with the ANOVA F-test, we use software to carry it out.

- If we are able to reject our Null Hypothesis, we continue on, conducting post-hoc analyses to discover where the difference in the sample means lies.

Part II: ANOVA, MANOVA
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MANOVA

- Multivariate analysis of variance (MANOVA) is simply an ANOVA with several dependent variables. ANOVA tests for the difference in means between two or more groups, while MANOVA tests for the difference in two or more vectors of means.
- For example, we may conduct a study where we try two different textbooks, and we are interested in the students' improvements in math and physics. In that case, improvements in math and physics are the two dependent variables, and our hypothesis is that both together are affected by the difference in textbooks.
- Instead of a univariate F value, we would obtain a multivariate F value based on a comparison of the error variance/covariance matrix and the effect variance/covariance matrix.

MANOVA (2)

- The main objective in using MANOVA is to determine if the response variables are altered by the observer’s manipulation of the independent variables.
- If the overall multivariate test is significant, we conclude that the respective effect is significant.
- However, our next question would of course be whether only math skills improved, only physics skills improved, or both. In other words, one would identify the specific dependent variables that contributed to the significant overall effect.
- Generally, unless the multivariate F-test is significant, the results of the univariate tests are disregarded.

Part II: ANOVA, MANOVA
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Applications

- ANOVA is designed to detect differences among the means from population subject to different treatments. ANOVA models can realistically be used in numerous industries and applications:

  - Studying whether advertisements of different kinds solicit different numbers of customer responses
  - Comparing the gas mileage of different vehicles, or the same vehicle under different fuel types, or road types.
  - Understanding the performance, quality or speed of manufacturing processes based on number of cells or steps they're divided into

Applications (2)

- MANOVA is useful in experimental situations where at least some of the independent variables are manipulated. By measuring several dependent variables in a single experiment, there is a better chance of discovering which factor is truly important.

- The MANOVA model can be used to test the following effects:
  - Main effects of the independent variables
  - Interactions between the independent variables
  - The degree of relationship between the dependent variables
  - How significant are the dependent variables, in terms of being affected by independent variables

Part II: ANOVA, MANOVA

Introduction

ANOVA

MANOVA

Applications

Code in R

CODE IN R: MANOVA

- Data on triathlon performance. Triathlon is a multi-sport race where competitors complete a swim course, bike course, and run course, in that order.

- We want to know if gender or age category has an effect on the times for the individual sports. So we have a multifactor MANOVA.

- The MANOVA analysis assumes both normality and homoscedasticity (equality of variance) of your experimental errors (residuals). We can check these graphically or with dedicated tests.

CODE IN R: MANOVA (2)

- Importing data:

```
dat <- read.csv("triathlon.csv")
```

- R needs each independent variable in its own vector of factors. It also needs all the continuous response variables together in a separate matrix. Not to worry, we can make those files easily:

```
gender <- as.factor(dat[,1])
cat <- as.factor(dat[,2])
times <- as.matrix(dat[,3:5])
```

CODE IN R: MANOVA (3)

- Checking for normality graphically by looking at the boxplots of residuals.
- In this code, we will make residuals for each treatment and plot them, separated by the treatment levels:

```
boxplot(lm(dat$SWIM~cat)$residuals~cat) # For CATEGORY
boxplot(lm(dat$SWIM~cat)$residuals~cat) # For GENDER
# Repeat for each sport
```

- Almost all the groups are normally distributed.

CODE IN R: MANOVA (4)

- Now, we'll look at homoscedasticity.
- We can look at the residual plots for all combinations of CATEGORY and GENDER for each sport:

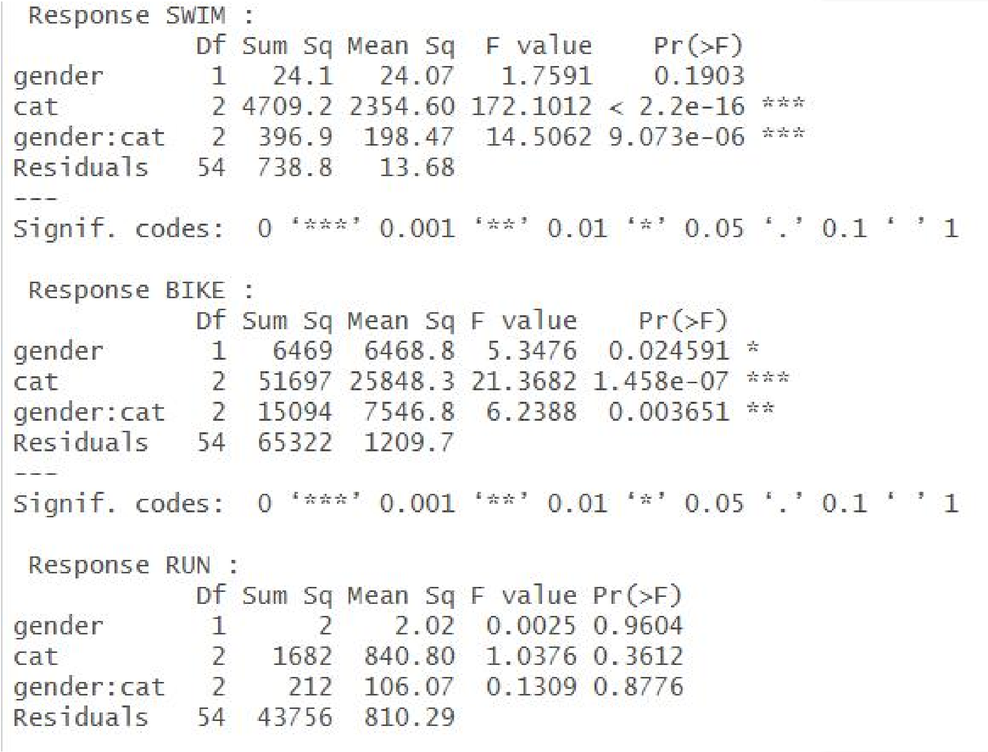
```
plot(lm(dat$SWIM~dat$CATEGORY*dat$GENDER))
plot(lm(dat$BIKE~dat$CATEGORY*dat$GENDER))
plot(lm(dat$RUN~dat$CATEGORY*dat$GENDER))
```

CODE IN R: MANOVA (5)

- We’re interested in the effect of both gender and age category on all three times, so we’ll include them both in the model. But we’re also interested in the interaction between them, so we’ll separate them with an asterisk (*):

```
output <- manova(times~gender*cat)
summary.aov(output)
```

CODE IN R: MANOVA (6)



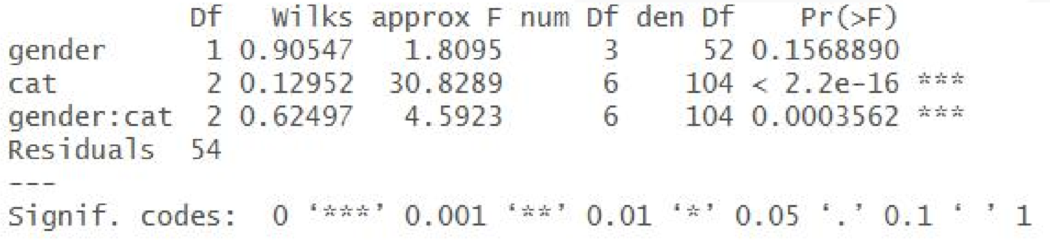
```
 Response SWIM :
            Df Sum Sq Mean Sq  F value    Pr(>F)    
gender       1   24.1   24.07   1.7591    0.1903    
cat          2 4709.2 2354.60 172.1012 < 2.2e-16 ***
gender:cat   2  396.9  198.47  14.5062 9.073e-06 ***
Residuals   54  738.8   13.68                       
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

 Response BIKE :
            Df Sum Sq Mean Sq F value    Pr(>F)    
gender       1   6469  6468.8  5.3476  0.024591 *  
cat          2  51697 25848.3 21.3682 1.458e-07 ***
gender:cat   2  15094  7546.8  6.2388  0.003651 ** 
Residuals   54  65322  1209.7                      
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

 Response RUN :
            Df Sum Sq Mean Sq F value Pr(>F)
gender       1      2    2.02  0.0025 0.9604
cat          2   1682  840.80  1.0376 0.3612
gender:cat   2    212  106.07  0.1309 0.8776
Residuals   54  43756  810.29               
```

- In this summary, we can see how each response variable relates to each treatment.
- We can see that the swim and bike times for at least one level of age category is significantly different. Gender doesn’t appear to have much effect on anything (maybe bike time).
- However, we have a significant interaction between gender and category for the swim and bike times.

CODE IN R: MANOVA

- In order to find out if the arrangement of responses (if ALL responses together) are significant as a whole Wilk’s lambda (Λ) test for MANOVA can be used, where 1-Λ is often interpreted as the proportion of the variance explained by the model.

```
summary(output, test="Wilks")
```



```
            Df   Wilks approx F num Df den Df    Pr(>F)    
gender       1 0.90547   1.8095      3     52 0.1568890    
cat          2 0.12952  30.8289      6    104 < 2.2e-16 ***
gender:cat   2 0.62497   4.5923      6    104 0.0003562 ***
Residuals   54                                              
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

CODE IN R: MANOVA

- Data on triathlon performance. Triathlon is a multi-sport race where competitors complete a swim course, bike course, and run course, in that order.
- We want to know if gender or age category has an effect on the times for the individual sports. So we have a multifactor MANOVA.

Thank you!

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