

# ✓ MATRIX - CHAIN MULTIPLICATION

Algorithm :-

Matrix-chain-Order ( $P$ )

- 1)  $n = P.length - 1$
- 2) for  $i=1$  to  $n$
- 3)  $m[i,i] = 0$
- 4) for  $l=2$  to  $n$  //  $l$  is a chain length
- 5) for  $i=1$  to  $n-l+1$
- 6)  $j = i + l - 1$
- 7)  $m[i,j] = \infty$
- 8) for  $k=i$  to  $j-1$
- 9)  $q = m[i,k] + m[k+1,j] + p_{i-1} p_k p_j$
- 10) if  $q < m[i,j]$
- 11)  $m[i,j] = q$
- 12)  $s[i,j] = k$
- 13) return  $m$  and  $s$ .

$$m[i,j] = \begin{cases} 0 & \text{if } i=j \\ \min_{1 \leq k < j} \{ m[i,k] + m[k+1,j] + p_{i-1} p_k p_j \} & \text{if } i < j \end{cases}$$

(2)

imp.

Q =

$$A_1 = 30 \times 35$$

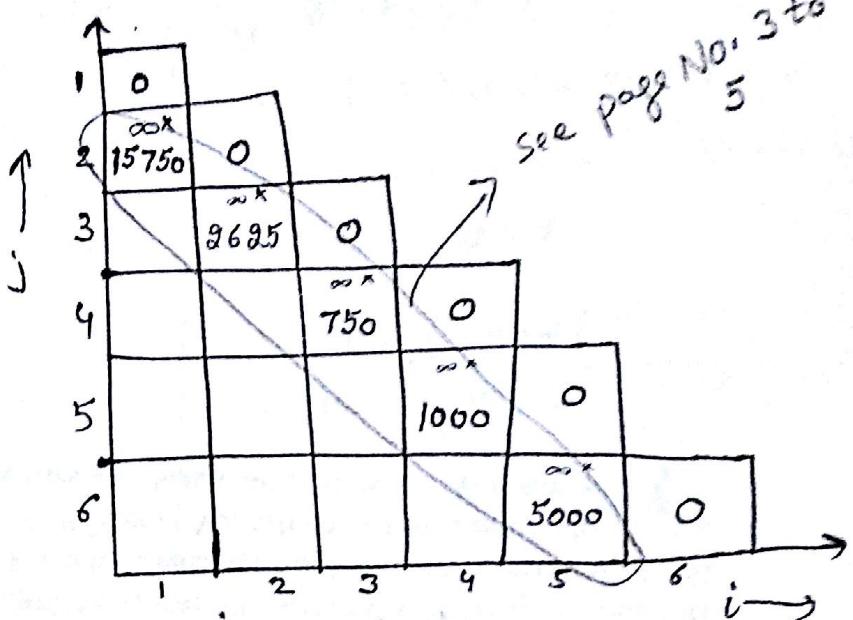
$$A_2 = 35 \times 15$$

$$A_3 = 15 \times 5$$

$$A_4 = 5 \times 10$$

$$A_5 = 10 \times 20$$

$$A_6 = 20 \times 25$$



$$\langle \underset{P_0}{\downarrow} 30, \underset{P_1}{\downarrow} 35, \underset{P_2}{\downarrow} 15, \underset{P_3}{\downarrow} 5, \underset{P_4}{\downarrow} 10, \underset{P_5}{\downarrow} 20, \underset{P_6}{\downarrow} 25 \rangle$$

$$\Rightarrow \boxed{P = n+1 \\ = 6+1 \\ = 7}$$

$\Rightarrow$  matrix chain order (P)

1)  $n = P.length - 1$

$$\boxed{n = 6}$$

2) for  $i = 1$  to  $6$  do  
 $m[i, i] = 0$

$$\left. \begin{array}{l} \text{e.g. } m[1,1] = 0 \\ m[2,2] = 0 \\ m[3,3] = 0 \\ m[4,4] = 0 \\ m[5,5] = 0 \\ m[6,6] = 0 \end{array} \right\}$$

3) for  $l = 2$  to  $6$

4)      for  $i = 1$  to  $n-l+1$   
 $= 1$  to  $6-2+1$   
 $i = 1$  to  $5$

(3)

i.e.

$$\boxed{l=2, i=1}$$

5)  $j = i + l - 1$   
 $j = 1 + 2 - 1$   
 $\boxed{j = 2}$

6)  $m[i, j] = \infty$

7) for  $k = 1$  to  $j-1$   
 $k = 1$  to 1  
 $\boxed{k=1}$

8)  $q = m[i, k] + m[k+1, j] + P_{i-1} P_k P_j$   
 $= m[1, 1] + m[2, 2] + P_0 P_1 P_2$   
 $= 0 + 0 + 30 \times 35 \times 15$   
 $= 15750$

9) if  $q < m[1, 2]$   
 $15750 < \infty$

10) then  $\boxed{m[1, 2] = 15750} \checkmark$   
 $\boxed{S[1, 2] = 1}$

11)  $\Rightarrow \boxed{l=2, i=2}$

5)  $j = 2 + 2 - 1$   
 $\boxed{j = 3}$

6)  $m[2, 3] = \infty$

7) for  $k = 1$  to  $(3-1)$   
 $k = 1$  to 2

8)  $\boxed{k=1 \rightarrow}$   
 $q = m[2, 1] + m[2, 3] + P_1 P_2 P_3$   
 $= - + \infty + 35 \times 35 \times 5$   
 $\approx 6125 + \infty$

9) if  $6125 < \infty$   $\boxed{No\ update}$

10)  $\boxed{m[2, 1] = \infty}$

Take min value of  $q$

$S[2, 1] = \infty$

8)  $\boxed{k=2 \rightarrow}$

$q = m[2, 2] + m[3, 3] + P_1 P_2 P_3$   
 $= 0 + 0 + 35 \times 15 \times 5$   
 $= 2625$

9) if  $2625 < \infty$

10)  $\boxed{m[2, 3] = 2625} \checkmark$

$\boxed{S[2, 3] = q}$

$\Rightarrow \boxed{l=2, i=3}$

5)  $j = 3 + 2 - 1$   
 $\boxed{j = 4}$

6)  $m[3, 4] = \infty$

7) for  $k = 1$  to  $(4-1)$   
 $k = 1$  to 3

$\boxed{k=1 \rightarrow}$

8)  $q = m[3, 1] + m[2, 4] + P_2 P_1 P_4$   
 $= - + \infty + 15 \times 35 \times 10$   
 $= 5250 + \infty$

9) if  $(5250 + \infty) < \infty$  No updates  
10)

$\boxed{k=2 \rightarrow}$

8)  $q = m[3, 2] + m[3, 4] + P_2 P_2 P_4$   
 $= - + \infty + 15 \times 15 \times 10$   
 $= 2250 + \infty$

9) if  $2250 + \infty < \infty$

10)  $\boxed{\text{X } (\text{No update})^2}$

8)  $q = m[3,3] + m[4,4] + P_2 P_3 P_4$   
 $= 0 + 0 + 15 \times 5 \times 10$   
 $= 750$

9) if  $750 < \infty$

10) then  $m[3,4] = 750$   
 $S[3,4] = 3$

$$\Rightarrow l = 2, i = 4$$

$$5) \quad j = 4 + 2 - 1$$

$$j = 5$$

$$6) \quad m[4,5] = \infty$$

7) for  $k = 1$  to  $(5-1)$   
 $= 1$  to 4

$k=1$

8)  $\eta_2 = m[4,1] + m[2,5] + P_3 P_1 P_5$   
 $= - + \infty + 5 \times 35 \times 20$   
 $= 3500 + \infty$

9) if  $3500 + \infty$   
No updates

o)

$k=9$

8)  $q = m[4, 2] + m[3, 5] + p_3 p_2 p_5$   
 $= - + \infty + 5 \times 15 \times 20$   
 $= 1500 + \infty$   
 9) if  $1500 + \infty < 3500$   
 No updates  
 10)

$$K=3 \rightarrow$$

$$\begin{aligned}
 8) q &= m[4,3] + \underline{m[4,5]} + p_3 P_3 P_5 \\
 &= - + \infty + 5 \times 5 \times 20 \\
 &= , \quad 500 + \infty \\
 9) \text{ if } & 500 + \infty < \infty - X \\
 10) & \text{ --- } \boxed{\text{No up}}
 \end{aligned}$$

k=4

$$8) q = m[4,4] + m[5,5] + P_3 P_4 P_5 \\ = 0 + 0 + 5 \times 10 \times 20 \\ = 1000$$

$$9) \text{ if } 1000 < \infty$$

$$10) \text{ then } \begin{cases} m[4,5] = 1000 \\ s[4,5] = 4 \end{cases}$$

$$\Rightarrow \boxed{l=2, i=5}$$

$$5) \quad j = 5 + 2 - 1$$

$$j = 6$$

$$6) \quad m[5, 6] = \infty$$

7) for  $k=1$  to  $(6-1)$   
 $k=1$  to 5

$k=1$  ]

$$8) q = m[5,1] + m[2,6] + p_4 p_1 p_6 \\ = - + \infty + 10 \times 35 \times 25 \\ = 8750 + \infty \\ 9) \text{ if } 8750 + \infty < \infty \\ \text{No update}$$

$\boxed{k=2} \rightarrow$

$$\left. \begin{array}{l} 8) q = m[5,2] + m[3,6] + P_4 P_2 P_6 \\ = - + \infty + 10 \times 15 \times 25 \\ = 3750 + \infty \end{array} \right.$$

- 9) if  $3750 < \infty$   
 10) No updates

$\boxed{k=3} \rightarrow$

$$\left. \begin{array}{l} 8) q = m[5,3] + m[4,6] + P_4 P_3 P_6 \\ = - + \infty + 10 \times 5 \times 25 \\ = 1250 + \infty \end{array} \right. \text{ till now not fix value } \therefore \text{ take } = \infty$$

- 9) if  $1250 + \infty < \infty$   
 10) No updates

$\boxed{k=4} \rightarrow$

$$\left. \begin{array}{l} 8) q = m[5,4] + m[5,6] + P_4 P_4 P_6 \\ = - + \infty + 10 \times 10 \times 25 \\ = 2500 + \infty \end{array} \right.$$

- 9) if  $2500 + \infty < \infty$   
No updates

$\boxed{k=5}$

$$\left. \begin{array}{l} 8) q = m[5,5] + m[6,6] + P_4 P_5 P_6 \\ = 0 + 0 + 10 \times 20 \times 25 \\ = 5000 \end{array} \right.$$

- 9) if  $5000 < \infty$

$$10) \text{ then } \boxed{m[5,6] = 5000} \\ S[5,6] = 5$$

Now  $\Rightarrow$

$$3) l = 3 \rightarrow 6$$

$$4) \text{ for } i = 1 \text{ to } n-l+1 \\ i = 1 \text{ to } 6-3+1 \\ i = 1 \text{ to } 4$$

i.e.

$$\Rightarrow \boxed{l=3, i=1}$$

$$5) j = i + l - 1$$

$$j = 1 + 3 - 1$$

$$\boxed{j=3}$$

$$6) m[1,3] = \infty$$

$$7) \text{ for } k = 1 \text{ to } (3-1) \\ k = 1 \text{ to } 2$$

$\boxed{k=1} \rightarrow$

$$\left. \begin{array}{l} 8) q = m[1,1] + m[2,3] + P_0 P_1 P_3 \\ = 0 + 2625 + 30 \times 35 \times 5 \\ = 2625 + 5250 \\ = 7875 \end{array} \right.$$

$$9) \text{ if } 7875 < \infty$$

$$10) \text{ then } m[1,3] = 7875 \\ S[1,3] = 1$$

$\boxed{k=2} \rightarrow$

$$\left. \begin{array}{l} 8) q = m[1,2] + m[3,3] + P_0 P_2 P_3 \\ = 15750 + 0 + 35 \times 15 \times 5 \\ = 18375 \end{array} \right.$$

$$9) \text{ if } 18375 < 7875$$

No updates

$$l=3, i=2$$

5)  $j = 2+3-1$   
 $j = 4$

6)  $m[2,4] = \infty$

7) for  $k=1$  to 3

$k=1 \rightarrow$

8)  $q = m[2,1] + m[2,4] + P_1 P_1 P_4$   
 $= - + \infty + 35 \times 35 \times 10$   
 $= 12250 + \infty$

9) if  $12250 + \infty < \infty$   
 No updates

$k=2 \rightarrow$

8)  $q = m[2,2] + m[3,4] + P_1 P_2 P_4$   
 $= 0 + 750 + 35 \times 15 \times 10$   
 $= 750 + 5250$   
 $= 6000$

9) if  $6000 < \infty$

10) then  $m[2,4] = 6000$   
 $s[2,4] = 2$

$k=3 \rightarrow$

8)  $q = m[2,3] + m[4,4] + P_1 P_3 P_4$   
 $= 2625 + 0 + 35 \times 5 \times 10$   
 $= 4375$

9) if  $4375 < 6000$

10) then  $m[2,4] = 4375$   
 $s[2,4] = 3$

$$\Rightarrow l=3, i=3$$

5)  $j = 3+3-1$   
 $j = 5$

6)  $m[3,5] = \infty$

7) for  $k=1$  to 4

$k=1 \rightarrow$

8)  $q = m[3,1] + m[3,5] + P_2 P_1 P_5$   
 $= - + \infty + 15 \times 35 \times 20$   
 $= 10500 + \infty$

9) if  $10500 + \infty < \infty$   
 No updates

$k=2 \rightarrow$

8)  $q = m[3,2] + m[3,5] + P_2 P_2 P_5$   
 $= - + \infty + 15 \times 15 \times 20$   
 $= 4500 + \infty$

9) if  $4500 + \infty < \infty$   
 10) No updates

$k=3 \rightarrow$

8)  $q = m[3,3] + m[4,5] + P_2 P_3 P_5$   
 $= 0 + 1000 + 15 \times 5 \times 20$   
 $= 2500$

9) if  $2500 < \infty$

10) then  $m[3,5] = 2500$   
 $s[3,5] = 3$



$K=4 \rightarrow$

$$\begin{aligned} 8) q &= m[3,4] + m[5,5] + p_3 p_4 p_5 \\ &= 750 + 0 + 15 \times 10 \times 20 \\ &= 3750 \end{aligned}$$

9) if  $3750 < 1500$   
No updates

9  $\Rightarrow l=3, i=4$

10)  $j = 4+3-1$

$j=6$

6)  $m[4,6] = \infty$

7) for  $k=1$  to 5

$K=1 \rightarrow$

$$\left\{ \begin{array}{l} 8) q = m[4,1] + m[2,6] + p_3 p_1 p_6 \\ = - + \infty + 5 \times 35 \times 25 \\ = 8750 + \infty \end{array} \right.$$

9) if  $8750 + \infty < \infty$   
No updates

$K=2 \rightarrow$

$$\begin{aligned} 8) q &= m[4,2] + m[3,6] + p_3 p_2 p_6 \\ &= - + \infty + 5 \times 15 \times 25 \\ &= 1875 + \infty \end{aligned}$$

9) if  $1875 + \infty < \infty$

10) No updates

$K=3 \rightarrow$

$$\begin{aligned} 8) q &= m[4,3] + m[4,6] + p_3 p_3 p_6 \\ &= - + \infty + 5 \times 5 \times 25 \\ &= 625 + \infty \end{aligned}$$

9) if  $625 + \infty < \infty$

No updates

$K=4 \rightarrow$

$$\begin{aligned} 8) q &= m[4,4] + m[5,6] + p_3 p_4 p_6 \\ &= 0 + 5000 + 5 \times 10 \times 25 \\ &= 6250 \end{aligned}$$

9) if  $6250 < \infty$

10) then  $m[4,6] = 6250$   
 $s[4,6] = 4$

$K=5 \rightarrow$

$$\begin{aligned} 8) q &= m[4,5] + m[6,6] + p_3 p_5 p_6 \\ &= 1000 + 0 + 5 \times 20 \times 25 \\ &= 3500 \end{aligned}$$

9) if  $3500 < 6250$

10) then  $m[4,6] = 3500$   
 $s[4,6] = 5$

Now  $\Rightarrow$

3)  $l=4-6$

4) for  $i=1$  to  $n-l+1$   
 $i=1$  to  $6-4+1$

$i=1$  to 3

$\Rightarrow$  i.e.  $l=4, i=1$

5)  $j = i+l-1$

$$\begin{cases} j = 1+4-1 \\ j = 4 \end{cases}$$

$$6) m[1,4] = \infty$$

7) for  $k=1$  to 3

$\boxed{k=1} \rightarrow$

$$8) q = m[1,1] + m[2,4] + p_0 p_1 p_4 \\ = 0 + 4375 + 30 \times 35 \times 10 \\ = 14875$$

9) if  $14875 < \infty$

$$10) \text{ then } m[1,4] = 14875 \\ S[1,4] = 1$$

$\boxed{K=2} \rightarrow$

$$8) q = m[1,2] + m[3,4] + p_0 p_2 p_4 \\ = 15750 + 750 + 30 \times 15 \times 10 \\ = 21000$$

9) if  $21000 < 14875$

No update

$\boxed{K=3} \rightarrow$

$$8) q = m[1,3] + m[4,4] + p_0 p_3 p_4 \\ = 7875 + 0 + 30 \times 5 \times 10 \\ = 9375$$

9) if  $9375 < 14875$

$$10) \text{ then } \boxed{m[1,4] = 9375} \\ \boxed{S[1,4] = 3}$$

$$\Rightarrow \boxed{l=4, i=2}$$

$$5) j = i + l - 1$$

$$j = 2 + 4 - 1 \\ \boxed{j=5}$$

$$6) m[2,5] = \infty$$

7) for  $k=1$  to 4

$\boxed{k=1} \rightarrow$

$$8) q = m[2,1] + m[2,5] + p_1 p_2 p_5 \\ = - + \infty + 35 \times 35 \times 20 \\ = 24500 + \infty$$

9) if  $24500 < \infty$

10) No updates

$\boxed{K=2} \rightarrow$

$$8) q = m[2,2] + m[3,5] + p_1 p_2 p_5 \\ = 0 + 2500 + 35 \times 15 \times 20 \\ = 13000$$

9) if  $13000 < \infty$

$$10) \text{ then } m[2,5] = 13000 \\ S[2,5] = 2$$

$\boxed{K=3} \rightarrow$

$$8) q = m[2,3] + m[4,5] + p_1 p_3 p_5 \\ = 2625 + 1000 + 35 \times 5 \times 20 \\ = 7125$$

9) if  $7125 < 10500$

$$10) \text{ then } \boxed{m[2,5] = 7125} \\ \boxed{S[2,5] = 3}$$

K=4 ↴

$$\begin{aligned} 8) q &= m[2,4] + m[5,6] + P_1 P_4 P_5 \\ &= 4375 + \infty + 35 \times 10 \times 20 \\ &= 11375 \end{aligned}$$

9) if  $11375 < 7125$   
No updates

$$\Rightarrow l = 4, i = 3$$

$$5) j = 3+4-1$$

$$j = 6$$

$$\begin{array}{l} 6) m[3,6] = \infty \\ 7) \text{for } k = 1 \text{ to } 5 \end{array}$$

K=1 ↴

$$\begin{aligned} 8) q &= m[3,1] + m[2,6] + P_2 P_1 P_6 \\ &= - + \infty + 15 \times 35 \times 25 \\ &= 13125 + \infty \end{aligned}$$

9) if  $13125 + \infty < \infty$   
No updates

K=2 ↴

$$\begin{aligned} 8) q &= m[3,2] + \underline{m[3,6]} + P_2 P_2 P_6 \\ &= - + \infty + 15 \times 15 \times 25 \\ &= 5625 + \infty \end{aligned}$$

9) if  $5625 + \infty < \infty$   
10) No updates

K=3 ↴

$$\begin{aligned} 8) q &= m[3,3] + m[4,6] + P_2 P_3 P_6 \\ &= 0 + 3500 + 15 \times 5 \times 25 \\ &= 5375 \end{aligned}$$

9) if  $5375 < \infty$ 

$$\begin{array}{l} 10) \text{then } m[3,6] = 5375 \\ S[3,6] = 3 \end{array}$$

K=4 ↴

$$\begin{aligned} 8) q &= m[3,4] + m[5,6] + P_2 P_4 P_6 \\ &= 750 + 5000 + 15 \times 10 \times 25 \\ &= 9500 \end{aligned}$$

9) if  $9500 < 5375$   
No updates

K=5 ↴

$$\begin{aligned} 8) q &= m[3,5] + m[6,6] + P_2 P_5 P_6 \\ &= 2500 + 0 + 15 \times 20 \times 25 \\ &= 10000 \end{aligned}$$

9) if  $10000 < 5375$   
No updates

Now,

$$3) l = 5 - 6$$

$$\begin{array}{l} 4) \text{for } i = 1 \text{ to } n-l+1 \\ \quad i = 1 \text{ to } 6-5+1 \\ \quad \quad i = 1 \text{ to } 2 \end{array}$$



i.e.

$$\Rightarrow \boxed{l = 5, i = 1}$$

5)  $j = 1+5-1$

$$\boxed{j = 5}$$

6)  $m[1,5] = \infty$

7) for  $k = 1$  to 4

$$\boxed{k=1} \rightarrow$$

8)  $q = m[1,1] + m[2,5] + P_0 P_1 P_5$   
 $= 0 + 7125 + 30 \times 35 \times 20$   
 $= 28125$

9) if  $28125 < \infty$

10) then  $m[1,5] = 28125$   
 $S[1,5] = 1$

$$\boxed{k=2} \rightarrow$$

8)  $q = m[1,2] + m[3,5] + P_0 P_2 P_5$   
 $= 15750 + 2500 + 30 \times 15 \times 20$   
 $= 27250$

9) if  $27250 < 28125$

10) then  $m[1,5] = 27250$   
 $S[1,5] = 2$

$$\boxed{k=3} \rightarrow$$

8)  $q = m[1,3] + m[4,5] + P_0 P_3 P_5$   
 $= 7875 + 1000 + 30 \times 5 \times 20$   
 $= 11875$

9) if  $11875 < 27250$

10) then  $\boxed{m[1,5] = 11875}$   
 $S[1,5] = 3$

$$\boxed{k=4} \rightarrow$$

8)  $q = m[1,4] + m[5,5] + P_0 P_4 P_5$   
 $= 9375 + 0 + 30 \times 10 \times 20$   
 $= 15375$

9) if  $15375 < 11875$   
 No update

$$\Rightarrow \boxed{l=5, i=2}$$

5)  $j = 2+5-1$   
 $\boxed{j = 6}$

6)  $m[2,5] = \infty$   
 7) for  $k = 1$  to 5

$$\boxed{k=1} \rightarrow$$

8)  $q = m[2,1] + m[2,6] + P_1 P_2 P_6$   
 $= - + \infty + 35 \times 35 \times 25$   
 $= 30625 + \infty$

9) if  $30625 < \infty$   
 No update

$$\boxed{k=2} \rightarrow$$

8)  $q = m[2,2] + m[3,6] + P_1 P_2 P_6$   
 $= 0 + 5375 + 35 \times 15 \times 25$   
 $= 18500$

9) if  $18500 < \infty$

10) then  $m[2,6] = 18500$   
 $S[2,6] = 2$

$K=3 \rightarrow$ 

$$8) q = m[2,3] + m[4,6] + P_1 P_3 P_6 \\ = 2625 + 3500 + 35 \times 5 \times 25 \\ = 10500$$

9) if  $10500 < 18500$ 

10) then  $m[2,6] = 10500$   
 $S[2,6] = 3$

 $K=4 \rightarrow$ 

$$8) q = m[3,4] + m[5,6] + P_1 P_4 P_6 \\ = 4375 + 5000 + 35 \times 10 \times 25 \\ = 18125$$

9) if  $18125 < 10500$   
No update $K=5 \rightarrow$ 

$$8) q = m[2,5] + m[6,6] + P_1 P_5 P_6 \\ = 7125 + 0 + 35 \times 10 \times 25 \\ = 15875$$

9) if  $15875 < 10500$   
No update

Now,

$$l = 6 - 6$$

- 3) for  $i=1$  to  $n-l+1$   
 $i=1$  to  $6-6+1$   
 $i=1 - 1$

i.e.  $\rightarrow l = 6, i = 1$

5)  $j = 1+6-1$   
 $j = 6$

6)  $m[1,6] = \infty$

7) for  $k=1$  to 5 $K=1 \rightarrow$ 

$$8) q = m[1,1] + m[2,6] + P_0 P_1 P_6 \\ = 0 + 10500 + 30 \times 35 \times 25 \\ = 36750$$

9) if  $36750 < \infty$ 

10) then  $m[1,6] = 36750$   
 $S[1,6] = 1$

 $K=2 \rightarrow$ 

$$8) q = m[1,2] + m[3,6] + P_0 P_2 P_6 \\ = 15750 + 5375 + 30 \times 15 \times 25 \\ = 32375$$

9) if  $32375 < 36750$ 

10) then  $m[1,6] = 32375$   
 $S[1,6] = 2$

 $K=3 \rightarrow$ 

$$8) q = m[1,3] + m[4,6] + P_0 P_3 P_6 \\ = 7875 + 3500 + 30 \times 5 \times 25 \\ = 15125$$

9) if  $15125 < 32375$ 

10) then  $m[1,6] = 15125$   
 $S[1,6] = 3$

$K=4$ 

$$8) q = m[1,4] + m[5,6] + P_0 P_4 P_6 \\ = 9375 + 5000 + 30 \times 10 \times 25 \\ = 21875$$

9) if  $21875 < 18125$ 

No update

 $K=5$ 

$$8) q = m[1,5] + m[6,6] + P_0 P_5 P_6 \\ = 11875 + 0 + 30 \times 20 \times 25 \\ = 26875$$

9) if  $26875 < 18125$ 

No update

1	0				
2	15750	0			
3	7875	2625	0		
4	9375	4375	750	0	
5	11875	7125	2500	1000	0
6	15125	10500	5375	3500	5000
	1	2	3	4	5
j					i →

⇒ matrix 'm'

1	0				
2	1	0			
3	1	2	0		
4	3	3	3	0	
5	3	3	3	4	0
6	3	3	3	5	5
	1	2	3	4	5
j					i →

⇒ matrix 's'

# SHORTCUT - METHOD

## Matrix-Chain Multiplication

$$A_1 = 5 \times 10$$

$$A_2 = 10 \times 20$$

$$A_3 = 20 \times 5$$

$$A_4 = 5 \times 100$$

$$\left( \begin{matrix} 5 \\ P_0 & 10 \\ P_1 & 20 \\ P_2 & 5 \\ P_3 & 100 \\ P_4 & \end{matrix} \right)$$

1	0			
2	1000	0		
3	12.50	1000	0	
4	3750	6000	10000	0
	i	2	3	4

$$m[i, j] = \begin{cases} 0 & \text{if } i=j \\ \min_{i \leq k < j} \{ m[i, k] + m[k+1, j] + p_{i-1} p_k p_j \} & \text{if } i \neq j \end{cases}$$

$$m[1, 2] = \begin{cases} m[1, 1] + m[2, 2] + p_0 p_1 p_2 \\ = 0 + 0 + 5 \times 10 \times 20 \end{cases}$$

$= 1000$

$\because i \leq k < j$   
 e.g.  $(1 \leq 0 < 2)$

1	0			
2	1	0		
3	1	2	0	
4	3	3	3	0
	1	2	3	4

$$m_{\substack{i \\ \downarrow \\ k=2}}^{[2,3]} = \begin{cases} m_{\substack{i \\ \downarrow \\ k=2}}^{[2,2]} + m_{\substack{i \\ \downarrow \\ k=2}}^{[3,3]} + P_1 P_2 P_3 \\ = 0 + 0 + 10 \times 20 \times 5 \\ = 1000 \end{cases} \checkmark$$

$$\Rightarrow m_{\substack{i \\ \downarrow \\ k=3}}^{[3,4]} = \begin{cases} m_{\substack{i \\ \downarrow \\ k=3}}^{[3,3]} + m_{\substack{i \\ \downarrow \\ k=3}}^{[4,4]} + P_2 P_3 P_4 \\ = 0 + 0 + 20 \times 5 \times 100 \\ = 10000 \end{cases} \checkmark$$

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$$\Rightarrow m_{\substack{i \\ \downarrow \\ k=1,2}}^{[1,3]} = \min \begin{cases} \boxed{k=1} \rightarrow \\ = m_{\substack{i \\ \downarrow \\ k=1}}^{[1,1]} + m_{\substack{i \\ \downarrow \\ k=1}}^{[2,3]} + P_0 P_1 P_3 \\ = 0 + 1000 + 5 \times 10 \times 5 \\ = 1250 \end{cases} \checkmark$$

$$\begin{cases} \boxed{k=2} \rightarrow \\ = m_{\substack{i \\ \downarrow \\ k=2}}^{[1,2]} + m_{\substack{i \\ \downarrow \\ k=2}}^{[3,3]} + P_0 P_2 P_3 \\ = 1000 + 0 + 5 \cancel{\times 10} \times 20 \times 5 \\ = 1500 \end{cases}$$

$$\Rightarrow m_{\substack{i \\ \downarrow \\ k=2,3}}^{[2,4]} = \min \begin{cases} \boxed{k=2} \rightarrow \\ = m_{\substack{i \\ \downarrow \\ k=2}}^{[2,2]} + m_{\substack{i \\ \downarrow \\ k=2}}^{[3,4]} + P_1 P_2 P_4 \\ = 0 + 10000 + 10 \times 20 \times 100 \\ = 30000 \end{cases}$$

$$\begin{cases} \boxed{k=3} \rightarrow \\ = m_{\substack{i \\ \downarrow \\ k=3}}^{[2,3]} + m_{\substack{i \\ \downarrow \\ k=3}}^{[4,4]} + P_1 P_3 P_4 \\ = 1000 + 0 + 10 \times 5 \times 100 \\ = 6000 \end{cases} \checkmark$$

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$$\Rightarrow m_{i,j} = \min_{k=1,2,3} \left[ \begin{array}{l} k=1 \\ \rightarrow m_{1,1} + m_{2,4} + P_0 P_1 P_4 \\ = 0 + 6000 + 5 \times 10 \times 100 \\ = 11000 \end{array} \right]$$

$$\left[ \begin{array}{l} k=2 \\ \rightarrow m_{1,2} + m_{3,4} + P_0 P_2 P_4 \\ = 1000 + 10000 + 5 \times 20 \times 100 \\ = 21000 \end{array} \right]$$

$$\left[ \begin{array}{l} k=3 \\ \rightarrow m_{1,3} + m_{4,4} + P_0 P_3 P_4 \\ = 1250 + 0 + 5 \times 5 \times 100 \\ = 3750 \quad \checkmark \end{array} \right]$$