Graph Definition

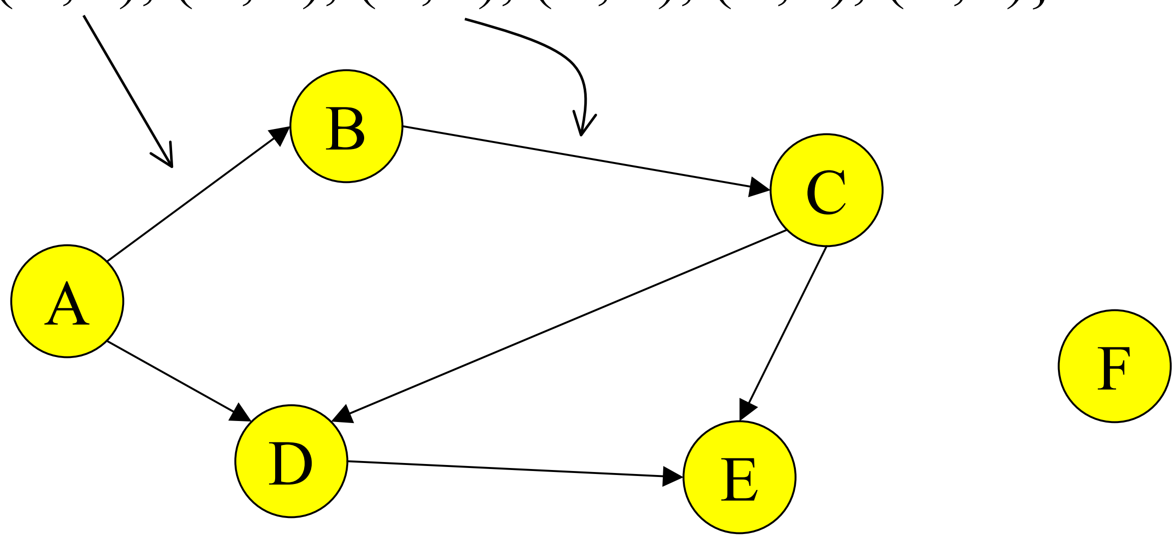
- A graph is simply a collection of nodes plus edges
  - Linked lists, trees, and heaps are all special cases of graphs
- The nodes are known as vertices (node = “vertex”)
- Formal Definition: A graph G is a pair (V, E) where
  - V is a set of vertices or nodes
  - E is a set of edges that connect vertices

Graph Example

- Here is a graph $G = (V, E)$
  - Each edge is a pair (v_1, v_2), where v_1, v_2 are vertices in V

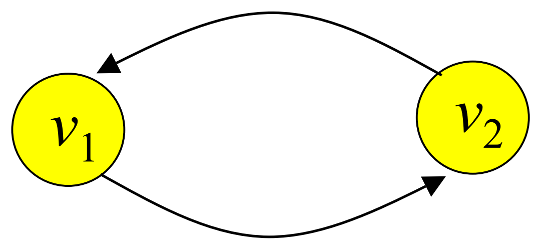
$V = \{A, B, C, D, E, F\}$

$E = \{(A,B), (A,D), (B,C), (C,D), (C,E), (D,E)\}$

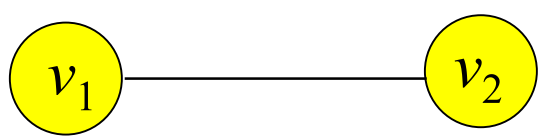


Directed vs Undirected Graphs

- If the order of edge pairs (v_1, v_2) matters, the graph is directed (also called a digraph): $(v_1, v_2) \neq (v_2, v_1)$



- If the order of edge pairs (v_1, v_2) does not matter, the graph is called an undirected graph: in this case, $(v_1, v_2) = (v_2, v_1)$



Undirected Terminology

- Two vertices *u* and *v* are *adjacent* in an undirected graph G if {*u*,*v*} is an edge in G
  - edge e = {*u*,*v*} is *incident with* vertex *u* and vertex *v*
- The degree of a vertex in an undirected graph is the number of edges incident with it
  - a loop counts twice (both ends count)
  - denoted with deg(*v*)

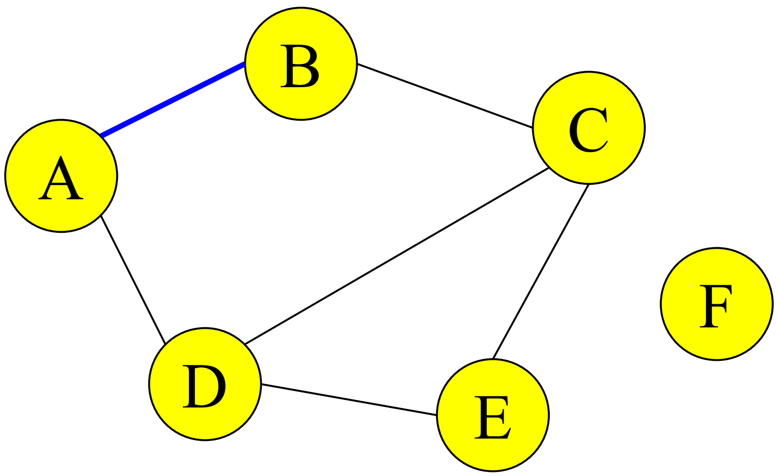
Directed Terminology

- Vertex *u* is *adjacent to* vertex *v* in a directed graph G if (*u,v*) is an edge in G
  - vertex *u* is the initial vertex of (*u,v*)
- Vertex *v* is *adjacent from* vertex *u*
  - vertex *v* is the terminal (or end) vertex of (*u,v*)
- Degree
  - *in-degree* is the number of edges with the vertex as the terminal vertex
  - *out-degree* is the number of edges with the vertex as the initial vertex
  - a loop adds 1 to in-degree and 1 to out-degree

Graph Representations

- Space and time are analyzed in terms of:
  - Number of vertices = $|V|$ and
  - Number of edges = $|E|$
- There are two ways of representing graphs:
  - The *adjacency matrix* representation
  - The *adjacency list* representation

Adjacency Matrix

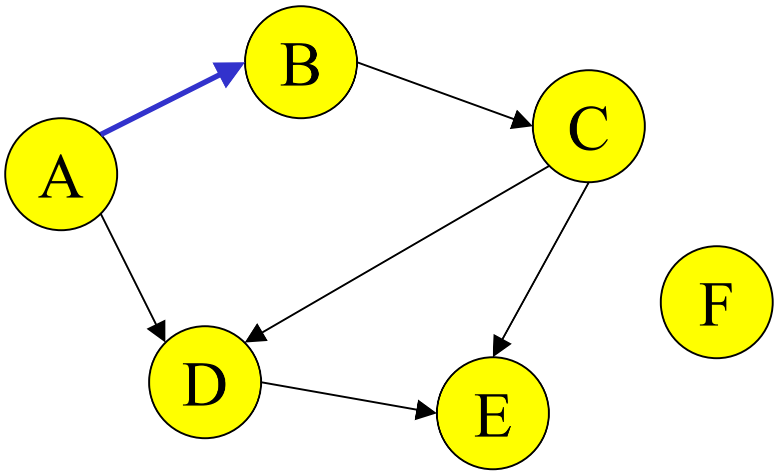


$$
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E \\ F
\end{array}
\begin{array}{c}
\begin{array}{cccccc} A & B & C & D & E & F \end{array} \\
\begin{pmatrix}
0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

$$M(v, w) = \begin{cases} 1 \text{ if } (v, w) \text{ is in E} \\ 0 \text{ otherwise} \end{cases}$$

Space = $|V|^2$

Adjacency Matrix for a Digraph



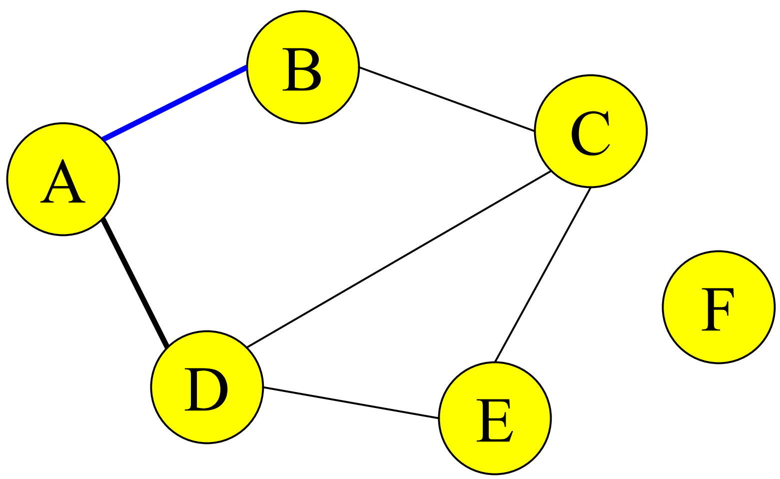
$$M(v, w) = \begin{cases} 1 \text{ if } (v, w) \text{ is in E} \\ 0 \text{ otherwise} \end{cases}$$

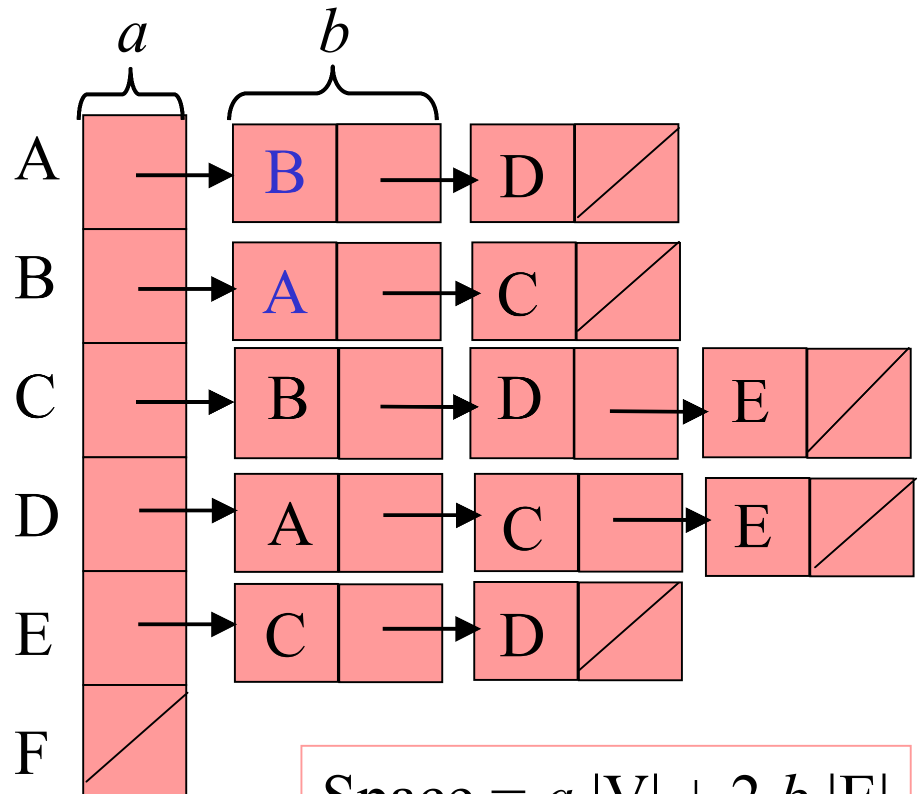
$$\begin{array}{c c} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} & \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Space = $|V|^2$

Adjacency List

For each v in V, $L(v)$ = list of w such that (v, w) is in E

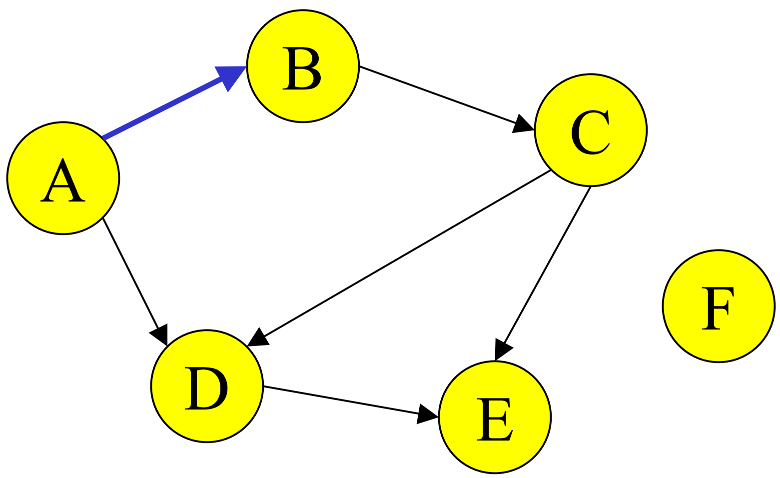


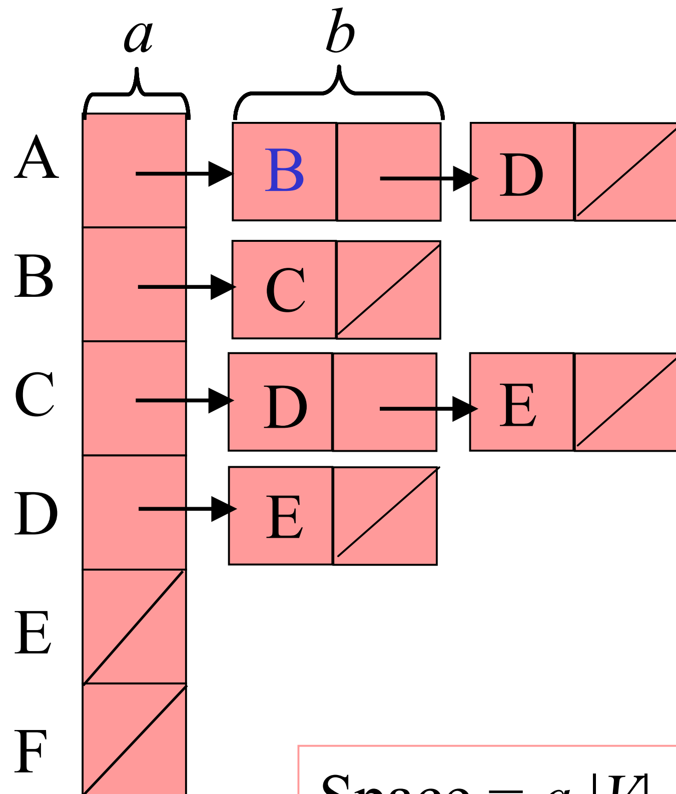


Space = $a\,|V| + 2\,b\,|E|$

Adjacency List for a Digraph

For each v in V, $L(v)$ = list of w such that (v, w) is in E





Space = $a\,|V| + b\,|E|$