UNIT-2
Wave Optics

Principle of Superposition: When two or more than two waves travel in a medium then the resultant displacement at any point in the medium is the algebraic sum of the displacements produced by individual wave. Let n number of waves travelling in a medium with the individual displacements y_1, y_2, y_3……… y_n then according to the principle of superposition the resultant displacement at any point in the medium will be given by:

$Y = y_1 + y_2 + y_3 \ldots\ldots\ldots\ldots + y_n$

Interference: When two or more than two waves travel in a medium then the modification in the intensity distribution in the region of superposition is called as interference. At the points where waves superpose with the same phase the intensity is maximum; and is called as constructive interference. While the points where waves superpose with the opposite phase the intensity is minimum; and is called as destructive interference.

Types of Interference: Interference can be classified on the basis of the way interference is produced. The interference is classified as:

Interference by Division of wavefront: When the incident wavefront is divided into two parts by the phenomenon of reflection refraction. When these two divided parts reunite then the interference obtained is called interference by division of wavefront. The examples are Fresnel bi prism, Lloyd's mirror.

Interference by Division of amplitude: When the incident amplitude is divided into two parts by the phenomenon of reflection refraction. When these two divided amplitudes reunite then the interference obtained is called interference by division of amplitude. The examples are interference in thin films, Newton's ring, and Michelson interferometer.

Analytical treatment of interference: Let us consider two plane waves in a medium in the same direction. The displacement of individual waves is given by y_1& y_2, while their amplitudes are a_1& a_2, the angular frequency of these waves is 'ω'. Mathematically the waves can be represented as

$y_1 = a_1 \sin \omega t$ 1

$y_2 = a_2 \sin(\omega t + \varphi)$ 2

Where φ is the phase difference between two waves.

According to the principle of superposition the resultant displacement at any point will be

$Y = y_1 + y_2$ 3

Therefore $Y = a_1 \sin \omega t + a_2 \sin(\omega t + \varphi)$ 4

Or $Y = a_1 \sin \omega t + a_2 \sin \omega t\, cos\varphi + a_2 \cos \omega t\, sin\varphi$ 5

$Y = (a_1 + a_2 cos\varphi) \sin \omega t + a_2 sin\varphi \cos \omega t$ 6

Let $(a_1 + a_2 cos\varphi) = Acos\theta$ 7

And $a_2 sin\varphi = Asin\theta$ 8

The equation 6 can be rewritten as-$Y = Acos\theta \sin \omega t + Asin\theta \cos \omega t$

Or $Y = A \sin(\omega t + \theta)$ 9

The equation 9 represents a simple harmonic plane wave with the amplitude . The amplitude can be calculated by squaring and adding the equations 6 & 7

$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 cos\varphi$ 10

From the equation above it is evident that the resultant amplitude depends on the phase difference between two waves. Let us consider the extremes that is the condition of constructive and destructive interference

Condition for Maxima or constructive interference: If the$\varphi = 2n\pi$ where $n = 0,1,2,3 \ldots \ldots$ then $A^2 = (a_1 + a_2)^2$ if $a_1 = a_2 = a$ then $A^2 = 4a^2$

Condition for Minima or destructive interference: If the$\varphi = (2n + 1)\pi$ where $n = 0,1,2,3 \ldots \ldots$ then $A^2 = (a_1 - a_2)^2$ if $a_1 = a_2 = a$ then $A^2 = 0$

Relation between phase and path difference:

$$Phase\ difference = \frac{2\pi}{\lambda}\ Pathdifference$$

Construction of Fresnel bi prism & formation of fringes: Fresnel Bi prism is a device to obtain two coherent sources to produced sustained interference. Fresnel Bi prism is constructed by polishing a glass plate such that one of its angles is about 179^0 while two acute angles are $30'$ each. The construction of Fresnel Bi Prism is shown in Figure 'a'.

Formation of fringes: Light from a narrow adjustable slit S is allowed to fall on symmetrically on the bi prism. When light from the source S falls on the lower portion (AC) of bi prism, after refraction it appears to come from the virtual image S_2. Similarly light falling on upper portion (AB), after refraction, appears to come from virtual image S1. Hence S1 & S2 acts as two coherent sources, when the light from these two sources superimposes on each other then the interference pattern is obtained on the screen. The interference pattern can be observed using an eye piece.

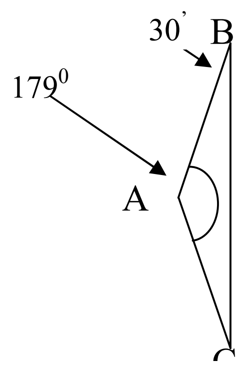


Figure a: Fresnel Bi prism

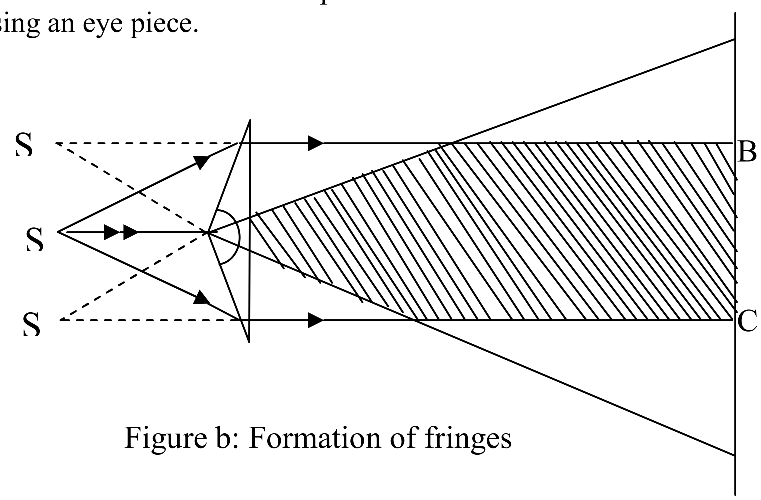


Figure b: Formation of fringes

Determination of fringe width in case of Interference by Fresnel Bi prism: Let us consider a monochromatic source of light. The light from the source is incident on Fresnel bi prism and is divided in to two coherent sources S1 & S2. The light from these sources interferes and an interference pattern is obtained on the screen as shown.

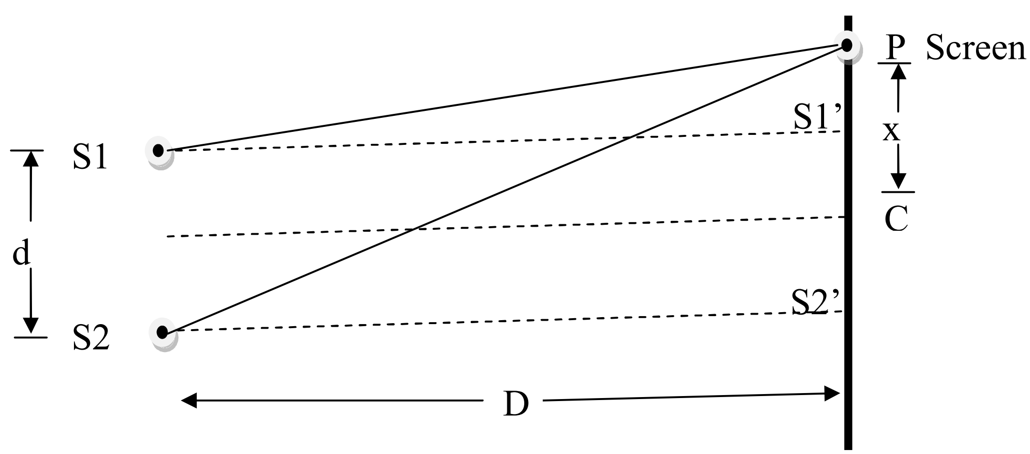


Figure 1: Interference by two virtual sources

To find out the fringe width we proceed with the determination of condition of constructive and destructive interference. Let us consider a point 'P' at the screen. The path taken by the wave from source S1 to the point P is S1P, while the path for wave from S2 will be S2P. The path difference at point P between two waves will be- S2P – S1P. Let the separation between two virtual sources be d, while the distance between the sources to screen is D.

The distance of the point P form the centre of screen is x.

In ΔS1S1'P according to Pythagoras theorem

$S1P^2 = S1S1'^2 + S1'P^2$ 1

$S2P^2 = S2S2'^2 + S2'P^2$ 2

Since as shown in figure 1

$$S1S1' = S2S2' = D$$

$S1P = x - \frac{d}{2}$ and $S2P = x + \frac{d}{2}$ Hence equations 1 and 2 can be rewritten as- $S1P^2 = D^2 + (x - \frac{d}{2})^2$ and $S2P^2 = D^2 + (x + \frac{d}{2})^2$

Therefore $S2P^2 - S1P^2 = 2xd \quad and \quad S2P - S1P = \frac{2xd}{2D}$

The path difference between two waves reaching at point P is-

$$S2P - S1P = \frac{xd}{D}$$

Condition of constructive Interference or Maxima- For constructive interference the path difference between two waves must be equal to integer multiple of wavelength 'λ' i.e. $\frac{xd}{D} = n\lambda$ or $x = \frac{n\lambda D}{d}$ for n^{th} bright fringe we can say $x_n = \frac{n\lambda D}{d}$ where x_n stands for the position of n^{th} bright fringe.

Condition of destructive Interference or Minima- For destructive interference the path difference between two waves must be equal to odd multiple of half of wavelength (λ/2) i.e. $\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$ or $x = \frac{(2n+1)\lambda D}{2d}$ for n^{th} bright fringe we can say $x_n = \frac{(2n+1)\lambda D}{2d}$ where x_n stands for the position of n^{th} dark fringe.

Fringe width: The distance between two consecutive dark and bright fringe is denoted by $\bar{X}$

The position of n^{th} bright fringe is given as: $x_n = \frac{n\lambda D}{d}$ while the position of $(n+1)^{th}$ bright will be given as $x_{n+1} = \frac{(n+1)\lambda D}{d}$ the distance between two will be $x_{n+1} - x_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d}$ we can say that the distance between two bright.

Determination of wavelength by Fresnel bi prism: The Fresnel bi prism, slit, sodium lamp, a convex lens, and eye-piece are arranged on the optical bench as shown in the figure. Following Procedure is adopted for obtaining the fringes with good contrast.

- The optical bench should be made parallel using spirit level.
- Widen the slit and set the cross wire of eye-piece vertical.
- The slit and edge of the bi prism is made parallel by removing 'lateral shift'.

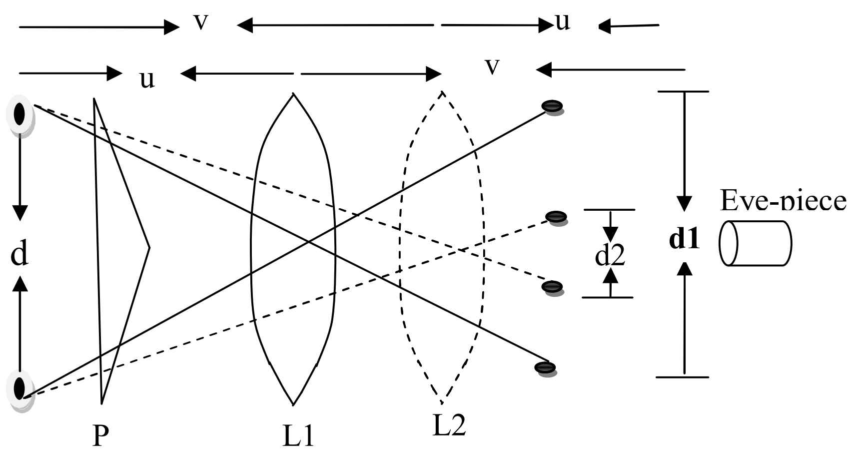
Following measurements are now made:

1. **Measurement of fringe width:** Adjust the vertical cross wire of eye-piece on a bright fringe. Take reading. Then eye-piece is moved laterally so that vertical crosswire coincides with the successive bright fringe and corresponding readings are noted. The difference of these readings gives $\bar{X}$**.**
2. **Measurement of D:** Take the reading of position of the slit and eyepiece. The difference between these readings gives **D**.
3. **Measurement of (d) the distance between two virtual sources in Bi prism:** The distance between two virtual sources can be determined by any of the two methods.

A. Displacement method B. Deviation method

A. **Displacement method:** In this method the distance between the slit and eye-piece is kept more than four times the focal length of convex lens used, so that the two positions of lens can be arranged for obtaining the images of sources S1 & S2.

As shown in the figure a convex is mounted between the bi prism P and the eye-piece. By moving the lens along the length of the bench, two positions L_1 and $L_2$are obtained such that the real images of two virtual sources S1 & S2 are obtained in the eye-piece.



For L1 position: $\frac{I}{O} = \frac{d_1}{d} = \frac{v}{u}$ 1

For L2 position: $\frac{I}{O} = \frac{d_2}{d} = \frac{u}{v}$ 2

Multiplying equations 1 and 2 we get:

$\frac{d_1 d_2}{d^2} = 1$Or $d = \sqrt{d_1 d_2}$

Thus measuring d_1 and d_2 , d can be calculated.

For accurate measurement of d the procedure of determining d_1 and d_2 is repeated at least three times by moving eye-piece in different positions.

Newton's Ring

Experimental arrangement & Formation of Newton's Rings: The experimental arrangement is shown in the figure below. A large Plano-convex lens is placed over a glass plate with its convex surface in contact with the glass plate. Then an air film of gradually increasing thickness is formed between the glass plate and Plano- convex lens. The light from a monochromatic source is allowed to fall normally over this assembly using a plane glass plate inclined at angle of 45^0.

The light rays reflected from the bottom of Plano convex lens and upper surface of glass plate superimpose on each other to produce interference pattern. The interference pattern is in the form of concentric bright and dark circles. These concentric bright and dark circles are called as Newton's rings. The fringes obtained are circular because the thickness of air film is constant in a circle around the point of contact. The obtained interference pattern is observed using a travelling microscope. The fringes obtained are called the fringes of equal thickness.

Theory: The path difference between two rays in case of Newton's rings will be $2\mu t \cos r + \frac{\lambda}{2}$ .The additional path difference of $\frac{\lambda}{2}$ is due to the fact that the ray reflected from the top surface of glass plate is reflected from the denser medium. Since the medium of film is air hence$\mu = 1$.

Also for normal incidence the value of $r = 0$ as a consequence the effective path difference is given by $2t + \frac{\lambda}{2}$

Condition of Maxima: For maxima the path difference must be an integral multiple of λ i.e. $2t + \frac{\lambda}{2} = n\lambda$or $2t = (2n - 1)\frac{\lambda}{2}$. Since at the point of contact the thickness of film is zero and the effective path difference is $\frac{\lambda}{2}$. Therefore the centre of Newton's rings is dark.

Condition of Minima: For minima the path difference must be an odd multiple of $\frac{\lambda}{2}$ i.e. $2t + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$ or $2t = n\lambda$

Newton's Rings in Transmitted light: The effective path difference between to interfering rays in the transmitted part of Newton's rings is$2t$. Therefore the condition of maxima is $2t = n\lambda$ and the condition of minima is $2t = (2n + 1)\frac{\lambda}{2}$ hence we can say that the interference pattern in the reflected and transmitted system is complimentary to each other. The centre in the transmitted system will be bright because the effective path difference will be zero at the point of contact.

Diameters of dark Newton's rings are proportional to the square root of natural numbers: Refer to the figure below let ADB be the lens placed on the glass plate MN, the point of contact being O. Let R be the radius of curvature of the curved surface of the lens. Let r_n be the radius of Newton's ring where the film thickness is *t*.

In triangle OPQ, $OP = R - t$ and $OQ = R$ and $PQ = R$ using Pythagoras theorem for triangle OPQ$OQ^2 = OP^2 + PQ^2$ Or $R^2 = (R - t)^2 + r_n{}^2$ or $R^2 = R^2 + t^2 - 2Rt + r_n{}^2$

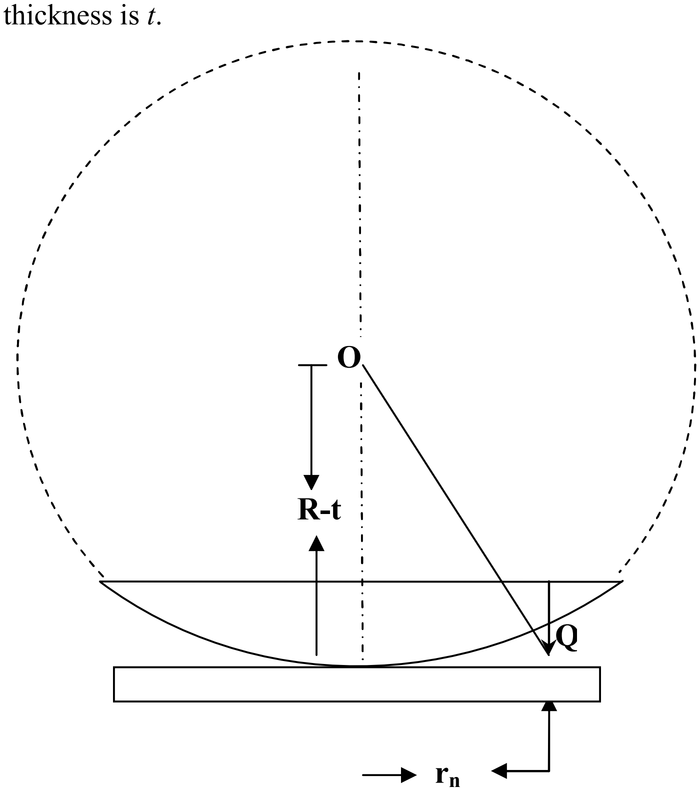
In the above equation t^2 can be neglected because t is very small.

Therefore $r_n{}^2 = 2Rt$ **t**

For dark rings in case of Newton's rings $2t = n\lambda$ where n=0, 1, 2…

Hence $r_n{}^2 = n\lambda R$ or $D_n{}^2 = 4n\lambda R$ where $D_n{}^2$ is diameter of nth dark ring. Or $D_n = 2\sqrt{n\lambda R}$ the diameter of dark rings is proportional to the square root f natural numbers.

Diameters of bright Newton's rings are proportional to the square root of odd natural numbers: Refer to the figure below let ADB be the lens placed on the glass plate MN, the point of contact being O. Let R be the radius of curvature of the curved surface of the lens. Let r_n be the radius of Newton's ring where the film thickness is *t*.



In triangle OPQ, $OP = R - t$ and $OQ = R$ and $PQ = R$using Pythagoras theorem for triangle OPQ

$$OQ^2 = OP^2 + PQ^2$$

Or
$$R^2 = (R - t)^2 + r_n{}^2$$
$$R^2 = R^2 + t^2 - 2Rt + r_n{}^2$$

In the above equation t^2 can be neglected because t is very small.

Therefore $r_n{}^2 = 2Rt$. For bright rings in case of Newton's rings $2t = (2n + 1)\frac{\lambda}{2}$ where n=0, 1, 2…

Hence $r_n{}^2 = (2n + 1)\frac{\lambda}{2}R$ or $D_n{}^2 = 4(2n + 1)\frac{\lambda}{2}R$ where $D_n{}^2$ is diameter of nth bright ring.

Or $D_n = 2\sqrt{(2n + 1)\frac{\lambda}{2}R}$ the diameter of dark rings is proportional to the square root of odd natural numbers.

Determination of wavelength by Newton's Ring Method:

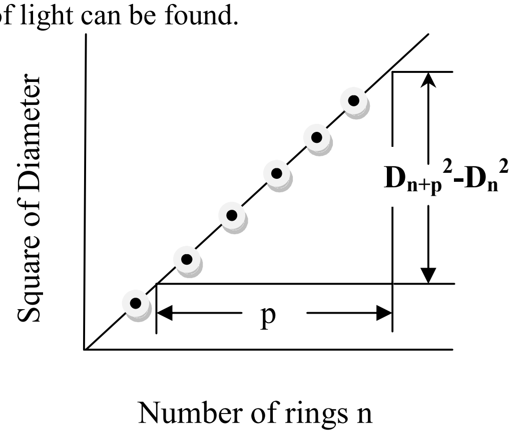
Experimental arrangement: The experimental arrangement is shown in the figure below. A large Plano-convex lens is placed over a glass plate with its convex surface in contact with the glass plate. Then an air film of gradually increasing thickness is formed between the glass plate and Plano- convex lens. The light from a monochromatic source is allowed to fall normally over this assembly using a plane glass plate inclined at angle of 45^0.

The light rays reflected from the bottom of Plano convex lens and upper surface of glass plate superimpose on each other to produce interference pattern. The interference pattern is in the form of concentric bright and dark circles. These concentric bright and dark circles are called as Newton's rings. The fringes obtained are circular because the thickness of air film is constant in a circle around the point of contact. The obtained interference pattern is observed using a travelling microscope. The fringes obtained are called the fringes of equal thickness.

Theory & Formula: As the square of diameter of nth dark ring is $D_n{}^2 = 4n\lambda R$ and that of (n+p)th dark ring will be $D_{n+p}{}^2 = 4(n + p)\lambda R$ where p is any integer. Therefore the formula of wavelength will be $\lambda = \frac{D_{n+p}{}^2 - D_n{}^2}{4pR}$.

Procedure for determination of diameter of dark rings:

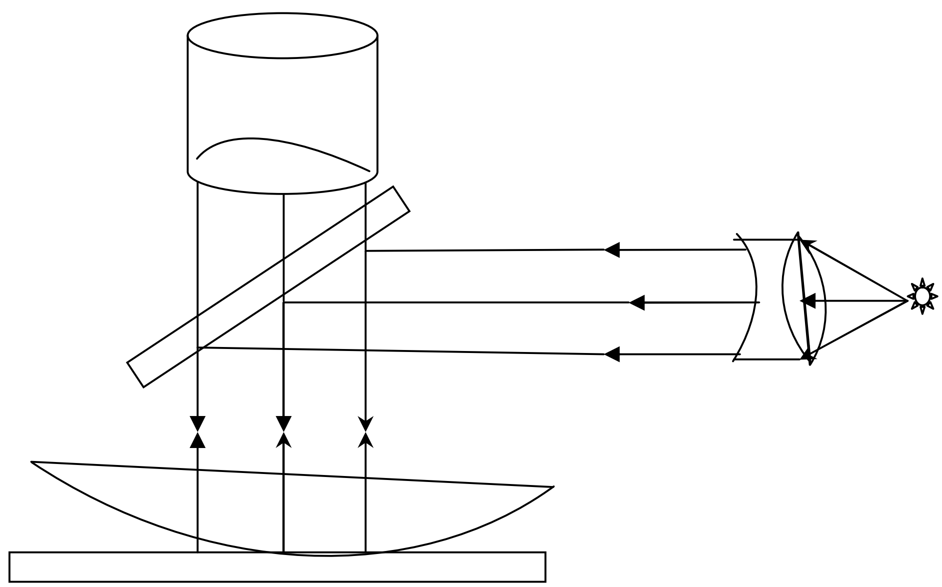
- The cross wire of microscope is focused, and then the crosswire is moved to extreme left to the circular interference pattern.
- Crosswire is tangentially adjusted to any dark ring (say 15th) at its outer edge and the reading of microscope is noted.
- Crosswire is moved towards right and the reading is recorded, when crosswire becomes tangential to the outer edge of every alternate dark ring. The procedure is continued till the center of interference pattern.
- On reaching the center crosswire is moved towards the right and the reading of alternate dark rings are noted till 15th dark ring. The subtraction of the reading of the left and right provides the diameter of the ring.
- The graph is plotted between D^2 and the number of ring n. The graph is a straight line with the slope $\frac{D_{n+p}{}^2 - D_n{}^2}{p}$. The radius of curvature of lens is measured and using the formula $\lambda = \frac{D_{n+p}{}^2 - D_n{}^2}{4p}$ the wavelength of light can be found.



Determination of refractive index of liquid by Newton's Ring Method:

Experimental arrangement: The experimental arrangement is shown in the figure below. A large Plano-convex lens is placed over a glass plate with its convex surface in contact with the glass plate. Then an air film of gradually increasing thickness is formed between the glass plate and Plano- convex lens. The light from a monochromatic source is allowed to fall normally over this assembly using a plane glass plate inclined at angle of 45^0.The light rays reflected from the bottom of Plano convex lens and upper surface of glass plate superimpose on each other to produce interference pattern. The interference pattern is in the form of concentric bright and dark circles. These concentric bright and dark circles are called as Newton's rings. The fringes obtained are circular because the thickness of air film is constant in a circle around the point of contact. The obtained interference pattern is observed using a travelling microscope. The fringes obtained are called the fringes of equal thickness.

Theory & Formula: As the square of diameter of n^{th} dark ring is $D_n{}^2 = 4n\lambda R$ when the film is of air, if a liquid of refractive index μ is inserted between the Plano convex lens and glass plate then the square of diameter of n^{th} dark ring is $D_{n\prime}{}^2 = \frac{4n\lambda R}{\mu}$. We can see that the diameter of rings decreases when a liquid is inserted between the Plano convex lens and glass plate. By taking the ratio of two diameters we obtain $\mu = \frac{D_n{}^2}{D_{n\prime}{}^2}$



Experimental procedure:

- Newton's ring experiment is arranged to obtain the fringes.
- Initially when the film between Plano convex lens and glass plate is of air.The diameter of any dark ring (Say n^{th}) is measured using the travelling microscope. The measured diameter is D_n
- Now a drop of the liquid is placed between the glass plate and Plano convex lens and again the diameter of the n^{th} ring is measured. The measured diameter is $D_{n\prime}$.
- Using the formula$\mu = \frac{D_n{}^2}{D_{n\prime}{}^2}$. Refractive index of liquid is obtained.

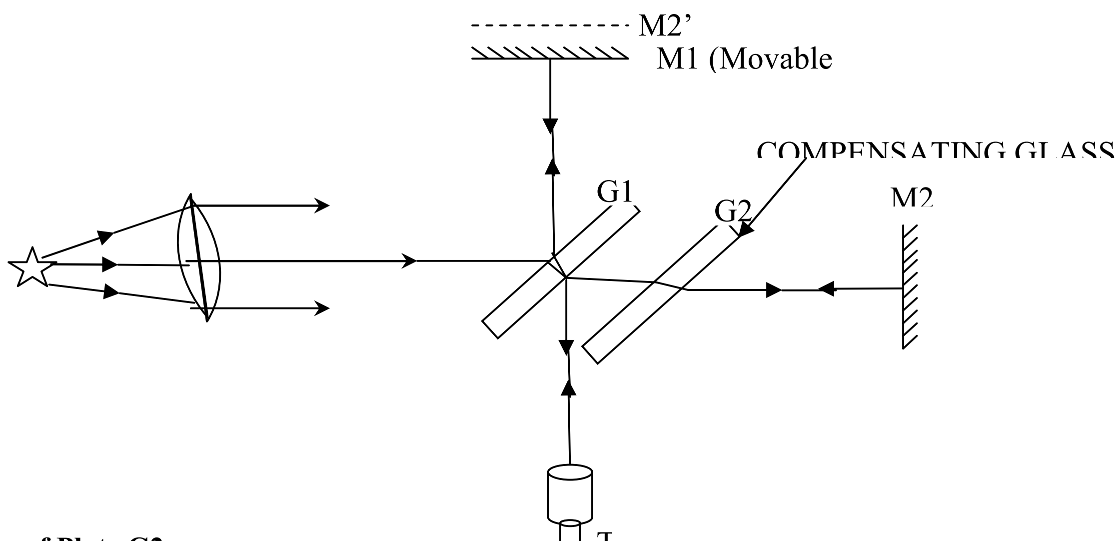
Michelson Interferometer: In Michelson's interferometer light from an extended source is divided into two coherent beams by partial reflection and partial refraction at semi-silvered glass plate. These beams are made to reunite at the same glass plate after reflection from two perpendicular placed mirrors.

Construction:

- The Michelson interferometer consists of two plane mirror M1 and M2 which are held perpendicular to each other as shown in figure.
- M2 is fixed while M1 can be moved backward and forward using micrometer screw capable of a measurement up to 10^{-5}cms.
- The mirrors M1 and M2 are provided with spring and screw arrangement at their backs. With the help of these screws the tilt of mirror can be adjusted.
- G1 and G2 are two identical glass plates i.e. having same thickness and the same refractive index. These glass plates are inclined at 45^0 to horizontal.
- The rear surface of plate G1 is partially polished. T is the telescope which receives the reflected light from the mirrors M1 and M2.

Working:

- A source of monochromatic light S is placed at the focus of a convex lens L, to obtain parallel light beam.
- The light from source is incident upon semi-silvered glass plate. The part of incident intensity is reflected towards mirror M1 and part of incident intensity is refracted towards mirror M2.
- Since the beams are incident normally on mirror M1 and mirror M2, therefore after reflection from the mirrors the beams retrace the same path.
- The reflected beams superpose on each other at glass plate to produce interference pattern. The produced interference pattern is observed by telescope T.



Function of Plate G2:

- In absence of plate G2 ray 1 travels through plate G1 twice while ray 2 does not travel through glass (G1) at all.
- Hence in absence of plate G2 the path of rays 1 and are not equal in glass.
- To equalize these paths plate G2 of same thickness and material as that of G1 is introduced in the path of ray no.2. Because of this nature plate G2 is called compensating glass plate.

Theory: The path difference between two rays in case of Michelson interferometer depends upon the distances of mirror M1 and M2 from the glass plate G1. The path difference can be altered by moving the mirror M1 in axial direction. The generalized expression for the path difference between two interfering rays is $2\mu t\cos r+\frac{\lambda}{2}$

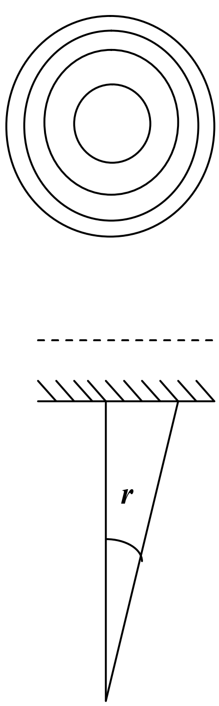
Condition of maxima: For maxima the path difference must be integral multiple of $\boldsymbol{\lambda}$i.e. $2\mu t\cos r+\frac{\lambda}{2}=n\lambda$

Condition of minima: For minima the path difference must be odd multiple of $\frac{\boldsymbol{\lambda}}{\mathbf{2}}$i.e. $2\mu t\cos r+\frac{\lambda}{2}=(2n+1)\frac{\lambda}{2}$

Types of fringes:

Circular fringes: The path difference between two rays in case of Michelson interferometer is given by $2\mu t\cos r+\frac{\lambda}{2}$. For a given wavelength λ and for air film $\mu=1$, when the two mirrors are exactly perpendicular to each other them *'t'* is also constant. In this situation path difference depends on the *'r'*. When both mirrors are exactly perpendicular to each other then the image of mirror M2 will be exactly parallel to the mirror M1and the value of *'r'* remains constant for a circular geometry; hence the circular fringes are obtained. The fringes are called the fringes of equal inclination.

Localized fringes: When the mirrors M1 and M2 are not exactly perpendicular to each other, then the air film between the image M2' and M1 is wedge shape. Since light is incident on the film at different angles, curved fringes with convexity towards the edge of wedge are seen as shown in figure 'a' and 'b'. When M1 and M2' intersect in the middle straight fringes are obtained as shown in figure 'c'.



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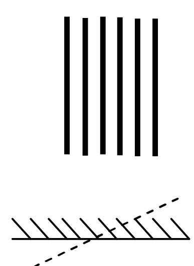

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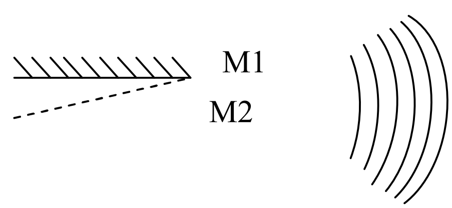

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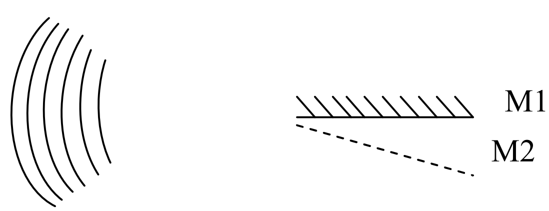

Applications of Michelson Interferometer:

Determination of wavelength:

- The Michelson interferometer is adjusted for circular fringes.
- Vertical crosswire of telescope is adjusted at the central fringe.
- The position $'x_1'$of mirror M1 is noted on the graduated scale.
- Looking through the eye-piece of the telescope, the mirror M1 is moved using micrometer screw, and the number of fringes crossing the field of view is counted.
- When 'N' number of fringes crosses the field of view then the position of $'x_2'$ of the Mirror M1 is noted.
- The displacement $'x'$ of the mirror M1 is obtained by subtracting $'x_1'$ from $'x_2'$ i.e. $x = x_1 - x_2$.
- By using the formula $\lambda = \frac{2x}{N}$ the wavelength of light is obtained.

Determination of thickness of a thin transparent film:

To measure the thickness of thin film Michelson interferometer is adjusted for localized white light fringes. This is accomplished using a white light source instead of monochromatic light and the tilt of mirrors is adjusted such that the straight fringes with central fringe dark appears in the field of view.

If the dark fringe is not at the centre of the vertical cross wire, then mirror M1 is moved to get the central dark fringe to coincide with the vertical cross-wire.

Let a film of thickness 't' and refractive index μ is inserted in the path of one of the beam. Then the central fringe will be displaced by an amount

$$S = (\mu - 1)t \qquad 1$$

Hence the mirror M1 will have to move the distance $'S'$ to get back the dark fringe at the centre.Now the white light source is replaced by monochromatic light and the mirror M1 is moved the distance $'S'$ and the number of fringes (N) crossing the field of view is counted. Then $\lambda = \frac{2S}{N}$

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Using equations 1 and 2 the thickness of the film will be$t = \frac{N\lambda}{2(\mu-1)}$

By knowing the refractive index the thickness of the film can be measured.

Determination of difference of two wavelengths or resolution of spectral lines: If a source of light consisting of two wavelengths λ_1 *and* λ_2 which differ slightly is used for production of fringes in Michelson interferometer. Then two sets of fringes are produced. When the bright fringes due one wavelength coincide with the bright fringe due to the other wavelength; then the distinctive interference pattern is produced and is called the position of maximum distinctness. If we start to move the mirror M1 then the bright and dark fringes due to two wavelengths gets displaced by different amount and a situation arises when the bright fringe due to one wavelength coincides with the dark fringe due to the other wavelength. In this position complete field of view becomes bright and this situation is called as the position of maximum indistinctness. If $\lambda_1 > \lambda_2$ and N number of fringes are crossing the field of view corresponding to the wavelength λ_1, then for the same displacement N+1 fringes will cross the field of view for wavelength λ_2. Say the displacement of the mirror for the maximum distinctness to maximum indistinctness is S then

$\lambda_1 = \frac{2S}{N}$ *and* $\lambda_2 = \frac{2S}{(N+1)}$ Or $N = \frac{2S}{\lambda_1}$ *and* $(N+1) = \frac{2S}{\lambda_2}$Therefore $1 = \frac{2S}{\lambda_2} - \frac{2S}{\lambda_1}$ or $\lambda_2\lambda_1 = 2S(\lambda_1 - \lambda_2)$ the $\lambda_2\lambda_1 = \lambda_{av}^2$Then $\lambda_1 - \lambda_2 = \frac{\lambda_{av}^2}{2S}$. By measuring the movement of mirror M1 for distance corresponding to the form the maximum distinctness to maximum indistinctness the difference of two wavelengths can be found by using the above formula.

Diffraction: The phenomenon of bending of light at the edges of obstacles is known as diffraction. Diffraction is divided into two classes Fresnel diffraction and Fraunhoffer diffraction.

Difference between Fresnel and Fraunhoffer diffraction:

Fresnel diffraction	Fraunhoffer diffraction
1. In the Fresnel class diffraction the distance between the source and obstacle is finite 2. The wavefronts can be spherical or cylindrical. 3. No lenses are required to produce Fresnel diffraction. 4. The image obtained at the screen is image of obstacle.	1. In the Fraunhoffer class diffraction the distance between the source and obstacle is infinite The wavefronts here are plane wavefronts only. Lenses are required to produce diffraction pattern inside laboratory. 4. The image obtained at the screen is image of source.

Diffraction at single slit: Let us consider a narrow slit with its width ‘d’. The light from a monochromatic source ‘S’ falls on this slit through a convex lens. The incident wavefront is diffracted by and angle θ and is focused at screen at point P.

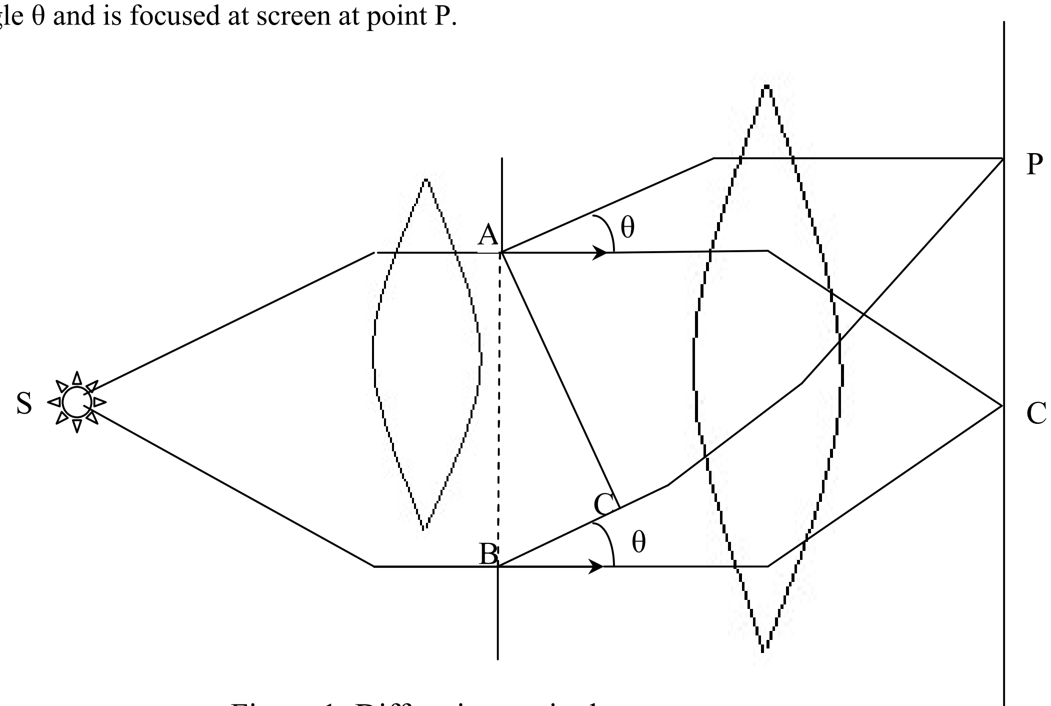

Figure 1: Diffraction at single

The path difference between two rays of parallel wavefront from the point A and point B will be equal to BC.

In triangle ABC $\sin\theta = \frac{BC}{AB}$ or $BC = AB\sin\theta$ since $AB = d$ therefore $BC = d\sin\theta$.

As $Phase\ difference\ = \frac{2\pi}{\lambda} *\ pathdifference$ therefore the phase difference between two rays from point A and B will be $\emptyset\ = \frac{2\pi}{\lambda} *\ d\sin\theta$ where $\emptyset = Phase\ difference$.

Since between A and B there will be large number of secondary sources generating the light wave. If we consider the ‘n’ number of sources between A to B, then the phase difference between two consecutive sources will be

$$\emptyset_n\ = \frac{2\pi}{\lambda} *\ d\sin\theta \qquad 1$$

To find out the resultant amplitude at point P by ‘n’ number of sources we use the vectorial addition by polygon method. As there are ‘n’ vectors and the phase difference between two consecutive vectors is $\emptyset_n$. We draw a polygon with its side length representing the amplitude ‘A’ of individual source. As the slit width is small therefore the amplitude of all sources can be considered as same.

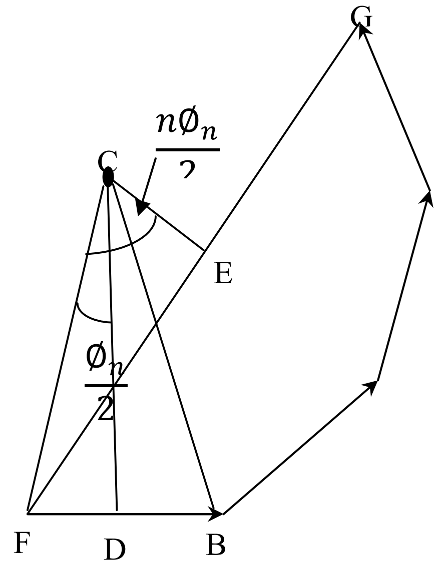

Figure 2: Polygon of vectors

According to polygon addition of vectors each vector subtends an angle at the centre of polygon, equal to the angle between two vectors, as shown in figure 2.

In figure 2 $FB = A$ the amplitude of individual vector, CD is perpendicular bisector at FB. Angle $FCB = \emptyset_n$ and $FCD = \frac{\emptyset_n}{2}$

$FD = DB = \frac{A}{2}$. Angle $FCE = \frac{n\emptyset_n}{2}$.

In triangle FCD $\sin\frac{\emptyset_n}{2} = \frac{FD}{FC}$ or

$$\sin\frac{\emptyset_n}{2} = \frac{R}{2FC} \qquad 2$$

As FG represents resultant amplitude and $FG = R$ since CE is perpendicular bisector at FG therefore $FE = EG = \frac{R}{2}$

Now in triangle FCE $\sin\frac{n\emptyset_n}{2} = \frac{FE}{FC}$

Or
$$\sin\frac{n\emptyset_n}{2} = \frac{R}{2F} \qquad 3$$

On dividing equation 3 by equation 2 we get

$$\frac{\sin\frac{n\emptyset_n}{2}}{\sin\frac{\emptyset_n}{2}} = \frac{\frac{R}{2FC}}{\frac{A}{2FC}} \qquad \text{Or} \qquad \frac{\sin\frac{n\emptyset_n}{2}}{\sin\frac{\emptyset_n}{2}} = \frac{R}{A}$$

Therefore $R = A\frac{\sin\frac{n\emptyset_n}{2}}{\sin\frac{\emptyset_n}{2}}$ by substituting the value of $\emptyset$ from equation 1 we get $R = A\frac{\sin(\frac{\pi}{\lambda}d\sin\theta)}{\sin(\frac{\pi}{n\lambda}d\sin\theta)}$

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let $\frac{\pi}{\lambda}d\sin\theta = \alpha$ then $R = A\frac{\sin\alpha}{\sin\frac{\alpha}{n}}$ as n is very large therefore $\frac{\alpha}{n}$ will be very small and hence $\sin\frac{\alpha}{n} = \frac{\alpha}{n}$

The equation 4 reduces to

$$R = nA\frac{\sin\alpha}{\alpha} \qquad 5$$

The equation 5 gives the resultant amplitude at point P. The resultant intensity at point P will be

$$I = n^2A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad 6$$

Principle Maxima: As $\alpha = 0$ the term$\left(\frac{\sin\alpha}{\alpha}\right) = 1$, therefore the intensity$\mathrm{I} = n^2\mathrm{A}^2$or $\mathrm{I} = I_{max}$. As $\alpha = 0$ indicates $\theta = 0$ i.e. the intensity is maximum at the centre of the screen.
Secondary Maxima: For other maximas apart from centre $\alpha = (2m+1)\frac{\pi}{2}$ where *m*= 1, 2, 3…
For first maxima *m*=1 and $\mathrm{I} \cong \frac{I_{max}}{22}$; for second maxima *m*=2 and $\mathrm{I} \cong \frac{I_{max}}{62}$as we can see that the intensity at secondary maximas decreases.
Condition of Minima: Minimum intensity is obtained when $\sin\alpha = 0$but $\alpha \neq 0$. I.e. $\alpha = \pm m\pi$ where *m*=1, 2, 3…. In this situation I=0 because the term$\left(\frac{\sin\alpha}{\alpha}\right) = 0$,
As $\alpha = \frac{\pi}{\lambda}\mathrm{d}\sin\theta$therefore the condition of minima can also be written as $\mathrm{d}\sin\theta = \pm\mathrm{m}\lambda$
7

Diffraction at two slits: Let us consider two narrow slits with slit width 'd' and the separation between the slits equal to 'e'. The light from a monochromatic source 'S' falls on this slit assembly through a convex lens. The incident wavefront is diffracted by an angle θ and is focused at screen at point P.

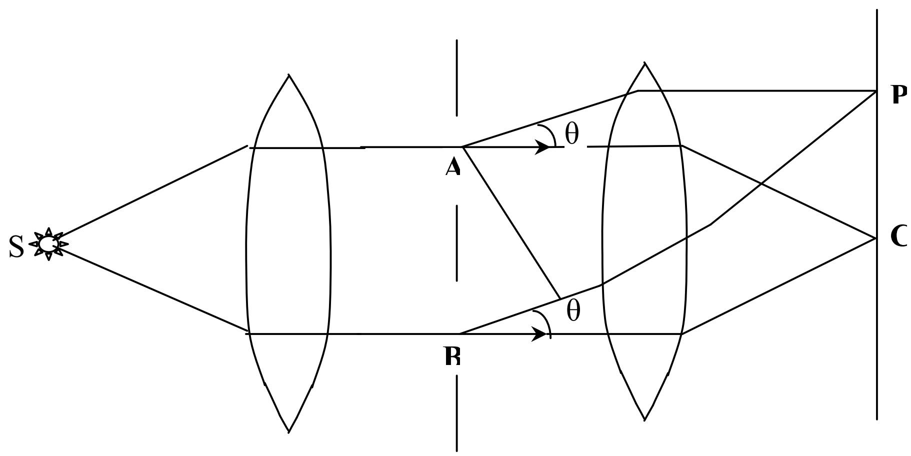

Figure 1: Diffraction at two

The path difference between two rays of parallel wavefront from the point A and point B will be equal to BC.
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Therefore $BC = (e+d)\sin\theta$.
As $Phase\ difference = \frac{2\pi}{\lambda} * \ pathdifference$ therefore the phase difference between two rays from point A and B will be $\emptyset = \frac{2\pi}{\lambda} * (e + \ d)\sin\theta$where $\emptyset = Phase\ difference$.
We are having two vectors with phase difference $\emptyset$ and superimposing each other at point P, the amplitude of individual vector $R = nA(\frac{\sin\alpha}{\alpha})$, here 'n' is number of secondary sources within single slit and *'A'* is amplitude of each vector within single slit. The resultant of two vectors will be $R_2 = \sqrt{R^2 + R^2 + 2R^2\cos\emptyset}$ or $R_2 = \sqrt{2R^2(1+\cos\emptyset)}$
Or $R_2 = \sqrt{2R^2(1 + 2\cos^2\frac{\emptyset}{2} - 1)}$

$R_2 = R\cos\frac{\emptyset}{2}$ Therefore intensity I

$I = n^2A^2(\frac{\sin\alpha}{\alpha})^2\cos^2\frac{\emptyset}{2}$ The intens

single slit and $\cos^2\frac{\emptyset}{2}$ the interfer

Condition of Maxima: For intensit

$\pm n\pi \cos^2\frac{\emptyset}{2} = 1$ and the int

Condition of Minima: For inten

$\frac{\emptyset}{2} = \pm(2n+1)\frac{\pi}{2}$ the $\cos^2\frac{\emptyset}{2} = 0$ in

Intensity Distribution due to term

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Effect of increasing the slit wid sharper, but fringe spacing remain diffraction maximum.

Effect of increasing the distance keeping the slit width (d) constant,

unchanged. Hence more int

Absent Order of Spectrum (Mis

screen due to two slit is the product

the condition of minima while other

will be absent from the diffraction

$d)\sin\theta = n\lambda$ where n= 0, 1, 2.

$d\sin\theta = m\lambda$; where m= 1,

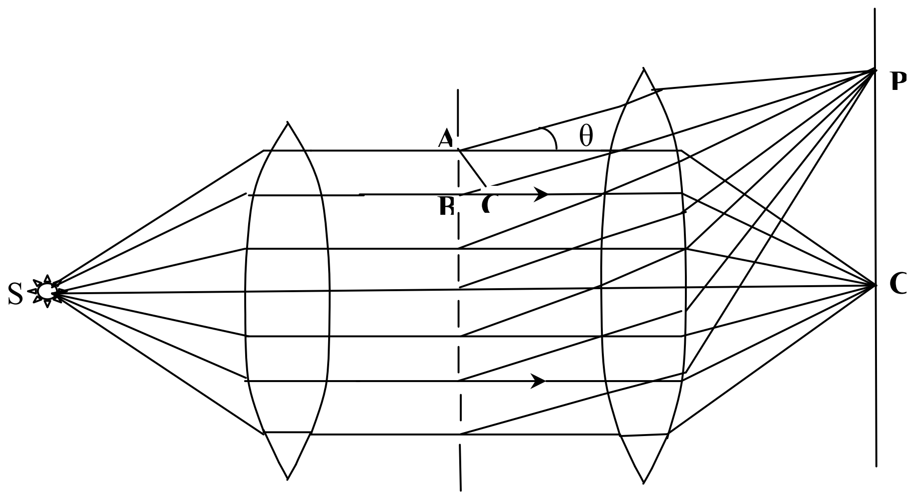
$$\frac{(e+\ d)\sin\theta}{d\sin\theta}=\frac{n\lambda}{m\lambda}$$

We get $\frac{(e+d)}{d}=\frac{n}{m}$ if e=d then $n=2m$ i.e. we can say that in such situation the 2, 4, 6….. Order maxima will be absent from diffraction spectrum.

Diffraction due to N-slits or diffraction grating:

- A plane diffraction grating is an arrangement consisting of a large number of close, parallel, straight, transparent and equidistant (d) slits. The slit width (e) is same for all the slits.
- A grating is made by drawing a series of very fine equidistant and parallel lines on optically plane glass plate by means of a fine diamond pen.

Theory of Plane Transmission Grating: Let us consider 'n' parallel slits with slit width 'e' and the separation between the slits equal to 'd'. The light from a monochromatic source 'S' falls on this slit assembly through a convex lens. The incident wavefront is diffracted by an angle θ and is focused at screen at point P.



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In triangle ABC $\sin\theta=\frac{BC}{AB}$ or $BC=AB\sin\theta$ since $AB=e+d$ Therefore $BC=(e+d)\sin\theta$ 1

As $Phase\ difference\ =\frac{2\pi}{\lambda}*\ path difference$ therefore the phase difference between two rays from point A and B will be $\emptyset\ =\frac{2\pi}{\lambda}*(e+\ d)\sin\theta$ 2 where $\emptyset=Phase\ difference$.

We are having 'n' vectors with phase difference $\emptyset$ and superimposing each other at point P, the amplitude of individual vector $R=nA(\frac{\sin\alpha}{\alpha})$, here 'n' is number of secondary sources within single slit and *'A'* is amplitude of each vector within single slit.

Using the polygon addition of 'n' vectors, the resultant at point P can be written as

$$I_p=R_N^2=N^2R^2\left(\frac{Sin\ N\frac{\phi}{2}}{N\ Sin\ \frac{\phi}{2}}\right)^2 \qquad 3$$

let $\frac{\phi}{2}=\beta$ now equation 3 can be rewritten as-

$$I_p=R_N^2=N^2R^2\left(\frac{Sin\ N\beta}{N\ Sin\ \beta}\right)^2 \qquad 4$$

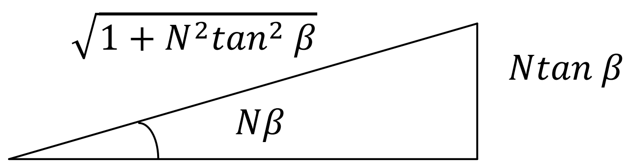
Condition of Maxima: For intensity to be maximum $\beta = \pm n\pi$where *n=1, 2, 3……* when $\beta = \pm n\pi$; $\left(\frac{Sin\ N\beta}{N\ Sin\ \beta}\right)^2 = \frac{0}{0}$ therefore by applying L-Hospitals rule $\left(\frac{\frac{d}{d\beta}(Sin\ N\beta)}{\frac{d}{d\beta}(\ N\ Sin\ \beta)}\right) = \frac{N\ \mathrm{Cos}\ N}{N\ Cos\ \beta}$ =1 when $\beta = \pm n\pi$ and the intensity is maximum. i.e. $I_p = R_N^2 = N^2R^2$is condition of maxima. By placing the value of β in $\beta = \pm n\pi$we get$(e+d)\sin\theta = \pm n\lambda$ this equation is known as grating equation.

Condition of Minima: For intensity to be minimum i.e. I=0 we must have $Sin\ \beta = 0$ but $Sin\ N\beta \neq 0$ i.e. $\beta = \pm m\pi$ where*m=1, 2, 3……N-1* . This indicates that there are N-1 minimas between two consecutive maximas, this suggest that there must be N-2 secondary maximas between two principle maximas.

Intensity of secondary maxima: To find the intensity of secondary maxima we differentiate the intensity with respect to β and equate it equal to zero. i.e. differentiate the equation 4 wrt to β and equate it equal to zero. $\frac{dI}{d\beta} = 2N^2R^2\left(\frac{Sin\ N\beta}{N\ Sin\ \beta}\right)^2 \frac{N\sin\beta\ N\cos N\beta - \sin N\beta\ N\cos\beta}{N^2\ sin^2\beta} = 0$

From above we get $\frac{N\sin\beta\ N\cos N\beta - \sin N\beta\ N\cos\beta}{N^2\ sin^2\beta} = 0$ or $N\tan\beta = \tan N\beta$ 5

To find the intensity we derive the bracket $\left(\frac{Sin\ N\beta}{N\ Sin\ \beta}\right)^2$ under the light of equation 5. Let us consider a triangle as shown below



Now using the triangle above we ca 1 $\beta = \frac{Ntan\ \beta}{\sqrt{1+N^2tan^2\ \beta}}$ or

$$\left(\frac{Sin\ N\beta}{N\ Sin\ \beta}\right)^2 = \frac{N^2tan^2\ \beta}{1+N^2tan^2\ \beta}\frac{1}{N^2sin^2\ \beta}$$

On simplifying the above equation we get

$$\left(\frac{Sin\ N\beta}{N\ Sin\ \beta}\right)^2 = \frac{N^2}{1+(N^2-1)sin^2\ \beta}$$

Therefore the intensity at secondary maxima can be written as

$$I_s = N^2R^2\frac{N^2}{1+(N^2-1)sin^2\ \beta}$$

Hence the intensity of the secondary maxima is proportional to $\frac{N^2}{1+(N^2-1)sin^2\ \beta}$ whereas the intensity ofprinciple maxima is proportional to N^2. Therefore

$$\frac{I_s}{I_p} = \frac{Intensity\ of\ secondary\ maxima}{Intensity\ of\ principal\ maxima} = \frac{1}{1+(N^2-1)sin^2\ \beta}$$

This indicates that greater the value of N weaker is the intensity of secondary maxima.

Dispersive power of a Grating: The dispersive power of a grating is defined as the rate of change of the angle of diffraction with the wavelength of the light. It is expressed as $\frac{d\theta}{d\lambda}$. As grating equation is $(e+d)\sin\theta = \pm n\lambda$ differentiating this with respect to λ we get $(e+d)\cos\theta\ \frac{d\theta}{d\lambda} = n$ or $\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$ the equation obtained is dispersive power of grating. This implies that the dispersive power is

1. Directly proportional to the order of spectrum ‘n’.
2. Inversely proportional to the grating element (e + d)
3. Inversely proportional to $\cos\theta$.

Angular Half Width: The n^{th} order principle maximum is obtained in the direction θ_n given by $(e+d)\sin\theta_n = \pm n\lambda$ 1

Let the first minimum adjacent to the n^{th} maximum be obtained in the direction $\theta_n + d\theta_n$ where $d\theta_n$ is called the angular half width of n^{th} maximum. The minima's are obtained in the directions given by

$N(e+d)\sin\theta_n + d\theta_n = \pm m\lambda$ 2

where$N$= total number of slits and m= integer except 0, N, 2N,…….nN. This gives $m = nN + 1$. Equation 2 now can be rewritten as

$$N(e+d)\sin(\theta_n + d\theta_n) = \pm(nN+1)\lambda$$

Or $N(e+d)\,(\sin\theta_n \cos d\theta_n + \sin d\theta_n \cos\theta_n) = \pm(nN+1)\lambda$

Since $d\theta_n$ is small therefore $\sin d\theta_n = d\theta_n$ and $\cos d\theta_n = 1$

The equation above reduces to

$$N(e+d)\sin\theta_n + N(e+d)(d\theta_n \cos\theta_n) = \pm(nN\lambda + \lambda) \qquad 3$$

using equation 1, equation 3 can be rearranged as –

$$d\theta_n = \frac{\lambda}{N(e+d)\cos\theta_n}$$

Using equation 1, above equation can be rearranged as

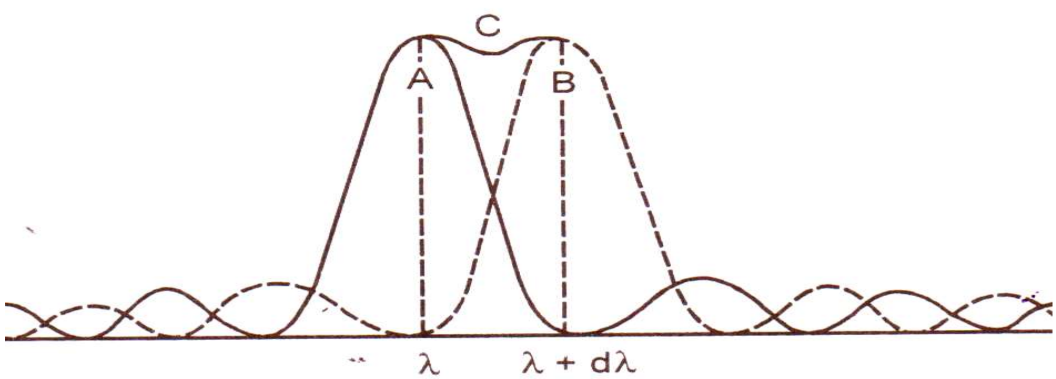
$$d\theta_n = \frac{\lambda}{Nn\cot\theta_n}$$

The expression above is known as angular half width.

Resolving power of Optical Instruments The ability of an optical instrument to resolve the images of two nearby points is termed as its resolving power.

Rayleigh's Criterion of Resolution:

Two spectral lines of equal intensities are said to be resolved if the principle maximum of diffraction pattern of one falls on the first minimum of the diffraction pattern of the other. Suppose two wavelengths λ and $\lambda + d\lambda$ are incident on an optical instrument then they will be said to resolved just if the diffraction pattern is as shown below.



Resolving Power of Diffraction Grating

The resolving power of a diffraction grating represents its ability to form separate lines for wavelengths very close together. It is given by $\frac{\lambda}{d\lambda}$. Let a parallel beam of wavelength λ and $\lambda + d\lambda$ is incident normally on a diffraction grating. The n^{th} principal maximum is formed in the direction θ_n then

$$(e+d)\sin\theta_n = n\lambda$$

1

The first minimum adjacent to n^{th} principal maximum be formed in the direction $\theta_n + d\theta_n$, then we have- $(e+d)\sin(\theta_n + d\theta_n) = n\lambda + \frac{\lambda}{N}$ 2

Because there are $(N-1)$ minimas between two principal maximas. According to Rayleigh's criterion the n^{th} maxima due to wavelength $\lambda + d\lambda$ must be formed in the direction $\theta_n + d\theta_n$ i.e we have

$$(e+d)\sin(\theta_n + d\theta_n) = n(\lambda + d\lambda) \qquad 3$$

By equations 2 & 3 we can write $n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}Or\frac{\lambda}{d\lambda} = nN$this is the desired expression for resolving power of grating.Thus the resolving power of grating is equal to the product of total number of rulings on the grating and the order of spectrum.